

Energy distribution controlled ballistic Josephson junction

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We report an experimental study on the tuning of supercurrent in a ballistic graphene-based Josephson junction by applying a control voltage to a transverse normal channel. In this four-terminal geometry, the control voltage changes the occupation of Andreev states in the Josephson junction, thereby tuning the magnitude of the supercurrent. As a function of gate voltage, we find two different regimes characterized by a double-step distribution and a hot-electron distribution, respectively. Our work opens opportunities to design highly controllable Josephson junctions for tunable superconducting quantum circuits.

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I. INTRODUCTION

The design of controlled Josephson junctions is a crucial aspect of engineering superconducting circuits and their potential application in quantum technologies and information [1]. This is particularly important in Josephson junctions based on superconducting weak links [2], especially in a multiterminal geometry, in which Weyl singularities in the Andreev bound state (ABS) spectra offer new platforms to observe topological effects [3–8] as well as the emergence of new correlated states [9–15]. Many different systems have been used to mediate superconductivity induced by proximity of superconducting leads such as metallic [16–20] or semiconducting [21–28] nanostructures, quantum point contacts [29,30], quantum dot [31], topological insulators [32–38], or graphene [39]. When superconducting leads connecting the weak link are close enough to each other, a dissipationless current can flow via Andreev bound states [40]. Dedicated low noise electronics and low temperature filtering is required to measure finite [41–44] and stable [45] maximum supercurrent or critical current I_c . A full control strategy strongly depends on the tunability of the superconducting weak link itself. Indeed, I_c depends on many parameters such as temperature, superconducting gap, contact transparency, or/and disorder.

Moreover, when the weak link carrier density is gate tunable, the critical current amplitude varies inversely to the normal-state resistance R_N , i.e., I_c increases when R_N decreases. Therefore, the electrostatic control remains the most straightforward way to tune the amplitude of the critical current [21,27,28,32–39], while local gating allows its spatial confinement [29–31]. An alternative way to control the amplitude of I_c was demonstrated by tuning the electron dis-

tribution in the weak link itself with a transverse controlled voltage [17–19,25,26]. In these experiments, the control voltage allowed one to tune I_c up to reversing its sign and therefore forming a so-called π junction [18,19]. In this work, we study the effect of such control voltage in a ballistic superconducting weak link, here a single layer graphene, in which the supercurrent is no longer carried by a continuum of ABS but instead a series of discrete states [40]. We show that this transverse voltage tunes the occupation of Andreev states in the Josephson junction, adding an additional knob to control the magnitude of the supercurrent. We have used a model accounting for the coupling of the control channel to the Josephson junction [46–49] with the coupling strength of the control channel to the Josephson junction as a single parameter. We obtain two distinct regimes, a double-step distribution and a hot-electron distribution, which fits the amplitude of I_c when the gate voltage tunes the Fermi level in the valence and the conduction band, respectively, corresponding to a high and low normal-state resistance.

II. EXPERIMENTAL RESULTS

A. Device fabrication and measurement setup

We have fabricated our device by encapsulating a single layer graphene between two hexagonal boron nitride (h-BN) crystallites, 28- and 18-nm thick for the top and bottom h-BN, respectively. Graphene and h-BN crystallites were mechanically exfoliated from natural graphite (NGS Naturgraphit GmbH) and commercially available h-BN powder (Momentive, gradePT110), respectively, on a Si substrate with 300-nm-thick SiO₂. The h-BN/graphene/h-BN heterostructure was prepared by following a dry transfer technique similar to Wang *et al.* [50]. The fabricated heterostructure was subsequently transferred to a different Si substrate with 1000-nm-thick SiO₂. Edge contacts to the h-BN encapsulated graphene were established in a similar manner as in Kraft *et al.* [30]. However, this process was adapted for the

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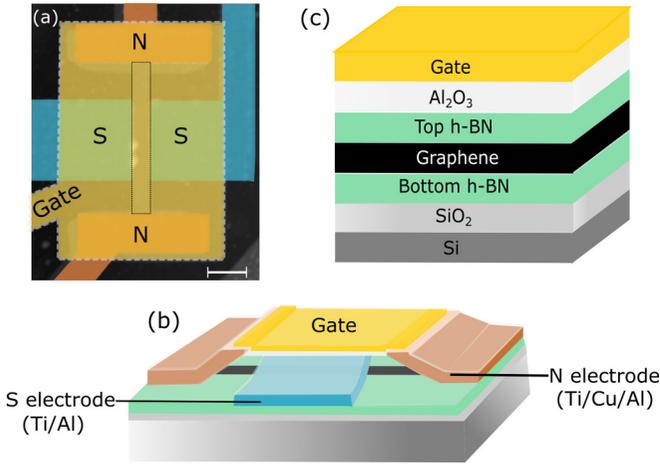


FIG. 1. (a) False color atomic force micrograph of the studied device showing the normal metal (N), superconducting (S), and top gate (Gate) electrodes. The region enclosed by the dashed black lines shows the h-BN encapsulated graphene channel. The scale bar is 1 μm . (b) Schematics of the device depicting the edge contact geometry. (c) Cross-section schematic of the central part of the device

deposition of two different types of metal electrodes as in Pandey *et al.* [51]. For the superconducting electrodes, we have used Ti (5 nm)/Al (60 nm), and for the normal-metal electrodes, Ti (5 nm)/Cu (80 nm)/Al (5 nm) was used. To control the charge-carrier density in the graphene channel, a top gate electrode with Ti (5 nm)/Cu (100 nm)/Al (5 nm) was fabricated. Here, a 25-nm-thick Al_2O_3 was deposited with atomic layer deposition method to act as the top gate dielectric in addition to the top h-BN. In all the electrodes, Ti serves as an adhesive layer, while the thin Al layer in the Cu contact and gate electrodes is used as a protective layer to prevent oxidation of these electrodes.

