## 8.3 Radiance

Radiance is a quantity used to describe a light field in the context of geometric optics. We introduce it step by step. We first choose a measure for the amount of light: energy. (The following considerations would be the same with other extensive quantities, such as the number of photons or entropy).

The light in the box in Fig. 8.7a has a well-defined energy. In Fig. 8.7b light is leaving the box through the hole, and thus an energy current of strength $P$ flows outwards. If this current is divided by the surface element $dA$ through which it flows, we obtain the magnitude $j_E$ of the energy current density. It is

$$P = \int \mathbf{j}_E \, d\mathbf{A}$$

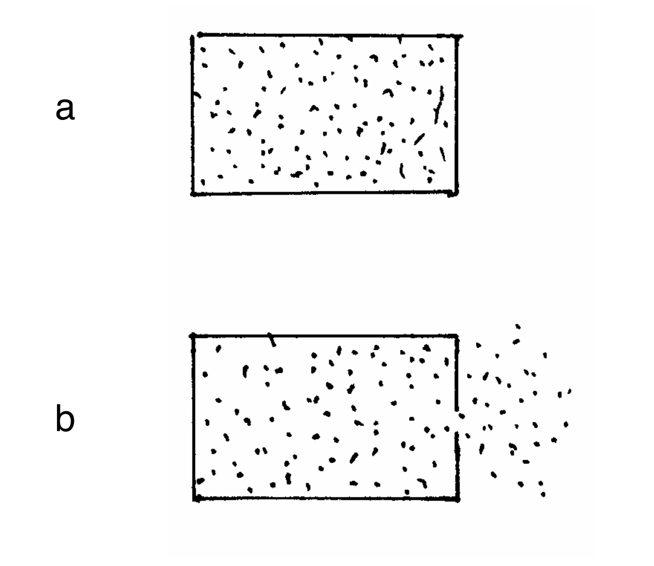


**Fig. 8.7**
(a) The box contains light. (b) Through the hole, light flows out.

Now the light rays pass through each surface element $dA$ in the various directions. We therefore divide $j_E$ by the solid angle element $d\Omega$ and obtain the energy flux density per solid angle, or in short, the radiance $L_E$. It is

$$P = \int_A \int_\Omega L_E \, d\Omega \, dA \tag{8.1}$$

The quantity $L_E$ depends

- on the position in the light field;
- at a fixed position on the direction.

So we have

$$L_E = L_E(\mathbf{r}, \vartheta, \Phi)$$

where the direction in space is characterized by the angles $\vartheta$ and $\Phi$.

Figure 8.8 shows a measuring device for $L_E$. The surface of the lens corresponds to the area $dA$ in (8.1), the photocell area defines $d\Omega$.

