The Karlsruhe Physics Course

for the secondary school A–level

Electromagnetism

The Karlsruhe Physics Course

A textbook for the secondary school A–level

- **Electromagnetism**
- Thermodynamics
- Oscillations, Waves, Data
- Mechanics
- Atoms Physics, Nuclear Physics, Particle Physics

Herrmann

The Karlsruhe Physics Course

Issue 2019

Edited by Prof. Dr. *Friedrich Herrmann* and *Dr. Holger Hauptmann*

Figures: *F. Herrmann* und *H. Schwarze*

Translation: *Kathrin Schilling*

Layout: *H. Schwarze*

TABLE OF CONTENT

1 The electric field

1.1 Review: the electric circuit 5
1.2 The electric potential 6
1.3 The zero-point of the electric potential 7
1.4 Electrotechnical problems 9
1.5 Characteristic curves – the electric resistance 11
1.6 The resistance of voltmeter and ammeter 12
1.7 The electric conductivity 13
1.8 Electric potential and energy 14
1.9 Charge and charge carriers 15
1.10 Charge current and charge carrier current 15
1.11 Accumulating electric charge 16
1.12 The electric field 17
1.13 The electric field strength 20
1.14 Graphical representation of electric fields 21
1.15 Rules for drawing electric fields 22
1.16 Four important electric fields 24
1.17 Calculation of electric field strengths 26
1.18 Several charged bodies – vector addition 27
1.19 Pressure and tension within the electric field 28
1.20 Capacitor and capacitance 32
1.21 Surfaces of constant potential 34
1.22 More about the capacitor 36
1.23 The energy of the electric field 37
1.24 Discharge curve of the capacitor 39
1.25 Fields and electric conductors 42
1.26 The electric current density – Ohm’s law locally 43
1.27 How to load electrically charged particles with energy – electron beams 45

2 The magnetic field

2.1 Magnetic charge and magnetic field 47
2.2 Magnetization 48
2.3 The magnetic field strength 49
2.4 Magnetization lines and field lines 51
2.5 Four important magnetic fields 51
2.6 Soft magnetic materials 53
2.7 Electric current and magnetic field 54
2.8 Calculation of magnetic field strengths 57
2.9 Measuring the magnetic charge 59
2.10 Pressure and tension within the magnetic field 60
2.11 Electromagnets 62
2.12 Magnetic field strength, magnetization and magnetic flux density 63
2.13 The coil – the inductance 64
2.14 The energy of the magnetic field 64
2.15 “Discharge” of the coil 65
2.16 How the magnetic field presses on an electric current 67

3 The interplay between electric and magnetic fields

3.1 Analogy in electrodynamics 73

3.2 Electromagnetic induction 74

3.3 The generator 78

3.4 Alternating voltage and alternating current 80

3.5 The transformer 81

3.6 A somehow peculiar generator – "magnetic currents" 83

3.7 Superconductors 84

3.8 Induced electric fields – the interplay between electric and magnetic fields 86

3.9 Electromagnetic waves 87

3.10 The square wave 88

3.11 Energy transmission with electromagnetic waves 89

3.12 Sine waves 90

Appendix

Physical constants and formulas 92

1 THE ELECTRIC FIELD

1.1 Review: the electric circuit

We start by remembering some things. In this part, you will only come across a few subjects that you are not already familiar with from earlier lessons.

Fig. 1.1a shows a light bulb that is connected to a battery via a switch. From the battery, the energy is transported to the lamp by electricity as a carrier. At the lamp, it is transferred to the energy carrier light. The energy comes from the battery, it reaches the lamp and leaves the lamp with the light. The battery slowly runs dry in the process, i.e. its energy content decreases.

The energy carrier, i.e. the electricity, takes a different way: it flows in a closed loop. It comes out of the battery on one of the two contacts, the plus contact, then flows through a wire to the lamp, subsequently through the filament of the lamp and the second wire and finally over the switch to the minus contact of the battery and back to the plus contact through the battery. As the electricity moves on a closed path without accumulating anywhere, the entire system is called *electric circuit*. The current of electricity also has a shorter synonym: electric current.

An electric circuit is quite similar to a hydraulic circuit that is, for instance, part of an excavator, Fig. 1.2a. Here, the energy carrier, i.e. the hydraulic liquid, also flows in a closed circuit. The flow diagrams, Fig. 1.1b and 1.2b illustrate the similarity.

Just as the pump makes the liquid flow in the hydraulic circuit, the battery in our electric circuit causes the electricity flow. We can therefore regard the battery as an *electricity pump*.

There are also other sources that release energy with electricity as a carrier, i.e. other electricity pumps. One of them is the bicycle dynamo. Very large dynamos such as the ones in power plants are called *generators*. Other types of electricity pumps are *solar cells* and *thermocouples*. While a generator gets its energy with the carrier angular momentum, a solar cell receives its energy with light and a thermocouple with entropy.

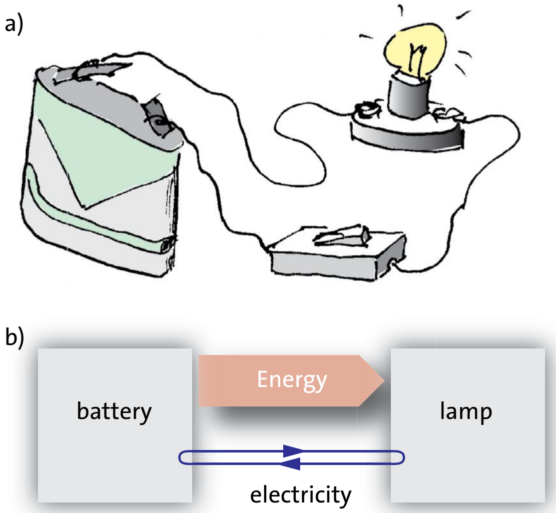


Fig. 1.1 Electric circuit with flow diagram

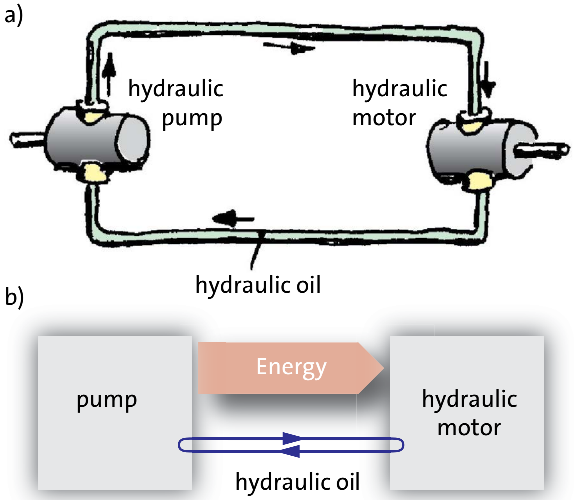


Fig. 1.2 Hydraulic circuit and flow diagram

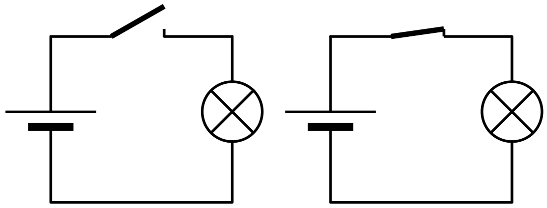


Fig. 1.3 Symbolic illustration of the electric circuit from Fig. 1.1 with an open and a closed switch.

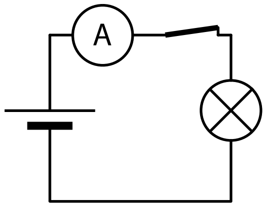


Fig. 1.4 Measurement of the electric current: the electric circuit is interrupted and the two newly formed ends of the wire are connected to the ampere meter.

Battery, generator, solar cell and thermocouple are electricity pumps.

When dealing with electric circuits, a symbolic illustration of the individual components is helpful. Fig. 1.3 shows a symbolic display of the electric circuit from Fig. 1.1.

The electricity or, as we also say, the *electric charge* is a physical quantity. Its symbol is Q, the measurement unit Coulomb, abbreviated as C.

The electric current intensity I, or electric current for short, at any point of an electric circuit is defined as the amount of electricity (amount of charge) ΔQ, that flows at that point through a cross-sectional area of the conductor in a given interval of time Δt, divided by this interval of time:

$$I = \frac{\Delta Q}{\Delta t}$$

The measurement unit of the electric current is ampere (A). We can write

$$\mathrm{A} = \frac{\mathrm{C}}{\mathrm{s}}.$$

The electric current is measured with an *ampere meter* (or ammeter, for short). An ampere meter is integrated in an electric circuit in a way that the current has to flow through the meter, Fig. 1.4.

The electric circuit on Fig. 1.3 is not closed at first. We now close the switch. Electric charge flows through the lamp. But where does this charge come from? We could think that it comes from the battery, just as the energy. It is actually different. Just as a water pump can only release as much water at its output as it takes up at its input, an electricity pump can only release as much electricity at its output, i.e. at the plus contact, as it absorbs at the minus contact. So where does the electricity come from?

It is contained from the start in the components of the electric circuit: in the battery, in the lamp and in the wires. This electricity, however, is not put into these devices by the manufacturer, but it is naturally contained in them. Every piece of wire, and every piece of metal, contains electricity. The electricity starts to flow when the wire or the piece of metal is integrated in an electric circuit.

1.2 The electric potential

A water pump ensures that the water at its output has a higher pressure than at the input, Fig. 1.5. It creates a pressure difference. This pressure difference can cause a water current.

Also a battery, i.e. an electricity pump, creates a driving force: a driving force for an electric current. And here, there is also a physical quantity that has a higher value on one terminal, i.e. on the plus terminal, than on the other one, the minus terminal, Fig. 1.6. This physical quantity is called *electric potential*. The electric potential in an electric circuit corresponds to the pressure in a hydraulic circuit.

A battery creates a potential *difference*, and this potential difference works as a driving force for an electricity current. A potential difference is also called *voltage*.

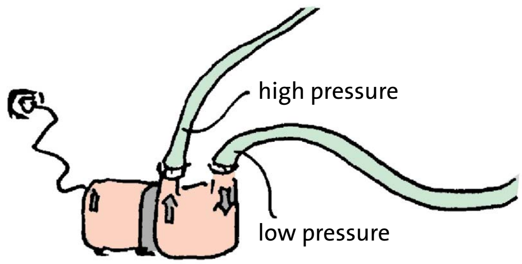


Fig. 1.5 The pressure at the output of the water pump is higher than at the input.

An electricity pump (battery, generator) creates a potential difference (= voltage). The potential difference is a driving force for the electric current.

The terminal with the higher potential is marked with a plus sign, that with the lower potential with a minus sign.

The measuring unit of the potential, and consequently also of the voltage, is the volt. Hence, a monocell creates a potential difference of 1.5 V, a larger battery makes a potential difference of 4.5 V and a car battery of 12 V.

The Greek letter φ (say: phi) is used as a symbol for the potential; U is the symbol for the voltage. Therefore, we obtain for our battery

$$\varphi_+ - \varphi_- = 4.5 \text{ V, or } U = 4.5 \text{ V.}$$

Voltages are measured with a voltmeter. For this purpose, the two terminals of the voltmeter are connected to the two points with a different potential, Fig. 1.7. Points that are connected to each other with a cable are on the same potential.

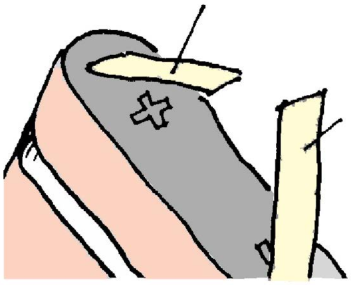


Fig. 1.6 The electric potential at the plus terminal of the battery (output) is higher than at the minus terminal (input).

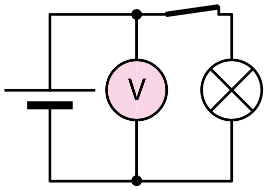


Fig. 1.7 Measuring the voltage: the terminals of the voltmeter are connected to the two points with a different potential

1.3 The zero-point of the electric potential

There is a full battery on the table in front of you. The potential difference between its terminals is 4.5 V, the potential at the plus terminal therefore exceeds the potential at the minus contact by 4.5 V. But what is the potential at the minus contact itself? And what is the potential at the plus contact?

These questions are not easy to answer. However, the problem will be easier to solve if we first clarify another question. Fig. 1.8 shows a pocket rule which is placed vertically on a table; we ask: what is the altitude of the upper end of the pocket rule?

For now, we can only say that the upper end is located 1 m above the lower one. But what is the altitude of the lower end? The answer to this question depends on our reference point: the floor of the room, the level of the soil outside the house or any other level. As you certainly know, the altitude of a piece of land is usually indicated with reference to the sea level. The altitude of the surface of the ocean is arbitrarily set as 0 m. Now, we could theoretically indicate the altitude of the upper end of our pocket rule in relation to sea level. The distance to the sea level, however, is actually not easy to determine.

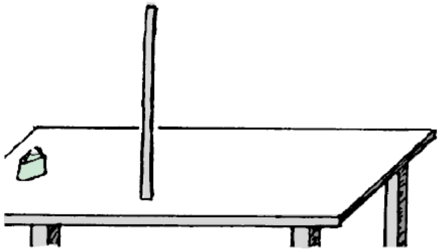


Fig. 1.8 What is the potential of the plus contact of the battery? What is the altitude of the upper end of the pocket rule?

The potential behaves in a way that is very similar to the altitude. At first, we would have choose an electric conductor to which we attribute the potential value 0 V. Starting from there, the potential values of all other wires, electric terminals, etc. could then be indicated. The conductor whose potential is used as a reference potential should of course be accessible to everyone. A conductor that fulfills these conditions is the Earth. Hence, the following was established:

The potential of the Earth is 0 V.

If any point of an electric circuit is connected to the Earth through a wire, this point will be at 0 V. We say that this point was *grounded*.

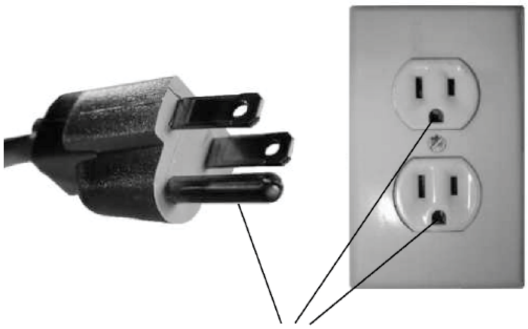


Fig. 1.9 The protective contact of the socket is on ground potential.

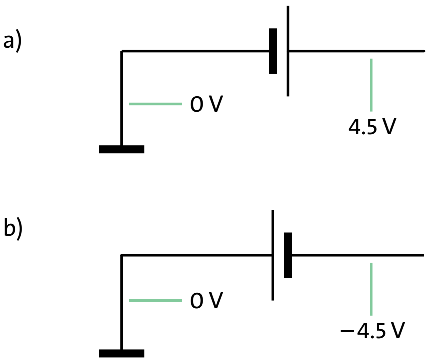


Fig. 1.10 (a) Minus contact of the battery grounded; plus contact on + 4.5 V. (b) Plus contact grounded; minus contact on – 4.5 V.

To ground something, it is not even necessary to lay a line to the Earth. The protective contact of the socket is connected to the so-called neutral conductor of the electric grid, and this neutral conductor is grounded. Also the protective contact of the socket is consequently at 0 V, Fig. 1.9.

Let's get back to the battery on the table in front of you. Based on what has been said so far, we do not know the individual potential values of the plus and minus contact, just as we do not know the altitudes of the ends of the pocket rule. For the battery, we can easily make things clear though: we simply ground one of the two terminals. Fig. 1.10a shows a battery whose minus contact is grounded, so we can say

$$\varphi_- = 0 \text{ V}.$$

For the plus contact, we therefore obtain

$$\varphi_+ = 4.5 \text{ V}.$$

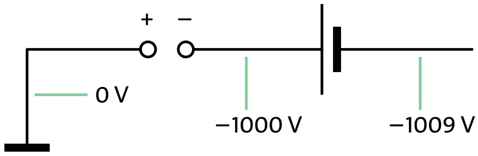


Fig. 1.11 The plus contact of the battery has a potential of –1000 V.

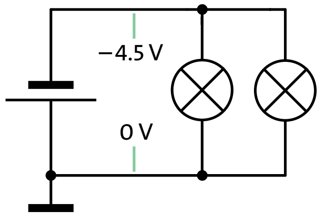


Fig. 1.12 Electric circuit that is grounded at one point.

In Fig. 1.10b, the plus contact is grounded. We thus have

$$\varphi_+ = 0 \text{ V}$$

and

$$\varphi_- = -4.5 \text{ V}.$$

Therefore, the potential of the minus contact is now negative. In both cases, i.e. in the Figures 1.10a and 1.10b, the following applies of course:

$$\varphi_+ - \varphi_- = 4.5 \text{ V}.$$

The words plus contact and minus contact (or plus terminal and minus terminal) are usually used although they are somehow confusing. They suggest that the plus contact is on a positive and the minus contact on a negative potential. Fig. 1.10 shows that this does not necessarily have to be the case. In Fig. 1.10a, the minus contact has the potential 0 V; its potential is consequently not negative, and in Fig. 1.10b the plus contact is not positive.

Fig. 1.11 shows it even more clearly.

Here, a 9 V battery and a 1000 V power supply are connected with each other. The plus contact of the power supply is grounded and its potential is consequently 0 V. Its minus contact is 1000 V lower, i.e. it is equal to –1000 V. As the plus contact of the battery is connected to the minus contact of the power supply, the plus contact of the battery also has the potential –1000 V. Hence, the potential of the plus contact of the battery is negative.

Fig. 1.12 shows an electric circuit that is grounded at one point.

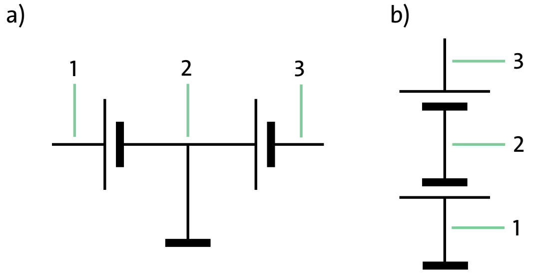


Fig. 1.13 For exercises 1 and 2

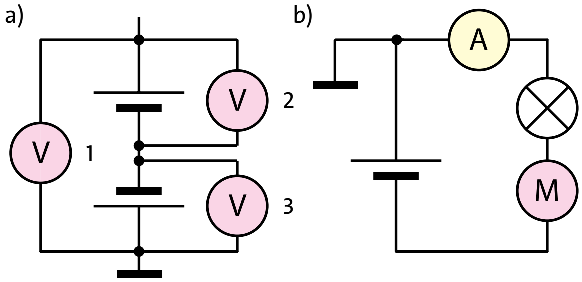


Fig. 1.14 For exercises 3 and 4

Exercises

1. Each of the batteries in Fig. 1.13a creates a voltage of 4.5 V. At which potentials are the points 1, 2 and 3?
2. Each of the batteries in Fig. 1.13b creates a potential difference of 12 V. At which potentials are the points 1, 2 and 3?
3. Each of the two batteries in Figure 1.14a creates 9 V. Which voltage is indicated by the three voltmeters?
4. Draw a voltmeter in Fig. 1.14b that measures the voltage between the connections of the lamp. Draw a voltmeter that measures the battery voltage.
5. Give examples of electric circuits that cannot be grounded.

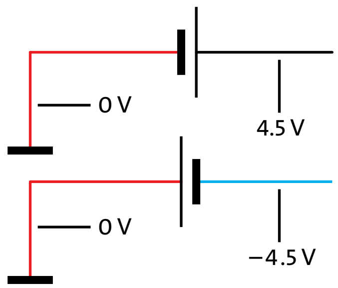


Fig. 1.15 Different colors of the conductors stand for different potentials.

1.4 Electrotechnical problems

We are going to talk about a method that facilitates the solution of electrotechnical problems.

Every time a "circuit diagram" is given, the conductors are at first highlighted in color in a way that all conductors that have the same potential are marked with the same color. It is clear that a continuous conductor will be marked in one single color. When passing through an electric device (lamp, electric motor, battery, generator, etc.) the color usually changes.

Figures 1.15 to 1.17 show examples.

Fig. 1.15 shows the battery from Fig. 1.10 with its connecting wires according to the new method.

Fig. 1.16 once again shows the lamps from Fig. 1.12, and Fig. 1.17 shows an electric circuit with four different potential values.

We would like to apply the coloring of the wires to two problems:

1. The lamps L_1 and L_2 in Fig. 1.16 are identical. At point P, there is a current of 3 A. What is the electric current in L_1 and in L_2?

As the junction rule applies for the branch points, we obtain

$$I_{L1} + I_{L2} = 3\ \text{A}.$$

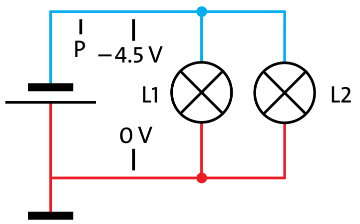


Fig. 1.16 As shown in Fig. 1.12. The potentials are marked with different colors.

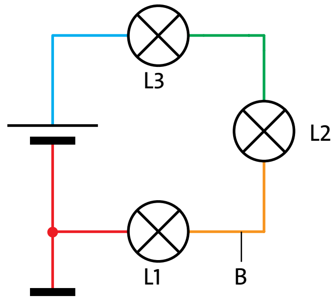


Fig. 1.17 There are four different potentials in this circuit.

(I_{L1} and I_{L2} are the currents in the lamps.) We can see from the color marking that there is the same voltage at both lamps (i.e. the same as at the battery). The electric current therefore has the same driving force in both lamps. As the lamps are identical, the currents must also be the same in the two lamps, i.e.

$$I_{L1} + I_{L2} = 1.5\ \text{A}.$$

2. Section B of the conductor in Fig. 1.17 is at a potential of 6 V. The lamps L_1, L_2 and L_3 are identical. Which voltage is created by the battery?

As the electric circuit is not branched, the current is the same everywhere. The voltage over lamp 1 is 6 V. It is the driving force for the current through L_1. As the same current flows through lamps L_2 and L_3 and as these lamps are identical with L_1, the electricity needs the same driving force as in lamp L_1, i.e. 6 V to flow through these lamps. Hence, if we move from the plus contact of the battery via the three lamps to the minus contact, the voltage is reduced to 0 V in 3 steps of 6 V. The plus contact must consequently be at 18 V.

In the two examples, the potential at the input of a lamp was different from that at the output. But this rule does not always apply. A lamp, through which no electric current is flowing, must have the same potential at the input and at the output because otherwise there would be a current. Fig. 1.18 shows two examples.

Exercises

1. The batteries in Fig. 1.19a are 4.5 V batteries. Mark the points of equal potential and indicate the potential values for all line sections.
2. The electric current that flows through the battery in Fig. 1.19b is 1.6 A. Indicate the points of equal potential. What is the electric current in the lamps?
3. The electric potential at point C in Fig. 1.20 is 20 V. The three lamps are identical. Mark the points of equal potential. Indicate the potential values for sections A, B and D of the conductors. Which voltage is supplied by the battery? What will happen to the potentials when the switch is opened?
4. The battery voltage in Fig. 1.21a and 1.21b is 12 V. The lamps are identical. Mark the points of equal potential. What is the value of the potential at point P? What are the potential differences at lamps L_1 and L_2? Is the current that flows through lamp L_1 greater when the switch is closed (Fig. 1.21a) or when it is open (Fig. 1.21b)? When is the current that flows through lamp L_2 greater: when the switch is open or when it is closed?
5. The voltage at the power supply in Fig. 1.22a and 1.22b is 150 V, the lamps are identical. Mark the points of equal potential. Indicate the potential values of all conductor sections. Which lamp will remain lit up if the switch is opened?

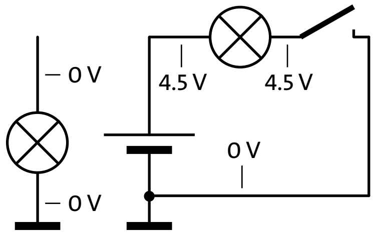


Fig. 1.18 As no electric current flows through the lamp, its terminals must be at the same potential.

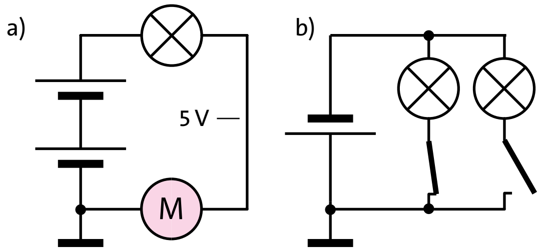


Fig. 1.19 For exercises 1 and 2

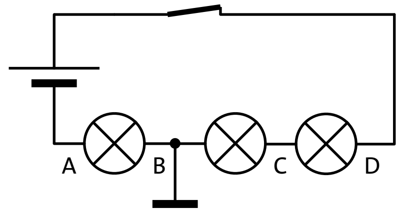


Fig. 1.20 For exercise 3

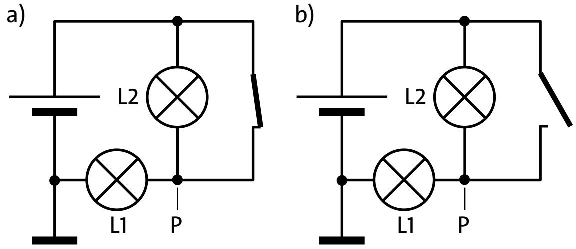


Fig. 1.21 For exercise 4

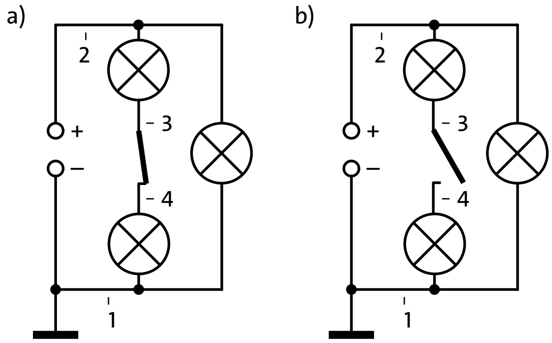


Fig. 1.22 For exercise 5

1.5 Characteristic curves – the electric resistance

If we want electricity to flow through an object, we apply a voltage: we create a driving force. Each object tends to hamper the flow. It counters the flowing electricity with resistance. We also say: it has a resistance.

Some objects have a high resistance, they conduct the electric current only poorly or not at all. Others have a low resistance; they are good conductors of electricity.

Electric cables, for instance, have a low resistance. This does not mean, however, that they do not have any resistance at all.

The way in which the electric current flows through an object and how it reacts to the applied voltage can be quite a complicated issue. If the voltage is increased, the current will usually also increase – but not always.

We would like to analyze the relationship between voltage and current for different electric devices. Fig. 1.23 shows how to do it: we connect the object to be analyzed to a power supply with an adjustable voltage. The voltage is read from the power supply. The electric current that causes the voltage is measured with an ammeter. If the values of the current are plotted over the voltage, we obtain the *characteristic curve* of the analyzed device.

Fig. 1.24 shows the characteristic curve of an incandescent lamp (top) and of a diode (bottom). In case you do not know what a diode is used for, you can figure it out with the characteristic curve: the curve shows that the diode lets the electric current flow in only one direction. It therefore works for the electric current in the same way as a bicycle valve works for the air current.

For some objects or devices, the current is proportional to the voltage:

$$I \sim U.$$

This simple relationship applies, for example, for a common wire, provided that the current is not as high as to heat it up. We say that the wire complies with *Ohm's law*. Fig. 1.25 shows the characteristic curves for two wires of different length. In case of an equal driving force, the current in one wire is greater than in the other one. The wire with the greater current has a lower resistance.

> For a normal electric conductor, the voltage between its ends is proportional to the current that flows through it:
> $I \sim U$ Ohm's law

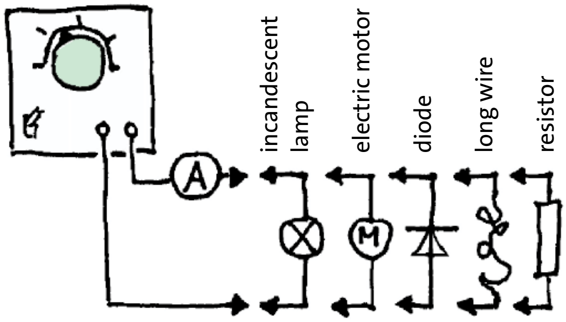


Fig. 1.23 Plotting characteristic curves: The electric current is measured for different given values of the voltage.

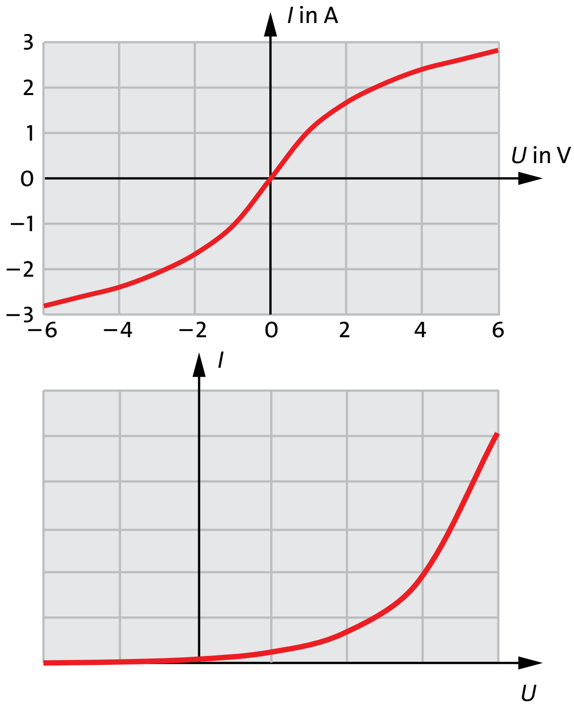


Fig. 1.24 Characteristic curves of an incandescent lamp (top) and a diode (bottom)

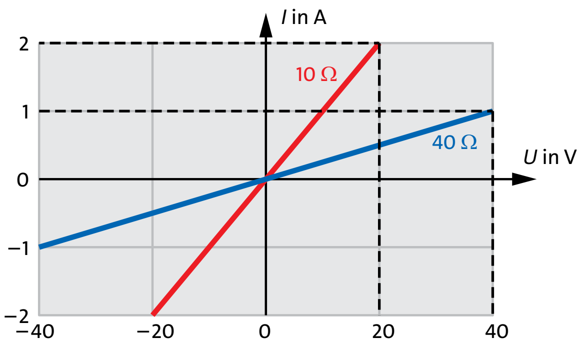


Fig. 1.25 Characteristic curves of two long wires. They comply with Ohm's law.

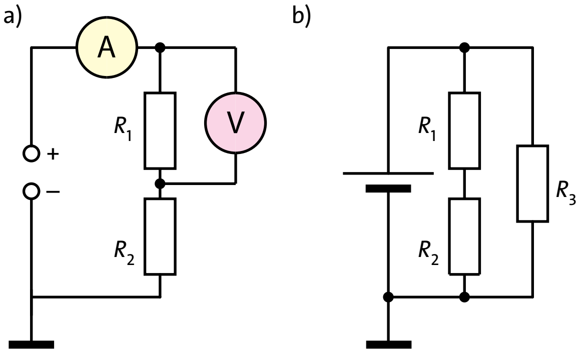


Fig. 1.26 (a) For exercise 4; (b) For exercise 5

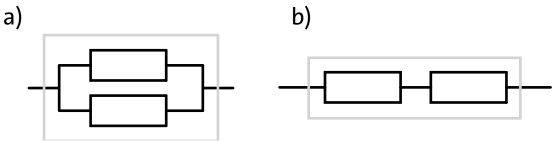


Fig. 1.27 For exercise 6

The resistance can be characterized by means of the quotient of voltage and current. The higher the resistance of the wire, the greater this quotient. Therefore, the quotient itself is called *resistance of the wire* and denominated with the letter R. Thus, we have

Electric resistance: $$R = \frac{U}{I}.$$

The resistance R is a physical quantity. As a measurement unit, we find volt/ampere (V/A). The Ohm, abbreviated with the Greek letter Ω (say omega), is usually used instead of the composed unit volt/ampere. Hence, we have

$$\Omega = \text{V/A} .$$

We can now indicate the resistance of our two wires: the two values are 10 Ω and 40 Ω.

If the characteristic curve is not a straight line, it does not make sense to calculate a quotient U/I since this quotient would have a different value for each point of the characteristic curve.

In electric technology and electronics, there are cases where we would like to "hamper" an electric current deliberately, i.e. where a resistance is desired. Therefore, devices or "components" are created with the only purpose of representing a resistance for a current. These components are called *resistors*. Resistors are built in a way that they have a linear characteristic curve. They obey Ohm's law and can be characterized by indicating a resistance value, i.e. a number of Ohms.

The symbol of a resistor is a rectangle as shown in Figures 1.23 and 1.26.

Exercises

1. A voltage of 20 V is applied to an unknown resistor. An electric current of 4 mA is measured. How many Ω does the resistor have?
2. A voltage of 120 V is applied to a 2 kΩ resistor. What is the electric current that flows through the resistor?
3. An electric current of 0.1 mA flows through a 1 MΩ resistor. What is the voltage on the resistor?
4. The power supply in Fig. 1.26a creates a voltage of 35 V. The ammeter indicates 5 A and the voltmeter 10 V. What is the resistance of R_1? What is the voltage at resistor R_2? What is the resistance of R_2?
5. The voltage of the battery in Fig. 1.26b is 12 V. Each of the resistors has 100 Ω. Give the potential values of all wire sections. What are the voltages at the three resistors R_1, R_2 and R_3? What values have the electric currents that flow through the three resistors? What is the electric current that flows through the battery?
6. (a) Two 100 Ω resistors are connected in parallel, Fig. 1.27a. What is the resistance of the whole arrangement? Formulate a rule. (b) Two 100 Ω resistors are connected in series, Fig. 1.27b. What is the resistance of the whole arrangement? Formulate a rule. Do the rules look familiar to you?

1.6 The resistance of voltmeter and ammeter

We need to go back to the question of how to deal with the meters for current and voltage, or more precisely: what is the resistance of an ammeter and what is the resistance of a voltmeter?

The ammeter is inserted in a wire through which an electric current is flowing. The current should not change due to the installation of the ammeter, Fig. 1.28. This means: the resistance of the ammeter should be as low as possible. In fact, the resistance is so low that it can be neglected in most cases, i.e. it can be assumed to be equal to zero.

Next the voltmeter: the electric current should flow in a circuit, i.e. through the battery and the lamp and the respective wires. There should be no "leaks". In order to avoid a leaking effect caused by the voltmeter, its resistance should be as high as possible. Voltmeters actually do have a resistance that is so high that the leak current can be neglected in relation to the current that flows through the lamp, i.e. we can assume that the resistance is infinitely high.

Ammeters have a very low, voltmeters a very high resistance.

Exercise

1. Comment on Figure 1.29.

1.7 The electric conductivity

Which properties of a wire does its resistance depend upon?

We connect a wire with the resistance R to a power supply, Fig. 1.30a. There is an electric current I. Next, we connect a second wire in parallel to the first one, Fig. 1.30b. Now, a current I is flowing in each of the two wires. Consequently, the total current is

$$I' = 2I\,.$$

This means that the resistance R' of the two parallel wires together is half that of a single wire:

$$R' = R/2.$$

We can now also look at the two wires together as a single one with the double cross-sectional area. In conclusion we can say: *If the cross-sectional area of a wire is doubled, the resistance will decrease to half of its initial value.*

Instead of connecting the second wire in parallel, we connect it with the first one "in series", Fig. 1.30c. Both wires now have to share the potential difference U, i.e. on each individual wire there is only a voltage U' = $U/2$. Hence, the current has only half of its initial value:

$$I' = I/2.$$

This means that the overall resistance R' of the two wires connected in series is twice that of a single wire:

$$R' = 2R.$$

Again, the two wires can be regarded as one, this time with the double length, and we can conclude: *If the length of a wire is doubled, the resistance will double as well.*

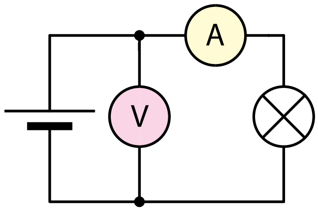


Fig. 1.28 The resistance of an ammeter is very low, that of a voltmeter is very high.

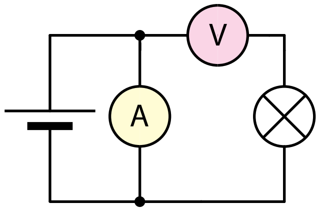


Fig. 1.29 For the exercise

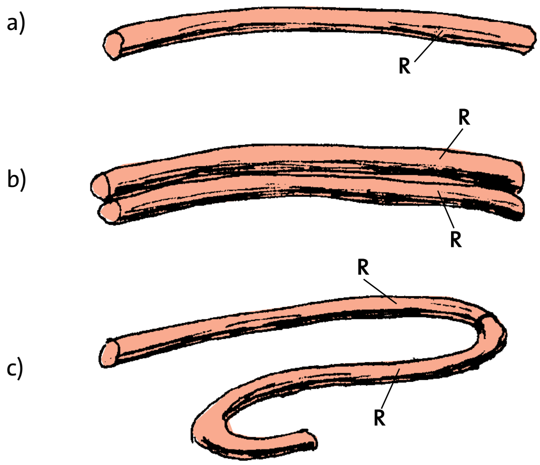


Fig. 1.30 (a) A current *I* flows through a wire with the resistance *R*. (b) Two parallel wires are equivalent to a wire with the double cross-sectional area. The resistance is half as high, the current twice as high as in (a). (c) Two wires in series are equivalent to a wire with the double length. The resistance is twice as high, the current half as high as in (a).

Both results can be summarized:

$$R \sim \frac{d}{A}.$$

Here, d is the length and A the cross-sectional area of the conductor.

The resistance also depends on the material the conductor is made of. This is accounted for by another factor, i.e. the *electric conductivity* σ:

$$R = \frac{1}{\sigma} \cdot \frac{d}{A}$$

d = length

A = cross-sectional area

σ = conductivity

The greater the conductivity of a material, the lower its resistance. Hence, the conductivity is in the denominator of the formula.

Table 1.1 shows the conductivity of some substances. It is remarkable that the best conductors differ from the weakest conductors by a very large factor, i.e. by approximately 10^{24}.

Material	σ in $1/(\Omega \cdot m)$
Copper	$5.59 \cdot 10^7$
Aluminum	$3.7 \cdot 10^7$
Iron	$1.02 \cdot 10^7$
Distilled water	$3.33 \cdot 10^{-5}$
Plexiglas	10^{-13}
PVC	10^{-13}
Fused silica	$2 \cdot 10^{-17}$

Table 1.1

Exercises

1. Estimate the resistance of the cable of a 50 m cable drum.
2. How long is a copper wire that has the same resistance as a 1 m PVC bar with an equal thickness? How far would the wire reach out from here?
3. The conductivity of a material is higher the more mobile charge carriers it contains. How can the conductivity of a salt solution be improved?

1.8 Electric potential and energy

We track the way of a small portion of electricity ΔQ in an electric circuit. We start at the low terminal of the electric energy source (the "electricity pump"), i.e. at its input. Within the source, the electricity portion ΔQ moves from the low potential φ_1 to the high potential φ_2. To get from φ_1 to φ_2, it must be supplied with energy in the source. (If the source is a generator, this energy comes from the drive shaft.) We call this amount of energy ΔE. The electricity portion then continues to flow through the wire on the high potential φ_2 until it arrives at the energy receiver. In the energy receiver (an electric motor or a light bulb, for example), it moves from the high potential φ_2 back to φ_1. Thereby it releases the energy ΔE. (If the receiver is an electric motor, this occurs through the motor shaft; if it is a light bulb, the energy goes away with light.) Then, ΔQ flows back to the input of the source through the return line on the low potential.

Thus, our electricity portion ΔQ absorbs the energy portion ΔE in the source and releases it in the receiver.

You might remember the relationship

$$P = U \cdot I = (\varphi_2 - \varphi_1) \cdot I$$

It allows us to calculate how much energy is transported from the source to the receiver per time.

If we insert

$$P = \frac{\Delta E}{\Delta t}$$

(i.e. the energy current is equal to energy per time) and

$$I = \frac{\Delta Q}{\Delta t}$$

(i.e. the electric current is equal to the amount of electricity per time) and multiply with Δt, we obtain

$$\Delta E = (\varphi_2 - \varphi_1) \cdot \Delta Q,$$

the energy that an electricity portion ΔQ absorbs in the source and releases in the receiver.

We will see later where exactly this energy is located during the time the electricity portion moves between the source and the receiver. The energy appears to be located in the same place as the electricity portion, since that energy has been supplied to it. For the time being, you can imagine it to be like this. However, we will see later that the actual energy storage system is not the electricity itself. (Maybe you can already guess where the energy is located if you think of another energy storage system: where will the energy be stored that is supplied to a body by lifting it up?)

Energy must be supplied to bring electricity from low to high potential. Energy is released while electricity moves from high to low potential. We have:

$$\Delta E = (\varphi_2 - \varphi_1) \cdot \Delta Q.$$

1.9 Charge and charge carriers

In cases where electricity moves in a wire from one side to the other, we talk about an electric current. So far, we have analyzed the effects of electric currents and the relationship between the value of the electric current and other physical quantities. We have never asked questions about the effects and characteristics of the electricity itself though. The electricity should be best analyzed while it is not moving, i.e. while no electric current is flowing.

We have to admit that the electricity in a copper wire that is not integrated in an electric circuit, cannot be noticed. Why? A possible answer would be: electricity at rest has no detectable properties. This answer, however, is not correct. Electricity can be felt very clearly, even if it comes only in very small quantities. Its description is dealt with in the field of *electrostatics*. The fact that we do not notice anything of the electricity in a piece of copper wire in front of us is due to a property of electricity that makes it different from other physical quantities: it can assume positive and negative values.

All material substances contain electricity, but usually they contain equal amounts of positive and negative electricity so that the total amount is zero. For example, 1 g of copper contains 44032 C of positive electricity and the same amount of negative electricity; hence, the total amount is 0 C. (In comparison: the mass, i.e. the quantity that is measured in kg, can only have positive values).

> Electricity can assume positive and negative values.

But what is the sense of saying that a body, whose electricity amounts to 0 C, actually has a well-defined amount of positive electricity and an equal amount of negative electricity? Doesn't 0 C mean that it has no electricity at all? We will see that it does indeed make sense to say that copper (or any other material) contains both positive as well as negative electricity by looking at the microscopic structure of the material.

