KPK

The Karlsruhe Physics Course

for the secondary school A-level

Atomic Physics
Nuclear Physics
Particle Physics

The Karlsruhe Physics Cours

A textbook for the secondary school A-level

- 🟩 Electrodynamics
- 🟥 Thermodynamics
- 🟪 Oscillations, Waves, Data
- 🟦 Mechanics
- 🟧 **Atomic Physics, Nuclear Physics, Particle Physics**

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1 MODELS IN PHYSICS

1.1 The description of a process with a model

Two methods can be used to describe a physical phenomenon. Either we use a model or we describe the phenomenon with a theory. We will explain the two methods by means of an easy example: an electric circuit, Fig. 1.1.

The object of our analysis is a physical process X that is not visible directly and that is at first perceived as unclear or abstract. However, X is similar to another process Y that everyone is familiar with. To understand the process X and to explain it to someone else, we can take advantage of this similarity.

Back to our example from Fig. 1.1a. We would like to explain what happens here to someone with absolutely no idea of electric processes. We could also say: imagine that something is flowing in the wires. We call it electricity. It is like water that is flowing in a tube. The thing on the left that we call battery can be imagined as a pump that drives the electric current, similar to a water pump that drives a water flow. The thing on the right is called "resistor". You can imagine that it hampers the electric current, just as a constriction in a water tube hampers the water flow, Fig. 1.1b. There are other similarities, but we would like to stop the comparison here.

If a process is described as done by us at this point, we can say that a *model* is used. The process X to be described is the electric circuit from Fig. 1.1a, the model Y is the water circuit from Fig. 1.1b. The model Y behaves in many respects like the system X. Many experiences made with Y can be transferred to X.

> A physical process for which we do not have a direct observation can be described by means of a model. The model is a system that we are familiar with.

Of course, the original system X and the model Y are not equivalent to each other in all respects. This is obvious if we compare the Figures 1.1a and 1.1b with each other. This also means that we cannot tell from a model whether it is wrong or right. A model is supposed to be practical. It is enough that Y and X are equivalent in relation to the properties that we are looking at. Hence, it is not a problem that the wires are massive while the pipes of the model systems are hollow. Also, we do not mind that there is negative electricity while the water quantity of the model system is always positive.

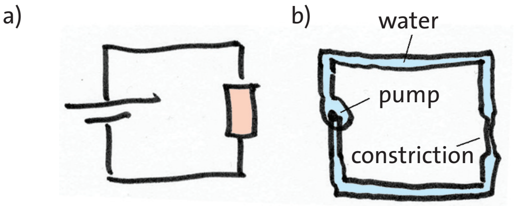

Fig. 1.1 (a) Experimenting with the items battery, resistor, cable, etc., we can conclude that what happens here is similar to what happens in the water circuit (b).

> Models are not wrong or right, but only more or less practical.

Exercises

1. Name further physical examples for the use of a model. Name examples from other sciences.

1.2 The description of a process by means of a theory

Physics tells us that the electric current will double by the resistor from Fig. 1.1a if the voltage of the battery is doubled. Such quantitative relationships are expressed through formulas. Thus, they are described with mathematical tools. Whether a formula is correct, i.e. whether it describes nature correctly, can be verified by means of experiments. You can take advantage of your skilled use of formulas to make correct predictions. In physics, a formula is the simplest case of a theory. However, we usually refer to a theory to describe a larger system of formulas and so-called differential equations. In other sciences, the term "theory" is used slightly differently.

Colloquially, the word "theory" is often used as a synonym for "assumption", or we say that something is only "theoretical" meaning that the actual situation is different. As we have seen, the physical meaning of the word "theory" is different. There are physical theories that apply well to nature. And there are other theories that reflect nature badly or not at all. If course, an inappropriate theory will not be used anymore if people become aware of its inappropriateness. Finally, there are theories whose usefulness is not yet known. An important goal of science is to create new theories and to verify how well they describe nature.

> A theory describes a physical process mathematically, i.e. by means of equations.

Exercises

1. Name theories that you know or that you have heard about, both from physics as well as from other sciences.

1.3 The limits of models

As explained, we have two methods to describe a physical system: with a model or with a theory. The two methods are basically quite similar. Both times we take advantage of the fact that nature behaves like something that we already know: either another, familiar system from real life or something from the abstract world of mathematics, Fig. 1.2.

In physics, only the second method can be used to make exact statements: mathematical description and experimental verification of the mathematical laws. In the way that you have gotten to know physics up to present, however, method 1 has been the dominant one. Probably you felt to have understood something every time you saw that something is similar to something you already know.

Unfortunately, the first method, i.e. the model method, does not always work. We cannot describe everything as „being just like that and what you already know“, because sometimes a physical phenomenon is different from everything we know from everyday life. And this is particularly the case for the topics that we will address in the following.

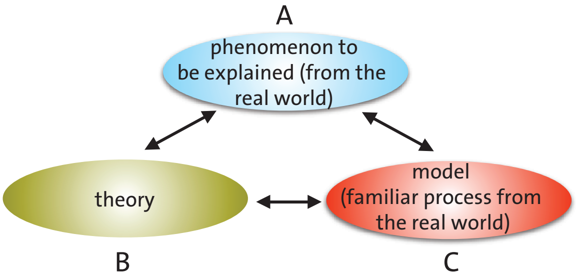

Fig. 1.2 Three areas have similar structures:
A: The physical phenomenon to be described;
B: the theory that is used to describe the phenomenon;
C: the model that is used to make the phenomenon plausible.

You will see that atoms sometimes behave in a way that we simply do not know from the things around us. This can even go as far as to make us believe that atoms would behave inconsistently. The interesting point is that atoms can indeed be described mathematically. Hence, their behavior cannot be inconsistent either. It is only not intuitive; we do not know any model, i.e. no familiar system, that behaves like the atom.

Fortunately though, this does not mean that we have to do completely without models, i.e. without „things being like ...“. We will still use models, but with restrictions. For example: we will say that you can imagine the electronic shell of an atom to be like a substance from which only portions of a very specific quantity can be extracted. Although you have never seen such a material, you will not have trouble imagining it.

Sometimes it becomes even more difficult. We look at an atom that has more than one electron, i.e. for example two or three or thirty. You would now have to imagine the electronic shell as a substance that is located in a space with more than three dimensions. The electronic shell of the helium atom, for example, would be a substance in a six-dimensional space. Our experiences from everyday life, however, are all related to a three-dimensional space. We can hardly imagine a six-, twelve- or thirty-dimensional space.

In sum: we will also use pictures and models in the following, i.e. we will say „it is like ...“, but we always have to bear in mind: do not take the models too seriously. The mathematical description is the only way that works and that characterizes physics. The text between the formulas only applies with reservations. We will sometimes refer to it in the following in order to remind you, but we will not do it in every sentence. Hence, you have to read the texts with the right attitude from the beginning. When it reads that „the atom is a small object with the following structure …“, it only means: in the context that we are currently talking about, you can imagine the atom as a small body. But you have to expect that, at a later point, we will say that „now the atoms behave very differently from small bodies. You should rather imagine them as a wave on an imaginary wave carrier“.

> For some physical processes, we have a working theory but no suitable model. Such processes appear unintuitive to us.

The fact that there is no model for some physical phenomena also implies that certain questions, that seem logical, may not be asked.

> A question can be pointless even if it seems logical.

2 PHOTONS AND ELECTRONS

2.1 Transports with waves and transports with matter

A "transport" is defined by us as a process in which something moves from one place to another: water in the Rhine river flowing from Karlsruhe to Mannheim, electric particles flowing in a wire from a switch to a lamp, a rocket moving from the Earth into an orbit, light moving from the Sun towards the Earth, radio waves moving from a mobile phone to the next antenna, sound moving from a speaker to your ears, etc.

These transports, as well as any others you might think of, can be divided into two categories. In case of transports of one category, some matter moves through a space: water, electrons, a rocket... in case of the other type, a wave moves through any wave carrier: light, sound...

In the following, we will deal with the two transport types time and again. To analyze them in greater detail, we will look at a representative example of each category. As an example of matter, we will use electrons; and as an example of waves, the light.

Hence, we compare a current of electrons (i.e. in the tube of an old TV) to a current of light (from a torch or a laser).

First the common aspects: both transport energy. In addition, both transport momentum and entropy, even if it is not obvious.

But there also seem to be essential differences:

- The electron beam consists of small portions or small bodies that are called electrons. We can also say that the matter is *quantized*. The light beam, in contrast, is a wave, i.e. a seemingly steady or "continuous" process.
- With the light beam interference patterns can be created: if two parts of the light beam overlap, at some places the wave will be reinforced and at others they will be attenuated or even eliminated. This proves that light is a wave process. We have not known this for matter up to present.

Hence:

> Matter is quantized, light shows interference.

It appears as if a light beam and an electron beam were two completely different transports. This conclusion could be generalized for the other representative examples of waves and matter. The interesting point of this conclusion is that it is wrong. We will see that both characteristics, i.e. the portion-like appearance and the interference property, are not only valid for one substance class but also for the other: light also consists of small portions, and electrons can also be used to create interference patterns.

> Not only matter but also light is quantized. Not only light but also matter shows interference.

2.2 Photochemical reactions

There are chemical reactions that involve light as a reactant. They are called *photochemical reactions* (from the Greek word ‘phos’ that means light). Reactions are described by means of chemical equations. If a substance A reacts with substance B to form a substance C, we can write

$$A + B \rightarrow C.$$

A specific example is the reaction of hydrogen and oxygen to water, i.e.

$$2\,H_2 + O_2 \rightarrow 2\,H_2O.$$

In this process, the substances hydrogen and oxygen disappear and water is formed.

But there are also reactions in which light emerges or disappears besides other substances. We will show a few examples in the following.

Glow sticks

So-called glow sticks are available at car accessory stores. A glow stick consists of a transparent plastic tube that is filled with two fluids

A and B, which are separated from each other at first. If the plastic tube is slightly bent, the partition wall between the two fluids will break. They will mix and start to react with each other. During the reaction, light is formed besides a fluid C. Hence, a glow stick is a light source that works without a battery or a socket. The Greek letter γ (Gamma) is used as a symbol for light in chemical equations. The equation for the reaction in a glow stick is therefore

$$A + B \rightarrow C + \gamma.$$

Here, we have not indicated the chemical expression for the three substances A, B and C. What matters is only the fact that light works like a usual substance in this context.

Photosynthesis

The photosynthesis in plants is a photochemical reaction that is very important for life on Earth. In this reaction, plants produce glucose ($C_6H_{12}O_6$) and oxygen (O_2) out of carbon dioxide (CO_2), water (H_2O) and light. Glucose is used by plants as a nutrient for their cells and as an energy storage material.

Photosynthesis is a complex reaction with many intermediate steps.

If we skip the intermediate steps, i.e. if we only look at the initial and the final substances, we can establish the following chemical equation

$$6\,CO_2 + 6\,H_2O + 48\,\gamma \rightarrow C_6H_{12}O_6 + 6\,O_2.$$

From the examples discussed, we can conclude:

> When describing photochemical reactions, light can be treated like a substance.

2.3 Portions of light in the course of time

Every substance is composed of tiny portions, the atoms or molecules. If light behaves like a common substance, we can expect that there are also tiny portions of light, a sort of "light atoms". In the following,

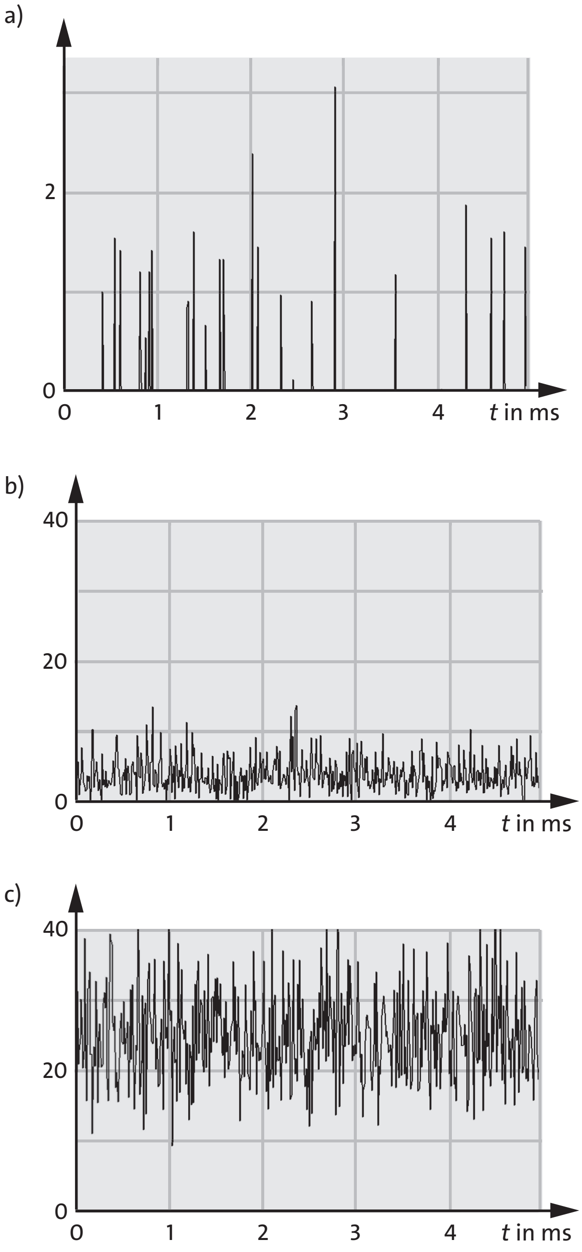

Fig. 2.1 The light arrives with irregular consecutive strokes. In the upper image (a) the light intensity is very low; in the central image (b) it is slightly higher and in the lower image (c) it is even higher.

we will describe an experiment that confirms this assumption.

The "atoms" of light cannot be perceived with the naked eye, just like the atoms of other substances. What we need is a very sensitive light meter, a so-called *photomultiplier*. When the sensor of the photomultiplier is held into light, it will react with the incident light, i.e. the light will be absorbed. We can read from an indicator device the quantity that is absorbed by the photomultiplier per unit of time.

We would like to analyze the light of a common light bulb. If we let this light fall directly onto the photomultiplier, the latter would be destroyed immediately. It would be like weighing an elephant with a letter scale. Rather, we have to attenuate the light of the light bulb before it falls onto the photomultiplier: by means of a neutral grey filter that is only permeated by a very small amount of light.

The light bulb is connected to an adjustable power supply. Then, we slowly increase the electric current so that the lamp will start to glow slightly. It sends out just as much light as to be visible with our eyes. Although the photomultiplier is located behind the gray filter, it already reacts to the light of the slightly glowing light bulb: the display of the photomultiplier starts deflecting.

Fig. 2.1a shows how the display unit reacts during a period of 5 ms.

Surprisingly, the deflection of the display is not constant over time, but strokes or pulses occur in irregular time intervals. Between the strokes, the deflection is zero. Hence, the light bulb is not lit continuously but it sends out small light portions in irregular but very short intervals. These light portions are called photons. If the intensity of the light is increased, the photon pulses will occur in shorter and shorter time intervals and finally the meter will no longer be able to detect them individually. Of course, we will not be able to distinguish them with our eyes either.

> Light is emitted and absorbed in portions, i.e. the photons.

2.4 Portions of light in space

An image in a digital camera is created on the CCD chip. The chip consists of a field of several millions of light-sensitive sensors, the pixels. If light with a very low intensity falls onto the chip so that the photons arrive at the chip in a well-separated way as shown in the previous experiment, we will make a peculiar observation: a photon will only be registered by a single pixel, i.e. the photon will be absorbed by a single pixel and will not spread over several pixels.

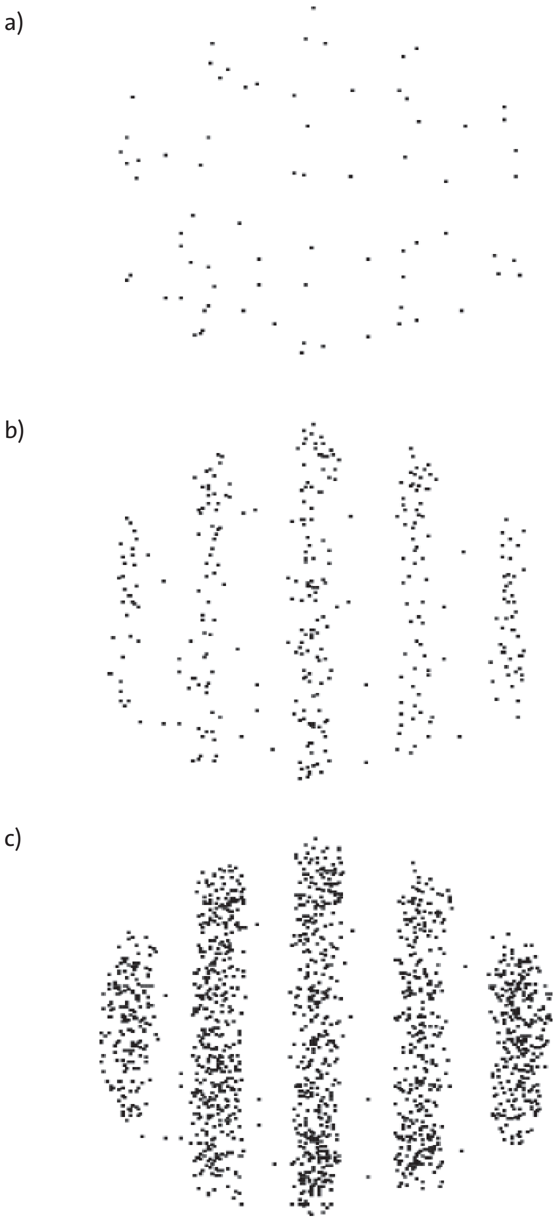

Fig. 2.2 An interference pattern is formed with a growing exposure to light.

> A photon is absorbed by only one pixel of a CCD chip.

2.5 Interference and statistics

Earlier, you learned about an experiment in which laser light falls onto two adjacent slits in an obstacle. The light is "diffracted" at the slits so that the light waves intersect with each other behind the slits. If a sheet of paper is put in the way of the light behind the slits, we will see an interference pattern: a sequence of light and dark stripes. If one of the slits is kept shut, the interference pattern will disappear.

Now we would like to replace the paper by a digital camera. Also the CCD chip registers the interference pattern. Earlier, we interpreted this phenomenon as follows: at the dark stripes, the light waves extinguish each other; at the light stripes, they intensify each other. However, here we see that it cannot be that simple.

One photon is absorbed by *one* pixel. Therefore, a single photon can certainly not produce an interference pattern, even if it is part of two intersecting laser beams. It only contributes to the overall interference pattern. The interference pattern can only be formed if many photons fall on the chip. They spread out *statistically*, i.e. in a way that the interference pattern is formed as an overall result. Fig. 2.2 shows how the interference pattern is formed gradually (a → b → c).

> A single photon cannot create an interference pattern on a CCD chip. The photons spread out statistically over the pixels whereby an interference pattern is created.

2.6 The size of photons

The question about the size of photons is one of the questions we have warned you about: it is a question to which there is no clear physical answer, i.e. a "forbidden" question. But still, photons behave under certain circumstances as if they had a well-defined size and shape. And this is what we can read from the experiments described before.

We will start with the digital camera. As a photon is absorbed by a single pixel, it should be very small, i.e. not larger than a pixel. A pixel of a CCD chip has a side length of approximately 5 µm. Hence, we conclude that a photon has a diameter –transversal to its direction of movement– that is smaller than 5 µm.

$d < 5\ \mu\text{m}.$

Now we look at the interference experiment. The interference picture is gradually formed by the photons falling onto the CDD chip one by one. This means that each photon "has to know" with which pixel it reacts, at least on a statistical average. But this means that each photon has passed both holes in the obstacles as interference can only occur if a wave passes through both gaps. Or in other words: transversally to its movement direction, a photon has to be at least as large as the space between the two holes in the obstacles, which amounts to approximately 1 mm, i.e.

$d > 1\ \text{mm}.$

How can we conciliate the two statements? Of course, they cannot apply at the same time. No object can be smaller than 5 µm and larger than 1 mm at the same time. We conclude that the photon can change its size and also its shape. We can imagine that it shrinks at least by a factor 200 (1 mm : 5 µm = 200) during the absorption process.

Consequently, we already have a process to determine the diameter of photons. We constantly increase the distance between the gaps in the obstacles. At some point, the interference pattern will disappear. We know that the space between the holes will then have exceeded the diameter of the photons.

Also the length of the photons can be determined in a similar way. We summarize:

> Photons do not have a uniform shape.

Shape and size depend on the light source but still change on the way after leaving the light source.

For instance, the photons of the laser from the school equipment have a length of around 10 cm and the same diameter as the laser beam itself, i.e. around 5 mm. Their shape and size is consequently similar to a pencil. There are lasers whose photons are much longer. Some lasers emit photons that are several kilometers long.

The photons of the sunlight are much shorter. Their length has approximately the same value as its wavelength, i.e. approximately 600 nm. Their diameter is around 0.06 mm (but only if the sunlight is not spread by clouds or fog – otherwise it will be even smaller). This is approximately the 100-fold length. Hence, the photons of the sunlight have a similar shape as pancakes.

You might have noticed that what we have called length and diameter of the photons here is equivalent to the coherence length and the coherence width of light.

Exercises

1. How could the length of a photon be determined experimentally?

2.7 Energy and momentum of photons

As light transports energy, each photon also transports energy. The transported amount of energy, however, is not equal for all photons. It is related to the frequency of the respective light in a simple way:

$$E_{\text{Photon}} \sim f.$$

The proportionality factor is called *Planck's constant h* (named after Max Planck who discovered the photons). It has the value

$$h = 6.6 \cdot 10^{-34}\ \text{J} \cdot \text{s}.$$

Hence,

$$E_{\text{Photon}} = h \cdot f. \tag{2.1}$$

This equation can be used to calculate the energy of photons if the frequency is known.

However, in many cases the wavelength of light but not the frequency is known. In these cases, the frequency is calculated at first by means of the wavelength. Let us recall the relationship between the wavelength λ, the frequency f and the velocity c:

$$c = \lambda \cdot f.$$

which leads to:

$$f = \frac{c}{\lambda}.$$

A high frequency means a small wavelength and vice versa. We insert the expression in equation (2.1) and obtain

$$E_{\text{Photon}} = \frac{h \cdot c}{\lambda}.$$

Hence: the larger the wavelength of the light, the lower the energy of its photons.

We calculate the energy of the photons of "red light" ($\lambda = 8 \cdot 10^{-7}$ m):

$$E_{\text{red}} = \frac{h \cdot c}{\lambda_{\text{red}}} = \frac{6.6 \cdot 10^{-34}\,\text{Js} \cdot 3 \cdot 10^{8}\,\text{m/s}}{8 \cdot 10^{-7}\,\text{m}}$$
$$\approx 2.5 \cdot 10^{-19}\,\text{J}$$

and of "blue light":

$$E_{\text{blue}} = \frac{h \cdot c}{\lambda_{\text{blue}}} = \frac{6.6 \cdot 10^{-34}\,\text{Js} \cdot 3 \cdot 10^{8}\,\text{m/s}}{4 \cdot 10^{-7}\,\text{m}}$$
$$\approx 5.0 \cdot 10^{-19}\,\text{J}.$$

These energies are extremely small compared to the amounts of energy that we are dealing with in daily life. Under usual lighting conditions, however, many more photons are involved. For example, a torch creates approximately 10^{15} per second. The sunlight that hits one square meter of the surface of the Earth consists of 10^{21} photons.

Also the momentum that is transported by a photon is interrelated to the wavelength in a simple way:

$$p_{\text{Photon}} = \frac{h}{\lambda}.$$

The momentum of a photon of red light is therefore

$$p_{\text{red}} = \frac{h}{\lambda_{\text{red}}} = \frac{6.6 \cdot 10^{-34}\,\text{Js}}{8 \cdot 10^{-7}\,\text{m}} = 8.25 \cdot 10^{-28}\,\text{Hy}.$$

The momentum of a photon of blue light is twice as high:

$$p_{\text{blue}} = 16.5 \cdot 10^{-28}\ \text{Hy}.$$

Consequently, for example a momentum current of $3 \cdot 10^{8}$ Huygens per second (= 300 million Newton) flows into the Earth with the sunlight that hits the Earth. Hence, the sunlight pushes against the Earth, just as a water beam that hits a ball and pushes against it.

We can observe the pressure of the sunlight on the tails of comets that always point away from the Sun. The dust particles the tail consists of are pushed away from the Sun by the light.

So far, we have only spoken about visible light and UV light. However, what applies for these two types of light applies in the same way to any other types of light, i.e. to radio and TV waves, microwaves, infrared light, X-rays and gamma rays: all these radiations do not only behave like waves, but they are also quantized, i.e. they

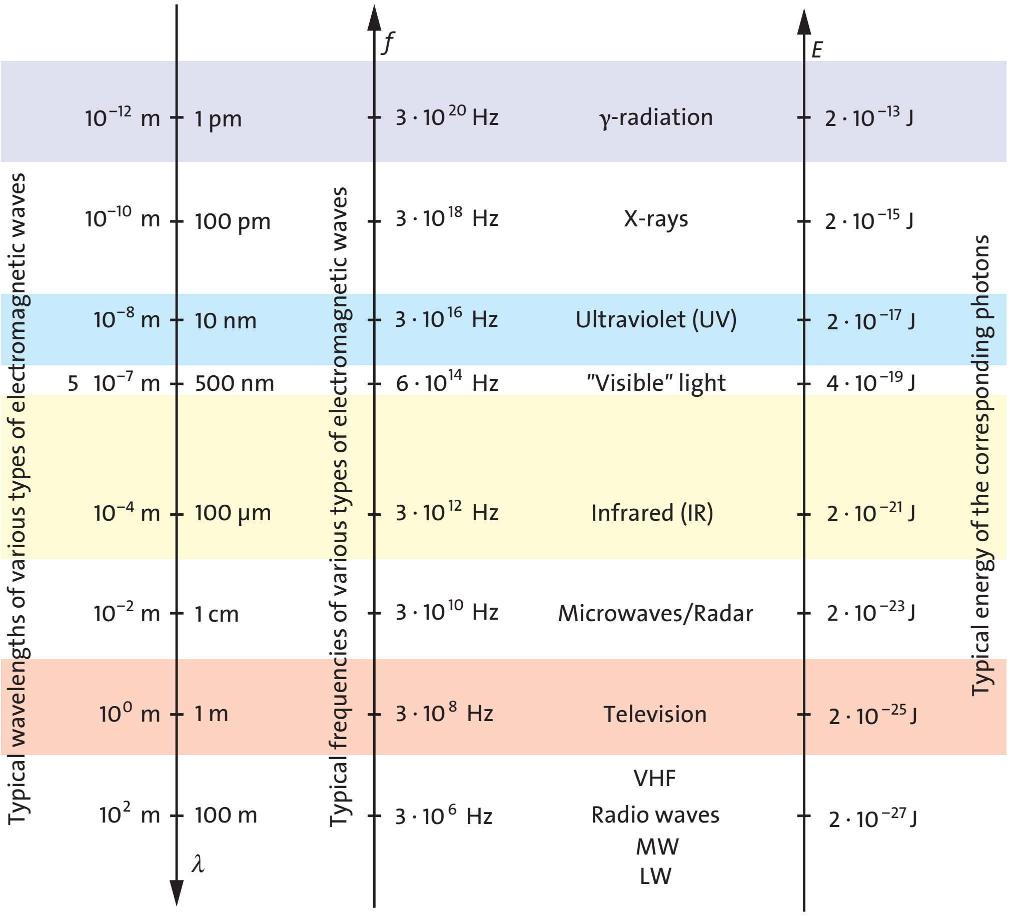

Fig. 2.3 The different types of electromagnetic waves with their wavelengths and frequencies and the energy of the associated photons

consist of photons and also the interrelations between the frequency of the waves and the energy and the momentum of the photons are the same as for visible light. Fig. 2.3 shows an overview of the different types of electromagnetic waves with their wavelengths and frequencies as well as the energy of the associated photons.

γ-radiation has the shortest wavelength and the highest frequency. Consequently, the photons of the γ-radiation are the most energetic ones. Radio waves, in contrast, have a long wavelength and a low frequency. A radio photon therefore has very little energy.

Notice that the energy values among the consecutive photon types in the Figure mostly differ by the factor 100.