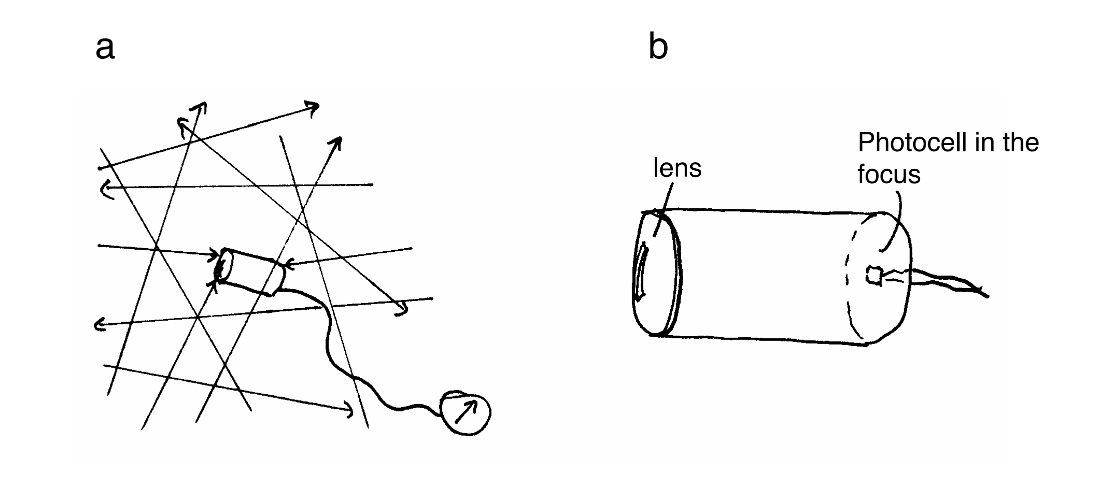


**Fig. 8.8**
(a) The radiance meter registers light that belongs to a position and direction. (b) Design of the instrument

Often the $L_E$ distribution is rotation-symmetric with respect to an *optical axis*. In this case, two coordinates in the positional space and one angular coordinate are sufficient.

Figure 8.9 shows an example of a radiance distribution. The light comes from a sharply delimited, uniformly radiating surface F, Fig. 8.9a. Fig. 8.9b shows the distribution of $L_E$ at the position $z_0$ of the $z$-axis over $x$ and the angle $\vartheta$ against the $z$-axis in perspective. Fig. 8.9c shows a projection onto the $x$-$\vartheta$ plane. In the hatched area $L_E$ has a finite, constant value, outside $L_E = 0$.

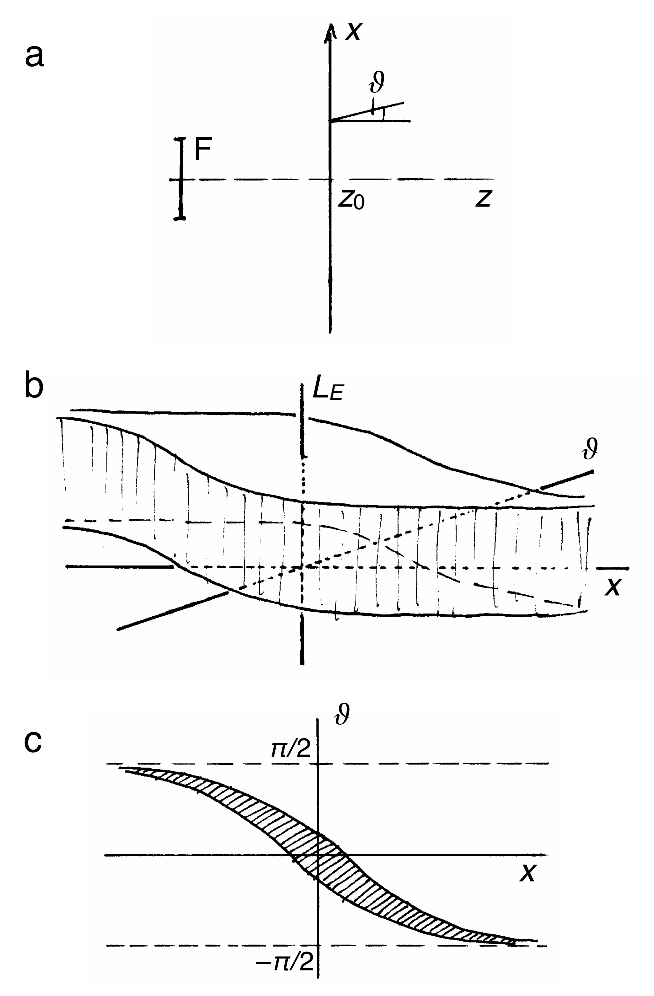


**Fig. 8.9**
Example of a radiance distribution. (a) The light comes from the uniformly radiating surface F. (b) The radiance is plotted over the position and the direction. (c) Projection into the $x$-$\vartheta$ plane

Figure 8.10 shows a sequence of representations of $L_E$ over $\vartheta$ for $x = 0$ at various distances from the light source on the $z$-axis.