All substances consist of atoms and groups of atoms, the molecules, and each atom consists of the protons and neutrons (located in the nucleus) and a shell of electron. Two of these atomic components carry electricity. The proton carries positive electricity, i.e.

$$Q_{\text{Proton}} = 1.602 \cdot 10^{-19}\ \text{C}.$$

The electron carries negative electricity, i.e.

$$Q_{\text{Electron}} = -\,1.602 \cdot 10^{-19}\ \text{C}.$$

Neutrons do not carry any electricity. Hence, we have

$$Q_{\text{Neutron}} = 0\ \text{C}.$$

As an atom has as many protons as electrons, the total amount of electricity of the atom is 0 C.

In some cases, an atom can have one or several electrons in excess or fall short of them. Such an entity is called a ion. Thus, the amount of electricity of an ion is not zero.

In this context, we have also learned another important characteristic of the electricity: it is always located on some particle. Besides protons and electrons, there are other electrically charged particles: positrons, muons, anti-protons and others. They do not exist under normal conditions but can be created artificially and only have a very short lifetime.

Particles on which electricity is located are said to be electrically charged. Hence, the electricity is usually referred to as *electric charge*. And electrically charged particles, i.e. electrons, protons, ions etc., are *called charge carriers*.

> Electric charge (= electricity) is always located on particles: the charge carriers.

1.10 Charge current and charge carrier current

We can now understand how electric conductors differ from nonconductors: conductors are materials that contain mobile charge carriers; in non-conductors or *insulators*, all charge carriers are immobile. The nature of the mobile charge carriers in an electric conductor can be different in each case. In some conductors, only positive charge carriers move, in some only negative ones and in others both positive and negative charge carriers.

In metals, the mobile charge carriers are electrons. However, not all electrons of the metal atoms can move but usually only one per atom. There are no mobile electrons in acids, bases and salt solutions. The electric conductivity is caused by mobile ions in this case. As there are both positive and negative ions, we also have charge carriers with both positive and negative charges here.

If an electric current flows in an electric circuit, the mobile charge carriers move past the remaining ones with an opposite charge so that the electric circuit remains neutral everywhere. The net charge of all wires, energy sources and energy receivers remains zero.

We see that an electric current can be realized in different ways. In all three sections of Fig. 1.31, we have an electric current of 2 A that flows from the left to the right. In part (a) of the figure, it is formed by positively charged carriers that move to the right, in (b), negative charge carriers flow to the left. In part (c), positive charge carriers move to the right and negative ones to the left at the same time; both charge carrier types contribute to the total current.

You will be surprised how slowly the charge carriers move in a conductor: if an electric current of 1 A flows in a copper wire with a cross-sectional area of 1 mm^2, the velocity of the mobile charge carriers (mobile electrons) is only 0.07 mm/s.

Exercises

1. Two electrodes are immersed in a salt solution in which positive ions flow from the left to the right. They transport 0.5 Coulomb per second. At the same time, negative ions flow from the right to the left. They transport 0.3 Coulomb per second from the right to the left. In which direction does the electric current flow? What is the value of the electric current?
2. An electric current of 2 A flows in a copper wire. How many electrons move per second through a cross-section of the wire?

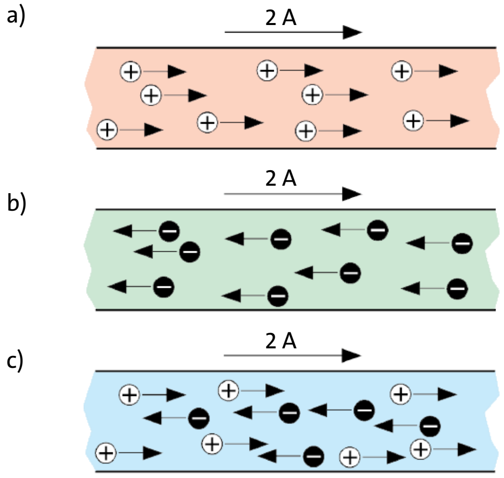


Fig. 1.31 An electric current flowing to the right is realized by charge carriers that move (a) to the right, (b) to the left and (c) in both directions.

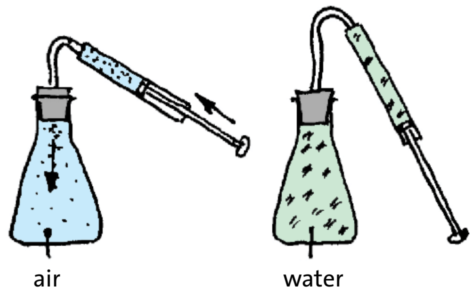


Fig. 1.32 The amount of air in the recipient on the left can be changed easily, the amount of water on the right only with great difficulty.

1.11 Accumulating electric charge

Our initial goal was to learn something about the properties of electricity. But then we explained why a normal electric circuit is electrically neutral everywhere, i.e. why the electric charge can usually not be noticed at all. We would now like to check if the neutrality of an electric conductor can be disturbed. We will try to accumulate electric charge on a conductor so that its total charge is different from zero. We will see that this comes with some difficulties.

To get a better understanding of the problem that is going to arise, let us have a look at Fig. 1.32.

The recipient on the left is filled with air under normal pressure. We would like to increase the amount of air in this recipient. Therefore, we simply pump air from the outside into the recipient. The pressure increases in this process. The recipient on the right in the figure is filled with water and we would like to increase the amount of water in the recipient. But this is clearly not as easy as for the air. Even with a pump that creates a very high pressure, the amount of water can only be increased to a very small extent. This is due to the fact that water cannot be compressed as easily as air.

The behavior of electricity is similar to that of water: it is very hard to realize in an object a deviation from the normal amount of electricity, i.e. 0 Coulomb.

How could electricity then be accumulated in an object? With an "electricity pump", of course, i.e. with a battery or a power supply. Fig. 1.33 shows an experiment that does not work: the plus terminal of a battery is connected to a wire, the minus terminal to the Earth. The battery should now pull electricity out of the Earth and push it into the wire. The wire should take on an electric charge and also remain charged when it is disconnected from the battery. If it is subsequently touched with the terminal of a small lamp whose other terminal is grounded, the lamp should light up because the accumulated electricity is supposed to flow back to the Earth through the lamp. But the lamp does

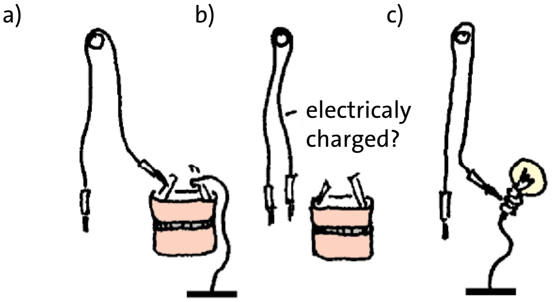


Fig. 1.33 (a) The battery pumps electricity from the Earth into the wire. (b) The wire is electrically charged. (c) The lamp is not lit because the charge of the wire is far too low.

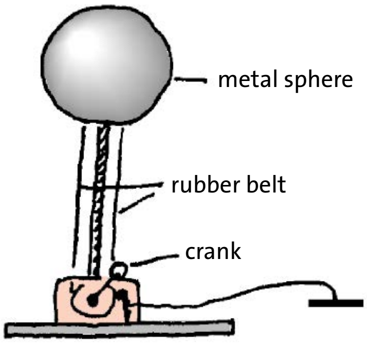


Fig. 1.34 Van de Graaff generator

not light up. Why not? Because the amount of electricity that we have pumped onto the wire is far too small.

To provide evidence of an accumulated charge on the wire, we need to improve the experiment in two ways:

(1) we use an electricity pump that "pushes much more", i.e. a power supply that creates a much higher voltage, possibly a conventional high-voltage power supply (with a transformer) or a Van de Graaff generator, Fig. 1.34. The Van de Graaff generator creates voltages of up to approximately 50 kV.

(2) To detect the charge on the wire, we use a device that is more sensitive, i.e. that reacts to smaller amounts of charge, than our light bulb: a glow lamp. The glow lamp has an additional advantage for our experiment: it allows us to see in which direction the current is flowing through it because it only glows on the side that is on the lower potential.

After taking these measures, our charge accumulation experiment is successful. Depending on which of the two terminals of the highvoltage power supply is grounded, the amount of electricity of the wire is either increased or reduced. If the minus contact of the power supply is grounded, the wire will be positively charged. As the mobile charge carriers of the wire are electrons, this means that electrons can be removed from the wire. Hence, it has fewer electrons than in the neutral state. If the plus contact is grounded and the minus contact is connected to the wire, the wire becomes negative. It has excess electrons.

The amount of charge that we accumulate grows the more we increase the potential of the wire. A high positive potential is related to a (relatively) large amount of positive charge; a high negative potential corresponds to a (relatively) large amount of negative charge. We can summarize this result:

> The higher the electric potential of a body, the more electric charge it contains.

The reverse situation is also true:

> The greater the electric charge that sits on a body, the higher the electric potential of the body.

These simple rules only apply as long as there are no other charged bodies around. We will see later that electrically charged bodies influence one another.

Please bear in mind that the amount of electricity that we finally accumulated in our experiment is still extremely small. It only amounts to a few μC. Compare this with the overall positive charge that is located on the respective metal parts but that is compensated by almost the same amount of negative charge: there are approximately 44 000 C in 1 g of copper (see section 1.9).

1.12 The electric field

We have managed to accumulate charge and also to provide evidence of this charge. But we have not yet noticed any particular properties of the electric charge. To analyze the properties of the electricity, we make the experiment illustrated in Fig. 1.35.

Two hollow metal spheres A and B are connected to a high-voltage power supply. Sphere B is very light. It is suspended on a thin wire so that it can move easily. If the power supply is switched on so that one sphere is charged positively and the other one negatively, B will be pulled towards A. If we charge the previously positive sphere negatively and the previously negative one positively, nothing will change: B will be pulled to A again.

Now we connect the spheres to the power supply in a way that their charges have the same +/– sign, Fig. 1.36.

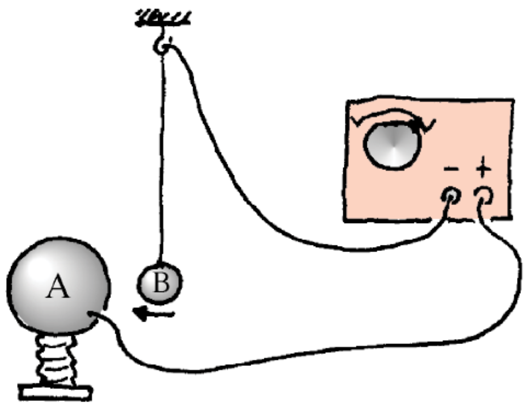


Fig. 1.35 Sphere B is pulled towards sphere A by means of the electric field.

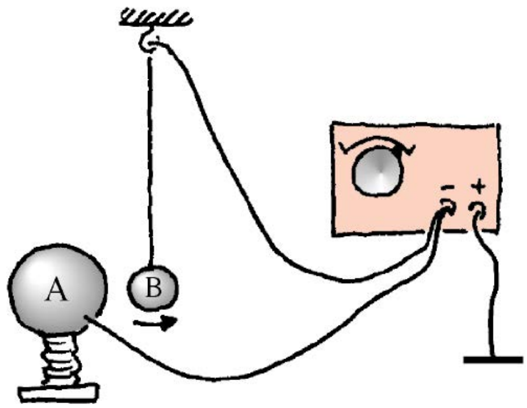


Fig. 1.36 The electric field pushes sphere B away from sphere A.

B is now pushed away from A, regardless of whether both spheres are charged positively or negatively.

We can conclude from the fact that one sphere is pulled towards the other or that one is pushed away from the other that there is a connection between the spheres.

This connection is called *electric field*. We call the invisible "material" the field. It consists of *field stuff*.

> An invisible entity is attached to electrically charged objects. This entity is called electric field. If the charges of two objects have the same +/– sign, the field pushes the objects away from each other; if they have different +/– signs, the field pulls them towards each other.

The fact that objects can pull and push each other by means of the electric field means that momentum flows from one to the other body. Hence, the electric field transports momentum: momentum currents are flowing within the electric field. This is equivalent to the statement that the field stuff is exposed to mechanical stress.

In the field between two bodies, momentum can flow both to the right as well as to the left. This means that there can be both compressive stress as well as tensile stress in the electric field. We will study this question in greater detail at a later time.

For now, we make another experiment that is even simpler than the previous ones, Fig. 1.37: only the fixed sphere A is electrically charged, sphere B is uncharged and insulated. Surprisingly, B is pulled towards A again, regardless of whether A is charged positively or negatively. How can that be explained? As sphere B is not connected to the power supply, no field should be attached to it.

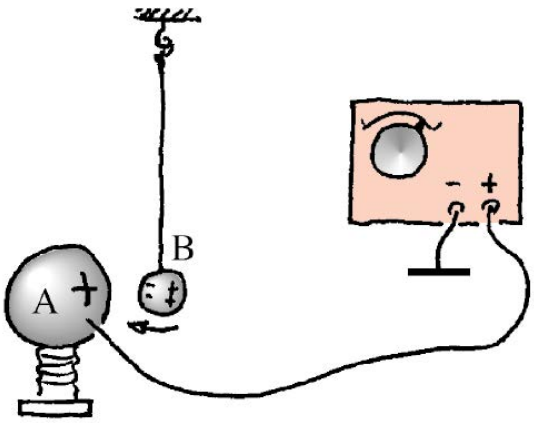


Fig. 1.37 The mobile charge carriers on B are displaced by the electric field. Electrically charged areas develop on the surface of B.

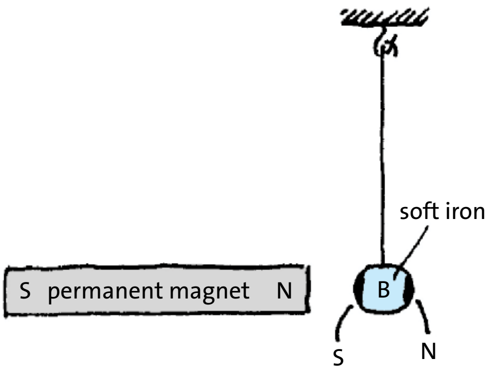


Fig. 1.38 B is magnetized by the field of the permanent magnet. Magnetic poles develop on the surface of B.

We can find the explanation if we recall a similar phenomenon of magnetism: a piece of soft iron, i.e. an object that is not magnetic at first, is pulled towards a magnetic pole, and that to both a magnetic north and a magnetic south pole. Here, the explanation is as follows: as soon as the soft iron comes close to a magnetic pole it develops poles itself. Close to a north pole, it develops a south pole on the side that faces the north pole and a north pole on the opposite side, Fig. 1.38.

It is very similar in or last experiment. The electric field pulls on the charge carriers of B and slightly displaces them so that B is charged positively on one side and negatively on the opposite side. The overall charge of B remains zero. If A is positively charged, B will become negative on the side that faces A and positive on the side that faces away from A. As the negative side of B has a shorter distance from A than the positive side, sphere B is pulled towards A.

If A is negative, the charges on B will move in the other direction and the charges of A and of the side of B that faces A have opposite +/– signs again so that B is pulled towards A.

This charge displacement process under the influence of the electric field of another body is called *electrostatic induction*.

To prove that an object is electrically charged, we used a small light bulb earlier. Another device to provide evidence of electricity is the *electroscope*. Now we can understand how it works.

Inside the metal ring, Fig. 1.39, there is a vertical bar. The bar is electrically insulated from the ring. On this bar, there is another bar attached in a rotatable way. This rotatable bar is very light. Both bars are connected to the upper terminal of the electroscope in an electrically conductive way. The ring is grounded.

We would like to use the electroscope in order to find out if charge is sitting on a sphere. Therefore, the upper terminal of the electroscope is touched with the sphere. Electric charge flows from the sphere to the two bars. The latter are now charged equally and the movable bar is pushed away from the fixed bar by the electric field. The more charge there is on the electroscope, the stronger the movable bar spreads away from the fixed one.

We use the electroscope to show the phenomenon of electrostatic induction once again in a simpler experiment, Fig. 1.40.

The big sphere was charged positively. We bring two neutral spheres B and C into the field area of the big one, Fig. 1.40a. B and C are put together so that they can touch each other, but they do not touch A, Fig. 1.40b. The field of A, causes the charges on B and C to separate (electrostatic induction). On the left, i.e. on B, negative charge accumulates. On the right, i.e. on sphere C, positive charge concentrates. Now, we separate B and C from each other while they are still close to A, Fig. 1.40c, and then we take them out of the area

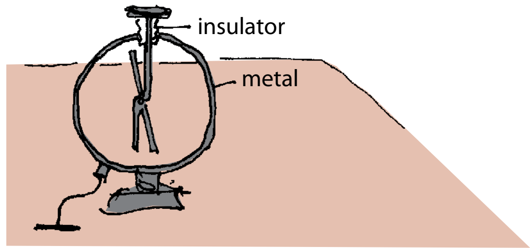


Fig. 1.39 Electroscope. The movable bar carries charge of the same +/– sign as the fixed one.

of the big sphere, Fig. 1.40d. Normally, the charges on B and C would compensate each other again. This is impossible, though, as the connection is interrupted.

We use the electroscope to show that B and C are charged. We touch the electroscope with one of the two spheres, for example with B. From B, negative charge flows to the electroscope, which sets off a signal. Then, we touch the electroscope with sphere C. Electricity is flowing onto the electroscope and neutralizes the negative charge so that the signal disappears.

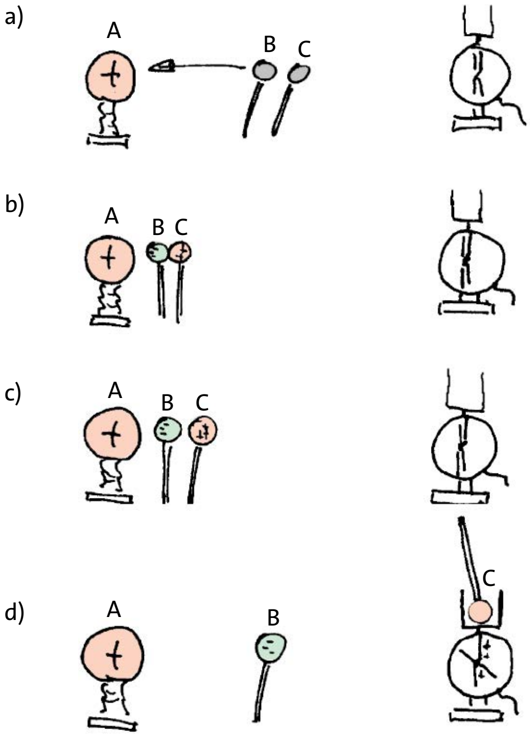


Fig. 1.40 (a) The neutral spheres B and C are brought close to A. They are put together so that they touch each other. (b) The charge on B and C is displaced (induction). (c) The contact between B and C is interrupted. (d) The charges of B and C are detected with the electroscope.

Exercises

1. On sphere B in Fig. 1.37, positive and negative charge carriers are separated through induction. Sphere B is pulled towards A by the field. As soon as it has touched A, however, it will be pushed away from A. How can this be explained?
2. How could we show that the field that is located close to electrically charged objects is not a magnetic field?
3. A light metal sphere A is suspended between two fixed spheres B and C, Fig. 1.41. Sphere A is brought in short contact with C and subsequently released. What happens?

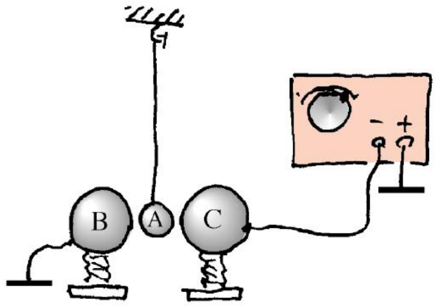


Fig. 1.41 For exercise 3

1.13 The electric field strength

Assume that there is an electric field in a given area of space (for example because this area is currently covered by a thundercloud). Imagine you wish to tell someone how much and what type of field stuff is there at a given place of the area. To do so you need a measure, a physical quantity. Such a measure is the *electric field strength*.

At first, one would probably expect that the electric field strength simply has a high value in close proximity to a charged body and a lower one farther away from the body. However, the field cannot be clearly described yet.

We have seen that there could be both compression as well as tensile stresses in the electric field. In fact, there is compressional and tensile stress at every point at the same time in a field. How is this possible?

For each little piece of field stuff, there is a well-defined direction in which the field is under tensile stress. It is called tensile direction. In all directions that are perpendicular to it, the field stuff is under compressive stress. Fig. 1.42 shows a small cylinder that was hypothetically cut out of a field in a way that the cylinder axis is oriented in the tensile direction of the field. In the directions that are perpendicular to it, there is a compressive stress. (The fact that a material has a well-defined direction at each point is not an uncommon characteristic. For example, the texture of a piece of wood also has a certain direction at each point. Which characteristic of the wood depends on the direction of the texture?)

We conclude: if we wish to characterize the field stuff at one point of the field, it will not be sufficient to say whether there is much or little field stuff. In addition, we need to indicate the tensile direction at that point. (The compressive directions can be derived from it unambiguously.) In other words, the electric field strength must be a vector.

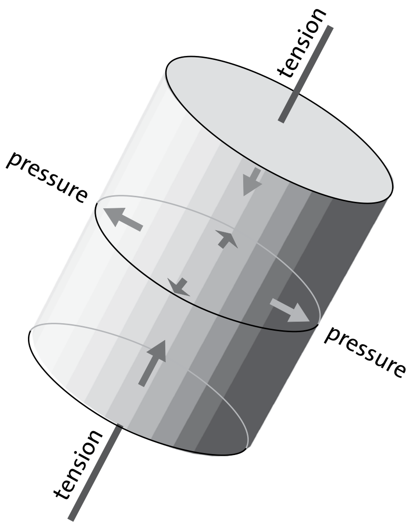


Fig. 1.42 At each point the field has a well-defined particular direction: the direction in which there is tensional stress.

> The electric field strength is a vector.
> *Magnitude of the vector:* measure for the density of the field stuff.
> *Direction of the vector:* tensile direction of the field stuff.

The magnitude of this vector provides evidence of the density of the field; its direction is that of the tensile direction of the field.

Back to our initial problem: we would like to tell someone about the electric field strength that a field has at a given point. Therefore, we need a measurement technique for the field strength vector. There are different methods for this purpose. We would like to look at one that, although it is so unhandy that it is practically not used, is particularly easy to understand – and this is what matters for us at the moment.

A small electrically charged "test body" is brought at the point at which the field strength is to be measured. Now, a momentum current flows into this test body. By means of a momentum current meter (= dynamometer), the amount and direction of the momentum current vector are determined. If we divide by the charge of the test body, we obtain the electric field strength:

$$\vec{E} = \frac{\vec{F}}{Q}$$

Then, we transform the equation:

$$\vec{F} = Q \cdot \vec{E} \qquad (1.1)$$

$\vec{F}$ = momentum current flowing into the body
Q = electric charge of the body
$\vec{E}$ = electric field strength

We thus have: The momentum current is equal to the electric charge multiplied by the electric field strength. This equation has the same form as one that we already know:

$$\vec{F} = m \cdot \vec{g}$$

i.e. the momentum current is equal to mass multiplied by the gravitational field strength.

If the charge of the test body is doubled, the momentum current will also double. The quotient of momentum current strength and charge therefore remains constant. Its value is independent of the charge of the test body, i.e. independent of the meter. And this is how it should be. Although the body changes the original field considerably, the equation still tells us the field strength of the field without the body.

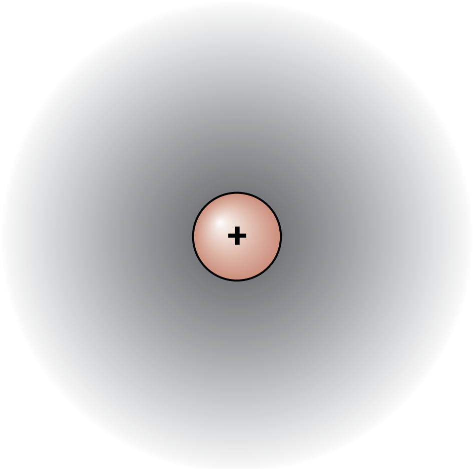


Fig. 1.43 Field stuff surrounding a charged sphere. High density: dark shade; low density: light shade

For the measuring unit of the electric field strength, we obtain Newton/Coulomb. This can be converted to the more common unit Volt/meter.

Later, you will get to know another measurement technique for the electric field strength.

Exercises

1. An electron is brought into an electric field to a point where the field strength is 80 000 V/m. How does the momentum of the electron change? (What is the rate of change of the momentum?)
2. An electrically charged body is brought to a location P. It carries a charge of 10 nC. The momentum current that flows into the body is found to be 0.02 N. What was the field strength of the field at point P before the charged body was brought there?

1.14 Graphical representation of electric fields

In the following, we will often need pictures of electric fields. Hence, we must examine possibilities to graphically display electric fields.

If the tensile and compressional stress in the field, i.e. the direction of the field strength vector, is not a crucial aspect, a very simple method can be used. The density of the field stuff is illustrated by means of gray shadings: black or dark gray where the magnitude of the field strength vector is great and lighter shades where it is small. Fig. 1.43 shows the field of a charged sphere. We see that the field has no clear external boundary – similar to the atmosphere above the surface of the Earth.

Another illustration method is applied in Fig. 1.44. There, the field strength vectors are represented by arrows at regular intervals. This picture does not only tell us the density for each point of the field, but also the tensional direction.

We will now look at the third and most important method. The field is illustrated by means of *field lines* and *field surfaces*.

Field lines: a line is drawn in a way that each point of it has the direction of the field strength vector associated with this point. This is how we obtain one field line. Then, we draw many of these field lines. Fig. 1.45 shows what we obtain for the case of a charged sphere.

It is a common practice to draw arrows on the field lines that point in the same direction as the field strength vectors.

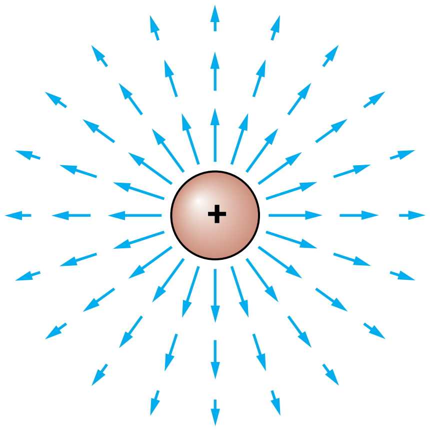


Fig. 1.44 Field of a charged sphere, illustrated with field strength vector arrows

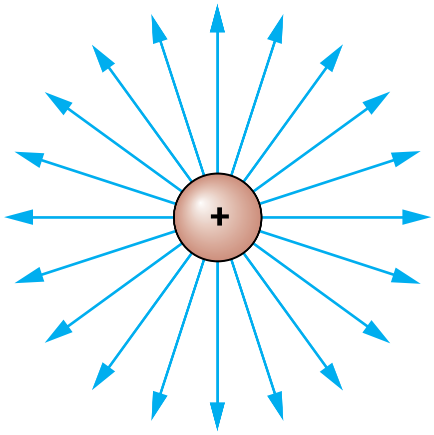


Fig. 1.45 Field of a charged sphere, illustrated with field lines

Field surfaces: we have drawn field lines on our two-dimensional paper. At first, you need to imagine that the field lines are actually located in three-dimensional space. Now, we can construct surfaces in such a way, that the field lines are crossing them perpendicularly in every point, Fig. 1.46. These surfaces are the field surfaces. If we make a two-dimensional sectional image, each surface will also be visible as a line. These lines are always perpendicular to the field lines. In the following, we will mostly look at such two-dimensional sectional images.

Fig. 1.47 shows the field of our charged sphere, represented by field lines and field surfaces in addition to gray shadings.

We have learned how a field can be represented graphically. Of course, this is only possible if we know the field, i.e. that we know the field strength vectors at the different points of space.

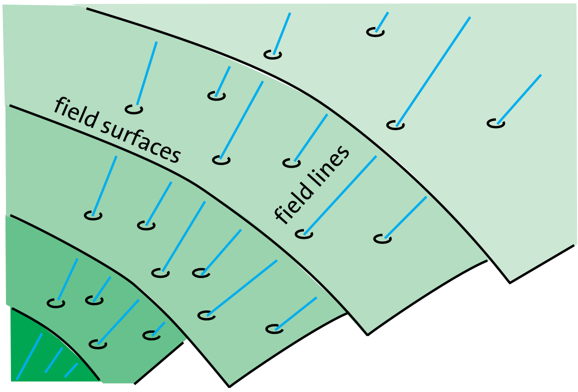


Fig. 1.46 Perspective view of a field section. The field lines cross the field surfaces perpendicularly.

1.15 Rules for drawing electric fields

We start from an arrangement of bodies that are electrically charged, i.e. at which an electric field is attached. How can we know what this field looks like? In other words, what is the shape of the field lines and field surfaces? There are different answers to this question.

The first answer: the field strength vectors for the different points in space can be calculated based on the distribution of the electric charge. The calculation method is quite complicated, but there are computer programs that can do the work for us.

The second answer: field lines and field surfaces can be visualized experimentally. A particularly simple ver-

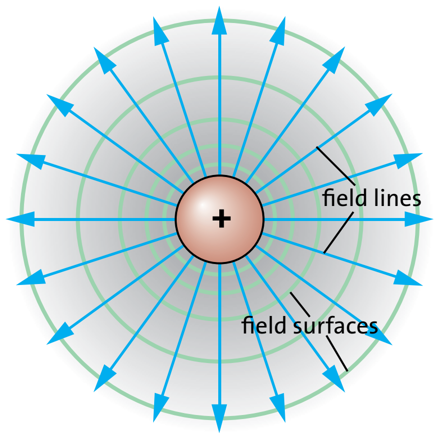


Fig. 1.47 Field of a charged sphere, illustrated with field lines, field surfaces and gray shading

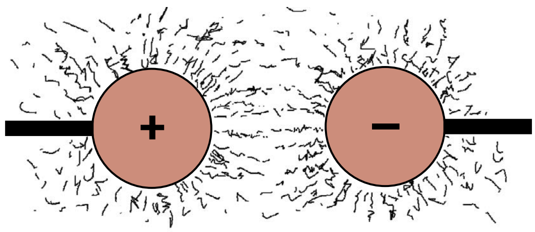


Fig. 1.48 The field lines are visualized by means of castor oil and semolina grains.

sion of such an experiment almost sounds like a kitchen recipe: castor oil is poured between the charged bodies whose field is to be analyzed. Then, some semolina is spread onto the oil. The semolina grains form small chains that are oriented at each point in the tensile direction of the field, Fig. 1.48 (the experiment is similar to the one in which the field lines of a magnetic field are visualized by means of iron fillings.)

The third answer: if the image does not have to be exact, it is sufficient to know a few rules. This method is the most important one for us at the moment because each of these rules tells us an important aspect of the nature of the field. We would like to provide an overview of these rules here.

You already know the first one: the field lines are always perpendicular to the field surfaces.

Another rule can be identified if we take a closer look at Figure 1.49: field lines end at electric charges; field surfaces, in contrast, are closed. Of the two charges that are connected by a field line, one is always positive and the other one negative. The arrow that we put on the field lines points from the positive to the negative charge. Hence, the rule can also be formulated like this: field lines start at positive and end at negative charges. (However, we will later see fields in which the field lines have no beginning or end, whereas the field surfaces are not closed.)

If possible, the field lines are drawn in a way that one field line corresponds to a given amount of charge. In Fig. 1.49, for example, the body at the top left carries 10 positive units of charge, the two others respectively 5 negative units. If the overall charge of the illustrated bodies is not equal to zero, field lines need to leave the picture. This is the case for instance in Fig. 1.47. Here, only a single positively charged body is shown. Hence, all field lines must run out of the picture.

Another rule says that the field lines never intersect. To understand this, we do not need to look at the figures: in one point, the field strength vector must have a well-defined direction. It cannot have two directions at the same time.

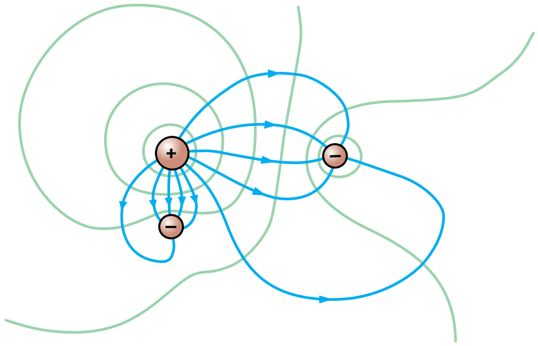


Fig. 1.49 Field lines start at positive and end at negative charges. Field surfaces are closed.

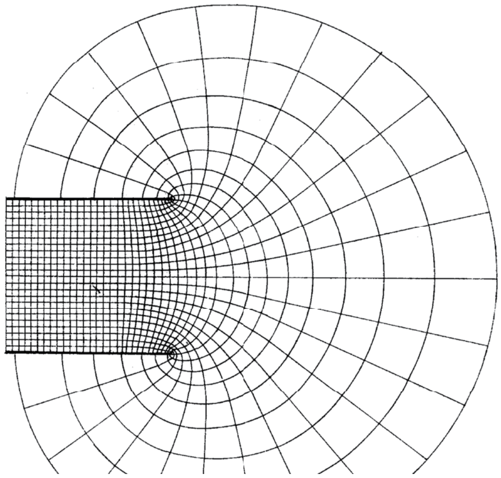


Fig. 1.50 Field lines and field surfaces at the edge of a charged capacitor from Maxwell's Treatise on Electricity and Magnetism

Another simple rule: except on the surface of bodies, field lines and field areas do not have any kinks.

Finally, another very "powerful" rule: if the arrangement of the charged bodies is somehow symmetric, the field line image will display the same symmetry.

We summarize:

> In every point, the field lines are perpendicular to the field surfaces.
>
> The field lines start on positively charged bodies and end on negatively charged ones. The greater the charge, the more field lines start or end on the body.
>
> Field lines do not intersect each other. Field surfaces do not intersect each other.
>
> Field lines and field surfaces do not have kinks.
>
> A field image has the same symmetry as the electric charges.

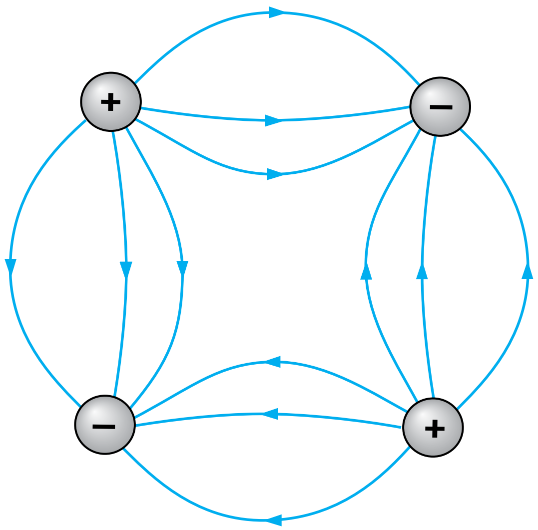


Fig. 1.51 For exercise 1

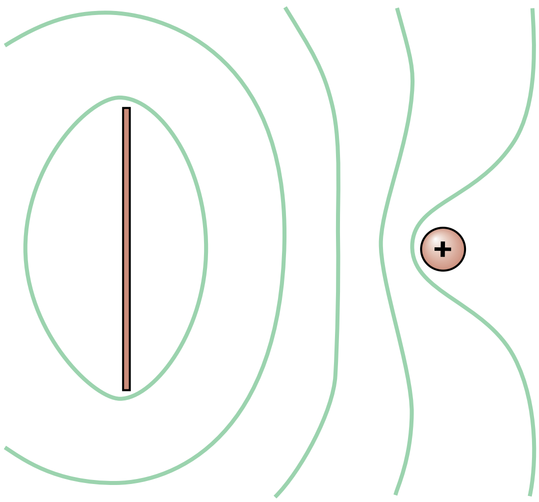


Fig. 1.52 For exercise 2

Fig. 1.50 shows field lines and field surfaces of the field at the edge of a capacitor from the original publication by James Clerk Maxwell from 1873. Maxwell formulated the theory of the electric and magnetic fields that is still used today.

Exercises

1. Draw the field surfaces in Fig. 1.51. (The figure shows the field lines.)
2. Draw the field lines in Fig. 1.52. (The figure shows the field surfaces.)
3. Fig. 1.53 shows the field surfaces of a field. Mark the points where charges are sitting. Draw the field lines.

Fig. 1.53 For exercise 3

1.16 Four important electric fields

We will repeatedly come across some particular charge distributions. Hence, it is worth memorizing the images of the corresponding fields.

1. A charged sphere

We already know the field. It is shown in Fig. 1.47. We would still like to convince ourselves that we would have also been able to draw the image by using our rules:

The field lines must start at the charged body. As there is no other body, they all need to leave the image in an outward direction.

Regarding the symmetry of the charge distribution: if the charged body is rotated by a random angle, we will not be able to distinguish the rotated sphere from the one that has not been rotated. The charge distribution is rotationally symmetric for any angle. The field must have the same symmetry.

Therefore, the field image is already defined unambiguously. The field lines run radially in an outward direction, the field surfaces appear as concentric circles.

2. The electric dipole

A dipole is a structure that consist of two adjacent bodies that have the same charges with opposite +/– signs: one is positively charged, the other one carries a negative charge of the same amount. The field image is shown in Fig. 1.54.

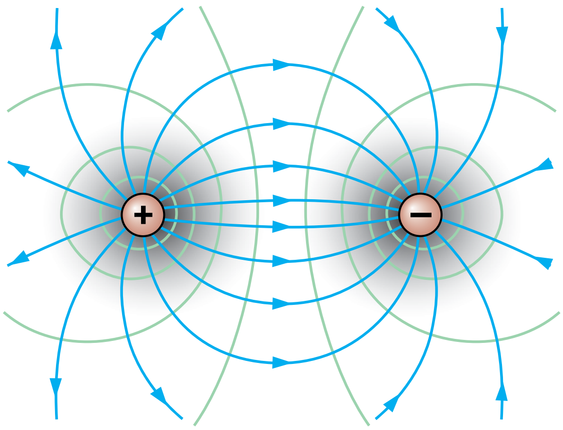


Fig. 1.54 Electric dipole with its field

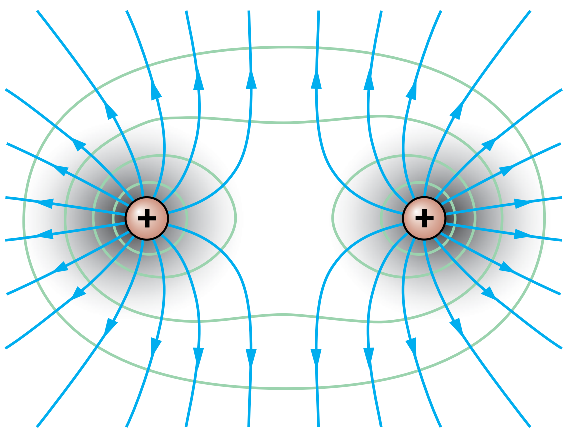


Fig. 1.55 Two positively charged bodies with their field

As the amount of negative charge is equal that of positive charge, all field lines run from one to the other body.

Exercise 2 asks for the symmetry of the charge distribution.

3. Two equally charged bodies

The charge distribution is similar to that of the dipole, but the two bodies now have the same charge. Let's assume both charges are positive, Fig. 1.55.

Since there are no negatively charged bodies, all field lines must run out of the picture.

Exercise 3 asks for the symmetry of the charge distribution.

4. Two plates with equal but opposite charges

Also in this case, the charge has the same amount but opposite +/– signs, and it is distributed equally on each of the two plates. Fig. 1.56 shows the picture for the case that the length of the plates is approximately three times their distance.

Just as for the dipole, all field lines run from one to the other body, i.e. from one to the other plate. It is remarkable that the field concentrates on the space between the plates. We can also see that the field strength in the central area is very even: both the direction and the magnitude of the field strength vector are almost not changing from one point to another. The field is nearly *homogeneous*.

"Homogeneous" means "the same everywhere". If, for example, the temperature has the same value at each point of a room, the temperature distribution is homogeneous.

A homogeneous field is the simplest field that we can imagine. We can easily realize a nearly homogeneous electric field with two plates. The greater the extension of the plates compared to their distance, the more even will be the field, the more similar it is to a homogeneous field. We often imagine that the plates would have an infinite length and width. In this case, the field between the plates would be perfectly homogeneous and outside of it, the field strength would be exactly zero. Fig. 1.57 shows a section of such a pair of plates, together with its field.

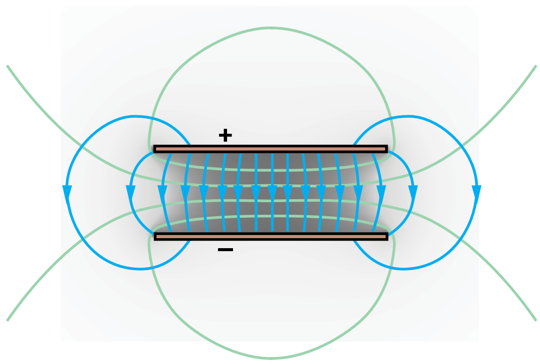


Fig. 1.56 Two plates with equal but opposite charges with their field

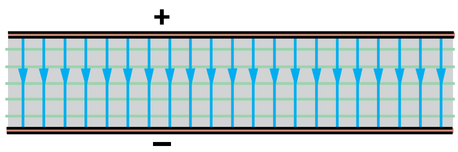


Fig. 1.57 Sectional view of two infinitely extended plates with equal but opposite charges

Exercises

1. A sphere carries the charge Q_0. The charge sits (a) on the surface of the sphere, or is (b) spread throughout the inside of the sphere. In what way is the field line image outside of the sphere different between case (a) and case (b)? Explain.
2. Which are the symmetries of the charge distribution of the electric dipole in Fig. 1.54? Check if the field has the same symmetry. Consider the +/– signs of the charge. Formulate an extension of the rule about the relationship of the symmetries of charge and field.
3. Which are the symmetries of the charge distribution from Fig. 1.55? Does the field have the same symmetry?

1.17 Calculation of electric field strengths

Calculating the field strength for a given charge distribution is generally difficult, but not so in a few important cases – for example if the charge distribution is spherically symmetric. We consider an electrically charged sphere whose charge is spread throughout the entire inside. The charge density is supposed to be the same at every point inside the sphere. The magnitude of the field strength vector outside of the body is then described by the following equation:

$$\left|\vec{E}(r)\right| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \qquad (1.2)$$

electric constant $\varepsilon_0 = 8.854 \cdot 10^{-12}$ C/(V · m)

where
Q = electric charge
r = distance from the center of the charge distribution
ε_0 = electric constant.