As can be seen in Fig. 1(a), the graphene channel has $L = 0.46 \mu\text{m}$ and $W = 2 \mu\text{m}$ across the SGS junction and $L = 4 \mu\text{m}$ and $W = 0.46 \mu\text{m}$ across the NGN junction. Note that L denotes the length of the channel between the electrodes, while W denotes the width of the channel along the electrodes. Figure 1(b) shows the schematic of the device with the edge contact geometry and Fig. 1(c) shows the cross-section schematic of the device. All of the transport measurements were conducted with standard low-frequency lock-in technique. In order to avoid any spurious effects, the measurement lines were fed through a series of filters. In all of the data shown, $I_{\text{bias}}(V_{\text{bias}})$ denotes the current (voltage) across the SGS junction, whereas V_{ctrl} denotes the voltage bias across the NGN junction.

B. Normal-state characterization and ballistic transport

Figures 2(a) and 2(b) show the gate voltage V_g dependent resistance of the device across the SGS and NGN junctions, respectively, when the device is in the normal state at $T = 6 \text{ K}$. It can be readily observed that the Dirac point for both of these junctions is shifted to negative gate voltages, which indicates n -type doping of the graphene sheet due to charge transfer from the contacts. It also results in the formation of potential barriers close to the metal/graphene contact inter-

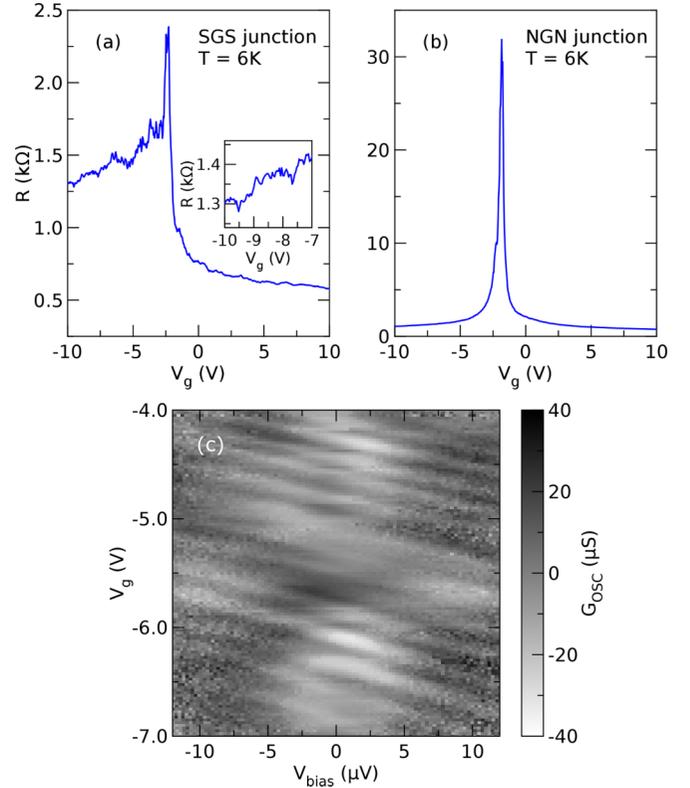


FIG. 2. Normal-state resistance R of the device as a function of gate voltage V_g as measured across: (a) Superconductor/graphene/superconductor (SGS) junction; the inset shows Fabry-Pérot resonances in the p -doped regime. (b) Normal metal/graphene/normal metal (NGN) junction. (c) Fabry-Pérot oscillations in the SGS junction as a function of gate voltage V_g and bias voltage V_{bias} across the SGS junction.

faces when the charge transport in the graphene channel is driven into the p -doped regime [52–54]. An important point to note is that the two junctions have different Dirac points where $V_{\text{CNP}} = -2.3 \text{ V}$ for the SGS junction and $V_{\text{CNP}} = -1.8 \text{ V}$ for the NGN junction. In addition, comparing the charge transport between the n -doped and p -doped regions, higher asymmetry is observed in the SGS junction while the Dirac curve looks symmetric across the NGN junction. This can be explained by considering the dimensions of the two junctions, where the SGS junction falls in the category of wide and short junction. NGN junction, on the other hand, falls in the narrow and long junction regime. Therefore, the doping from the contacts is more prominent in the SGS junction leading to a higher shift in the V_{CNP} and charge transport asymmetry in the Dirac curve as compared to the NGN junction. It also results in the formation of a coherent electronic Fabry-Pérot (FP) cavity, a signature of ballistic transport, in the SGS junction when the Fermi level in graphene is tuned in the valence band [55–64].

