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SMEFT truncation effects in Higgs boson pair production at NLO QCD

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Abstract. We present results for Higgs boson pair production in gluon fusion at next-to-leading order in QCD, including effects of anomalous couplings within Standard Model Effective Field Theory (SMEFT). In particular, we investigate truncation effects of the SMEFT series, comparing different ways to treat powers of dimension-six operators and double operator insertions.

1. Introduction

Higgs boson pair production in gluon fusion offers the possibility to measure the trilinear Higgs boson self-coupling and therefore to verify whether the form of the Higgs potential assumed in the Standard Model (SM) is correct. Deviations from this form, manifesting themselves in anomalous Higgs boson self-couplings, would be a clear sign of new physics and most likely would come along with other non-SM Higgs couplings. Therefore it is important to control the uncertainties of the theory predictions in simulations that include anomalous couplings. The theoretical uncertainties have various sources, the dominant ones in the SM being uncertainties related to the top quark mass renormalisation scheme. Theory predictions with full top quark mass dependence are available at NLO QCD [1, 2, 3, 4] and have been included in calculations where higher orders have been performed in the heavy top limit [5, 6, 7], thus reducing the scale uncertainties and the uncertainties due to missing top quark mass effects, such that the top mass scheme uncertainties currently constitute the main uncertainties [8] of the SM predictions.

Going beyond the SM description of the process $gg \to HH$, considering in particular effective field theory (EFT) parametrisations of new physics effects, new uncertainties arise, coming mainly from the truncation of the EFT expansion.

In the following we will present results at NLO SMEFT for this process, including also double operator insertions. Our implementation allows us to investigate various scenarios of truncation and to assess the related uncertainties. For more details we refer to Ref. [9].

2. Effective field theory descriptions of Higgs boson pair production

2.1. HEFT and SMEFT

In this section we introduce the two effective descriptions of unknown new physics that appears at the high energy scale Λ.

In the Standard Model Effective Theory (SMEFT) [10, 11] the SM field content and symmetries are assumed for the construction of higher order operators, such that the physical

Higgs boson is part of a linearly transforming $SU(2)_L \times U(1)$ doublet. The terms in the Lagrangian are ordered by an expansion in canonical dimension of the higher order operators $\mathcal{O}_i$ where the suppression in inverse powers of Λ is typically made explicit in the notation of the Wilson coefficients $\frac{C_i}{\Lambda^{d-4}}$, i.e.

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{C_i^{(6)}}{\Lambda^2}\mathcal{O}_i^{\mathrm{dim6}} + \mathcal{O}(\frac{1}{\Lambda^3}) \,. \tag{1}$$

The complete set of non-redundant dimension 6 (dim-6) operators is commonly used in the Warsaw basis [10], where the relevant (CP-even) terms to $gg \to hh$ are

$$\begin{aligned}\Delta\mathcal{L}_{\mathrm{Warsaw}} = &\frac{C_{H,\Box}}{\Lambda^2}(\phi^\dagger\phi)\Box(\phi^\dagger\phi) + \frac{C_{HD}}{\Lambda^2}(\phi^\dagger D_\mu\phi)^*(\phi^\dagger D^\mu\phi) + \frac{C_H}{\Lambda^2}(\phi^\dagger\phi)^3 \\ &+ \left(\frac{C_{uH}}{\Lambda^2}\phi^\dagger\phi\bar{q}_L\phi^c t_R + h.c.\right) + \frac{C_{HG}}{\Lambda^2}\phi^\dagger\phi G^a_{\mu\nu}G^{\mu\nu,a} \,.\end{aligned} \tag{2}$$

In this work the dipole operator $\mathcal{O}_{tG}$ is not included, since in SMEFT we consider a heavy sector given by renormalizable quantum field theories that couple weakly to the SM, for which it can be shown that the $\mathcal{O}_{tG}$ contribution carries an additional $\frac{1}{16\pi^2}$ factor with respect to the listed operators [14, 15]. In the case of strong coupling to the SM fields an expansion in the canonical dimension only would not be the appropriate description.