The whole factor $1/(4\pi\varepsilon_0)$ is also a constant.

Does this equation look familiar to you? It has the same structure as the equation for the gravitational field strength of spherically symmetric bodies:

$$\left|\vec{g}(r)\right| = G \cdot \frac{m}{r^2}$$

We already know the direction of the field strength vectors: if the charge is positive, they point radially outwards.

The $1/r^2$ law of equation (1.2) is more commonly used than we would think at first.

Hence, it is not only valid for the field of a homogeneously charged sphere, but also for any other spherically symmetric charge distribution; however, only outside of the space in which the charge is located. Fig. 1.58 shows three spherically symmetric charge distributions. The total charge, i.e. Q is the same for all of them. Equation (1.2) applies for all three, but only for radii that are larger than R.

The $1/r^2$ law, however, can also be used if the charge distribution is not spherically symmetric because it applies for any charge distributions if we are at a sufficient distance. Figure 1.59 shows a charge distribution whose cross-section has the shape of an acute-angled triangle. From a certain distance, the field lines are almost straight lines and the equipotential areas are almost spheres. When this is the case, the $1/r^2$ law can also be applied.

Exercises

1. In Fig. 1.58c, the charge $Q/2$ on the inner sphere is replaced by $-Q/2$. What does the field for $r > R$ now look like?
2. Insert the expression for the field strength of equation 1.2 in equation 1.1. Be well aware of what you are doing. What is the meaning of Q in equation 1.1 and what is the meaning of Q in equation 1.2? What is the meaning of r? In case you have done everything correctly, you have obtained Coulomb's Law. Search for an analogous law in mechanics.

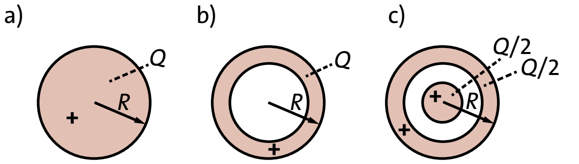


Fig. 1.58 For each charge distribution, the $1/r^2$ law applies outside of the distance R from the center.

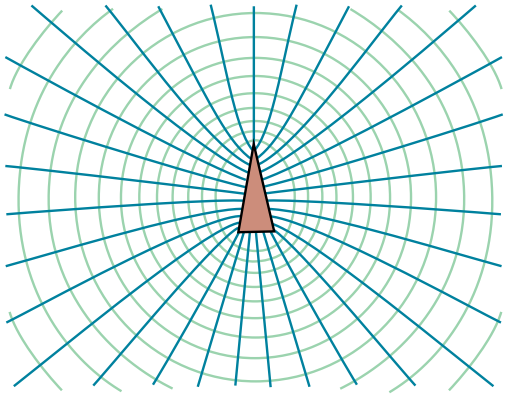


Fig. 1.59 Field lines (blue) and field surfaces (green). At a long distance from a charge distribution, the field becomes spherically symmetric and the $1/r^2$ law applies.

1.18 Several charged bodies – vector addition

We will try to bring two fields to the same point, for example the fields of two charged spheres A and B. A and B carry the same charge.

But how can a field be brought from one point to another? By moving the charged body to which it is attached. Hence, let us move one of the charged spheres closely towards the other.

What happens? Does the field of A prevent the field of B from merging into it? Or can two fields be located at one point at the same time? Neither the former nor the latter is the case. Something else and something surprisingly simple happens. A field results, whose field strength is obtained by vector addition of the field strengths of the individual fields, Fig. 1.60.

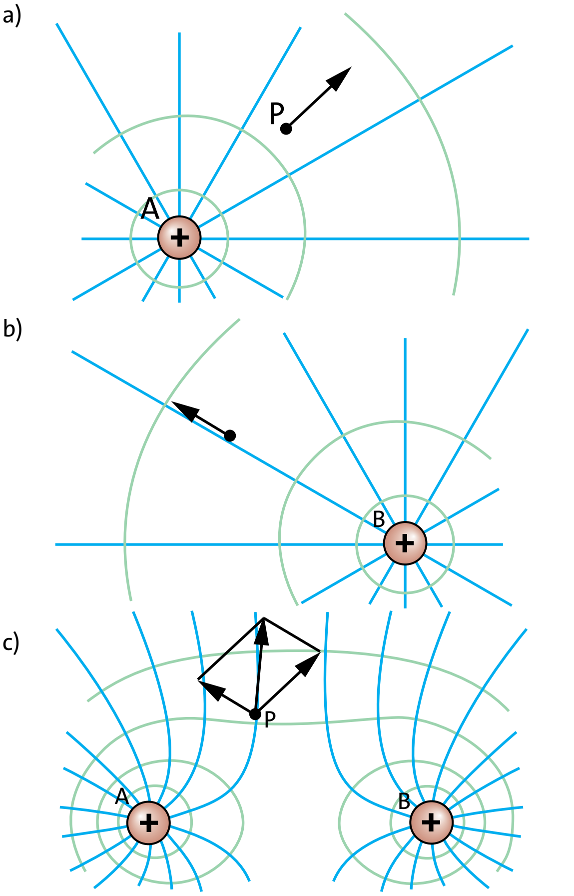


Fig. 1.60 (a) Sphere A with its field. (b) Sphere B with its field. (c) Spheres A and B with their field. The field strength is obtained through vector addition.

Fig. 1.60a shows sphere A with its field. There are no other charged bodies close to it. Fig. 1.60b shows sphere B alone with its field. Fig. 1.60c finally shows the case that there are both spheres. We are interested in the field strength in point P. We assume that we know the field strength vectors in the case that there is only one sphere, i.e. either only A or only B. These vectors are shown in Figures 1.60a and 1.60b. We obtain the field strength in P for the case that there are both spheres by adding the two field strength vectors, Fig. 1.60c. This is how the field strength at each point can be obtained based from the field strengths of the individual charges.

We would now like to get a useful, general rule by means of our new knowledge. We will derive it on the basis of an easy example and subsequently generalize it. We look at the field of a sphere that carries the electric charge Q_0. The field strength of the field in a randomly chosen point P shall be $\vec{E}_0$, Fig. 1.61a.

We now bring another charge Q_0 to the point of the original charge, Fig. 1.61b. In other words: the sphere that had the charge Q_0 before now carries the charge $2\,Q_0$. What is the new field strength? How does the electric field strength change at a point when the electric charge that the field creates is doubled? We know how to calculate the new field strength, namely by vector addition of the contributions of the individual charges: field strength created by the first charge Q_0 plus field strength created by the second charge Q_0. As the two vectors are equal, the result is a vector that has the same direction as $\vec{E}_0$ but twice the length. Hence, the new field strength is simply $2\,\vec{E}_0$.

This result can be generalized:

> If all charges of a charge distribution are multiplied by a factor k, the magnitudes of all field strengths will increase by the factor k. The field strength directions will not change.

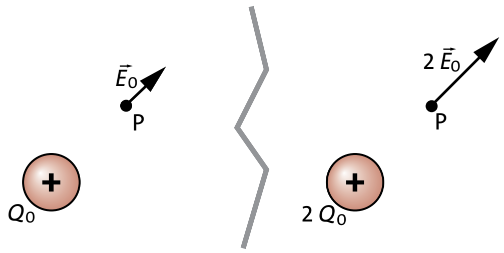


Fig. 1.61 If the charge is doubled, the electric field strength will double at each point of the field.

Vector addition is also used by the computer to calculate field distributions. We imagine the charge distribution to be composed of many tiny point charges. Each individual one creates a field whose field strength can be calculated according to equation (1.2). The computer calculates the field strength contribution of all point charges for a given point P1 and adds them up according to the vector addition rule. The result is the field strength in P1. The same is done for all other points P2, P3 ... for which we want to have the field strength (usually for all pixels of the computer screen).

Exercises

1. A small pair of plates with equal but opposite charges, is brought transversally between the plates of a large pair of plates, Fig. 1.62. Also the large plates have equal but opposite charges. The magnitudes of the field strengths shall be equal (before bringing the small pair of plates into the large one). What is the field strength in the space between the small plates?
2. The field of the infinitely extended pair of plates from Fig. 1.57 can be regarded as composed of the field of the upper plate and the one of the lower plate. How does the field of the upper plate alone look like? How does the field of the lower plate alone look like? How does the field of both plates result?
3. The absolute value of all the charges in Fig. 1.63a and 1.63b is the same. Determine the direction of the field strength vector in points A, B, C and D.

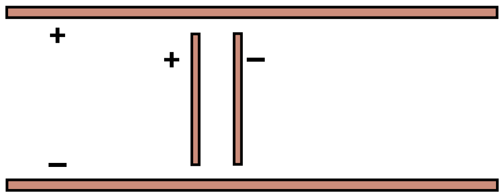


Fig. 1.62 For exercise 1

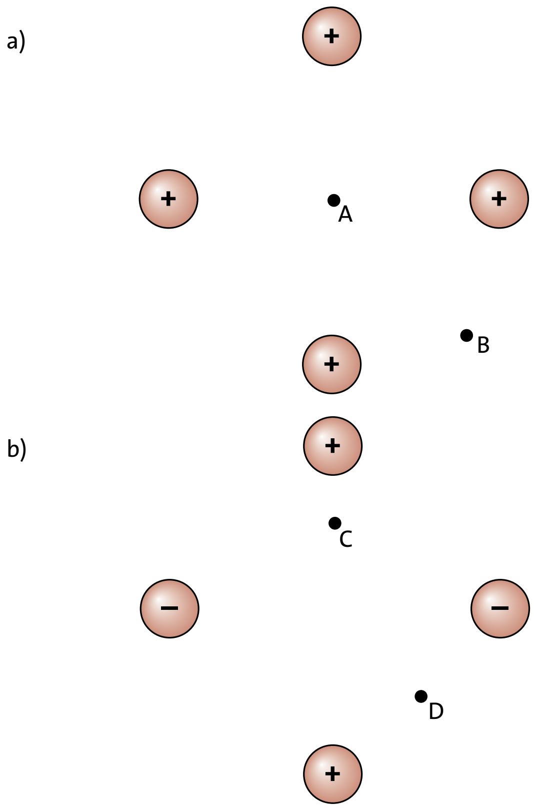


Fig. 1.63 For exercise 3

1.19 Pressure and tension within the electric field

We have seen that the electric field is in a state of mechanical stress. At each point, it is under tensile stress in one direction and under compressive stress in the transversal directions. In places where the field strength is high, i.e. where the field stuff is dense, the stresses are strong. Mechanical stress is a physical quantity that can be described with numbers. We will see how this is done with an example that is slightly easier than the electric field.

A block K1 is clamped between two walls by means of a spring, Fig. 1.64a. A momentum current is flowing through the arrangement. We now compare it with block K2 in Fig. 1.64b. As the springs are identical, the momentum currents are the same in both cases. However, as the block K2 has a greater cross-section than K1, the momentum current spreads across a larger surface. In K2, the momentum current per unit area is smaller than in K1; in other words: the *momentum current density* is lower in K2.

The momentum current density is abbreviated with σ.

$$\sigma = \frac{F}{A} \quad \text{momentum current density}$$

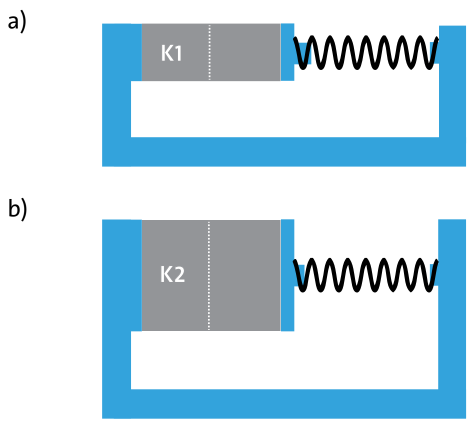


Fig. 1.64 The same momentum current flows through the blocks K1 and K2. The momentum current density (= mechanical stress) in K1 is higher than in K2.

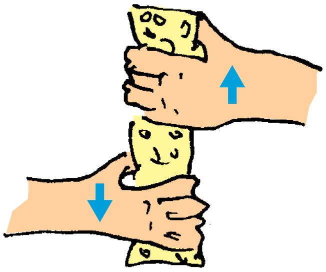


Fig. 1.65 The inside of the sponge is exposed to tensile stress in a vertical direction and to compressive stress in a horizontal direction.

"Mechanical stress" is just another name for the momentum current density:

> σ = mechanical stress = momentum current density

We will now expose an object to compressive and tensile stress at the same time; for example a blackboard sponge, Fig. 1.65. We grab it with both hands and compress it from the side; at the same time, we pull in a longitudinal direction. The inside of the sponge is now exposed simultaneously to compression and tension: compression in the direction from right to left, and tension in the direction from top to bottom. It could of course also be exposed to tension or to compression in both directions respectively. In any case, the two compressive or tensile stresses can have different values.

Finally, we can also expose the sponge to any compressive or tensile stress in the third spatial direction, the direction from the front to the back, by pushing or pulling accordingly.

You might now imagine that we could continue like this; that further different compression values could be produced in other spatial directions. Why not five different compression values (or tensile stress values) in five different directions? This is not possible, though. As soon as we try to change the stress in a fourth direction, the stress will change automatically in the first three directions.

Hence, we have the following result:

> Mechanical stress can have different values in three directions that are perpendicular to each other.

In solid bodies, also in bodies such as a sponge, the three directions and the pertaining stress values can be set arbitrarily (at least as long as they are not as high as to break the body).

However, there are systems in which the three stresses are related to each other in a particular way.

In liquids or gases, for example, the three stresses are always equal. They are then just called *pressure*.

> Liquids and gases: $\sigma_1 = \sigma_2 = \sigma_3 = p$

Coming back to the electric field, we already know: there is tensile stress in one direction and compressive stress in the respective perpendicular direction. A simple rule applies for the values of these stresses: the tensile stress has the same magnitude as the compressive stress, i.e.:

> Electric field: $\sigma_{\|} = -\frac{\varepsilon_0}{2}\left|\vec{E}\right|^2 \qquad \sigma_{\perp} = \frac{\varepsilon_0}{2}\left|\vec{E}\right|^2$

$\sigma_{\|}$ is the stress in the direction of the field lines; it is negative, i.e. it is a tensile stress. $\sigma_{\perp}$ is the stress perpendicular to the field lines; it is positive, i.e. a compressive stress. $|\vec{E}|$ is the magnitude of the electric field strength. (In order to avoid confusion with the energy E, we added a vector arrow and bars to the field strength symbol.) ε_0 is the electric constant:

> Electric constant $\varepsilon_0 = 8.854 \cdot 10^{-12}$ C/(V · m)

We can now understand why and how bodies attract and repel each other. We would like to predict, only by looking at the field image, in which direction the field pushes or pulls a charged body.

All we need to know is that there is tensile stress in the direction of the field lines and compressive stress in the respective transversal direction, i.e. in the direction of the field surfaces.

1. A pair of plates

Fig. 1.66 once again shows the field image. We look at the field in the area that is contoured by the dotted line. The field lines leave the upper plate on the lower side. Since there is tensile stress in the direction of the field lines and since the field lines begin on the plate, the field pulls the plate downwards. Accordingly, the lower plate is pulled upwards.

One question remains open: perpendicularly to the field lines, i.e. in the direction of the field surfaces, there is compressive stress. If the field presses outward in the horizontal plane, it will press against what? Where does the field adhere to on the sides? It can be seen by considering the model of the field in Fig. 1.67. Here the field is replaced with many small springs. Some of them are oriented parallel to the field lines. These are under tensile stress. The others are parallel to the field surfaces. They are under compressive stress.

What is now the answer to our question? Where do the horizontal springs keep hold at the sides? At the springs that are at the far left and the far right in a skew orientation; those, in turn, adhere to the plates so that the plates are exposed to a tensile stress in their longitudinal direction. This implies for the field that the pressure inside the field works in a way that the field pulls the plates in a longitudinal direction.

We summarize:

> The field between two plates with opposite charges
> - pulls the plates towards each other;
> - pulls each plate in a longitudinal direction.

We thereby have found a new method to measure the strength of the field between the plates, Fig. 1.68: we measure the momentum current that flows through the field from the upper to the lower plate (in the negative *z*-direction). This momentum current comes from above, flows through the meter and the upper plate into the field and leaves the field at the lower plate (and then through the table and the rods back to the top so that the circuit is closed).

We know that $F_{el} = \sigma \cdot A$.

With

$$\sigma_{||} = -\frac{\varepsilon_0}{2}\left|\vec{E}\right|^2$$

we obtain

$$F_{el} = -\frac{\varepsilon_0}{2}\left|\vec{E}\right|^2 \cdot A$$

and thereof

$$\left|\vec{E}\right| = \sqrt{\frac{-2F_{el}}{\varepsilon_0 \cdot A}}.$$

The meter indicates the sum $F_{el} + F_{grav}$, i.e. the momentum current that flows into the ground through the gravitational field and that corresponds to the weight of the upper capacitor plate, in addition to the momentum current that flows away through the electric field. To obtain F_{el}, F_{grav} must be subtracted from the measured value.

Since in common electric fields the momentum currents are very weak, the momentum current meter must be very sensitive.

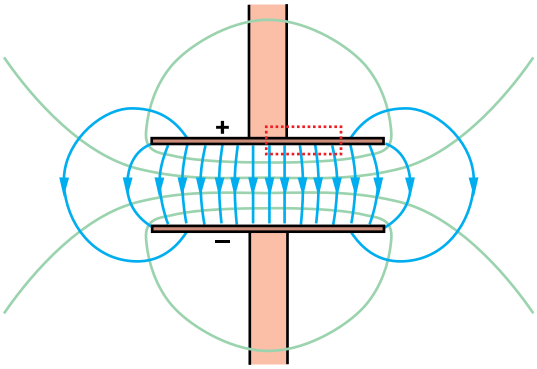


Fig. 1.66 The field pulls downwards on the part of the upper plate that is framed by a dotted line.

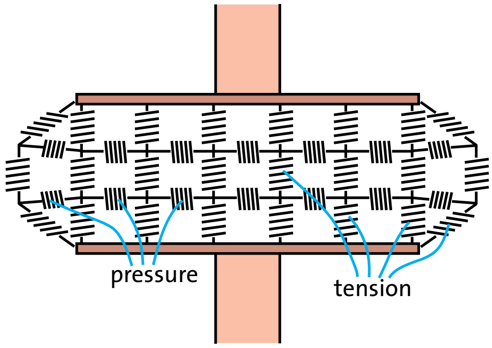


Fig. 1.67 Model of the electric field between two parallel plates. The springs, that are oriented in the direction of the electric field lines are under tensile stress. The springs that are parallel to the field surfaces, are under compressive stress.

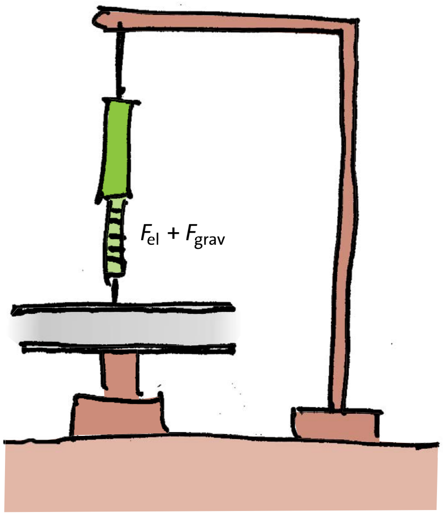


Fig. 1.68 The momentum current meter indicates the sum $F_{el} + F_{grav}$ of the momentum current, that arrives in the upper plate through the electric field, and the one that comes through the gravitational field.

Example

We assume to have measured $F_{el} = -0.05$ N .

If the plate area is $A = 500$ cm², we get the electric field strength:

$$|\vec{E}| = \sqrt{\frac{-2F_{el}}{\varepsilon_0 \cdot A}}$$

$$= \sqrt{\frac{2 \cdot 0.05 \text{ N}}{8.85 \cdot 10^{-12} \text{ C/(V} \cdot \text{m)} \cdot 0.05 \text{ m}^2}}$$

$$= 4.8 \cdot 10^5 \text{ V/m}$$

If we know the electric field strength from another source (we will soon learn about a more practical method to measure it), the experiment can also be used to determine the electric constant ε_0.

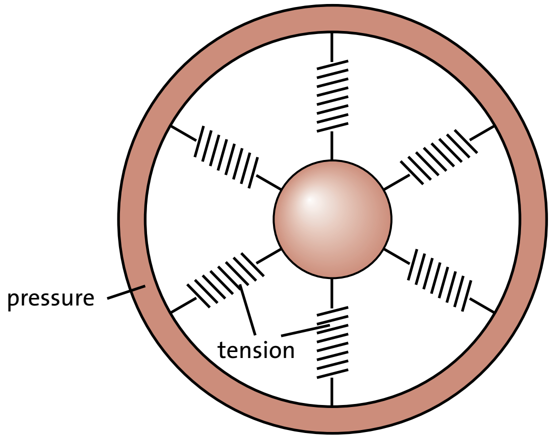


Fig. 1.69 Model of the electric field of a charged sphere. The springs pull at the surface of the sphere outwards. There is compressive stress in the ring to which the springs are attached.

2. A spherical body

We assume the charge to be sitting on the surface of a sphere. You know how the field image looks like. As the field lines extend radially outwards, the field pulls outwards on the surface of the sphere – with the same strength in all directions. It is best to think of the sphere as a hollow, elastic globe, as a balloon or a soap bubble for example. Such a globe will increase in size when electric charge is put on it. Also here, we wonder where the electric field holds at the outside. This time, the answer is: at itself. Fig. 1.69 again shows a material model. It consists of radial springs and a ring. This structure also pulls at the central body in all directions. The springs hold on the ring at the outside. Therefore, compressive stresses develop in the ring in the direction of the circumference, i.e. perpendicularly to the direction of the springs. In the electric field, the stress is similar indeed, since there is compressional stress perpendicular to the field lines.

> The field of a charged sphere pulls outward on the surface.

3. The electrically charged "test body"

If a body that carries electric charge Q is brought to any point of an electric field of field strength $\vec{E}$, a momentum current will flow into the body whose current strength $\vec{F}$ is calculated according to:

$$\vec{F} = Q \cdot \vec{E} .$$

Measuring the momentum current strength and the electric charge allows us to determine the electric field strength.

Now we can understand the origin of this momentum current. Fig. 1.70 shows on the left a sectional view of the original field A without any additional body.

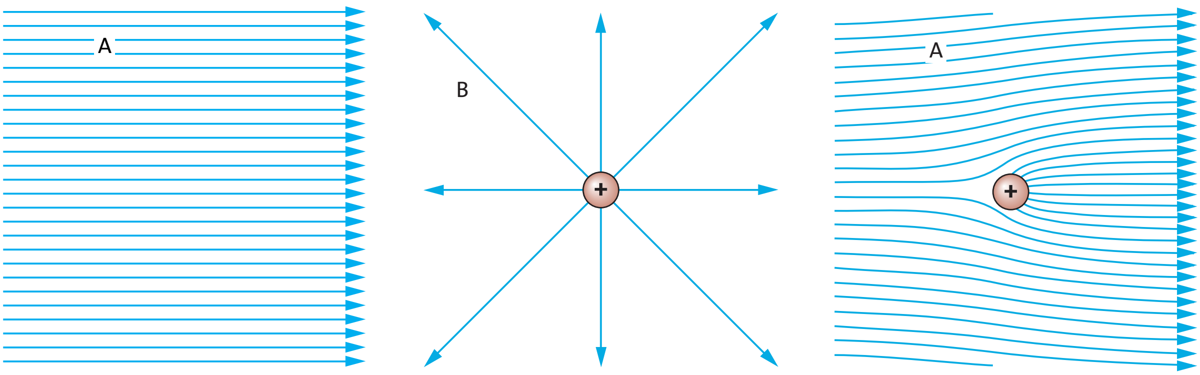


Fig. 1.70 The field strengths of fields A and B are added. The resulting field C pulls more at the right side than on the left. A net momentum current flows into the body.

The picture in the middle shows the body with its field B. If the body is then brought into field A, a new field C, on the right, will result. We obtain C by combining the field strength vectors of A and B point by point according to the rules of vector addition. (This can of course be done by the computer.) You see: on the right side of the body, the field strength has increased compared to B; on the left side, it has decreased. On the right, the field pulls more than on the left, i.e. in total, the field pulls the body to the right. In other words: a net momentum current flows into the body. This is exactly the momentum current that we can calculate according to the above-mentioned equation.

Exercises

1. As we know, the charged bodies of a dipole are pulled towards each other by the field. How can that be seen from the field picture of Fig. 1.54?
2. Two equally charged spheres are pulled apart by the field. How is this read from the field picture in Fig. 1.55?
3. There is a homogeneous electric field between two parallel plates with equal but opposite charges (plate surface: 2400 cm^2). The field pulls the plates towards each other. The respective momentum current is measured and we find 0.0025 N. What is the electric field strength between the plates?
4. Fig. 1.71 shows two concentric spheres. The inner one is negatively charged. The outer one carries positive charge of the same amount as the inner one. The field is only located in the space between the two spheres. Sketch the field lines and surfaces.

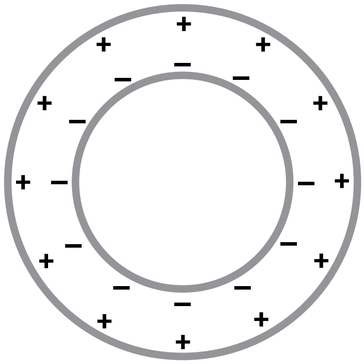


Fig. 1.71 For exercise 4

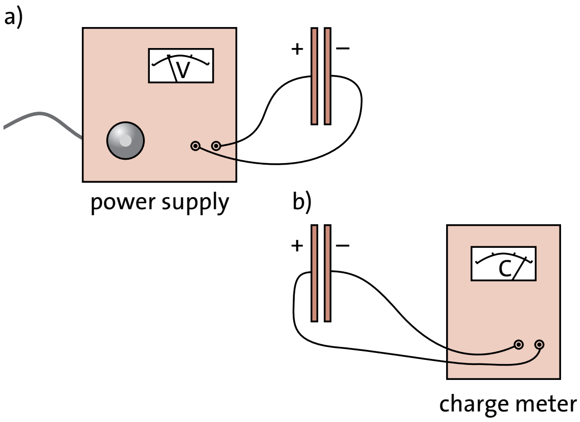


Fig. 1.72 (a) Charging the capacitor: the power supply "pumps" electric charge from one plate onto the other. (b) The capacitor is discharged through the meter.

1.20 Capacitor and capacitance

Capacitors are used to store electric charge and to store energy. A capacitor consists of two thin metal plates or layers with a very small distance between them and that are electrically insulated against each other. The stored charge sits on the plates, the energy in the field between the plates. The field of a capacitor is the same as the one of Fig. 1.57. During the charging process of the capacitor, electric charge is "pumped" from one plate onto the other. One plate will then carry just as much negative charge as the other one carries positive charge. Hence, the capacitor is neutral in total. The technical symbol of a capacitor are two short parallel thick lines as shown in the following figures.

As long as a capacitor is charged, there is an electric potential difference between its plates. The greater the charge Q, the higher the potential difference U. We would like to analyze the relationship between Q and U. We charge a capacitor by means of a power supply, Fig. 1.72a. The charging process is very fast. The terminals of the power supply have to be brought in short contact with the feed lines of the capacitor. The voltage between the plates of the capacitor is now equal to the voltage of the power supply. Then, we measure Q by discharging the capacitor by means of a charge meter, Fig. 1.72b. The charge of the positive plate flows through the meter to the negative one until both plates are uncharged.

We repeat the process "charging-measuring" with other voltages and thereby obtain a respective charge value for each voltage. With these values we draw a Q-U-diagram and notice that the charge is proportional to the voltage,

$Q \sim U.$

This statement is equivalent to saying that the quotient Q/U is constant. This quotient is called the capacitance C of the capacitor:

$$C = \frac{Q}{U}$$

If the capacitance of a capacitor B is three times as high as that of another capacitor A, there will be three times as much charge on B than on A in case of a given voltage, Fig. 1.73.

Please bear in mind that Q is not the total charge of the capacitor; the total charge is always zero. Q is the charge of the positively charged plate; U is the difference "high potential minus low potential".

The value of the capacitance is printed on technical capacitors. From the last equation we can conclude that the measurement unit is Coulomb/Volt. This unit is abbreviated Farad (F). Hence, we have

$$1\ \text{C/V} = 1\ \text{F}.$$

One Farad is a very large unit. The capacitance of technical capacitors is often in the range of nanofarad to millifarad.

> The electric charge Q that sits on one of the plates of a capacitor is proportional to the voltage U between the plates.
>
> $$Q = C \cdot U$$
>
> C is the capacitance of the capacitor.

What does the capacitance of the capacitor depend upon? How does a capacitor have to be designed to have a high capacitance? These questions are not hard to answer. We start our reasoning based on a given capacitor that we try to improve, i.e. whose capacitance we try to increase.

How can we change a capacitor so that it carries more charge at the same voltage? First, we increase the plate surface. To understand that this must lead to an increased capacitance, we insert an intermediate step. It is logical that two capacitors "connected in parallel", Fig. 1.74a, can store twice as much charge as a single one. But we can also consider the two parallel capacitors as a single one with a double-sized plate surface, Fig. 1.74b.

With a similar reasoning we obtain the dependency of the capacitance on the distance between the plates.

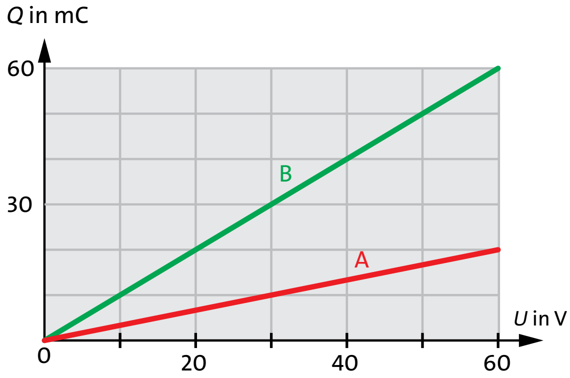


Fig. 1.73 The voltage between the plates of a capacitor is proportional to the charge that sits on the plates. The capacitance of capacitor B is three times that of A.

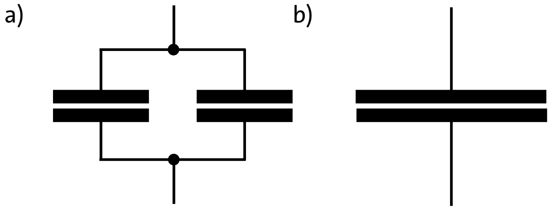


Fig. 1.74 (a) Two capacitors connected in parallel store twice as much electric charge as a single one (at an equal voltage). (b) A capacitor with twice the plate surface stores twice as much electric charge as a capacitor with the original plate surface.

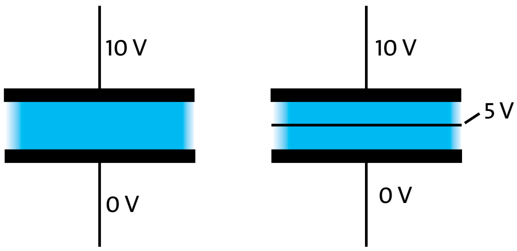


Fig. 1.75 A capacitor is decomposed in two capacitors with half the distance between the plates. On each of these partial capacitors there is only half of the total voltage.

We can decompose the capacitor of Fig. 1.75 in our mind in two capacitors that are "connected in series". The voltage on each of them is only half of the total voltage, but each of them stores the same charge as the whole capacitor at the left side of the figure. We can conclude that the capacitor with half of the distance between its plates has a capacitance that is twice that of the capacitor at the left side.

We summarize the two results:

The capacitance of a capacitor is proportional to the plate surface A and inversely proportional to the plate distance d:

$$C \sim \frac{A}{d}$$

To get an equation, we need to introduce a proportionality factor:

$$C = \varepsilon_0 \frac{A}{d}$$

A = plate surface
d = plate distance
$\varepsilon_0 = 8.854 \cdot 10^{-12}$ C/(V · m) = electric constant

The value of the electric constant is obtained through measurement.

Exercises

1. A capacitor is charged ten milliseconds with 20 µA. Thereby it reaches a voltage of 60 volts. What is its capacitance?
2. A capacitor of 16 µF is charged until its voltage amounts to 10 volts. Which charge will then sit on its plates?
3. Which plate distance does a capacitor with a plate surface of 0.5 m^2 need to have so that its capacitance will be 1 µF?
4. Two capacitors of 8 µF each are connected in parallel. What is the overall capacitance? Formulate a rule.
5. Two capacitors of 8 µF each are connected in series. What is the overall capacitance? Formulate a rule.

1.21 Surfaces of constant potential

Each point of an electric circuit is at a well-defined electric potential. If we go from a point that is at the potential φ_1 to a point whose potential is φ_2, we move past all intermediate values. We start, for example, at point P_1 in Fig. 1.76 and move through the two resistors to P_2. The potential in P_1 shall be 0 V, the one of P_2 10 V. We come past a point where the potential is 5 V, i.e. exactly between the two resistors. But there is also a place where the potential is 1 V and another one where it is 2 V or 2.5 V or 2.6 V or 7.344 V etc., i.e. within one of the resistors. Hence the place with the potential 2.5 is located somewhere inside the lower resistor.

The fact that we find all intermediate values of the potential on the way from P_1 to P_2 does not only apply for a path within the electric circuit. Also, each point

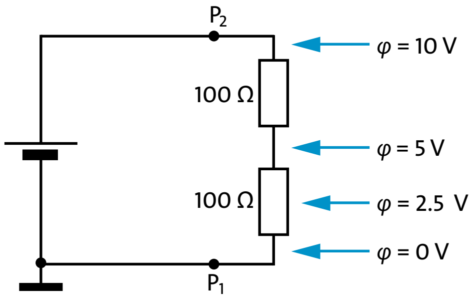


Fig. 1.76 If we move from P_1 (0 volt) to P_2 (10 volt), we will come past all potential values between 0 and 10 volt.

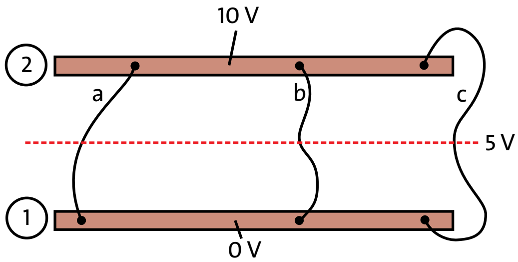


Fig. 1.77 Regardless on which way we move from plate 1 to plate 2: we come past a point where the potential is 5 volts.

outside has a well-defined potential and, when moving outside of the electric circuit from P_1 to P_2, we move past all potential values between φ_1 and φ_2.

How this is to be understood is best explained by means of a simple example. We look once again at a capacitor, Fig. 1.77.

Suppose plate 1 (bottom) is at a potential $\varphi_1 = 0$ V, plate 2 (top) at $\varphi_2 = 10$ V. If we move from plate 1 to plate 2, we necessarily come past all values between φ_1 and φ_2. Hence, if we choose path a, we will somewhere come to a point with the potential 5 V. But also if we chose path b or c, we will pass by a point with 5 V. Consequently, there must be a line between the two plates of our figure on which all points with $\varphi = 6$ volt are located and another one with all points of $\varphi = 2$ volt etc.

It is much like when we climb up a mountain that is 200 m high. Regardless of the path we choose, we will reach an altitude of 100 m at some time. On a map, there is a line that connects all places of the altitude 100 m to each other – a contour line.

Fig. 1.77 shows only a two-dimensional cross-section through the arrangement of the plates. In three-dimensional space, the 5-volt points are not located on a line but on a surface, just as the 6-volt points, the

2-volt points etc. In other words: there are surfaces between the plates on which the potential has a uniform value. Each point of the field has a well-defined electric potential and is located on one of these surfaces.

We know that each point of a field is located on a field surface. With the surface of constant potential, we have come back to a concept we already know because they are identical to the field surfaces.

On a field surface the electric potential has a constant value.

Fig. 1.78 shows a capacitor. The potential of the lower plate is –200 V, that of the upper plate is +400 V. The field surfaces corresponding to integer multiples of hundred volts are also shown.

As the field is homogeneous inside the capacitor, they all have the same distance from each other. Hence, from such a picture the field strength can be read. To see how that works, we compare two homogeneous fields. Fig. 1.79 shows two capacitors that have the same structure: same plate distance, same plate area. But they are differently charged: there is twice as much electric charge on the plates of the right capacitor than on the plates of the left one. Therefore, also the voltage between the plates is twice as high for the right capacitor (why?), and as a consequence also the field strength (why?).

We now look at a small section of each of the two capacitors – indicated with dotted lines in the figure– in an enlarged view, Fig. 1.80. The field surfaces are illustrated in 1-volt steps.

Now the problem that we will analyze: how can the field strength be read from the field surface images of Fig. 1.80?

The question has two parts:

1. How can the direction of $\vec{E}$ be seen in the images?

2. How can the magnitude of $\vec{E}$ be read from the images?

The first question has already been answered: the field strength vector is perpendicular to the field surfaces.

The question about the magnitude is more interesting. Actually, the magnitude can be read from the images as well. The shorter the distance between the field surfaces, the higher the field strength. We choose any two field surfaces. They correspond to a well-defined potential difference $\Delta\varphi$ and they have a well-defined distance d.

The quotient $\Delta\varphi/d$ is a measure of the density of the field surfaces. Its value is equal to the magnitude of the electric field strength:

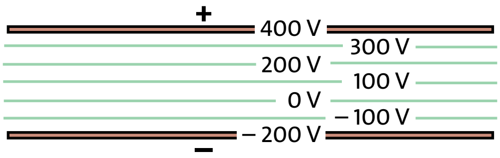


Fig. 1.78 The field surfaces whose potentials differ from each other by 100 V are indicated. As the field is homogeneous, these surfaces all have the same distance from each other.

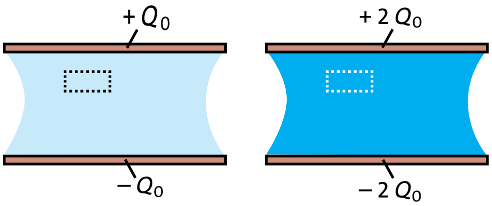


Fig. 1.79 There is twice as much charge on the capacitor plates on the right as on that on the left. Voltage and field strength on the right are also twice as high as on the left.

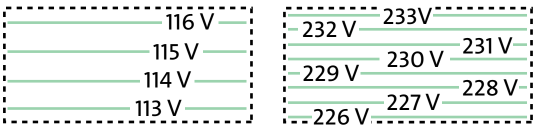


Fig. 1.80 Sections of the capacitor fields from Fig. 1.79. The density of the field surfaces in the picture on the right is twice as large as in the picture on the left.

$$|\vec{E}| = \frac{\Delta\varphi}{d}$$

How to read the electric field strength from a field surface picture:

direction of $\vec{E}$: perpendicular to the field surfaces

magnitude of $\vec{E}$: quotient of potential difference and distance of two field surfaces

As a measuring unit for the field strength, we obtain V/m from the equation, i.e. the unit that we have already used earlier.

We have derived our equation by means of the homogeneous field of a capacitor. The formula is important because it is also valid for any other field, i.e. also for inhomogeneous fields. In an inhomogeneous field as the one shown in Fig. 1.81, the field strength changes from point to point. To determine the field strength at any point P, we look at two field surfaces that are close to P. Also here, the magnitude of the field strength can be obtained as a quotient of the potential difference of the two field surfaces and their distance.

In a homogeneous field, the field strength is the same everywhere. Hence, the field strength can be determined also by using field surfaces that are far away from each other. In the case of a capacitor, we can simply use the plates themselves. Therefore, the field strength of the field in the capacitor is obtained by dividing the voltage *U* between the plates by the plate distance *d*.

Strength of the electric field in a capacitor:

$$|\vec{E}| = \frac{U}{d} \qquad (1.3)$$

Exercises

1. There is a voltage of 2000 volt on a capacitor with a plate distance of $d = 0.5$ cm. What is the electric field strength of the field between the plates?
2. What is the value of the field strength in points P and Q of the capacitor field from Fig. 1.82? (The scale is 10 : 1, i.e. the image is 10 times as large as the actual capacitor.)

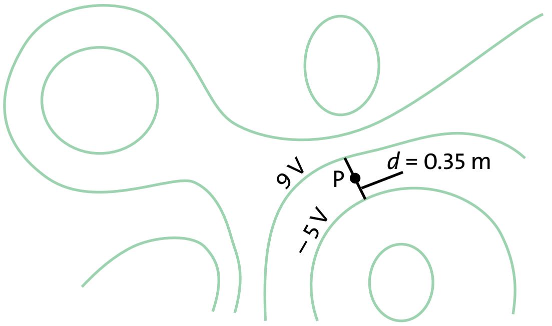


Fig. 1.81 Finding the field strength in point P of an inhomogeneous field: we choose two adjacent field surfaces and divide the corresponding potential difference (here 14 V) by the distance (here 0.35 m, not $d = 0.35$ m drawn to scale).

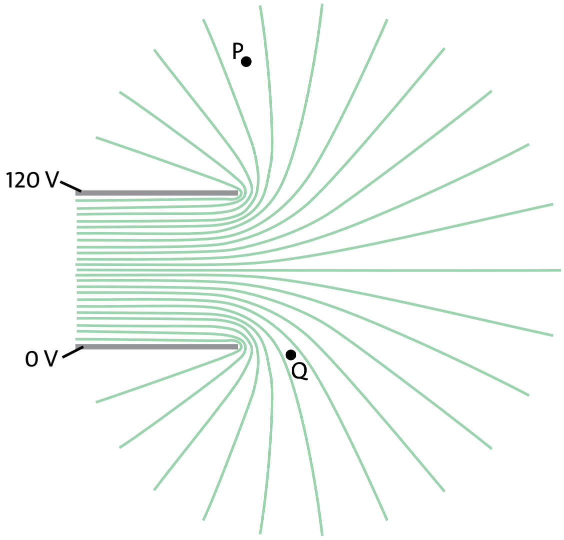


Fig. 1.82 For exercise 2

1.22 More about the capacitor

We increase the plate distance of a capacitor from its original value *d* to $d' = 3d$ and wonder what is going to happen to the capacitance, the charge, the voltage and the electric field strength, Fig. 1.83.

First, let us look at the capacitance. Because of

$$C = \varepsilon_0 \frac{A}{d}$$

the capacitance will decrease from the original value *C* to

$$C' = C/3.$$

Concerning the charge and the voltage, we need to distinguish between two possibilities of realizing the process: we either keep the charge constant or the voltage while changing *d*.