> The following applies to the photons of electromagnetic waves with the frequency f and the wave length λ:
>
> $$E_{\text{Photon}} = h \cdot f \text{ and } p_{\text{Photon}} = \frac{h}{\lambda}.$$

Exercises

1. a) Calculate the energy and momentum of a photon of the SWR-3 radio station (frequency of SWR 3: 98.4 MHz). b) Calculate the energy and momentum of the photons of X-rays with a wavelength of 150 pm. c) Compare the values calculated in a) and b) with the energy and momentum values of a photon of visible light.
2. a) A ball is lifted up by the water jet of a fountain. Why does the ball not fall down although a momentum current from the gravitational field of the Earth is flowing into the ball? b) A small bead is lifted up by an upward-facing light beam. Why does the bead not fall down although a momentum current from the gravitational field of the Earth is allegedly flowing into the bead? c) How many photons will have to hit the bead from exercise part b) per second if the momentum current from the gravitational field into the bead is $7 \cdot 10^{-11}$ N? (Wavelength of the light: 800 nm).

2.8 Electrons

We recall once again our starting point: we knew that light is a phenomenon made up of waves, i.e. that there are interference effects. And we knew that an electron is a small portion of matter, for example in a beam of electrons. However, we have just convinced ourselves that this distinction is not applicable to light: light also consists of small portions, i.e. the photons. Now you will certainly not be surprised anymore that the electrons can also be described as a wave process and that an interference image can also be produced with a beam of electrons.

First, the experimental finding: a beam of electrons can be used to carry out an interference experiment that is very similar to the interference experiment with light. It is only a bit more difficult in technical terms. A beam of electrons is divided into two partial beams with only a minimal directional difference. Then, the partial beams are merged at an acute angle. If a screen that is sensitive to electrons is put in the intersection area of the beams, we will obtain an interference image, Fig. 2.4.

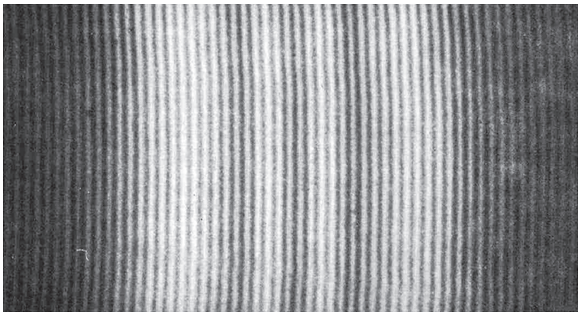

Fig. 2.4 Interference image that was created with an electron beam

We can conclude from such experiments that the following equations also apply for electrons:

$$E = h \cdot f$$

$$p = \frac{h}{\lambda}. \qquad (2.2).$$

Here, E is the total energy of the electrons (i.e. not only the kinetic energy), p the amount of the momentum, f the frequency and λ the wavelength of the electron wave.

Such a wave is called a *matter wave*, because interference patterns can be produced not only with electrons, but also with beams of neutrons, protons, entire atoms and even molecules.

We would like to calculate the wavelength for a beam of electrons in a television tube. The acceleration voltage U in such a tube is approximately 20 kV.

The kinetic energy of the electrons is

$$E_{\text{kin}} = e \cdot U.$$

Here, e is the elementary charge. The kinetic energy is related to the momentum as

$$E_{\text{kin}} = \frac{p^2}{2m}.$$

Hence, we can write for the momentum:

$$p = \sqrt{2mE_{\text{kin}}}.$$

If we insert $e \cdot U$ for the kinetic energy, we will obtain

$$p = \sqrt{2m \cdot e \cdot U}.$$

By means of equation (2.2) we obtain for the wavelength of the matter wave:

$$\lambda = \frac{h}{\sqrt{2m \cdot e \cdot U}}.$$

The electron mass is

$$m = 9.1 \cdot 10^{-31}\text{ kg}.$$

Also,

$$h = 6.6 \cdot 10^{-34}\text{ Js}$$

and

$$e = 1.6 \cdot 10^{-19}\text{ C}.$$

For electrons that were accelerated with 20 kV, we therefore obtain:

$$\lambda = \frac{6.6 \cdot 10^{-34}\,\text{Js}}{\sqrt{2 \cdot 9.1 \cdot 10^{-31}\,\text{kg} \cdot 1.6 \cdot 10^{-19}\,\text{C} \cdot 2 \cdot 10^{4}\,\text{V}}}$$
$$= 0.86 \cdot 10^{-11}\,\text{m}.$$

> The following applies for electrons with the energy E and the momentum p:
>
> $$E_{\text{electron}} = h \cdot f \quad \text{and} \quad p_{\text{electron}} = \frac{h}{\lambda}.$$

3 THE QUANTUM THEORY

3.1 Data goes in and data comes out

We have already seen what physicists understand by a theory: the mathematical equations and statements to describe a natural phenomenon. The use of a theory is simple in principle. We insert data that is known and we obtain data that we would like to know. The calculation processes in between can be very entangled. Fig. 3.1 illustrates the theory as a coffee mill.

A variety of problems can be solved by means of a theory:

- We insert the values of certain physical quantities and calculate the values of other quantities, e.g.: we insert electric current and resistance and the theory provides the electric voltage. (Here, the theory is Ohm's law.)
- We insert the values of certain quantities and the theory provides the values of the same quantities at a later time. Example: we insert the location and velocity coordinates of a space probe at present and obtain the coordinates of the space probe at a future instant of time.
- We insert the values of certain quantities and the theory provides the values of the same quantities at an earlier time. For example: we insert the location and velocity coordinates of stars at present and find out that there was a big bang 15 billion years ago. Hence, we calculate a result that lies in the past.

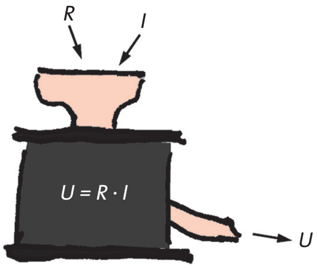

Fig. 3.1 The theory as a mathematical mill: we insert numbers and obtain other numbers.

> Theory: system of equations and statements.
> We insert known data and obtain the desired data.

3.2 A theory with two steps

The theory that can be used to describe the behavior of atoms, electrons, photons and other systems is the quantum theory. It is not overly complicated, but the mathematics you have learned is not yet sufficient to understand it. Therefore, we would like to address only a general structural feature of this theory.

Concretely speaking, we look at a specific problem: the hydrogen atom or, more precisely, the electronic shell of the hydrogen atom. The data that we put into the theory includes:

- Data regarding the characteristics of the nucleus. We know that, for our purposes, the nucleus can be regarded as a point-like charged particle. The electric potential in the environment of such a particle is known to us.
- The data that characterizes the electron, i.e. its mass and its electric charge.

The calculation of the quantum theory that leads from the initial data to the final data can now be divided in two parts that are illustrated by two mills, Fig. 3.2.

The initial data is inserted in mill 1. As an intermediate result, mill 1 provides a function

$$\psi(x, y, z),$$

the so-called *psi-function* or *wave function*. ψ is a function of the position coordinates. Next, the wave function is inserted into mill 2 which will only then provide the numbers that we are actually interested in and that we can prove experimentally.

The first step, i.e. the calculation of the wave function, is the more complex one. Therefore, complicated calculation processes need to be applied. A *differential equation*, the so-called *Schrödinger's equation*, has to be solved. This is why physicists also say that they have "solved the problem" as soon as they have calculated the respective wave function. The second step is not a big deal anymore: everything that is needed can be obtained from the wave function relatively fast and easily: the shape or form of the electronic shell of the atom, the energy of the electron, the momentum, the angular momentum, the magnetism...

However, some aspects of what we might have wanted to obtain are not provided. And this means: who asks for it asks a wrong or pointless question.

The quantum theory solves a problem in two steps:

- calculation of the wave function based on the inserted data;
- calculation of the desired data out of the wave function.

3.3 The wave function

The wave function has x, y and z as independent variables. Hence, their value depends on the position. We have already seen such functions earlier. We have described electric and magnetic fields with the field strengths whose independent variables have also been the position coordinates. Therefore, the wave function is comparable with a field strength.

We would like to take the comparison between the wave function of the electron and the field strengths of the electromagnetic field a bit further.

The electric and magnetic field strengths contain everything that we can know about an electromagnetic field. The values of other physical quantities can be calculated from them: energy, energy current, momentum and momentum current. Earlier, you learned how to calculate the energy density of an electric field:

$$\rho_E(x,y,z) = \frac{\varepsilon_0}{2}\left|\vec{E}(x,y,z)\right|^2 .$$

This expression tells us how the energy is distributed in space. Thus, it also tells us how the field substance is distributed.

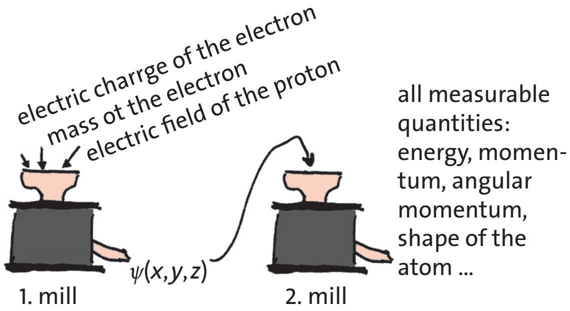

Fig. 3.2 The mathematical mill of quantum mechanics divided in two partial mills. The first one provides the wave function based on the problem description, the second one calculates verifiable numbers out of the wave function.

Things are very similar for the wave function. It also contains everything that we can know about the matter that it describes. The values of other physical quantities can be calculated from it. This is illustrated by the second mill in Fig. 3.2.

At first, we would like to look at a very simple example of the working of mill 2: the calculation of the square

$$\rho(x,y,z) = \psi^2(x,y,z)$$

The result is a function of x, y and z. In contrast to the wave function itself, it can only take on positive values (or be equal to zero). Multiplying it with the mass m of the electron

$$\rho_m = m \cdot \rho = m \cdot \psi^2,$$

we obtain a mass density. We can interpret this result as follows: the electron is a portion of a substance. We call this substance *electronium*. The quantity ρ_m represents the mass density of the electronium:

$$\rho_m = m \cdot \rho = m \cdot \psi^2.$$

Multiplying ρ with $-e$, i.e. the negative elementary charge

$$\rho_Q = -e \cdot \rho = -e \cdot \psi^2,$$

we obtain the charge density of the electronium.

These considerations were quite abstract. In the next section, we will see how the mass and/or charge distribution of the electronium looks like in specific cases.

The square of the wave function is a measure for the mass density and the charge density of the electronium.

4 THE ATOM – STATIONARY STATES

4.1 The density of the electronium

An atom consists of a small, heavy nucleus and a light and relatively large electronic shell. The diameter of the nucleus is approximately one 50 000 th of the atomic diameter, i.e. of the diameter of the shell. The mass of the nucleus amounts to approximately 99.99 % of the total mass of the atom. The nucleus and the shell are electrically charged: the nucleus positively and the shell negatively. The charges of the nucleus and the shell are equal in terms of their magnitude. This means that the atom as a whole is electrically neutral, i.e. its total charge is zero.

> Atoms are composed of a small, heavy, positively charged nucleus and a large, light, negatively charged shell.

The atomic shell consists of electronium. We have seen that its density is described by the square of the wave function.

If an atom is left alone, i.e. if it is not disturbed from outside in any way, the electronium distribution is almost spherically symmetric (for some atom types even exactly spherically symmetric). It is densest in direct proximity of the nucleus. At a growing distance from the nucleus, the density decreases at first fast and then more and more slowly. Hence, the atom does not have a sharp boundary surface, Fig. 4.1.

An inflated air balloon has a defined shape. This shape can be modified by pressing against the balloon. If it is released again, it will return to its initial state.

The atom behaves in a similar way. Nothing that could deform the atom may be located near the atom: no other atoms and no electric or magnetic fields. Only when these preconditions are met, the atom has its natural spherical shape.

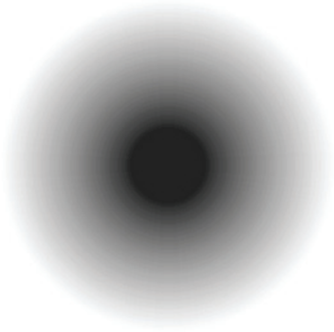

Fig. 4.1 Atom in the ground state. The mass and charge density of the electronium become apparent from the gray shading.

The condition of no other atom being near is met in case of gaseous substances as the individual atoms are located at longer distances from each other. In the following, we will look at such free atoms. In case of liquids and solid substances, the atoms are very close to each other. We will look at the shape of the atoms in these substances in a later chapter.

The electronium of the atom has a certain similarity to the atmosphere of the Earth: also the density of the air depends on the position and it becomes increasingly thinner at a growing distance from the surface of the Earth. Also, no sharp boundary at which the atmosphere ends can be indicated.

To indicate a radius of the atom, we arbitrarily take the distance from the nucleus at which the electronium density amounts to only 10 % of the maximum value. The radii of the different types of atoms are different. A typical value is 10^{-10} m.

> The atomic shell consists of electronium. The electronium is distributed in a spherically symmetric way around the nucleus. Its density decreases from the center to the outside of the atom.

A surprising feature of the atom becomes apparent when we try to extract electronium from the shell. For now, it is not relevant how this can be done. We simply imagine the atom were very big so that we could reach into it with our hands. Hence, we reach into the atom – in an imaginary way – in order to extract a portion of electronium. We obtain a certain amount of electronium, deposit it somewhere and repeat the operation: we reach in once again and extract a second portion. Thereby, we see that – surprisingly – this portion has exactly the same size as the first one. We repeat the process once again and, again, the portion of electronium is equal to the two previous ones. Now we make a larger effort in order to see whether there is a way to extract a bit more. We reach in much more vigorously – with success: we have more electronium in our hands. This time, however, exactly twice the amount of the portions that we had obtained before. If we reach in even more forcefully, we will obtain the threefold, four-fold ... quantity, but never quantities in between. Then, we also try to extract a smaller amount of electronium from the atom. It is impossible. Either we obtain one of the portions that we have already had or nothing at all.

In other words, only portions of electronium of a very specific size, the so-called *elementary portions*, or a multiple of them can be extracted from the atom. Such a portion of electronium is exactly what we have already called an *electron* up to now. In general, such elementary portions are also called *particles*.

Here is the fact sheet of the electron: it has a mass of approximately 10^{-30} kg, an electric charge of minus $1.6 \cdot 10^{-19}$ C and behaves like a small permanent magnet of a very specific strength.

> An electron is an elementary portion of the electronium. The elementary portions of substances are also called particles.

Just as the atomic shell, the atomic nucleus consists of a substance. In contrast to the density of the electronium in the shell, the density of this nuclear matter inside the nucleus is about the same everywhere, similar to the density of the steel in a massive steel ball.

Also from the nucleus, only the extraction of specific portions is possible. However, there are two slightly different portions whose names you probably know: *protons* and *neutrons*.

Protons and neutrons have a much larger mass than electrons. They are approximately 1800 times heavier. The electric charge of the proton has the same magnitude as that of the electron, but an opposite sign, i.e. it is $+1.6 \cdot 10^{-19}$ C. The neutron is not charged. Protons and neutrons are also magnetic, but substantially less than the electrons. The values of the physical quantities that characterize the electrons, protons and neutrons are summarized in Table 4.1.

	mass	electric charge	magnetism
electron	$0.9 \cdot 10^{-30}$ kg	$-1.6 \cdot 10^{-19}$ C	strong
proton	$1700 \cdot 10^{-30}$ kg	$+1.6 \cdot 10^{-19}$ C	weak
neutron	$1700 \cdot 10^{-30}$ kg	0 C	weak

Table 4.1 Properties of the electron, proton and neutron

The nucleus of each atom contains positively charged protons; the shell consists of negatively charged electrons. Atoms as a whole, however, are not charged. As the charge of electrons and protons is equal but opposite, we can conclude that the number of the electrons in the shell of an atom must be equal to the number of protons in the atomic nucleus.

The number of protons in the nucleus is the distinctive criterion for the different types of atom: hydrogen atoms have one proton in the nucleus, helium atoms two, lithium atoms three, etc. In the periodic table of elements, the atoms are sorted according to the number of protons in the nucleus, i.e. the *atomic number.*

> The nuclei of the atoms contain a number of protons that is typical for each type of atom.

Electrons, protons and neutrons are also magnetic. Now we know that practically all substances that exist in our environment are nonmagnetic. And even iron is only magnetic in particular cases.

The reason is that the effects of magnets can compensate each other. Hence, in approximately one quarter of all atom types –among them all inert gas atoms– the magnetic effects of the electrons in the atoms neutralize each other. (The magnetic effects of protons and neutrons are so low that they cannot contribute to the magnetism of matter in a noticeable way.) Therefore, such atoms are non-magnetic, and so are the substances that consist of such atoms.

Although the individual atoms are magnetic in the remaining three quarters of the atom types, also substances that consist of such atoms are mostly nonmagnetic: the atoms bind to form molecules and thereby compensate their magnetic effects. For example hydrogen, whose atoms have each only one electron in the shell and are consequently magnetic, only exists in a molecular form in nature, i.e. it consists of H_2 molecules. However, the magnetic effects of the two electrons of each molecule compensate each other so that the molecule as a whole is non-magnetic.

Molecules in which the magnetic effects of the involved atoms do not compensate each other often bond to larger, non-magnetic molecule groups so that the magnetic effects of these groups offset each other and can therefore not be perceived from outside. Only in exceptional cases, i.e. if magnetic atoms or molecules in substances are arranged in a way that their magnetic effects do not compensate each other, there will be a magnetization of the material. In nature, such materials are very rare; but they can be produced artificially. Permanent magnets are made of such materials.

> The magnetic effects of the electrons can compensate themselves
> - within the atomic shell
> - within a molecule
> - within molecule groups.

Exercise

1. If an atom were magnified up to the size of the Earth (diameter of the Earth: approx. 12000 km), what would be the diameter of the nucleus?

4.2 Size and density of the atomic shell

The quantity of electronium in the shell of an atom increases with a growing number of protons. For example, the shell of a mercury atom (atomic number 80) has eighty times as much electronium as the shell of a hydrogen atom. It seems logical to assume that mercury atoms are much bigger than hydrogen atoms. In fact, however, the hydrogen and the mercury atom have almost the same size.

This means that the density of the electronium in the mercury atom must be much higher than in the hydrogen atom.

We would like to build the atoms of the different chemical elements step by step with protons, neutrons and electrons in an imaginary way and thereby look how the size and the density of the atoms change. (As the neutrons do not affect the size of the atoms, we will not mention them anymore in the following.) We start with the hydrogen atom that consists of one proton as a nucleus and an elementary portion of electronium as a shell.

We increase the number of protons by one and obtain a positively charged helium ion. As two positively charged protons are now pulling on the negatively charged electronium, the electronium moves closer to the nucleus. Hence, the helium ion is smaller than the hydrogen atom. This means that the density of the electronium is higher in the helium ion.

To obtain the helium *atom*, we add an electron in the next step. Due to this, the size of the shell increases again, like an air balloon that is inflated with additional air. The two electronium portions in the helium atom take up more space as the single portion in the helium ion; consequently, the helium atom is larger than the ion.

The addition of a further proton results in a positively charged lithium ion, Fig. 4.2, and again, the electronium shrinks in the process. The completion of the shell to build the lithium atom has the same effect as in the case of helium: the shell with three electrons needs more space; the lithium atom is larger than the lithium ion.

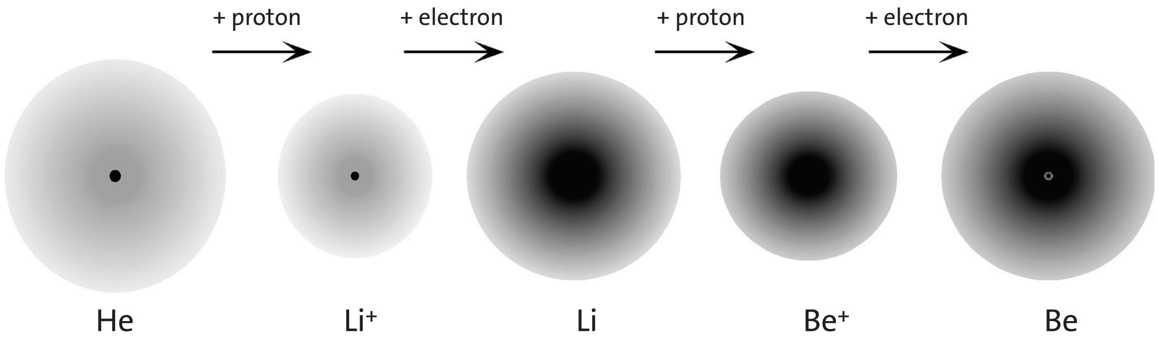

Fig. 4.2 We build up heavy atoms by adding respectively a proton and an electron in an alternating way. (Neutrons are also added but they do not affect the size of the atoms and/or ions.)

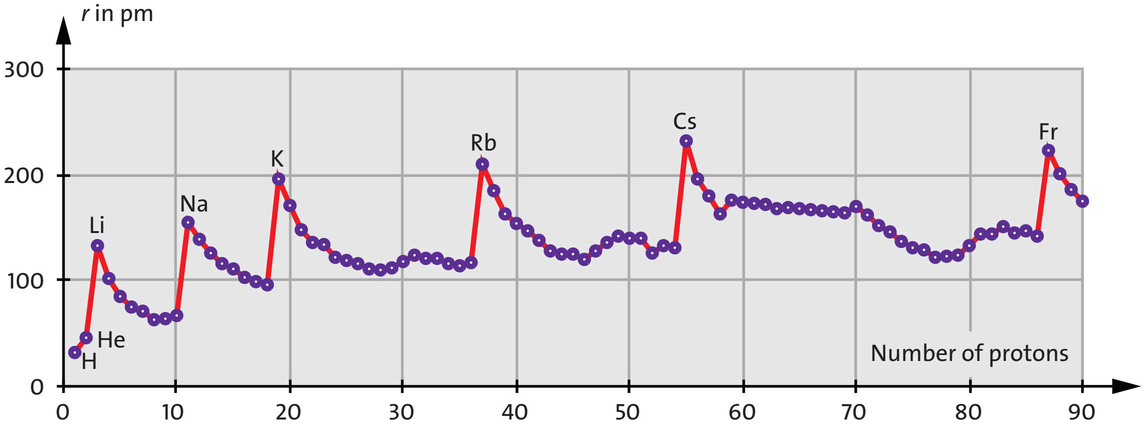

Fig. 4.3 The radii of the different atoms

In the process of building the atoms step by step, we observe two trends in opposite directions: adding a proton leads to shrinkage of the shell while adding an electron leads to an increase.

In Fig. 4.3, the radii of the atoms are indicated above the number of protons. We can see that sometimes the trend to shrinkage is dominant, and sometimes the trend to increase. The radius of most atoms, however, does not diverge very strongly from the typical value of slightly more than 10^{-10} meters. The largest atoms are approximately twice as large, the smallest ones half as large.

> The radius of most atoms diverges only slightly from 10^{-10} meters. In contrast, there are very strong differences in the density of the electronium in the atomic shells.

Exercise

1. Calculate the approximate average density of the electronium in the hydrogen atom and in the lead atom. Estimate the density of atomic nuclei. Compare with the density of normal matter. Only use information from this text.

4.3 The excitation of atoms

An atom that is left undisturbed has its "natural", (almost) spherical shape. We say that it is in the *ground state*.

However, an atom can also be deformed. Therefore, it has to be provided with energy, i.e. by shooting an electron or a photon against it. In general, the atom usually returns to its old, natural shape immediately after, just like a tennis ball that is deformed for a short time through the contact with the racket immediately returns to its old shape.

However, there are deformations that the atom keeps for a longer time. In those cases, we say that the atom is *excited* or that it is in an *excitation state*.

Fig. 4.4 shows the density $\rho = \psi^2$ for some states of the hydrogen atom: in the ground state on the left, and in different excitation states next to it on the right. The density is illustrated in a section plane through the center of the atom. Red stands for a high, blue for a low density. The function is rotationally symmetric for all states. The rotary axis in the figure extends from the top to the bottom.

Fig. 4.5 shows the atom in one of these states in an illustration in which the three-dimensional shape can be seen.

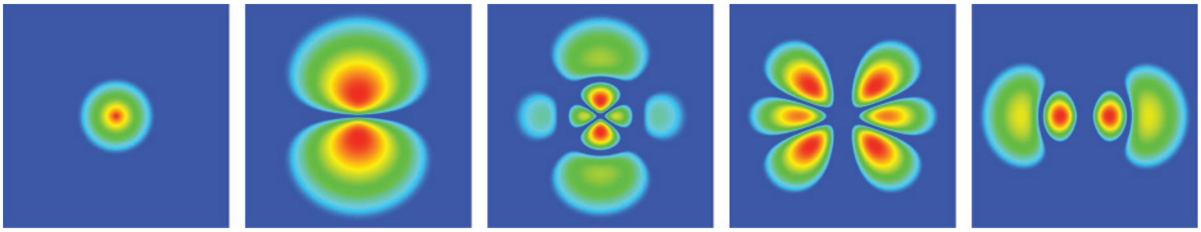

Fig. 4.4 Electronium density of the hydrogen atom in several stationary states

The ground state and the excitation states are called the *stationary states* of the atom. Why stationary? If the atom is in such a state, the density of the electronium will not change over time.

However, an atom can also be forced into another irregular shape. But in such a state, the electronium performs fast oscillations. Oscillating charge leads to the emission of an electromagnetic wave. Therefore, the atom loses energy. Due to these oscillations, such states are called *non-stationary*. We will come back to them at a later time. At the moment, we will only look at the stationary states.

> *Stationary state of an atom:*
>
> The electronium distribution is constant over time.
>
> *Non-stationary state:*
>
> The electronium distribution performs fast oscillations.

A few more words regarding the wording: when an electron is in one of the stationary states, we also say that the electron occupies a certain *orbital*. Hence, the orbital is the place defined by the respective wave function that can be occupied by the electronium. This defines how much electronium is located in the different places (if the orbital is occupied).

The orbital can be compared to a cinema or theater auditorium. When the auditorium is crowded, the people are distributed in it in a very specific way. They do not have to sit at the same distance to each other everywhere though. In the royal box of a theater, people sit at a larger distance to each other than in the area of the economic seats.

> When an electron is in a stationary state, we say that it occupies a certain *orbital*.

Now we can also explain more precisely which results are generated by the mill on the left side of Fig. 3.2. Earlier we said that the wave function comes out at its output. This is not quite true as the Schroedinger equation, i.e. the calculation of mill 1, supplies a series of wave functions: the wave function of all stationary states. In fact, there are even an infinite number of wave functions, and hence stationary states, for an atom.

Many phenomena and many features of the atoms can be explained by means of the wave function. Therefore, it is useful to put the large number of wave functions or orbitals in an order at first. A possibility would be to simply number them. The respective list would still be very confusing, though. There is a better process: a wave function or an orbital can be characterized by three features and we can describe them by respectively one number how strongly these features are pronounced.

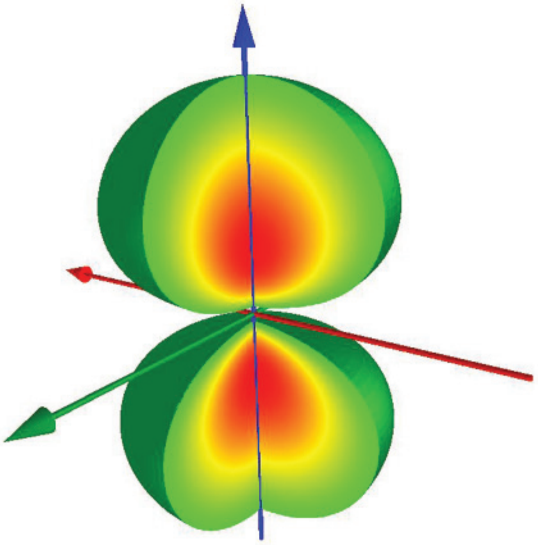

Fig. 4.5 3D illustration of a hydrogen atom in an excitation state (equivalent to the second image part of Figure 4.4)

A simple example shows how the procedure works: in a large clothing factory, the manufactured pants should be characterized: the kind of pants should be visible on a label. Pants are produced in 12 different colors (c), of 4 fabric types (f) and in 5 different sizes (s). Hence, there are $12 \cdot 4 \cdot 5 = 240$ different types of pants. Although we could number them in some way from 1 to 240, there is another more practical method. Each of the three features is numbered individually: the colors from $c = 1$ to $c = 12$, the fabric types from $f = 1$ to $f = 4$ and the sizes from $s = 1$ to $s = 5$. Now, each type of pants can be described by 3 smaller number, e.g. $c = 9, f = 2, s = 3$.

