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PREFACE: SPECIAL ISSUE ON ADVANCES IN THE MATHEMATICAL STUDY OF PATTERN FORMATION

Natural physical mechanisms are capable of forming an incredible array of self-organized and regular spatio-temporal structure. Often referred to as patterns, such structures have elicited interest by scientists, engineers, and mathematicians alike. Inspired by pioneers like Liesegang, Haeckel, Wentworth Thompson, and Turing, researchers seek to understand how the combination of simple mechanisms, such as diffusion, reaction, convection, or transport, combine to create and mediate regular periodic patterns. Classic examples (for which many interesting questions are still unanswered) arise in the formation of roll states in convective fluids, stripes and spots in animal coats, ordered cell differentiation in biological morphogenesis, and spiral arrangements of primordia in flowers. As such diverse systems often produce qualitatively similar forms, it is of interest to distill any universal properties of patterns across these systems.

This special issue presents recent developments in the study of patterns in the following areas:

- Identification of patterns and their dynamics in novel important real-world applications.
- Development and application of new mathematical techniques.
- Characterization and explanation of complex, physically relevant dynamical phenomena in pattern-forming systems and models.

In recent times, a wealth of new applications and examples of pattern-forming phenomena has proliferated, in areas ranging from the formation of vegetation patterns in semi-arid climates, large-scale condensates in turbulent fluid flows, the formation of periodic waves on fluid surfaces, the propagation of electro-chemical pulses through a nerve axon or cardiac tissue, interfaces in amphiphilic bi-layers, ripples and dots in material erosion, and various patterns in social and biological swarming. Additionally, researchers have sought to harness natural pattern-forming mechanisms to fabricate novel and functional materials at various scales from the macro-scale to the nano-scale.

In parallel with this explosion of experimental and scientific study, mathematical theory has attempted to keep pace by applying, combining, and developing novel tools and techniques from various theoretical sub-domains to rigorously establish the existence, stability, bifurcation, and dynamics of patterned solutions in relevant models. Areas of recent interest include the further application of ideas from dynamical systems theory, such as spatial dynamics, where one views a PDE model as a dynamical system with an unbounded spatial variable playing the role of the time-like, or evolutionary variable, and a space of functions in other system variables playing the role of the (often infinite-dimensional) phase space. Another current area of focus consists of viewing the system in an abstract functional analytic setting, where tools from Fredholm theory, operator theory, and analytic continuation can be used to examine systems where spatial dynamics tools are difficult to apply. A third avenue of recent focus lies in geometric singular perturbation theory, where coherent structures and patterns are constructed by making use of the presence of multiple spatio-temporal scales in a system and unfolding dynamics at singular points using geometric "blow-up" or de-singularization techniques.

Further areas of development include the leveraging of additional structure in a model to understand patterns. For example, the presence of a free-energy in a given system suggests the use of tools from the calculus of variations to realize a pattern as a local minimizer, or the existence of a comparison principle in a model allows one to construct solutions of interest using sub- and super-solutions. Another interesting area lies in rigorous validation, where one wishes to use multiple-scales/modulational analysis to approximate solutions of a system in simpler, more tractable models. Finally, the use, enhancement, and development of novel computational approaches, including topological data analysis, data-assimilation, computer-assisted proofs, and numerical continuation, have provided many impactful insights into complicated phenomena.

In addition to the standard program where one establishes "existence, stability, and bifurcation" of pure patterned states, much recent work has focused on their dynamics and steered towards answering questions relevant to more complex and physically relevant dynamical phenomena, which typically involves the interaction of patterns of either the same or different types, and/or some symmetry breaking of the system. Examples include the characterization of defects, invasion fronts, and spatially localized patches of patterns. The aforementioned tools have also been recently used to rigorously understand patterns in the presence of spatial heterogeneities and domain impurities, spatio-temporal noise and random forcing, complex domain geometries, discrete media, non-local interaction, or in spatially multi-dimensional domains, all of which cause significant difficulty in the application of standard approaches.

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