1. Constant charge, $Q' = Q$, Fig. 1.83a.

We make sure that no charge can flow to or from the plates while they are being pulled apart. With $Q =$

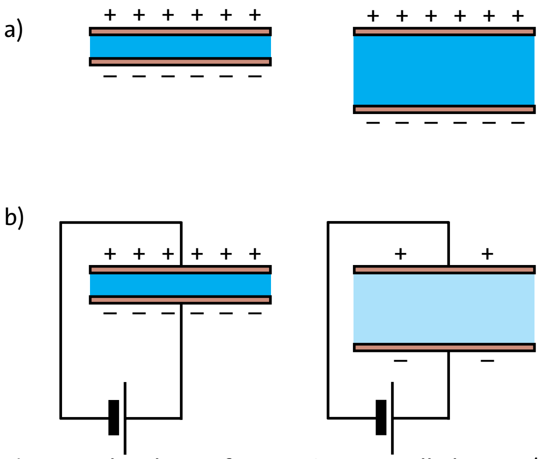


Fig. 1.83 The plates of a capacitor are pulled apart, (a) at constant charge, (b) at constant voltage.

$C \cdot U$ we obtain $U' = 3\,U$. With equation (1.3), we have

$$|\vec{E}'| = |\vec{E}|$$

If the plates of a charged capacitor are pulled apart, the voltage increases and the field strength remains constant.

2. Constant voltage, $U' = U$, Fig. 1.83b.

The capacitor remains connected to the power supply as the plates are pulled apart. With $Q = C \cdot U$ we get $Q' = Q/3$ and with equation (1.3) we obtain

$$|\vec{E}'| = |\vec{E}|/3\ .$$

If the plates are pulled apart at a constant voltage, the charge and the field strength will decrease.

One of our findings is particularly interesting:

> In the case of constant charge, the field strength in the capacitor is independent of the plate distance.

This result can also be expressed this way: if the plate distance is increased while Q remains constant, the quantity of field stuff will increase while its density remains equal.

Exercises

1. Examine how capacitance, charge, voltage and field strength change if the plate area of the capacitor is increased three-fold (while the plate distance is left constant). Distinguish between the cases Q = const and U = const.

1.23 The energy of the electric field

We charge the capacitor in Fig. 1.84 by connecting it briefly to a 6-V power supply. Then, it is connected to a small electric motor. The motor runs for a few seconds.

We see from this experiment how a capacitor can be used: as an energy storage device. While the plates are charged with electricity, an electric field is building up between the plates which contains energy just as a magnetic field. During the charging process, energy flows from the power supply into the capacitor. When discharging, the capacitor releases the energy to the motor.

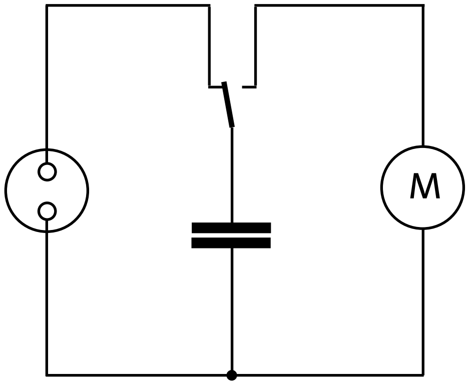


Fig. 1.84 Capacitor as an energy storage device

Therefore, the capacitor can have a similar function as a storage battery (accumulator), because the battery is an energy storage device as well. Just as the battery, the capacitor receives energy during charging with electricity as an energy carrier and releases it in the process of discharging with the same energy carrier. However, it is different from the battery with regard to two characteristics: first, the energy can be brought into the capacitor and removed from it much faster compared to the battery. On the other hand, the storage capacity of a capacitor (for the energy) is much lower than that of a battery with the same size. Some watches that are supplied with energy by means of solar cells have a capacitor as an energy storage device for the time during which the watch is not exposed to light.

In the following, we will calculate the energy that an electric field contains.

We charge a capacitor of capacitance C by letting a constant electric current I flow for a well-defined period of time: from the time $t = 0$ until $t = t_0$. Thereby the charge on the plates of the capacitor increases linearly with time:

$$Q = I \cdot t\ .$$

At the end, i.e. at time t_0, the electric charge is

$$Q_0 = I \cdot t_0$$

on each of the two plates.

In addition, an energy current

$$P = U \cdot I$$

flows into the capacitor during the charging process. While the electric current I remains constant, the voltage U increases during the charging process

$$U = \frac{Q}{C} = \frac{I \cdot t}{C}$$

Hence, we obtain for the energy current

$$P = \frac{1}{C} \cdot I^2 \cdot t.$$

Also the energy current P increases linearly with time, Fig. 1.85.

However, what we look for is not the energy current P, but the energy E_0 at the end of the charging process, i.e. at time t_0. If the energy current was constant in terms of time, we would have the relation

$$E = P \cdot t$$

between the energy current P and the energy E. Thus we would also have

$$E_0 = P \cdot t_0.$$

Since in reality P is not constant in time, we have to use the time average $\overline{P}$ of P to calculate E_0.

$$E_0 = \overline{P} \cdot t_0.$$

As we can read from Fig. 1.85, the average value of P is

$$\overline{P} = \frac{1}{2} \cdot \frac{1}{C} \cdot I^2 \cdot t_0.$$

We thus obtain the energy at the end of the charging process

$$E_0 = \overline{P} \cdot t_0 = \frac{1}{2} \cdot \frac{1}{C} \cdot I^2 \cdot t_0^2.$$

We insert $Q_0 = I \cdot t_0$

$$E_0 = \frac{Q_0^2}{2C}.$$

We had needed the index 0 only to distinguish between the variable and the final value. We now can drop it since the equation is valid for any Q. We thus have the important equation

$$E = \frac{Q^2}{2C}.$$

Notice that this equation has a similar structure as

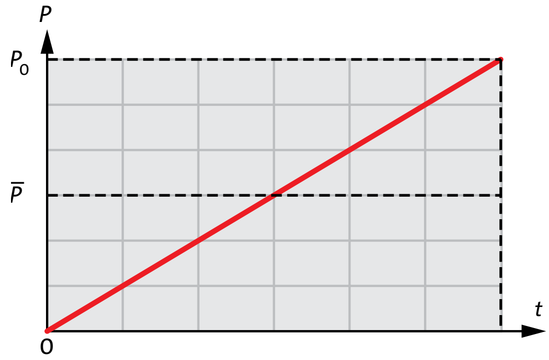


Fig. 1.85 During the charging process of the capacitor with a constant electric current, the energy current increases linearly with time.

$$E = \frac{p^2}{2m}.$$

By using of $Q = C \cdot U$ we can also write:

$$E = \frac{C}{2} U^2. \tag{1.4}$$

This is the total energy contained in the field. We can also calculate the energy density ρ_E of the field, i.e. energy divided by volume:

$$\rho_E = \frac{E}{V}.$$

In (1.4) we insert

$$U = |\vec{E}| \cdot d$$

and

$$C = \varepsilon_0 \frac{A}{d}$$

and obtain

$$E = \frac{\varepsilon_0}{2} \cdot \frac{A}{d} \cdot \left(|\vec{E}| \cdot d\right)^2 = \frac{\varepsilon_0}{2} \cdot |\vec{E}|^2 \cdot V.$$

In the last step, we have replaced the volume V by $A \cdot d$. The energy density now becomes

$$\rho_E = \frac{\varepsilon_0}{2} \cdot |\vec{E}|^2.$$

This equation is not only valid for the field of a capacitor but for any other electric field.

The energy in the field of a capacitor can be calculated from the capacitance and the voltage:

$$E = \frac{C}{2} U^2.$$

The energy density of an electric field can be calculated from the field strength:

$$\rho_E = \frac{\varepsilon_0}{2} \cdot |\vec{E}|^2.$$

Exercises

1. A capacitor with capacitance 16 µF is charged for 8 seconds with an electric current of 10 mA. (a) Which charge will sit on its plates at the end of the charging process? (b) How much energy is contained in the field?
2. An 80-µF capacitor is connected to a 300-volt power supply. How much energy will go into the field of the capacitor in the process? The plate distance is 8 µm. What is the energy density in the field of the capacitor?
3. An energy of 1.6 joule is needed to charge a capacitor to a voltage of 10 000 volt. (a) What is the capacitor's capacitance? (b) What charge sits on the capacitor's plates?
4. The cylindrical plates of a "cylinder capacitor" have the radii 24 mm and 25 mm. The cylinder length is 120 mm. The capacitor was charged to 2000 volt. (a) Calculate the capacitor's capacitance. (b) What is the charge on the capacitor? (c) How much energy is in the capacitor's field? (d) What is the energy density?
5. How will the energy in the field of the capacitor from Fig. 1.83a and b change if the plate distance is increased three-fold? Distinguish between the cases Q = const and U = const. Examine how the energy will change if the area of the capacitor is increased three-fold (while leaving the plate distance constant). Also distinguish between the cases Q = const and U = const.
6. The terminals of a charged capacitor A are connected to those of an uncharged capacitor B. A and B have the same capacitance. What happens? What are the individual values of electric charge, electric voltage, field strength and energy for A and B? Do you notice anything? Try to explain. We came across a similar phenomenon in mechanics (MECHANICS, Ch. 5.4, Exercises 3 and 8).

1.24 Discharge curve of the capacitor

We imagine the capacitor in Figure 1.86 to be charged. (The charging circuit is not shown.) If the switch is closed, it will discharge through the resistor.

We are interested in the discharge process: How fast does the capacitor discharge? What is the temporal variation $Q(t)$ of the capacitor's charge?

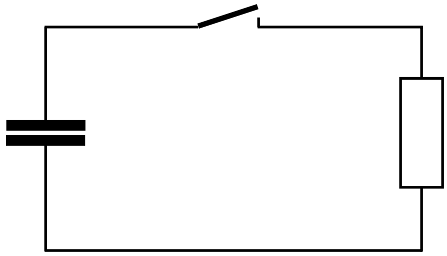


Fig. 1.86 When the switch is closed, the capacitor will discharge through the resistor. What does the function $Q(t)$ look like?

To answer these questions, we need a mathematical method that we have not yet seen. We need to solve a *differential equation*.

It is always a good idea to think about what the solution could look like before starting to calculate. In our case, this means: What do we expect with regard to $Q(t)$?

After we have closed the switch, an electric current flows through the resistor. At the beginning, the voltage is still high, and so is the current because of

$$I = \frac{U}{R}.$$

The charge of the capacitor decreases because the electric current is flowing. Due to

$$U = \frac{Q}{C}$$

the voltage decreases as well. Hence, this also leads to a decreasing current. When the current is reduced, the charge decreases more slowly than before etc. etc. We can see: The less charge there is still sitting on the capacitor, the slower the discharge process.

Before we continue our analysis and calculation, we would like to make a more general observation. There are many processes like the one just mentioned. For the moment, we would like to highlight the particularity of these processes: The more of "something" there is, the faster the amount of this "something" changes.

Here are two examples:

Example: rabbits

We assume that some rabbits are abandoned in an area where there is abundant food and where they are not threatened by enemies. The abandoned rabbits will reproduce, Fig. 1.87. They will have baby rabbits so that there will be more rabbits in the next generation. The children will reproduce themselves and, again, there will be more rabbits. The increase in the second generation is higher than in the first because there are more parents. Hence: *the higher the number of rabbits, the higher the growth rate.*

Example: light in the sea

Sunlight hits the surface of the sea, Fig. 1.88. The water is not completely pure but partially absorbs the light. We assume that half of the light arrives in a depth of 10 m, i.e. half of it was absorbed. In the course of the next 10 m, again half of the remaining light is absorbed, etc., hence: *the lower the quantity of light, the lower the absorbed quantity*.

We use the same method to calculate how the charge on the capacitor decreases, how the population of rabbits increases or how the intensity of the light in the water decreases. We will get to know this method by means of a discharging capacitor.

In the resistor, the electric current I flows from the high to the low potential, in Fig. 1.89 from the top to the bottom. This current causes the positive charge Q of the capacitor (= charge on the upper, positively charged plate) to decrease. Hence:

$$I = -\frac{dQ}{dt}$$

We now replace

$$I = \frac{U}{R}.$$

and

$$Q = C \cdot U$$

and obtain

$$\frac{U}{R} = -C\frac{dU}{dt}.$$

We slightly reformulate the equation and obtain the differential equation for the discharge of the capacitor through a resistor:

$$U + RC\frac{dU}{dt} = 0.$$

What can we do with such an equation? Apart from the resistance R and the capacitance C, whose values we assume to be known, the equation contains the voltage U and its time derivative dU/dt. U is a function of time. What we look for is the function $U(t)$. We would have to solve the differential equation somehow for U. This, however, is not possible with the old methods that you know because the equation not only contains U directly but also its time derivative.

Such an equation can be solved in different ways. We would like to address two methods here:

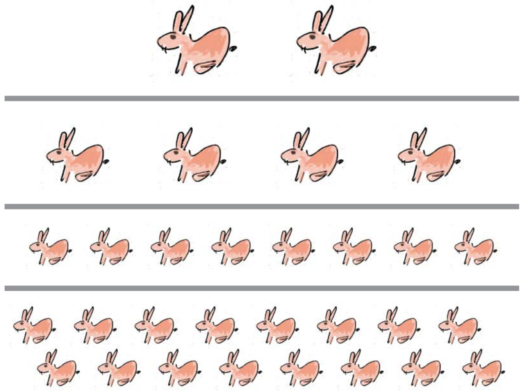


Fig. 1.87 Four generations of rabbits: the higher their number, the faster their population grows.

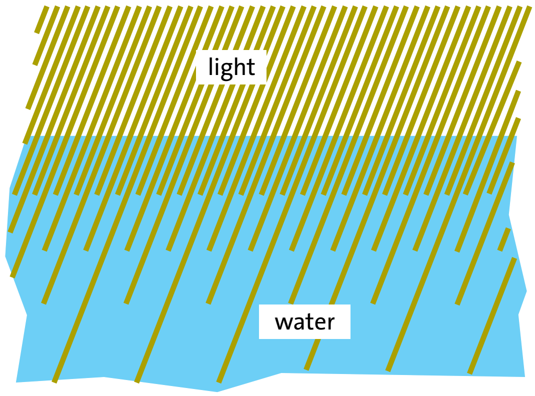


Fig. 1.88 The weaker the intensity of the light, the lower its decrease as a function of the water depth.

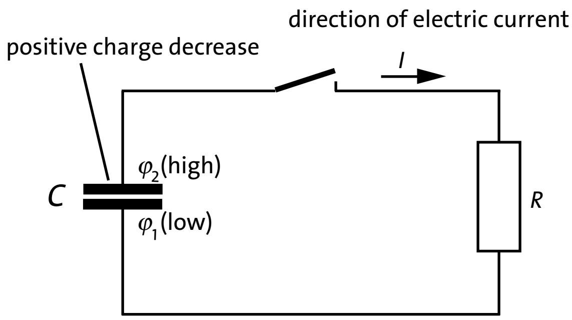


Fig. 1.89 A positive current causes a reduction of the capacitor's charge.

1. Numerical solution

There are computer programs to perform this task. They calculate the function $U(t)$ point by point on the time axis. As a result, we will not obtain a functional expression but a table of values or a diagram.

2. Guessing of a solution and inserting

In many cases, we already have a rough idea of how the solution could look like. This is also the case here. We know: the lower the charge, the more slowly it decreases. Such a behavior is described by an exponential function

$$y(x) = e^{-x}.$$

The greater x, the smaller the change dy/dx. Hence, we try the following solution:

$$U(t) = U_0 \cdot e^{-t/\tau}. \qquad (1.5)$$

The factor U_0 is needed to ensure equal measuring units on the right and on the left. This applies accordingly for the exponent: it must be "dimensionless", i.e. without a measuring unit. We therefore divide the time t by the physical quantity τ that is also measured in seconds.

To be able to check whether our trial function solves the differential equation, we also need the derivative of $U(t)$:

$$\frac{dU}{dt} = -\frac{1}{\tau} \cdot U_0 \cdot e^{-t/\tau}. \qquad (1.6)$$

We insert (1.5) and (1.6) in the differential equation:

$$U_0 \cdot e^{-t/\tau} - \frac{RC}{\tau} \cdot U_0 \cdot e^{-t/\tau} = 0.$$

We divide by

$$U_0 \cdot e^{-t/\tau},$$

reformulate and obtain:

$$\tau = R \cdot C\,.$$

What does this mean? Is our trial solution correct? Yes, it is. This becomes evident by the fact that the time dependency has vanished. The differential equation is solved by the function that we had guessed:

$$U(t) = U_0 \cdot e^{-t/\tau}.$$

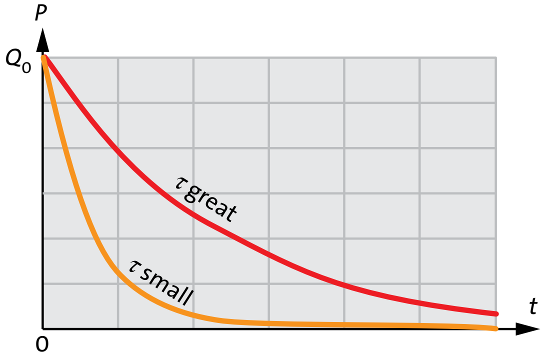


Fig. 1.90 A great time constant τ means: the charge on the plates of the capacitor decreases slowly.

In addition, the calculation has shown that U_0 can have any value and that τ is equal to $R \cdot C$. The meaning of τ is illustrated in Fig. 1.90: the greater τ, the more slowly the function levels off, the more slowly the capacitor discharges.

This is also in line with our expectation: the capacitor discharges slowly when its capacitance is high and when the resistance is high. τ is called *time constant*.

> *Discharge of a capacitor through a resistor*
> *Voltage decreases exponentially:*
>
> $$U(t) = U_0 \cdot e^{-t/\tau}.$$
>
> $\tau = R \cdot C =$ time constant

Exercises

1. Use a quadratic function as a trial solution for our differential equation. Is the differential equation satisfied? Comment.
2. So-called conductor spheres are often used for electrostatic experiments. A conductor sphere consists of a metallic conductor and a Perspex shaft – you will remember. If the sphere is electrically charged, the charge will remain on it for a while. Estimate how fast it will flow away, i.e. what the time constant will be. (Help: estimate the capacitance of the sphere against the Earth, as well as the resistance of the Perspex shaft, by means of the respective formulas.) The result has to be exact only to within a factor 1000; in other words: is the time constant approximately:
10^{-9} s, 10^{-6} s, 10^{-3} s, 1 s, 10^{3} s, 10^{6} s or 10^{9} s?

1.25 Fields and electric conductors

A long wire is connected to a power supply so that an electric current is flowing through it. If we move alongside the wire, starting from the high potential terminal, the potential will decrease with the distance traveled, Fig. 1.91. The potential changes linearly with the position on the wire.

Fig. 1.92 shows an enlarged section of the wire. The field surfaces, i. e. the surfaces of constant potential, intersect the wire perpendicularly to its longitudinal direction. This implies that the field lines inside the wire run parallel to the direction of the wire. At places where the wire is straight, the field is exactly homogeneous. In places where the wire is bent, the field lines follow the curvature.

Hence, the electric charge that flows through the wire follows the potential slope, always in the direction of the field lines.

We can also reverse the conclusion: when an electric current flows in a conductor, there must be a potential slope and consequently an electric field. When there is no electric current in a conductor, the field strength in the conductor must be zero and there is no potential slope – the potential is the same everywhere.

> In a conductor in which no electric current is flowing, the potential is the same everywhere.

Now we bring a conductor, a metal sphere for instance, to a place where there used to be an electric field, Fig. 1.93a. While being put at that place, electric charge will be displaced in the sphere. We referred to this process as induction. The movement of the charge, however, comes to an end after a very short time. There will be no electric current anymore. This means that the inside of the ball is now field-free. The whole sphere is at the exact same potential.

Especially the surface of the sphere is a surface of constant potential, i.e. a field surface. This implies in turn that the field lines on the outer surface of the sphere are perpendicular to the surface.

Nothing of these statements will change in cases where the electric conductor is hollow, Fig. 1.93b. This has an interesting consequence: electric fields can be shielded by means of metal walls. Therefore, the walls even need not to be tightly sealed. A wire netting is often sufficient.

Fig. 1.94 shows a cylinder made of wire netting that is connected to a Van de Graaff generator. Its potential is therefore several 10 000 V above the potential of the Earth, and there is a strong electric field between the wire netting and the Earth. It can be detected thanks to

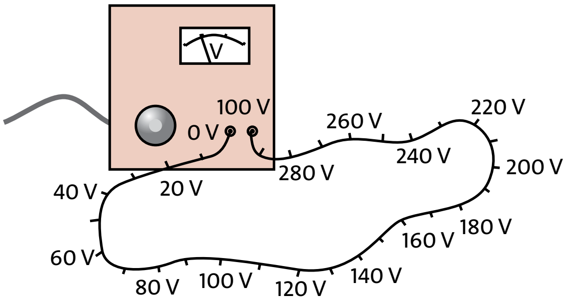


Fig. 1.91 The potential in the wire decreases linearly on the way from one to the other terminal of the power supply.

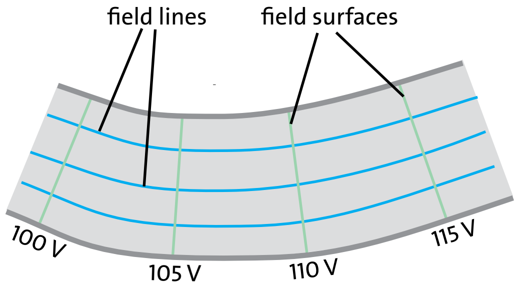


Fig. 1.92 The field surfaces are perpendicular to the direction of the wire, the field lines follow the wire.

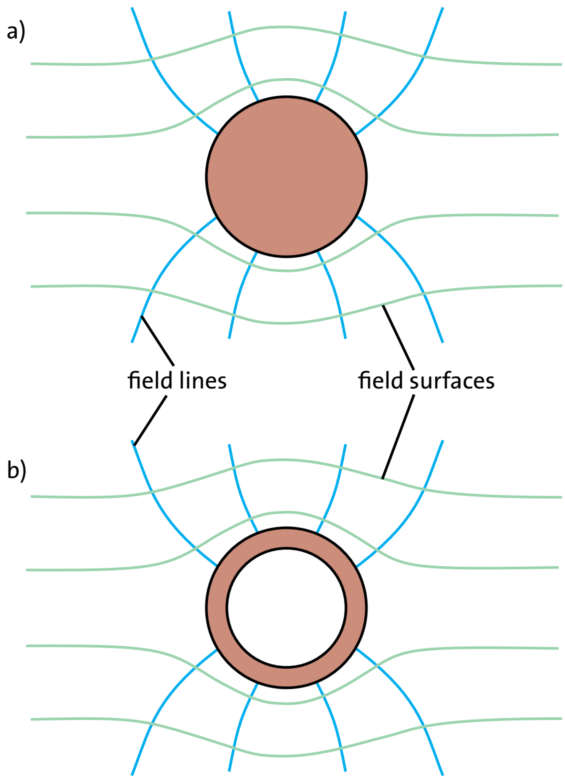


Fig. 1.93 (a) The electric potential is the same at all points inside the metal sphere. (b) Also a hollow sphere is field-free inside.

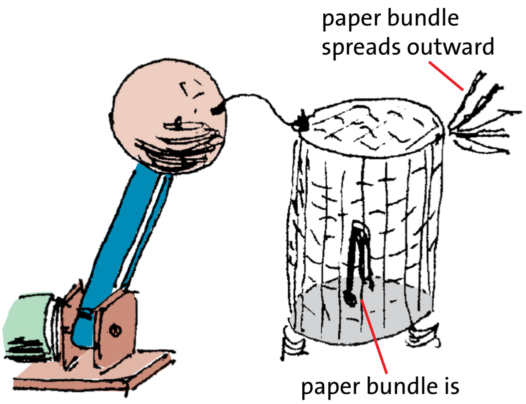


Fig. 1.94 The inside of the wire cage is field-free.

the paper bundle whose ends are drawn outwards. A paper bundle inside the wire netting, in contrast, does not move. Although this space is on a very high potential, it is field-free. There, the potential is the same everywhere.

Exercises

1. A conductor, through which an electric current is flowing, becomes thicker at one place, Fig. 1.95. Sketch field surfaces and field lines inside the conductor. Pay attention to the distances between the field surfaces.
2. A metal plate is brought between the plates of a charged capacitor in parallel to the capacitor plates, Fig. 1.96. Draw field surfaces and field lines. What is the mechanical stress in the metal plate: tension or compression? Which direction does it have?
3. A metal sphere with a diameter of about 1 cm is brought to the point P of the field from Fig. 1.97. How will the field surfaces and field lines change?
4. A thin metal plate is put to the place that is marked with a dotted line between the two charge carriers in Fig. 1.98. Sketch field surfaces and field lines before and after.

1.26 The electric current density – Ohm's law locally

Fig. 1.99 is an enlarged view of a wire through which an electric current is flowing. We assume the current to be 6 A. At point P, the cross-sectional area of the wire increases from 2 mm^2 to 8 mm^2. We look at a small area within the wire, located perpendicularly to the direction of the wire, at position A where the wire is thin, as well as at position B where it is thick. How does the electric current differ between the two areas? The current flowing through the area at A is four times the current flowing through the area at B. Only in this

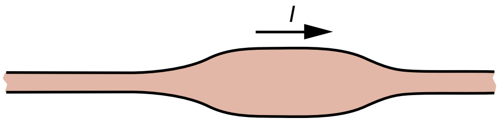


Fig. 1.95 For exercise 1

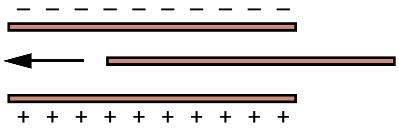


Fig. 1.96 For exercise 2

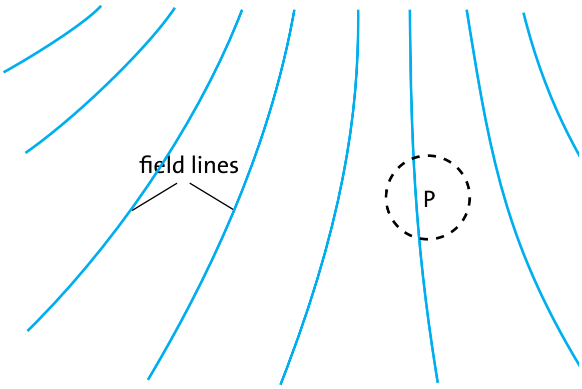


Fig. 1.97 For exercise 3

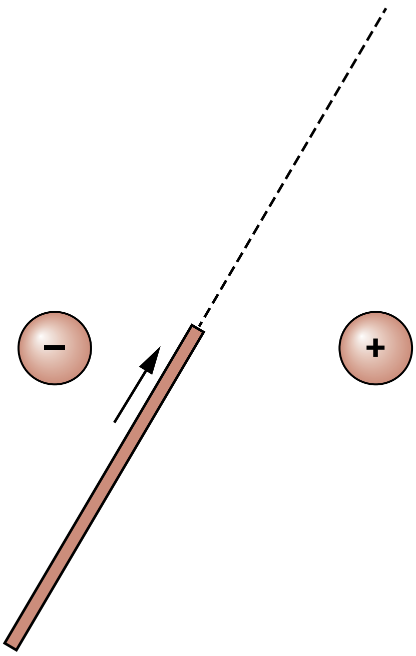


Fig. 1.98 For exercise 4

way the same current can flow through the total cross-sectional area at A and at B.

What we have just compared is the *electric current density j*. The electric current density is the electric current in a conductor divided by the cross-sectional area of the conductor:

$$j = \frac{I}{A}$$

(The momentum current density was defined in an analogous way: momentum current divided by the cross sectional area of the momentum conductor; see section 1.19.)

Hence, the electric current density in the thin area of our wire is

$$j_{\text{thin}} = \frac{6\ \text{A}}{2\ \text{mm}^2} = 3\ \text{A/mm}^2$$

and in the thick part

$$j_{\text{thick}} = \frac{6\ \text{A}}{8\ \text{mm}^2} = 0.75\ \text{A/mm}^2.$$

While the current tells us the total electric charge that is flowing through a wire cross-section per time, the current density tells us how much is flowing "at one point".

We would now like to change the form of Ohm's law

$$R = \frac{U}{I}.$$

First, we bring the current to the left side of the equation:

$$I = \frac{U}{R}$$

Next, we insert the expression for the resistance

$$R = \frac{1}{\sigma} \cdot \frac{d}{A}$$

and obtain:

$$I = \frac{\sigma A}{d} \cdot U.$$

A simple rearrangement leads to

$$\frac{I}{A} = \frac{\sigma}{d} \cdot U.$$

This equation can be simplified though: on the left side, there is the electric current density I/A, and on the right there is the electric field strength U/d besides the conductivity σ. Thus, we can write: $\vec{j} = \sigma \cdot |\vec{E}|$.

Now, not only the electric field strength, but also the current density is a vector. Its direction is equal to the direction of the electric current. Our final equation is therefore:

$$\vec{j} = \sigma \cdot \vec{E} \qquad \text{Ohm's law locally}$$

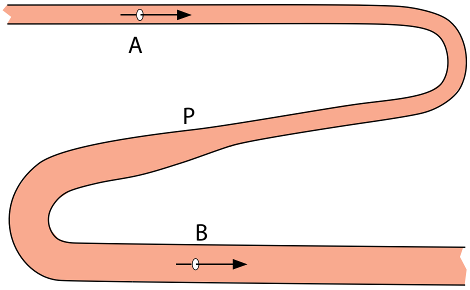


Fig. 1.99 Wire in which an electric current is flowing. The cross-sectional area increases four-fold at P. The electric current density is thereby four times as high at A as at B.

What we have derived with some effort is basically nothing new. We can say that it is the "local" form of Ohm's law. Ohm's law in its usual form tells us: the electric current is proportional to the voltage, or in short: the current is proportional to the driving force. Exactly the same is expressed by our new equation, too. Only that here, we use the voltage per length, i.e. the electric field strength, as a measure for the driving force, and the current per area, i.e. the current density, as a measure for the current. While Ohm's law in its old form provides a statement about the whole wire, our new, local law gives evidence about a given place within the wire.

Exercises

1. A copper wire with a length of 100 m and a cross-section of 1 mm^2 is connected to a 1.5 V battery with both of its ends. What is the field strength in the wire? What is the current density? What is the current in the wire?
2. An electric current of 5 Ampere flows through a copper wire of 3 mm^2. What is the field strength in the wire? Compare with the field strength in a capacitor with the plate distance of 5 mm that was charged with 1 kV.
3. A copper wire with a length of 2 m is connected to an iron wire with a length of 1 m (the cross-section of both wire pieces is 1 mm^2). An electric current of 0.5 A flows through the entire wire of 3 m length. What is the field strength in the copper wire and in the iron wire? What is the potential difference between the ends of the copper wire and between the ends of the iron wire?
4. The filament of a light bulb is a bit thicker than normal at a place A and a bit thinner than normal at a place B. What can you say about the current densities at places A and B while the lamp is lit up? At which point will the filament burn through at a given time?

1.27 How to load electrically charged particles with energy – electron beams

We remember an earlier result: To bring a portion of electric charge ΔQ in a conductor from a place of low potential φ_1 to a place of high potential φ_2, we need energy. If the charge portion moves from the high to the low potential, it receives energy. The amount of this energy is:

$$\Delta E = (\varphi_2 - \varphi_1) \cdot \Delta Q$$

We can now omit the restriction of the charge having to move in an electric conductor. The equation is also valid if a charged body or particle moves outside of electric conductors. The charged particle in Fig. 1.100 can be a paper snippet that was charged through the contact with plate 2.

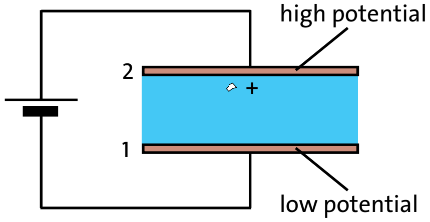


Fig. 1.100 The charged particle absorbs energy on its way from the high to the low potential. This energy is supplied by the battery.

Where does the energy that is supplied to the paper snippet on the way from plate 2 to plate 1 come from? Plate 2 has lost a charge portion and plate 1 has received this charge. Due to the movement of the charge portion, the capacitor would lose a part of its charge. The missing charge, however, is supplied in compensation by the battery. Hence, the energy that the charge portion receives comes from the battery.

> To bring a positively charged body from the low to the high potential, it must be provided with energy. When it moves from the high to the low potential, it will recover the energy.

(In case of a negative charge, the statement of the sentence will be reversed: to bring a negatively charged body from the high to the low potential, it has to be provided with energy. When it moves from the low to the high potential, it will recover the energy.)

Fig. 1.101 shows nearly the same experiment as Fig. 1.100. The only difference: the charged plates are not connected to the battery. However, the experiment still works the same way. The charged paper snippet moves from the plate with the high potential to the one with the low potential. Where does the energy now come from? The moving particle transports electric charge from one to the other plate. The capacitor is thereby discharged to a certain extent. However, this means that the field between the plates becomes a little weaker. Hence, the energy comes from the electric field.

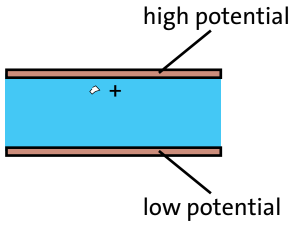


Fig. 1.101 The charged particle absorbs energy on its way from the high to the low potential. This energy is taken from the electric field between the plates.

When a charged body moves from the high to the low potential and when it cannot give away the energy that it receives, it needs to keep the energy. It will become faster, its *kinetic energy* increases.

Therefore, we have found a possibility to load a body with kinetic energy. Although this method is not suitable for macroscopic, i.e. very large, bodies, it is extremely effective for microscopic bodies, i.e. the so-called particles. This is shown by the following example: How much energy does an electron gain when it moves from an "electrode" on Earth potential (0 V) to an electrode on +20 000 V? (As the electron is negatively charged, it will gain energy when it moves from the low to the high potential.)

With $Q = -1.6 \cdot 10^{-19}$ C we obtain

$$E = (0\text{ V} - 20\,000\text{ V}) \cdot (-1.6 \cdot 10^{-19}\text{ C}) = 3.2 \cdot 10^{-15}\text{ J}$$

This value does not appear to be very high at first. When calculating the associated speed, however, we can see that it is indeed very high for the small electron. From

$$E = \frac{m}{2} v^2$$

we obtain

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \cdot 3.2 \cdot 10^{-15}\text{ J}}{0.91 \cdot 10^{-30}\text{ kg}}} \approx 8.4 \cdot 10^7\text{ m/s}.$$

This is approximately a quarter of the speed of light, i.e. the highest existing transportation speed.

When charging particles with energy in this way, their energy is usually not indicated in the measurement unit joule but in electron volt, abbreviated eV.

Here, the "e" simply stands for the elementary charge, i.e.

$$1\ \text{eV} = -1.6 \cdot 10^{-19}\ \text{C} \cdot \text{V} = -1.6 \cdot 10^{-19}\ \text{J}.$$

> 1 eV = energy that a particle with the charge $1.6 \cdot 10^{-19}$ C absorbs while going through a potential difference of 1 V.
>
> $1\ \text{eV} = 1.6 \cdot 10^{-19}\ \text{J}$

What is the reason for this deviation from the SI system? When dealing with particles that all carry the positive or negative elementary charge such as electrons or protons or positrons, the eV is simply more convenient. If, for example, electrons are accelerated over a potential difference of 8 kV, their kinetic energy at the end will be 8 keV. Hence, no calculation is needed to obtain the energy.

Exercises

1. A water drop charged with 50 negative elementary charges (mass: 10 mg) falls through a potential difference of 20 million volt in a thundercloud that is 1000 meters thick. (The potential decreases towards the bottom.) How much energy does it take from the gravitational field? How much energy does it release to the electric field? Its kinetic energy has not changed while falling. Why? Where did the excess energy go?
2. In a Van de Graaff generator, electric charge is continuously "pumped" from Earth potential up to 50 000 volt. We suppose that the electric current that is generated by the generator is 50 μA. What is the resulting energy consumption of the Van da Graaff generator? (In fact, it needs much more. Most of the energy is lost through friction.)
3. The electrons in an electron microscope are accelerated with a voltage of 1.2 MV. (a) What is the energy of the electrons? (b) Calculate the speed of the electrons with the formula for the kinetic energy. How can we see that the result must be wrong? Which step of the calculation was wrong?

2 THE MAGNETIC FIELD

2.1 Magnetic charge and magnetic field

Magnets can attract or repel each other. The attraction and/or repulsion is due to the magnetic charge Q_m. The places of the magnet where the magnetic charge is sitting are called *poles* of the magnet. The measurement unit of the magnetic charge is the Weber (Wb).

Just as the electric charge, the magnetic charge can take on both positive and negative values. Areas with a positive magnetic charge are called *north poles*, areas with a negative magnetic charge *south poles*.

A small magnet as the one we use to pin something on an iron wall has a charge of approximately 10^{-4} Wb at its positive pole.

If a bar-shaped magnet, whose poles are located on its ends, is suspended horizontally on a thin thread so that the magnet can rotate easily, it will align itself in a north-south direction. The positive pole points to the North, the negative one to the South.

The overall magnetic charge of a magnet is always zero, i.e. the positive charge has the same absolute value as the negative one.

> The total magnetic charge of a magnet is zero.

This is different from in case of the electric charge. A body can be given an electric net charge (albeit only a very small one). This difference between electric and magnetic charge is very important. It means that there are electric currents (flowing electric charge) but no magnetic currents (flowing magnetic charge).

An invisible entity is attached to the magnetic charges: the *magnetic field* (just as the electric field is attached to electric charges). We call the "substance" that the magnetic field consists of *magnetic field stuff*. If there is no risk of confusion with the electric field stuff, we can also call it just field stuff.

> Magnetic poles are surrounded by magnetic fields. If the charge of two poles has the same plus/minus sign, the field will pull the poles apart; if it has different plus/minus signs, the field draws them towards each other.

In conventional magnets, the magnetic charge usually sits on the surface. This can be shown in the following way, Fig. 2.1.

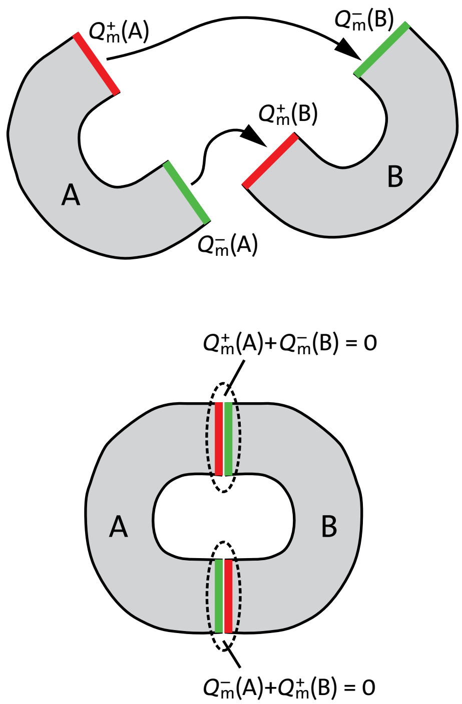


Fig. 2.1 If the two identical magnets A and B are brought together in a way that the north pole of A joins the south pole of B, and the south pole of A joins the north pole of B, the magnetic charge at the two contact areas will add up to zero.

We take two identical horseshoe magnets A and B. For each magnet individually, the charge Q_m^- of the south pole is equal to the negative value of the charge Q_m^+ of the north pole:

$$Q_m^- = -Q_m^+ \qquad (2.1)$$

As the two magnets are identical, the positive charge at the north pole of one is equal to the positive charge at the north pole of the other:

$$Q_m^+(A) = Q_m^+(B) \qquad (2.2)$$

In addition, we have $Q_m^-(A) = Q_m^-(B)$. If the two magnets are held together in a way that the north pole of A joins the south pole of B and that the south pole of A joins the north pole of B, the charges at the contact areas will add up to zero.

The charge at one contact area is $Q_m^+(A) + Q_m^-(B)$.

By means of equation (2.1), we can replace $Q_m^-(B)$ by $-Q_m^+(B)$:

$$Q_m^+(A) - Q_m^+(B)$$

This expression, however, is zero according to equation (2.2). Thus, the new, ring-shaped magnet that we created by combining the two horseshoe magnets has no magnetic field. Can the ring be even be regarded as a magnet? We will answer this question in the following section.

2.2 Magnetization

There are many different magnetic materials: chemical elements, compounds, alloys and ceramic materials. The most well-known and most frequently used material, albeit not the best, is iron.

There is a simple explanation for the fact that a magnet always carries equal amounts of positive and negative magnetic charge: some atoms are magnetic, i.e. each atom behaves like a small permanent magnet with two poles. If an object consists of such atoms and if, in addition, the atomic magnets do not have a completely irregular orientation, Fig. 2.2a, but if they are regularly aligned, a large magnetic pole will emerge at each of the two ends of the magnet.

Of course, the small atomic magnets do not have to be all parallel to each other. They could, for instance, have an orientation as illustrated in Fig. 2.3.

When the atomic magnets are aligned, we refer to the material as being *magnetized*.

In Figures 2.2 and 2.3, we have illustrated the state of magnetization by means of many small arrows. A somewhat clearer graphical illustration method is shown in Fig. 2.4a. Here, continuous lines, the *magnetization lines*, were drawn instead of the arrows. The figure shows the relationship between magnetization and magnetic charge: the magnetic charge sits where the magnetization lines end at the surface.

The magnetization lines, however, can also be located inside the magnet without start and end points. The respective magnet has no poles in that case. The ring, that we had created out of two horseshoe magnets, Fig. 2.1., exemplifies this case.

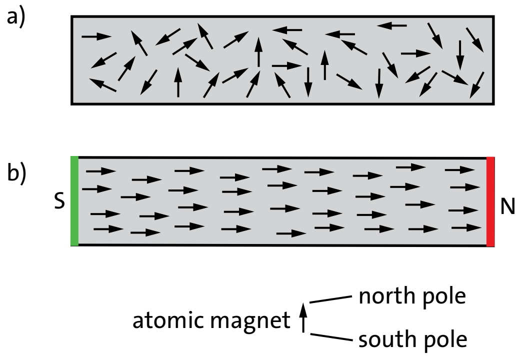


Fig. 2.2 (a) In a non-magnetized piece of iron, the directions of the small atomic magnets are not aligned. (b) In a magnetized piece of iron, the atomic magnets are aligned. There is a negative charge at the left end of the magnet and a positive charge at the right end.