This effect can be readily seen in the inset of Fig. 2(a) as the conductance oscillates with the gate voltage. These conductance oscillations can be tuned by V_g as well as by an applied bias V_{bias} across the SGS junction. Figure 2(c) shows these conductance oscillations mapped with respect to V_g and V_{bias} . As the conductance of the device varies strongly with V_g and V_{bias} , a nonoscillating background conductance

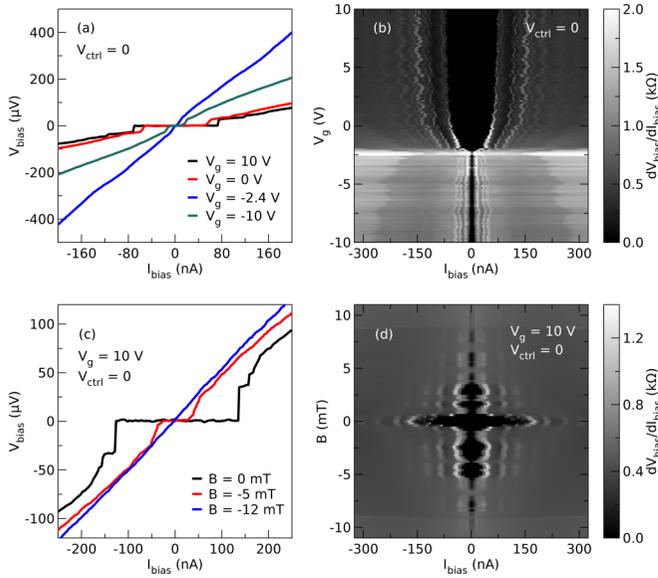


FIG. 3. Characterization of the supercurrent at $T = 50$ mK and $V_{\text{ctrl}} = 0$. (a) $I_{\text{bias}} - V_{\text{bias}}$ characteristics at different gate voltages V_g . (b) Gate voltage dependence of the differential resistance $dV_{\text{bias}}/dI_{\text{bias}}$. (c) $I_{\text{bias}} - V_{\text{bias}}$ characteristics in the presence of perpendicular magnetic field B at $V_g = 10$ V. (d) Measured Fraunhofer pattern (B vs I_{bias}) under the same conditions as in (c). Please note that a magnetic field offset correction was made for (c) and (d).

was subtracted to enhance the visibility of the conductance oscillations. As can be seen, instead of the usual checkerboard pattern for symmetric contacts, we observe an asymmetric interference pattern which suggests an asymmetric coupling of the two S contacts to the graphene channel [51,65]. Despite this asymmetric coupling, clear interferences can be observed in Fig. 2(c). For normal incidence of charge carriers, the cavity length L_C that gives rise to this pattern can be calculated by using the relation $k_F L_C = N\pi$ where N is an integer and $k_F = \sqrt{n\pi}$ is the Fermi wave vector with $n = \alpha_g V_g$ denoting the charge-carrier density. Here $\alpha_g = 3.496 \times 10^{11} \text{ V}^{-1} \text{ cm}^{-2}$ is the gate coupling efficiency which was extracted from Shubnikov–de Haas oscillations at high field (not shown). Inserting α_g into the expression to calculate L_C results in $L_C \sim 450 \pm 5$ nm. Since L_C is in agreement with the length of the graphene channel across the SGS junction, it can be concluded that the observed interference pattern is indeed arising from the FP oscillations. This serves as the proof that the charge transport across the SGS junction is in the ballistic regime.

C. Gate-dependent supercurrent and Fraunhofer pattern

Here, we characterize the SGS Josephson junction in the absence of a control voltage across the NGN channel at $T = 50$ mK, far below the critical temperature of Al (~ 1 K). Supercurrent in ballistic graphene-based Josephson junctions can be efficiently tuned by an applied gate voltage which tunes the charge-carrier density in the weak link [30,66–69]. Figure 3(a) shows the $I_{\text{bias}} - V_{\text{bias}}$ characteristics of the SGS junction. It is to be noted that due to a slight shift in the zero magnetic field, the device was in a nonzero magnetic field during this set of

measurements which resulted in a reduced supercurrent across the SGS junction. The $I_{\text{bias}} - V_{\text{bias}}$ curves represent the high p -doped regime ($V_g = -10$ V), the vicinity of the Dirac point ($V_g = -2.4$ V), and the n -doped regime ($V_g = 0$ V and $V_g = 10$ V). Figure 3(b) shows the complete map of the differential resistance $dV_{\text{bias}}/dI_{\text{bias}}$ across the SGS junction as a function of I_{bias} and V_g under the same measurement conditions as in Fig. 3(a). It can be clearly seen that the device has a noticeable supercurrent (~ 80 nA) in the entire n -doped regime while it is considerably smaller even in the highly p -doped regime. Close to the Dirac point, the device is very close to the normal state with slight nonlinearity in the current-voltage relation. As can be seen in Fig. 3, the gate tuning of the supercurrent in our device agrees with the commonly observed gate-tunable supercurrent in graphene-based Josephson junctions [66–68].

