# Heaviside's Gravitoelectromagnetism: What is it good for and what not?

Friedrich Herrmann [1][0000-0002-6456-4601] and Michael Pohlig [2][0000-0003-1654-3598]

[1] Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany
[2] Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany
f.herrmann@kit.edu

**Abstract.** We learn and teach classical mechanics essentially as it was developed by Newton. The theory is more than 300 years old, but still useful for many purposes. However, even in contexts where it produces correct results, it has a flaw: it uses actions at a distance. In addition, it is not able to describe the transport and storage of energy in the gravitational field locally.
The general theory of relativity not only eliminated the actions at a distance of Newtonian mechanics, but also predicted phenomena which the Newtonian theory of gravitation could not explain. However, for solving many problems and for an introduction of gravitation in a standard lecture, general relativity is too complicated, mainly because of the tensor calculus.
To bridge the gap between these two theories we propose to use Heaviside's theory of gravitoelectromagnetism. This theory has the same structure as Maxwell's electromagnetism. It has the advantage that it does not describe forces as actions at a distance and that it allows to establish a local energy balance.
We discuss the limits of applicability of Heaviside's theory. It turns out that besides the well-known condition low field/slow motion, another condition must be satisfied. The theory is only applicable to quasi-stationary processes. In particular, it cannot describe gravitational waves. Nevertheless, it is useful for teaching, because some major shortcomings of the Newtonian theory are avoided.

**Keywords:** Gravitation, Gravitoelectromagnetism, Gravitational waves, Second law, Action-at-a-distance.

## 1 Introduction

We learn and teach classical mechanics essentially as it was developed by Newton. The theory is more than 300 years old, but still useful for many purposes. However, even in contexts where it produces correct results, it has a flaw: it uses actions at a distance, which were already disapproved by Newton [1]. Also the more modern formulations by Hamilton, Lagrange, Jacobi have not changed anything essential in this respect.

Faraday and Maxwell showed how to describe electromagnetic interactions without actions at a distance, i.e. by means of fields. After Maxwell had described the electric and magnetic interactions in one of the most elegant theories known to us, it was natural to put also an end to the actions at a distance of the gravitational interaction, and indeed the attempt was made - first by Maxwell himself. Such a description of gravitation has the consequence that the energy density of gravitational fields is negative. This was reason enough for Maxwell to reject this kind of theory and he did not publish it [2]. Nevertheless, 20 years later a similar theory was published by Heaviside [3].

Heaviside was not deterred by Maxwell's uneasiness about the negative energy density from proposing a theory of gravitation analogous to electromagnetism. In fact, later developments proved him right. From the general theory of relativity (GTR) (together with the special theory of relativity) follows that the total energy of the system body + field always remains positive.

The Heavisidian "gravitoelectromagnetism" (in the following HGEM) is analogous to Maxwell's electromagnetism (EM): Mass is the source of the gravistatic field (field strength **g**), a mass flow is a source of the gravinetic field (field strength **b**). (We adopt the terms gravistatic and gravinetic from Krumm and Bedford [4].) However, there was still a problem: the gravinetic part of the theory could not be experimentally confirmed (or refuted). The gravinetic forces, i.e. the forces analogous to the Lorentz forces of electromagnetism, are too weak to be measured.

22 years later, a new theory of gravitation was established which not only eliminated the actions at a distance of Newtonian mechanics, but also predicted phenomena which neither Newton's nor Heaviside's theory could explain. It was, in a sense, the final solution that was soon confirmed experimentally: the general theory of relativity (GTR).

So why should we still be interested in HGEM today? The answer has to do with our teaching at school and university. The GTR is not suitable for school, and as far as the college is concerned, students learn about it at best in a special lecture. The difficulties of the GTR are not in the first place conceptual ones. They are mainly mathematical difficulties, namely, first, the four-dimensional tensors, and second, the nonlinearity of the Einstein equation. The GTR is mathematically too complex to serve as an everyday tool for solving simple problems of gravitation.

Thus we have a reason to ask whether HGEM could fill the gap between Newtonian actions-at-a-distance mechanics and Einstein's theory, which is mathematically too demanding.

Heaviside's theory is not to be confused with another theory which goes under the name of gravitoelectromagnetism (GEM). Shortly after the appearance of the GTR, Thirring [5] showed that one can construct a theory by approximation of the GTR which shows a "formal analogy" to EM. It allows to describe GTR effects locally by tidal forces. It achieves more than the original Heaviside theory, but it is considerably more complicated than the latter, since it needs the tensor calculus. It is fair to say that it can only be understood if the GTR is already understood [6 - 9].