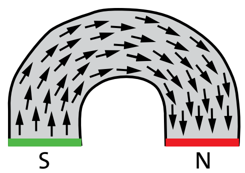


Fig. 2.3 Orientation of the small atomic magnets in a horseshoe magnet.

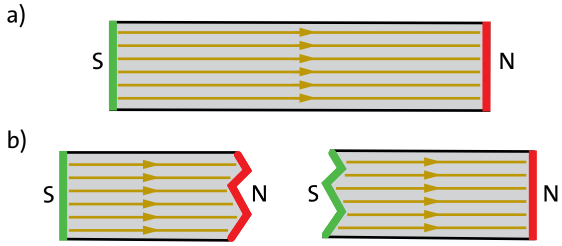


Fig. 2.4 (a) Graphical display of the state of magnetization by means of magnetization lines. (b) New poles are formed when the magnet is broken.

Magnetization lines start on negative and end on positive magnetic charge, or they run within a magnet without start and end points.

The magnetization lines are usually marked with a directional arrow. By convention, they are oriented from the negative to the positive charge.

Now we can also understand another interesting phenomenon. When breaking a bar-shaped magnet, two new poles emerge at the fracture surfaces, Fig. 2.4. This is the only way to ensure that the overall magnetic charge of each of the two pieces will be zero again.

If the magnetization is known, we can tell unambiguously where the magnetic charge sits: at the point where the magnetization lines start and end.

If, by contrast, only the poles of a magnet are known, i.e. the distribution of the magnetic charge on its surface, no clear conclusion can be drawn about magnetization yet. The charge distribution from Fig. 2.5a is compatible with both the magnetization distribution from Fig. 2.5b as well as with that of Fig. 2.5c. The actual magnetization status cannot be seen from the outside of the magnet. A method to differentiate between the two possibilities consists in breaking the magnet. Breaking of the magnet from Fig. 2.5b does not lead to the formation of new poles, Fig. 2.5d. If, however, the magnet from Fig. 2.5c is broken, new north and a south poles will emerge, Fig. 2.5e.

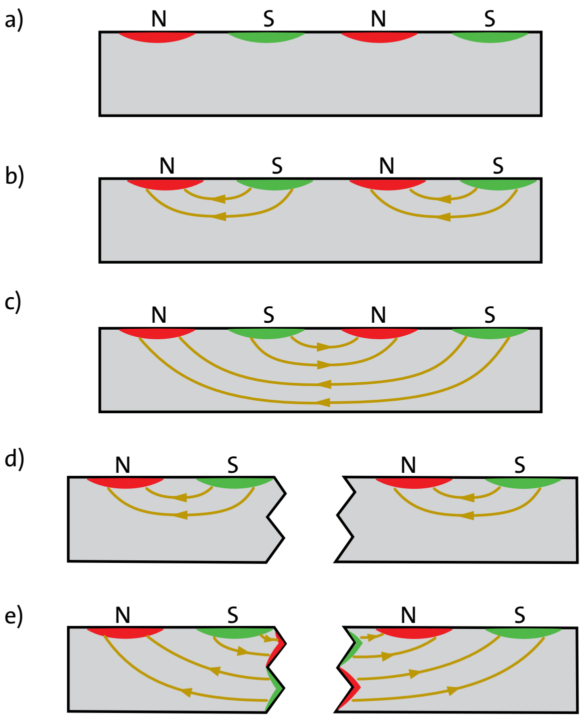


Fig. 2.5 (a) A magnet with 4 poles on one side. (b) and (c) There are several possibilities for the magnetization. (d) and (e) The difference between b and c can be seen if the magnet is broken in the middle.

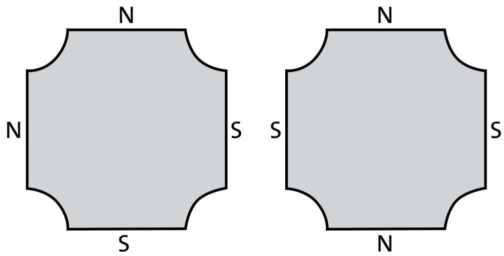


Fig. 2.6 For the exercises 1 and 2. What is the course of the magnetization lines?

Exercises

1. How could be the course of the magnetization lines in the magnet from Fig. 2.6a?
2. How could be the magnetization lines in the magnet from Fig. 2.6b? Indicate two solutions.
3. A magnet has the shape of a cylindrical slice. On its cylindrical surface, the magnet has 3 north and 3 south poles. North and south poles are alternating and are distributed regularly over the circumference of the cylinder. What could the magnetization of the cylinder be like? Indicate two solutions.
4. Someone gives you a steel ring and tells you that the ring is magnetized in a way that the magnetization lines follow the ring shape. Hence, the magnet has no poles. How can you find out whether the statement is true?

2.3 The magnetic field strength

We will now introduce a measure for the magnetic field stuff: the *magnetic field strength*. We use a method which is analogous to that for the introduction of the electric field strength. Also in a magnetic field, there is tensile stress at each point in a given direction, and compressive stress in all directions that are perpendicular to it. Thus, the magnetic field strength is a vector that is oriented in the direction of the field's tensile stress. Its magnitude tells us how dense the field is at the respective point.

The magnetic field strength is a vector.
Magnitude of the vector: measure for the density of the field stuff.
Direction of the vector: tensile direction of the field stuff.

Also the measurement process is analogous to that of the electric field strength. A magnetic "test charge", i.e. one pole of a very thin and long bar magnet, is brought to the point P at which the field strength is to be measured. (The other pole of the magnet is so far away that it does not feel the field anymore.) Then, a momentum current flows into this magnetic pole. By means of a momentum current meter, we determine magnitude and direction of the momentum current vector. Then, we divide by the magnetic charge of the pole and obtain the magnetic field strength:

$$\vec{H} = \frac{\vec{F}}{Q_m}.$$

We transform and obtain:

$\vec{F} = Q_m \cdot \vec{H}$
$\vec{F}$ = momentum current into the pole
Q_m = magnetic charge of the body
$\vec{H}$ = magnetic field strength

The equation has the same structure as two other equations we are already familiar with:

$$\vec{F} = Q \cdot \vec{E} \text{ and } \vec{F} = m \cdot \vec{g}.$$

If the magnetic charge of the pole is doubled, the momentum current will also double. The quotient of the momentum current and the charge is independent of the charge of the pole. And this is how it should be. Although the pole changes the originally existing field considerably, the equation still tells us the field strength of the field without it.

As a measurement unit of the magnetic field strength we obtain Newton/Weber. The unit can be transformed into the more common unit ampere/meter.

The measurement process for the magnetic field strength that we have just described is practically used as rarely as the corresponding method to measure the electric field strength.

Magnetic field strengths can be measured in a particularly convenient way by using a device whose functionality we can not yet understand right now but whose way of use can be easily described. The flat sensor equipped with a handle is connected to the display unit through a cable. To measure the magnetic field strength at a given point of a field, the sensor is held at that point. The instrument displays a certain value. This value, however, still depends on the orientation of the sensor surface. Hence, the sensor is turned in the different directions until the display has reached its highest value, Fig. 2.7. This is how we do not only know the magnitude of the field strength, but also the direction: the field strength vector is perpendicular to the sensor surface.

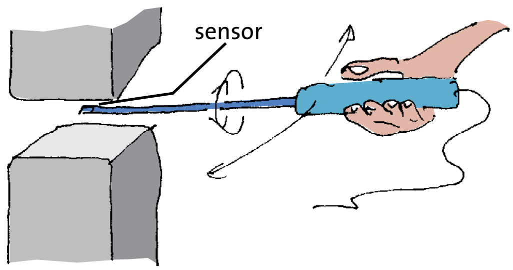


Fig. 2.7 The sensor is turned until the value displayed by the meter is highest. This is how the direction of the magnetic field strength can be determined in addition to its magnitude.

For the graphical display of a magnetic field we can use the same methods as for the illustration of electric fields: with different gray shadings, with vector arrows or with field lines and field surfaces. For the drawing of field lines and field areas of magnetic fields, the same rules apply as for the drawing of field lines and field surfaces of electric fields:

The field lines are perpendicular to the field surfaces at each point.

The field lines start at positive and end at negative poles.

The greater the magnetic charge, the more field lines start or end at a pole.

Field lines do not intersect.

Field surfaces do not intersect.

Field lines and field surfaces do not form any kinks.

A field image has the same symmetry as the magnetic charges.

Also the vector addition of magnetic field strengths has the same meaning as that of electric field strengths. Consider two magnets A and B. We suppose that the field strength of the field of A alone is known for each point, just as that of the field of B alone. If we place the two magnets next to each other, there will be a field that is different from the field of A alone and different from the field of B alone. The field strength of the resulting is obtained by means of vector addition of the field strengths of the individual magnets.

Exercises

1. A compass needle with a length of 5 cm is hold perpendicularly to the magnetic field of the Earth. The positive pole of the needle carries a magnetic charge of 10^{-5} Wb, the negative one carries -10^{-5} Wb. (a) Which momentum current flows over the magnetic field to the positive pole; which momentum current flows to the negative pole? (The magnetic field strength of the Earth field is 6.4 A/m.) (b) How does the compass needle react?
2. Draw the field lines in the Figures 2.8a and 2.8b. (The Figures show the field surfaces.)

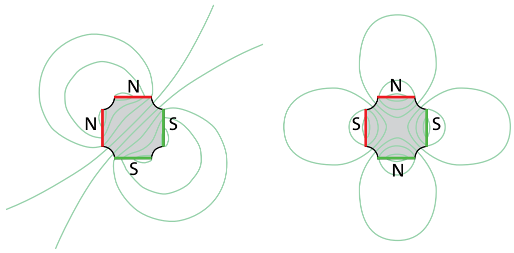


Fig. 2.8 For exercise 2

2.4 Magnetization lines and field lines

We have seen that both the state of magnetization of a material as well as the magnetic field can be graphically displayed with lines: the material by means of magnetization lines, the field through field lines. We would now like to display both methods in one single figure. We therefore recall the rules: magnetization lines start at the negative pole and end at the positive one, magnetic field lines start at the positive pole and end at the negative one. We can summarize these rules:

> Magnetic field lines start where magnetization lines end, and vice versa.

As an example, we look at an individual positive magnetic pole, Fig. 2.9, which can be the end of a long bar magnet. The negative pole of the magnet shall be located so far away that its field cannot be felt at the place of the positive pole anymore.

Please bear in mind that the field lines run through the hard magnetic material as they run through an empty space.

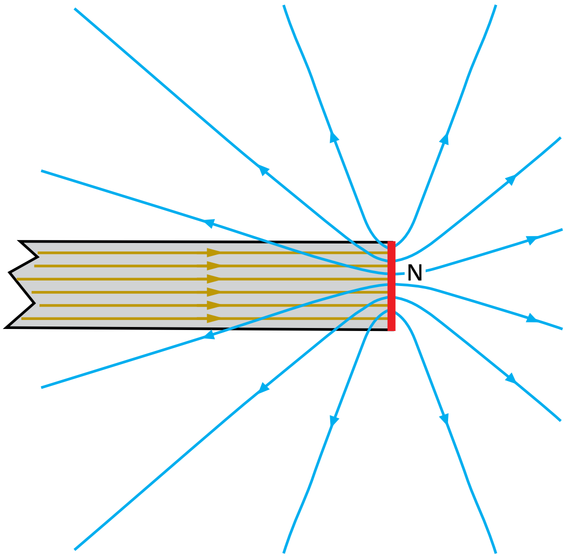


Fig. 2.9 The positive end of a long bar magnet with magnetization lines and field lines

2.5 Four important magnetic fields

Each figure consists of two parts. The first part shows (besides the magnet) the magnetization lines, the second one shows field lines and field surfaces. It would be nicer to illustrate magnetization lines, field lines and field surfaces in one single image. As, however, there are generally both magnetization lines as well as field lines and field surfaces at the same point inside magnetized bodies, such a display would be confusing.

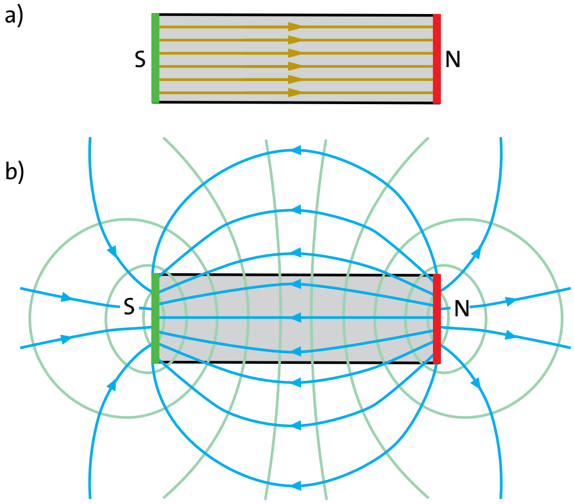


Fig. 2.10 Bar magnet. The magnetic charge sits on the end surfaces. (a) Magnetization lines; (b) Field lines and field surfaces

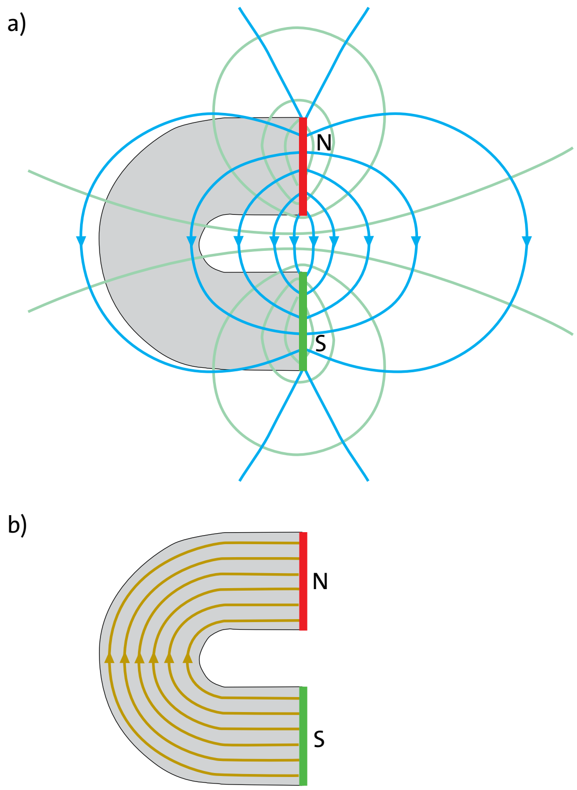


Fig. 2.11 Horseshoe magnet. The magnetic charge sits on the end areas. (a) Magnetization lines; (b) Field lines and field surfaces

1. Bar magnet

The magnetic charge sits on the end surfaces. The magnetization, Fig. 2.10a, is homogenous; the field, Fig. 2.10b, is not. Such a magnet is also called a magnetic dipole.

2. Horseshoe magnet

The magnetization lines follow the shape of the magnet; the magnetic charge sits on the end surfaces again, Fig. 2.11.

3. Magnetic ring with gap

The magnetic charge sits on the plane surfaces on both sides of the gap, Fig. 2.12a. In figure 2.12b, the gap area is displayed with field lines and field surfaces in an enlarged view. The field is essentially limited to the gap area and it is almost homogeneous. Field lines and field surfaces of this magnetic field have the same shape as the field lines and field surfaces of the electric field of a capacitor.

4. Disc-shaped magnet

It has precisely the shape of the missing ring piece of the ring magnet from Fig. 2.12, Fig. 2.13. The mag-

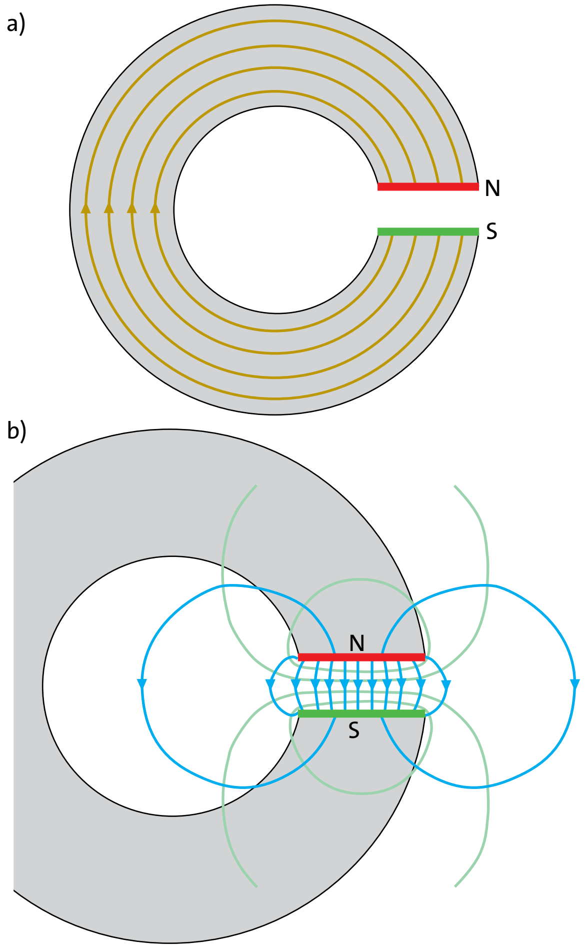


Fig. 2.12 Ring magnet. (a) Magnetization lines; (b) Field lines and field surfaces

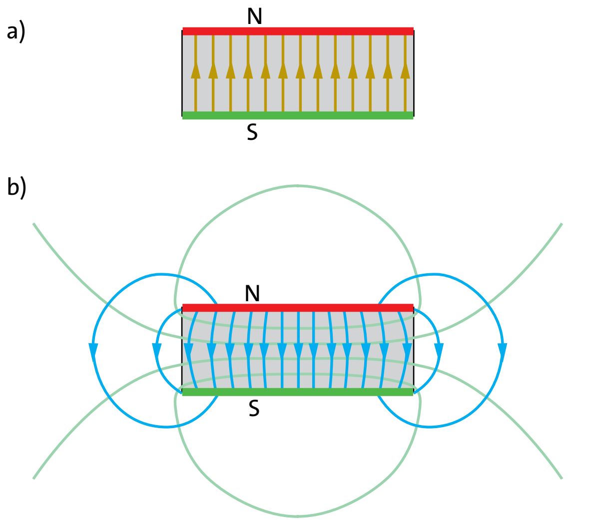


Fig. 2.13 Disc-shaped magnet. The magnetic charge sits on the top and the bottom side of the disc. (a) Magnetization lines; (b) Field lines and field surfaces

netic charge sits on the top and the bottom side of the disc. Hence, the charge distribution is the same as that of the ring magnet with a gap. This implies that also the fields of the two magnets look equal.

Exercises

1. Fig. 2.14a shows the ends of two bar magnets. The two other ends are located beyond the drawing at a long distance. (a) Sketch (with different colors) magnetization lines, field lines and field surfaces. (b) How can we read from the field image that the two magnets attract each other?
2. Fig. 2.14b shows the ends of two bar magnets. The two other ends are located beyond the drawing at a long distance. (a) Sketch (with different colors) magnetization lines, field lines and field surfaces. (b) How can we read from the field image that the two magnets repel each other?
3. Fig. 2.15 shows a somehow strange magnet. It has the shape of a hollow sphere. The outer surface carries positive magnetic charge, the inner surface carries the same amount of negative charge. Draw magnetization lines, field lines and field surfaces in different colors. What is the magnetic field like beyond the sphere and how is it in the hollow internal space?
4. A magnet shall be ring-shaped as the one from Fig. 2.12, but this time without a gap. The magnetization lines follow that shape of the ring. What can be said about the magnetic poles and what about the magnetic field lines and field surfaces?

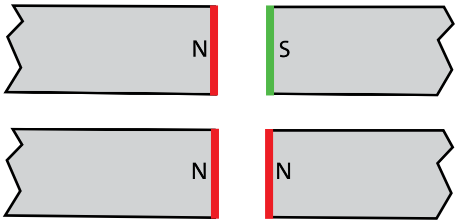


Fig. 2.14 For exercises 1 and 2

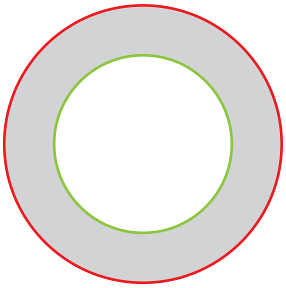


Fig. 2.15 For exercise 3

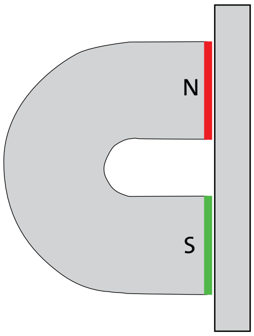


Fig. 2.16 What is the course of the magnetization lines in the horseshoe magnet and in the soft iron body?

2.6 Soft magnetic materials

A magnet does not only attract another magnet, but also bodies made of certain other materials, the magnetically soft materials or *soft magnetic materials*. The materials of which the permanent magnets, that we have analyzed in the previous sections, are made, are called *hard magnetic materials*. Just as hard magnetic materials, soft magnetic materials consist of magnetic atoms. While the alignment of the atomic magnets of the hard magnetic materials does not change under normal circumstances, the atomic magnets of the soft magnetic materials can be easily rotated. A typical soft magnetic material is "soft iron". (There are different types of iron, depending on which other substances were added to the alloy.)

In soft magnetic materials, the atomic magnets are oriented without any regularity at first so that there is no net charge on the surface. If, however, a soft magnetic body is brought into a magnetic field, the atomic magnets will align. This means that the inside of the body is magnetized and that the surface is magnetically charged. If the soft magnetic body is removed from the field, the atomic magnets will scramble again. It loses its magnetization and the magnetic charge disappears.

This behavior is similar to that of metals that are brought into an electric field. Here, electrically charged areas develop on the surface, whereby the overall charge of the body remains zero. We have referred to this phenomenon as electrostatic induction. The corresponding magnetic phenomenon is called *magnetostatic induction*.

Just as no electric field can be maintained inside a metal, no magnetic field can exist in a soft magnetic material. And just as the electric field lines merge perpendicularly into metal surfaces, magnetic field lines merge perpendicularly into soft magnetic bodies. Fig. 2.16 shows a piece of soft iron that is located in close proximity to a horseshoe magnet. What is the course of the magnetization lines in the horseshoe magnet and in the soft iron body?

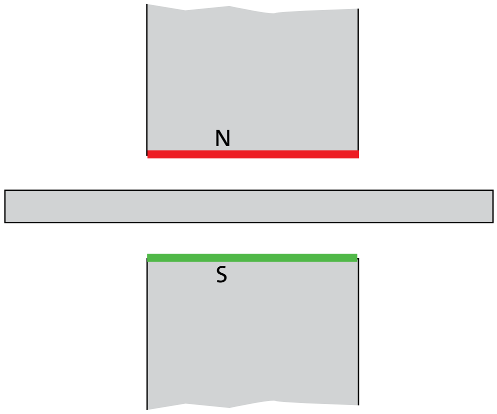


Fig. 2.17 For exercise 1

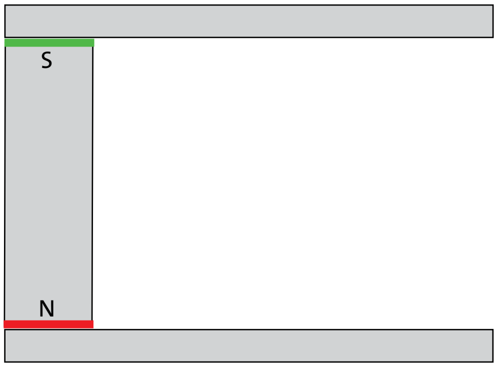


Fig. 2.18 For exercise 2

Magnetically soft bodies are magnetized when they are brought into a magnetic field. Magnetically charged areas develop on their surface. The magnetic field is displaced from their inside.

The magnetic field lines merge perpendicularly into the surface and they end at the surface.

Exercises

1. A plate made of a soft magnetic material is brought between the poles of a magnet, Fig. 2.17. Draw the field surfaces and field lines. Which mechanical stresses exist in the metal plate: compression or tension? What are their direction?
2. You probably have a magnet that looks like the one shown in Fig. 2.18 in your school equipment. It consists of a bar magnet and two soft iron parts. Where are magnetic poles formed in the soft iron? What is the course of the magnetization lines? What is the course of the field lines? What is the advantage of the magnet? What is its disadvantage?

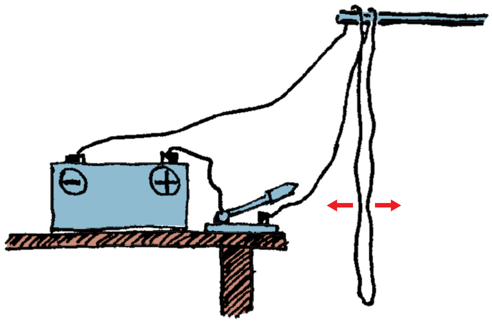


Fig. 2.19 When the electric circuit is closed, the wires bounce apart.

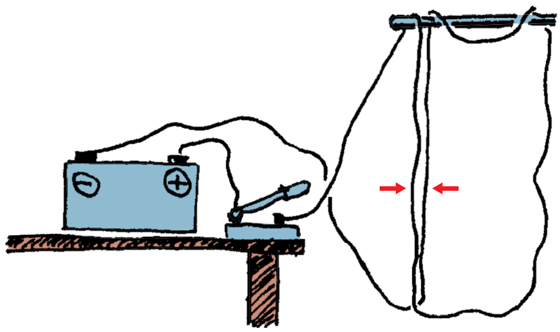


Fig. 2.20 When the electric circuit is closed, the wires bounce towards each other.

2.7 Electric current and magnetic field

A long wire is suspended as shown in Figure 2.19. The wire can be connected with a car battery so that an electric current can flow in it: in the right section downwards and in the left section upwards. The electric circuit may only be closed for a short time as the resistance of the wire is very low and a current of over 50 A is flowing. While closing the switch, we look at the dangling sections of the wire. The wire sections bounce apart. Something has pushed them apart.

We repeat the experiment but we lay the wire in a way that the electricity in the wire sections, that are suspended vertically and next to each other, flows in the same direction, Fig. 2.20. This time, the wire sections bounce towards each other when the electric circuit is closed.

What is the connection through which one wire pulls or pushes the other? The answer is easy to find. We bring a compass needle in close proximity to an individual wire through which a strong electric current can flow. As soon as the electric current is switched

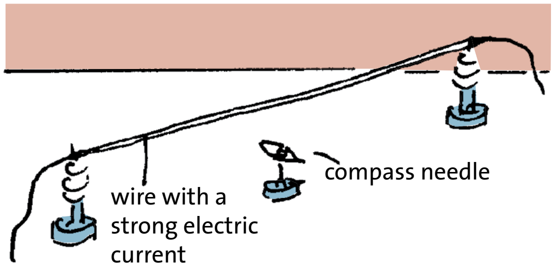


Fig. 2.21 When the electric current is switched on, the compass needle changes its direction.

on, the compass needle orientates itself in a certain direction, Fig. 2.21. When the electric circuit is interrupted again, the needle oscillates back into its original direction. The wire is obviously surrounded by a magnetic field as long as an electric current is flowing in it.

The compass needle, or iron filings, can be used to determine the direction of the field lines and hence also of the field surfaces: for a single wire the field lines are circular. They surround the wire in a way that the center of the circle is located in the wire. Therefore, they neither have a start nor an end point – in contrast to the field lines of the field of a permanent magnet.

The field surfaces are planes that end in the wire, Fig. 2.22.

Fig. 2.23 shows a section through the field that is perpendicular to the direction of the wire. (An electric current that flows into the image plane is marked by a cross; a current that flows out of the image plane is marked by a dot.)

> Each electric current is surrounded by a magnetic field. The field lines enclose the current. The field surfaces end on the current.

From the fact that the field surfaces merge perpendicularly into the wire we can conclude that the field pushes onto the wire.

A simple rule can be formulated for the direction of the field line arrows, Fig. 2.24. Curl the fingers of your right hand around the conductor in a way that the thumb points in the direction of the electric current. Then, the fingers point in the direction of the magnetic field strength vector.

Fig. 2.25a shows the field of two wires in which the electric current is flowing in opposite directions. In Fig. 2.25b, the electric current flows in the same direction.

The magnetic field caused by electric currents can be made much denser by means of a simple trick: the same wire is led many times past the same place, or

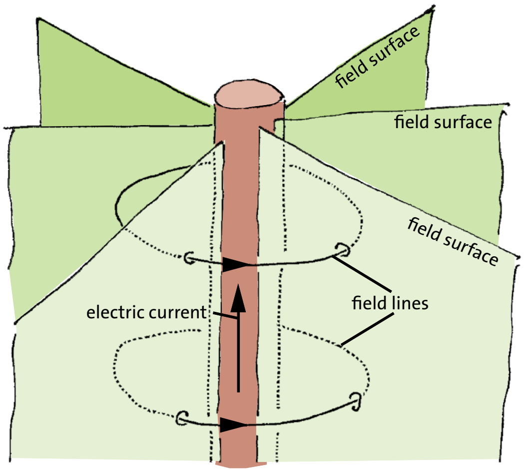


Fig. 2.22 Perspective view of field surfaces and field lines of the magnetic field of a wire in which an electric current is flowing.

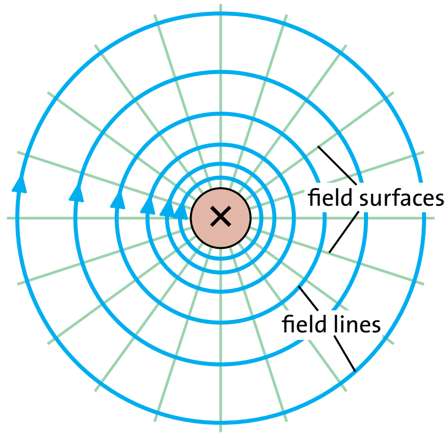


Fig. 2.23 Cross-section through the wire and the field of Fig. 2.22. The cross in the wire means that the current flows into the drawing plane.

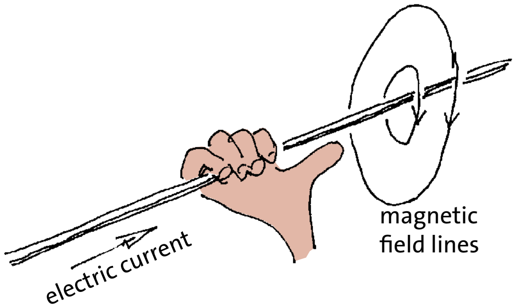


Fig. 2.24 When the thumb of the right hand has the direction of the electric current, the other fingers point in the direction of the magnetic field strength.

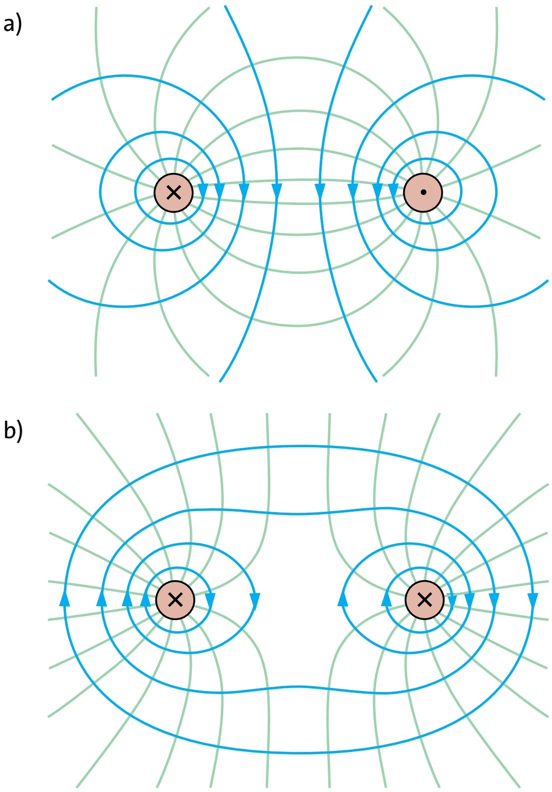


Fig. 2.25 Field surfaces and lines of the field of two parallel wires. (a) The currents flow in opposite directions. (b) The currents flow in the same direction.

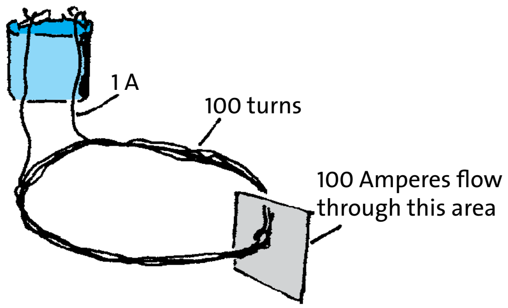


Fig. 2.26 The field of a bundle of wires is as dense as the one of a wire in which a current of 100 A is flowing.

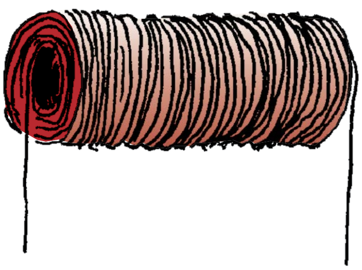


Fig. 2.27 Cylindrical coil

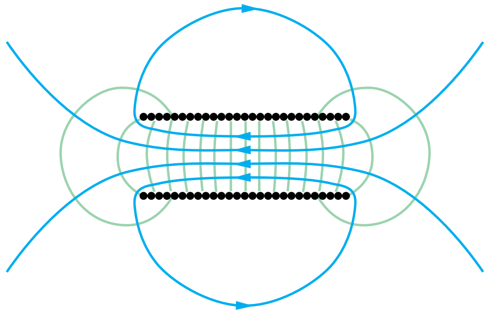


Fig. 2.28 Field surfaces and field lines of the field of a cylindrical coil

better, it is rolled up to form a coil. In Fig. 2.26, the wire runs 100 times in a circle. When a current of 1 A is flowing in the wire, an overall current of 100 A flows through the indicated cross-section. Therefore, there is a magnetic field around this bundle of wires which is as dense as that of a single wire in which an electric current of 100 A is flowing.

A cylindrical coil provides a very useful arrangement. Here, the wire is rolled up in many layers, Fig. 2.27. (Of course, the wire must be insulated because the current could take a shorter way otherwise.)

Fig. 2.28 shows a cross-section of the field of a coil. It is mainly located inside the coil and is almost homogeneous (just as the field of the capacitor is located mainly between the plates where it is homogeneous). The field lines are parallel to the axis of the coil.

Exercises

1. A copper pipe is used as an electric conductor whereby the electric current is flowing in a longitudinal direction. Draw a cross-sectional image of the pipe and sketch field lines and field surfaces of the magnetic field outside of the pipe. Try to draw field lines and field surfaces inside the pipe. What can you say in conclusion? What are the compressive and tensile stresses “felt by the pipe“?
2. Electric currents of the same magnitude but not the same directions flow through four parallel wires, Fig. 2.29a. Sketch field surfaces and field lines.
3. Electric currents of the same magnitude but not the same directions flow through four parallel wires, Fig. 2.29b. Sketch field surfaces and field lines.

Fig. 2.29 (a) For exercise 2; (b) For exercise 3

2.8 Calculation of magnetic field strengths

The magnetic field strength is a vector. It follows the same rules as the electric field strength. If an electric current I_1 taken alone creates a magnetic field with field strength H_1 a point P, and if another current I_2, also taken alone, creates a field of field strength H_2 in the same point P, both currents together will create a field with the field strength

$$\vec{H} = \vec{H}_1 + \vec{H}_2 \,.$$

In analogy to the electric field, the following applies:

> If all electric currents are multiplied by a factor *k*, the values of all magnetic field strengths will increase by the same factor *k*. The field strength directions will remain equal.

We need these rules to calculate the magnetic field strength in a coil. We know that the field is homogeneous and that the field strength vector has the same direction as the axis of the coil. The mnemonic tells us that the field strength is proportional to the electric current. Hence, we can say

$$|\vec{H}| \sim I \,.$$

But there must be more than that. Not the current in a wire is solely responsible for the field, but rather the overall current that flows around the inside of the coil. In terms of the respective total current I_t, we obtain

$$I_t = n \cdot I$$

(n = number of turns of the coil). If the number of turns is doubled while the rest of the coil is left as it was, the field strength inside the coil must also double, Fig. 2.30.

It looks as if two coils were slid on top of each other. Hence, we complete our proportionality to obtain

$$|\vec{H}| \sim I_t = n \cdot I \,.$$

Also this relationship is not yet complete. Fig. 2.31 shows two coils that differ from each other with regard to their length *l*: the second one is twice as long as the first one.

A current of 1 A shall flow through the wire of each coil. The overall current strength in the first coil is

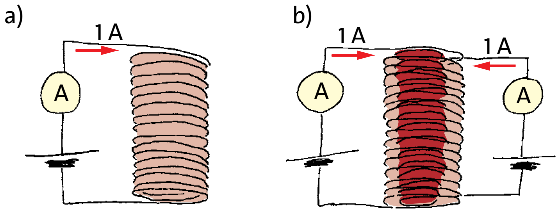


Fig. 2.30 (a) Coil with 200 turns through which a current of 1 A is flowing. (b) A coil with 400 turns through which a current of 1 A is flowing is equivalent to two coils rolled on top of each other whereby each one has 200 turns. The magnetic field strength is twice as high.

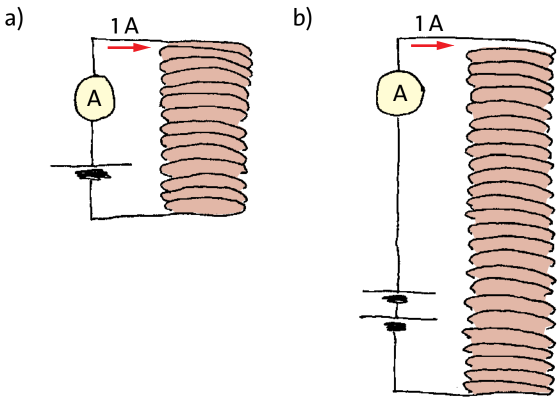


Fig. 2.31 (a) Coil with 200 turns, the current is 1 A. (b) Coil is twice as long with 400 turns, current is 1 A. The coil is equivalent to two adjacent coils with 200 turns each. The field strength is the same as in the coil from (a).

therefore

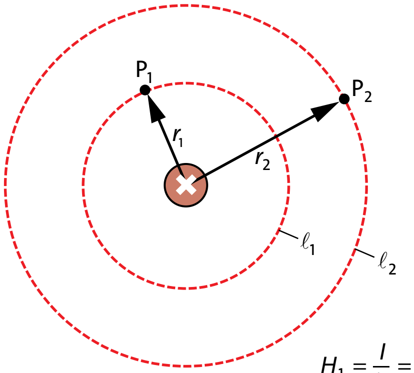
$$I_t = 1\ \text{A} \cdot 200 = 200\ \text{A}$$

and

$$I_t = 1\ \text{A} \cdot 400 = 400\ \text{A}.$$

in the second one.

The second coil, however, can be regarded as equivalent to two adjacent coils through which overall currents of respectively 200 ampere are flowing. In each of the individual coils, we have the same field strength as in the single coil on the left of the picture.



$$H_1 = \frac{I}{\ell_1} = \frac{I}{2\pi r_1}$$

$$H_2 = \frac{I}{\ell_2} = \frac{I}{2\pi r_2}$$

Fig. 2.32 To calculate the magnetic field strengths H_1 and H_2 in the points P_1 and P_2 around a straight wire

A comparison of the short coil on the left with the long one on the right shows that the quotient of the overall current strength and the coil length l

$$I_t / l = n \cdot I / l,$$

on the left and on the right is equal. Hence, what matters for the field strength is the quotient of the total current and the length l, or the number of turns multiplied by the current divided by the coil length:

$$|\vec{H}| \sim \frac{I_t}{l} = \frac{n \cdot I}{l}.$$

The measurement unit of the magnetic field strength has been chosen in a way that the equation contains an equal sign instead of a proportionality sign.

Magnetic field strength of the coil:

$$|\vec{H}| = \frac{I_t}{l} = \frac{n \cdot I}{l}. \quad (2.3)$$

Compare this formula with equation (1.3) for the electric field strength in the capacitor.

Example

An electric current of 4 ampere flows through a coil with 1500 turns and a length of 30 cm. What is the magnetic field strength inside the coil?

$$|\vec{H}| = \frac{n \cdot I}{l} = \frac{1500 \cdot 4 \text{ A}}{0.3 \text{ m}} = 20000 \text{ A/m}.$$

The calculation of the field strength for other electric conductors is generally more complicated than for a coil. But there is yet another case in which the field strength can be calculated easily: the field of a straight long wire.

Magnetic field strength around a straight wire

$$|\vec{H}| = \frac{I}{l}$$

l = circumference of the circle

Here, I is the current in the wire and l is the circumference of the circle on which the point for which the field strength should be calculated is located, Fig. 2.32. You see that the formula is very similar to that for the coil. But bear in mind the different meanings of l in the two cases.

Exercises

1. Compare equation (2.3) with equation (1.3). Why does equation (1.3) not contain the area of the capacitor plates? Why does equation (2.3) not contain the cross-sectional area of the coil?
2. A coil with a length of 60 cm has 3000 turns. An electric current of 0.8 A is flowing through the coil. What is the magnetic field strength inside the coil?
3. You find a coil and would like to know the number of its turns. The coil has a length of 15 cm. You send an electric current of 500 mA through it and you measure a field strength of 3000 A/m inside with the magnetic field meter.
4. A torus-shaped coil (a cylinder that was bent to form a tire-shaped ring) has 1000 turns. The ring diameter is 0.5 m. An electric current of 2.5 A is flowing through the coil. (a) Sketch field lines and field surfaces. (b) What is the field strength inside the coil?
5. An electric current of 16 A is flowing in a wire with a thickness of 2 mm. (a) What is the magnetic field strength on its surface? (b) What is the magnetic field strength at a distance of 1 cm from the center of the wire?
6. A coaxial cable consists of a flexible hollow metallic cylinder and a wire that is located in the cylinder axis. The cylinder and the wire are electrically insulated from each other. They form the forward and return line of the cable. A current of 0.5 A shall flow in such a cable (i.e. in the wire in one direction, in the cylinder in the other direction). The outer diameter of the cable is 10 mm, the diameter of the wire 1 mm. (a) What is the magnetic field strength outside of the cable? (b) What is the magnetic field strength on the surface of the wire?

2.9 Measuring the magnetic charge

We would like to measure the magnetic charge of a magnetic pole of a bar magnet.

We transform our well-known equation

$$\vec{H} = \frac{\vec{F}}{Q_m}$$

and obtain

$$Q_m = \frac{F}{H} \qquad (2.4)$$

To determine F, we suspend the magnet on a momentum current meter so that one of the poles is located completely inside a coil while the other one is as far outside that its contribution to the field within the coil can be neglected, Fig. 2.33. Now we let an electric current flow through the coil and check by which amount the momentum current increases. The increase corresponds to the momentum current that flows from the coil through the field into the magnetic pole.