In a wide and short Josephson junction with uniform current density, the critical current is expected to exhibit a Fraunhofer interference pattern according to the relation [70]

$$I_c(B) = I_c(0) \frac{\sin(\pi \Phi / \Phi_0)}{\pi \Phi / \Phi_0}. \quad (1)$$

Here $I_c(B)$ is the critical current at a perpendicular magnetic field B , $I_c(0)$ is the critical current at zero magnetic field, $\Phi = BA$ is the magnetic flux through the Josephson junction with junction area A , and Φ_0 is the magnetic flux quantum. The Fraunhofer pattern bears a strong dependence on the current density distribution in the Josephson junction. Figures 3(c) and 3(d) show the $I_{\text{bias}} - V_{\text{bias}}$ characteristics and the differential resistance, respectively, in the presence of a perpendicular magnetic field B (offset-corrected), at $V_g = 10$ V. We observe a maximum critical current of ~ 130 nA at zero magnetic field and an oscillatory pattern as a function of field is seen in Fig. 3(d). A clear asymmetry in the interference pattern for positive and negative magnetic fields is due to the flux trapping in the magnet itself which could not be corrected in the measurement and analysis. The interference pattern resembles a Fraunhofer pattern, though slightly distorted, and the field scale of the oscillation is consistent with the expected $\Delta B \approx 2$ mT for the geometry of our device. As our four-terminal geometry has extended graphene parts on each side of the junction, the width of the junction appears to be not well defined, while a gradient of supercurrent density may also flow within these side superconducting weak-link parts and therefore, the regular Fraunhofer pattern observed in a short and wide Josephson junction with constant supercurrent density across it, as predicted in the seminal work of Dynes and Fulton [71], cannot be observed in our geometry.

D. Tuning the supercurrent with a transverse normal channel

Following the characterization of the Josephson junction, we now turn to the control of the occupation of Andreev states in the SGS channel by applying a control voltage V_{ctrl} across the NGN channel. Figures 4(a)–4(d) show the $I_{\text{bias}} - V_{\text{bias}}$ curves across the SGS junctions at various values of V_{ctrl} at 50 mK under zero magnetic field. The charge transport in the p -doped regime corresponds to Figs. 4(a) and 4(b), while Figs. 4(c) and 4(d) correspond to the n -doped regime. Figures 4(e)–4(h) show the complete map of $dV_{\text{bias}}/dI_{\text{bias}}$ across the SGS junction with respect to I_{bias} and V_{ctrl} under

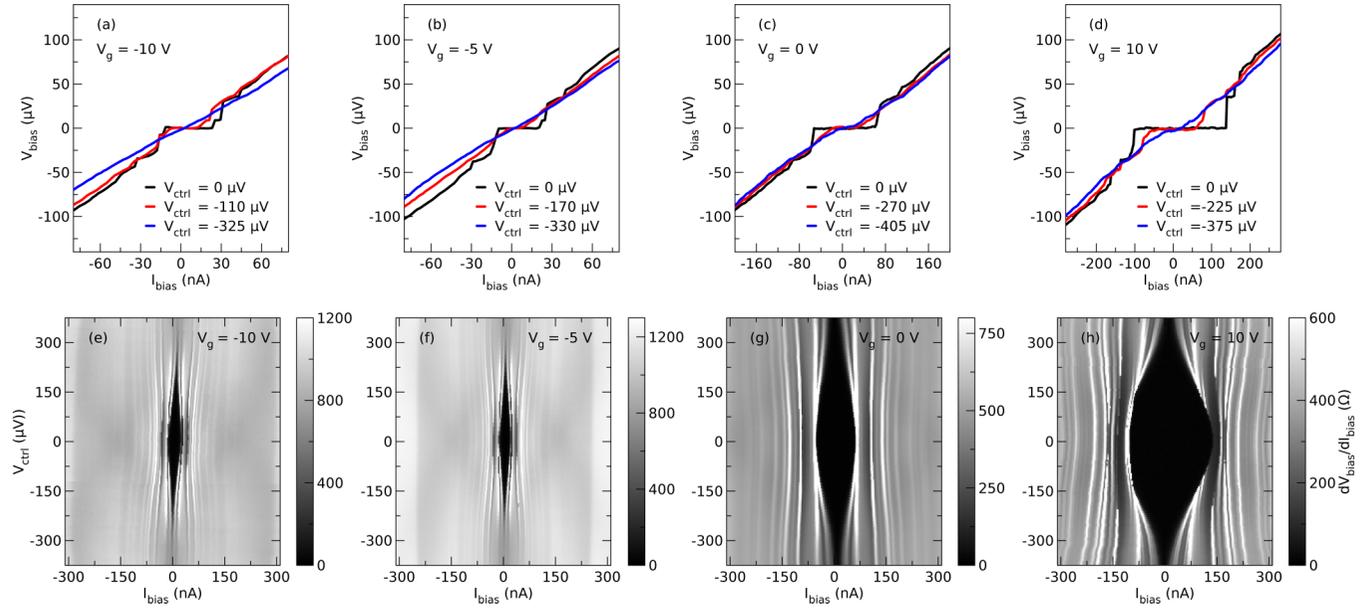


FIG. 4. $I_{\text{bias}} - V_{\text{bias}}$ characteristics for different control voltages V_{ctrl} at (a) $V_g = -10$ V, (b) $V_g = -5$ V, (c) $V_g = 0$ V, (d) $V_g = 10$ V. Differential resistance $dV_{\text{bias}}/dI_{\text{bias}}$ map as a function of I_{bias} and V_{ctrl} at (e) $V_g = -10$ V, (f) $V_g = -5$ V, (g) $V_g = 0$ V, and (h) $V_g = 10$ V. All data were taken at $T = 50$ mK and $B = 0$.