In contrast to SMEFT, the governing principle of Higgs Effective Field Theory (HEFT) is an expansion in loop orders, which can be equivalently formulated by using a counting of chiral dimensions [12, 13]. The Lagrangian has the form

$$\mathcal{L}_{d_\chi} = \mathcal{L}_{(d_\chi=2)} + \sum_{L=1}^{\infty}\sum_i \left(\frac{1}{16\pi^2}\right)^L c_i^{(L)} O_i^{(L)} \,, \tag{3}$$

where the loop factor appears as the expansion parameter $\frac{f^2}{\Lambda^2} \simeq \frac{1}{16\pi^2}$, with f being the typical energy scale of the EFT (as compared to the pion decay constant in chiral perturbation theory). In $gg \to hh$ the relevant Lagrangian terms are parametrised by five anomalous couplings [14]

$$\Delta\mathcal{L}_{\mathrm{HEFT}} = -m_t\left(c_t\frac{h}{v} + c_{tt}\frac{h^2}{v^2}\right)\bar{t}\,t - c_{hhh}\frac{m_h^2}{2v}h^3 + \frac{\alpha_s}{8\pi}\left(c_{ggh}\frac{h}{v} + c_{gghh}\frac{h^2}{v^2}\right)G^a_{\mu\nu}G^{a,\mu\nu} \,, \tag{4}$$

with no a priori relation between the c_i, since the phyical Higgs h enters as an EW singlet.

In order to write the relevant parts of the SMEFT Lagrangian in eq. (2) in a more convenient form, we expand the Higgs doublet around its vacuum expectation value and apply a gauge-dependent field redefinition for the physical Higgs boson

$$h \to h + v^2\frac{C_{H,kin}}{\Lambda^2}\left(h + h^2 + \frac{h^3}{3}\right) \,, \tag{5}$$

with $\frac{C_{H,kin}}{\Lambda^2} := \frac{C_{H,\Box}}{\Lambda^2} - \frac{1}{4}\frac{C_{HD}}{\Lambda^2}$. Afterwards, the Higgs kinetic term is canonically normalized (up to $\mathcal{O}\left(\Lambda^{-4}\right)$) and the anomalous couplings can be related between SMEFT and HEFT by a comparison of the coefficients of the corresponding terms in the Lagrangian, which is shown in Table 1.

It is important to note, however, that a translation between the coefficients should only be considered with care. EFT descriptions have a limited validity range due to unitarity constraints and, in addition, SMEFT relies on a small expansion parameter C_iE^2/Λ^2 (with E being the probed energy scale) for a convergence of the operator series. Moreover since in SMEFT the Higgs doublet field is used, there are relations between the anomalous couplings that are not present in HEFT. Hence HEFT is the more general theory and a naive translation of a perfectly valid point in its parameter space can lead out of the validity range of SMEFT.

HEFT	Warsaw
c_{hhh}	$1 - 2\frac{v^2}{\Lambda^2}\frac{v^2}{m_h^2}\,C_H + 3\frac{v^2}{\Lambda^2}\,C_{H,kin}$
c_t	$1 + \frac{v^2}{\Lambda^2}\,C_{H,kin} - \frac{v^2}{\Lambda^2}\frac{v}{\sqrt{2}m_t}\,C_{uH}$
c_{tt}	$-\frac{v^2}{\Lambda^2}\frac{3v}{2\sqrt{2}m_t}\,C_{uH} + \frac{v^2}{\Lambda^2}\,C_{H,kin}$
c_{ggh}	$\frac{v^2}{\Lambda^2}\frac{8\pi}{\alpha_s}\,C_{HG}$
c_{gghh}	$\frac{v^2}{\Lambda^2}\frac{4\pi}{\alpha_s}\,C_{HG}$

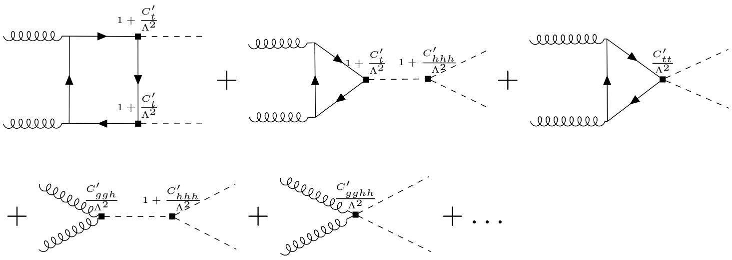
Table 1. Leading order translation between different operator basis choices.

