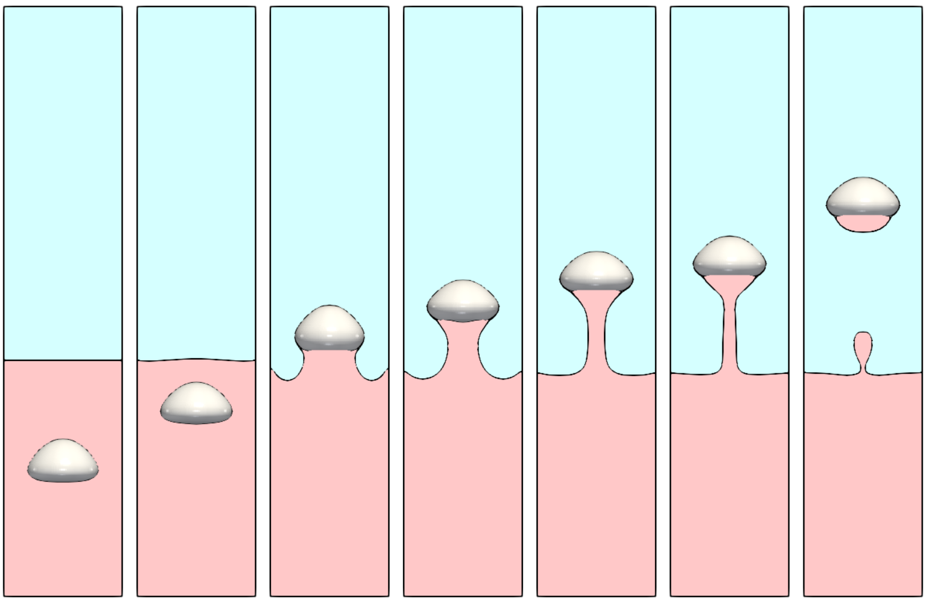
A UNIFIED FINITE VOLUME FRAMEWORK FOR PHASE-FIELD SIMULATIONS OF ARBITRARY NUMBER OF IMMISCIBLE INCOMPRESSIBLE FLUIDS

MILAD BAGHERI[1,4], ABDOLRAHMAN DADVAND[2], MARTIN WÖRNER[3], and HOLGER MARSCHALL[1,4]

[1]*Computational Multiphase Flow, Department of Mathematics, TU Darmstadt, Germany,*
milad.bagheri@tu-darmstadt.de & holger.marschall@tu-darmstadt.de
[2]Urmia University of Technology, Faculty of Mechanical Engineering, Urmia, Iran
[3]*Karlsruhe Institute of Technology (KIT), Institute of Catalysis Research and Technology, Germany*
[4]*Mathematical Modeling and Analysis, Department of Mathematics, TU Darmstadt, Germany*

Keywords: *Phase-Field; Cahn-Hilliard Navier-Stokes; Immiscible multiphase flows*

We have developed a unified Finite Volume framework for simulating N ($N > 2$) generic immiscible, incompressible and isothermal fluids employing a diffuse interface phase-field interface capturing method in FOAM-extend 4.1. The development is considered an extension of our previously validated two-phase Cahn-Hilliard-Navier-Stokes framework [1–6]. The phase-field method is originally based on the work of Cahn and Hilliard [7] where the interface is composed of a region of finite thickness and the composition profile across the interface changes rapidly but continuously from one phase to the other.



(a) $t = 0.1$s (b) $t = 0.2$s (c) $t = 0.32$s (d) $t = 0.36$s (e) $t = 0.4$s (f) $t = 0.42$s (g) $t = 0.5$s

Figure 1: Phase distribution. Blue: light fluid, Red: heavy fluid, Gray: air bubble.

The implemented methodology follows closely [8–10] and honors thermodynamic and reduction consistency. A comprehensive set of benchmark cases such as the floating liquid lens problem [10, 11], rising bubble in two stratified layers problem [12, 13] (depicted in Fig. 1), and the four-phase fluid mixture problem [14] will be presented for validation. Moreover, numerical simulations of the drop impact onto a thin immiscible liquid film will be shown and compared to in-house experimental results.