the same measurement conditions as in Figs. 4(a)–4(d). The asymmetry with respect to the positive and negative I_{bias} in Figs. 4(d) and 4(h) can be attributed to the self-heating of the device at high critical current. While the supercurrent is continuously tuned by the control voltage in all of the measurements, a clear trend can be observed in all of the maps, that is, the magnitude of the control voltage required to diminish the superconductivity depends on the magnitude of the critical current. A comparatively smaller control voltage can clearly suppress the supercurrent in the p -doped regime as seen in Figs. 4(a), 4(b), 4(e) and 4(f), while the supercurrent cannot be completely suppressed in the n -doped regime as observed in Figs. 4(c), 4(d), 4(g), and 4(h) with the observable nonlinearity even at high control voltages. It can be understood by considering the fact that the device is weakly superconducting in the p -doped regime as compared to the n -doped regime, and therefore, it is more sensitive to the change caused by the control voltage in the p -doped regime.

III. MODEL

Supercurrent is carried across a ballistic Josephson junction by discrete ABS below the gap and additional continuum contributions from “leaky” Andreev resonances above the gap [40,46]. The magnitude and direction of the supercurrent is controlled by the phase difference across the junction and the occupation of the states. In our case, the Josephson junction is strongly coupled to the control channel, since they are both part of the same graphene sheet. This coupling leads to dephasing of the Andreev levels with a concomitant reduction of the critical current. We use the model of a strongly coupled control channel [47] generalized to the four-terminal geometry of our experiment [48].

The model and experimental setup are illustrated in Fig. 5(a). A bias current I_{bias} is driven through the Josephson

junction, while an independent circuit applies a control voltage V_{ctrl} to the control channel. Therefore, the control channel does not inject any net current into the Josephson junction, and only controls the occupation of the Andreev bound states. Both the SGS Josephson junction and the NGN control channel are modeled as single-channel one-dimensional conductors, coupled by a junction of transparency τ . Note that the SGS Josephson junction itself is modeled with perfect transparency. τ is the coupling to the control channel, and therefore only controls the dephasing. The length $L = 460$ nm of the SGS junction was taken from the geometry of the device. The coherence length of the Andreev pairs in the graphene sheet is $\xi_G = \hbar v_F / 2\Delta = 2.4$ μm , where $v_F = 10^6$ m/s is the Fermi velocity of graphene, and $\Delta = 135$ μeV is the experimentally observed gap of the aluminum leads. Therefore, our junction is in the short limit, and we expect a single ABS below the gap. As an illustration, we show the spectral supercurrent $I(E)$ at the critical current for three different values of τ in Fig. 5(b). As can be seen, the single ABS is significantly broadened by the coupling to the control channel. In addition, there is a small negative contribution from the continuum of leaky states above the gap.

The supercurrent is controlled by the applied voltage V_{ctrl} . The distribution functions for ideal reservoirs are given by

$$f_{\pm}(E) = f_0\left(E \pm \frac{eV_{\text{ctrl}}}{2}, T\right), \quad (2)$$

where f_0 is the Fermi distribution, and \pm corresponds to the upper/lower control terminal. In the absence of inelastic scattering, the distribution function in the middle of the control wire is the superposition

$$f_d(E) = \frac{1}{2}[f_+(E) + f_-(E)], \quad (3)$$

i.e., a double-step distribution function [72]. The double-step distribution assumes perfect cooling of the dissipated electric