The wave functions of the stationary states can be characterized in the same way. Although we are not yet able to name the three features by which they are classified, we would already like to point out that the wave functions can also be classified by means of three numbers. These numbers are called *quantum numbers*, and they are denominated with the symbols n, l and m.

The quantum numbers can take on the following values:

- $n = 1, 2, \ldots, \infty$
- $l = 0, 1, \ldots, n - 1$
- $m = -l, -(l-1), -(l-2), \ldots, -1, 0, 1, \ldots, l-2, l-1, l$

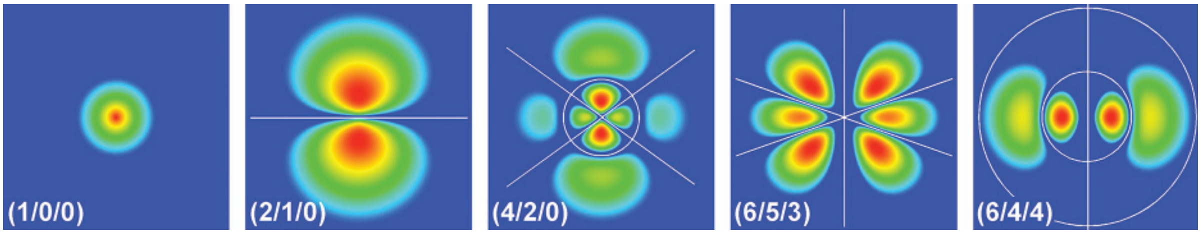

Fig. 4.6 Nodal surfaces for the states from Fig. 4.4 (The surfaces are visible as lines in the section view.)

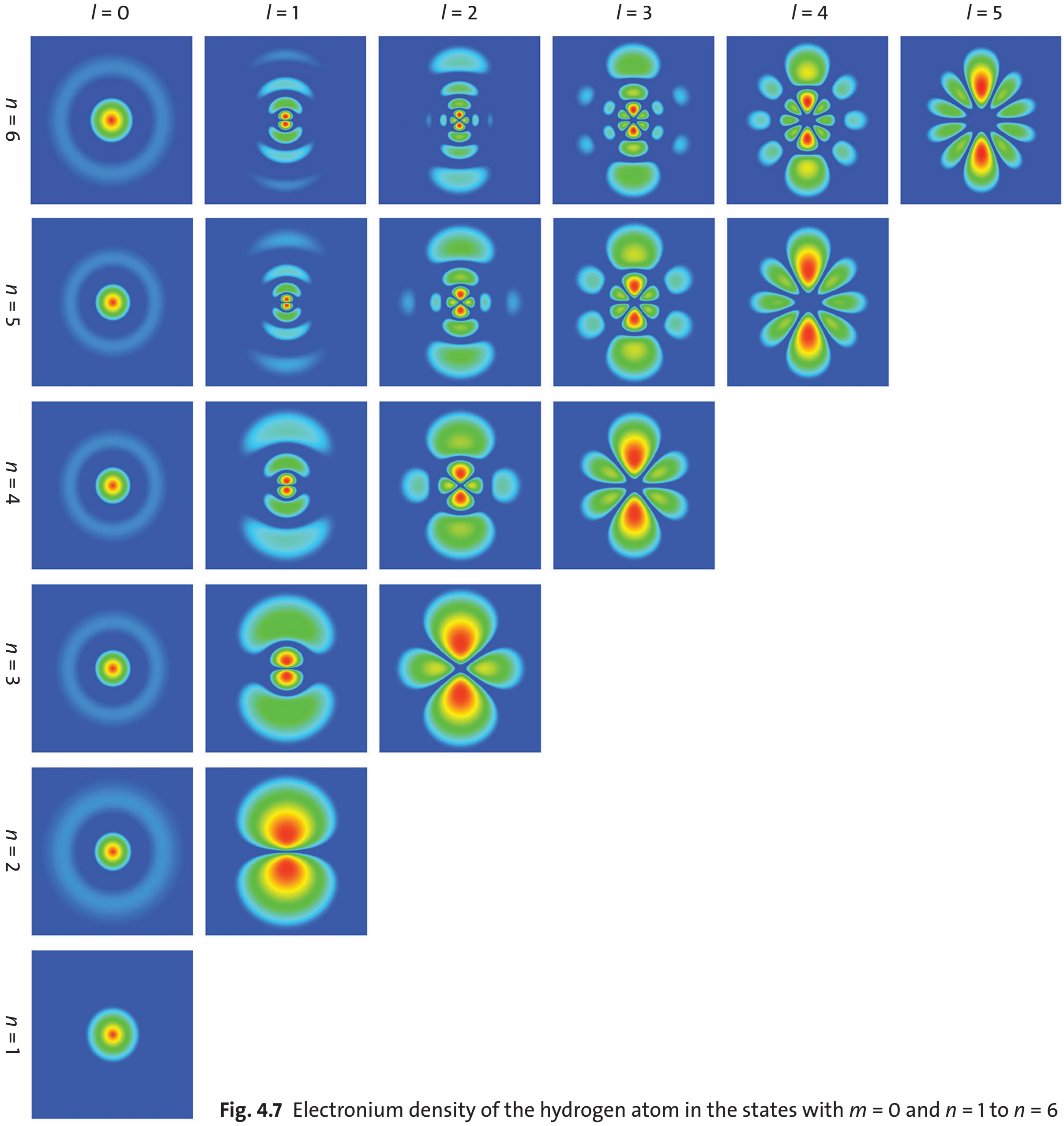

Fig. 4.7 Electronium density of the hydrogen atom in the states with $m = 0$ and $n = 1$ to $n = 6$

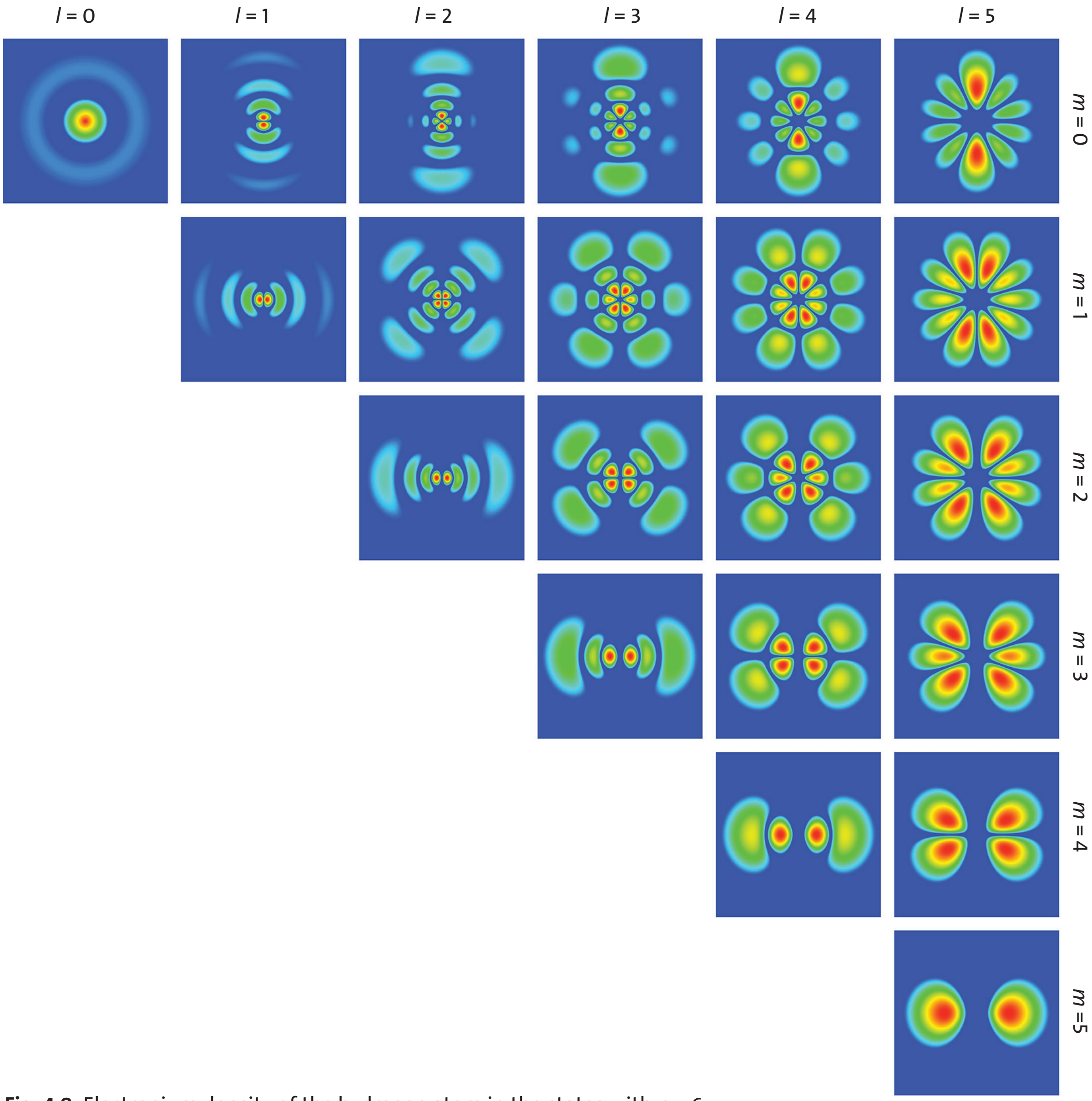

Fig. 4.8 Electronium density of the hydrogen atom in the states with $n = 6$

The range of values of l depends on the value of n, and the range of values of m depends on l. The higher n, the more different values can be taken on by the quantum number l and the larger l, the higher the number of different m-values. For the ground state, the following applies: $n = 1$, $l = 0$ and $m = 0$.

In Fig. 4.6, the quantum numbers are indicated for each illustrated orbital. Hence, (2/1/0) means: $n = 2$, $l = 1$ and $m = 0$.

You will learn step by step which features are expressed by the three quantum numbers. At first, we can already see that they must have something to do with the shape of the orbitals. Fig. 4.7 shows all orbitals for $n = 1$ to $n = 6$ in which $m = 0$.

The image of the ground state is in the bottom left corner. In all states (except from the ground state), there are surfaces in which the electronium density is equal to zero, i.e. spherical surfaces, conical surfaces and planes. (In the two-dimensional section view, the spherical surfaces appear as circles and the conical surfaces as straight lines). Such surfaces are called nodal surfaces. In Fig. 4.6, the nodal surfaces of the orbitals from Fig. 4.4 are illustrated.

Now you can see that the quantum numbers have something to do with the nodal surfaces. In Fig. 4.7, the number of the spherical nodal surfaces increases from the bottom to the top, and the number of the conical nodal surfaces from left to right. (In the left column, the condition that the number of the spherical nodal surfaces increases towards the top is not visible as the electronium density on the outside is so low that nothing can be seen in case of the applied color code.)

Fig. 4.8 shows another selection of orbitals: here, $n = 6$ for all images. The illustrated orbitals differ from each other in the value of l and in the value of m. Again, we can recognize a system: with a decreasing m value, the number of nodal surfaces increases.

> The stationary states are numbered by means of the three quantum numbers n, l and m.

4.4 The energy of the atom in stationary states – the energy ladder

When an atom passes to an excited state, it stores the energy that was used for the change of shape. In Fig. 4.9, the energy values of the different stationary states of the hydrogen atom are marked on the energy axis. We call this illustration *energy ladder*. The energy of a stationary state only depends on the quantum number n. In states that only differ in l or m, the atom has the same energy.

As a measuring unit for the energy, the eV (electron volt) has been used as it is customary in nuclear physics and chemistry: the energy that a particle with the elementary charge e absorbs when it passes through a potential difference of 1 volt. We have

$$1\ \text{eV} = 1.6 \cdot 10^{-19}\ \text{J}.$$

The energy has the lowest value when the atom is in the ground state. The energy values in the excitation states are increased by the amount that is absorbed during the excitation process. Here, we do not consider the fact that an atom can also be set in motion whereby its energy can be changed as we wish. Hence, we look at resting atoms.

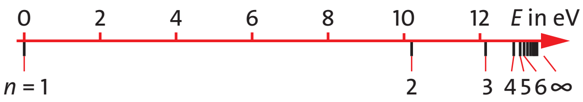

Fig. 4.9 Energy of the hydrogen atom in its stationary states

The Figure shows that the differences of neighboring energy values become increasingly smaller with a growing energy. The highest energy corresponds to the ionization of the atom: when the energy of the atom exceeds this value, the electron frees itself from the atom. In case of the hydrogen atom, only a proton is left behind.

> A resting atom can only store very specific amounts of energy.
>
> The energy of the atom only depends on the quantum number n, but not on l and m.

4.5 The current density of the electronium

We have seen that the density of the electronium can be calculated from the wave function.

With a bit more effort, mill 2 in Fig. 3.2 can calculate another interesting function: $\vec{j}(x,y,z)$. $\vec{j}(x,y,z)$ not only depends on $\psi(x,y,z)$ itself, but also on the derivative of $\psi(x,y,z)$ with respect to x, y and z. We are not interested in the specific structure of the formula at this point but only in what can be understood by $\vec{j}$. If $\vec{j}$ is multiplied with the mass m of the electron, we obtain the mass current density of the electronium:

$$\vec{j}_m(x,y,z) = m \cdot \vec{j}(x,y,z).$$

If we multiply with the charge $-e$ of the electron, we will obtain the electric current density:

$$\vec{j}_Q(x,y,z) = -e \cdot \vec{j}(x,y,z).$$

Remember:

$$\text{mass current density} = \frac{\text{mass current}}{\text{passed through area}}$$

and

$$\text{electric current density} = \frac{\text{electric current}}{\text{passed through area}}$$

So much for the new physical quantity $\vec{j}$ and its calculation. However, what is the calculation good for and how can $\vec{j}$ be used?

4.6 Angular momentum and magnetism of atoms

If $\vec{j}$ is calculated for different states of the electron, we will make some striking observations:

- for the states with $m = 0$, $\vec{j}$ is equal to zero at every point (x, y, z).
- for the states with $m \neq 0$, $\vec{j}$ is different from zero. The electronium flows on circular lines around an axis that passes through the nucleus, Fig. 4.10. In Figure 4.8, $m = 0$ for the upper image line; the electronium does not flow. In the other image parts, it forms a current. For two states, for which m only differs in the sign, the electronium density distribution is equal but the flow directions are opposite.

> In states with $m \neq 0$, the electronium forms a current with circular flow lines. A different +/- sign of m indicates a different flow direction.

As the electronium flows, the atom behaves like a turning gyro and has an angular momentum. This angular momentum can be calculated (with mill 2), which leads to another interesting result. The value is an integer multiple of $h/2\pi$. We have

$$L = m \cdot h/2\pi\ .$$

This result might be surprising, but it basically fits in the overall picture as we already know the same behavior from the electric charge. Also the electric charge only exists in integer multiples of a defined value. We have said that it is quantized. Hence, we see that also angular momentum is quantized. Now we also understand the physical meaning of the quantum number m: it tells us the angular momentum of the atom.

This angular momentum that is related to the flow of the electronium around the core is also called *orbital angular momentum* in order to distinguish it from another angular momentum that we will get to know at a later time.

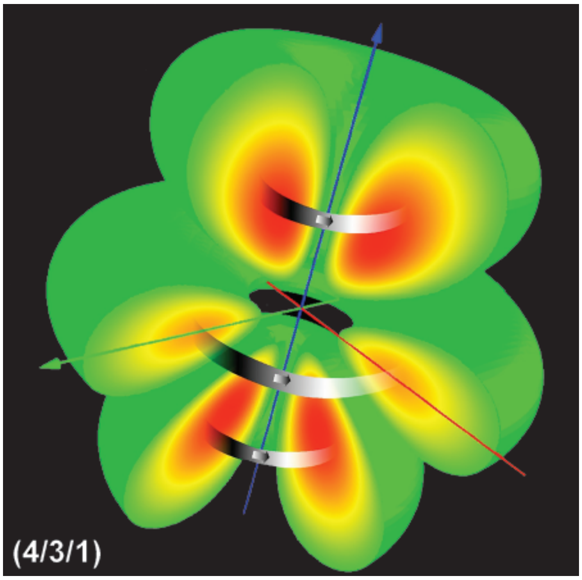

Fig. 4.10 The flow of the electronium is indicated by arrows.

> In states with $m \neq 0$, the atom has angular momentum. The value of this orbital angular momentum is an integer multiple of $h/2\pi$. The angular momentum is quantized.

The fact that the electronium flows also means that there is a circular electric current. An electric current is surrounded by a magnetic field. The field of a circular current is similar to the field of a small pill magnet.

> In states with m ≠ 0, the atom is magnetic.

5 TRANSITION IN THE ATOM – NON-STATIONARY STATES

5.1 Transitions between stationary states

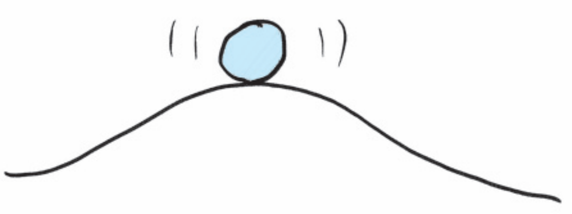

Fig. 5.1 The state of an excited atom can be compared to the state of unstable equilibrium of the ball.

A stationary state of excitation is a precarious situation. You can compare it to the state of the ball in Fig. 5.1.

The center of mass of the ball is located exactly above the high point of its surface. The ball is in an unstable equilibrium. Normally, it would have no reason to roll down. However, the slightest disturbance is sufficient to end its balanced position. When this happens, it rolls down into a state in which the gravitational field has less energy. An atom in an excited state has a similar behavior: the slightest disturbance is enough to trigger a transition to a state with less energy. It releases the corresponding energy in the process, mostly by means of emitting a photon. Then, the energy will be carried away by the photon.

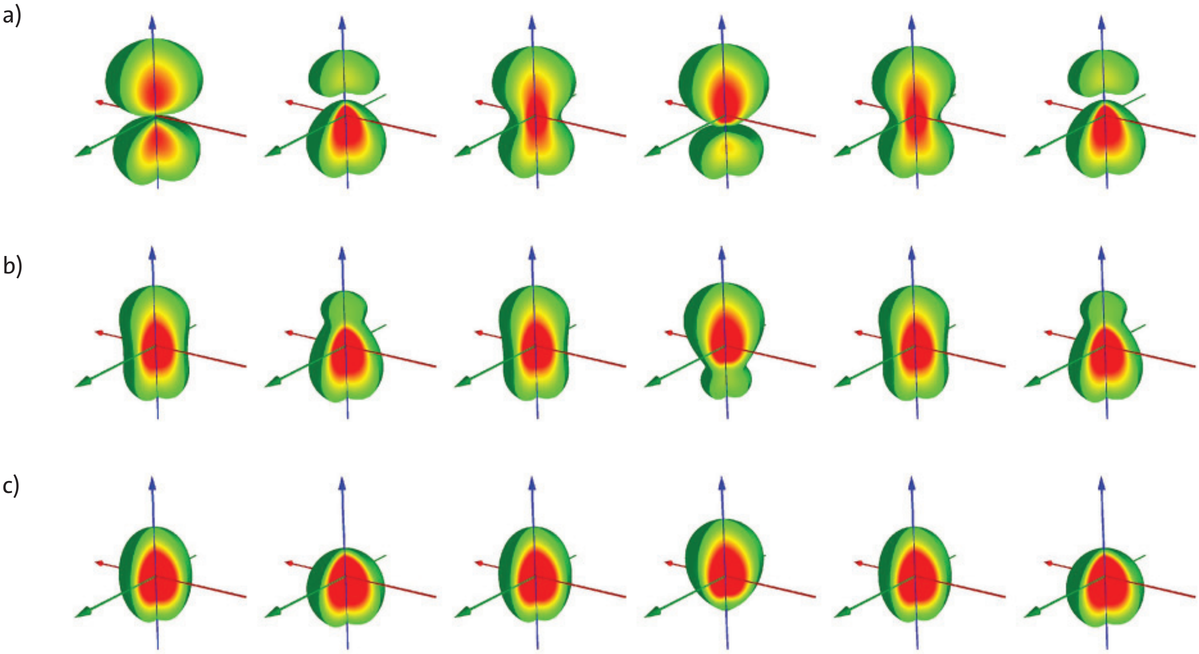

Fig. 5.2 Transition from the stationary state (2/1/0) to the ground state (1/0/0). The electronium oscillates up and down. (a) beginning, (b) middle, (c) end of the transition

We assume the atom to go from state i to state k. The energy of the atom in these states shall be E_i and E_k. Then, the photon must have the energy

$$E = E_\mathrm{i} - E_\mathrm{k}.$$

As $E = h \cdot f$ applies for the photon, we have:

$$f = \frac{E_\mathrm{i} - E_\mathrm{k}}{h}$$

With our second mill, Fig. 3.2., we can also calculate how the electronium behaves during the transition process. If the transition is from state i to state k, the wave function of the initial state ψ_i and the one of the final state ψ_k has to be put into the mill. From that, the wave function $\psi_{\mathrm{i} \to \mathrm{k}}$ that describes the atom during the transition is calculated. Again, we are interested in the electronium density and let mill 2 calculate the square

$$\rho_{\mathrm{i} \to \mathrm{k}} = (\psi_{\mathrm{i} \to \mathrm{k}})^2$$

of the transition wave function. The result is interesting: the electronium densitye $\rho_{\mathrm{i} \to \mathrm{k}}$ does not only depend on the position coordinates, but also on the time. When the transition starts, the density distribution described by $\rho_{\mathrm{i} \to \mathrm{k}}$ starts to oscillate. Fig. 5.2 shows three sections of this process: the image sequence from Fig. 5.2a belongs to the beginning of the process; the transition has not yet significantly advanced. In Fig. 5.2b, approximately half of the transition has passed; shortly before the ground state is reached, it looks the way it is illustrated in Fig. 5.2c.

The transition process can be seen much better in an animation, for example at www.physikdidaktik.uni-karlsruhe.de/Atom/Start.html

Two trends can be identified from the movement of the electronium:

- The electronium makes a fast oscillatory movement. We can see it in each of the three image sequences of Fig. 5.2.
- The global shape of the electronium slowly moves from that of the initial state to that of the final state. We can see this if we compare the three image sequences with each other.

A typical transition takes approximately 10^{-8} s. During this time, the electronium performs around 10^7 oscillations. Of course, it is not possible to illustrate ten million oscillations in an animation. Hence, a trick had to be used for the production of the videos: we assume that only around 15 oscillations take place during the overall transition.

The states that the atom passes during the transition are called *nonstationary states*.

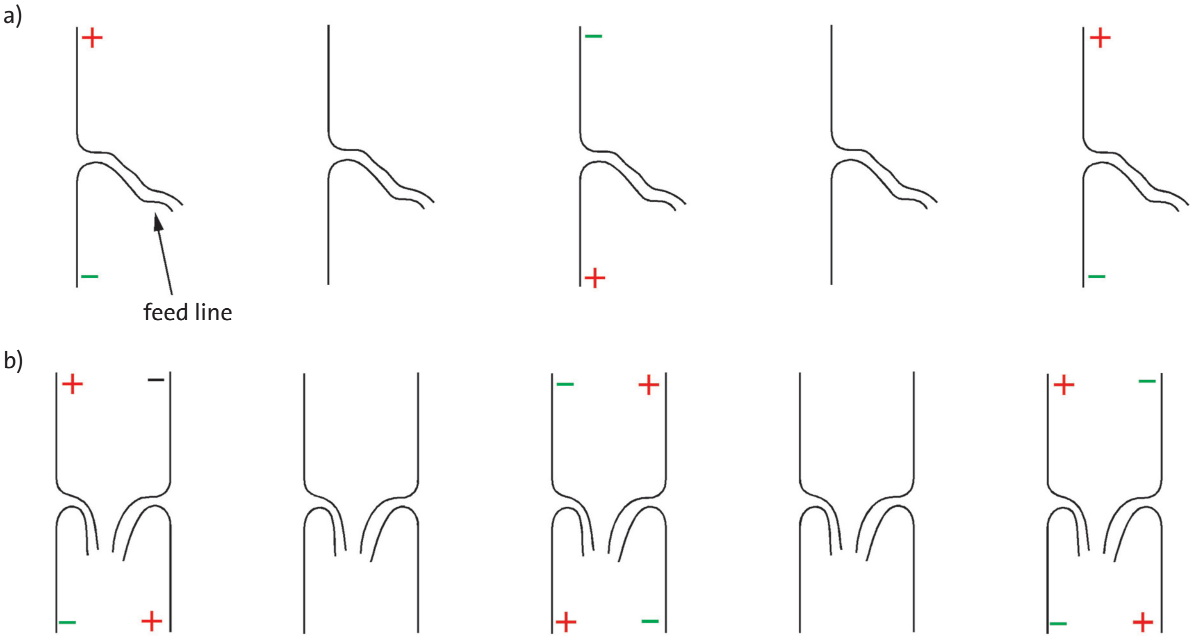

Fig. 5.3 (a) In a dipole antenna, the electric charge oscillates up and down. (b) A quadrupole antenna consists of two dipole antennae that oscillate in opposite directions.

5.2 The emission process

The "animation" from Fig. 5.2 shows $\rho = \psi^2$ as a function of the position and the time. Now, we have

$$\rho_Q = e \cdot \rho \; .$$

The images therefore tell us that also the charge density of the electron oscillates up and down. This means that the atom behaves like an antenna. In an antenna for radio waves, electric charge oscillates up and down. Thereby, an electric and a magnetic field are formed.

Both fields change quickly, resulting in an electromagnetic wave. An antenna, in which the electric charge oscillates up and down, is also called *dipole antenna*, Fig. 5.3a.

As the electric charge in the atom oscillates quickly up and down, the atom behaves like a dipole antenna: it emits an electromagnetic wave and therefore releases energy. Now we understand two things:

- As the atom releases energy during the oscillation process, it gradually moves into the state of low energy.
- The frequency of the oscillation of the electronium is equal to the frequency of the emitted light.

> During the transition of an atom from a state of higher to a state of lower energy, the electric charge of the electronium performs oscillations. The atom works as an antenna. The oscillation frequency is equal to the frequency of the emitted light.

5.3 Fast and slow transitions

A typical transition takes approximately 10^{-8} s. But there are also transitions that run more slowly, and there are others that are so slow that we could practically say that no transition takes place anymore. We will understand why some transitions are so slow if we look at the relevant videos. Fig. 5.4 shows a very slow transition. Also here, the charge distribution oscillates. However, the movement is not simply up and down. It is not equal to the oscillation of a dipole antenna but it is similar to the one of a *quadrupole antenna*.

A quadrupole antenna can be obtained by placing two dipole antennas, which oscillate in opposite directions, closely next to each other or on top of each other, Fig. 5.3b. The fact that such an antenna radiates only weakly is understandable: the field that is created by one antenna is almost compensated by the field of the other one. In Fig. 5.4, the upper half of the electro-

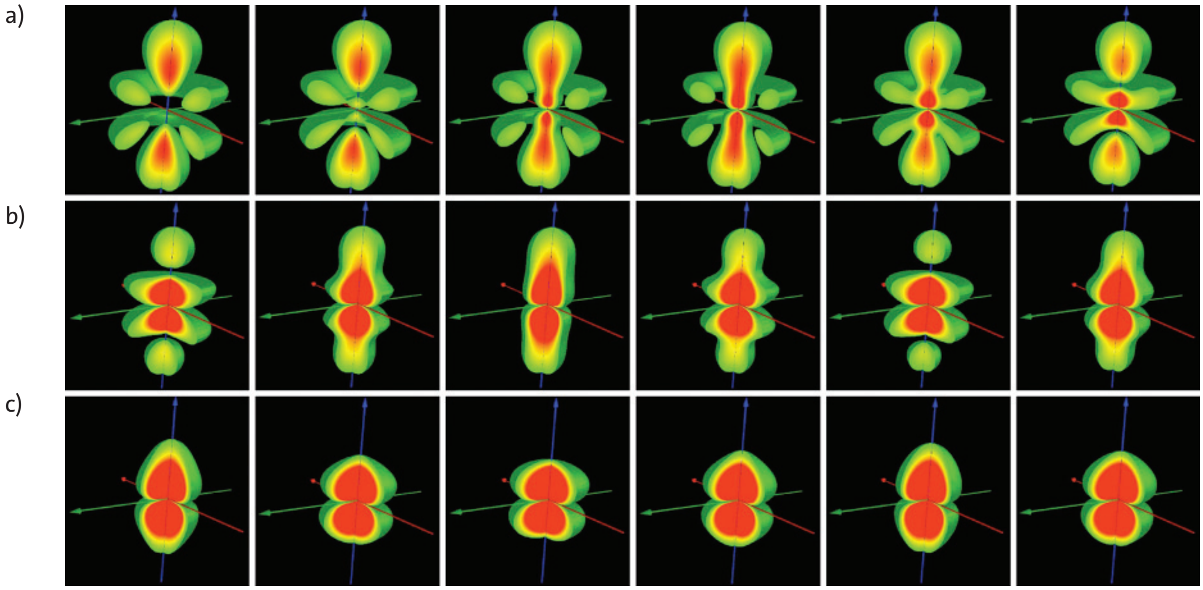

Fig. 5.4 Quadrupole transition (4/3/0) → (2/1/0). The upper and the lower half of the electronium oscillate alternatingly.