Now we only need to calculate the magnetic field strength H in the coil and will then obtain the magnetic charge with equation (2.4).

Example

Length of the coil: l = 8 cm
Number of turns: n = 500
Electric current (measured): 1.2 A
Momentum current: 0.15 N
Magnetic field strength of the field in the coil:

$$H = \frac{n \cdot I}{l} = \frac{500 \cdot 1.2\ \text{A}}{0.08\ \text{m}} = 7500\ \text{A/m}$$

Charge of the magnetic pole:

$$Q_m = \frac{F}{H} = \frac{0.15\ \text{N}}{7500\ \text{A/m}} = 2 \cdot 10^{-5}\ \text{Wb}$$

This experiment can also be used to confirm that the overall magnetic charge of a magnet is equal to zero: a magnet is suspended in a way that the other pole is located inside the coil and we let flow the electric current in the opposite direction. We find the same increased value for the momentum current as in the first experiment.

We could also suspend a smaller magnet from the momentum current meter in a way that the magnet is located entirely within the coil. Now the increase of the momentum current is zero.

Just for fun, we would like to look at another variant of the experiment: we put the coil on a balance (if possible, on a balance whose weighing pan does not move during weighing, e.g. the analytical balance from the chemistry laboratory) and switch on the electric current in the coil once again, Fig. 2.34.

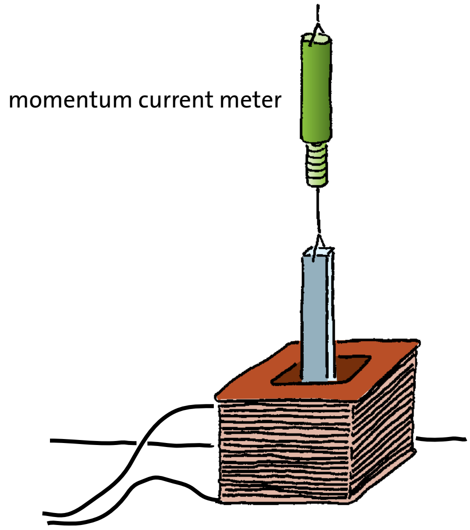


Fig. 2.33 Measuring the magnetic charge of a magnetic pole momentum current meter

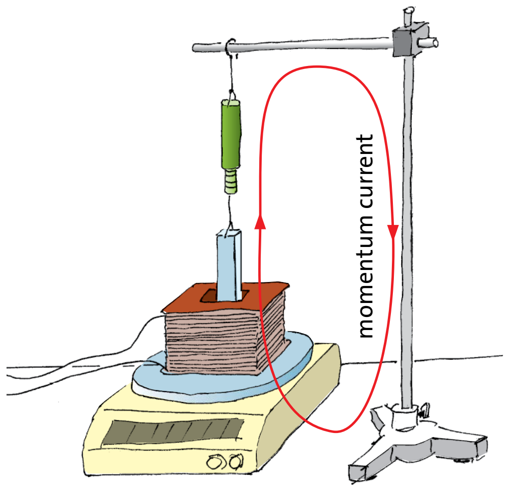


Fig. 2.34 The momentum current is measured at two places: at one with a momentum current meter and at the other with an analytical balance.

An increased momentum current is now flowing from the table over the balance, the coil, the magnetic field, the magnet, the upper momentum current meter and its suspension back into the table. As the balance is nothing else than a momentum current meter, the same increase of momentum current flows successively through two meters. Of course, both indicate the same increased value. (To read the momentum current from the balance, the indicated value must be multiplied by the gravitational field strength as the scale is calibrated in units of mass.)

2.10 Pressure and tension within the magnetic field

Just as in the electric field, there are mechanical stresses in the magnetic field. In the direction of the field strength vector, the field is under tensile stress, and in the perpendicular direction it is under compressive stress. These stresses are used for many technical applications and have an impact on nature.

The values of the stresses (= momentum current densities) are calculated with formulas that have the same structure as those for the electric field.

Magnetic field: $\sigma_{\parallel} = -\frac{\mu_0}{2}|\vec{H}|^2 \quad \sigma_{\perp} = \frac{\mu_0}{2}|\vec{H}|^2$

$\sigma_{\parallel}$ is again the stress in the direction of the field lines; it is negative (tensile stress). $\sigma_{\perp}$ is the stress perpendicular to the field lines; it is positive (compressive stress). $|\vec{H}|$ is the magnitude of the magnetic field strength. μ_0 is the *magnetic constant*:

Magnetic constant $\mu_0 = 1.257 \cdot 10^{-6}$ Wb/(A·m)

We can now understand why and how magnetized bodies attract or repel each other. Again, we would like to predict by simply looking at the field image in which direction the field pushes or pulls the bodies.

1. The ring magnet

Fig. 2.12 shows an image of the field. The field lines merge from below into the upper pole in a practically vertical direction. As there is a tensile stress in the direction of the field lines and as the field lines end on the pole surface, the field pulls the upper pole downwards. On the lower pole, it draws upwards accordingly. A ring with a pivot, Fig. 2.35a, would not be stable; it would collapse, Fig. 2.35b.

2. The single wire

Fig. 2.23 shows field lines and field surfaces for a wire in which an electric current is flowing. The field surfaces merge from all sides into the wire where they end. Hence, the field compresses the wire from the outside. This is similar to the electric field of a charged sphere, Fig. 1.47. When the field pushes inwards, it must also push outwards. Where does it adhere to on the outside? Again, the answer can be found by looking at a material model of the field, Fig. 2.36.

Just as the field, the radial springs are under compressive stress. They adhere to the ring on the outside.

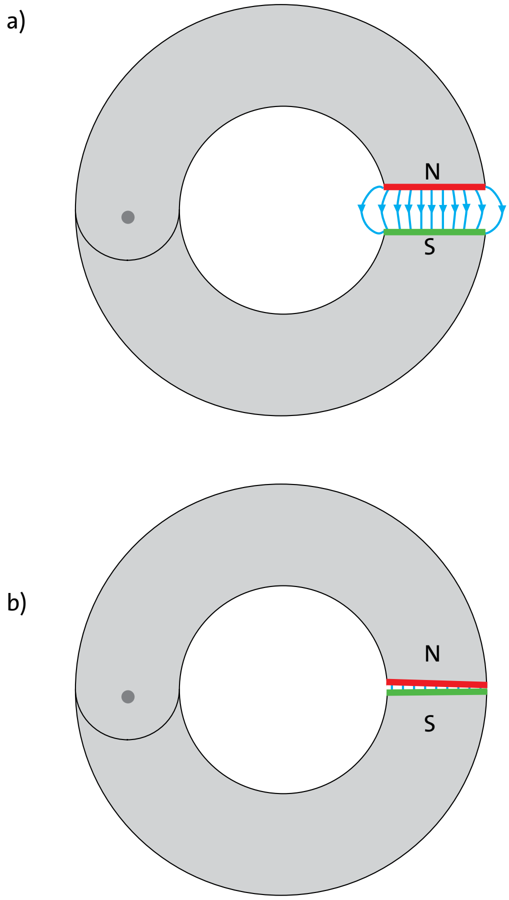


Fig. 2.35 Ring magnet with a pivot. (a) In this position , the magnet is not stable; (b) it collapses in a new position.

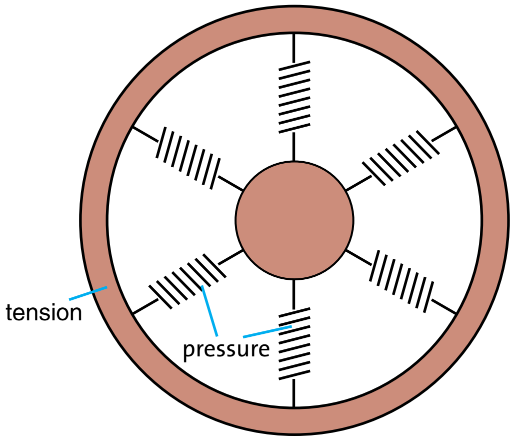


Fig. 2.36 Model of the magnetic field of the wire in which an electric current is flowing. The springs push on the wire in an inward direction and on the ring in an outward direction. Therefore, the ring is exposed to tensile stress.

Therefore, the ring is under tensile stress. The same applies to our field: also the field is exposed to tensile stress in the direction of the circular circumferences, i.e. in the direction of the field lines.

For a typical electric conductor made of a solid material such as copper with a typical electric current, e.g. of 20 A, the pressure of the magnetic field is hardly noticeable (see also Exercise 1).

If, however, a very strong electric current flows in a liquid conductor or in a gaseous conductor (a plasma), the pressure can have strong effects.

In a nuclear fusion reactor, a so-called *Tokamak*, a plasma must be maintained in a torus-shaped recipient at a temperature of approximately 100 million Kelvin. The plasma must not come in contact with the walls of the recipient. Therefore, one ensures that a strong electric current flows in the gas. Its magnetic field pushes from the outside onto the plasma and holds it together. Thus, the gas is locked in the magnetic field.

3. The coil

Its field is shown in Figure 2.28. What are the effects of the compressive and tensile stress on the coil? What does the coil feel? Again, we look at a material model, Fig. 2.37.

The springs that are perpendicular to the axis of the coil (vertical in the figure), are compressed. They push on the coil wires from the inside. If the coil was made of a very soft material, the individual turns would expand.

The horizontal springs, that are under tensile stress, adhere each to the ends of two inclined springs which push from the outside on the coil ends. Hence, the coil is exposed to compressive stress in the longitudinal direction. In fact, a loosely twisted coil is compressed by the field when a strong current is switched on, Fig. 2.38.

The x momentum current flows inside the coil from the right to the left (tensile stress). It branches off at the left end of the coil and flows through the coil material back to the right (compressive stress) and then back to the field, Fig. 2.39.

We can calculate the respective momentum current. Suppose, the following values are known:

Length of the coil: $l = 40$ cm

Cross-sectional area of the coil: $A = 100\ \text{cm}^2$

Number of turns: $n = 50$

Electric current: $I = 120$ A

At first, we look at the magnetic field strength:

$$|\vec{H}| = \frac{n \cdot I}{l} = \frac{50 \cdot 120\ \text{A}}{0.4\ \text{m}} = 15000\ \text{A/m}$$

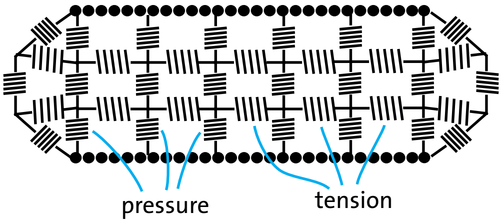


Fig. 2.37 Model of the magnetic field of a coil. The springs push from the inside on the individual wires and from the sides in a longitudinal direction on the coil.

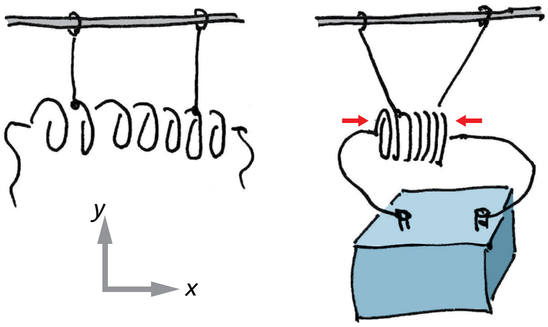


Fig. 2.38 (a) Loosely winded, elastic coil. (b) The coil is connected briefly to a car battery. It is immediately compressed by the field.

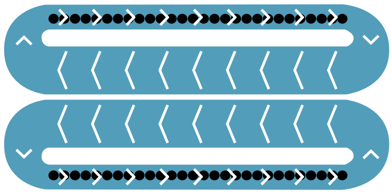


Fig. 2.39 The x momentum current flows in the field inside the coil from the right to the left and through the coil material back to the right.

Therewith we obtain the tensile stress inside the coil

$$\sigma_{||} = -\frac{\mu_0}{2}|\vec{H}|^2$$
$$= -\frac{1.257 \cdot 10^{-6}\ \text{Wb/(A} \cdot \text{m)}}{2} \cdot (15000\ \text{A/m})^2$$
$$= -141\ \text{Pa}$$

and finally the momentum current:

$$F = \sigma_{||} \cdot A = -141\ \text{Pa} \cdot 0.01\ \text{m}^2 = -1.41\ \text{N}$$

Exercises

1. A copper wire shall have a cross-sectional area of 1.5 mm^2. An electric current of 16 A is flowing in the wire. What is the pressure of the magnetic field on the surface of the wire?
2. A flash of lightning consists of ionized air (i.e. a plasma) through which a strong electric current is flowing from the Earth into the thundercloud. A typical lightning has a diameter of 1 cm, the electric current is 10 000 A. (a) What is the pressure of the magnetic field at the surface of the plasma? (b) The plasma has a temperature of 10 000 K. It is heated up so fast that it does not have time to expand at first. What is the pressure of the gas generated in the process? Compare with the result from part (a).
3. As we know, two parallel wires, in which electric currents flow in opposite directions, are moved away from each other by the magnetic field, Fig. 2.19. How can this be seen in the field image from Fig. 2.25a?
4. Two parallel wires, in which electric currents flow in the same direction, are moved towards each other by the magnetic field, Fig. 2.20. How can this be seen in the field image from Fig. 2.25b?

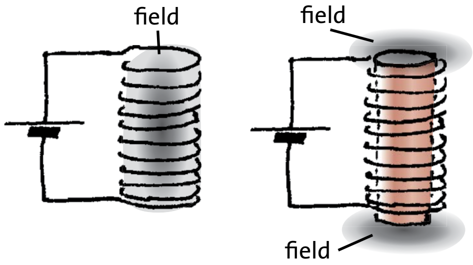


Fig. 2.40 The magnetic field is displaced from the coil by the soft iron core.

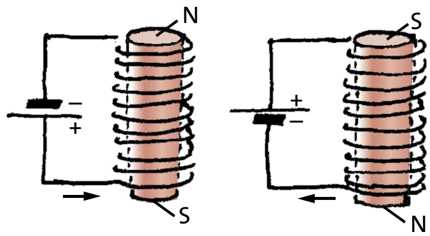


Fig. 2.41 If the direction of the electric current in the coil is reversed, the magnetic poles will swap positions.

2.11 Electromagnets

We remember that soft magnetic materials, for example soft iron, displace the magnetic field from their inside. The soft iron is magnetized in the process and magnetic poles develop on its surface. If the inside of a coil, i.e. the area where the largest part of the field of the coil is located, is filled with soft iron, the field will be displaced from there. It is now located outside of the coil, at the ends, Fig. 2.40. Such a coil with a soft iron core is called electromagnet.

The poles are located at the ends of the soft iron core. In relation to the coil, the electromagnet has the advantage that the magnetic field is no longer hidden inside, but located at an easily accessible place. In contrast to the permanent magnet, it can be switched on and off, be set to a high and low strength and its polarity can be reversed. On Fig. 2.41, we can see on which end the positive and on which end the negative magnetic pole form.

Fig. 2.42 shows a particularly interesting variant of an electromagnet: the iron core is ring-shaped and has a gap. If the gap width d is small, compared to the lateral extension of the gap area, the field is essentially restricted to the gap space and it is nearly homogeneous. The poles are located on the surface of the gap. Hence, the field is similar to the electric field of a capacitor.

The figure shows both the magnetization lines as well as the field lines. We remember the rule according to which the field lines start where the magnetization lines

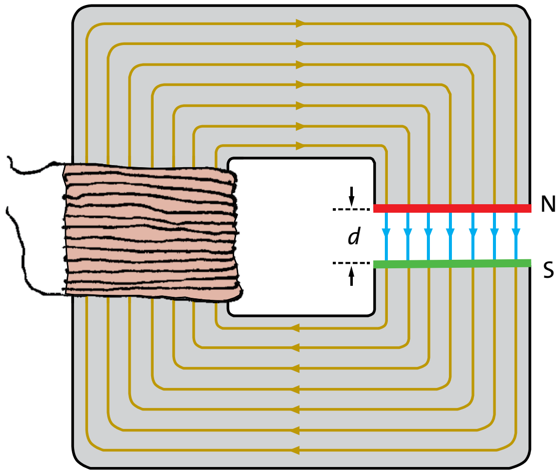


Fig. 2.42 Ring-shaped electromagnet

end, and magnetization lines start where field lines end.

Surprisingly, we can use our old formula once again to calculate the field strength:

$$|\vec{H}| = \frac{n \cdot I}{d}$$

Instead of the coil length l, however, the gap width d is in the denominator. We can see that this electromagnet has an interesting property: the field strength is increased by reducing the gap.

Electromagnets are used in many different ways. The most important one is probably the electric motor.

Exercises

1. Name devices that contain electromagnets. How do these devices work?
2. Invent an electric motor.
3. An electromagnet with a gap (as in Fig. 2.42) shall have the following dimensions: gap width 1 cm, cross-sectional area of the iron core 100 cm^2. The coil of the magnet shall have 1000 turns and the electric current shall be 2 A. (a) Calculate the magnetic field strength in the gap. (b) How much energy is contained in the field? (c) By which factor will the field strength change if the gap width is reduced to 1 mm? By which factor will the energy change?
4. Sometimes an electromagnet has a peculiar behavior. Fig. 2.43 shows an electromagnet and a permanent magnet that are located at a long distance from each other. The two are brought together, Fig. 2.43b (without flipping them in the process). Will they attract or repel each other? The denomination of the pole of the electromagnet was omitted deliberately. Why?

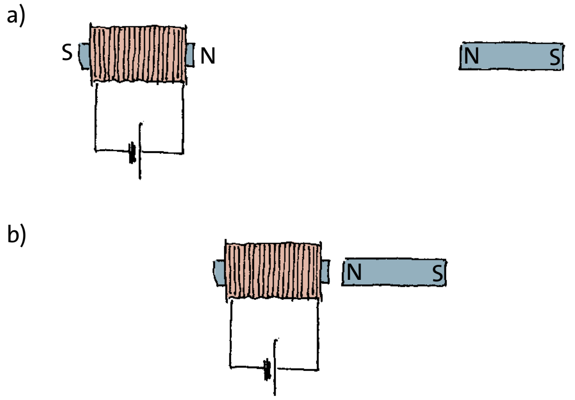


Fig. 2.43 For exercise 4

2.12 Magnetic field strength, magnetization and magnetic flux density

We have not yet learned to describe the magnetization of a material quantitatively, i.e. by means of numbers. This shall be done now.

Just as the magnetic field strength, the magnetization can be represented by lines. This means that magnetization as a physical quantity –just as the field strength– is a vector quantity. The symbol for this vector is $|\vec{M}|$.

If the electric current in the coil from Fig. 2.42 is increased, both the magnetization in the iron core as well as the field strength of the field will increase. Therefore, we can use for the magnetization in the iron core the same measurement unit as the field strength in the gap. We define: at the pole areas of the iron core from Fig. 2.42, the magnetization on the inside is equal to the field strength outside:

$$\vec{M}_{\text{inside}} = \vec{H}_{\text{outside}}$$

At first, it looks as if this definition was only applicable to a very specific magnet. The magnetization, however, can actually be defined this way any time the magnetization lines continue as field lines at a limit area without kinks.

Let's get back to the ring magnet once again:

Here, a single closed line can be followed in a closed path. In one part of the path, the line is a magnetization line, in another one it is a field line. Later, we will learn about phenomena that do not require a distinction between magnetization and magnetic field strength. In this case it is useful to summarize magnetization and magnetic field strength as one single physical quantity. We define:

$$\vec{B} = \mu_0 (\vec{M} + \vec{H}) \qquad (2.5)$$

The new quantity is called *magnetic flux density*. Due to the factor μ_0, $\vec{B}$ does not have the same measurement unit as $\vec{H}$ and $\vec{M}$. We obtain

$$\frac{\text{Wb}}{\text{A} \cdot \text{m}} \cdot \frac{\text{A}}{\text{m}} = \frac{\text{Wb}}{\text{m}^2} = \text{Tesla}$$

We can already see an advantage that comes with the application of the flux density. The theorem that says that field lines start where magnetization lines end, and vice versa, can now be formulated in a simpler way:

> Flux density lines have no beginning and no end.

Let's compare this to an analogue situation. Look at a closed loop that is made of a pieces of wire. Some pieces are made of copper and some of aluminum. The following rule applies for the circuit:

An aluminum wire starts where a copper wire ends, and a copper wire starts where an aluminum wire ends.

The rule can be formulated more briefly if the material does not matter: the metal wire has no beginning and no end.

From equation (2.5) we can conclude that outside of magnetizable substances (where the magnetization is zero) we have:

$$\vec{B} = \mu_0 \vec{H}$$

and within soft magnetic materials (where the magnetic field strength is zero) we obtain:

$$\vec{B} = \mu_0 \vec{M}.$$

2.13 The coil – the inductance

We have already compared the coil with the capacitor several times. Both devices are important electric components. In electronic devices, there are both coils and capacitors. Now, we would like to take the comparison even further.

A capacitor is characterized by its capacitance. The capacitance depends on the geometrical dimensions of the capacitor, but not on the applied voltage or the charge on the plates. If we want to install a capacitor somewhere, we must know the capacitor's capacitance. If we were to buy a capacitor, we would have to indicate the capacitance.

There is a physical quantity that is characteristic for a coil in a similar way: the inductance L. The inductance also depends on the geometrical dimensions of the coil (the number of turns, inter alia) and not, for example, the current in the coil. The measurement unit of the inductance is the Henry (H). It is

$$\mathrm{H} = \frac{\mathrm{Wb}}{\mathrm{A}} = \frac{\mathrm{V \cdot s}}{\mathrm{A}}.$$

To define the inductance, we first need to introduce another physical quantity: the *magnetic flux*:

$$\Phi = \mu_0 \cdot H \cdot A = B \cdot A$$

Hence, the magnetic flux in a coil is the product of the magnetic flux density B and the cross-sectional area A of the coil. The flux is needed to define the inductance L:

$$L = \frac{n \cdot \Phi}{I}$$

The magnetic flux Φ in a coil is proportional to the electric current that flows through the coil.

$$\Phi = \frac{1}{n} \cdot L \cdot I$$

L is the inductance of the coil.

Hence, the inductance of the coil tells us how a strong magnetic flux can be created with a given electric current. To calculate how L is related to the geometrical data of the coil, we replace in the penultimate equation in the numerator

$$\Phi = B \cdot A$$

and in the denominator

$$I = \frac{l \cdot H}{n}$$

and obtain

$$L = \frac{n \cdot \Phi}{I} = \frac{n \cdot B \cdot A}{(l \cdot H)/n} = \mu_0 \cdot n^2 \cdot \frac{A}{l}$$

$$L = \mu_0 \cdot n^2 \cdot \frac{A}{l}$$

A = cross-sectional area of the coil
l = length of the coil
$\mu_0 = 1.257 \cdot 10^{-6}$ Wb/(A·m) = magnetic constant

Compare the result with the formula for the capacitance of the capacitor. The technical symbol of the coil are four semicircles in a row, see also Fig. 2.44.

Exercises

1. A coil with 500 turns has a cross-sectional area of 10 cm^2 and a length of 8 cm. Calculate its inductance.
2. A loosely winded coil is extended to twice its length. How does its inductance change?

2.14 The energy of the magnetic field

In a coil, through which an electric current is flowing, there is magnetic field stuff which contains energy just as the electric field stuff. We had developed a formula that can be used to calculate the energy content of the electric field in the capacitor:

$$E = \frac{C}{2} U^2.$$

The corresponding formula for the coil and its magnetic field shall not be developed but just indicated. As you might have expected, it is very similar to the capacitor formula. We simply replace the capacity by the inductance and the voltage by the electric current:

$$E = \frac{L}{2} I^2.$$

Also the formula for the energy density in the magnetic field shall be indicated without any calculation. It also has the same structure as the one for the electric field:

$$\rho_E = \frac{\mu_0}{2} \left| \vec{H} \right|^2 .$$

The energy in the magnetic field of a coil can be calculated from the inductance and the electric current:

$$E = \frac{L}{2} I^2.$$

The energy density of any magnetic field can be calculated from the magnetic field strength:

$$\rho_E = \frac{\mu_0}{2} \left| \vec{H} \right|^2 .$$

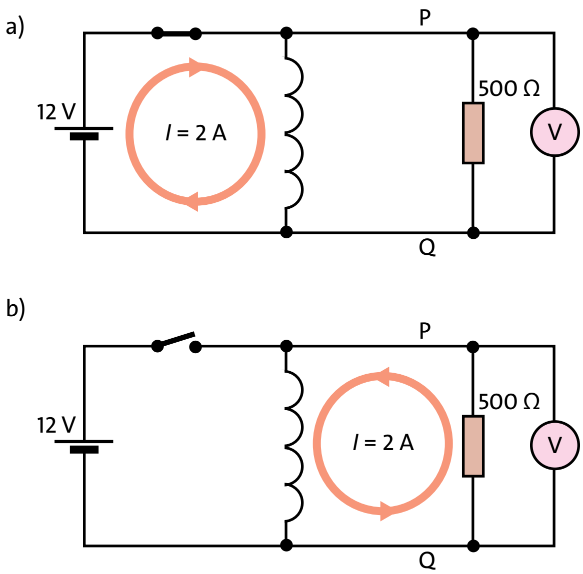


Fig. 2.44 (a) An electric current of 2 A flows through the coil. (The current through the resistor is very weak.) (b) Immediately after opening the switch, the electric current of 2 A has to flow through the resistor. This creates a high voltage, but the electric current decays very fast.

Exercises

1. The field strength of the magnetic field of the Earth is approximately 20 A/m. How much energy is contained in 1 m^3 of this field?
2. We assume in an approximative way that the field between the poles of the two bar-shaped magnets in Fig. 2.14a is homogeneous and restricted to the space between the two pole areas. The surface area of the pole shall be 4 cm^2, the distance between the magnets 0.5 cm and the magnetic field strength 120 000 A/m. How much energy is contained in the field?
3. An electric current of 2.5 A flows through a coil with an inductance of 0.01 mH. The coil is 10 cm long and has a cross-sectional area of 4 cm^2. (a) How much energy is contained in the field of the coil? (b) What is the energy density inside the coil?
4. The wire of a coil normally has an electric resistance. A coil with an inductance of 0.2 mH and a resistance of 500 Ω is connected to a voltage source of 200 V. How much energy will be stored in the magnetic field of the coil in the process?
5. (a) You connect a capacitor to a power supply. Thereby, a field is created in the capacitor. The energy of the field is provided by the power supply. Now you disconnect the capacitor from the power supply. The field will be maintained, at least for a while. Hence, the energy remains stored in the capacitor. Now you connect a coil to a power supply whereby a field is created in the coil. The energy of the field is provided by the power supply. You would now like to disconnect the coil from the power supply unit in a way that the field in it does not disappear. How can you do that? (b) The capacitor detached from the power supply slowly loses its energy. What is the defect of the capacitor that causes this loss? The coil disconnected from the power supply unit loses its energy very quickly. Which defect of the coil causes this loss? There are coils that do not have this defect. What type of coils are they?

2.15 "Discharge" of the coil

An electric current flows through the coil from Fig. 2.44a; we assume the current to be 2 A. Therefore, the coil contains a certain amount of magnetic field stuff and hence a certain amount of energy.

We now open the switch, Fig. 2.44b. Right after the opening, the field is as it was shortly before because its energy cannot just disappear from one moment to the other. However, the fact that the field is still there also means that the electric current still has to flow as no field is possible without a current, and no current can exist without a field. The current must be the same as before opening the switch, i.e. 2 A. As the battery circuit is interrupted, these 2 A are now flowing through the resistor.

We can further conclude: if an electric current is flowing through the resistor, the voltage at the resistor must have the respective value; in our case

$$U = R \cdot I = 500\ \Omega \cdot 2\ \text{A} = 1000\ \text{V}.$$

We consequently see: a voltage emerges between the ends of the resistor and hence between the upper conductor P and the lower conductor Q when the switch is opened. This voltage is much higher than the voltage between P and Q before opening the switch.

We can also say: the current that needs to continue flowing after opening the switch, and that can only flow through the resistor, creates this voltage.

What we have just said is only valid for the first moment though, i.e. for the “time zero”. What happens after? As an electric current flows through the resistor, entropy is generated there. This requires energy which comes from the coil. The respective energy current is related to the electric current via

$$P = R \cdot I^2.$$

(The equation results from $P = U \cdot I$ and $U = R \cdot I$.)

Hence, the energy in the coil decreases. This means, however, that also the electric current decreases due to

$$E = \frac{L}{2} I^2.$$

When the electric current decreases, the energy outflow of the coil decreases as well since

$$P = R \cdot I^2.$$

As a consequence, the electric current decreases more slowly than before etc. etc.

You see the logic: *The lower the electric current, the more slowly it decreases.*

This phrase might sound familiar to you. We examined similar phenomena earlier. In the case of a capacitor discharge we had:

The lower the voltage, the more slowly it decreases.

Also here, we could develop a differential equation and look for a solution. We save the time as this calculation would be the same as for the capacitor. Only a few physical quantities would have to be replaced by others. For the coil “being discharged” we obtain:

$$I(t) = I_0 \cdot e^{-t/\tau}$$

i.e. the electric current “decays” exponentially, Fig. 2.45. I_0 is the current immediately after opening the switch.

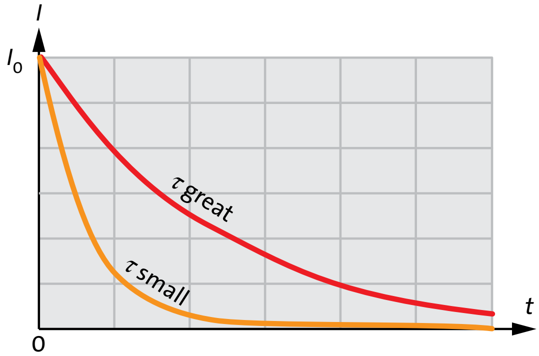


Fig. 2.45 A long decay time τ means: the current in the coil decreases slowly.

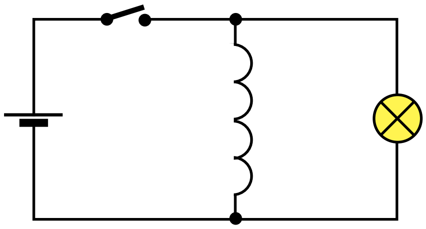


Fig. 2.46 The battery voltage is not sufficient to light up the light bulb. The bulb only lights up for a short time when the switch is opened. The required energy comes from the magnetic field of the coil.

The decay time τ is:

$$\tau = \frac{R}{L}$$

We see that in order to get a slow decay, the inductance of the coil must be high and the resistance of the resistor low.

> *“Discharge” of a coil through a resistor*
> The electric current decreases exponentially:
>
> $$I(t) = I_0 \cdot e^{-t/\tau}$$
>
> $$\tau = \frac{R}{L} = \text{decay time}$$

The fact that the electric current, that flows in the battery circuit, evades into the resistor when the switch is opened can be seen by using a light bulb instead of the resistor, Fig. 2.46. After the switch is opened, it will light up for a short time. (A light bulb that does not yet light up at a battery voltage of 12 V shall be chosen.)

We would now like to derive an equation that looks a little boring at first, but that we will see again in the next chapter. Only then we will understand its importance.

We establish the energy balance for the electric circuit from Fig. 2.44. The switch has just been opened. We have:

$$P = \frac{dE}{dt}$$

In words: the energy current P flowing into the resistor is equal to the rate of change of the energy content of the coil.

Now we replace on the left side:

$$P = U \cdot I.$$

On the right, we replace by means of

$$E = \frac{L}{2} I^2.$$

For this purpose, we first need to differentiate E with respect to the time. To do so, we apply the chain rule:

$$\frac{dE}{dt} = \frac{L}{2} \cdot 2I \frac{dI}{dt} = L \cdot I \cdot \frac{dI}{dt}.$$

We insert and obtain:

$$U \cdot I = L \cdot I \frac{dI}{dt},$$

and thereof:

$$U = L \frac{dI}{dt}$$

The right side can then be transformed by means of

$$n \cdot \Phi = L \cdot I$$

and we finally obtain

$$U = n \frac{d\Phi}{dt}$$

Hence: the voltage at the resistor is equal to n times the rate of change of the magnetic flux in the coil. We will see later that this is a famous law: *Faraday's law of induction.*

Exercises

1. Measurement of the inductance of a coil: The coil is built into a circuit as shown in Fig. 2.44. The resistance of the resistor is to 500 Ω. We find that the voltage decreases to one tenth of its initial value in 4 ms. What is the value of L?
2. According to which time function does the energy in the coil decay?

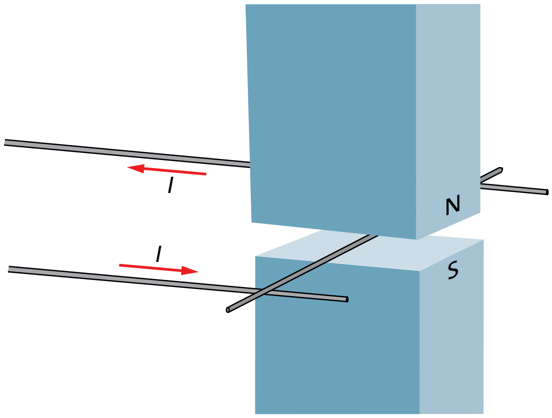


Fig. 2.47 The mobile conductor with the electric current is pushed to the left by the magnetic field.

2.16 How the magnetic field presses on an electric current

We know already: if a magnetic pole is brought to a point P where there is a field (field strength H), it will be drawn in the direction of the vector H by the field.

Something similar happens if an electric current, that flows in a right angle in relation to the field lines, is brought to the point P instead of the magnet. We place an electric conductor within the nearly homogeneous field between the poles of a magnet, Fig. 2.47, and find: the conductor is pushed to the side by the field.

An image of the field, Fig. 2.48, shows us the reason. Fig. 2.48a illustrates the field of the magnet alone. The image b shows the field of the electric current alone. c shows the resulting field. Finally, d shows, besides the field lines, the field surfaces. The resulting field is more dense on the right side of the wire than on the left. The pressure (in the direction of the field surfaces) is therefore higher on the right side than on the left so that the wire is pushed to the left.

Deriving the formula for the respective momentum current is a bit difficult and shall be skipped here. The result itself, however, is simple. The following applies for an electric current that flows in a right angle to the field vector of the field without a current:

$$F = I \cdot \Delta s \cdot B$$

Here, I is the electric current, B is the magnetic flux density of the field without the conductor and Δs is the length of that part of the conductor that is located in

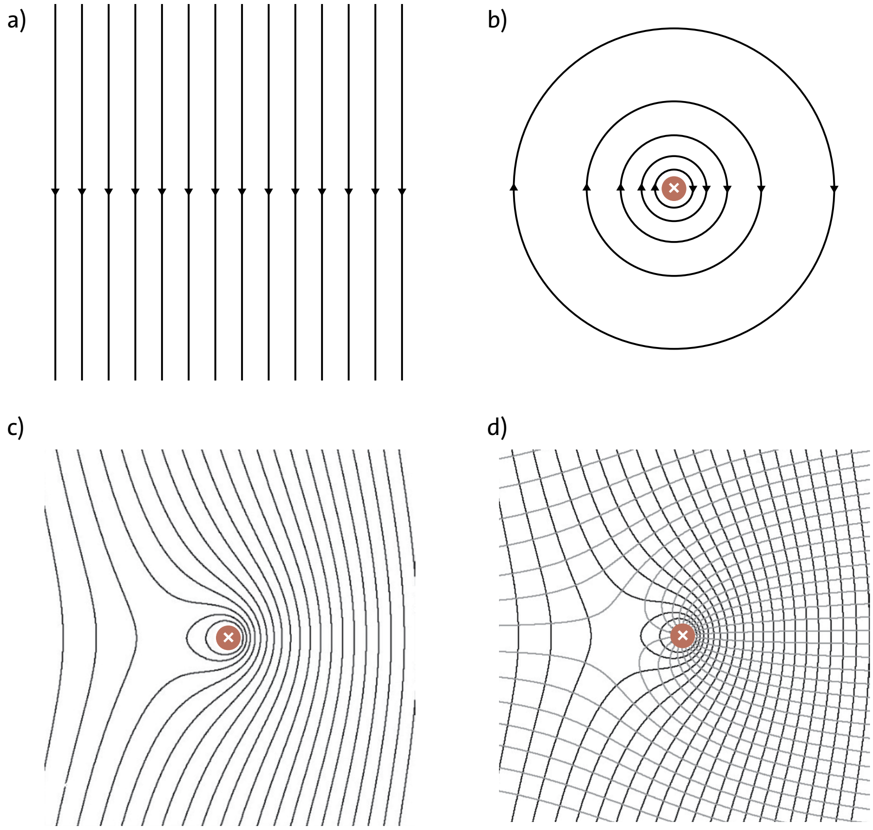


Fig. 2.48 (a) homogeneous field; (b) electric current with its field; (c) resulting field; (d) resulting field with field surfaces

the area of the magnetic field. F is the magnitude of the momentum current vector. This vector is perpendicular to the magnetic flux density (and field strength) and perpendicular to the electric conductor.

Bear in mind that the initial field is strongly changed by the conductor. Still, the flux density of the initial field needs to be inserted in the formula.

> A conductor with an electric current is pushed perpendicularly to the direction of the field without the conductor.
>
> The respective momentum current is
>
> $$F = I \cdot \Delta s \cdot B . \quad (2.6)$$

There are three relevant directions in this law:

1. Direction of the electric current (= direction of the current density vector $\vec{j}$)
2. Direction of the flux density vector $\vec{B}$ (= direction of the field strength vector $\vec{H}$)
3. Direction of the momentum current vector $\vec{F}$

The relationship between these directions can be memorized by means of the three-finger-rule of the right hand, Fig. 2.49: when the thumb points in the direction of the electric current and the index finger in

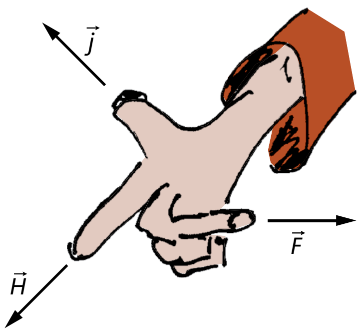


Fig. 2.49 Three-finger-rule of the right hand

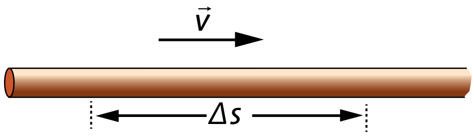


Fig. 2.50 Section of a beam of charged particles

the field strength direction, the flexed middle finger indicates the direction of the momentum current vector.

Orbits of charged particles

We apply equation (2.6) to a beam of charged particles, such as electrons, that fly through a magnetic field in a direction perpendicular to the field direction. We look at a section of length Δs of the beam, Fig. 2.50.

We calculate the momentum current that flows into this bundle of particles, as a function of its charge and its speed. Therefore, we express the electric current in equation (2.6) by means of

$$I = \frac{\Delta Q}{\Delta t}.$$

Here, ΔQ is the charge of the bundle of particles and Δt is the time that the charge needs to pass through Δs. In addition, we replace

$$\Delta s = v \cdot \Delta t,$$

where v is the velocity of the particles. We obtain:

$$F = I \cdot \Delta s \cdot B = \frac{\Delta Q}{\Delta t} \cdot v \cdot \Delta t \cdot B = \Delta Q \cdot v \cdot B$$

Usually, the particles carry the elementary charge e. In this case, we can replace ΔQ by e.

> Charged particles that move in a magnetic field receive a transversal momentum through the field:
> $F = e \cdot v \cdot B$
> e = electric charge of the particles
> v = velocity of the particles

Again, the three-finger-rule applies for the directions, whereby the velocity vector has to be used instead of the current density vector.

In vector form, the relationship can be expressed as

$$\vec{F} = e \cdot (\vec{v} \times \vec{B})$$

This rule is interesting. We imagine a particle that moves in a homogeneous magnetic field at a right angle to the field lines. Hence, its orbit is located on a field surface. The particle is constantly provided with a transversal momentum through the field: the direction of the momentum that it receives is always transversal to the one of the momentum that it currently has. Hence, it is deviated.

We have already seen that such a process leads to a circular movement, Fig. 2.51. Hence, the particles describe a circular orbit in a homogeneous magnetic field.

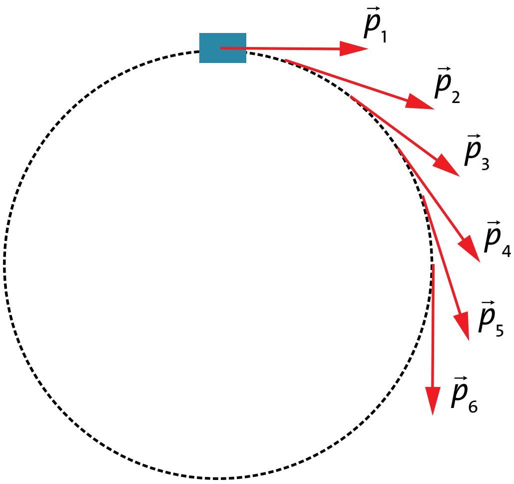


Fig. 2.51 The particle receives new momentum, whose direction is perpendicular to the direction of the momentum that it currently has, in every moment. The image shows the momentum vectors at six different times.

We have expressed the rate of change of the momentum of a body, that makes a circular movement, by its speed v, its mass m and the radius r of its orbit:

$$\frac{dp}{dt} = m\frac{v^2}{r}$$

Since the rate of change of its momentum is equal to the momentum current, i.e.

$$\frac{dp}{dt} = F,$$

we obtain

$$m\frac{v^2}{r} = e \cdot v \cdot B.$$

We divide both sides of the equation by v and bring r on one side.

> Particles within magnetic field
> $$r = \frac{m \cdot v}{e \cdot B} \qquad (2.7)$$
> The radius of the orbit is large when the particle is heavy and fast; it is small when the field strength is high.

If the magnetic field is not homogeneous, r is the radius of curvature of the orbit in every moment.

The equation tells us that particle beams can be "manipulated" by means of magnetic fields. In particular electron beams can be focussed and deviated just as light beams can be focussed by means of lenses and deviated in prisms.

This process is applied in electron microscopes and in particle accelerators.

Measuring *e/m*

Equation (2.7) contains two quantities that characterize the particle: its mass m and its electric charge e. Both values are very small, and hence difficult to measure. However, the equation allows us to measure the ratio of the two.