Acknowledgments

The authors acknowledge the financial support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) project number 237267381 SFB/Transregio 150. The authors thank all those involved in the organisation of OFW17 and to all the contributors that will enrich this event.

References

[1] X. Cai, H. Marschall, M. Wörner, and O. Deutschmann, "Numerical simulation of wetting phenomena with a phase-field method using OpenFOAM®," *Chemical Engineering & Technology*, vol. 38, no. 11, pp. 1985–1992, 2015.

[2] X. Cai, M. Wörner, H. Marschall, and O. Deutschmann, "Numerical study on the wettability dependent interaction of a rising bubble with a periodic open cellular structure," *Catalysis Today*, vol. 273, pp. 151–160, 2016.

[3] V. Fink, X. Cai, A. Stroh, R. Bernard, J. Kriegseis, B. Frohnapfel, H. Marschall, and M. Wörner, "Drop bouncing by micro-grooves," *International Journal of Heat and Fluid Flow*, vol. 70, pp. 271–278, 2018.

[4] M. Wörner, N. Samkhaniani, X. Cai, Y. Wu, A. Majumdar, H. Marschall, B. Frohnapfel, and O. Deutschmann, "Spreading and rebound dynamics of sub-millimetre urea-water-solution droplets impinging on substrates of varying wettability," *Applied Mathematical Modelling*, vol. 95, pp. 53–73, 2021.

[5] N. Samkhaniani, A. Stroh, M. Holzinger, H. Marschall, B. Frohnapfel, and M. Wörner, "Bouncing drop impingement on heated hydrophobic surfaces," *International Journal of Heat and Mass Transfer*, vol. 180, p. 121777, 2021.

[6] M. Bagheri, B. Stumpf, I. V. Roisman, C. Tropea, J. Hussong, M. Wörner, and H. Marschall, "Interfacial relaxation – crucial for phase-field methods to capture low to high energy drop-film impacts," *International Journal of Heat and Fluid Flow*, vol. 94, p. 108943, 2022.

[7] J. W. Cahn and J. E. Hilliard, "Free energy of a nonuniform system. I. Interfacial free energy," *The Journal of Chemical Physics*, vol. 28, no. 2, pp. 258–267, 1958.

[8] F. Boyer and S. Minjeaud, "Hierarchy of consistent n-component Cahn–Hilliard systems," *Mathematical Models and Methods in Applied Sciences*, vol. 24, no. 14, pp. 2885–2928, 2014.

[9] S. Dong, "Multiphase flows of N immiscible incompressible fluids: a reduction-consistent and thermodynamically-consistent formulation and associated algorithm," *Journal of Computational Physics*, vol. 361, pp. 1–49, 2018.

[10] Z. Huang, G. Lin, and A. M. Ardekani, "A consistent and conservative phase-field method for multiphase incompressible flows," *Journal of Computational and Applied Mathematics*, vol. 408, p. 114116, 2022.

[11] S. Dong, "An efficient algorithm for incompressible N-phase flows," *Journal of Computational Physics*, vol. 276, pp. 691–728, 2014.

[12] F. Boyer and C. Lapuerta, "Study of a three component Cahn-Hilliard flow model," *ESAIM: Mathematical Modelling and Numerical Analysis-Modélisation Mathématique et Analyse Numérique*, vol. 40, no. 4, pp. 653–687, 2006.

[13] F. Boyer, C. Lapuerta, S. Minjeaud, B. Piar, and M. Quintard, "Cahn–Hilliard/Navier–Stokes model for the simulation of three-phase flows," *Transport in Porous Media*, vol. 82, no. 3, pp. 463–483, 2010.

[14] S. Dong, "Physical formulation and numerical algorithm for simulating N immiscible incompressible fluids involving general order parameters," *Journal of Computational Physics*, vol. 283, pp. 98–128, 2015.