The two theories, HGEM and GEM, have in common that they are valid only in the approximation of weak fields and slow motion. In both of them a quantity is introduced which is known from classical mechanics as gravitational field strength **g**. In addition, both introduce a quantity which corresponds to the magnetic field strength. But here we come to the dissimilarity. This second field quantity is not identical in the two theories, although it is often designated by the same letter, namely **b**.

In this paper we want to discuss the usefulness of HGEM, or, more precisely, we want to explore the limits of its usefulness. Apart from the weak-fields-slow-motion condition, are there any other limits? The main question will be whether the theory predicts gravitational waves, and if not, why not.

We first recall in Section 2 some characteristics of HGEM fields, in particular the consequences of the reversed signs.

While time-constant gravistatic and gravinetic fields have been discussed in [10], in Section 3 we investigate the HGEM description of some effects with time-dependent fields.

In section 4, we address gravitational waves. We will show that HGEM gravitational waves cannot exist. We will thus define an additional limit for the validity of HGEM: HGEM only describes quasi-stationary processes.

Do we have to conclude that HGEM should be discarded? In section 5 we argue that this should not be the consequence. If one knows the range of validity of the theory, it is a description of gravitation which is simple and superior to Newtonian mechanics.

## 2 Properties of the HGEM fields

We first contrast the most important equations of EM (equations 1 to 7) and HGEM (equations 8 to 14).

$$\nabla \cdot \mathbf{E} = \frac{1}{\varepsilon_0}\rho_Q \quad (1) \qquad \nabla \cdot \mathbf{g} = -\frac{1}{\varepsilon_\mathrm{g}}\rho_m \quad (8)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (2) \qquad \nabla \cdot \mathbf{b} = 0 \quad (9)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (3) \qquad \nabla \times \mathbf{g} = -\frac{\partial \mathbf{b}}{\partial t} \quad (10)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}_Q + \frac{1}{c^2}\cdot\frac{\partial \mathbf{E}}{\partial t} \quad (4) \qquad \nabla \times \mathbf{b} = -\mu_\mathrm{g}\mathbf{j}_m + \frac{1}{c^2}\cdot\frac{\partial \mathbf{g}}{\partial t} \quad (11)$$

$$\rho_E = \frac{\varepsilon_0}{2}\mathbf{E}^2 \quad (5) \qquad \rho_E = -\frac{\varepsilon_\mathrm{g}}{2}\mathbf{g}^2 \quad (12)$$

$$\rho_E = \frac{1}{2\mu_0}\mathbf{B}^2 \quad (6) \qquad \rho_E = -\frac{1}{2\mu_\mathrm{g}}\mathbf{b}^2 \quad (13)$$

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} \quad (7) \qquad \mathbf{S} = -\frac{1}{\mu_\mathrm{g}}\mathbf{g}\times\mathbf{b} \quad (14)$$

We have abbreviated

$$\varepsilon_\mathrm{g} = \frac{1}{4\pi G} \qquad \mu_\mathrm{g} = \frac{4\pi G}{c^2}$$

The differences between Maxwell's equations and their analogs are easy to see: a sign in the sources $\rho$ and $\mathbf{j}$. This difference has far-reaching consequences and will be of concern to us in what follows. It propagates to the energy densities and into the energy current densities.

The equations look reasonable and indeed they allow a consistent description of many phenomena [10]. The fact that the energy density of the **g** and **b** fields is negative means that energy is needed to create field-free space, a fact that is easy to see in the case of **g**. When a white dwarf collapses to a neutron star in the form of a supernova, new field is produced (because the neutron star is smaller than the white dwarf), and gigantic amounts of energy are released.

Usually, the **b**-field is so weak that “Lorentz forces” are immeasurably small. But it clearly manifests itself via the energy flow, for example in the gravitational field of the earth, if a body is moved up or down, so that “its” potential energy changes, which is actually the energy of the gravitational field.