We create a beam of electrons in a homogeneous magnetic field. The orbit of the electrons can be visualized easily. From equation (2.7) we can conclude

$$\frac{e}{m} = \frac{v}{r \cdot B}$$

All quantities on the right side of the equation can be easily measured. If the elementary charge is also measured by means of another method, the mass of the electron can be calculated. Remember:

$e = 1.602 \cdot 10^{-19}$ C
$m = 0.911 \cdot 10^{-30}$ kg

Charged particles follow magnetic field lines

We have assumed so far that the electrons fly transversally to the field lines in a homogeneous magnetic field. But what will they do if they start in another direction? Let's imagine a homogeneous field once again. It should fill out a large space; in the space, there should be a vacuum so that the electrons can fly freely. If an electron starts in any direction now, we can break down its velocity in two components: one that is transversal to the field strength and another one that is parallel to it. In other words: a component parallel to a field surfaces and one parallel to a field line. To see how the electron moves, we look at each of the two components separately. A circular movement is associated to a velocity perpendicular to the field lines, and a normal linear movement to the velocity parallel to the field lines. Hence, we have a circular movement and at the same time a normal movement in a transversal direction to the circle. The result is a helical movement, Fig. 2.52.

There are situations in which the movement can be described as follows: the electrons follow the magnetic field lines, which is particularly appropriate when the field strengths are very high.

This is the case, for example, in a fusion reactor where we have magnetic field strengths of approximately $5 \cdot 10^6$ A/m. The particles are electrons, deuterons and tritons. (A deuteron is composed of a proton and a neutron, a triton of a proton and two neutrons.) There is a temperature of approximately 100 million Kelvin in the reactor. This means that the electrons move with a speed of around $4 \cdot 10^7$ m/s. We calculate the radius of their helical path:

$$r = \frac{m \cdot v}{e \cdot B} = \frac{10^{-30}\,\text{kg} \cdot 4 \cdot 10^7\,\text{m/s}}{1.6 \cdot 10^{-19}\ \text{C} \cdot 5\ \text{T}}$$
$$= 5 \cdot 10^{-5}\ \text{m}$$
$$= 50\ \mu\text{m}$$

Hence, the helix is very thin compared to the overall size of the reactor (several meters) and we can reasonably say that the electrons follow the field lines.

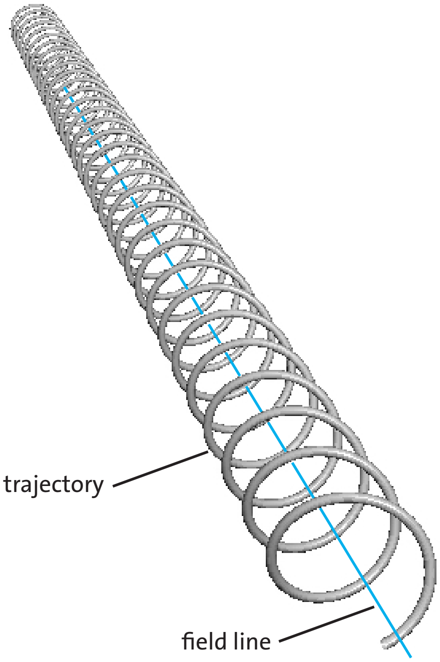


Fig. 2.52 The particle moves on a helical path around a field line.

The Hall effect

A flat electric conductor, through which an electric current is flowing, is brought in a magnetic field; the field strength vectors are perpendicular to the conductor surface and perpendicular to the current direction, Fig. 2.53.

In the figure, the current direction (direction of the current density vector) is from the bottom to the top. We first assume to deal with positive mobile charge carriers, Fig. 2.53a. Their direction of movement is the same as the direction of the electric current. The magnetic field now pushes the charge carriers to the left (three-finger-rule of the right hand). They accumulate on the left side so that some of them are missing on the right whereby the right side takes on a negative charge. Thereby, an electric field develops which pulls the charge carriers to the right. Shortly after the current is switched on, the push of the magnetic field to the left becomes equal to the pull of the electric field to the right. That means that the charge carriers can now move in a straight direction.

The momentum current, that comes over the electric field, i.e.

$$F_{el} = e \cdot E\,,$$

is now equal to the momentum current

$$F_{mag} = e \cdot v \cdot B$$

that comes over the magnetic field. Hence, we have:

$$E = v \cdot B$$

This creation of an electric charge at the surface of the conductor is called *Hall effect*.

The relation between the electric field strength E in the conductor and the voltage U_H between the two faces is

$$U_H = E \cdot d\,.$$

Hence,

Hall effect

$$U_H = v \cdot B \cdot d \qquad (2.8)$$

U_H can be easily measured.

There is a variety of applications for the Hall effect. We would like to introduce two of them.

Plus/minus sign of the charge carriers

We have seen: if the mobile charge carriers are positively charged, the left side in Fig. 2.53a will become positively charged. Now we assume that the same electrical current is created by negative charge carriers, Fig. 2.53b. The charge carriers in the figure must flow from the top to the bottom so that the current direction remains the same as before. In relation to their direction of movement, they are now deviated to the right, i.e. to the left from our perspective. (When applying the three-finger-rule we need to reverse the direction of deviation of the particles because they carry negative charge.) The left side (from our perspective) will therefore take on a negative charge, the right side a positive charge. We can therefore tell from the charge which plus/minus sign the charge carriers have. For most metals, the mobile charge carriers are electrons, i.e. negative particles. For some metals and for many semiconductors, the mobile charge carriers are positive particles, the so-called holes. A hole can be imagined as a missing electron in a large “lake of electrons”, similar to the bubbles in mineral water that can be regarded as missing water. The bubble behaves like a body with a negative mass (as it raises to the top instead of falling down). A hole in the lake of electrons of an electric conductor behaves accordingly like a particle with a positive electric charge.

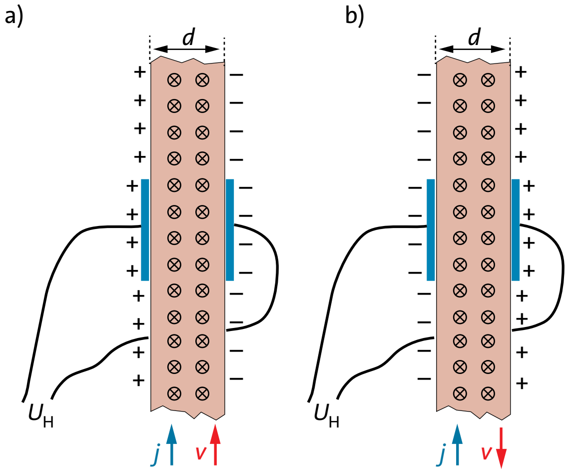


Fig. 2.53 Hall effect. Direction of the magnetic field strength: into the image plane.

The Hall sensor

The arrangement of Fig. 2.53 can be used to measure the magnetic flux density or field strength. Then, it is called a Hall sensor. When a constant electric current is sent through a sensor, the voltage U_H is proportional to the magnetic flux density and field strength.

Measuring U_H therefore means measuring the magnetic field strength at the same time. Most magnetic field meters take advantage of this effect. As a Hall sensor is very cheap and robust, it is also used as a sensor for the position of anything in the car: are the doors closed? Is the seatbelt worn? How fast does the crankshaft turn? A small magnet is always fastened somewhere, and the Hall sensor detects whether the magnet is positioned in the desired place.

Exercises

1. A straight wire, in which an electric current of 200 A is flowing, is placed perpendicularly to the field lines of the magnetic field of the Earth (field strength 40 A/m). Which momentum current flows into a piece of the wire with a length of 1 m?
2. A beam of electrons enters a homogeneous magnetic field with H = 2400 A/m from a field-free space, Fig. 2.54. Which kinetic energy (in eV) must the electrons have in order to leave the field area at a right angle to the input direction (to the bottom in the figure)?
3. An electric current of 2.5 A flows through a coil with an inductance of 0.01 mH. The coil is 10 cm long and has a cross-sectional area of 4 cm^2. (a) How much energy is contained in the field of the coil? (b) What is the energy density inside the coil?
4. Two small material samples are equipped with contacts in a way that an electric current can be sent through them in a longitudinal direction and that the Hall voltage U_H can be measured in the respective transversal direction. Both samples have a width of 5 mm (distance of the contacts between which the Hall voltage is measured). They are brought in a magnetic field with the flux density 0.2 T and a current of 200 mA is let flow. On one, a Hall voltage of 0.12 mV is measured, on the other 0.36 μV. What is the speed of the charge carriers in the two cases? What could be the cause of the great difference?

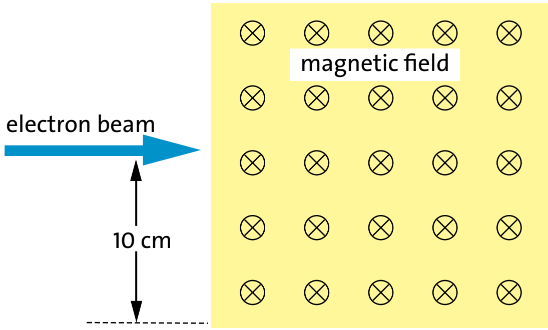


Fig. 2.54 For exercise 2

3 THE INTERPLAY BETWEEN ELECTRIC AND MAGNETIC FIELDS

3.1 Analogy in electrodynamics

We have learned a lot about two categories of natural phenomena: first about electricity, then about magnetism. You have certainly seen that there are similarities between the two. We say that there is an *analogy*. The two categories have the same conceptual and mathematical structure.

The two columns of Table 3.1 contain elements that are corresponding: names of concepts, physical quantities and formulas. The table also contains some aspects that have no corresponding term in the other

Electric field	Magnetic field
electric charge Q	magnetic charge Q_m
electric potential φ	magnetic potential φ_m
electrically charged particle (electron, proton,..)	Magnetically charged particles do not exist.
electric current I	Magnetic currents do not exist.
polarization $\vec{P}$	magnetization $\vec{M}$
electric field strength $\vec{E}$	magnetic field strength $\vec{H}$
$\vec{F} = Q \cdot \vec{E}$	$\vec{F} = Q_m \cdot \vec{H}$
spherically symmetric charge distribution $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$	spherically symmetric charge distribution $H = \frac{1}{4\pi\mu_0}\frac{Q_m}{r^2}$
inside an electric conductor: $\vec{E} = 0$	inside a soft-magnetic material: $\vec{H} = 0$
mechanical stress $\sigma_{\parallel} = -\frac{\varepsilon_0}{2}\lvert\vec{E}\rvert^2 \quad \sigma_{\perp} = \frac{\varepsilon_0}{2}\lvert\vec{E}\rvert^2$	mechanical stress $\sigma_{\parallel} = -\frac{\mu_0}{2}\lvert\vec{H}\rvert^2 \quad \sigma_{\perp} = \frac{\mu_0}{2}\lvert\vec{H}\rvert^2$
energy density $\rho_E = \frac{\varepsilon_0}{2}\lvert\vec{E}\rvert^2$	energy density $\rho_E = \frac{\mu_0}{2}\lvert\vec{H}\rvert^2$
Capacitor	**Coil**
$Q = C \cdot U$ capacitance C $\quad C = \varepsilon_0 \frac{A}{d}$	$\Phi = (1/n) \cdot L \cdot I$ inductance L $\quad L = \mu_0 n^2 \frac{A}{l}$
energy $E = \frac{C}{2}U^2$	energy $E = \frac{L}{2}I^2$
decay of the voltage: $U = U_0 e^{-t/\tau} \quad \tau = RC$	decay of the electric current: $I = I_0 e^{-t/\tau} \quad \tau = \frac{L}{R}$

Table 3.1 Analogy between electric and magnetic phenomena

Electricity	Magnetism	Gravitation
electric charge Q	magnetic charge Q_m	mass m
electric potential φ	magnetic potential φ_m	gravitational potential ψ_m
electric field strength $\vec{E}$	magnetic field strength $\vec{H}$	gravitational field strength $\vec{g}$
$\vec{F} = Q \cdot E$	$\vec{F} = Q_m \cdot \vec{H}$	$\vec{F} = m \cdot \vec{g}$
spherically symmetric charge distribution $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$	spherically symmetric charge distribution $H = \frac{1}{4\pi\mu_0}\frac{Q_m}{r^2}$	spherically symmetric mass distribution $g = G\frac{m}{r^2}$

Table 3.2 Analogy between electricity, magnetism and gravitation

column. Finally, there are also concepts – with a blue background – that have not been relevant in our context and have therefore not been addressed yet.

Although it is not our topic at the moment, we would also like to recall another analogy at this point. A third area can be added to these two: gravitation, Table 3.2. Here, the analogy does not have the same extent – for a simple reason: while both the electric and the magnetic charge can be positive and negative, there are only positive masses. Therefore, there is no gravitational analogue for some effects of electricity and of magnetism. We will only be able to understand the fact that gravitation is the more extensive field of physics if we know the complex *General Theory of Relativity.*

One important phenomenon is not listed in the table: an *electric* current is the cause of a *magnetic* field. This phenomenon shows that electricity and magnetism are not only structured analogously, but that they are also closely interrelated. By applying our analogy, we could formulate the following expectation:

Just as an electric current creates a magnetic field, a magnetic current could possibly create an electric field.

But we know that magnetic currents do not exist. So is there no inversion of this phrase either? We will see in this chapter that this inversion does indeed exist.

3.2 Electromagnetic induction

A voltmeter is connected to a coil. If one pole of a permanent magnet is moved into the coil, Fig. 3.1, the pointer of the voltmeter deflects, but only as long as the magnetic pole is moving. When the magnetic pole is removed from the coil, the meter deflects again, this time in the opposite direction.

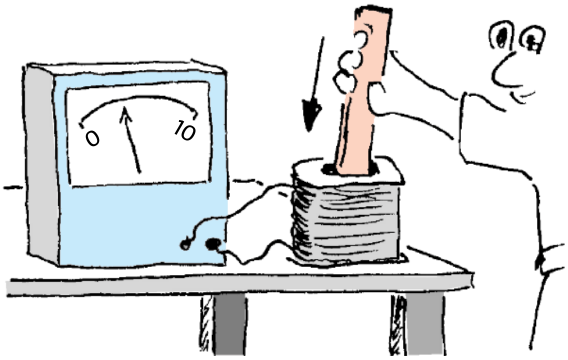


Fig. 3.1 The pointer of the voltmeter deflects as long as the permanent magnet is moving.

The direction of the voltmeter deflection also depends on whether the north or the south pole are moved into the coil.

We examine what will happen if the coil is short-circuited in the experiment and we build an ammeter into the circuit. Also the ampere meter deflects when the magnetic pole is moved into the coil and again when it is removed (you might have expected that).

These phenomena are called *electromagnetic induction.* We say that an electric voltage or an electric current is induced while the magnet is being moved.

A voltage (or a current) can also be induced in another way and without moving anything: by placing an electromagnet next to the coil so that its field reaches into the coil, Fig. 3.2. If the electromagnet is switched on or off, a voltage will be induced in the coil again.

> When the magnetic field strength changes within the coil, a voltage is created between the connections of the coil. In case of a closed circuit, an electric current is flowing. This process is called electromagnetic induction.

Finally, we realize another variant of the induction experiment. We put a soft iron core into the coil and extend the ends of the core in a way that the entire core forms a “U”. Then, no magnetic field can enter the coil anymore. Will the induction stop as well? We move a permanent magnet closely to the ends of the soft iron core, Fig. 3.3, until the poles of the magnet touch these ends and we will observe a deflection of the voltmeter. How is this possible? The iron in the coil has been magnetized, its magnetization has changed.

> If the magnetization of the material in the coil changes, a voltage is also induced.

We had seen that with the magnetic field strength $\vec{H}$ and the magnetization $\vec{M}$ we can form one single physical quantity, i.e. the magnetic flux density $\vec{B}$:

$$\vec{B} = \mu_0(\vec{M} + \vec{H})$$

Hence, we can summarize our observations as follows:

> If the magnetic flux density in a coil changes, a voltage is created between the connections of the coil.

We will now examine what the value of the induced voltage depends upon. Therefore, we choose a particularly simple arrangement: a small, flat coil is brought into the homogeneous field of a large, long coil. A voltmeter is connected to the small coil. If the electric current in the large coil is changed, the magnetic field strength will change and a voltage is induced between the connections of the small coil.

We now connect the large coil to a power supply that supplies a current which grows linearly with the time. Therefore, we obtain a magnetic field whose flux density grows linearly with the time, Fig. 3.4.

Of course, this is only possible for a limited time, but still long enough for our observation. We can see that the voltage induced at the small coil is constant over time. Hence,

$$\frac{dB}{dt} = \text{const} \;\Rightarrow\; U = \text{const}.$$

If the flux density is changed faster, i.e. if dB/dt is greater, the induced voltage will also be greater. More precisely: if dB/dt doubles, the induced voltage will also double. Thus, U is proportional to dB/dt:

$$U \sim \frac{dB}{dt}. \qquad (3.1)$$

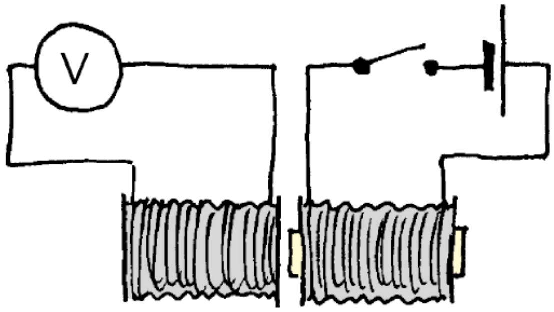


Fig. 3.2 If the electromagnet is switched on or off, the magnetic field strength in the coil will change and a voltage will be induced.

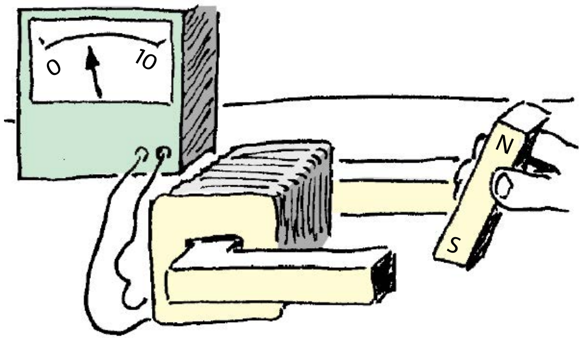


Fig. 3.3 Also the change of the magnetization inside the coil causes an induced voltage.

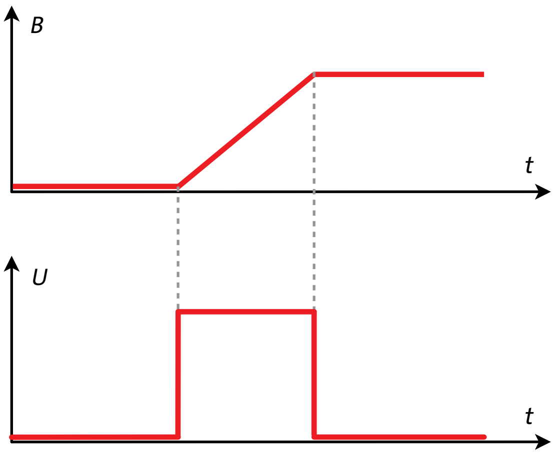


Fig. 3.4 If the flux density changes linearly with the time, the induced voltage is constant over time.

We will now examine another way in which the induced voltage can be influenced.

If the small coil is replaced by another one whose only difference is the cross-sectional area A (the number of turns shall remain the same), we find that the induced voltage is proportional to this area:

$$U \sim A \qquad (3.2)$$

This result could have been predicted. A coil with the double surface is equal to two coils that are placed next to each other and that are connected in series, Fig. 3.5.

If finally the coil is replaced by another one whose only difference is the number of turns, we will find that the induced voltage is proportional to the number of turns n:

$$U \sim n \qquad (3.3)$$

Also this result is not surprising: a coil with the number of turns $2\,n$ is equivalent to two coils connected in series that have a number n turns each.

The results (3.1), (3.2) and (3.3) can be summarized to one single relationship:

$$U \sim nA\frac{dB}{dt}\,. \qquad (3.4)$$

Now, the proportionality sign may be replaced by an equal sign because the flux density has been defined in such a way that there is no other factor in equation (3.4). Hence:

$$U = nA\frac{dB}{dt}\,. \qquad (3.5)$$

Equation (3.5) is almost our final result. Earlier, we had abbreviated the product $A \cdot B$:

$$A \cdot B = \Phi$$

Φ is the magnetic flux. Therefore, from equation (3.5) we obtain *Faraday's law of induction*:

$$U = n\frac{d\Phi}{dt} \qquad \text{Faraday's law of induction} \quad (3.6)$$

However, it contains more in this formulation than we have originally put in. To induce a voltage, we have changed the magnetic flux Φ, and in order to change the flux, we have changed the flux density B. If equation (3.6) is correct, there should also be an induced

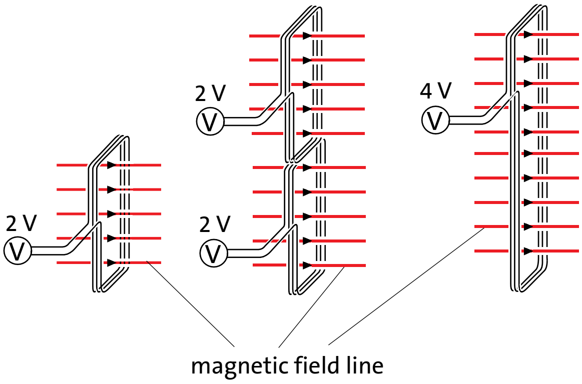


Fig. 3.5 The flux density grows over time. If the coil area is doubled, the induced voltage will also double.

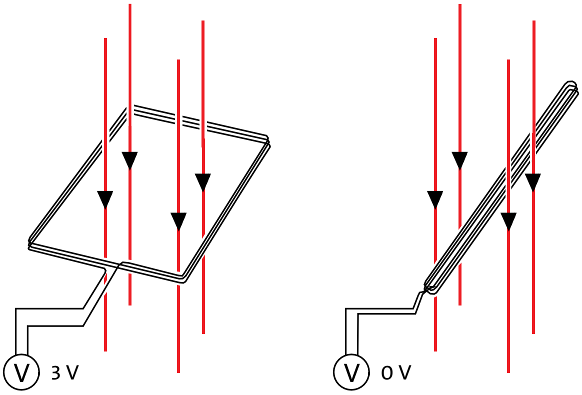


Fig. 3.6 The flux density is constant over time. The coil is deformed in a way that the area, that is crossed by the field lines, is reduced. Also in this process, a voltage is induced.

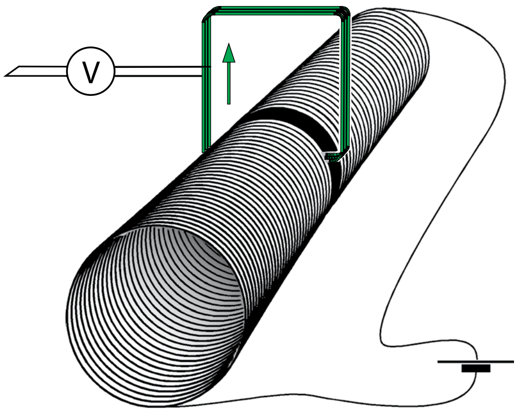


Fig. 3.7 If the small, quadratic coil is moved out of the large one, the area that is crossed by the field lines will decrease.

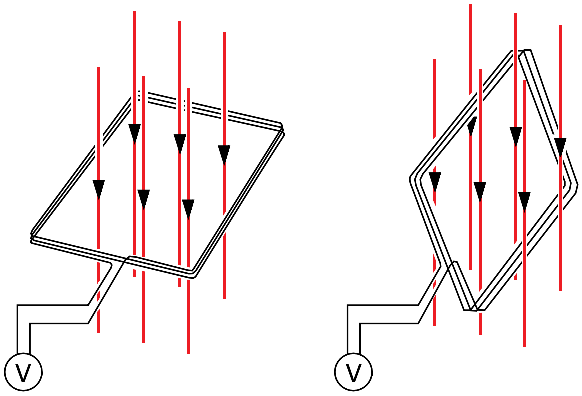


Fig. 3.8 Rotating the coil also leads to a change of the magnetic flux through it.

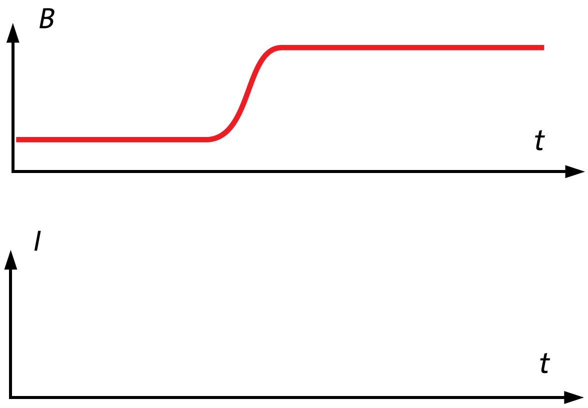


Fig. 3.10 For exercise 2

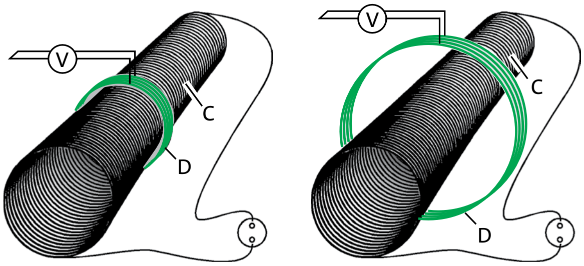


Fig. 3.9 The induced voltage is equal on the left and on the right because the changing flux through D is equal both times.

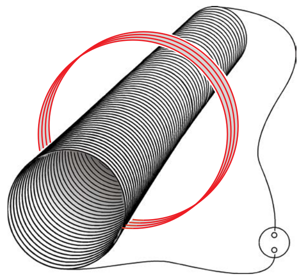


Fig. 3.11 For exercise 4

voltage in the case that B is left constant while the area A is changed. Fig. 3.6 shows a method to achieve it.

But there is an even simpler way: the coil is moved out of the constant magnetic field, Fig. 3.7. The area that is crossed by the field lines decreases, and therefore also the magnetic flux. Again, a voltage is induced, which is also the case, of course, when the coil is moved back into the field.

A particularly practical way to change the magnetic flux is shown in Fig. 3.8: the coil is rotated. This also leads to a change of the flux and to the creation of an induced voltage. The induction in an electric generator is realized in this way.

An interesting variant of an induction experiment is shown in Fig. 3.9.

We change the electric current and consequently also the flux density in coil C. Coil D, between whose connections a voltage is induced, is completely outside of C. The cross-sectional area in the experiment on the left is smaller than on the right. However, the value of the induced voltage does no longer depend on the crosssectional area of D as the magnetic flux is limited to that of C.

Exercises

1. The flux density of a homogeneous magnetic field increases linearly within 2 seconds from 0 T to 0.3 T. There is a flat coil with 200 turns in the field. The coil area is parallel to the field surfaces of the magnetic field, the surface area is 8 cm^2. What is the voltage that is created between the connections of the coil?
2. The flux density of a homogeneous magnetic field changes over time in a way shown in Fig. 3.10. There is a closed metal ring in the field. The ring area is perpendicular to the field lines. Qualitatively sketch the temporal course of the electric current in the ring in an I-t-diagram.
3. Inside a large coil with a length of 0.5 m and 2000 turns, there is a small, flat coil with n = 500 and A = 15 cm^2. Both coils have the same orientation.
(a) Calculate the magnetic field strength in the large coil while an electric current of 10 A is flowing.
(b) Calculate the magnetic flux density.
(c) In which time does the electric current in the large coil have to increase from 0 A to 10 A so that a voltage of 100 V is induced in the small coil?
4. In a long, thin coil with a cross-sectional area of 2 cm^2 the flux density increases with 0.2 T/s, Fig. 3.11. The coil is enclosed by a ring that is not a good conductor. The resistance of the ring is 200 Ω. What is the current of the current induced in the ring?

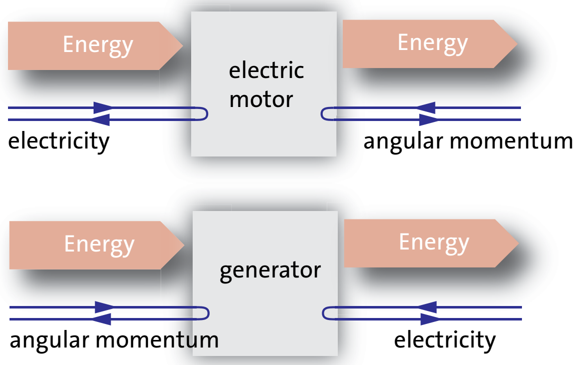


Fig. 3.12 Flow charts of electric motor and generator

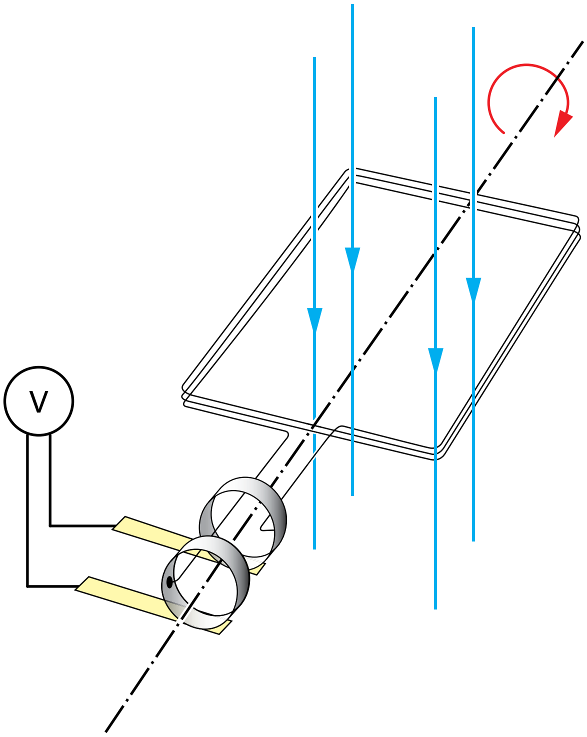


Fig. 3.13 Simple generator: a rectangular, flat coil is rotated in a homogeneous magnetic field. The magnetic flux through the coil thereby changes periodically.

3.3 The generator

The generator is a machine that is part of all power plants. It does the opposite of an electric motor. While an electric motor is supplied with energy with the energy carrier electricity (electric charge) and releases this energy with angular momentum as a carrier, Fig. 3.12a, the generator is provided with energy carried by angular momentum and releases it with electricity, Fig. 3.12b.

Small generators often have a different name: in bicycles, they are called dynamo; in cars alternator.

The structure of a generator is generally not different from that of an electric motor. Some electric motors can even be operated directly as generators. This only requires the replacement of the electric energy source by an electric energy receiver, e.g. a light bulb. If the shaft is turned, the lamp will light up.

To understand the functioning, we look at a particularly simple version of a generator: a rectangular, flat coil is rotated in the homogeneous field of a permanent magnet, Fig. 3.13. The magnetic flux through the coil is changing continuously in the process. We would like to calculate how it changes, i.e. how the function $\Phi(t)$ looks like. Then, we can determine the induced voltage as a function of the time by means of Faraday's law of induction.

Fig. 3.14 shows the arrangement in the cross-section. One side length of the coil is l, the other one b. Hence, the coil area A_0 is

$$A_0 = l \cdot b$$

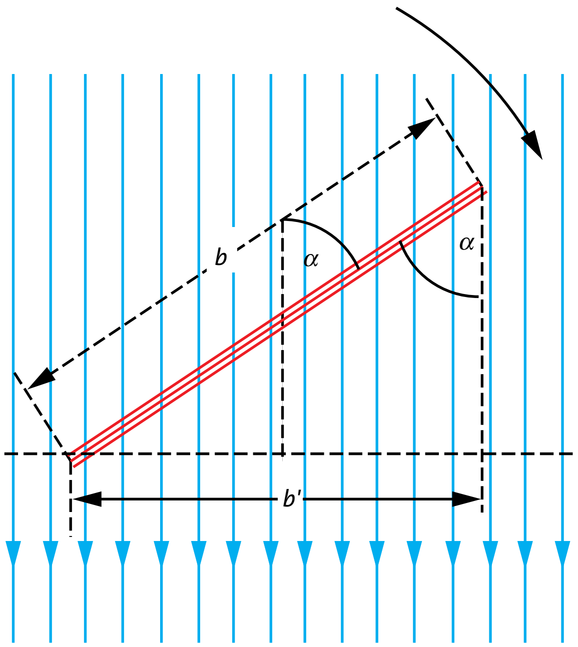


Fig. 3.14 Cross-section through the arrangement from Fig. 3.13. The magnetic flux through the coil is calculated by projecting the coil area on a plane perpendicular to the field lines.

In Fig. 3.14 only b can be seen.

The coil is turned with the angular velocity

$$\omega = \frac{\alpha}{t}.$$

The angle α must be measured in radian, i.e. the value for a complete circle is 2π. The angular velocity is constant over time. Consequently, α increases linearly with time:

$$\alpha = \omega \cdot t \qquad (3.7)$$

To calculate the magnetic flux

$$\Phi = B \cdot A$$

we may not insert the area A_0 of the coil here. The area that is traversed by the magnetic flux is only the projection of that area on a plane perpendicular to the field lines of the magnetic field. This projection changes over time. It reaches its maximum when the coil is perpendicular to the field lines. It is zero when the coil is parallel to the field lines.

From Fig. 3.14, we can see how to calculate this area. With

$$\sin\alpha = \frac{b'}{b}$$

we obtain

$$A = l \cdot b' = l \cdot b \cdot \sin\alpha = A_0 \cdot \sin\alpha\,.$$

With equation (3.7), we get

$$A(t) = A_0 \cdot \sin(\omega t)$$

Hence, we obtain the magnetic flux as a function of time:

$$\Phi(t) = B \cdot A_0 \cdot \sin(\omega t) \qquad (3.8)$$

The magnetic flux through the coil therefore changes with time in accordance with a sine function. To calculate the induced voltage, we insert equation (3.8) in Faraday's law of induction:

$$U(t) = n \cdot \frac{d\Phi}{dt} = n \cdot B \cdot A_0 \cdot \frac{d(\sin(\omega t))}{dt}.$$

With

$$\frac{d(\sin(\omega t))}{dt} = \omega \cdot \cos(\omega t)$$

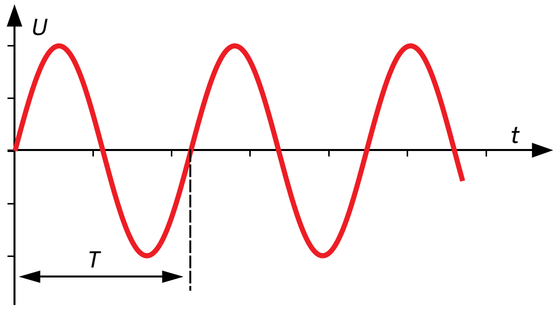


Fig. 3.15 Voltage as a function of time for the generator from Fig. 3.13.

we obtain

$$U(t) = n \cdot B \cdot A_0 \cdot \omega \cdot \cos(\omega t)$$

We summarize the constant factors ahead of the cosine function:

$$U_0 = n \cdot B \cdot A_0 \cdot \omega$$

and obtain for the induced voltage:

$$U(t) = U_0 \cdot \cos(\omega t)$$

In our calculation, we have obtained a cosine function. However, it can be transformed into a sine function by displacing the zero point of the time. We would then obtain

$$U(t) = U_0 \cdot \sin(\omega t)$$

The voltage as a function of time is shown in Figure 3.15.

Real technical generators have a more complicated structure than the one we have just seen. The underlying physical principle, however, is the same as in our rectangular coil generator. And the reason for them generating a sine voltage is also the same.

Exercises

1. Invent a generator that creates a voltage whose plus/minus sign does not change.
2. Will the sine voltage remain a sine voltage if the rotating coil is not rectangular but circular?
3. Will the sine voltage remain a sine voltage if the magnetic field is not homogeneous?

3.4 Alternating voltage and alternating current

A voltage that has a sine-shaped time-dependence is called *alternating voltage*. If an alternating voltage is applied to a resistor, there will be an *alternating current*.

With

$$U = R \cdot I$$

we obtain

$$I(t) = \frac{U_0 \cdot \sin(\omega t)}{R} = I_0 \cdot \sin(\omega t).$$

Here, we abbreviated U_0/R with I_0. The pre-factor ahead of the sine (or cosine) function is called *amplitude*. Hence, U_0 is the amplitude of the voltage, I_0 is the amplitude of the current.

Fig. 3.15 also shows the period T. It is the time interval in which a full sine oscillation is passed. For $t = 0$, the sine function has a zero crossing, for $t = T$ it has another one, after having passed a full oscillation. Hence, the period is the time interval between two adjacent points in time that are equivalent to each other. For $t = T$, the argument of the sine function is equal to 2π. Therefore, we obtain

$$\omega T = 2\pi$$

or

$$\omega = \frac{2\pi}{T}. \qquad (3.9)$$

The frequency f, that is a measure for the number of oscillations per time interval, is related to the period according to

$$f = \frac{1}{T}. \qquad (3.10)$$

Equations (3.9) and (3.10) can be combined to

$$\omega = 2\pi f.$$

Hence, the factor ω in the argument of the sine function is equal to the frequency except for a factor 2π. As it is used in physics very often, it has a proper name: *angular frequency*.

Here some more data about the alternating voltage of the socket.

You know that the frequency is 50 Hertz (in the US 60 Hz):

$$f = 50 \text{ Hz}$$

Consequently, the alternating voltage passes through 50 full oscillations every second.

One of the two contacts of the socket is grounded, its potential φ1 is 0 V. The respective wire is called *neutral conductor*.

We therefore obtain

$$U = \varphi_2 - \varphi_1 = \varphi_2 - 0 \text{ V} = \varphi_2 .$$

Thus, the value of the voltage is equal to the potential value of the socket contact that is not grounded.

The voltage of the socket is, as we know, 230 volt (in the US 120 V). Or at least this is what it is said to be. But what do these 230 volt mean when the voltage is constantly changing? To understand the meaning of this information, we look at an electric resistor, i.e. an energy consumer that is connected to an alternating voltage source. The energy current that flows to the resistor is

$$P = U(t) \cdot I(t) = U_0 \cdot \sin(\omega t) \cdot I_0 \cdot \sin(\omega t)$$
$$= U_0 \cdot I_0 \cdot \sin^2(\omega t)$$

The energy current changes over time as well. The function graph of $P(t)$ is shown in Fig. 3.16. Fig. 3.16a shows the voltage once again. The function $P(t)$ essentially has the same shape as the sine function of the voltage, but it oscillates

- between 0 and +1, instead of between –1 and +1
- twice as fast.

In maths class, you get to know the relationship

$$\sin^2 \alpha = \frac{1 - \cos(2\alpha)}{2}.$$

The fact that the energy current $P(t)$ always remains positive can be easily understood in physical terms: it always flows to the resistor, regardless of whether the electric current flows in one or in the other direction.

We are now interested in the time average $\overline{P}$ for the energy current. From Fig. 3.16 we see:

$$\overline{P} = \frac{U_0 \cdot I_0}{2}.$$

Now we define the *effective voltage* (or root mean square voltage):

$$U_{\text{eff}} = \frac{U_0}{\sqrt{2}};$$

and in addition the *effective current* (or root mean square current):

$$I_{eff} = \frac{U_{eff}}{R} = \frac{U_0}{R \cdot \sqrt{2}} = \frac{I_0}{\sqrt{2}}.$$

and express the average energy current through U_{eff} and I_{eff}:

$$\bar{P} = \frac{U_0 \cdot I_0}{2} = \frac{U_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} = U_{eff} \cdot I_{eff}.$$

When U_{eff} and I_{eff} are multiplied, we obtain the average energy current. This means that U_{eff} and I_{eff} can be used in the same way as a direct voltage and a direct current since the following applies for them:

$$\bar{P} = P = U \cdot I$$

Back to the socket: the 230 V of the socket are the effective voltage. Also, the value displayed by the voltmeter when measuring an alternating voltage is the effective voltage, and an ammeter indicates the effective current.

You have seen how the alternating voltage is created. We could think that such a voltage is not very practical and that it would be better to rectify it right after the generator or to use a direct current generator from the beginning. In fact, there are somehow more complicated generators that supply a direct voltage.

The fact that it is not done is due to a significant advantage of alternating voltages compared to direct voltages: they can be increased and reduced in a convenient way by means of a transformer.

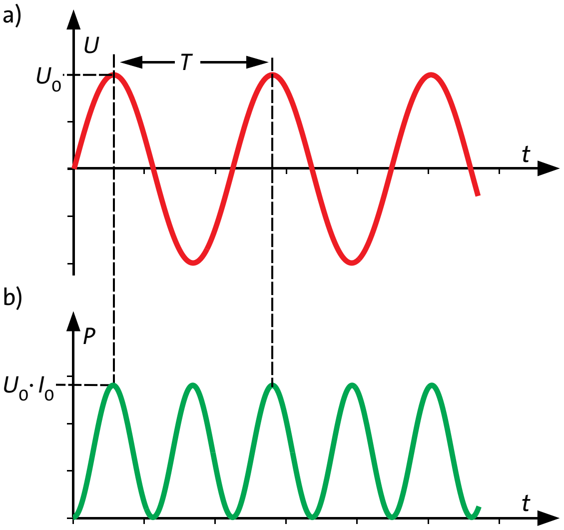


Fig. 3.16 (a) Alternating voltage, applied to a resistor; (b) Energy current flowing to the resistor; both as a function of time

Exercises

1. What is the value of the amplitude of the alternating voltage of the socket?
2. Explain on the basis of Fig. 3.16 why the average value of P is equal to $U_0 \cdot I_0/2$.
3. Someone thinks that the average value of the energy current strength P is equal to the product of the average value of the voltage U and the average value of the electric current I. Is this correct?

3.5 The transformer

The TV, the computer and any other electronic devices require a much lower voltage than the 230 V of the socket. If these devices are to be connected, the voltage must be transformed from 230 volt down to a lower value. For this purpose, a transformer is installed between the device and the socket.

To transport energy electrically over long distances, it is reasonable to use a high voltage. The energy losses during transportation are lower than in case of a low voltage. Hence, a transformer, that changes the voltage to a higher value, is installed directly behind the generator. The energy is subsequently transported over a long distance with a high voltage power line. At the destination, the voltage is transformed back down by another transformer.

Of course, no energy should be lost in the transformer, which is almost achieved.

Hence, we have

$$P_1 = P_2.$$

The index 1 refers to the input, the index 2 to the output of the transformer.

With

$$P = U \cdot I$$

we have

$$U_1 \cdot I_1 = U_2 \cdot I_2. \qquad (3.11)$$

When the transformer raises the voltage by a given factor, the current will be smaller by the same factor. If the voltage is increased ten-fold, the electric current will be one tenth so that the energy current remains equal.