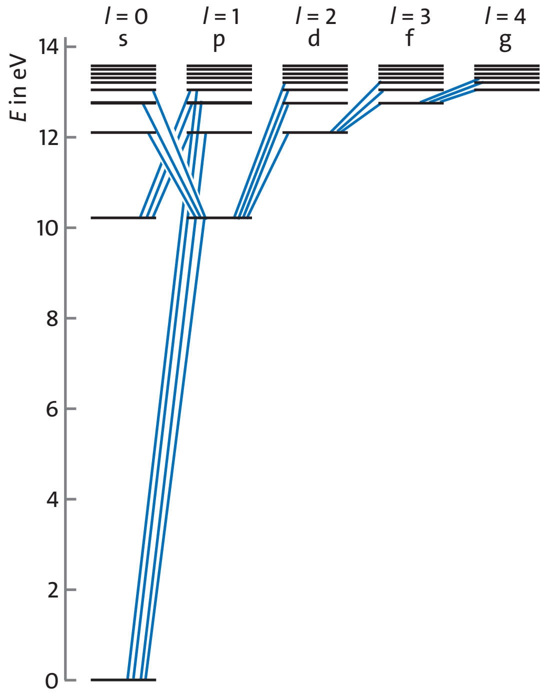

Fig. 5.5 Some allowed transitions (dipole transitions) of the hydrogen atom.

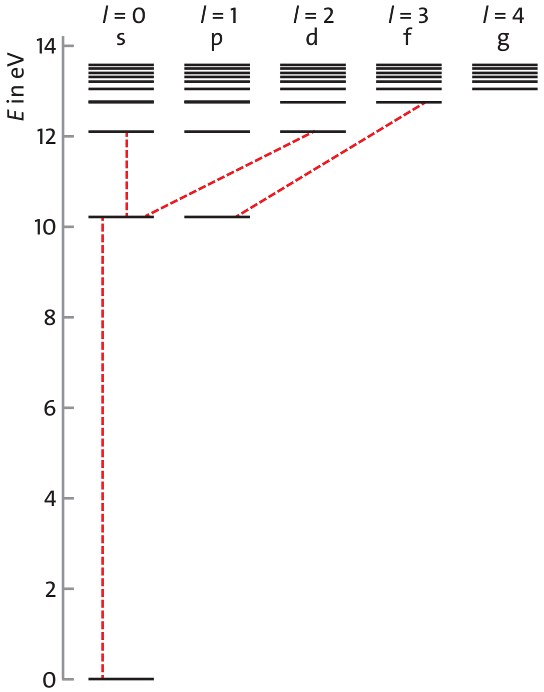

Fig. 5.6 Transitions as the ones illustrated with dotted lines do not take place or are so weak that they cannot be observed.

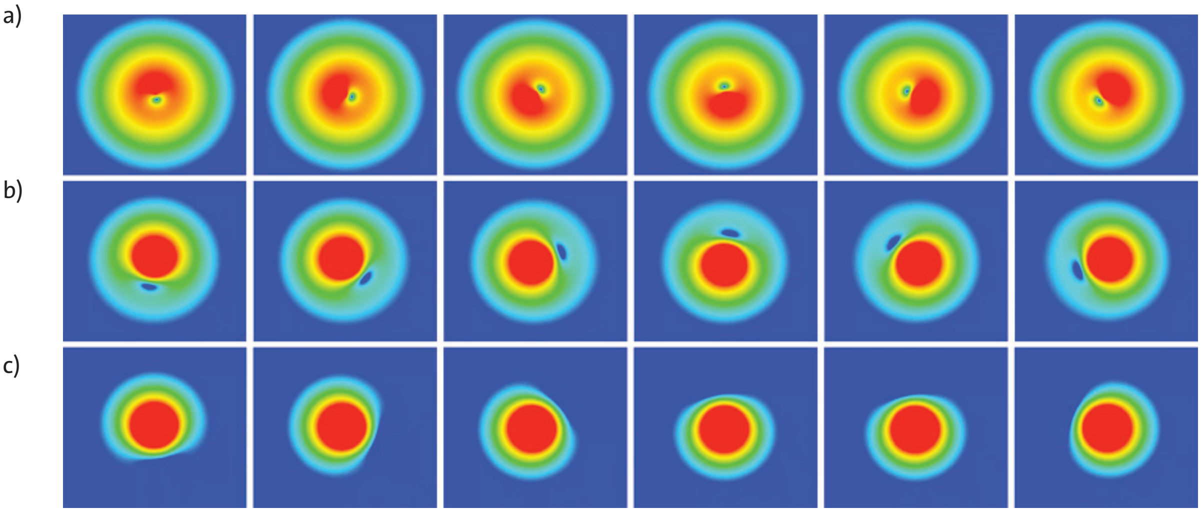

Fig. 5.7 Transition (2/1/1) → (1/0/0). The electronium oscillates in a circular way. (a) beginning, (b) middle, (c) end of the transition

nium oscillates in the opposite direction of the lower one. Hence, it is not surprising that the atom only radiates weakly during such a transition. As this atom radiates weakly, it also takes long until it has released the energy that belongs to the transition.

Finally, there are transitions in which the electronium radiates even less than a quadrupole antenna.

In different transitions, the atom radiates with different intensities.

Fig. 5.5 once again shows the energy ladder of the hydrogen atom, but this time in a special way. For states with a different quantum number *l*, the energy ladders were drawn separately next to each other. The figure also shows the *dipole transitions*, i.e. the transitions in which the atom radiates strongly.

We see that in these transitions the quantum number l changes by just 1. For dipole transitions, we can say that $\Delta l = \pm 1$.

Fig. 5.6 shows "transitions" for which we have $\Delta l = 0$ or $\Delta l = \pm 2$. They (almost) do not take place.

> For fast transitions $\Delta l = \pm 1$. The other transitions are very slow or do not take place at all.

When the emission spectra of the atoms were discovered, the quantum theory did not even exist. It was found that some transitions do not take place but it was not known why. Therefore, it was simply said that they were *forbidden*. The ones that take place were called *allowed transitions*. These expressions have been maintained up to present.

An interesting transition is also illustrated in Fig. 5.7. Here, the electronium makes a circular movement. This results in the emitted light being *circularly polarized*: the tip of the electric field strength vector in a fixed place of the wave field does not make any up-and-down movement as we are used to, but it rotates around the direction of movement of the wave.

6 ATOMS WITH SEVERAL ELECTRONS

6.1 Atoms with several electrons

We have looked at –imaginary– ways to gradually build larger atoms out of smaller ones: by putting a proton into the core and an electron into the shell in an alternating manner. We can now describe this process of putting together a larger atom in greater detail after having seen what the shape of the orbitals of the electrons looks like. A pretty simple system appears in this process.

Each of the electrons in an atom has to be in a stationary state because otherwise the atom would radiate and consequently release energy – until all electrons are in stationary states. We therefore see: each of the electrons of an atom has to occupy one of the orbitals that we learned about in the previous chapter. However, certain particularities appear in this process.

We start with the hydrogen atom in the ground state.

The electron occupies a spherically symmetric (1/0/0) orbital. Now, we imagine to put a second proton into the core (and also two neutrons that are not relevant at the moment). In this process, the orbital of our electron shrinks but keeps its spherically symmetric shape. Then, we add a second electron that, again, occupies a (1/0/0) orbital. The orbitals of the two electrons have the same shape and are also described by the same density distribution. However, both of the new orbitals have slightly grown once again.

Up to now, nothing noteworthy has happened – or at least it appears that way.

Things only become peculiar as soon as we add a third electronium portion (and a third proton) so that we obtain a lithium atom. This electron does no longer occupy the (1/0/0) orbital of the ground state but it occupies a (2/0/0) orbital that is still spherically symmetric but clearly different from the (1/0/0) orbital. It appears as if there was no more space in the (1/0/0) orbital of the ground state. And indeed, this is a very general rule:

> An orbital can only be occupied by two electrons.

We add a fourth electron (and a fourth proton) so that we obtain a beryllium atom. The fourth electron occupies the same orbital as the third, i.e. the (2/0/0) orbital. All four atoms –hydrogen, helium, lithium and beryllium– are spherically symmetric.

Only if we build up the fifth atom, the boron atom, we will obtain an orbital that is no longer spherically symmetric: a (2/1/0) orbital. The boron atom is therefore no longer spherically symmetric. However, the deviation from the spherical shape is low as the four occupied atoms are all spherically symmetric.

We can continue building the atoms this way by adding one portion of electronium after the other. Each of these portions occupies a specific orbital in the atom or in other words: it takes on a specific shape. The occupied orbitals thereby fit together like the pieces of a puzzle.

During the occupation of new orbitals, distributions that are exactly spherically symmetric emerge time and again.

We would like to exemplify this fact by means of a strongly simplified model, Fig. 6.1. The three puzzle pieces of a puzzle A, B and C fit together in a way as to form a black ring. However, the pieces do not have a sharp edge as it would be the case for pieces of a normal puzzle. This means that they have to be placed in a way as to overlap a little bit. Thereby, grey is placed on grey so that black emerges as an overall color.

The electron orbitals show a similar behavior. When overlapping, the electronium densities are added and a

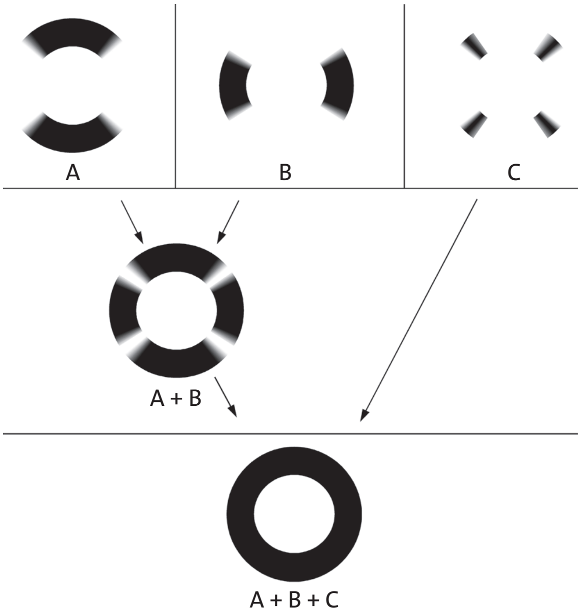

Fig. 6.1 Puzzle with blurred contours. When put together, all three parts A, B and C form a black ring. Thereby, the following applies: grey + grey = black (equally: light grey + dark grey = black etc.).

finished "puzzle" will emerge at some point if new orbitals ("puzzle pieces") are added.

If a further electron is added, a new puzzle will start. The new electron is bonded only weakly. It can easily be removed from the atom.

Through the addition of further electrons, this puzzle will be finished and the next stable atom will be reached. This is how it will go on.

We can also say that the orbitals, which are part of one of these finished puzzles, form a *shell*. We can imagine very roughly that the electronium shell would be built in subshells: one subshell after the other. The *shell model* provides a rough orientation. However, we need to bear in mind that the subshells (in a radial direction) do not have a sharp edge. Just as the orbitals that are part of a subshell, they overlap.

Fig. 6.2 shows how a shell is formed of 5 orbitals.

Atoms with a full subshell are especially stable. In order to remove an electron, i.e. an elementary portion of the electronium, from such a stable atom, a particularly high amount of energy is necessary. These stable, spherically symmetric atoms include the atoms of the inert gases helium, neon, argon, krypton, xenon and radon.

The subshell structure of the atomic shell is a reason for the – at least approximate – periodic repetitions of the properties of the atoms with the number of electrons. And therefore it is also useful to list the atoms in the *periodic table of elements.*

6.2 The spin

The rule that we have just learned seems to have a flaw. On one hand, it tells us that the electrons of an atom have to occupy different orbitals and therefore have to be described by different wave functions. We could also say that they have to be in different states. On the other hand, nature does not seem to take this rule very seriously: a defined orbital may be occupied by two electrons and not by only one. Now we will see that the state of an electron is not yet characterized unambiguously by indicating only the orbital.

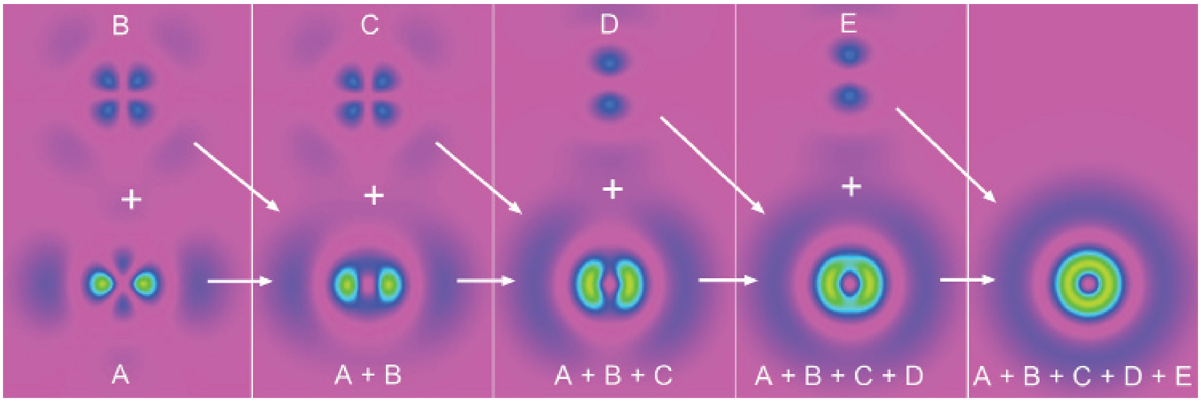

Fig. 6.2 Structure of a shell of orbitals A, B, C, D and E. The orbitals B and C are identical in the picture although they are different in the flow direction; the same is true for the orbitals D and E. The complete subshell – on the right – is spherically symmetric.

This is because the wave function contains another variable besides x, y, z: the *spin*. The word *spinning* is a synonym for rotating, turning.

The spin variable s of the wave function can only take on two values. Hence, we can imagine the spin variable as a parameter of the wave function that can take on one of two values. The physical meaning of the quantity s is simple in the first place. It tells us that the electron has an angular momentum, i.e. in addition to the orbital angular momentum that we have already addressed and that is related to the flow of the electronium around the nucleus. The spin angular momentum can have one of two values, i.e. $+h/4\pi$ and $-h/4\pi$. The values differ by exactly one angular momentum quantum $h/2\pi$.

But how can the spin angular momentum be imagined to be like? To what type of rotary movement is it related?

The electronium seems to have yet another rotary option besides the circular flow around the core. You can imagine it as a sort of intrinsic rotation at each point of the electronium, Fig. 6.3. But caution! This image represents a model – but not a very good one. If you do not understand it, you are not the only one. Even professional physicists have problems with a clear interpretation of the spin.

But what are we getting out of the spin? We come back to our old problem: we would like to build up an atom step by step by gradually adding electrons to the atomic shell (and the respective number of protons to the core). The two electrons of the helium atoms that both occupy a (1/0/0) orbital are not in the same state, they differ in the spin; all higher atoms behave in a similar way. While we had at first concluded that one orbital can be occupied by two electrons, we can now say that all electrons have to be in different states. In this context, a state is characterized by the orbital and by the spin.

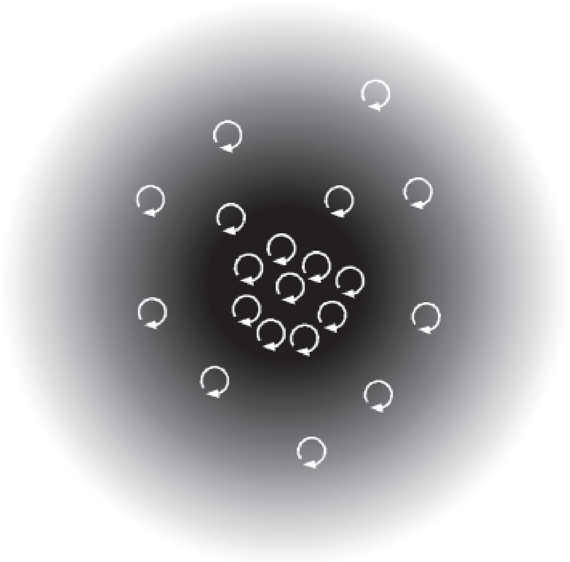

Fig. 6.3 A quite mediocre model for a rotational movement that belongs to the spin angular momentum: the electronium forms many very small swirls.

> The electrons of an atom are in different states. Two electrons that occupy the same orbital are different in the spin.

6.3 Back to the wave function

We have described the structure of atoms with several electrons by indicating which orbitals are occupied. This provides us with a good general overview. However, the procedure is not very precise because when a new orbita is occupied the old ones are deformed, and also the newly occupied one do not have exactly the same structure as the hydrogen orbitals.

If we are interested in the exact structure of the electron shell, we have to rely once again on the theory, i.e. our mathematical mill. However, the work that our mill has to do is much more difficult now. We look at the helium atom, i.e. the second simplest one after the hydrogen atom. The wave functions that are provided by mill 1 now have the following form:

$$\psi_i\,(x_1, y_1, z_1, x_2, y_2, z_2).$$

The index i is used to number the different functions in a logical order as, like in case of the hydrogen atom, we will obtain several or many results that correspond to the various stationary states of the atom.

But what exactly does such a function mean? It assigns a value not to one point in space but to two points in space. We choose the values of six independent variables and obtain a function value. Mathematically, this can be expressed as follows: to each point in a six-dimensional space a value of the wave function is associated. In case of the lithium atom, the space is nine-dimensional; in case of boron, it is twelve-dimensional; for oxygen 24-dimensional and for lead 246-dimensional, etc. It is very hard to get a clear idea of such functions. Fortunately, however, our mill 2 can process these functions without any problem and provides us once again with results that we can understand well: it gives us the electronium density, the current density, values of energy, momentum, angular momentum, etc.

7 SPECTRA

7.1 The excitation of atoms with light

To excite an atom with a photon, the energy of the photon has to be exactly equal to the energy that the atom needs to go from the ground state to the excitation state. If the photon has less energy, the atom cannot be excited because the energy is not sufficient. If the photon has more energy, however, the excitation will not be possible either.

We will describe a specific experiment, taking sodium atoms as atoms to be excited. With sodium, the observation of what happens works particularly well. Under normal conditions, sodium is a solid substance. It can also be transformed into sodium vapor by means of heating.

We obtain the photons with a lamp that creates a beam of white light, i.e. a mix of light with wavelengths from 400 to 800 nm. This means that we have a mix of photons with a variety of energies.

The light beam from the lamp is sent upon the sodium gas, Fig. 7.1. The largest part of the photons now has an energy that does not correspond to any excitation. The photons fly from the atoms through the gas without being disturbed. Only a very small fraction of the photons has a suitable energy, and only those photons are absorbed by the atoms. The atoms change over into an excited state.

After passing through the gas, the absorbed photons are missing in the light beam. However, as this is only true for a very small part of the light, nothing can be seen at first. Therefore, we further analyze the light after having passed through the sodium gas: we perform a spectral decomposition.

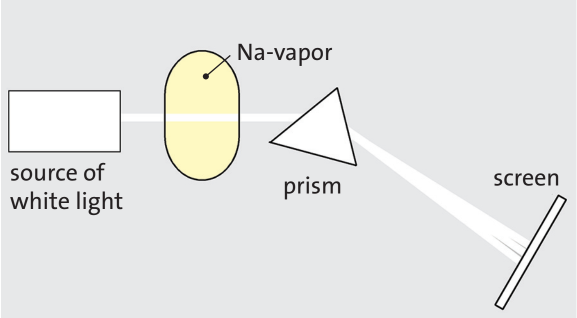

Fig. 7.1 Light of a light bulb passes through sodium gas and is spectrally decomposed afterwards.

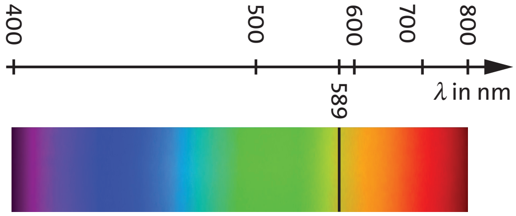

Fig. 7.2 Light of the wavelength 589 nm is missing in the spectrum

We let a thin beam of the light that has passed the sodium gas fall onto a prism. Behind the prism, a white screen is set up at a longer distance. Now, the light that is sorted by wavelengths falls upon this screen. As light with different wavelengths appears to our eyes as different color effects, we see the colors of the rainbow on the screen.

Taking a closer look, however, we see that the screen remains dark at a defined position in the range of the yellow light, Fig. 7.2: the light whose photons are absorbed by the sodium atoms is missing. We can determine the wavelength of the missing light by means of a spectrometer: $\lambda = 589$ nm.

From this, the energy of the absorbed photons can be calculated:

$$E = \frac{h \cdot c}{\lambda} = \frac{6.6 \cdot 10^{-34}\,\text{Js} \cdot 3 \cdot 10^{8}\,\text{m/s}}{589 \cdot 10^{-9}\,\text{m}}$$
$$\approx 0.3 \cdot 10^{-18}\,\text{J}.$$

This is at the same time the energy difference between the ground state and the lowest excitation state of the sodium atoms.

Photons of the energy $E = 0.3 \cdot 10^{-18}$ J are not the only ones that are absorbed by the sodium atoms. The sodium atoms can also be brought into many other excitation states. For this reason, there are also other photon types that can be absorbed. The energies of these photons are higher than of the ones analyzed before and the wavelengths are shorter.

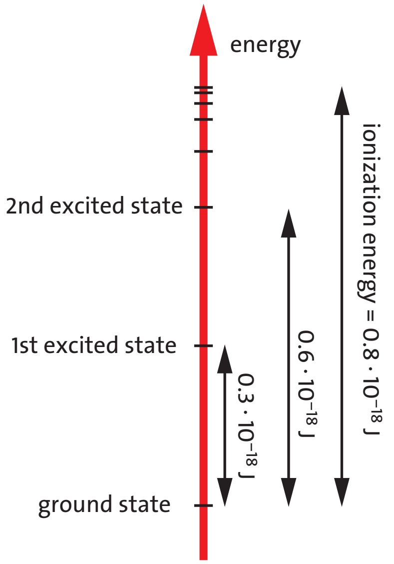

Fig. 7.3 Energies of the excitation states of the sodium atom

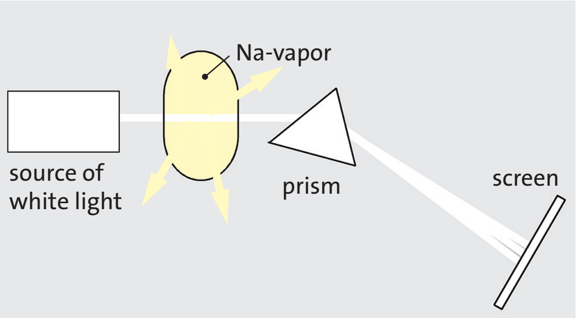

Fig. 7.4 The excited sodium radiates in all directions.

Hence, sodium atoms absorb for example photons with an energy of $0.6 \cdot 10^{-18}$ J, Fig. 7.3. This is equivalent to a wavelength of 330 nm; i.e. to ultraviolet light.

We summarize:

> Photons with a suitable energy can excite atoms. The photons are absorbed in the process.

We have seen that most photons of the visible light are not absorbed by the sodium atoms. The same is true for all other gaseous substances. Their atoms and/or molecules only absorb very few or no visible photons at all. This explains a feature of gases that you know quite well:

> Gases are transparent.

A photon is emitted when an excited sodium atom returns to the ground state. In the experiment just described, we can see that this is actually the case. Taking a closer look at the sodium glass recipient penetrated by the light beam, we can see a slightly yellowish light. An analysis with the spectrometer shows that it has the same wavelength as the light that is missing on the shade. Hence, the sodium atoms release the energy, which they have obtained during the absorption of a photon, by generating an equivalent photon.

However, the emission does not only take place in the direction of movement of the absorbed photons but to all sides, Fig. 7.4. This is why the photons that correspond to the excitation are missing almost completely in the passing beam.

Exercises

1. Sodium atoms can be brought from the ground state into the third excitation state by means of light of the wavelength $\lambda = 285$ nm. Calculate the energy difference between the ground state and the third excitation state.
2. What is the maximum wavelength that light, which should be used to ionize sodium atoms, can have? What type of light is that?

7.2 The excitation of atoms with electrons

Apart from photons, other particles such as electrons, ions, protons, atoms and molecules can be used for the excitation of atoms. As electrons are used very frequently, we will look at this possibility in the following.

To be able to excite an atom, an electron needs –in addition to its rest energy– the amount of energy that is required for the excitation. The energy of an electron can be increased by setting it in fast motion, i.e. by charging it with momentum, as it is provided with energy together with the momentum. This process of charging the electron with momentum and energy is not difficult.

Electrons are put into an electric field. As electrons carry electric charge, the field pulls on them. Momentum and energy of the electrons increase. If such fast electrons hit atoms, the atoms will be deformed and can pass over into a state of excitation.

If the electron has received more energy than required for the excitation, it will take this excess energy along after the excitation. Hence, it is different from the photons in this respect. The photons disappear in the excitation process and consequently nothing can absorb the remaining energy.

> Atoms can be excited with fast electrons.

If the energy of the electron is high enough, it can ionize the atom.

7.3 Gases as light sources

Gas-discharge lamps – not to be confused with *fluorescent lamps* – are frequently used artificial light sources.

A light bulb contains a glowing metal wire that is lit up by means of an electric current. In a gas-discharge lamp, a gas is emitting light. We can use our knowledge about the excitation of atoms to explain how the gas is lit up.

The gas is located in a closed glass tube. At each end of the glass tube, there is a metal electrode, Fig. 7.5. One of the electrodes is on a low, the other one on a high electric potential. The electrode with the low potential is heated up in addition. It releases electrons as a consequence. (A large part of solid substances also consists of electronium. Through heating, the electronium can be deformed up to a point where the electrons detach from the solid substance.) As the electrons have a negative charge, they are pulled towards the electrode with the high potential by the electric field between the two metal electrodes.

On their way to that electrode, the electrons come hit atoms. Thereby two things happen:

1. Atoms that are hit by fast electrons are ionized, i.e. further free electrons are produced.
2. Atoms that are hit by slower electrons are excited.

The additional electrons that we obtain in case 1 are in turn accelerated by the field and can ionize further atoms etc. Starting from the electrons that are released by the metal electrode, large quantities of free electrons are therefore created in the whole gas-discharge lamp.

Due to this, also the frequency of process 2 increases, i.e. atoms are excited by the impact of electrons. The excited atoms emit photons while returning to the ground state: the gas is emitting light.

> In gas-discharge lamps, atoms are excited with electrons. Atoms emit photons in the process of returning to the ground state.