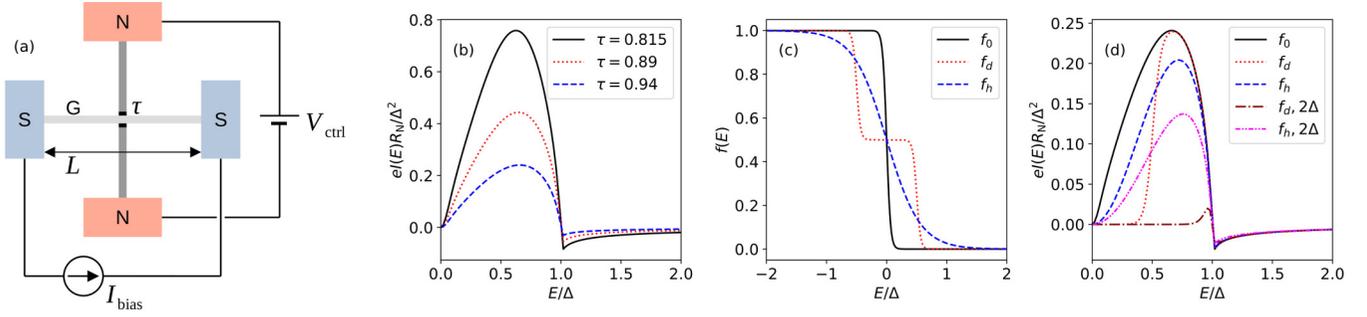


FIG. 5. (a) Schematic view of the model. (b) Spectral supercurrent at the critical current I_c for $V_{\text{ctrl}} = 0$ for three different transparencies τ . (c) Distribution functions f_0 , f_d , and f_h for $eV_{\text{ctrl}} = \Delta$. (d) Spectral supercurrent for $\tau = 0.94$ and different distribution functions. Three plots correspond to panel (c), whereas the remaining two plots correspond to $eV_{\text{ctrl}} = 2\Delta$.

power by the reservoirs. In the n -doped regime, the lower resistance of the graphene sheet leads to increased power dissipation and self-heating of the device, as already seen in the asymmetry of the switching and retrapping currents in Fig. 4 [73]. As an alternative scenario, we therefore also consider the limiting case of complete thermalization by Coulomb scattering, i.e., the hot-electron regime. In this case, the double-step distribution is replaced by a distribution

$$f_h(E) = f_0(E, T^*), \quad (4)$$

with an increased effective temperature T^* . Here, we neglect electron-phonon cooling, which leaves only cooling by electronic heat transport to the reservoirs. In this case, for a voltage-biased wire, the Wiedemann-Franz law gives the temperature [72,74]

$$T^* = \sqrt{T^2 + \frac{V_{\text{ctrl}}^2}{4\mathcal{L}}} \quad (5)$$

in the middle of the control wire, where \mathcal{L} is the Lorenz number. We assume $\mathcal{L} = \mathcal{L}_0 = \pi^2 k_B^2 / 3e^2$ for graphene at low temperatures and sufficiently far from the Dirac point [75]. f_d and f_h are plotted for $eV_{\text{ctrl}} = \Delta$ in Fig. 5(c), along with the equilibrium distribution f_0 for the base temperature $T = 50$ mK of the experiment.

Figure 5(d) illustrates the effect of the control voltage on the spectral supercurrent for a phase difference of $\pi/2$ across the junction and $\tau = 0.94$. The distribution functions for the first three plots are the same as in Fig. 5(c). Compared to the equilibrium case, the double-step distribution selectively removes the low-energy part, while the hot-electron distribution leads to an overall reduction. We also show the comparison for $eV_{\text{ctrl}} = 2\Delta$. At this point, the double-step distribution eliminates almost the entire positive subgap contribution, effectively reducing the critical current to zero. For the hot-electron distribution, a considerable positive contribution remains, and much higher control voltages are required to completely suppress the supercurrent. Note that this argument does not depend much on details of the model (e.g., multiple transport channels) as long as the junction is short, such that the spectral supercurrent is mostly positive below the gap and small above the gap.

Figure 6 shows the comparison of the experimental data to the model for different gate voltages. Here, the current is normalized as $eI R_N / \Delta$, so that the different resistance of the

SGS junction as a function of doping is already accounted for by the normalization. The coupling strength τ is the only free parameter, which we have adjusted to reproduce the observed $I_c R_N$ at $V_{\text{ctrl}} = 0$. We find a larger $I_c R_N$ product in the n -doped regime, corresponding to a smaller τ . This reflects the fact that $R_{\text{SGS}} \approx R_{\text{NGN}}$ on the p -doped side, while $R_{\text{SGS}} < R_{\text{NGN}}$ on the n -doped side. We therefore expect a weaker dephasing of the ABS and increased $I_c R_N$ on the n -doped side.

As a function of V_{ctrl} , the double-step distribution (solid lines) gives a good description of the data for small control voltage in the p -doped regime. In the n -doped regime, the hot-electron distribution (dashed lines) gives a good description, reflecting the higher power dissipation. Due to the negative spectral supercurrent at high energy, there should be a zero to π transition of the Josephson current at $eV_{\text{ctrl}}/2\Delta \lesssim 1$ for the double-step distribution. This is visible as a node in the model plots, where the critical current vanishes and then reappears. The transition is absent in the hot-electron regime. The fact that we observe a monotonic decrease of the critical current without 0 - π transition can be attributed to partial thermalization at higher control voltage even in the p -doped regime. A simple phenomenological discussion of partial thermalization is given in Appendix B. A more detailed model would require a realistic treatment of Coulomb scattering in the device, which is beyond the scope of our experimental work.