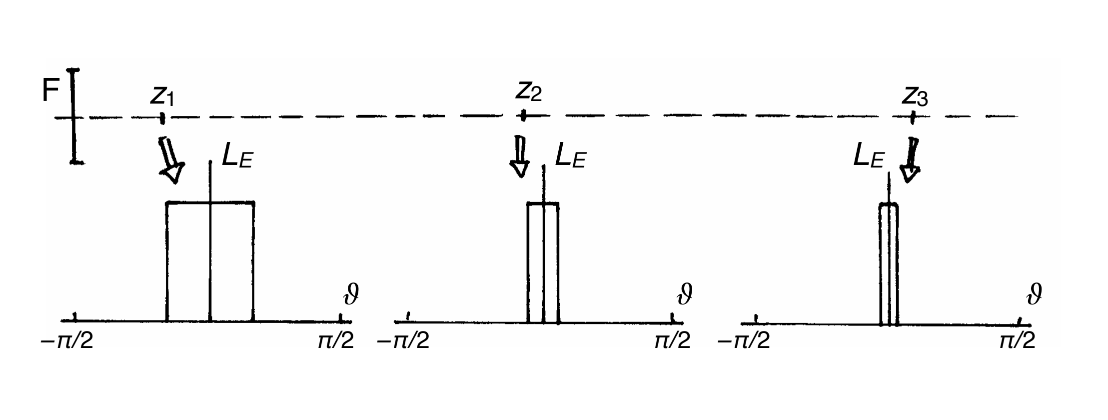


**Fig. 8.10**
Radiance versus direction for different distances from the illuminating surface F

One can see that the single images differ only in the width of the distribution. The value of $L_E$ in the direction of the $z$-axis ($\vartheta = 0$) does not change with increasing distance. This is an effect of the following rule: The radiance has the same value at all points of a beam in the direction of the beam.

In this form, however, the rule only applies as long as $n$ is the same everywhere on the beam. The rule can be generalized:

> The quantity $L_E/n^2$ has the same value at all points on a beam in the direction of the beam.

Here is another consequence of this rule:

One might expect that with a sufficiently large lens one could concentrate as much light from the sun as one wants in one point. If one places an object at this point, one could thus bring it to an arbitrarily high temperature. But this contradicts the 2nd law of thermodynamics. Our theorem $L_E/n^2$ = const shows us immediately that this is not possible.

The sequence of images in Fig. 8.11 shows that at best it is possible to arrive at a situation where $L_E$ in P has the same value $L_{E0}$ over the entire solid angle. If one has achieved this, however, the point is in an environment identical to the one directly at the solar surface. Because also on the sun $L_E$ has the same value $L_{E0}$ according to our rule. The point can therefore at most assume the temperature of the surface of the sun; it is then in thermal equilibrium with the sun – and by the way, it radiates back to the sun via the lens and the mirror as much light as it receives from there.