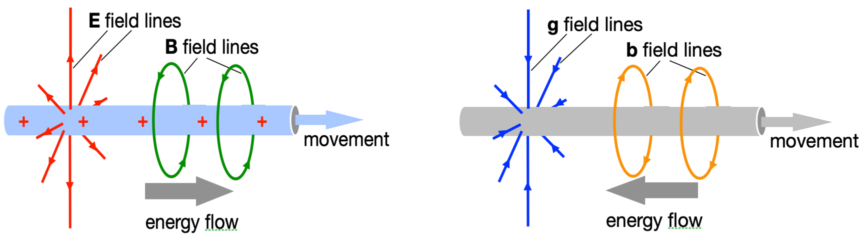
## 3 Energy flows in time-varying HGEM fields

Before we come to our main topic, gravitational waves, let’s get familiar with HGEM by using a simple setup to study the energy current distribution in a HGEM field.

We start with the EM situation and then translate into the HGEM version.

A long rod carries electric charge distributed uniformly along its length. It moves longitudinally to the right, at first with constant velocity, figure 1a. The **E**-field lines point radially outward, and we get the **B**-field lines using the right-hand rule. This results in an energy current that flows parallel to the direction of the charge current. Energy and charge are flowing in the same direction.

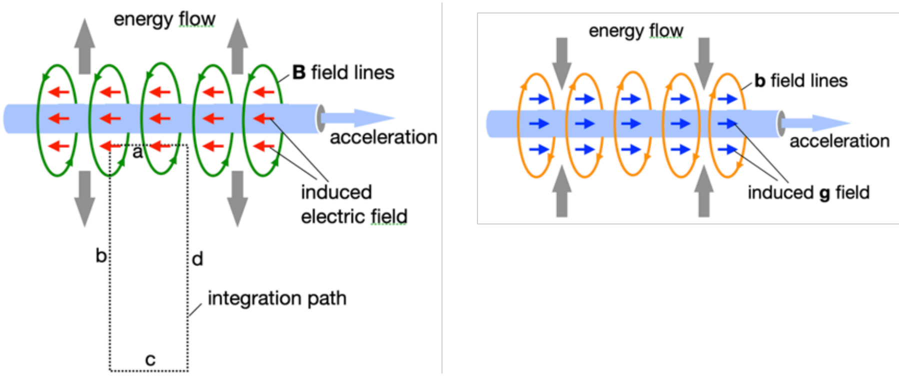
The HGEM analog is shown in figure 1b. The rod with its mass moves to the right. The directions of both field strengths are reversed. Because of the minus sign in equation (14), the energy now flows to the left, in the direction opposite to the motion of the rod.




(a) (b)

**Fig. 1** (**a**) A moving electrically charged rod is accompanied by an energy current within the field. The energy current density vector and the charge current density vector point in the same direction. (**b**) HGEM analog. The energy current density vector and the mass current density vector point in opposite directions.

Next, we are interested in the buildup of the magnetic and the gravinetic field. Starting from rest our rod is accelerated uniformly. Thereby the magnetic or gravinetic field builds up gradually. We ask for the way the energy gets into or out of this field, i.e. for the corresponding energy flux density distribution. We no longer ask for that part of the total energy current which flows parallel to the rod.




a b

**Fig. 2** (**a**) A moving electrically charged rod is accelerated uniformly. This causes a magnetic field to build up. The energy flows away from the rod into the field. (**b**) HGEM analog. When the field is built up, energy flows from the field into the rod.

Again we begin with the EM case, figure 2a. Since the electric current increases at a constant rate, the **B**-field strength also grows linearly with time and so does the magnetic flux through the area inside the dashed line. We apply Faraday's law of induction (i.e., equation (3) in its integral form) to this area. The flux through the area is equal to the line integral of **E** over the boundary line. The contributions of sides b and d to the integral cancel each other, and we can neglect the contribution of c because this section of the path is far away from the rod. So the change of the magnetic flux causes a field with the field strength vector **g** directed to the left. We now ask for the energy current density vector (Poynting vector). It is orthogonal to the rod and directed away from it. This corresponds to our expectation: We are building up a magnetic field. Since this field has energy, energy must flow into the field. We can also see where the energy comes from. The induced electric field also exists at the location of the moving charge. It pulls on the charge carriers in the direction opposite to the movement. It therefore acts as a brake on the movement. This energy must be supplied to the rod.