IV. DISCUSSION AND PERSPECTIVE

We have demonstrated that supercurrent can be tuned by a transverse voltage in a ballistic graphene-based Josephson junction. By using this control voltage, we continuously modify the occupation of the discrete Andreev levels and therefore the magnitude of I_c . We highlight two different regimes with our model, i.e., a double-step distribution and a hot-electron regime. The first corresponds to the case where the Fermi level, tuned by the gate voltage V_g , sits in the valence band and I_c amplitude is the lowest while the normal-state resistance is the highest due to the additional p - n junction formed in the vicinity of the leads by charge transfer (forming an electronic FP cavity, and FP interferences are observed via the conductance/resistance oscillations [see Fig. 2(a)] and the checkerboard pattern when the system is set out of equilibrium [see Fig. 2(c)]). As mentioned before, this double-step distribution implies a full heat dissipation by the leads. The second corresponds to the case where the Fermi level

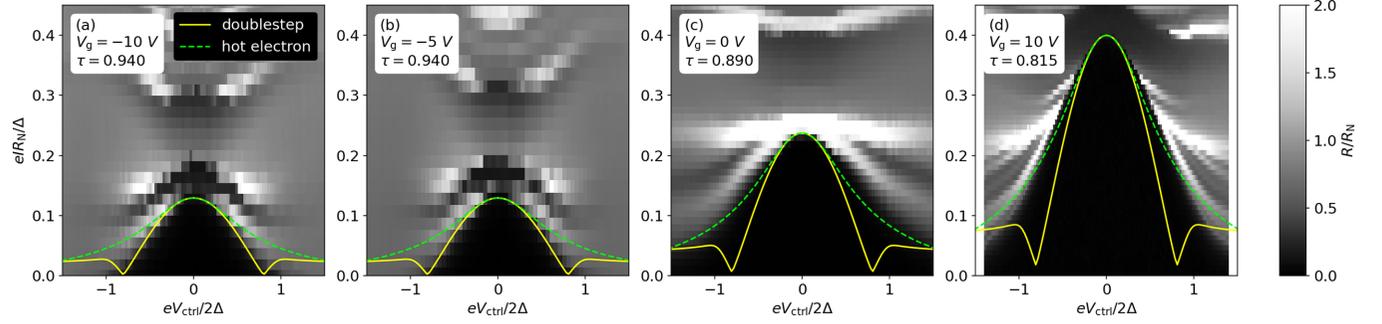


FIG. 6. Normalized resistance R/R_N as a function of normalized bias current eIR_N/Δ and normalized control voltage $eV_{\text{ctrl}}/2\Delta$ for different gate voltages V_g . Solid lines are model results for the double-step distribution and dashed lines are for the hot-electron regime (see text for details).

sits in the conduction band and I_c amplitude is the highest, while the normal-state resistance is the lowest at large charge-carrier density, in turn, leading to increased power dissipation and self-heating effect. We note that this is also reflected in the hysteretic behavior of I_c [73] [see the asymmetry of I_c in the differential resistance map in Figs. 4(g) and 4(h)]. While we observe two different regimes, we actually do not observe the sign reversal of the critical current implying the formation of a controllable π junction [76,77] as reported in metallic nanostructures [18,19] but not in semiconducting systems [25,26]. We have interpreted the absence of a $0-\pi$ transition in our samples by the partial charge-carrier thermalization at large control voltages. This could be prevented by optimizing the geometry of our system and enlarging the size of the reservoirs. We suggest that these experiments could be performed by using double-gated bilayer graphene for an even more tunable weak link in a Josephson junction [30], as the combination of gates not only allows the control of the charge-carrier density and the contact resistance, but also can be used to break the lattice inversion symmetry of the bilayer graphene system, opening a band gap and modifying the band structure itself. This would add another knob to explore many possible configurations of a Josephson junction.

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APPENDIX A: MODEL

Our model is based on the four-terminal generalization [48] of the three-terminal model originally developed in [47], as sketched in Fig. 5(a) in the main text. The Josephson junction is modeled as an ideal one-dimensional conductor (i.e., with perfect coupling to the superconducting reservoirs). Two control channels are connected to the Josephson junction by contacts with identical transmission probability τ . We also assume a symmetric control voltage $\pm V_{\text{ctrl}}/2$ applied to the two control terminals.

In the following, it is convenient to express all distribution functions in terms of $h_0(E) = 1 - 2f_0(E) = \tanh(E/2k_B T)$, which obeys $h_0(-E) = -h_0(E)$. In the main text and Fig. 5(c), the conventional definition of the distribution functions based on the Fermi function $f_0 = [\exp(E/k_B T) + 1]^{-1}$ is chosen.

The current of the four-terminal system can be calculated as the superposition of the currents of two three-terminal systems [48]. Consequently, the superposition of currents can be replaced by the superposition of distribution functions $h_{\text{ctrl}}(E) = [h_+(E) + h_-(E)]/2$, where h_{\pm} are the distribution functions at the two control terminals. For symmetric bias, $h_{\text{ctrl}}(E)$ obeys the same particle-hole symmetry as h_0 . Using this symmetry, we can express the supercurrent as an integral over positive energy only,

$$I(\phi) = \int_0^{\infty} I(E, \phi) dE, \quad (\text{A1})$$

which accounts for the balance of particle and hole contributions. Here, ϕ is the phase difference across the junction, and the spectral supercurrent is given by

$$I(E, \phi) = \frac{e}{h} \begin{cases} h_{\text{ctrl}}(E) j_a(E, \phi), & E < \Delta \\ h_{\text{ctrl}}(E) j_b(E, \phi) + h_0(E) j_c(E, \phi), & E > \Delta. \end{cases} \quad (\text{A2})$$