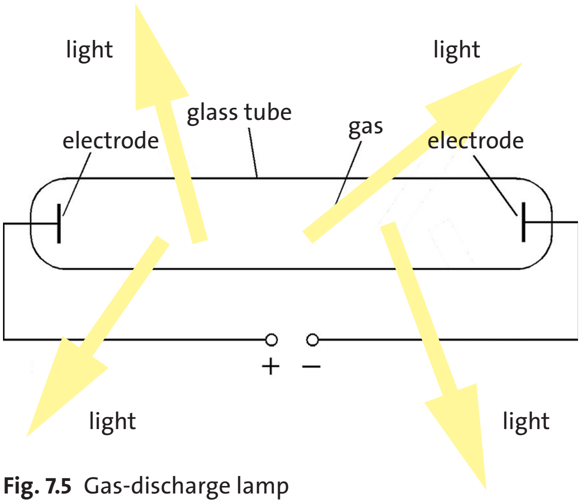

Fig. 7.5 Gas-discharge lamp

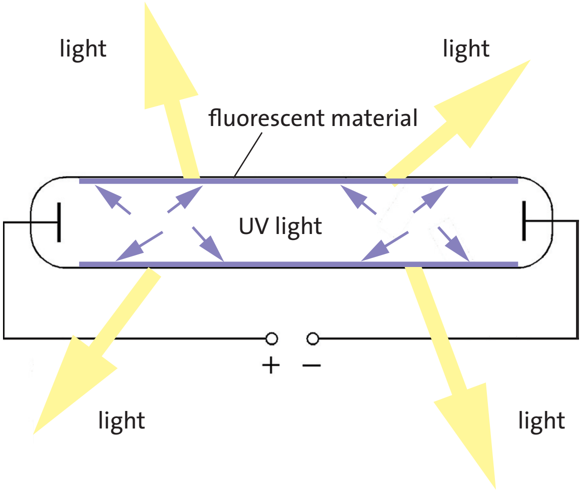

Fig. 7.6 Fluorescent lamp

Pedestrian crosswalks, tunnels and crossroads are often lit by means of gas-discharge lamps that contain sodium vapor. As sodium atoms emit "yellow" photons, the typical yellow light of the sodium lamps is created.

Compared to other gas-discharge lamps, sodium lamps consume the least energy. The disadvantage of these lamps is their spectrum. As they only emit light of a single wavelength, the objects illuminated by them appear all in the same color: everything appears in a more or less bright yellow. To attenuate this effect, the lamps often contain supplements of other gases that emit light of other wavelengths. This improves the color-reproducing effect.

A slightly different type of light sources are the fluorescent lamps. You know them well because they are certainly also used at your school to light the classrooms. Also, energy-saving lamps are fluorescent lamps.

The gas that is excited with electrons in this case is mercury vapor. The energy difference between the ground state and the first excitation state is approximately $0.8 \cdot 10^{-18}$ J. Photons of this energy are equivalent to light with a wavelength of approximately 250 nm. Hence, the mercury vapor emits at first invisible UV light. The glass tube in which the mercury is located is coated with a white substance, the *fluorescent substance*, on the inside. The UV light emitted by the mercury reacts with the fluorescent substance whereby the desired visible light emerges, Fig. 7.6.

The main advantage of these lamps in contrast to incandescent lamps is their low energy requirement. A usual light bulb needs four to five times more energy than a fluorescent lamp in order to create the same quantity of light. In addition, the lifespan of the fluorescent lamps is longer.

7.4 The spectra of gases

In a gas-discharge lamp, the atoms are not only brought into the first excited state, but also in higher excited states.

For the return from a higher excited state to the ground state, the atom has different possibilities as we have already seen. It can return to the ground state directly or descend in stages.

Excited atoms can therefore emit many different photons and the light that is radiated by a gas-discharge lamp consequently consists of a mix of light types with different wavelengths.

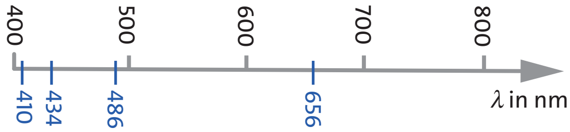

Fig. 7.7 Emission spectrum of hydrogen

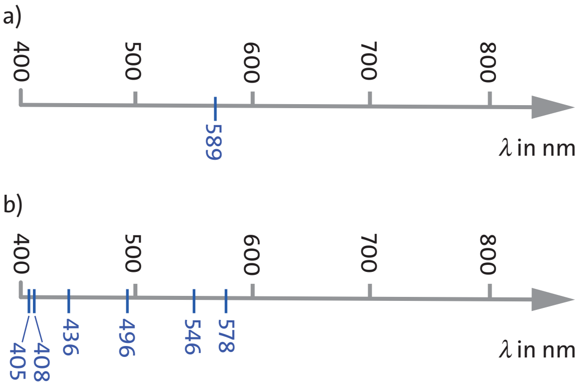

Fig. 7.8 Emission spectra of sodium (a) mercury (b)

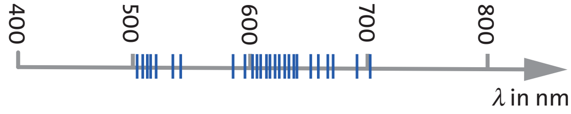

Fig. 7.9 Emission spectrum of neon

Fig. 7.7 illustrates the emission spectrum of hydrogen, however not in a complete manner. In order to display the complete spectrum of the light emitted by hydrogen, the wavelength scale would have to be extended as hydrogen also radiates light in the UV and infrared range.

We already know the emission spectrum of sodium vapor for the wavelength range from 400 to 800 nm: here, sodium only emits a single type of light, Fig. 7.8a.

We have already mentioned in the previous section that mercury vapor mostly emits UV light. However, as we can see in Fig. 7.8b, mercury also emits in the visible range. We can find light of the wavelengths 405 nm, 408 nm, 436 nm (all violet), 496 nm (turquoise), 546 nm and 578 nm (both yellow). We perceive this mix of light as blue-green light.

The spectrum of the reddishly glowing fluorescent tubes that are used for advertising is shown in Fig. 7.9. Fluorescent advertising tubes of a different color can be obtained by using other gases or gas mixtures.

A comparison of the figures 7.7, 7.8 and 7.9 shows that the spectra of different gases can easily be distinguished from each other. The spectrum is a typical characteristic feature, i.e. something like a "fingerprint" of the gas.

> Each gas emits light with certain wavelengths that are typical for the gas. The spectra of gases can therefore be easily distinguished from each other.

In fact, the spectral analysis is frequently used by chemists to identify substances. The atoms of the substance to be analyzed are thereby excited and the emitted light is examined by means of a spectrometer.

Another procedure of spectral analysis is the examination of the *absorption spectrum* of a substance. It involves sending light from a broad range of wavelengths through the gas and the subsequent analysis with the spectrometer to see which wavelengths are missing in the spectrum. As the atoms absorb the same photons as they emit, the absorption spectrum provides the same information as the emission spectrum.

The spectral analysis plays a particularly important role in astrophysics. The stars are much too far away as to simply go there by spaceship in order to take substance samples. What we do have access to is the electromagnetic radiation emitted by the stars. Hence, we can draw conclusions about the material composition of the gaseous atmosphere of the Sun based on the absence of light with defined wavelengths in the visible part of the sunlight: the atoms in the Sun's atmosphere absorb photons of the white light generated by the Sun. Hence, it is the absorption spectrum of the solar atmosphere.

All the knowledge we have about the chronological development and the material composition of stars are based on spectroscopic examinations of their radiation. Of course, not only the visible part of the radiation is analyzed in this process but also radiation of all other wavelength ranges, from radio waves up to gamma radiation.

7.5 Why flames shine

The flame of a candle, a lighter, a wood fire or a Bunsen burner under conditions of restricted air supply shines in bright yellow. When opening the air supply of the Bunsen burner so that the flame burns with a roaring sound, the yellow shining effect disappears and the flame only burns in a dim, bluish way, similar to the flames of a gas stove. A hydrogen flame does almost not shine at all. If, however, some sodium (or common salt, which is sodium chloride) or lithium is put into the hydrogen flame, it will shine in a strong yellow or red color. How do these different shining effects of the different flames happen?

The atoms and molecules of a gas fly around back and forth with a high speed and hit each other time and again in the process. When the atoms or molecules collide, their electronic shells are deformed. Normally, the energy is not sufficient to cause excitation. When gases are heated up, the atoms become faster; their energy increases. If the temperature is high enough, excitation processes can eventually take place.

However, in many gases, for example in hydrogen and the other gases contained in a hydrogen flame such as oxygen and nitrogen, much energy is needed to excite the atoms or molecules in a way as to emit visible photons. The required temperature is not reached in the flame and consequently a hydrogen flame does not shine.

The energy of the atoms and molecules in the flame is though sufficient for example to bring the atoms of lithium or sodium into the first excitation state. Hence, when some lithium is put into a hydrogen flame, its atoms are brought into the first excitation state and emit "red" photons when returning to the ground state: the flame shines red. Accordingly, the flame shines yellow when some common salt, i.e. sodium, is put into the flame. By means of analyzing the emitted light with a spectrometer, we can find out which substance in the flame is shining.

> When colliding with other atoms or molecules, for example in a flame, atoms can be excited.

The dim bluish light that a Bunsen burner flame sends out when provided with a sufficient quantity of oxygen comes from CO_2 molecules that pass over from excited states to the ground state. However, the CO_2 molecules are not excited through collisions with other atoms or molecules in this case. Hydrocarbons (mostly methane, CH_4) burn in the Bunsen burner flame into carbon dioxide (CO_2) and water (H_2O). The overall reaction

$$CH_4 + 2\,O_2 \rightarrow CO_2 + 2\,H_2O$$

runs in several steps. One of these steps is the combustion of carbon monoxide:

$$CO + O \rightarrow CO_2^*.$$

Carbon dioxide in an excited state emerges in this reaction. When returning to the ground state, the CO_2^* molecules emit photons:

$$CO_2^* \rightarrow CO_2 + \gamma.$$

These photons make up the dim bluish light of the flames of Bunsen burners and gas stoves.

> Excited atoms and molecules can be formed in chemical reactions.

Fireflies use the same principle to create the light that they emit. A firefly does not glow like a piece of hot iron; rather, chemical reactions take place in the abdomen of a firefly, in which excited molecules emerge that subsequently emit visible photons.

When reducing the air supply of a Bunsen burner, there is not enough oxygen available for the complete oxidation. Carbon atoms are left over, which join together to form carbon granules. These granules are the components of soot. Due to the high temperature in the flame, the soot granules shine, i.e. they emit light.

Hence, the relatively strong yellow glow of the flame of a Bunsen burner running at reduced air supply, a candle or a lighter does not come from the involved gases but from a glowing soot particle. The spectrum of these flames is therefore similar to the one of a incandescent lamp: the emitted light is composed of light of all wavelengths from 400 to 800 nm. We will learn later how this glowing effect can happen.

8 SOLID MATTER

Before looking at the solid substances, we will summarize the essential differences and common features with regard to the arrangement of the atoms in gases, liquids and solids.

8.1 The arrangement of the atoms in gases, liquids and solid matter

The atoms or molecules of a gas fly around back and forth – the higher the temperature, the faster. In the process, they collide time and again; most of the time, however, they are far away from each other.

In liquids and solid matter, by contrast, the atoms are arranged closely next to each other, just like tennis balls in a basket.

There are two possibilities of packing tennis balls closely together. They can either be piled up in regular layers, Fig. 8.1, or they can be laid next to or on top of each other in a disorganized way, Fig. 8.2. The same applies for the atoms in liquids and solid matter.

In solid substances, the atoms are mostly arranged regularly. Such substances are known as *crystalline* solids or crystals. Metals, most stone types, salts, sugar, sulfur and diamond are crystalline.

But there are also many solid substances in which the atomic arrangement is more similar to the one of the tennis balls in Fig. 8.2, for example glass and most plastics. The structure of such substances is called *amorphous*.

In liquids, the amorphous structure is the common standard. Almost all liquids are amorphous, also the most widespread liquid, i.e. water. Crystalline liquids are used in liquid crystal displays of calculators and flat screens. You may know the acronym LCD for the name *liquid crystal display*.

Table 8.1 provides an overview of the differences and common features in the arrangement of the atoms or molecules in gases, liquids and solid substances.

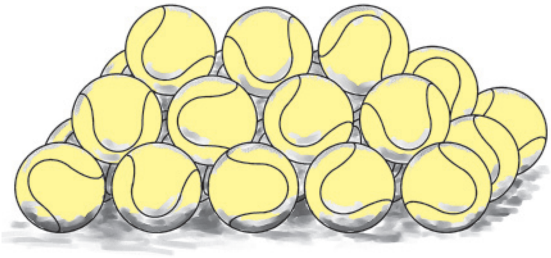

Fig. 8.1 Tennis balls can be piled up in regular layers ...

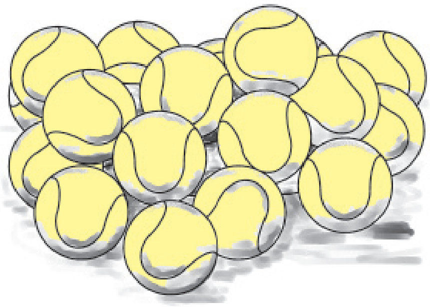

Fig. 8.2 ... or gathered in a random way.

8.2 The distribution of the electronium in solids

Fig. 8.3 shows what the inside of a solid substance, in this case of a crystal, looks like. The darker the grey hue in the Figure, the higher the density of the electronium. The atomic nuclei are located at the points of maximum density. With a growing distance from the nucleus, the density of the electronium decreases at first – up to the point where we come close to the next nucleus. Then, it increases again.

Just as in individual atoms, the largest part of the electronium is located close to the nuclei. There are no hollow spaces between the individual atoms like in the pyramid of tennis balls in Fig. 8.1. The whole space between the nuclei is filled with electronium. Also, there are no clear boundaries between the atoms. The electronium of all atoms is merged to a continuous "lake".

Fig. 8.4 shows the calculated density distribution of the electronium in a solid substance, i.e. in a common salt crystal, NaCl. Here, the density of the electronium is illustrated by means of lines of constant density. The higher the number in the Figure, the higher the density of the electronium at the place of the line. These lines can be compared to the lines of elevation on a map. The higher the number on a line of elevation, the higher the altitude of the respective places in the landscape.

Fig. 8.4 shows four areas in which the density of the electronium is particularly high: at the top left and at the bottom right in the area of the sodium nuclei, as well as at the top right and at the bottom left in the area of the chlorine nuclei. In between, there are areas in which the density of the electronium is quite low but still different from zero.

> In solid substances, the electronium fills up the whole space between the nuclei; it forms a continuous "lake".

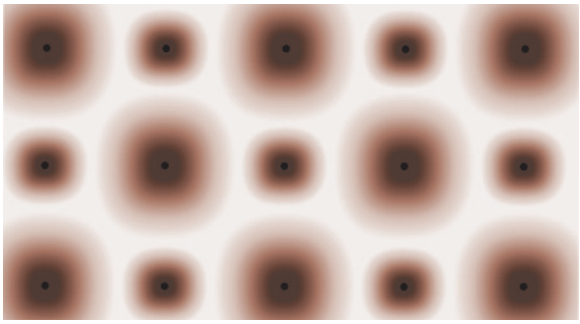

Fig. 8.3 Distribution of the electronium in a solid substance. The small black circles represent the atomic nuclei (not drawn to scale).

	distance between atoms or molecules	arrangement of atoms or molecules
gases	long	fly around in a disorganized way
liquids	densely packed	mostly amorphous, rarely cristalline
solids	densely packed	cristalline or amorphous

Table 8.1 provides an overview of the differences and common features in the arrangement of the atoms or molecules in gases, liquids and solid substances.

The atoms in a gas can fly around almost freely. The higher the temperature of the gas, the faster they move.

The atoms of solid substances, in turn, are bound to a fixed place. They cannot leave their place in the crystal grid. But still, they are not immobile: they perform oscillations around a central position and this oscillatory movement is faster the higher the temperature.

Exercise

1. How many atoms are contained in a common salt crystal with an edge length of one millimeter?

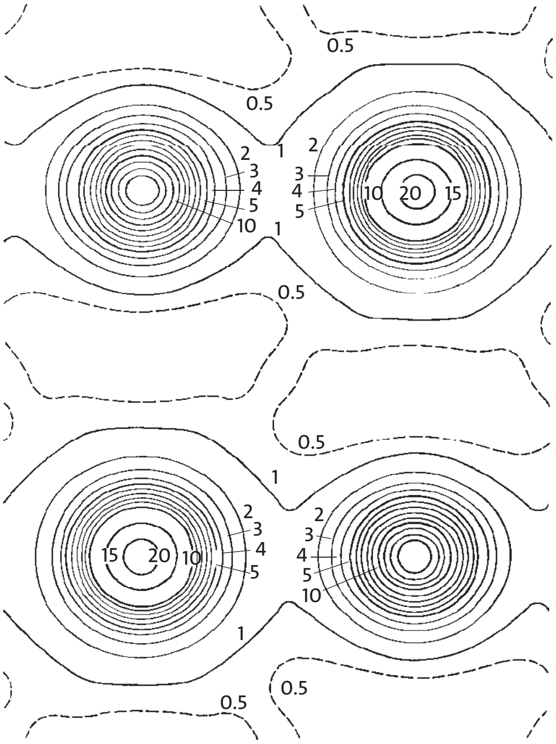

Fig. 8.4 The distribution of the electronium in common salt

8.3 The energy ladder of solid substances

We have seen that the electronium in the shell of an atom can be deformed and that certain forms correspond to a stationary state. The same applies for the electronium lake in solid substances as well.

Here, deformation means – just as in case of the single atom – that an elementary portion of the electronium is displaced from one place to another. Therefore, the electronium distribution in the lake changes: at one point, its density decreases while it increases accordingly at another point.

The electronium in solid matter can be excited in many different ways and the deformation in case of a single excitation is not limited to the area of an individual atom; it extends over many atoms.

Just as for the atoms, the stationary state with the lowest energy is called ground state. The states of higher energy are called excitation states.

The energy ladders of solid substances differ clearly from those of individual atoms: while the rungs of the energy ladder of an atom are very narrow because the electronium in an atom can only store very specific amounts of energy, the rungs of the energy ladder of a solid substance are wide. The energies that the electronium of a solid substance can store comprise entire ranges of values on the energy scale. We call them *allowed ranges* in the following. In between, there are the *forbidden ranges*.

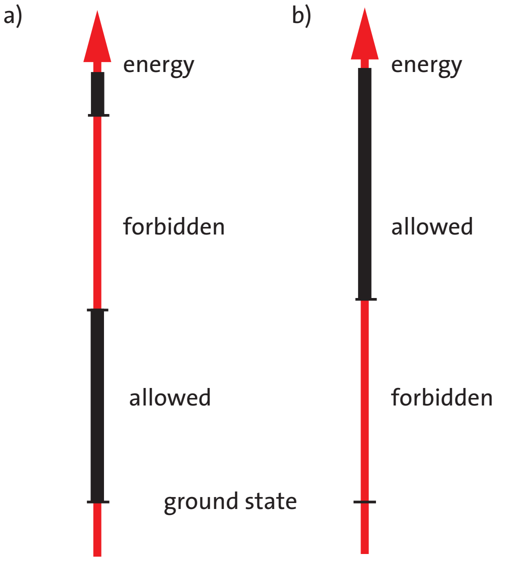

Fig. 8.5 The energy ladders of metals (a) and non-metals (b)

> The energies of the stationary electron states in solids form ranges.

The energy ladders of solid substances can be divided in two groups.

In substances of one group, the first allowed range starts at the ground state, Fig. 8.5a. Metals have energy ladders of this type. Very small amounts of energy are sufficient to excite the electronium.

In case of the second energy ladder type, there is a forbidden range between the ground state and the first allowed range, Fig. 8.5b. Such an energy ladder is typical for non-metals. Here, excitations will only be possible if the excitation energy is sufficient to overcome the gap between the ground state and the first allowed range.

8.4 When light hits metals

When light falls onto a very smooth metal surface, most of it is reflected. For example, the reflecting layer of a mirror is made of aluminum. But not all of the light is indeed reflected. We can observe that very well if we make the experiment illustrated in Fig. 8.6.

Two larger mirrors are opposite to each other at a short distance. When looking past the edge of mirror A in mirror B, we see the picture of mirror A. In the latter, we can see the picture of mirror B, and in it once again the picture of A, and so forth. Altogether, we see a sort of corridor whose walls are formed by mirror edges. Light that seems to come out of this corridor far from behind is the light that has been reflected back and forth between the mirrors many times. It is inter-

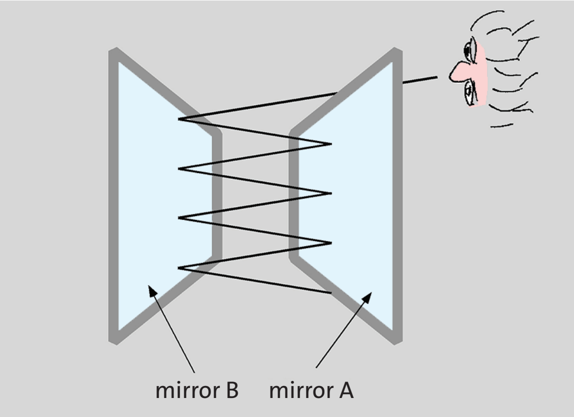

Fig. 8.6 The observer receives light that was reflected back and forth many times between the mirrors.

esting for us that it becomes increasingly dark towards the back of this corridor, which means that some of the light is lost in each reflection process. In fact, approximately 10 % of the light is absorbed during each reflection.

We can conclude that the mirror – just like any other metal – does not only reflect but also absorb light. The light is not absorbed to a greater extent because only a very small amount of light penetrates the metal due to reflection.

The absorption of the light in metals occurs in accordance with its energy ladder, Fig. 8.7. The three bars next to the energy ladder correspond to the energy of the photons of red, green and blue light. They are representative for all photons from the spectral range of visible light. All three bars do no extend beyond the lower allowed energy range. This means that the photons of visible light can excite the electronium in metals.

> Photons of visible light can excite the electronium of metals. Thereby they are absorbed.

It is not difficult to make a metal absorb not only 10 % but all of the incident light. It is sufficient to disassemble the metal into fine particles. Every metal powder absorbs light almost completely: metal powders are black. How can that be explained?

When light hits a metal powder, it is reflected not only once but many times – from one powder particle to the next, Fig. 8.8. After the first reflection, only 90 % of the original amount of light are left, after the second reflection only 81 %, after the third one only 73 %, and so on. After 25 reflections, only 7 % are left and after 50 reflections only approximately 0.5 %, i.e. almost nothing. The finer the metal powder, the higher the number of reflections and the darker the black color of the powder. The light dies off in the powder.

The fact that powdery metals absorb light is shown by an experience everyone has already made: when a clean, shiny aluminum surface is polished with a white metal cleaning agent, the cleaning agent turns black. The reason for this blackening process is the rubbed-off aluminum powder.

Also, another very simple experiment shows how light can gradually disappear completely in the course of many reflections. A small hole is cut into a cardboard box, for example a shoebox, Fig. 8.9.

The inner surfaces of the box should be white so that they only absorb a small amount of light. Now we can see that the hole is completely black. Light can enter the box through the hole. However, it does not find the way out of the box. As some light is absorbed every time it hits the wall of the box, nothing is left after some back-and-forth reflections.

Everything in our environment that we perceive as black is formed this way, for example the black color of printed letters. If we take a closer look at the paper, we will see that it has a rough surface. The incident light moves on a zigzag path in the paper before it comes

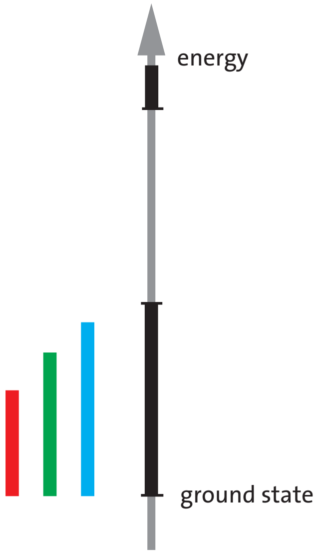

Fig. 8.7 Metal: the energy of the photons of visible light corresponds to the lowest allowed range.

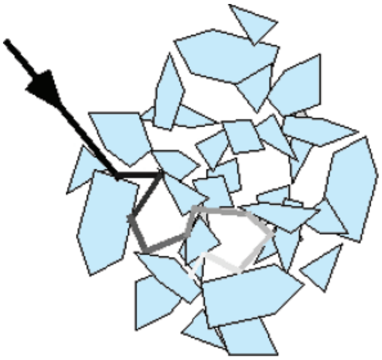

Fig. 8.8 In a metal powder, light is reflected back and forth several times.

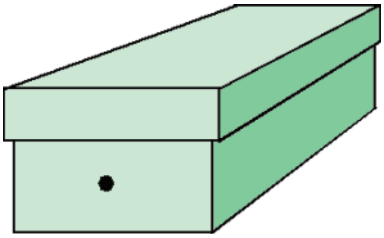

Fig. 8.9 The hole in the shoebox is black.

back out and arrives for example at your eyes. If there is a substance at the relevant place of the paper that absorbs a part of the light with each reflection, the paper will appear black at that place.

> Light dies off in black objects.

8.5 When light hits non-metals

Figure 8.10 shows the energy ladder for non-metals. Again, the bars on the left correspond to the photons of visible light. All three bars end within the forbidden energy range. Hence, the energy of the photons of the visible light is not sufficient to excite the electronium. The light is not absorbed but passes through the material. Substances with such an energy ladder are therefore transparent. Examples for such substances are glass, salts, sugar, ice, cellulose (the main component of paper), most minerals (e.g. quartz and feldspars), diamond, and plastics.

Maybe you can think of objects that are not transparent but white in spite of being made of one of the mentioned substances: for example refined salt, powdered sugar, paper, pulverized glass or snow. All these substances or things consist of small particles: granules or fibers. The smaller the particles, the stronger the white color. Powdered sugar is "whiter" than refined sugar.

What is white here, however, are not the small individual particles. In case of snow crystals, salt grains, sugar crystals and glass splinters, we can see it with our naked eye. Paper, in contrast, has to be contemplated with a microscope: the individual grains or fibers are transparent. The white color impression only emerges when many transparent particles gather.

> White objects are composed of many small transparent particles.

Light that hits such an object from one side is not absorbed by the transparent particles, but refracted and reflected. The direction of the light is changed in the process, i.e. nothing is lost. After several refraction and reflection processes, the light comes back out at the place where it has entered the object. Hence, it does not pass through the substance – the substance is opaque.

Also the white color of clouds is formed this way. A cloud consists of many small, transparent particles, i.e. water droplets. In case of milk, the fat droplets cause the white color.

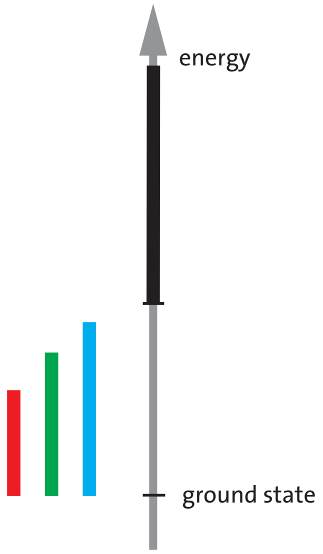

Fig. 8.10 Non-metal: the energy of the photons of visible light is in the lowest forbidden range.

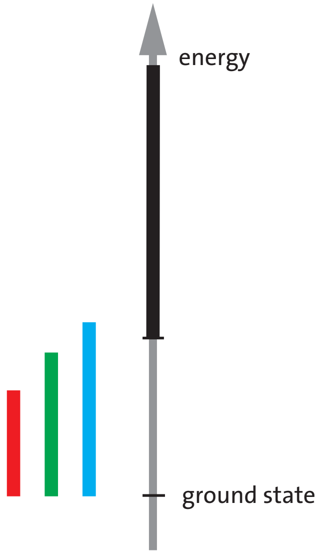

Fig. 8.11 The energy ladder of cadmium sulfide

But not all non-metals are transparent or white. For example, there is colored glass and paper, colored stones and colored plastics. In these cases, the color is caused by the fact that the transparent nonmetals are mixed with other substances that absorb visible light with specific wavelengths.

However, there is yet another cause for non-metals being colored or even black: in the energy ladder displayed in Fig. 8.11, the lower forbidden range is smaller than the energy of the photons of the blue light. An example for a substance with such an energy ladder is cadmium sulfide. The longest of the three bars (it corresponds to the photons of the blue light) extends into the range of the allowed energies. The energy of these photons is consequently sufficient to excite the electronium. The photons of the blue light are absorbed in the process.

All other photons are not absorbed. In the light that was left pass, the blue light is missing and the remaining mix forms a yellow color. A piece of cadmium sulfide is therefore transparent and slightly yellow. Cadmium sulfide powder is strongly yellow.

In case of silicon, germanium and graphite, the gap between the ground state and the first allowed range is even smaller than for cadmium sulfide, Fig. 8.12. The energies of all photons of the visible light are sufficient to excite the electronium. Consequently, visible light is not left pass. The part that is not reflected is absorbed. As a larger part of the light than in case of metals seeps away in the material, smooth surfaces do not have a mirror effect but show a shiny black. These substances are called semiconductors.