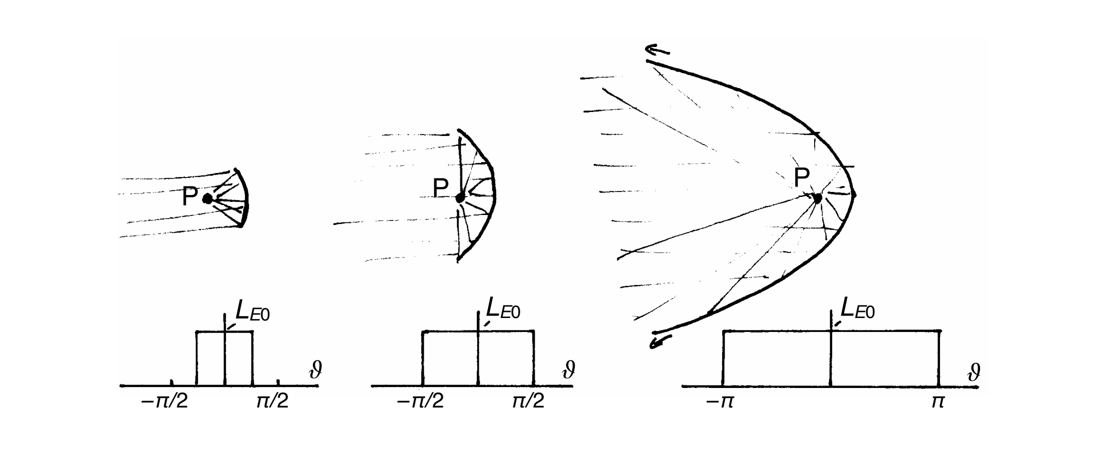


**Fig. 8.11**
By enlarging the parabolic mirror, light from all directions arrives at point P. The radiance is not changed by the mirror.

Fig. 8.12 finally shows qualitatively what happens to the $L_E$ distribution when the sky is cloudy. On the way from $z_1$ to $z_2$ through the clouds the narrow $L_E(\vartheta)$ distribution is smeared over the whole half space.

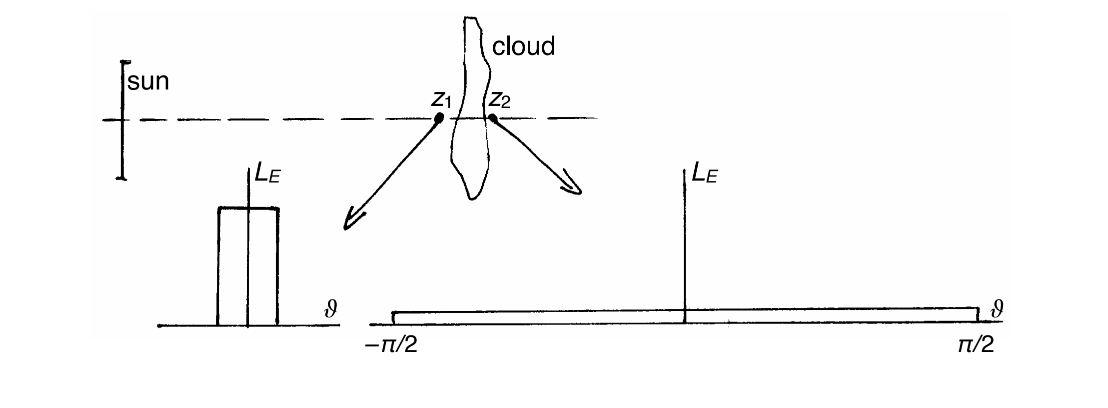


**Fig. 8.12**
Change in the radiance distribution of sunlight caused by a cloud

## 8.2 Fermat's principle

We assume that the conditions for operating with light rays are fulfilled and turn to the rules of geometric optics.

If one brings into a distribution of light that comes from the left, Fig. 8.3a, two pinholes mounted one behind the other, a so-called collimator, Fig. 8.3b, a narrow beam of light is created that propagates similarly to a ray. Therefore, such a light beam is often called a ray – in accordance with the colloquial use of the word ray. Such a light beam makes it possible to examine the rules that apply to light rays.