The subgap contribution is entirely controlled by h_{ctrl} , whereas the continuum contribution above the gap is affected by both the control voltage and the equilibrium distribution of quasiparticles in the superconductor. The spectral functions in Eq. (A2) are given by

$$j_a(E, \phi) = 2\tau(2 - \tau) \left(\frac{1}{F(E, \phi)} - \frac{1}{F(E, -\phi)} \right), \quad (\text{A3})$$

$$j_b(E, \phi) = \tau[u^2 + (1 - \tau)v^2] \left(\frac{1}{D(E, \phi)} - \frac{1}{D(E, -\phi)} \right), \quad (\text{A4})$$

$$j_c(E, \phi) = \frac{(2 - \tau)U(E)}{|u^2 - v^2|} \left(\frac{1}{D(E, \phi)} - \frac{1}{D(E, -\phi)} \right), \quad (\text{A5})$$

where D , F , and U are given by

$$D(E, \phi) = u^4 + v^4(1 - \tau)^2 - 2u^2v^2(1 - \tau) \cos \left(\frac{EL}{\Delta\xi_0} + \phi \right), \quad (\text{A6})$$

$$F(E, \phi) = 1 + (1 - \tau)^2 - 2(1 - \tau) \cos \left[-2 \arccos \left(\frac{E}{\Delta} \right) + \frac{EL}{\Delta\xi_0} + \phi \right] \quad (\text{A7})$$

$$U(E) = u^4 + v^4(1 - \tau) - u^2v^2((1 - 4v^2)\tau + 2) \quad (\text{A8})$$

and the coherence factors u and v are defined by

$$u^2 = \frac{1}{2} \left(1 + \frac{\sqrt{E^2 - \Delta^2}}{|E|} \right), \quad (\text{A9})$$

$$v^2 = \frac{1}{2} \left(1 - \frac{\sqrt{E^2 - \Delta^2}}{|E|} \right). \quad (\text{A10})$$

The model plots are normalized to the normal-state resistance $R_N = h/2e^2$ of the ideal single-channel Josephson junction to make them comparable to the experimental data.

APPENDIX B: PARTIAL THERMALIZATION

In addition to the fits shown in the main text, we have also experimented with phenomenological descriptions of partial thermalization. First of all, one might suspect that the effective T^* in the hot-electron regime is lower than the temperature due to the full Wiedemann-Franz-law heating. This might be the case if one includes electron-phonon cooling, or it may also occur in graphene in the hydrodynamic regime [75] (which is not relevant to our experiment). However, a hot-electron distribution with reduced heating will produce a weaker $I_c(V_{\text{ctrl}})$ dependence, and therefore does not give a good description in either doping regime. Nevertheless, one can construct simple phenomenological interpolations between the two limiting cases:

(1) A hot double step

$$f_{\text{hd}}(E) = \frac{1}{2} [f_+(E, T^*) + f_-(E, T^*)] \quad (\text{B1})$$

with enhanced T^* , but with reduced heating power compared to the Wiedemann-Franz law, i.e.,

$$T^* = \sqrt{T^2 + a \frac{V_{\text{ctrl}}^2}{4\mathcal{L}}} \quad (\text{B2})$$

with $a < 1$. This corresponds to nonideal reservoirs, where the temperature of the electrons in the copper leads is slightly increased.

(2) A superposition of double-step and hot-electron distribution

$$f_{\text{sup}}(E) = r f_{\text{h}}(E) + (1 - r) f_{\text{d}}(E) \quad (\text{B3})$$

with the distributions given in the main text and $0 < r < 1$. This is a very crude description of partial thermalization of electrons in the graphene sheet itself. These intermediate scenarios are illustrated in Fig. 7 for $V_g = -10$ V. Both can give a better fit of $I_c(V_{\text{ctrl}})$ than f_{d} , and the $0-\pi$ transition is nearly invisible. However, the distribution functions and the $I_c(V_{\text{ctrl}})$ dependencies are very similar, and a clear distinction regarding which one is better is not possible. Also, both scenarios introduce more parameters and assumptions, and provide little additional insight beyond the fact that partial thermalization must happen somewhere in the structure. We have therefore restricted ourselves to the well-defined limiting cases in the main text, and leave a full description of thermalization in these structures as a question to theory.

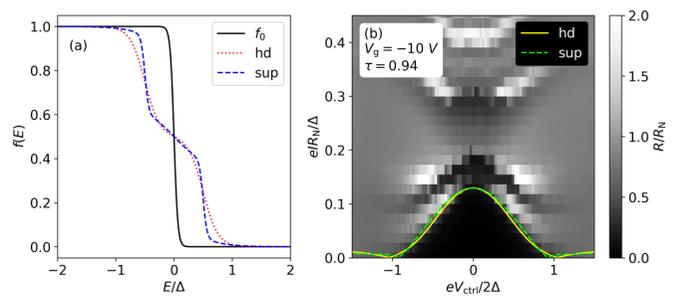


FIG. 7. Comparison of two scenarios of partial thermalization. (a) Distribution functions and (b) critical current. In both panels, $a = 0.2$ for f_{hd} and $r = 0.3$ for f_{sup} .

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