Also the semiconductors in a powdery form have a deep black color as the light dies off during the reflections on the grains of the powder. Soot is such a substance. It mainly consists of tiny graphite particles and is used, inter alia, for the production of printing ink and for blackening of plastics.

As the energy of infrared photons does not reach up to the first allowed range, semiconductors are permeable for infrared light – in contrast to metals.

Silicon is used for the production of lenses for infrared cameras. Infrared light is let through these lenses and refracted. Visible light, in turn, that would also light the film and that must therefore not enter the camera, is not let through.

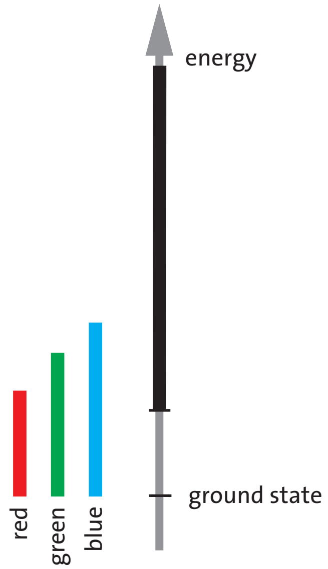

Fig. 8.12 Energy ladder of a semiconductor

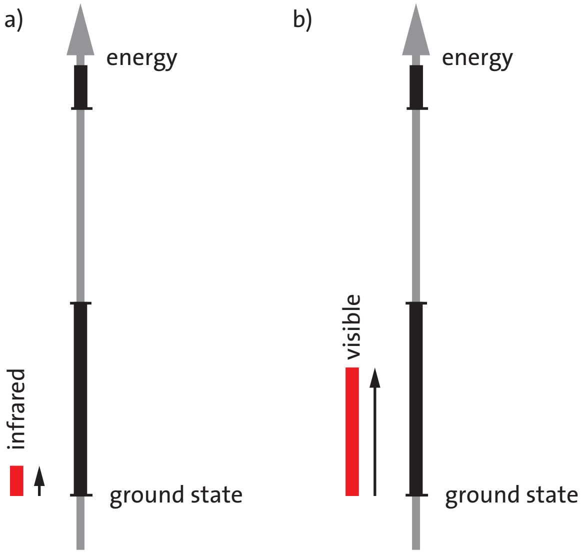

Fig. 8.13 (a) At a normal temperature, the energy of the movement of the atoms in metals is only sufficient for slight deformations of the electronium. The metal only emits infrared photons. (b) At a higher temperature, the electronium is excited more strongly. It emits photons of the visible light range. The metal glows.

8.6 Solid substances as light sources

The light of a light bulb comes from a glowing tungsten wire, the light of a candle flame comes from glowing soot particles. How does this light come into existence?

In a solid substance, the nuclei and the surrounding electronium move around an average position. The higher the temperature, the faster the movement and the higher the energy of the atoms.

In a metal, the energy of this movement is sufficient to excite the electronium already at a normal temperature. With the excitation processes, however, the electronium only absorbs a small amount of energy. Also the photons that are emitted during return to the ground state therefore have low energies. They are infrared photons and not photons of visible light, Fig. 8.13a.

When the metal is heated, the movement of the atoms becomes stronger; their energy increases. The electronium is more deformed and photons with a higher energy are emitted during return to the ground state. At an even higher temperature, photons of visible light are finally formed – the metal glows, Fig. 8.13b.

Also all black non-metals, i.e. semiconductors, start to glow when they are heated up.

> Metals and semiconductors can glow: at a high temperature, they emit visible light.

Transparent and colorless non-metals behave differently. Their energy gap is so large that the electronium is not excited even at high temperatures. Consequently, no photons of visible light can be produced.

> Objects made of transparent materials do not glow.

8.7 How solid substances conduct electricity

Cables that are made of metal – mostly copper – wires and that are coated with plastic are used as electricity lines. The wire is made of metal because metals are good conductors of electricity, and the coating is made of plastic as plastics (and generally most non-metals) do not conduct the electricity and therefore prevent the electricity from flowing out of the line.

Electricity always sits on particles, specifically on the negatively charged electrons and the positively charged protons of the atomic nuclei. When electricity is to flow through a copper wire, either the nuclei or the electronium have to move. However, the nuclei are bound firmly to their place in the crystal grid; they cannot move through the copper wire. Therefore, what remains is only the electronium.

We could imagine that an electric current is formed by the electronium of a solid substance flowing as a whole past the nuclei through the solid. However, the electronium is attached to the nuclei so firmly that it cannot be moved this way.

The situation is similar as for the following problem: a large, heavy carpet should be displaced, Fig. 8.14.

We grab the carpet on one end and try to pull it over the floor. As it is large and heavy, however, it does not move. Therefore, we try a trick. We push a pleat into the carpet on one side, Fig. 8.15. This pleat can now be pushed conveniently through the carpet – and at the end the whole carpet is displaced quite a bit.

The electronium in a solid substance behaves in a similar way as the carpet.

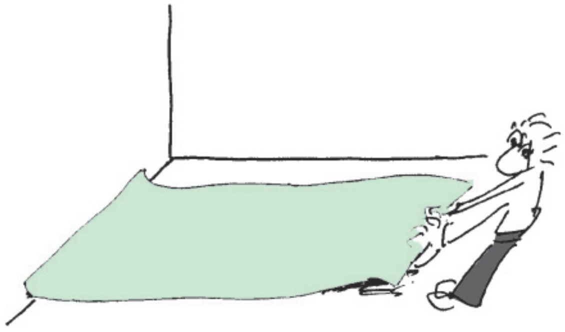

Fig. 8.14 One possibility to move a carpet ...

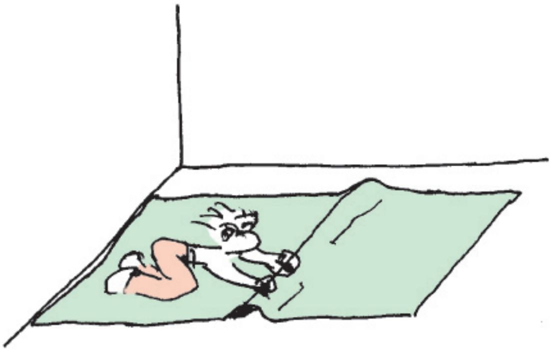

Fig. 8.15 ... and a second possibility

At first, we imagine the "electronium lake" to be in its ground state. There, it has a very specific distribution as shown for example in Fig. 8.3 and 8.4. When it is excited, an electron, i.e. an elementary portion of the electronium, is brought into an excitation state. The density distribution of the electronium lake changes somehow in the process. (This change takes place in a smaller area of space but is not limited to the area of an atom.) Compared to the ground state, we thus obtain an increase of the density at some places and a decrease at other places. If we apply a voltage and therefore create an electric field in the material, the densified area will move through the material and so does also the area of reduced density.

Hence, we see that a material will only be electrically conductive when the electronium can be excited easily. This is the case in metals, Fig. 8.7. Already the smallest electric field that is created in the conductor is sufficient for the excitation. Non-metals cannot be excited this way, Fig. 8.10; therefore, they do not conduct the electric current.

An additional elementary portion of the electronium, i.e. an electron, is exactly equivalent to a mobile densified area. We can also say: metals contain mobile electrons.

A density reduction is equivalent to a missing elementary portion in the electronium lake. Such a missing portion is also called a *hole*. Also the missing elementary portion behaves like a plait in the carpet and can pass through the material. It also behaves, as far as the charge balance is concerned, like a positively charged mobile charge carrier.

Metals contain mobile electrons and holes. Therefore, they are electric conductors.

Fig. 8.16 schematically illustrates the conduction process. The electronium lake is –in simplified terms– shown as an even grey area. The mobile electrons (densified areas) are darker, the holes (areas of reduced density) brighter. The electric field draws the electrons to the left (towards the high potential) and the holes to the right (towards the low potential).

The movement of the holes can be compared to the movement of an air bubble that rises from the bottom of a water glass to the top. Where there was air before the rise of the bubble, i.e. at the bottom, there is water after the rise. While the bubble moves from the bottom to the top, water moves from the top to the bottom, Fig. 8.17.

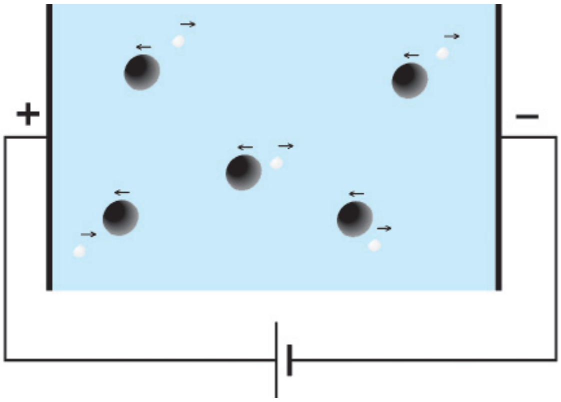

Fig. 8.16 When a battery is connected, the mobile electrons (densified areas) move towards the plus connection and the holes (areas of reduced density) towards the minus connection.

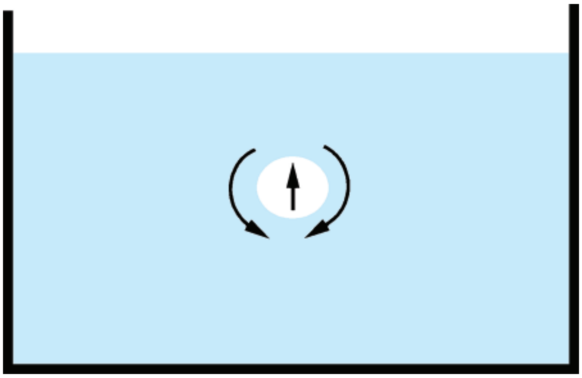

Fig. 8.17 While the air bubble moves from the bottom to the top, water moves from the top to the bottom.

8.8 How non-metals can be made conductive

We look at a non-metal whose energy gap is not too large, e.g. cadmium sulfide. Photons of blue light supply enough energy to overcome the forbidden range. When such a non-metal is illuminated with blue light (or with white light, which also contains blue light), the electronium is excited; mobile electrons are created. If the cadmium sulfide crystal is part of an electric circuit, an electric current can flow. The crystal has become conductive due to the excitation. As soon as the illumination stops, no mobile electrons will be created anymore and the electric conductivity will disappear again.

Substances that become conductive when illuminated are called *photoconductors*. The more light falls on a photoconductor, the lower is its resistance. Photoconductors are used to measure the light intensity, for example in cameras.

Non-metals with a narrow energy gap can be made conductive by illuminating them.

In case of *semiconductors*, heating is already sufficient to excite the electronium and to increase conductivity. The technically most important semiconductors are silicon and germanium.

There is another, more effective possibility to increase the conductivity of semiconductors: a small part of the atoms of the material is replaced by atoms of another substance. These "extraneous" atoms are chosen in a way that their shell has either one electron more or one electron less than the atoms of the material in which they are integrated. This integration of foreign atoms in an otherwise pure metal is called *doping*.

For example, germanium is doped with arsenic atoms. The additional electron of each arsenic atom behaves like a densification of the electronium atom, i.e. it is mobile. Hence, the germanium becomes conductive through the introduction of the arsenic atoms. As the new mobile charge carriers are negative, we also say the germanium was *n-doped*.

8.8 How non-metals can be made conductive

When atoms whose shell has an electron less than the germanium atoms, e.g. gallium atoms, are integrated in the crystal grid, areas of reduced density are created. Each gallium atom supplies a mobile hole. Also this way, the germanium becomes conductive. As the holes are positive, we say that the germanium was *p-doped.*

> Semiconductors can be made conductive by doping them with foreign atoms.
>
> The mobile charge carriers can be negative electrons or positive holes.

To achieve a good conductivity, it is already sufficient to replace approximately one of every million of germanium atoms by an extraneous atom.

The most important electronic components – diodes and transistors – are made of doped germanium or silicon, i.e. of germanium or silicon that was made conductive. We will look at them in the following sections.

9 DIODE AND TRANSISTOR

9.1 The semiconductor diode

A semiconductor diode is an electronic device with two connections. Integrated in an electric circuit, it lets the electricity pass in only one direction. Hence, a diode is for the electricity what a bicycle valve is for the air.

It consists of a small piece of a semiconductor material, mostly germanium or silicon, with two contacts, Fig. 9.1. On one side, the material is n-doped (on the right in the Figure). On the other side, it is pdoped (left side). The n-area and the p-area touch each other in the middle. Both areas are electrically conductive but the charge carriers are not the same. On the right, there are electrons; on the left, there are holes.

> One part of the diode is conductive for electrons, the other one for holes.

In the following, we will denominate the mobile electrons with e and the holes with h, just as sodium is denominated with Na, carbon with C or light with γ.

We connect the diode to a battery: the p-doped part with the holes to the high potential and the n-doped part with the electrons to the low potential. The positive holes now move towards the low potential, i.e. to the middle of the diode, the negative electrons towards the high potential, i.e. also to the middle, Fig. 9.2. At the same time, further holes are supplied through the left contact of the diode, and electrons through the right contact, because enough of both charge carrier types are available in the metal wires of the feed lines. Electrons and holes meet each other at the boundary surface in the middle of the diode. Here, the electrons

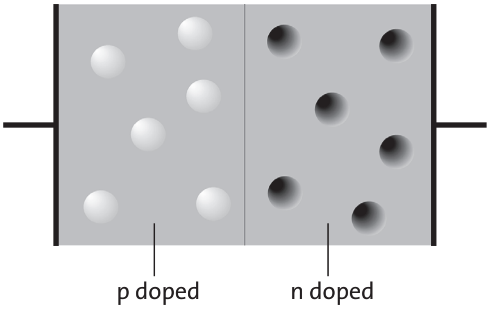

Fig. 9.1 In the right part of the diode, there are mobile electrons; in the left part, there are mobile holes.

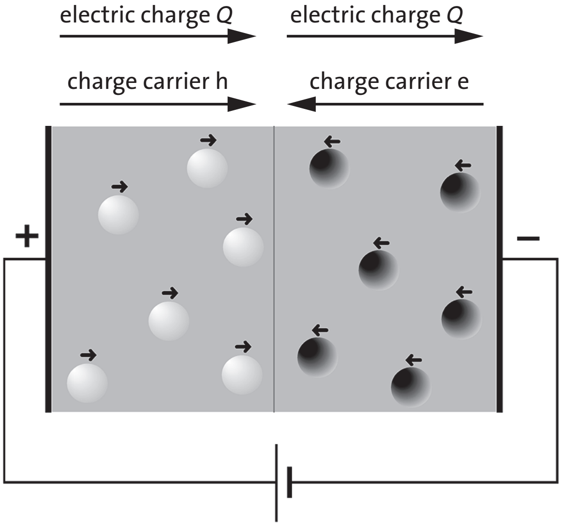

Fig. 9.2 Electrons and holes move towards the boundary surface. In both parts, an electric current is flowing from the left to the right.

"fall" into the holes and photons are created. We can describe this process as a photochemical reaction:

$$e + h \rightarrow \gamma \,.$$

This reaction runs as long as the diode is connected to the battery. Thereby an electric current is flowing. Notice that in the left area positive charge carriers (holes) flow to the right, and in the right area negative charge carriers (electrons) flow to the left. Both charge carrier currents correspond to a net current of electric charge to the right.

Now we connect the diode inversely to the battery, i.e. we connect the p-doped side to the low potential and the n-doped side to the high potential. The holes in the p-doped part are now moving away from the middle of the diode to the left, the electrons in the n-part move to the right, Fig. 9.3.

As no additional electrons and holes are supplied from the boundary surface in the middle of the diode, the process comes to an immediate halt. In spite of the battery, no electric current flows through the diode.

This results in several application possibilities.

9.2 The semiconductor diode as a rectifier, a light source and a solar cell

The diode as a rectifier

We have seen that the diode is a conductor for electricity in one direction but not in the other. If a diode is integrated in an electric circuit that is supplied by an alternating voltage source, Fig. 9.4, no alternating *current* will flow. The diode lets the current flow in only one direction. The current is *rectified*. The diode works as a *rectifier*.

Rectifier diodes exist in many different sizes. Some are several cm large and can withstand currents of 50 A, others can be found in millions on a computer chip and are so tiny that they cannot be perceived with the naked eye. Normal average diodes from the electric store are small, unspectacular cylinders with two connections whose appearance is barely distinguishable from a resistor or a capacitor, Fig. 9.5.

The diode as a light source

When a diode is connected to a battery as shown in Fig. 9.2, it emits light. The light comes from the immediate surroundings of the boundary between the n- and the p-material, i.e. from the place where electrons and holes react with each other. A diode that is optimized as a light source is called *light-emitting diode* or LED.

The diode as a solar cell

The reaction $e + h \rightarrow \gamma$ (9.1)

can also be made to run inversely, i.e.

$$e + h \leftarrow \gamma \,. \qquad (9.2)$$

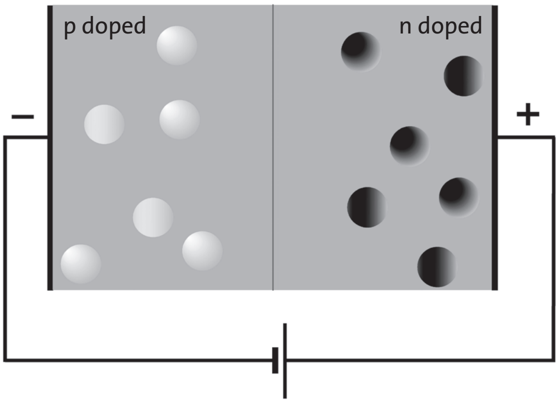

Fig. 9.3 Electrons and holes move slightly away from the boundary area. They cannot go far as no new, additional charge carriers are supplied from the boundary area.

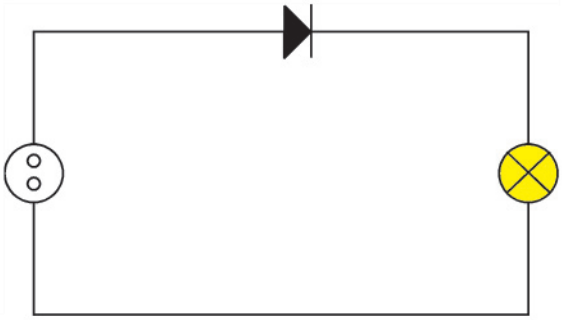

Fig. 9.4 The rectifier diode lets the electric current pass in only one direction.

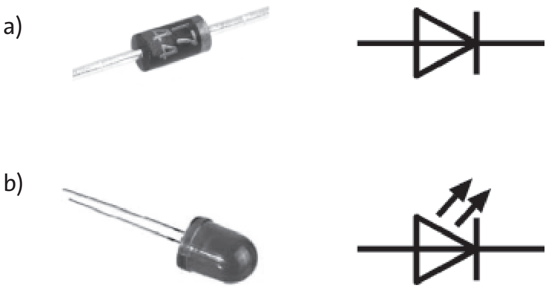

Fig. 9.5 Rectifier diode (a) and light emitting diode (b) with their symbols

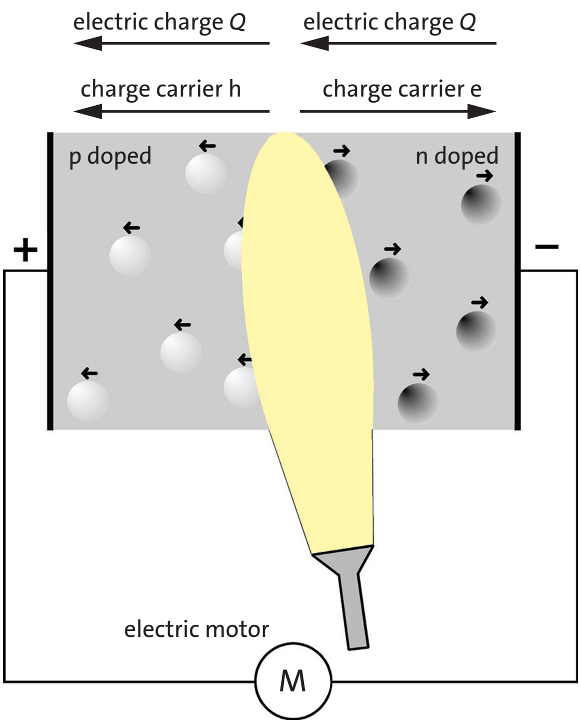

Fig. 9.6 At the boundary surface between the p- and the n-area, the light creates mobile electrons and holes. The electrons can flow away only to the right, the holes only to the left. Therefore, an electric current results, which flows to the left.

How can this be achieved? We apply a rule that every chemist knows.

To drive a reaction

$$A + B \rightleftharpoons C + D$$

towards the right, i.e. to get the substances A and B to transform themselves into C and D, it is sufficient to increase the concentration of one of the substances A and B. To drive the reaction to the left, we reverse the procedure: we increase the concentration of C and/ or D. (The quantitative version of this rule is called the *law of mass action.*)

To ensure that reaction (9.1) runs towards the left, we have to send light with a sufficient intensity ("concentration") to the boundary area between the p- and the n-region, Fig. 9.6. Here, mobile electrons and holes are created by the light. The electrons can only flow away to the right via the n-area because the p-area is not a conductor for electrons. Accordingly, the holes can only flow away to the left.

The electron and the hole current are driven by the excess concentration of charge carriers that are created by the light in the boundary area.

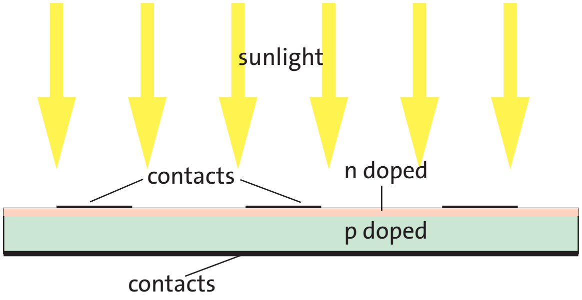

Fig. 9.7 Schematic cross-sectional view of a solar cell

As far as the directions are concerned, the same as what we have already seen for the rectifier applies: the particle currents in the pand in the n-area have opposite directions while the electric current has the same direction.

Hence, the semiconductor diode works as an "electricity pump" or an electric energy source.

> Light creates electrons and holes in the boundary region between the p- and the n-area. The electrons push towards one side through the n-region, the holes towards the other side via the p-region.

The technical realization of this type of diode is the *solar cell.*

Figure 9.7 shows a cross-sectional view of a solar cell. Its thickness is equal to just a fraction of a millimeter. The contacts on the side that faces the light have to be narrow so that they do not hamper the incident light too strongly.

Exercises

1. Graph the electric current as a function of time for the electric circuit from Fig. 9.4.
2. A resistor is connected to a transformer in Fig. 9.8. In between, there are four rectifier diodes. The arrangement of these diodes is called bridge rectifier. What is its effect? How does it work?

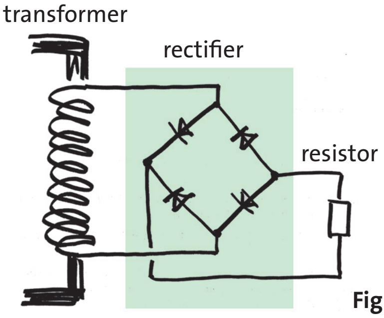

Fig. 9.8 For exercise 2

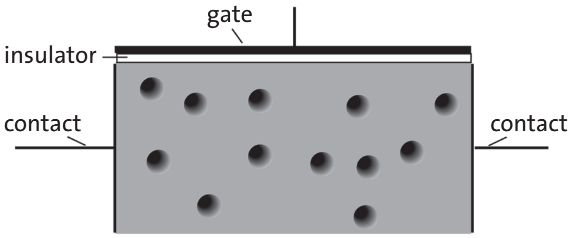

Fig. 9.9 Design of a transistor. The electric current in the semiconductor channel is controlled by means of the gate.

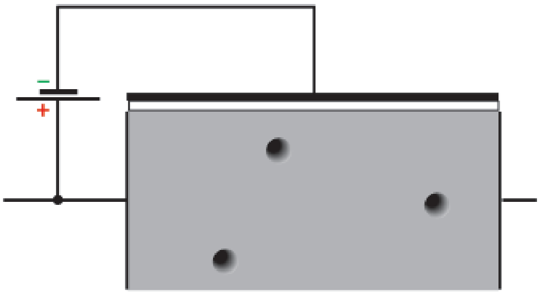

Fig. 9.10 The battery has pulled almost all electrons out of the semiconductor channel. The channel is no longer a conductor.

9.3 The transistor

A transistor is a switch for the electric current that can be electrically controlled, i.e. opened or closed. Hence, it essentially does the same as a relay (a switch that is controlled by means of an electromagnet). Compared to the relay, it has several advantages: first, a much lower current is neede for the control; second, it reacts much faster; third, it is much smaller; and fourth, this switch can not only be opened and closed but its "permeability" for the electric current can also be set to intermediate values.

Transistors are used in all electric devices: in computers, in TVs, in MP3 players, in smartphones, etc.

Fig. 9.9 shows the design of a transistor. The current to be controlled is sent through an n-doped "semiconductor channel".

On one side of the channel (at the top in the Figure), there is a metal layer, the *gate*. The gate is insulated electrically from the semiconductor channel. Together with the semiconductor, it forms a capacitor. This capacitor can be charged by means of applying a voltage between the gate and the semiconductor.

When the semiconductor channel is on a high potential (and the gate on a low potential), Fig. 9.10, the

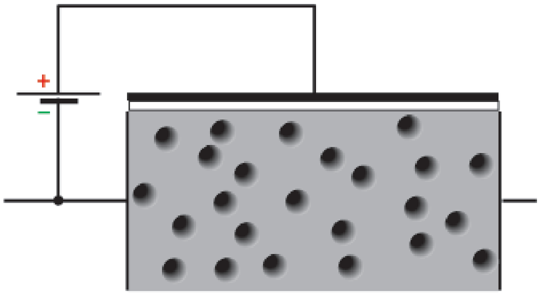

Fig. 9.11 The battery has pushed electrons into the semiconductor channel. The channel is a conductor.

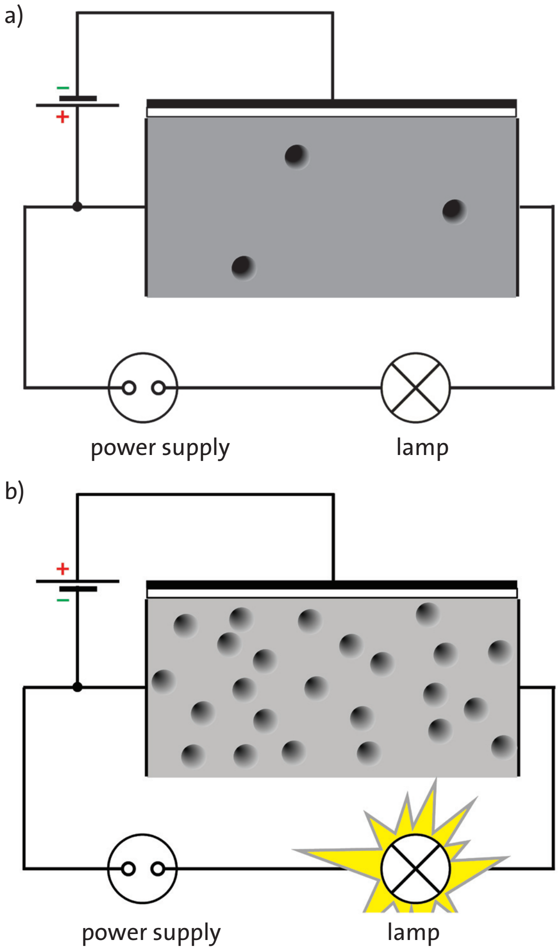

Fig. 9.12 (a) The gate has pushed the mobile (negative) electrons out of the channel; the channel does not conduct anymore and the lamp does not glow. (b) The gate has pulled the electrons into the channel; the channel conducts and the lamp glows.

semiconductor is positively charged. It contains fewer electrons than in the uncharged state. In other words, it contains fewer mobile charge carriers and is therefore also less conductive. The conductivity can thereby decrease up to a point where the channel is practically insulating for the electric current. It behaves like an open switch.

If the semiconductor channel is on the lower potential (and the gate on the higher one), Fig. 9.11, it contains more mobile charge carriers; its conductivity is now high. It behaves like a closed switch.

Consequently, the channel can be opened and closed by means of a varying voltage between the channel and the gate.