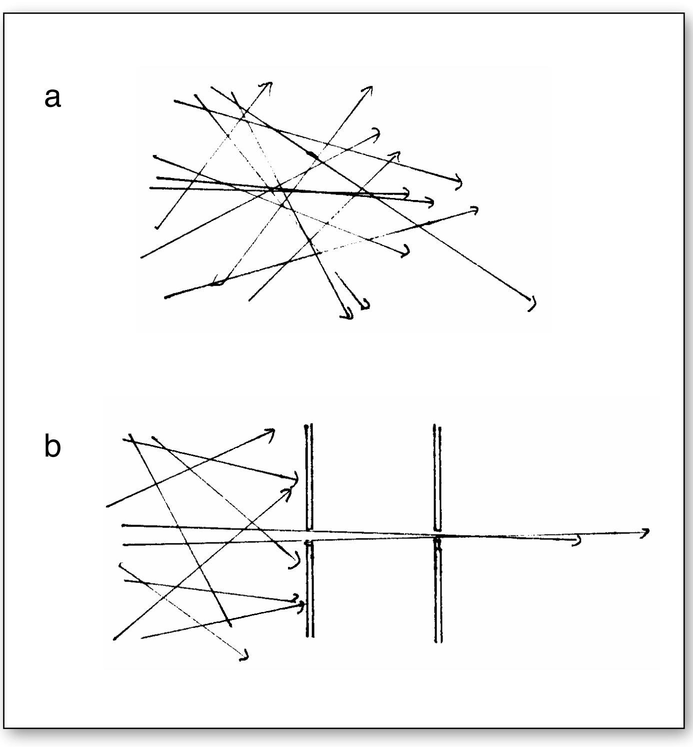


**Fig. 8.3**
(a) Light without a well-defined direction of propagation. (b) A light ray is generated with the collimator.

The typical task of geometrical or ray optics is as follows:

Given is a point P at the position ***r*** and a direction $\vartheta$, $\Phi$. What is the further path of the beam passing the point P in the direction $\vartheta$, $\Phi$? Fig. 8.4 illustrates this task for the case that the ray travels in the drawing plane.

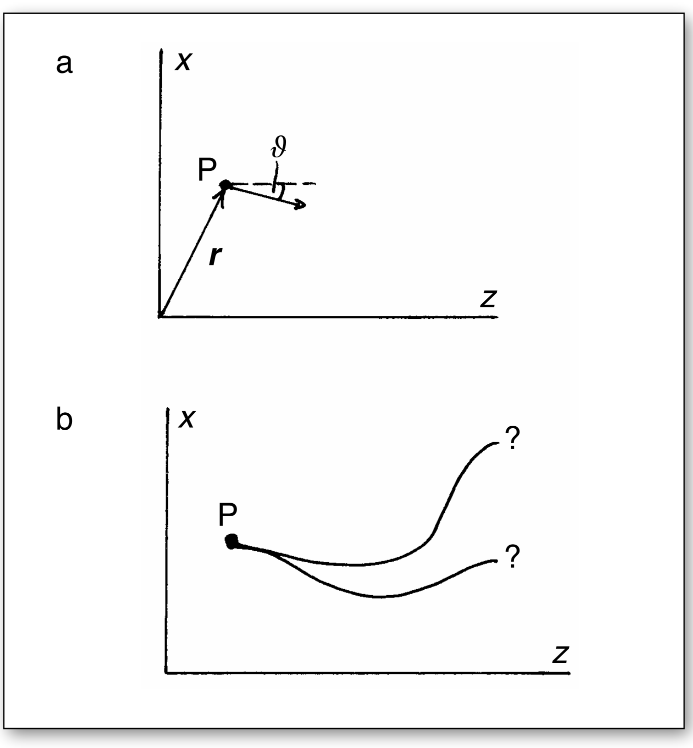


**Fig. 8.4**
(a) The light starts in a certain direction. (b) Where does it go next?

As long as the refractive index is spatially constant and changes step-wise only at well-defined interfaces, the following three known rules can be applied:

- Light propagates in a straight line.
- Law of reflection ($\alpha = \alpha'$)
- Law of refraction ($n_a \sin \alpha = n_b \sin \beta$)

These rules are sufficient for the treatment of many optical devices. Tracking a beam through a sequence of refractive and reflective interfaces is called *ray-tracing*.

Now, the three rules can be replaced with a single, generally valid rule: *Fermat's principle*. For this purpose, we first define the *light path w* between two points A and B:

$$w_{AB} = \int_A^B n\, ds$$

Here *ds* is an infinitesimal section of the light beam and *n* is the refractive index. Fermat's principle states that the actual light path between two given points A and B is minimum compared to hypothetical neighboring paths between these points:

$$\delta(w_{AB}) = 0$$

Now, we allow the refractive index to change continuously in space.