Figure 2b shows the HGEM analog. We do not need to describe the processes in detail, because they are essentially the same as those in figure 2a. Again, they differ only in signs and directions. The directions of the field strengths are reversed, and so is that of the energy current because of the minus sign in equation (14). This result is also in agreement with our expectations. While the gravinetic field is built up, energy flows out of the field. We remember: Energy is needed to create field-free space, and energy is released when field is created. And where does the energy go in this case? Into the rod. So the acceleration of the rod is not hindered, but supported. This observation will concern us later.

## 4 HGEM waves

So far, everything has gone well. Everything was consistent. We now come to our main topic: gravitational waves. Since Maxwell's equations of EM have electromagnetic waves as solutions, the "Maxwell equations" of HGEM should also have waves as solutions. One difference is due to the fact that there is no negative mass. But this has only the consequence that one cannot realize a dipole antenna for gravitational waves. The simplest antenna is a quadrupole antenna. Figure 3 shows a particularly simple version of it: two bodies of equal mass are connected by an elastic spring.

**Fig. 3** Quadrupole antenna for gravitational waves.

When the bodies oscillate against each other, according to HGEM the antenna should emit a gravitational wave composed by a gravistatic and a gravinetic field.

We now remember that the two fields have a negative energy density. Therefore, while a wave travels away from the antenna, energy must flow towards the antenna. The antenna must receive energy. This is coherent insofar as it fits to our rule: energy is needed to generate field-free space and energy is obtained when fields disappear.

But now the question arises: Are we dealing with a transmitting antenna because a wave travels away from it, or with a receiving antenna because it absorbs energy?

This question leads us, perhaps surprisingly, to thermodynamics. More precisely: to the problem of the reversibility of a process.

We first return to EM and consider a dipole antenna emitting an EM wave. However, we do not simply consider the antenna with its environment in which the emitted wave travels away undisturbed, having the shape we all know from the textbooks. Rather, we try to imagine how the field looks like in the environment of a real antenna. Thereby, fields coming from other sources will also be present. And now we ask the question: Could what we see also run backwards? Obviously not. As a reason, one would perhaps spontaneously argue: It would be completely improbable that the wave flurry contains a portion corresponding to a backward running Hertzian oscillator field. The reasoning is correct; however, it can be formulated in more physical terms: Entropy is generated during emission. Therefore, the process cannot run backwards. It is forbidden by the second law.

Let's now come back to our antenna for gravitational waves. We explore two possibilities:

- The antenna absorbs a quadrupole wave, thereby emitting energy.
- The antenna emits a quadrupole wave, thereby absorbing energy.

As our previous discussion has shown, the first possibility is forbidden by the 2nd law.

However, the second possibility is not more enjoyable. It would lead to a fundamental instability. Not only our antenna would be unstable, but also every other arrangement of masses, and that means that the whole universe would be unstable.

Thus, we have to conclude: HGEM waves cannot exist. However, the GTR teaches us that gravitational waves exist. It also tells us how to detect them, and they have indeed been detected. What is the character of these waves? In any case they are not waves describable by HGEM.

The fact that the HGEM waves we have just discussed have nothing to do with the gravitational waves predicted by GTR can also be seen by how a detector would have to react. As is well known, when a real gravitational wave arrives, the mutual distance of the two mirrors of the detector is changing. If a HGEM wave arrives, the mirrors would oscillate back and forth in phase to each other, see also [11].

## 5 The limits of HGEM – Conclusion

Let's summarize. HGEM describes consistently:

- stationary gravistatic and gravinetic fields, including the momentum flows (forces) and energy flows that occur.
- slowly changing gravistatic and gravinetic fields including the resulting momentum currents and energy currents. 'Slowly' means: the appearing fields are those which one gets without retardation, known as quasi-stationary fields.

And of course, low-field-slow-motion is assumed.

Thus, HGEM is not suitable for describing processes in which fields detach from their sources, i.e., processes that are no longer quasi-static.

Is this a reason to discard HGEM? Certainly not. The same restrictions apply to Newtonian mechanics as we teach it every day. And HGEM can do more than Newtonian mechanics [4, 10]. In particular it solves the problem of actions at a distance on which one depends with the Newtonian representation.

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*„That gravity should be innate inherent & essential to matter so that one body may act upon another at a distance through a vacuum without the mediation of any thing else by & through which their action or force may be conveyed from one to another is to me so great an absurdity that I beleive no man who has in philosophical matters any competent faculty of thinking can ever fall into it. Gravity must be caused by an agent acting constantly according to certain laws, but whether this agent be material or immaterial is a question I have left to the consideration of my readers."*


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