2.2. SMEFT truncation

Another delicate point in the EFT expansion is the question how to treat terms with inverse powers of Λ higher than two at cross section level, i.e. when squaring the amplitude. These are terms related to squared dim-6 operators, double operator insertions in a single diagram and combinations thereof. Related issues have been reviewed recently in Ref. [16].

We now introduce a Monte Carlo program which allows us to study the truncation effects systematically. In order to set up the different truncation options, we first decompose the amplitude into three parts: the pure SM contribution (SM), single dim-6 operator insertions (dim6) and double dim-6 operator insertions (dim6^2):

$\mathcal{M} =$



$$= \mathcal{M}_{\text{SM}} + \mathcal{M}_{\text{dim6}} + \mathcal{M}_{\text{dim6}^2}\ , \tag{6}$$

where C' denotes the respective combination of Wilson coefficients in Table 1. For the squared amplitude forming the cross section, we introduce four options to select the parts of eq. (6) entering $|\mathcal{M}|^2$:

$$\sigma \simeq \begin{cases} \sigma_{\text{SM}} + \sigma_{\text{SM}\times\text{dim6}} \\ \sigma_{(\text{SM+dim6})\times(\text{SM+dim6})} \\ \sigma_{(\text{SM+dim6})\times(\text{SM+dim6})} + \sigma_{\text{SM}\times\text{dim6}^2} \\ \sigma_{\left(\text{SM+dim6+dim6}^2\right)\times\left(\text{SM+dim6+dim6}^2\right)}\ . \end{cases} \tag{7}$$

The first line is the first order of an expansion in Λ^{-2} of the cross section, the second term is the first order of an expansion in Λ^{-2} of the amplitude. The third line adds all dim-6 operator contributions of $\mathcal{O}\left(\Lambda^{-4}\right)$ in the cross section (including single and double insertions), however it misses dim-8 operator contributions and $\mathcal{O}\left(\Lambda^{-4}\right)$ terms following the field redefinition of eq. (5) that enter at the same order. The fourth line is the naive translation from HEFT to SMEFT using Table 1. Typically, only the first two options are used for predictions and measurements using SMEFT, since both are unambiguous wrt. basis change and gauge invariance, however there is still a debate about the recommendations for their application to experimental analyses [16]. Thus, we include all of the presented options in our calculation, which can serve to contrast different outcomes of the predictions.

3. NLO Implementation into the event generator program POWHEG and results

3.1. Parametrisation of the $gg \to HH$ total cross section

The implementation of our calculation, combining numerically evaluated 2-loop results and NLO real radiation of a loop-induced process with SMEFT operators, used the publicly available NLO HEFT code presented in Refs. [17, 18] as a starting point. The formalism was converted to the SMEFT framework and extended such that the different options described in the previous section can be calculated, including NLO QCD corrections with full top quark mass dependence.

To calculate the contributions from real emission, a modified `GoSam` [19] version was constructed, splitting the amplitude evaluation according to eq. (6). The code is able to evaluate the squared amplitude with a truncation option that can be set by the user via an input variable. For the generation of the `GoSam` files containing the matrix elements used by the `POWHEG-BOX` [20], model files in UFO format [21] have been produced, containing the anomalous couplings such that `GoSam` is able to evaluate the contributions corresponding to the chosen truncation option. The existing interface to `POWHEG` has been modified to hand over parameters which depend on a scale to `GoSam`, such that the factor α_s between the effective Higgs-gluon couplings in HEFT and SMEFT is evaluated at the correct energy scale.