How does a transformer work? The phenomenon, which is taken advantage of also in this case, is induction.

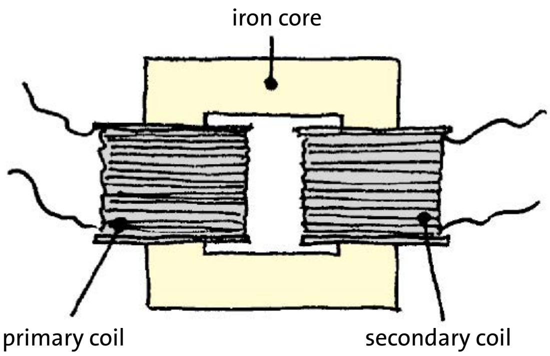


Fig. 3.17 A transformer consists of a soft iron core iron core and two coils.

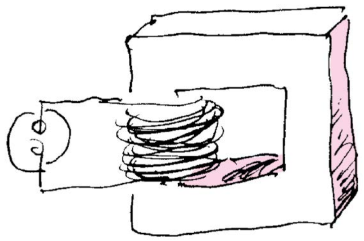


Fig. 3.18 "Transformer" without secondary coil. If an alternating current flows in the (primary) coil, the magnetization in the iron core will change in a sine function shaped way.

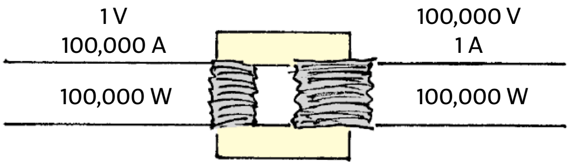


Fig. 3.19 For exercise 4

A transformer consists of an iron core with two coils, Fig. 3.17. The terminals of the primary coil form the input for the energy, the terminals in the secondary coil form the output.

First, we look at a "transformer" that does not have a secondary coil, Fig. 3.18. This structure can also be regarded as an electromagnet in which no gap has been built in the iron core by accident; compare with Fig. 2.42.

If the (primary) coil is connected to an alternating voltage source, an alternating current will flow in the coil. This leads to a magnetization of the iron core. The magnetization follows the electric current in the process: it changes its direction periodically. We add the secondary coil once again. As long as an alternating current is flowing in the primary coil, the magnetization inside the secondary coil is constantly changing, and so is the magnetic flux density, which leads to the creation of a voltage between the terminals of the secondary coils. When there is a sine voltage on the primary coil, the induced voltage at the secondary coil also has a sine-shaped time dependence.

How can we then raise or reduce a voltage by means of a transformer? The value of the induced voltage depends on the number of turns of the two coils. We would like to examine in what way.

Therefore, we build transformers with coils with different numbers of turns. We find at first that, when the number of turns of the primary and the secondary coil are equal, the primary and the secondary voltage are also equal. If the secondary coil has twice as many turns as the primary coil, the secondary voltage is also twice as high as the primary voltage. In general, we have:

$$\frac{U_1}{U_2} = \frac{n_2}{n_1} .$$

With equation (3.11) we obtain

$$n \cdot I_1 = n_2 \cdot I_2 .$$

Exercises

1. The two coils of a transformer have 1000 and 5000 turns, respectively. There is an alternating voltage of 230 V. Which voltages can be created with the transformer?
2. The primary coil of a transformer is connected to the socket. A voltage of 11.5 V is measured at the secondary coil. What can be said about the number of turns of the transformer coils? There is an electric current of 2 A in the secondary circuit. What is the current in the primary circuit?
3. A transformer has a primary coil with 1000 turns and a secondary coil with 10 000 turns. The primary coil is connected to the socket. There is a primary current of 100 mA. What are the values of the secondary voltage and the secondary current?
4. An energy current of 100 kW is flowing through the transformer in Fig. 3.19. What are the requirements with regard to the inputs and to the outputs?
5. A power plant supplies an energy current of 60 MW for an industrial area. The transmission line has an overall resistance (feed and return line) of 0.1 Ω. Look at two cases: the voltage that is used for the transmission is (1) 3000 V or (2) 300 000 V. (a) What is the electric current in the two cases? (b) Which voltage is created between the start and the end of the wires? (c) Which voltage is left for the consumer? (d) How much energy is lost?

3.6 A somehow peculiar generator – "magnetic currents"

We need a copper pipe and a strong bar-shaped magnet whose outer diameter is a bit smaller than the inner diameter of the pipe.

We hold the pipe in a vertical position and let small objects fall through it. This way, we can see that the pipe is not obstructed. We then insert the magnet in the upper pipe aperture, Fig. 3.20. Surprisingly, it does not simply fall through the pipe. Although it finally appears at the lower pipe end, it has taken much time for the movement.

While the non-magnetic objects come out with a high speed, i.e. have much energy (from the gravitational field), the magnet has almost no kinetic energy. Why does it fall so slowly? Where has the energy gone that it should normally have?

While moving to the bottom, there is a changing magnetic field in the material of the pipe. Hence, an electric current is induced in the copper. The respective current lines are circles around the pipe axis. The current only flows where the magnetic field changes, i.e. in close proximity to the poles. There is consequently an electric current close to one pole and another one close to the other pole.

The current close to the positive magnetic pole (the north pole) flows in one direction, the current close to the negative pole in the other direction. There is a simple rule for the direction of this current, Fig. 3.21: when the thumb of the left hand points in the direction of movement of the positive pole, the flexed fingers indicate the direction of the induced electric current. Caution! You really have to use your left hand (it is not a misprint).

We can reformulate this rule to make it easier to memorize.

When a current is flowing in the copper pipe, there must be an electric field in the pipe. We remember the following relation:

$$\vec{j} = \sigma \vec{E}.$$

An electric current will only flow if the electric field strength is different from zero. The electric field is a driving force for the electric current.

We could also say:

> A changing magnetic field creates an electric field.

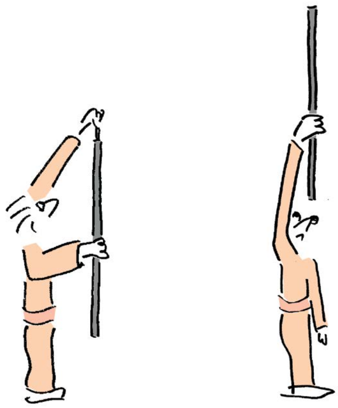


Fig. 3.20 Left side: Willy lets a magnet fall into an open copper pipe; right: it takes some time until the magnet comes out again at the bottom.

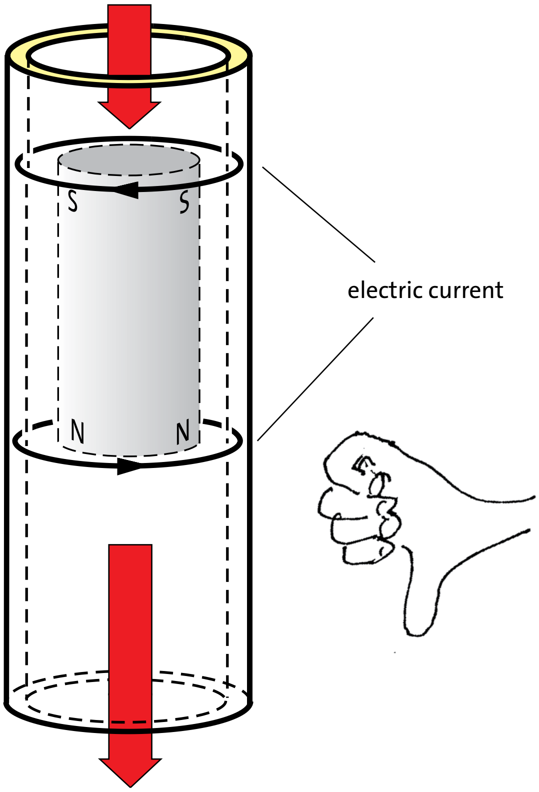


Fig. 3.21 The thumb of the left hand points in the direction of movement of the positive magnetic pole (the direction of the "magnetic current"), the flexed fingers indicate the direction of the induced current.

This phrase is nothing but a new formulation of Faraday's law of induction.

Finally, there is a third way to describe the matter: a magnetic charge moves from the top to the bottom. Just as we describe a moving electric charge as an electric current, we could interpret the moving magnetic charge as a magnetic current. Therefore, we could also formulate the rule as follows:

> Each magnetic current is surrounded by an electric field. The field lines embrace the current. When the thumb of the left hand points in the direction of the magnetic current, the flexed fingers point in the direction of the electric field strength.

Normally, we do not talk about magnetic currents; we even say that there are no magnetic currents at all due to the fact that magnetically charged particles do not exist. However, this statement could be mitigated as we can see from our example: there is a magnetic current for a short time. Shortly after, however, there is a magnetic current of the opposite direction: the negative magnetic pole, that moves downwards, is equivalent to a positive current that moves upward. Hence, we could also say: after every magnetic current, there is a current flowing in the opposite direction. This means that, although there are no direct magnetic currents, there are alternating magnetic currents.

3.7 Superconductors

There are materials that lose their electric resistance when they are cooled down below a certain temperature, Table 3.3. While in their resistance-less state, these materials are called *superconductors*. The transition temperature from the normal to the superconducting state is relatively high for some of these substances: approximately –180 °C. Such substances can be brought to the superconducting state relatively easily by cooling them with liquid nitrogen.

But superconductors are not only interesting because they do not have an electric resistance. They also have surprising magnetic properties.

We build an arrangement of permanent magnets as shown in Fig. 3.22.

We approach the magnet from above with a small piece of superconducting material that we let off. The superconductor does not fall down but it floats above the magnets, Fig. 3.23. It can be turned or pushed slightly aside: it remains in a floating state (of course

material	transition temperature
Zn	0.875 K
Al	1.2 K
Pb	7.2 K
Nb_3Sn	18 K
Nb_3Ge	22.3 K
$Bi_2Sr_2Ca_2Cu_3O_{10}$	110 K
$YBa_2Cu_3O_7$	92 K

Table 3.3

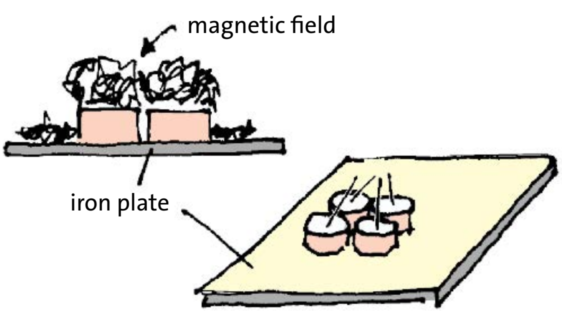


Fig. 3.22 Several magnets on a soft iron plate

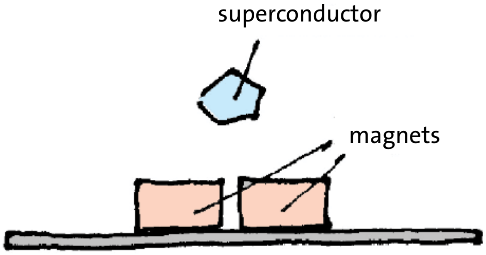


Fig. 3.23 The superconductor is kept in a floating state by the magnetic field.

soft magnetic material	superconducting material
displaces the magnetic field from its inside	displaces the magnetic field from its inside
forms magnetic poles on its surface (magnetic induction)	forms electric currents on its surface
magnetic field lines end on the poles	magnetic field surfaces end on the currents
magnetic field lines merge perpendicularly into the surface	magnetic field surfaces merge perpendicularly into the surface
The magnetic field pulls on the surface	The magnetic field presses onto the surface

Table 3.4 Analogy between soft magnetic and superconducting materials

only until it warms up and returns to the normal state).

Obviously, the superconductor is repelled by the magnets. It therefore behaves just the opposite way as a piece of soft iron which is always attracted. How can this be explained? When the superconductor approaches a magnet, currents start to flow in it, which are oriented in a way as to cause repelling effects. In a normally conducting (i.e. not superconducting) body, these induced currents would immediately stop to flow, they would be slowed down by the resistance of the material. In the superconductor, the currents once induced continue to flow because there is no resistance that could slow down these currents.

A more precise examination, that we will not do at this point, further shows

- that the currents only flow very closely under the surface of the superconductor;
- that the magnetic field does not penetrate the superconductor;
- that the field surfaces merge perpendicularly into the superconductor from the outside.

> If a superconductor is brought in a magnetic field, electric currents will start to flow at its surface. The field is displaced from its inside.
>
> The field surfaces merge perpendicularly into the surface and they end at the surface.

Do these statements sound familiar to you? Go back to section 2.6. Superconductors have properties that are very similar to the characteristics of magnetically soft materials. Just as the magnetically soft materials, superconductors are not permeable for the magnetic field. But they achieve the same effect through another "trick" than the magnetically soft materials: not by forming magnetic poles, but by inducing electric currents.

Fig. 3.24 shows the fields of two related arrangements:

- in the upper image a magnetic pole that is located closely above the plane surface of a magnetically soft body; (the second magnetic pole is so far away that it does not disturb us);
- in the lower image a magnetic pole above a superconducting body.

A magnetic charge has accumulated on the surface of the soft iron. The charge distribution has a rotational symmetry. The center of symmetry and the maximum of the charge distribution are located vertically under the magnetic pole. The field lines end at the magnetic charges in a way that they are perpendicular to the surface. Hence, the magnetic pole draws on the piece of soft iron.

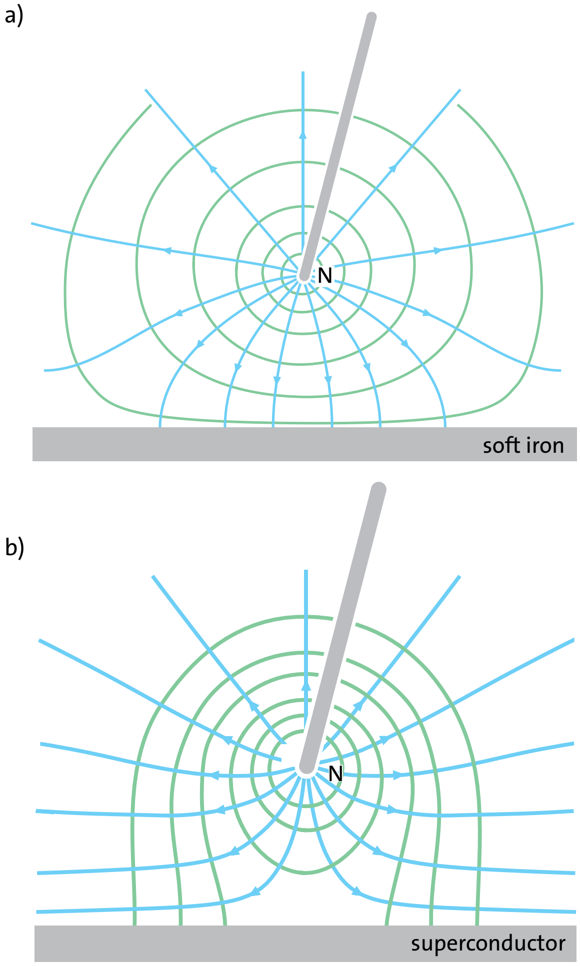


Fig. 3.24 (a) Single magnetic pole above a soft iron body. Another magnetic pole has been formed on the surface of the soft iron. (b) Single magnetic pole above a superconductor. Ring-shaped electric currents have developed on the surface of the superconductor.

Ring-shaped currents have emerged on the surface of the superconductor. The center of the rings is located under the magnetic pole. The field surfaces of the field end on these currents in a way that they are perpendicular to the surface. Hence, the magnetic field presses onto the superconductor.

> The magnetic field presses onto the surface of the superconductor.

Table 3.4 provides a comparative overview of the two materials.

Exercises

1. A cylindrical bar-shaped magnet is located in a long, superconducting pipe, Fig. 3.25a. Draw the magnetic field lines. In Fig. 3.25b, the ring fits exactly in the pipe. How are the magnetic field lines? Which electric currents are flowing in the pipe?
2. Willy, Fig. 3.20, wants to let the bar-shaped magnet fall through a superconducting pipe. What happens?

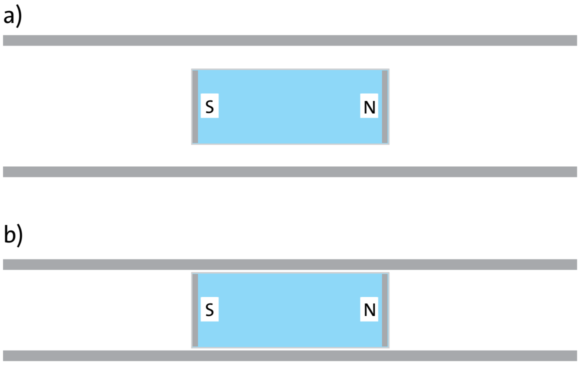


Fig. 3.25 For exercise 1

3.8 Induced electric fields – the interplay between electric and magnetic fields

We look again at the basics of electromagnetic induction and consider the induction process displayed in Fig. 3.26. The changing magnetic field causes an electric current in the ring-shaped, closed conductor.

An electric current is always caused by an electric field that drags the mobile charge carriers in the direction of the conductor. If this is also the case for an induced current, an electric field must have been created in the conductor. We draw the following conclusion:

> The change of the field strength of a magnetic field leads to the creation of an electric field in its surroundings.

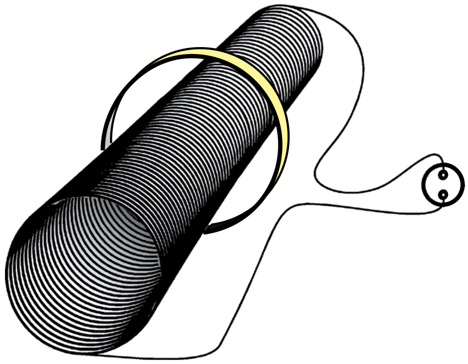


Fig. 3.26 The changing magnetic field causes an electric current in the ring-shaped electric conductor.

This statement does not only apply for the case in which an electric conductor is brought into the magnetic field. It is also true when the conductor is not present. Every time a magnetic field changes, an electric field is created.

In general, calculating the distribution of the electric field strength of such a field is a complicated task. But in the case of certain simple magnetic field changes, there will also be a simple electric field. Fig. 3.27 shows an electromagnet whose current increases so that the flux density will also increase and an electric field will be induced. The figure shows the flux density lines and the field lines of the induced electric field.

The field lines of the electric field loop around the flux density lines.

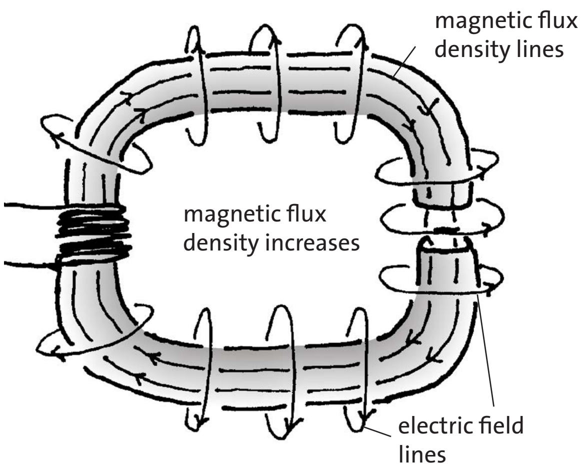


Fig. 3.27 The electric field lines loop around the changing magnetic field lines and the changing magnetization lines.

You have by now gotten used to the analogy between the electric and the magnetic field. Hence, you will not be surprised that the phrase we are currently dealing with also has an analogue:

> By changing the field strength of an electric field, a magnetic field is created in its surroundings.

The direct experimental proof of this statement is not as simple as in the case of the preceding statement about the change of the magnetic field. However, there is a lot of indirect evidence that we will address in the following. Fig. 3.28 shows how the magnetic field looks like in a simple case.

An electric current is flowing in the conductor. As long as the current is flowing, the electric charge on the plates of the capacitor increases and leads to an increase of the electric field strength between the capacitor plates. Here, the field lines of the magnetic field wind around the field lines of the electric field. We had seen before that also the feed lines to the capacitor are surrounded by a magnetic field whose field lines loop around the conductor. You see that the magnetic field behaves as if we had a closed electric circuit in front of us. The interruption by the capacitor has no influence on the magnetic field.

A simple rule can be read from Fig. 3.28. It is a variant of a well-known rule: "Align the thumb of your right hand with the direction of the electric current. The flexed fingers will then point in the direction of the field lines of the magnetic field."

Fig. 3.28 shows how the new rule can be formulated:

> Align the thumb of the right hand with the direction of the increasing electric field strength. The flexed fingers will then point in the direction of the field lines of the magnetic field.

Fig. 3.27 shows that an analogous rule holds for the induction, i.e. the creation of electric fields:

> Align the thumb of the left hand with the direction of the increasing magnetic flux density. The flexed fingers will then point in the direction of the field lines of the electric field.

Bear in mind that you need to use the right hand in one case and the left hand in the other, and that the field strength has to increase. If it decreases, the direction of the thumb must be inverted so that the fingers indicate the correct field line directions.

The calculation of the spatial distribution and the temporal change of the electric and the magnetic field strength is an subject of *Maxwell's theory of electromagnetism*.

James Clerk Maxwell published his theory in 1873. It is one of the most important physical theories. It does not only allow for a calculation of the electric and magnetic field strength distributions, but it also tells us how the energy and the mechanical stresses of the fields can be obtained based on the field strengths.

Maxwell's theory is mostly derived from ideas by Michael Faraday. Faraday discovered, among other

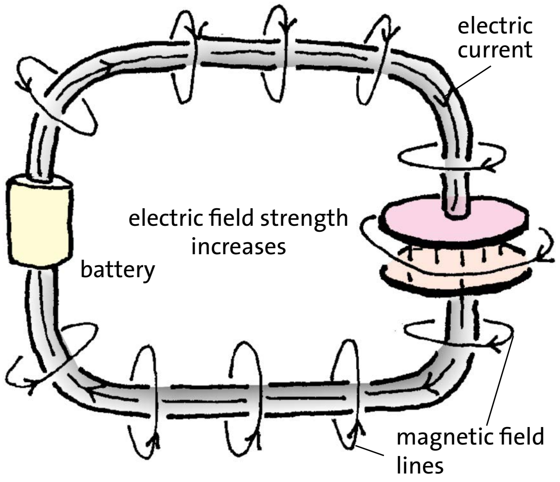


Fig. 3.28 The magnetic field lines loop around the changing electric field lines and the electric current lines.

things, the electromagnetic induction. His greatest achievement, however, was the discovery of the electric and the magnetic field. Before Faraday, the electric and magnetic force effects had been explained as "actions at a distance": an electrically charged body exerts a force on another body although there is no physical connection between the two bodies. Although this concept had been considered as unsatisfactory from the beginning, no evidence had been provided about the existence of what Faraday later called "field".

3.9 Electromagnetic waves

We would now have sufficient knowledge to make a discovery if Maxwell had not already made it more than a hundred years ago.

We look once again at the two rules that we formulated in the previous section:

- *By changing the field strength of a magnetic field, an electric field is created in its surroundings.*
- *By changing the field strength of an electric field, a magnetic field is created in its surroundings.*

We look at a magnetic field whose field strength changes. According to statement 1, an electric field is created around it. When the magnetic field strength changes linearly with time, the field strength of the emerging electric field is constant in time. But if the change of the magnetic field strength is not linear with time, an electric field results whose field strength is also changing.

According to statement 2, however, a changing electric field strength leads to a magnetic field. The change of the magnetic field strength can now cause another electric field, the change of its field strength another magnetic field, and so on.

What we have just described is the formation of an *electromagnetic wave.*

Electric and magnetic fields move through the space in a way that one creates the other while it vanishes.

The appearance of the fields, that are mutually creating themselves, can be very complicated. But there are cases in which the spatial distribution of the field strengths and their temporal changes are very simple. We will examine such special cases.

Therefore, we only look at what the wave is like and do not ask how it is created. The explanation of the creation is more difficult than the explanation of the wave itself. (It is similar to looking at water waves on the sea and to examining only how the waves come from one place to the other and how they change their shape in the process without asking how the waves have been created.)

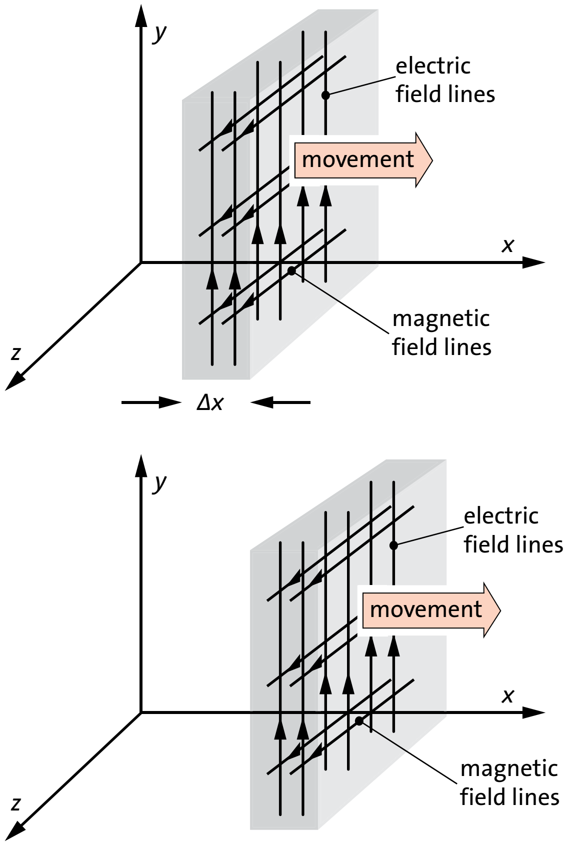


Fig. 3.29 "Snapshots" of a simple electromagnetic wave. The images correspond to two moments in quick succession.

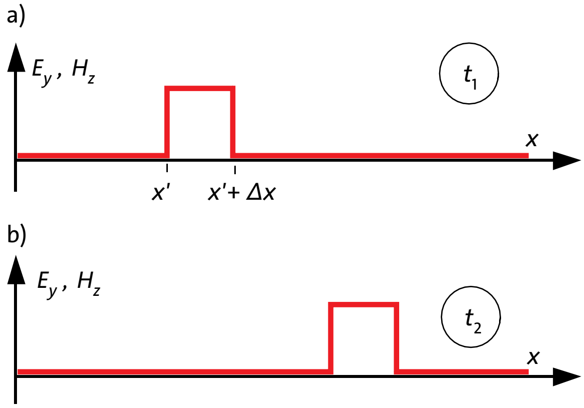


Fig. 3.30 Electric and magnetic field strength of the wave from Fig. 3.29 as a function of the spatial coordinate x at two different times t_1 and t_2.

3.10 The square wave

Our first example is a square wave. It is neither technically interesting, nor significant in nature, but it has the advantage of illustrating a few general characteristics of electromagnetic waves in a particularly clear way. Fig. 3.29 shows the wave at two different times, i.e. two "snapshots". The electric and magnetic field stuff is located in a plate-shaped area with the thickness Δx that extends infinitely in the *y*- and *z*-direction. The direction of movement is the *x*-direction. The electric field lines are perpendicular to the direction of movement of the wave, in Fig. 3.29 in the *y*-direction. The magnetic field lines are also perpendicular to the direction of movement but, and in addition, perpendicular to the electric field lines, in Fig. 3.29 in the *z*-direction.

We look at any straight line that is parallel to the *x*-axis. When we move on this straight line from the left to the right, we first reach an area in which the electric and the magnetic field strength are zero. At a defined point x', we enter the interior of the wave where the field strengths have constant values. At the position case of $x' + \Delta x$, the field strengths go back to zero. Fig. 3.30a. shows the field strengths as a function of *x*. More exactly: the *y*-component of the electric field strength (the only component that is different from zero) and the *z*-component of the magnetic field strength. Now you understand why we have chosen the name "square wave". Notice that the image is the same for each straight line that is parallel to the *x*-axis. Fig. 3.30b shows the respective image for a later instant of time. We see in these illustrations that the fields moves from the left to the right.

Other electromagnetic waves can be different from this particular example in many ways, but they have some characteristics in common with the wave examined here, for example the following one:

> At any point of an electromagnetic wave, the direction of movement, the electric field strength and the magnetic field strength are perpendicular to each other.

There is yet another characteristic property of electromagnetic waves: the energy density of the electric field is equal to that of the magnetic field

$$\frac{\varepsilon_0}{2}\left|\vec{E}\right|^2 = \frac{\mu_0}{2}\left|\vec{H}\right|^2 . \qquad (3.12)$$

This means that the values of electric and magnetic field strength are standing in a particular relationship to each other. From equation (3.12) we conclude:

> In an electromagnetic wave, we have
>
> $$\sqrt{\varepsilon_0}\left|\vec{E}\right| = \sqrt{\mu_0}\left|\vec{H}\right|.$$

The velocity of electromagnetic waves in the vacuum is designated with c. It is equal to

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}. \qquad (3.14)$$

By inserting the values of the electric and the magnetic constant, we obtain approximately 300 000 km/s.

> The velocity of an electromagnetic waves is 300 000 km/s.

The velocity can be measured quite easily. Hence, the validity of the relationship (3.14) can be verified with such a measurement.

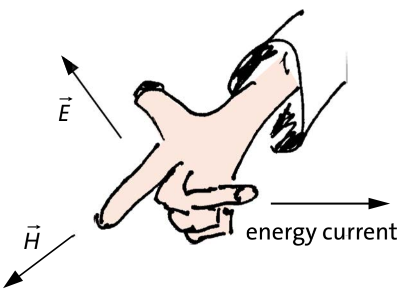


Fig. 3.31 Three-finger-rule for the energy current in the electromagnetic field

3.11 Energy transmission with electromagnetic waves

Studying the square wave, we can find yet another general property of electromagnetic waves. How is the direction of movement of the wave related to the direction of the electric and magnetic field strength? We have already seen that it is transverse to the two — in Fig. 3.29 in the direction of the x-axis. But we do not yet know a rule that allows us to decide whether the wave moves in the positive or in the negative x-direction. We would like to read this rule from Fig. 3.29.

We spread the first three fingers (thumb, index finger and middle finger) of the right hand in a way that they form a rectangular tripod, Fig. 3.31. We align the thumb with the direction of the electric field strength and the index finger with the direction of the magnetic field strength. The middle finger then points in the direction in which the wave propagates. This is also the direction in which the energy of the wave flows. Hence, the middle finger indicates the direction of the energy current density vector.

> *Three-finger-rule of the right hand:*
> Thumb – electric field strength
> Index finger – magnetic field strength
> Middle finger – direction of the energy current of the wave

The magnitude of the energy current density is obtained by multiplying both field strengths:

$$\left|\vec{j}_E\right| = \left|\vec{E}\right| \cdot \left|\vec{H}\right| \qquad (3.15)$$

The equation can be generalized so that it does not only apply for the case in which the electric and the magnetic field strength vectors are perpendicular to each other.

To understand this more general equation, we need to know another mathematical definition: the definition of the vector product, Fig. 3.32.

$$\vec{c} = \vec{a} \times \vec{b}$$

The product vector $\vec{c}$ is defined by the following rules:

Direction of $\vec{c}$: perpendicular to the plane in which lie the vectors $\vec{a}$ and $\vec{b}$;

Magnitude of $\vec{c}$: $|\vec{a}| \cdot |\vec{b}| \cdot \sin\gamma$
Here, γ is the angle between $\vec{a}$ and $\vec{b}$.

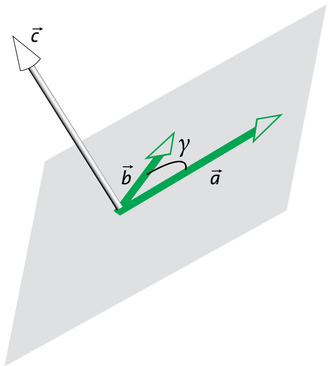


Fig. 3.32 The product vector $\vec{c}$ is perpendicular to the plane in which lie the vectors $\vec{a}$ and $\vec{b}$.

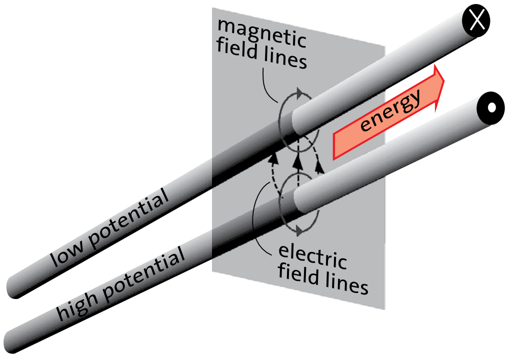


Fig. 3.33 Only outside of the conductors, the electric and the magnetic field strength are different from zero; only there, energy is flowing.

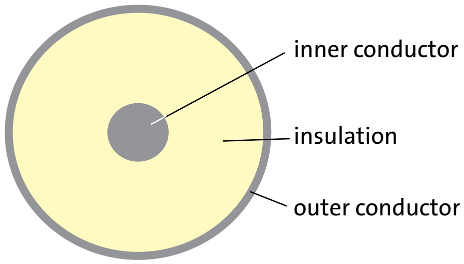


Fig. 3.34 Cross-sectional view of a coaxial cable. The outer conductor is at ground potential.

For the energy current density in the electromagnetic field, we have:

$$\vec{j}_E = \vec{E} \times \vec{H} \qquad (3.16)$$

This equation does not only provide a statement about the magnitude of $\vec{j}_E$; it also tells us the direction of the "product vector".

The formula does not only apply for such electromagnetic fields that we call "wave" or "radiation"; it applies for any field.

Now we can also answer a question that we have avoided so far. Where exactly does the energy flow when we use a normal, bifilar cable? A logical answer would be: "in the cable, where else?" But this answer is not correct. The electric charge is flowing in the cable, and the electrons are moving in the cable. Only the energy does not flow there. We see in Fig. 3.33 where it flows.

As an electric current is flowing, we have a magnetic field whose field lines loop around the conductor; and as there is an electric voltage between the conductors, we have an electric field whose field lines reach from one to the other conductor. Outside of the conductors, we thus have an electric and a magnetic field. The field lines of both fields are lying in planes that are perpendicular to the conductors. According to equation (3.15) or (3.16) we therefore have an energy current parallel to the conductors.

Exercises

1. A radio station emits an electromagnetic wave with 10 kW. The antenna is built in a way that the radiation only goes in a certain angular range. At a distance of 10 km, the beam has a width of 2.5 km and an altitude of 1 km. (These figures are only rough indications. In a real antenna, the intensity decreases towards the sides.) Calculate the energy current density at this point, i.e. at a distance of 10 km from the antenna. Calculate the electric and the magnetic field strength. Compare with the energy current of the sunlight.
2. Fig. 3.34 shows a cross-sectional view of a coaxial cable: one of the conductors is a conventional wire; the other conductor encloses the wire cylindrically at a certain distance. Coaxial cables are suitable for the transmission of high frequency alternating electric current signals. The outside conductor is at ground potential. Draw magnetic and electric field lines. Where and in which direction does the energy flow?

3.12 Sine waves

We look at a more realistic example for an electromagnetic wave: the plane sine wave. Fig. 3.35 shows a simplified "snapshot" of a sine wave that moves in the x-direction. Also here, the field strengths only depend on x, but not on y and z.

The x-dependence is shown in Fig. 3.36a. It is a sine function. A short time later, the sine wave line will have moved forward, Fig. 3.36b. The distance between

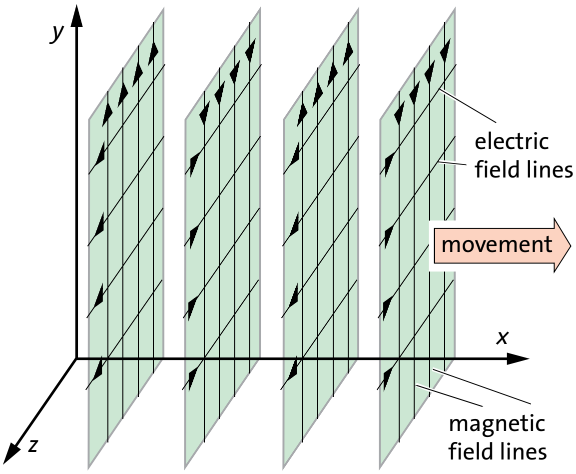


Fig. 3.35 "Snapshot" of an electromagnetic sine wave

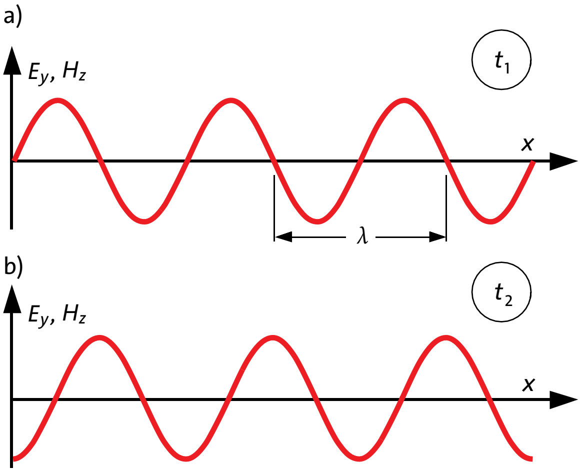


Fig. 3.36 Electric and magnetic field strength of the wave from Fig. 3.35 as a function of the spatial coordinate x at two different instants of time t_1 and t_2.

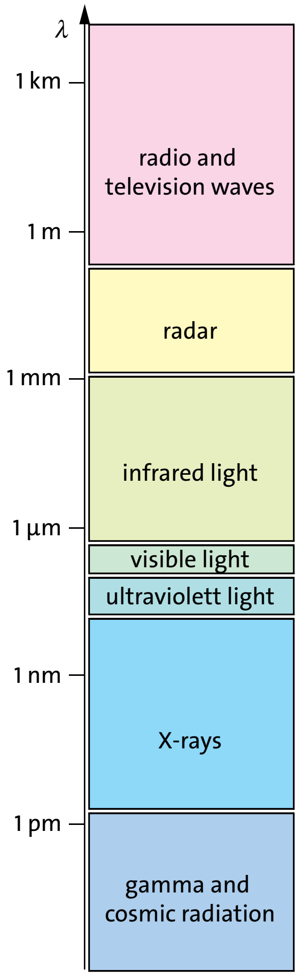


Fig. 3.37 The various wave length ranges of electromagnetic waves

two successive maximum or minimum values is called wave length λ of the wave.

Also for this wave, the following rules apply:

- electric field strength, magnetic field strength and direction of movement are perpendicular to each other;
- the magnitudes of the electric and magnetic field vectors are related at all points according to equation (3.13);
- the wave moves with the speed $c = 300\,000$ km/s.

The wave also consists of flat areas with high field strengths that move to the right and that are separated from each other by areas with low field strengths. As the areas with a high field strength also contain a high amount of energy, also the energy is transported in these "packages" that move to the right.

Electromagnetic waves are omnipresent in our lives, both as a natural phenomenon and as waves that are created with technical devices. It is interesting that there are electromagnetic waves with very different wave lengths. The shortest waves that have been observed, the so-called hard gamma radiation, have a wave length of 10^{-22} m. On the other hand, waves of up to a wave length of 10^4 m are used for radio transmission. Between these extremes (that are actually no extremes as there is no reason that keeps us from producing even shorter or even longer waves), there are the X-rays, the ultraviolet, the normal, "visible" light, the infrared radiation, the microwaves and the waves that are used for VHF transmission and television, Fig. 3.37.

Physical constants and formulas

Physical constants	
$\mu_0 = 1.257 \cdot 10^{-6}$ (V s)/(A m)	magnetic constant
$\varepsilon_0 = 8.854 \cdot 10^{-12}$ (A s)/(V m)	electric constant
$m_{el} = 9.11 \cdot 10^{-31}$ kg	mass of the electron
$e = 1.60 \cdot 10^{-19}$ C	elementary charge

Energy and energy currents	
$P = U \cdot I$	energy current for an electric energy transport
$E = \frac{C}{2} U^2, \quad E = \frac{L}{2} I^2$	energy in the electric field of a capacitor/in the magnetic field of a coil
$\rho_E = \frac{\varepsilon_0}{2} \lvert\vec{E}\rvert^2, \quad \rho_E = \frac{\mu_0}{2} \lvert\vec{H}\rvert^2$	energy density in the electric/magnetic field
$\Delta E = (\varphi_2 - \varphi_1) \cdot \Delta Q$	energy change of a charged body while passing through a potential difference

Momentum currents	
$F = Q \cdot \lvert\vec{E}\rvert$	momentum current entering a charged particle in an electric field
$F = Q_m \cdot \lvert\vec{H}\rvert$	momentum current entering a magnetic pole in a magnetic field
$F = I \cdot \Delta s \cdot B$	momentum current entering a conductor with an electric current in a magnetic field
$\sigma_\perp = \frac{\varepsilon_0}{2} \lvert\vec{E}\rvert^2, \quad \sigma_\parallel = -\frac{\varepsilon_0}{2} \lvert\vec{E}\rvert^2$	mechanical stress in the electric field
$\sigma_\perp = \frac{\mu_0}{2} \lvert\vec{H}\rvert^2, \quad \sigma_\parallel = -\frac{\mu_0}{2} \lvert\vec{H}\rvert^2$	mechanical stress in the magnetic field

Calculation of field strengths	
$\lvert\vec{E}\rvert = \frac{U}{d}$	electric field strength of the field of a capacitor
$H = I \cdot \frac{n}{\ell}$	magnetic field strength of the field of a coil (ℓ = length of the coil)
$H = \frac{I}{\ell}$	magnetic field strength of the field of a straight wire (ℓ = circumference of circle around the wire)

Physical constants and formulas (cont.)

Rates of change	
$\frac{\Delta Q}{\Delta t} = I$	rate of change of the electric charge is equal to electric current (conservation of electric charge)
$n \cdot \frac{\Delta \Phi}{\Delta t} = U$	rate of change of the magnetic flux is equal to induced voltage (Faraday's law of induction)

Equations that characterize a component	
$U = R \cdot I, \quad R = \frac{1}{\sigma} \cdot \frac{\ell}{A}$	characterizes resistor
$Q = C \cdot U, \quad C = \varepsilon_0 \cdot \frac{A}{d}$	characterizes capacitor
$n \cdot \Phi = L \cdot I, \quad L = n^2 \mu_0 \cdot \frac{A}{\ell}$	characterizes coil
$\vec{j} = \sigma \cdot \vec{E}$	local version of Ohm's law

Period	
$T = 2\pi\sqrt{L \cdot C}$	Period of an oscillating circuit