The variational calculus deals with the general solution of such an expression.

That the law of reflection follows from Fermat's principle can be seen easily, Fig. 8.5. Besides B, the point B', which is mirror-symmetrical to B, is marked in the figure. One can see that the path APB is equal to the path APB'. It is obvious that APB' is minimal if $\alpha = \alpha'$ is chosen.

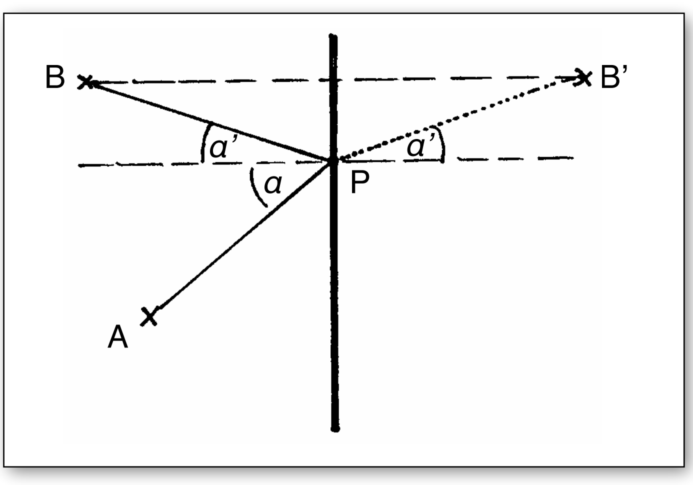


**Fig. 8.5**
The length of the path APB is minimal when $\alpha = \alpha'$.

The derivation of the law of refraction from Fermat's principle is somewhat more complicated.

A ray always starts at a light source or a scattering object and ends on an absorbing or scattering object. One notices the special role that scattering objects play, for example white surfaces or ground glass: the rays of the incident light end here, and new rays of light begin, but their directions cannot be determined according to Fermat's principle from the directions of the incident rays.

As an example, we consider a lens, Fig. 8.6. Here, we refer to a lens as a body of glass whose surface shape is such that all light rays emanating from a point A merge into a point B.

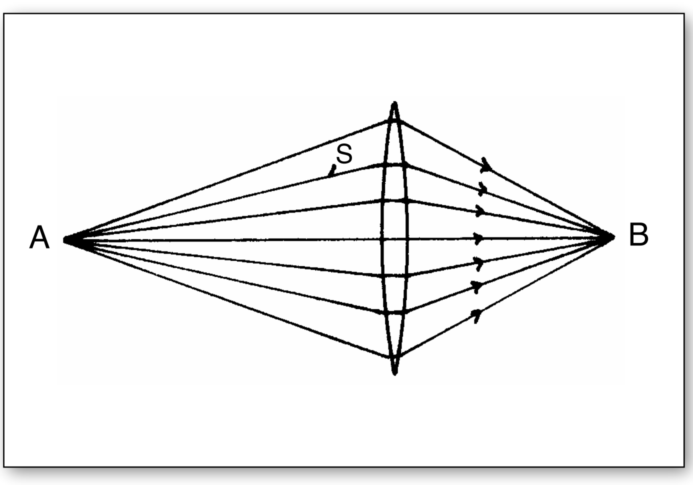


**Fig. 8.6**
The optical lengths of all rays are the same.

Notice that although such a glass body can be manufactured exactly within the framework of geometric optics, one should not expect the surfaces to be spherical surfaces, as is the case with most technical lenses.

The fact that in our arrangement not only one ray S but also many other rays adjacent to S run from A to B means that the light paths of all these rays are equal because of Fermat's principle. This is an important property of every optical imaging: If one passes from an object point A to an image point B, the light path is the same on all rays.

It should be mentioned already here that, if the lens is designed to image A into B, there is generally no other point A' that is imaged into any point B'. The rays emanating from A' do not intersect at a common point.

## 8.1 Light rays

We all know the description of light by rays. Rays are imaginary lines. Light moves along these lines like particles on a trajectory.