Figures 9.12a, 9.12b and 9.13 show a complete electric circuit with a transistor. If the gate potential is lower than the potential of the channel, Fig. 9.12a, the semiconductor will not conduct and the lamp will not glow. If the gate potential is higher, the semiconductor will conduct and the lamp will glow, Fig. 9.12b.

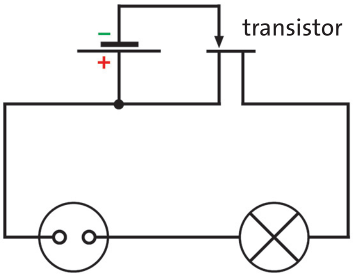

Fig. 9.13 Same as Figure 9.12a but with the technical symbol of the transistor

Exercise

1. There are transistors whose channel consists of a p-doped material. How does a battery have to be connected to the transistor so that the transistor behaves like an opened switch?

10 NUCLEAR PHYSICS

During our examination of atoms, we have mainly been interested in the electronium of the atomic shell; the atomic nucleus was needed to hold the electronium together. There are processes in nature that we can only understand if we have further knowledge of the atomic nucleus: processes in which the atomic nuclei change. Such processes are called nuclear reactions.

Natural nuclear reactions take place to a small extent on Earth; the ones in the Sun are much more important. They ensure that the Sun emits light and therefore provides the Earth with energy. But nuclear reactions are also applied technically – for the benefit and to the detriment of humans: in nuclear reactors, in fusion reactors, in atomic bombs and in hydrogen bombs.

When studying the nuclear structure, you will come across many aspects that you have already learned in connection with the atomic shell. And you will be able to apply much of what you have learned regarding the description of common chemical reactions in chemistry class.

10.1 The structure of the atomic nuclei

As we have already seen, the diameter of the atomic nucleus is approximately equal to 1/50 000 of the diameter of the atom. In spite of this tiny size, practically the whole mass of the atom is located in the atomic nucleus. The mass of the shell, in turn, only amounts to approximately 1/4000 of the nuclear mass.

Besides mass and size, the shell and the nucleus are different with regard to another feature.

You might remember that the density of the electronium decreases from the inside to the outside so that the atomic shell does not have a sharp edge – similar to the atmosphere of the Earth. In addition, we have seen that the shells of heavy and light atoms are not very different from each other in terms of size, but rather in terms of density. A gold and a lithium atom have approximately the same size. However, the density of the electronium of the gold atom is much higher than of the lithium atom.

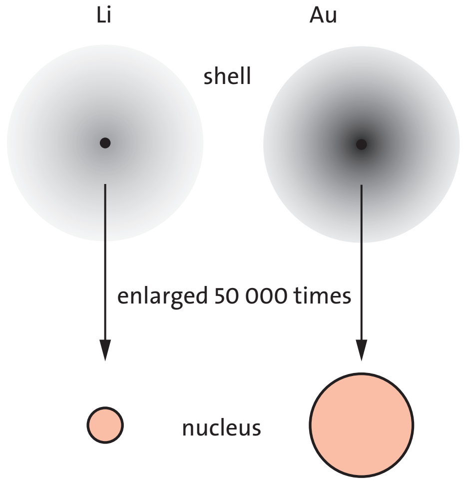

Fig. 10.1 The shell of the gold atom is approximately as large as the shell of the lithium atom. However, it has a much higher density. The nucleus of the gold atom has approximately the same density as the nucleus of the lithium atom. It is much larger though.

The atomic nuclei behave completely differently. The density in the atomic nucleus is the same everywhere, i.e. not like in the atmosphere of the Earth but for example like in a glass sphere. In addition, the density of the nuclear matter is the same in any atomic nucleus. The nucleus of the lithium atom has approximately the same density as the nucleus of the gold atom. Instead, the nuclei of the lithium and of the gold atom differ in their size, Fig. 10.1.

> The density in an atomic nucleus is the same everywhere. The densities of different atomic nuclei are equal.

As far as the shape is concerned, the nucleus and the shell are very similar. Also the nuclei are nearly spherical. Some are slightly divergent from the spherical shape: some are a bit flattened like a mandarin orange, others are slightly prolonged like a kiwi.

The density is measured in g/m^3. For example, water has a density of

$$\rho = 1 \text{ g/cm}^3,$$

i.e. the weight of 1 cm^3 of water is 1 g, or the weight of 1 liter is 1 kg.

The fact that almost the whole mass of an atom is concentrated in the nucleus results in a huge density. Nuclear matter has a density of

$$\rho = 10^{14} \text{ g/cm}^3.$$

If there were 1 cm^3 of nuclear matter, it would have a mass of

$$10^{14} \text{ g} = 100\,000\,000 \text{ tons.}$$

To learn more about that peculiar material the nuclei are made of, we will make a thought experiment that we have already made during the examination of the shell: we "reach into it" to take out a portion of it. Again, you should not understand this literally as we can reach neither into the shell nor into the nucleus with our hands. However, we can do something that is quite similar to reaching in: we can use other particles to shoot against the nucleus in order to catapult some of the nuclear matter out. At the moment, it is not relevant to know how this works in detail.

So let’s reach into the nucleus, take out a portion and look at "what we are holding in our hand". You will not be surprised that, just as in the case of the electronium of the shell, we will always obtain only very specific portions. But one aspect is different: there are two kinds of smallest portions. Sometimes, we hold something that is electrically charged and something we obtain something electrically neutral. If we reach in more strongly, it is possible that our portion will be twice the charged elementary position or twice the neutral elementary position or just as much as a charged and a neutral portion together. You can imagine what will possibly happen if we reach in even more strongly.

In addition, you already know how these portions are called: a positive elementary portion is called *proton*, a neutral one is a *neutron*. Hence, a nucleus contains an integral number of positive and generally another integral number of neutral portions. In other words: a nucleus contains a well-defined number of protons and a well-defined number of neutrons.

	electric charge	**mass**
proton	$1.6 \cdot 10^{-19}$ C	$1.6725 \cdot 10^{-27}$ kg
neutron	0 C	$1.6748 \cdot 10^{-27}$ kg
electron	$-1.6 \cdot 10^{-19}$ C	$9.1 \cdot 10^{-31}$ kg

Table 10.1 Data of the proton, the neutron and the electron

Notice however that the protons and the neutrons in the nucleus are not spatially separated from each other. If you imagine the proton material to be like a red pudding and the neutron material like a white one, the whole nucleus pudding would not be composed of red and white areas but it would be continuously pink.

Table 10.1 lists the electric charge and the mass of the proton, the neutron and the electron. The masses of the proton and the neutron are almost but not exactly equal. The neutron is slightly heavier. We will see later that this difference is important for certain nuclear reactions. Remember that each of the two particles is approximately 1800 times heavier than the electron.

The electric charge of the electron is exactly equal but opposite to the charge of the proton. An atom contains as many electrons in its shell as there are protons in its nucleus. This results in the atom not having an electric charge as a whole; it is electrically neutral.

> The nuclear matter can be divided into
> - portions that are positively charged and
> - portions that are electrically neutral.
>
> A positive portion is called proton, a neutral charge is called neutron.

Exercises

1. The overall number of protons and neutrons in a nucleus A is eight times higher than that in a nucleus B. By which factor is the volume of A greater than that of B? By which factor is the radius of A greater than the one of B?
2. There are celestial bodies that consist of nuclear matter: the *neutron stars*. A typical neutron star has a diameter of 10 km. What is its mass?

10.2 Elements, nuclides, and isotopes

The atoms of different chemical elements vary from each other by the number of protons in their nuclei. The atomic number indicates this proton number and, as long as the atom is not ionized, the atomic number also indicates the number of electrons.

> Atomic number = number of protons in the nucleus

When evaluating nuclei only by their number of protons, there are as many different nuclei as there are different chemical elements, i.e. approximately a hundred. However, an atomic nucleus with a given number of protons can contain more or less neutrons. For example, there are atomic nuclei of potassium (i.e. atomic nuclei with 19 protons) with numbers of neutrons from 16 to 33. Hence, there are different types of potassium, or isotopes as we say. The different isotopes of an element are almost not different from each other in chemical terms. Normal, natural potassium is a mixture of different isotopes. There are also different isotopes of all other elements.

> The isotopes of an element differ in the number of neutrons in their atomic nuclei.

When counting the different types of each chemical element individually, we do not obtain approximately 100 different basic substances but around 2000. As you know, the 100 different basic chemical substances are called elements. The 2000 different basic substances that we will obtain if we distinguish in addition by the number of neutrons are called *nuclides*. The following notation is used to denominate a nuclide:

- the letter symbol of the element as known from chemistry;
- the overall number of the protons and neutrons next to it on the left and at the top, and the atomic number, i.e. the number of protons, at the bottom.

Here are some examples of this form of notation

$$\left.\begin{array}{l} {}^{1}_{1}\mathrm{H} \\ {}^{2}_{1}\mathrm{H}\ \text{(Deuterium)} \\ {}^{3}_{1}\mathrm{H}\ \text{(Tritium)} \end{array}\right\}\text{three hydrogen isotopes}$$

$$\left.\begin{array}{l} {}^{4}_{2}\mathrm{He} \\ {}^{3}_{2}\mathrm{He} \end{array}\right\}\text{two helium isotopes}$$

$$\left.\begin{array}{l} {}^{35}_{19}\mathrm{K} \\ {}^{39}_{19}\mathrm{K} \\ {}^{40}_{19}\mathrm{K} \\ {}^{41}_{19}\mathrm{K} \\ {}^{50}_{19}\mathrm{K} \end{array}\right\}\text{five potassium isotopes}$$

${}^{56}_{26}\mathrm{Fe}$ the most frequent isotope of iron

${}^{238}_{92}\mathrm{U}$ the most frequent isotope of uranium

${}^{239}_{94}\mathrm{Pu}$ isotope of the artificial element plutonium

As you can see, the two heavy isotopes of hydrogen even have their own names. The isotope with one neutron is called *deuterium*, and the isotope with two neutrons is called *tritium*.

When nuclear reactions are investigated, usually only the nuclei are of interest. It is therefore practical for the nucleus of a nuclide to be given its own symbol. Therefore, the letter K is added to the nuclide symbol as an index. While the symbol ${}^{25}_{12}\mathrm{Mg}$ represents the complete atom of a certain magnesium nuclide, the symbol ${}^{25}_{12}\mathrm{Mg}_{\mathrm{K}}$ represents only the nuclei of this nuclide.

For some purposes, another symbolic representation of the nuclides is more appropriate. It is based on the representation that is used in chemistry to characterize molecules. The formula $\mathrm{Al_2O_3}$ indicates the aluminum and the oxygen content of the aluminum oxide. Accordingly, nuclides can also be characterized by indicating the content of protons p and of neutrons n. Here you see a few examples for the translation to this notation:

$${}^{1}_{1}\mathrm{H}_{\mathrm{K}} = \mathrm{p}$$
$${}^{2}_{1}\mathrm{H}_{\mathrm{K}} = \mathrm{pn}$$
$${}^{3}_{1}\mathrm{H}_{\mathrm{K}} = \mathrm{pn}_2$$
$${}^{4}_{2}\mathrm{He}_{\mathrm{K}} = \mathrm{p}_2\mathrm{n}_2$$
$${}^{3}_{2}\mathrm{He}_{\mathrm{K}} = \mathrm{p}_2\mathrm{n}$$
$${}^{35}_{19}\mathrm{K}_{\mathrm{K}} = \mathrm{p}_{19}\mathrm{n}_{16}$$
$${}^{39}_{19}\mathrm{K}_{\mathrm{K}} = \mathrm{p}_{19}\mathrm{n}_{20}$$
$${}^{40}_{19}\mathrm{K}_{\mathrm{K}} = \mathrm{p}_{19}\mathrm{n}_{21}$$
$${}^{41}_{19}\mathrm{K}_{\mathrm{K}} = \mathrm{p}_{19}\mathrm{n}_{22}$$
$${}^{50}_{19}\mathrm{K}_{\mathrm{K}} = \mathrm{p}_{19}\mathrm{n}_{31}$$
$${}^{56}_{26}\mathrm{Fe}_{\mathrm{K}} = \mathrm{p}_{26}\mathrm{n}_{30}$$
$${}^{238}_{92}\mathrm{U}_{\mathrm{K}} = \mathrm{p}_{92}\mathrm{n}_{146}$$
$${}^{239}_{94}\mathrm{Pu}_{\mathrm{K}} = \mathrm{p}_{94}\mathrm{n}_{145}$$

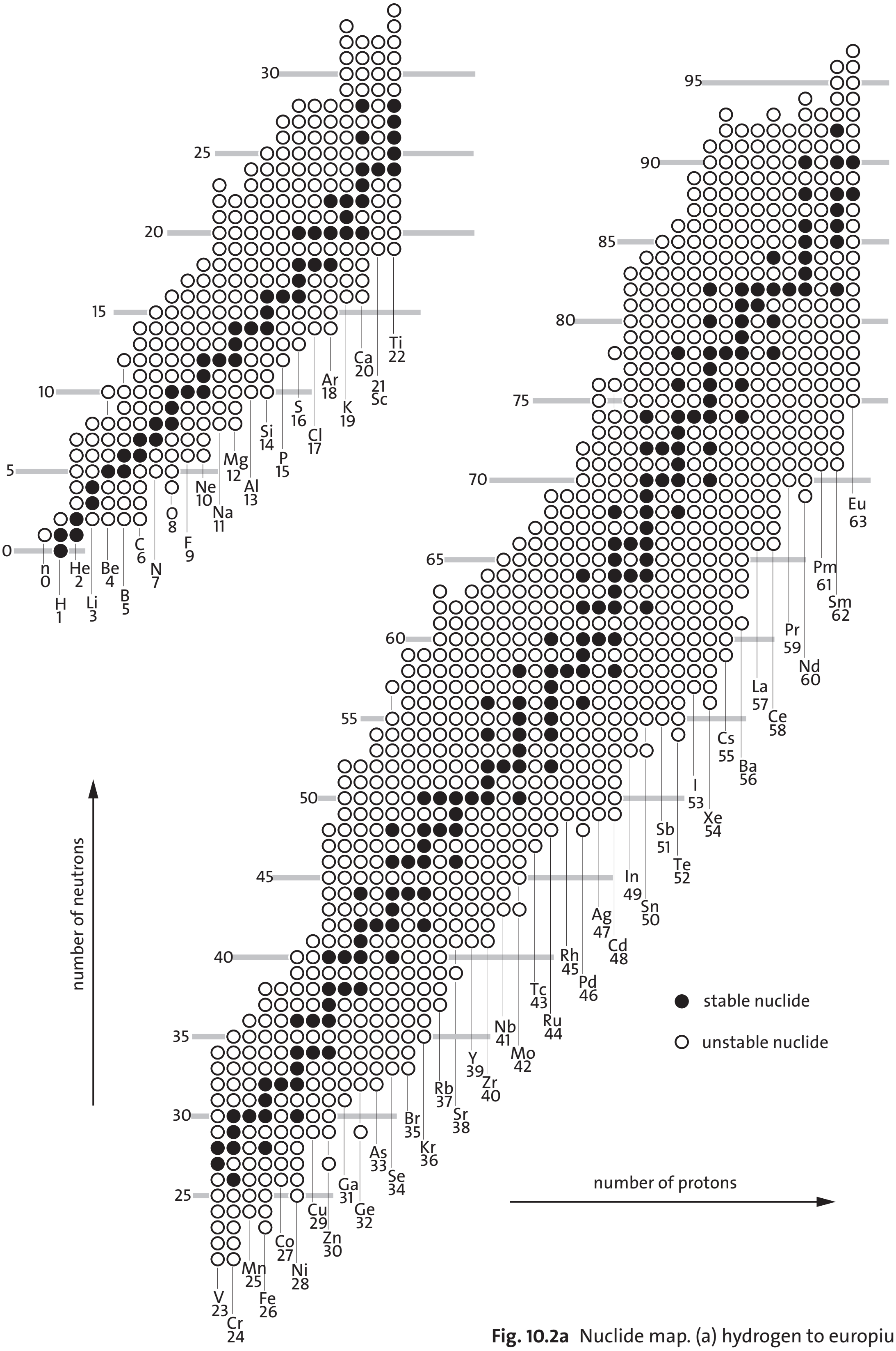

Fig. 10.2a Nuclide map. (a) hydrogen to europium

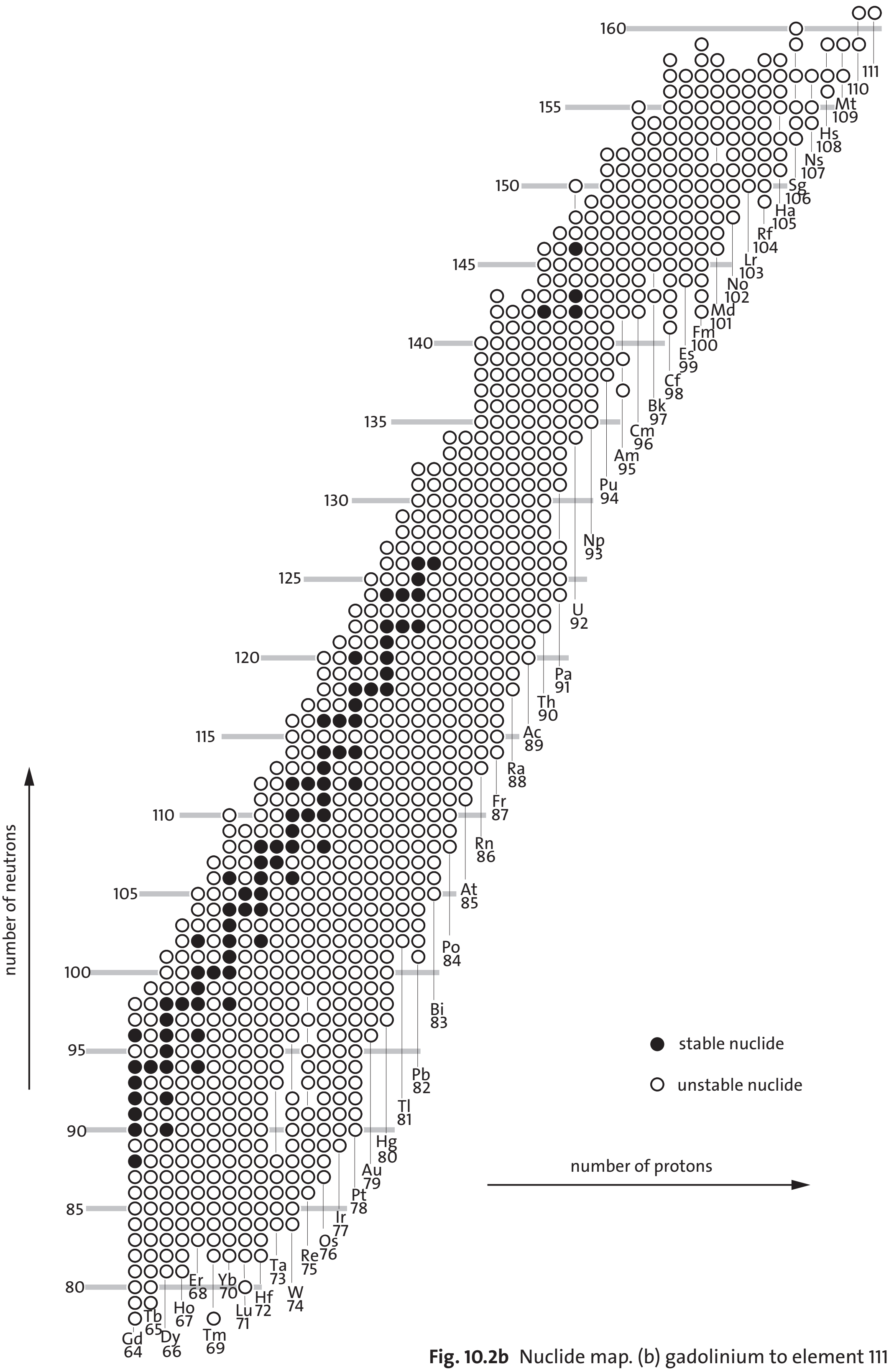

Fig. 10.2b Nuclide map. (b) gadolinium to element 111

But what types of nuclides do actually exist? We have already said that there is a total of 2000 nuclides. This means that not every arbitrary "chemical" formula corresponds to an existing nuclide. Hence, there is for example no pn_3, i.e. hydrogen with 3 neutrons, or the oxygen isotope p_8n_{17} or the zinc isotope $p_{30}n_{50}$ do not exist.

The question of which nuclides exist, i.e. which combinations of the nuclear matter p and the nuclear matter n are possible, is similar for example to the question about the existing compounds of the substances hydrogen and oxygen. Which of the compounds of hydrogen and oxygen listed in the following line do exist?

O, H, H_2, O_2, HO, H_2O, HO_2, O_3, H_2O_2, H_3O, H_3O_2, HO_3, …

Some of them you already know: molecular hydrogen H_2, molecular oxygen O_2, water H_2O. Regarding others of the listed substances, you certainly know that they do not exist. You might also know hydrogen peroxide H_2O_2. Although this substance exists, it is not stable.

The same is true for ozone O_3. And atomic hydrogen H is much more unstable. This applies to nuclides in a similar way. Most of the possible combinations p_xn_y do not exist. And only a small fraction of those that could exist is stable.

Fig. 10.2 shows the *nuclide map*, i.e. a diagram with all nuclides that are known. The number of protons is indicated towards the right, the number of neutrons in an upward direction. Each circle corresponds to one of the nuclides that have been found or produced up to present. However, only the full circles represent stable nuclides. The nuclides corresponding to the empty circles are unstable, i.e. they gradually decay on their own. While the lifespan for some is very short, it is very long for others.

	typical excitation energy	**typical transition time**
atomic shell	10^{-19} J	10^{-8} s
atomic nucleus	10^{-13} J = 0.1 pJ	10^{-14} s

Table 10.2

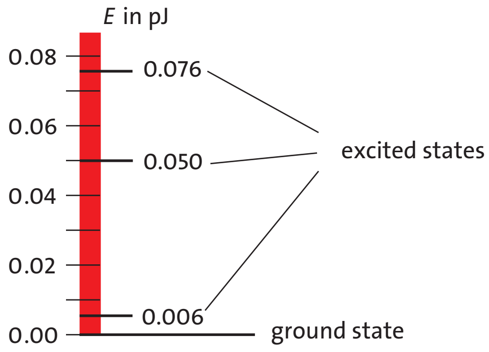

Fig. 10.3 Energy ladder of a thallium isotope

> There are about 100 different elements and 2000 different nuclides.

Exercises

1. Choose six nuclides with different atomic numbers from the nuclide map, Fig. 10.2, and represent them symbolically using the two methods described in the text.
2. Formulate a general rule for the ratio of the number of protons and neutrons for existing nuclides of light elements (of up to 20 protons). Do the same for heavy elements as well. Use the nuclide map of Fig. 10.2.
3. How many stable nuclides exist?
4. Which is the heaviest stable nuclide?
5. Which are the stable isotopes of neon?
6. Which is the lightest element with no stable isotope?
7. Which element has the most isotopes? How many does it have? How many of these are stable?

10.3 The excitation of nuclei

Just as the atomic shell, the nucleus can also be deformed; and the nucleus locks into defined forms in the same way as the shell. The most important differences: first, approximately a million times more energy is needed to deform an atomic nucleus than to deform the shell, and second, the duration of a transition between two states of the nucleus only amounts to approximately a millionth part of the transition time in the shell, Table 10.2.

In the following, we will often come across energies of the order of magnitude 10^{-12} to 10^{-15} Joule. Therefore, it is convenient to use the abbreviation "pico" for 10^{-12}: 10^{-12} J = 1 pJ. Fig. 10.3 shows the energy ladder of a thallium isotope.

Also the methods of excitation are in principle the same as for the shell.

For example, the nucleus can be excited with photons, which need to have exactly the energy that corresponds to the transition from the ground state to the excitation state. The energy may not be too low, but it may not be too high either. As mentioned before, we need much more energy than for the excitation of the shell. The photons consequently need to be γ radiation. An example for such a process is the excitation of a thallium nucleus

$$^{208}_{81}\mathrm{Tl}_{\mathrm{K}} + \gamma \rightarrow {}^{208}_{81}\mathrm{Tl}^{*}_{\mathrm{K}} \,.$$

However, the nuclei can also be excited with particles that have a rest mass, for example electrons, protons or neutrons. (Also this process corresponds to an excitation process of the shell.) In this case, the exciting particle has to provide enough energy; it may have even more energy than required for the excitation. The particle that flies away can take the remaining energy along. Again, we look at the thallium nucleus as an example:

$$^{208}_{81}\mathrm{Tl}_{\mathrm{K}} + \mathrm{p}' \rightarrow {}^{208}_{81}\mathrm{Tl}^{*}_{\mathrm{K}} + \mathrm{p} \,.$$

Here, p' stands for a fast proton, p for a slow one.

Also the third method to produce an excited nucleus has a counterpart in the atomic shell: some chemical reactions lead to reaction products in an excitation state. A gas flame is an example. Likewise, the reaction products of nuclear reactions often exist in an excited state. We will discuss nuclear reactions in great detail at a later time.

Here, let's just mention a first example: a certain carbon isotope reacts with a proton to form an excited nitrogen isotope:

$$^{13}_{6}\mathrm{C}_{\mathrm{K}} + \mathrm{p} \rightarrow {}^{14}_{7}\mathrm{N}^{*}_{\mathrm{K}} \,.$$

> The atomic nucleus can be excited just like the atomic shell. In an excitation state, the nucleus has another shape than in the ground state. A certain amount of energy is stored in the nucleus during the excitation.

You remember that mass and energy are the same physical quantity. When the quantity is measured in the unit kg, it is called mass. We call it energy when it is measured in Joule. Also, one measurement unit can be converted easily into the other one. In other words: mass can be expressed in units of energy.

The conversion is done according to the formula

$$E = k \cdot m$$

$k = 9 \cdot 10^{16}$ J/kg is the constant conversion factor.

If we supply energy to a system, it will become heavier. In many cases, however, the energy increase is so small compared to the total energy of the system that the increase of the weight cannot be noticed. For example, the mass of the atom changes so little during the excitation of the shell that it can almost not be perceived.

The excitation of atomic nuclei is different. We look at the excitation of the thallium isotope $^{208}_{81}\mathrm{Tl}$, whose energy ladder is shown in Fig. 10.3, to the second excited state. For this purpose, it has to be provided with an amount of energy of 0.05 pJ. We convert into kg:

$$m = \frac{E}{k} = \frac{0.05 \cdot 10^{-12}\,\mathrm{J}}{9 \cdot 10^{16}\,\mathrm{J/kg}} = 0.56 \cdot 10^{-30}\,\mathrm{kg}.$$

Compared to the overall mass of the thallium atom, this is still a small amount. However, it is almost as much as the mass of an electron ($0.9 \cdot 10^{-30}$ kg).

> A nucleus becomes heavier when it is excited.

Exercises

1. The mass of a thallium atom is $3.50 \cdot 10^{-25}$ kg. By what fraction of the total mass will the atom become heavier a) in the case of a typical excitation of the atomic shell, b) in the case of a typical excitation of the nucleus?
2. Is it possible to distinguish the mass of 1 mol of substance whose nuclei are excited from the mass of the same substance with the nuclei in the ground state by means of a scale?

10.4 The separation energy

Energy is needed to separate two magnets that are sticked together from each other. Hence, the separated magnets together have more energy than the joined ones. As energy is equal to mass, this means that the separated magnets are heavier than the joined ones, Fig. 10.4. However, the effect is so small that it cannot be proven in an experiment as the one from Fig. 10.4.

To extract a proton from a nucleus, energy is required. The nucleus and the proton have consequently less energy than the nucleus and the proton separately.

If a proton is removed from a $^{14}_{7}\mathrm{N}$ nucleus, a $^{13}_{6}\mathrm{C}$ nucleus is created. The $^{14}_{7}\mathrm{N}$ nucleus is lighter than the $^{13}_{6}\mathrm{C}$ nucleus and the proton together, Fig. 10.5. Here, the effect is much more significant than in the example with the magnet, and it can definitely be proven.

Of course, the same is true for a nucleus from which a neutron is extracted. Also during the separation of a neutron, the system becomes heavier because energy is needed for the process.