The virtual part is based on grids encoding the virtual 2-loop amplitudes. These grids can be used to reconstruct the amplitude for any given combination of anomalous couplings. The a_i listed below are defined as the coefficients in the representation of the sqared amplitude as a linear combination of all coupling combinations possible in HEFT at NLO QCD.

$$\begin{aligned}
|\mathcal{M}_{BSM}|^2 =& a_1 \cdot c_t^4 + a_2 \cdot c_{tt}^2 + a_3 \cdot c_t^2 c_{hhh}^2 + a_4 \cdot c_{ggh}^2 c_{hhh}^2 + a_5 \cdot c_{gghh}^2 + a_6 \cdot c_{tt} c_t^2 + a_7 \cdot c_t^3 c_{hhh} \\
&+ a_8 \cdot c_{tt} c_t c_{hhh} + a_9 \cdot c_{tt} c_{ggh} c_{hhh} + a_{10} \cdot c_{tt} c_{gghh} + a_{11} \cdot c_t^2 c_{ggh} c_{hhh} + a_{12} \cdot c_t^2 c_{gghh} \\
&+ a_{13} \cdot c_t c_{hhh}^2 c_{ggh} + a_{14} \cdot c_t c_{hhh} c_{gghh} + a_{15} \cdot c_{ggh} c_{hhh} c_{gghh} + a_{16} \cdot c_t^3 c_{ggh} \\
&+ a_{17} \cdot c_t c_{tt} c_{ggh} + a_{18} \cdot c_t c_{ggh}^2 c_{hhh} + a_{19} \cdot c_t c_{ggh} c_{gghh} + a_{20} \cdot c_t^2 c_{ggh}^2 \\
&+ a_{21} \cdot c_{tt} c_{ggh}^2 + a_{22} \cdot c_{ggh}^3 c_{hhh} + a_{23} \cdot c_{ggh}^2 c_{gghh} \,.
\end{aligned}$$

Since the first and the third truncation options of eq. (7) are expansions at cross section level, and the fourth option is the direct translation from HEFT to SMEFT, for those cases the application of the translation of Table 1, including all terms at the desired order in inverse Λ, is sufficient. In the case of the second truncation option, some of the grids can be reused as well, but the determination of the coefficients needs more care, as there are additional combinations. Note that we do not include RGE running of the couplings as we only consider NLO QCD corrections to the amplitude, which factorise.

In Fig. 1 we show that the results for the total cross sections (normalised to the SM case) are substantially different between option 1 (linear dim-6, top) and option 2 (quadratic dim-6, bottom). The white areas come from the fact that taking into account only linear dim6-contributions leads to negative cross sections over large parts of the parameter space. Furthermore, in the linear dim-6 case, there appears to be a completely flat direction in the observed parameter range for a combined variation of the respective Wilson coefficients in the diagrams. Flat directions are apparent in option 2 as well, however they correspond to an elliptic shape of equipotential lines due to the quadratic terms in the cross section.

3.2. Investigation of truncation effects for the Higgs boson pair invariant mass distribution

Now we turn to differential results, showing the effects of the different truncation options on the Higgs boson pair invariant mass distribution m_{hh}. We present results at two benchmark points,