The description of light by rays is only possible under certain conditions. In order to formulate these conditions, we have to investigate what the special feature of the description by rays is, if we assume that light should actually be described by waves.

The description by rays implies, on the one hand, that light casts a sharp, "geometric" shadow. In Fig. 8.1, light is emitted from the small pinhole L1, and creates a sharp shadow of the large hole L2 on the screen. The shape of the shadow is obtained by the construction known to everyone. It is also said that light propagates in straight lines. But the sharp shadow is only obtained if the diffraction of the light at L2 can be neglected, and this is the case if the hole is large compared to the wavelength. Thus, our first condition for the validity of ray optics is:

*The wavelength must be small compared to the dimensions of obstacles.*

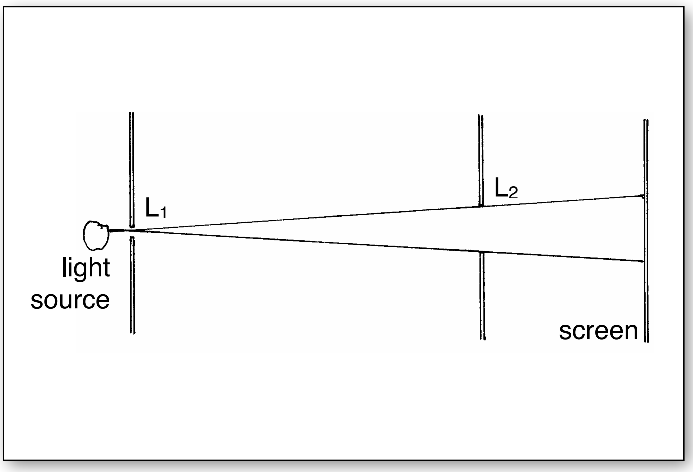


**Fig. 8.1**
If the wavelength of the light is small against the hole $L_2$ , a sharp shadow of the hole appears on the screen.

On the other hand, the description by rays implies that it is possible to add up the energy flows corresponding to two rays, Fig. 8.2, but this is only valid if the light is sufficiently incoherent. The light whose energy currents are added must not originate from the same elementary beam, otherwise interference patterns will occur. Our second condition for the validity of ray optics is therefore

*The light must be sufficiently incoherent.*

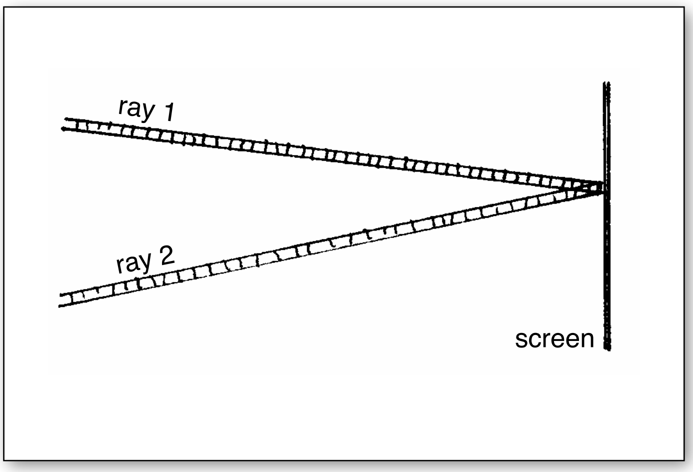


**Fig. 8.2**
The energy fluxes can only be added if the light is sufficiently incoherent.

The approximation of ray optics behaves to wave optics in the same way as the approximation of classical point mechanics to quantum mechanics. The concept of a light ray in ray optics corresponds to the concept of the path of a mass point in Hamiltonian mechanics.

If one applies ray optics, one only asks for the path of the light. One does not ask for the velocity at which the light travels on the rays. One also does not care about the polarization and therefore one should not ask about which part of the light is reflected and which part is refracted on a glass surface.

# 8

# Ray optics