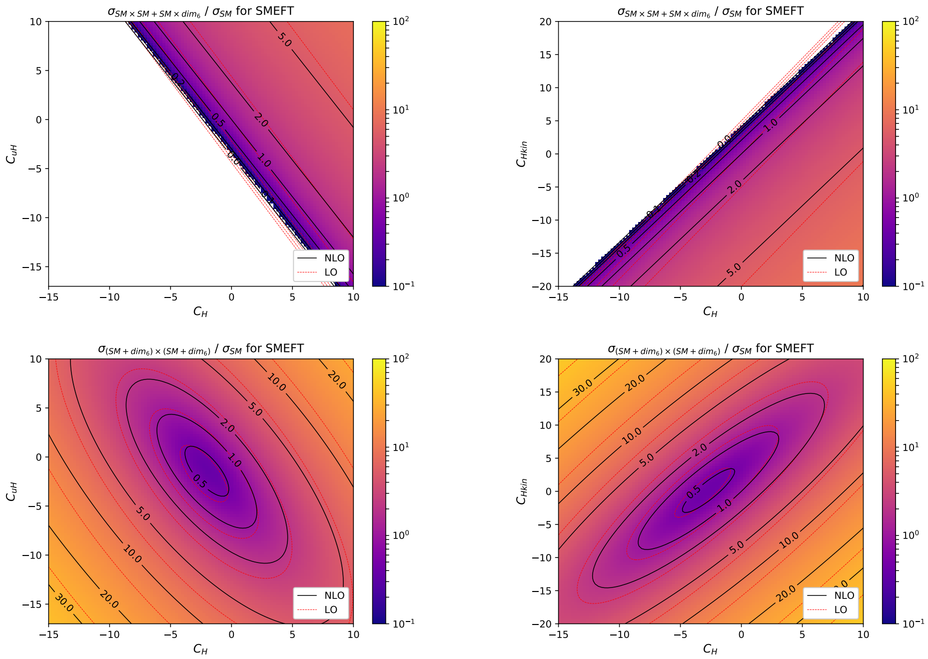


Figure 1. Heat maps showing the dependence of the cross section on the couplings C_H, C_{uH} (left) and C_H, $C_{H,kin}$ (right) with $\Lambda = 1\,\text{TeV}$ for different truncation options. Top: option 1 (linear dim-6), bottom: option 2 (quadratic dim-6). The white areas denote regions in parameter space where the corresponding cross section would be negative.

given in Table 2, which were derived analogously to [22] based on an analysis of characteristic shapes of the m_{hh} distribution, but with the inclusion of current experimental constraints. The upper panels show results for $\Lambda = 1\,\text{TeV}$, the lower panels show results for the same point for $\Lambda = 2\,\text{TeV}$, for the different truncation options. One can clearly see that (a) the negative differential cross section values in the linear dim-6 case (blue) indicate that parameter points in anomalous coupling parameter space which are valid in HEFT can lead, upon naive translation, to parameter points for which the SMEFT expansion is not valid, (b) destructive interference between different parts of the amplitude (e.g. box- and triangle-type diagams) can be enhanced or diminished depending on the truncation option, (c) increasing Λ reduces the differences between the results as they are smaller deformations of the SM parameter space. In addition, we observe that the contribution from the interference of double dim-6 operator insertions with the SM appears to be subdominant in the example of benchmark point 1*, as can be seen by comparing truncation option 2 (orange) with option 3 (red), the latter including the double operator insertions. We also should point out that the difference between HEFT (cyan) and SMEFT with truncation option 4 (green) is entirely due to the scale dependence of α_s, coming from the definition of C_{HG} in the Warsaw basis, see Table 1.

4. Summary

We have presented NLO QCD corrections to Higgs boson pair production in combination with a Standard Model Effective Field Theory (SMEFT) parametrisation of effects of physics beyond the Standard Model. The calculation has been implemented into the `GoSam+POWHEG` Monte Carlo program framework in a way which allows to choose different options for the truncation of the EFT series and to compare to results in (non-linear) Higgs Effective Field Theory (HEFT).

benchmark (* = modified)	c_{hhh}	c_t	c_{tt}	c_{ggh}	c_{gghh}	$C_{H,\text{kin}}$	C_H	C_{uH}	C_{HG}	Λ
SM	1	1	0	0	0	0	0	0	0	1 TeV
1*	5.105	1.1	0	0	0	4.95	−6.81	3.28	0	1 TeV
6*	−0.684	0.9	$-\frac{1}{6}$	0.5	0.25	0.561	3.80	2.20	0.0387	1 TeV

Table 2. Benchmark points used for the invariant mass distributions. The benchmark points were derived analogously to [22], but are slightly modified compared to the ones given in [22], to take into account current experimental constraints. The value of C_{HG} is determined using $\alpha_s(m_Z) = 0.118$.

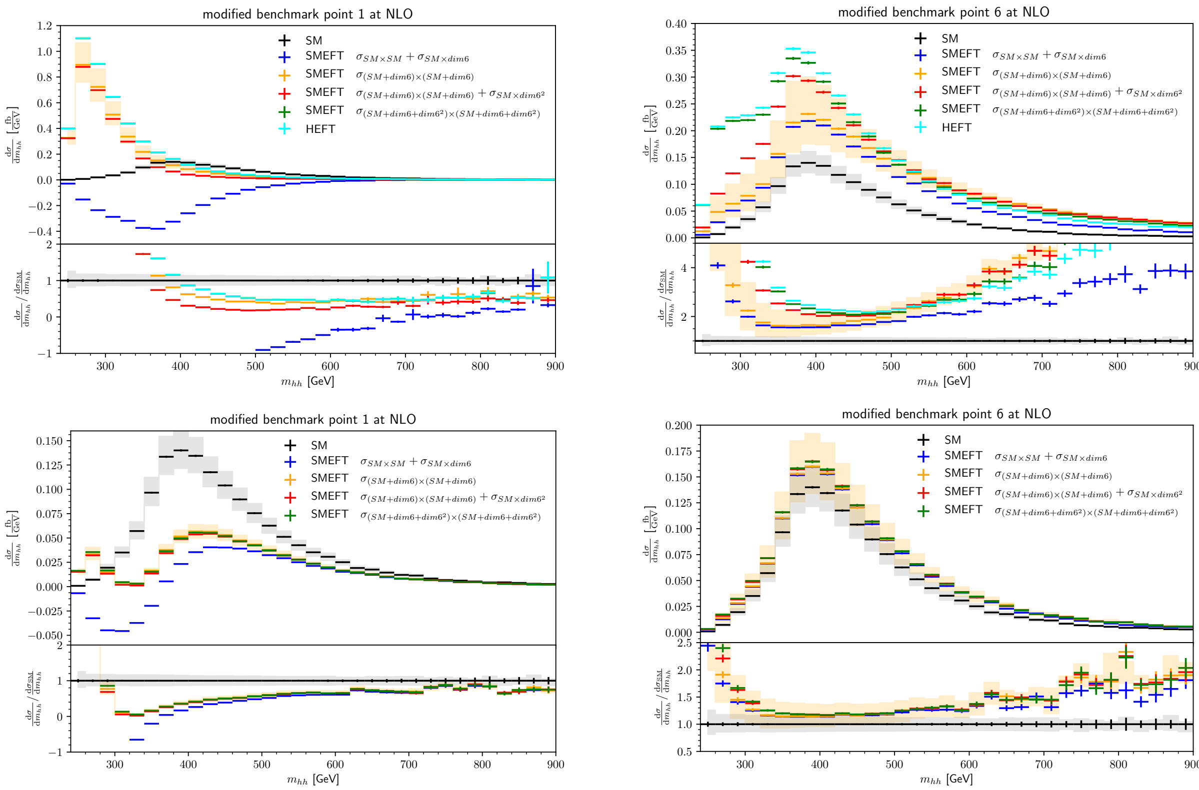


Figure 2. Differential cross sections for the Higgs boson pair invariant mass. Top row: $\Lambda = 1\,\text{TeV}$, bottom row: $\Lambda = 2\,\text{TeV}$. Left: benchmark point 1*, right: benchmark point 6* of Table 2. The gray and orange bands show the uncertainty from 3-point scale variations $\mu_R = \mu_F = c \cdot m_{hh}/2$, with $c \in \{\frac{1}{2}, 1, 2\}$, for the SM and SMEFT $\sigma_{(SM+dim6)\times(SM+dim6)}$, respectively.

The results show that a naive translation between HEFT and SMEFT has pitfalls and that the various truncation options can lead to large differences in the theory predictions.

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