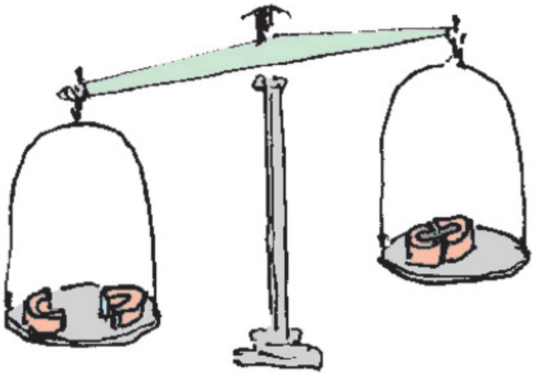

Fig. 10.4 The separated magnets have more energy than the magnets that are in contact. However, the effect is so small that it cannot be proven by means of weighing.

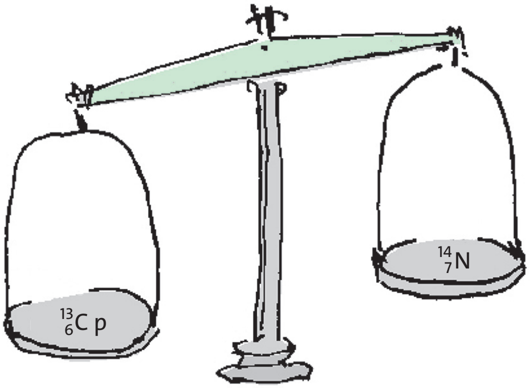

Fig. 10.5 A $^{14}_{7}$N nucleus is lighter than a $^{13}_{6}$C nucleus and a proton together.

We would now like to dismantle an imaginary atomic nucleus step by step into individual protons and neutrons. For this complete dismantling process of a nuclide nucleus, a very specific amount of energy – the *separation energy* – is required. It has a different value for each nuclide. We will need these energy values later in order to decide in which direction a nuclear reaction runs. The table in the appendix lists the separation energies of approximately 1000 nuclides.

> Energy (the separation energy) is needed to break down a nucleus into protons and neutrons. Therefore, the components together are heavier than the nucleus.

Exercises

1. The separation energy is approximately proportional to the total number of protons and neutrons. Choose 5 nuclides from the Table in the Appendix and graphically represent the separation energy as a function of the total number of protons and neutrons.
2. If a neutron is separated from a $^{15}_{6}$C nucleus, a $^{14}_{6}$C nucleus is obtained. If a neutron is again removed from it, a $^{13}_{6}$C nucleus will be formed; after separating a third neutron, a $^{12}_{6}$C nucleus will emerge, and so forth. How much energy is needed for the respective separation processes of a neutron? Graph this energy over the total number of protons and neutrons of the respective carbon isotope. Do the exercise accordingly for the helium isotopes. What can you observe?

10.5 Conserved quantities

Our goal is to predict which nuclear reactions are possible. In principle, you know the process that we are going to use. We will take advantage of the fact that certain substance-like quantities may not change their value during a reaction.

You have already learned about some of such quantities. They are called *conserved quantities*.

Energy is a conserved quantity. In each process the total amount of energy does not change. The same is true for the electric charge.

Now have a look at the reaction that is described by the following chemical equation:

$$\mathrm{NaCl + H_2O \rightarrow Na + Cl^- + H_2O}.$$

Such a reaction cannot take place because it does not take account of the conservation of the electric charge. The reactants (left side) as a whole are neutral, the products (right side) as a whole carry a negative electric charge. An allowed reaction is in contrast:

$$\mathrm{NaCl + H_2O \rightarrow Na^+ + Cl^- + H_2O}.$$

Here, the charge balance is correct.
Another forbidden process is:

$$\mathrm{2\,Al + 3\,O_2 \rightarrow 2\,Al_2O_3}.$$

Here, another conservation law is violated: in chemical reactions, the numbers of the atoms are conserved quantities. And the atom numbers on the right and on the left side of the chemical equation are not equal. A possible reaction is in turn:

$$4\ \mathrm{Al} + 3\ \mathrm{O_2} \rightarrow 2\ \mathrm{Al_2O_3}.$$

> In chemical reactions, the numbers of the atoms are conserved quantities.

Hence: to set up a chemical reaction equation, we make use of the conservation of the electric charge and of the conservation of the atom numbers.

Now, there is yet another issue regarding the conservation of the atom numbers: the conservation law does not always apply. We have emphasized that it applies for "chemical reactions". In the following, we will see that it does not apply for example for nuclear reactions. Therefore:

> Some conservation laws only apply under certain conditions.

We have already seen an example of a nuclear reaction:

$${}^{13}_{6}\mathrm{C}_\mathrm{K} + \mathrm{p} \rightarrow {}^{14}_{7}\mathrm{N}^{*}_\mathrm{K}.$$

You can see that there is a carbon nucleus on the left side of the equation but not on the right side. Also, there is a proton on the left side, i.e. a nucleus of a hydrogen atom, but not on the right side. On the right, in contrast, there is a nitrogen nucleus – but not on the left. The number of the atoms or atomic nuclei is consequently not conserved in the course of this nuclear reaction.

Now there is a problem: we do not have a rule for the establishment of the reaction equation anymore. Or is any reaction allowed? Experience shows that this is by no means the case. The examination of nuclear reactions has shown that there are other physical quantities for which a conservation law applies and that had not been known earlier: the *baryonic charge* and the *leptonic charge*. For each nucleus and in general for each particle, the baryonic charge and the leptonic charge has a specific value. The baryonic charge of the particles on the left side of the reaction equation (the reactants) always has to be equal to that of the right side (the products). Likewise, the leptonic charge of reactants and products has to be equal.

> Baryonic charge can be neither created nor destroyed.
>
> Leptonic charge can be neither created nor destroyed.

Prior to studying the establishment of a nuclear reaction equation in connection with this new aspect, we have to develop our knowledge of the elementary portions a bit further.

Exercise

1. Which substance-like quantities do you know? Which of them are conserved quantities and which are not?

10.6 Particles and antiparticles

In the process of dismantling the shell and the nucleus, we have come across three different types of elementary matter or of *elementary particles*, so far: electrons, protons and neutrons. Apart from these three particle types, others have been discovered. However, the greatest part of nature that surrounds us only con-

Name, Symbol	Rest energy	Charge		
		Electric*	Baryonic	Leptonic
Photon γ	0 pJ	0	0	0
Electron e	0.0819 pJ	-1	0	1
Antielectron $\bar{\mathrm{e}}$	0.0819 pJ	1	0	-1
Neutrino ν	nearly 0 pJ	0	0	1
Antineutrino $\bar{\nu}$	nearly 0 pJ	0	0	-1
Proton p	150.3277 pJ	1	1	0
Antiproton $\bar{\mathrm{p}}$	150.3277 pJ	-1	-1	0
Neutron n	150.5349 pJ	0	1	0
Antineutron $\bar{\mathrm{n}}$	150.5349 pJ	0	-1	0

* Multiples of the elementary charge

Table 10.3 Rest energy, electric, baryonic and leptonic charge of some particles sists of electrons, protons and neutrons. The other particles are mostly created artificially.

The existence of these new particles seems to make the discussion of nuclear reactions more complicated at first. In reality, however, all these particles can be summarized in a very systematic and clear way. In Table 10.3 they are listed with some of their characteristics. We would like to discuss this table in detail.

What we already know

The left column contains the names of the particles. We are already familiar with four of these particles: the photon, the electron, the proton and the neutron.

The neutrino

Actually, we should also be familiar with it as there is a significant quantity of it on Earth. Many of these particles are flying around freely: approximately 10^{13} particles per square centimeter and second. They mostly come from the Sun. There, they are formed in the reaction that also causes the Sun to send light to us. Very little is perceived of these neutrinos as they cross all matter almost unhindered. Therefore, there is almost no reaction that could be used to detect them. Most of these neutrinos penetrate the whole Earth without reacting with any other particles and even without being diverted.

By means of the neutrinos we can learn something about the reactions that take place deeply inside the Sun. However, as the neutrinos are so weakly reactive, experiments to measure them are very time-consuming. The detection devices have to be very large so that the probability that a neutrino will get stuck in them is not too low. In addition, the experiments have to be shielded against all other particles that are formed in any nuclear reactions or that come from space. Therefore, such experiments are made deeply under the surface of the Earth: in tunnels or in old mines.

The existence of neutrinos had already been assumed at a time when there was not yet any possibility to make such sophisticated experiments. How was that possible? Thanks to a method that has already proven to be effective in microphysics several times: analyzing a defined reaction, it can be found that any conservation law whose validity we are convinced of seems to be infringed. In case of the neutrinos, it was found that the energy was not conserved in a defined reaction. Therefore, it was concluded that a further particle, which had been unknown and undetected up to then, must have been involved in the reaction.

> Neutrinos only weakly react with other particles.

Antiparticles

The table contains four other particles. The names of these particles start all with the qualifying expression "anti": the antielectron, the antineutrino, the antiproton and the antineutron. You can convince yourself easily that they are very similar to the respective particle without the "anti": they are completely identical except for the +/- sign of the three charges. This means that the proton has a positive electric and a positive baryonic charge and the antiproton has a negative electric and baryonic charge.

We say that the antiproton is the antiparticle to the proton, and the proton is the antiparticle to the antiproton. Hence, the particle and the *antiparticle* form a pair of two very similar partners.

> Particles and antiparticles differ in the +/- sign of the electric, baryonic and leptonic charge.
>
> Electrons and neutrinos as well as their antiparticles carry leptonic charge.
>
> Protons and neutrons as well as their antiparticles carry baryonic charge.
>
> There is no antiparticle to the photon.

10.7 Charge balances

We are now able to establish a nuclear reaction. At first, we will look at a few very simple examples.

Is there is a reaction

$$n \overset{?}{\longleftrightarrow} p + e \qquad (10.1)$$

in which a neutron is transformed into a proton and an electron? We set up the balances of electronic, baryonic and leptonic charge:

	n	p + e
Electric charge	0	1 + (–1)
Baryonic charge	1	1 + 0
Leptonic charge	0	0 + 1

The electric charge of reactants and products is the same, i.e. zero. Therefore, the conservation principle of the electric charge is respected. Also the baryonic charge has the same value on the left and on the right, i.e. 1. The leptonic charge, in turn, would not be conserved in the reaction: it is zero for the left and one for the right side. The reaction can therefore not take place, neither from the left to the right nor from the right to the left.

However, it is easy to repair the reaction equation, for example by adding an antineutrino on the right side:

$$\mathrm{n} \overset{?}{\longleftrightarrow} \mathrm{p} + \mathrm{e} + \bar{\nu}$$

This time, the balance is correct:

	n	p + e + $\bar{\nu}$
Electric charge	0	1 + (–1) + 0
Baryonic charge	1	1 + 0 + 0
Leptonic charge	0	0 + 1 + (–1)

Also the leptonic charge is now equal on the left and on the right, i.e. zero. This reaction actually takes place: a free neutron decays by itself into a proton, an electron and an antineutrino.

Equation (10.1) can also be repaired by adding a neutrino on the left side:

$$\mathrm{n} + \nu \overset{?}{\longleftrightarrow} \mathrm{p} + \mathrm{e}.$$

Also this reaction is allowed as shown in the balance table:

	n + ν	p + e
Electric charge	0 + 0	1 + (–1)
Baryonic charge	1 + 0	1 + 0
Leptonic charge	0 + 1	0 + 1

Exercise

1. Establish an equation that describes the reaction of a proton with an antiproton. There are several possibilities.

10.8 The direction of a reaction

One important question is still not answered: in which direction does the reaction that we have established run?

Again, we can make our decision by applying a conservation law. We have not yet considered the energy conservation so far. Also, the energy balance of a reaction has to be correct of course. The energy of reactants and products has to be equal. If we compare the rest energy of the particles on one side of a nuclear equation to the one on the other side, the energies will never have exactly the same value though. In fact, however, the rest energies do not need to be equal.

When the initial nuclei or particles are at rest, the energy of the initial particles is the sum of the rest energy of these particles. The total rest energy of the emerging particles may certainly not be higher than the one of the initial particles. However, it may indeed be lower. In this case, something would be left of the energy of the initial particles. This remaining energy, however, can always be used: it can simply be supplied to the emerging particles by making them move fast. The faster the product particles move, the more energy they have in addition to their rest energy. Hence: a nuclear reaction occurs in a way that the rest energy of the involved nuclei and particles decreases.

To distinguish in which direction a reaction runs, we consequently have to compare the total rest energy of the particles on the left side to the one of the particles on the right side. The reaction runs from the high to the low rest energy.

> A nuclear reaction in which the initial particles are at rest always runs towards the decreasing rest energy.

We will now put our new tools to work. We would like to examine in which direction a reaction runs

$$A + B + \ldots \overset{?}{\longleftrightarrow} U + V + \ldots$$

Are neutrons stable?

The reaction

$$\mathrm{n} \overset{?}{\longleftrightarrow} \mathrm{p} + \mathrm{e} + \bar{\nu}$$

is correct as far as the balance of electric, baryonic and leptonic charge is concerned. We have already convinced ourselves of this fact. However, the energy balance is still missing. We add it to the table with the charge balances: the sum of the rest energies of the left side and the sum of the rest energies of the right side. We can take the rest energies of the individual particles from Table 10.3.

	n	p + e + $\bar{\nu}$
Electric charge	0	1 + (–1) + 0
Baryonic charge	1	1 + 0 + 0
Leptonic charge	0	0 + 1 + (–1)
Rest energy in pJ	n 150.5349	p 150.3277
		e 0.0819
Sum in pJ	150.535	150.410

The rest energy of the neutrons is higher than that of the proton, the electron and the antineutrino together. Hence, the reaction runs from left to right. We subtract the smaller sum from the larger one and obtain:

$$\Delta E = 150.535 \text{ pJ} - 150.410 \text{ pJ} = 0.125 \text{ pJ}.$$

In fact, we can observe that free neutrons, i.e. neutrons that are not bonded in a nuclei, decay by themselves. Neutrons can be produced artificially. However, they will decay within a few minutes if we store them carefully. Actually, their lifespan is much shorter as they are very reactive and bond quickly with the diverse atomic nucleus they encounter.

Are protons stable?

You already know that protons do not decay. We would like to convince ourselves that our new tool also leads us to this result. Regarding the balances of the three charges, the following reaction would be allowed:

$$p \rightarrow n + \bar{e} + \nu$$

We create a table with the balances:

	p	**$n + \bar{e} + \nu$**
Electric charge	1	0 + 1 + 0
Baryonic charge	1	1 + 0 + 0
Leptonic charge	0	0 + (–1) + 1
Rest energy in pJ	p 150.3277	n 150.5349
		$\bar{e}$ 0.0819
Sum in pJ	150.328	150.617

The rest energy of the proton is lower than that of the neutron and the antielectron together. Hence, the reaction can only run from right to left: the proton cannot decay.

Is tritium stable?

We will try it with a decay in a helium nuclei, an electron and an antineutrino:

$$^{3}_{2}\text{H}_K \overset{?}{\longleftrightarrow} {}^{4}_{2}\text{He}_K + e + \bar{\nu}$$

For the sake of clarity, we will always insert the reaction equation as a content formula in the table.

$$pn_2 \overset{?}{\longleftrightarrow} p_2n + e + \bar{\nu}$$

The charge balances are correct

	pn_2	**$p_2n + e + \bar{\nu}$**
Electric charge	1	2 + (–1) + 0
Baryonic charge	3	3 + 0 + 0
Leptonic charge	0	0 + 1 + (–1)
Rest energy in pJ	p 150.3277	p 2 · 150.3277
	n 2 · 150.5349	n 150.5349
		e 0.0819
$-E_S$ in pJ	– 1.359	– 1.2364
Sum in pJ	450.0385	450.0358

We calculate the rest energy of a nucleus by subtracting the separation energy from the rest energy of the particles of which it consists.

$E_S(pn_2)$ and $E_S(p_2n)$ were read from the table. The sum on the left is larger than that on the right; hence, the reaction can run from the left to the right.

In fact, tritium is unstable. Normal water contains a very small quantity of tritium. This tritium decays according to the reaction that we have just analyzed. In 1 kg of water, approximately 45 tritium nuclei decay per minute. But how can it be possible that water contains tritium? Would the total amount of tritium that has once fallen into the water for some reason not have to be decayed long ago?

Tritium does not only decay, but it is constantly produced anew in a reaction with the so-called cosmic radiation. The Earth is hit by a constant current of particles with a very high energy – primarily protons and helium nuclei. These particles cause numerous nuclear reactions in the atmosphere. This way, many unstable nuclides are formed.

The decay of $^{14}_{6}C_K$

Also the unstable carbon isotope $^{14}_{6}C_K$ is produced through cosmic radiation. It decays according to

$$^{14}_{6}\text{C}_K \rightarrow {}^{14}_{7}\text{N}_K + e + \bar{\nu}$$

or

$$p_6n_8 \rightarrow p_7n_7 + e + \bar{\nu}. \tag{10.2}$$

From now, we will no longer write down all those contributions of the energy balance that cancel in the equation anyway: the rest energies of those particles that appear both on the left and on the right side of the content formula. Hence, in the case of equation (10.2), we will no longer take into account:

- 6 protons
- 7 neutrons.

Then, the balance table will look as follows:

	p_6n_8	$p_7n_7 + e + \bar{\nu}$
Electric charge	6	7 + (–1) + 0
Baryonic charge	14	14 + 0 + 0
Leptonic charge	0	0 + 1 + (–1)
Rest energy in pJ	n 150.5349	p 150.3277
		e 0.0819
$-E_S$ in pJ	−16.87	−16.77
Sum in pJ	133.66	133.64

The sum on the left is larger than that on the right, i.e. the reaction can take place.

The decay of uranium

The isotopes of uranium can decay in many different ways. We will look at one of the many possibilities of decay of the uranium isotope $^{235}_{92}\mathrm{U_K}$, the decay into barium and krypton:

$$^{235}_{92}\mathrm{U_K} \rightarrow {}^{141}_{56}\mathrm{Ba_K} + {}^{92}_{36}\mathrm{Kr_K} + 2\,\mathrm{n}.$$

This reaction is one of many that are taken advantage of for the generation of energy in nuclear reactors.

	$p_{92}n_{143}$	$p_{56}n_{85} + p_{36}n_{56} + 2\,n$
Electric charge	92	56 + 36 + 0
Baryonic charge	235	141 + 92 + 2
Leptonic charge	0	0 + 0 + 0
$-E_S$ in pJ	−285.80	−188.09 – 129.78
Sum in pJ	−285.80	−317.87

The synthesis of carbon

We will analyze a reaction that takes place in stars and that is responsible for the formation of carbon in the universe:

$$3\,{}^{4}_{2}\mathrm{He_K} \rightarrow {}^{12}_{6}\mathrm{C_K}.$$

The fact that this reaction does normally not take place on Earth is due to the high reaction resistance. The resistance can be reduced by means of the usual methods used in chemistry: by means of a temperature increase or the use of a catalyzer. Since it is a nuclear and not a normal chemical reaction, the temperature at which the reaction starts running is very high: it has to be higher than 100 million Kelvin. This temperature is reached in stars.

	$3p_2n_2$	p_6n_6
Electric charge	3 · 2	6
Baryonic charge	3 · 4	12
Leptonic charge	0	0
$-E_S$ in pJ	−3 · 4.53	−14.77
Sum in pJ	−13.59	−14.77

Exercises

1. Check if deuterium can decay in one of the following ways:

   $\mathrm{pn} \rightarrow \mathrm{p} + \mathrm{n}$

   $\mathrm{pn} \rightarrow 2\,\mathrm{p} + \mathrm{e} + \bar{\nu}$

   $\mathrm{pn} \rightarrow 2\,\mathrm{n} + \bar{\mathrm{e}} + \nu$

2. The potassium isotope $^{40}_{19}\mathrm{K}$ is unstable. According to which reactions can its decay take place?

   $^{40}_{19}\mathrm{K_K} \rightarrow {}^{20}_{9}\mathrm{F_K} + {}^{20}_{10}\mathrm{Ne_K}$

   $^{40}_{19}\mathrm{K_K} \rightarrow {}^{4}_{2}\mathrm{He_K} + {}^{36}_{17}\mathrm{Cl_K}$

   $^{40}_{19}\mathrm{K_K} \rightarrow {}^{40}_{20}\mathrm{Ca_K} + \mathrm{e} + \bar{\nu}$

   $^{40}_{19}\mathrm{K_K} \rightarrow {}^{40}_{18}\mathrm{Ar_K} + \bar{\mathrm{e}} + \nu$

3. Check which of the following two reactions can take place:

   $^{14}_{6}\mathrm{C_K} \rightarrow {}^{4}_{2}\mathrm{He_K} + {}^{8}_{4}\mathrm{Be_K}$

   $^{14}_{6}\mathrm{C_K} \rightarrow 2\,{}^{7}_{3}\mathrm{Li_K}$

4. In the Sun, four protons react into a $^{4}_{2}\mathrm{He}$ nucleus. Which particles are formed in addition? Establish the energy balance of the reaction.

10.9 Nuclear radiation

When looking at a specific nuclide to decide whether it is stable or not, all possibilities of decay would have to be examined. In this process, we would observe that there is a variety of possibilities for many of the nuclides.

However, observation has shown many times that the nuclide is indeed stable. It does not decay although it would be allowed with regard to the balance of the three charges and the energy. This is due to the reaction resistance being too high – a phenomenon that we also know from chemistry.

And we also make another observation: most of the decay reactions that actually take place can be classified into a few groups. We will look at these classification in the following. The common point of these typical reactions is the fact that one of the reaction products is a very small particle: an electron, an antielectron, a neutrino or a $^{4}_{2}\mathrm{He}$ nucleus.

> In the most frequent decay reactions, one of the reaction products is a very small particle.

Formation of an electron

An example is the decay of the carbon isotope $^{14}_{6}\mathrm{C}_\mathrm{K}$ that we examined in the previous section; see equation (10.2).

Formation of an antielectron

In another frequent reaction type, one of the reaction products is an antielectron. The $^{40}_{19}\mathrm{K}_\mathrm{K}$ can decay in this way:

$$^{40}_{19}\mathrm{K}_\mathrm{K} \to {}^{40}_{18}\mathrm{Ar}_\mathrm{K} + \bar{e} + \nu \,.$$

We check whether this reaction is actually allowed:

	$p_{19}n_{21}$	$p_{18}n_{22} + \bar{e} + \nu$
Electric charge	19	18 + 1 + 0
Baryonic charge	40	40 + 0 + 0
Leptonic charge	0	0 + (–1) + 1
Rest energy in pJ	p 150.3277	n 150.5349
		$\bar{e}$ 0.0819
$-E_S$ in pJ	– 54.72	– 55.08
Sum in pJ	95.61	95.54

The sum on the left side is larger than the one on the right; hence, the reaction is allowed.

Formation of a $^{4}_{2}$He nucleus

An example for this reaction type is the decay of the radium isotope $^{226}_{88}\mathrm{Ra}_\mathrm{K}$ into a radon isotope $^{222}_{86}\mathrm{Rn}_\mathrm{K}$ and the nucleus of a helium isotope $^{4}_{2}\mathrm{He}_\mathrm{K}$:

$$^{226}_{88}\mathrm{Ra}_\mathrm{K} \to {}^{222}_{86}\mathrm{Rn}_\mathrm{K} + {}^{4}_{2}\mathrm{He}_\mathrm{K} \,.$$

	$p_{88}n_{138}$	$p_{86}n_{136} + p_2n_2$
Electric charge	8	86 + 2
Baryonic charge	226	222 + 4
Leptonic charge	0	0 + 0
Rest energy in pJ	0	0
$-E_S$ in pJ	−277.43	−273.68 – 4.5334
Sum in pJ	−277.43	−278.21

In most nuclear reactions that we are dealing with, only very small quantities react. The conversion rate is tiny compared to that of a typical chemical reaction. Usually, there is such a small quantity of the reaction products that they cannot even be detected with the most sensitive scales. We could think that the proof of a nuclear reaction having taken place is very difficult. However, the process of a nuclear reaction is clearly noticeable. If one of the emerging particles is a light particle, it seizes the biggest part of the energy ΔE that is left from the reaction. It flies away with a very high velocity and can therefore be detected in many different ways.

While at least a billion molecules are typically needed to detect the progress of a reaction with chemical methods, the mentioned nuclear decay reactions can already be detected if only a single nucleus decays.

> While a very large number of particles are needed to detect the progress of a reaction with chemical methods, the decay of a single nucleus can often be detected in case of a nuclear reaction.

The fact that there are indeed reactions of the nuclei was first discovered in the following way. It was found that certain substances emit an unknown "radiation": some very fast-moving particles. These substances were also said to be *radioactive*. As the fast particles are formed in nuclear reactions, we call them *nuclear radiation*. Also, it is often referred to as "radioactive radiation". The word radioactive means radiatively active (radius = beam). However, what is radioactive is the decaying substance, not the radiation.

At first, only three different types of nuclear radiation that are different in the electric charge had been discovered. (The charge of the particles was easy to determine.) Prior to knowing what exactly it was, the three radiation types discovered were named: the particles that were later identified as helium nuclei were called α-radiation, the fast electrons were called β-radiation. In addition, photons with a high energy were released during many reactions, i.e. always in cases where the reaction product is in an excited state. These photons were called γ-radiation. The names α-, β- and γ-radiation have been maintained up to present although we know that they are just any normal reaction products and that there are also many other reaction products that have not been given an additional name.

We have said that in most decays of unstable nuclides one of the reaction products is an electron, an antielectron or a helium nucleus. This behavior is illustrated in Fig. 10.6. The figure shows a simplified version of the nuclide map from Fig. 10.2.

Nuclear radiation with a too high intensity is dangerous for humans and other organisms. Hence, there are strict safety rules for the handling of radioactive substances. But there are also radioactive substances in our environment so that we are constantly exposed to slight nuclear radiation.

Exercises

1. Which of the three particles e, $\bar{e}$ and ${}^{4}_{2}\mathrm{He}_{K}$ is formed during the decay of
   a) ${}^{61}_{29}\mathrm{Cu}$ b) ${}^{66}_{29}\mathrm{Cu}$, c) ${}^{228}_{90}\mathrm{Th}$.
2. Demonstrate that the balance of the three charges is correct for the reaction
   ${}^{55}_{26}\mathrm{Fe} \rightarrow {}^{55}_{25}\mathrm{Mn} + \nu$.
   Please bear in mind that not the symbols of the nuclei but of the whole atoms, including the shell, are listed here. The reaction actually occurs. Describe in words what happens in the process. Establish the energy balance.
3. (a) We look at the decay reaction of an unstable nuclide A, in which a nuclide B and an electron are formed:
   $A_K \rightarrow B_K + e + \bar{\nu}$.
   Where is the nuclide in the nuclide map? Where is A in relation to B?
   (b) Where are the nuclides C and D that emerge in the decay reaction
   $C_K \rightarrow D_K + \bar{e} + \nu$?
   (c) Where are the nuclides E and F that emerge in the decay reaction
   $E_K \rightarrow F_K + p_2n_2$?

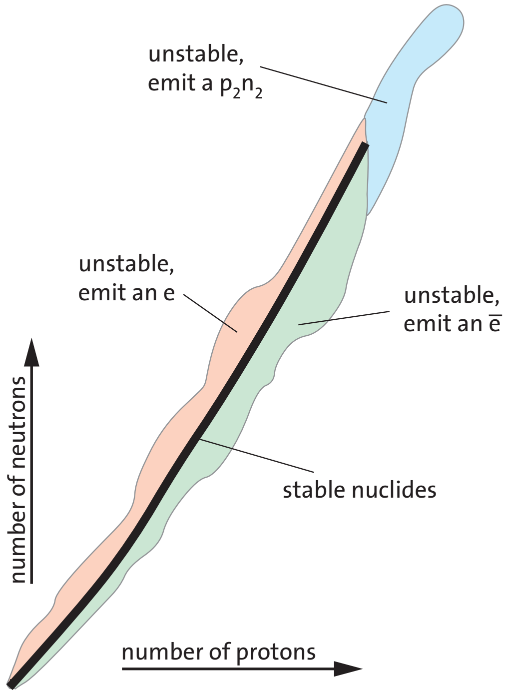

Fig. 10.6 Strongly simplified nuclide map

10.10 The conversion rate of nuclear reactions

We notice so little of nuclear reactions in daily life because they generally occur very slowly. The conversion rate of the reaction

$$\text{conversion rate} = \frac{\text{conversion}}{\text{time inteval}}$$

or in symbols

$$I_n = \frac{n}{t}.$$

tells us how fast a reaction occurs.

The conversion n is a measure for the amount of the reacting substances. In chemical reactions, it is measured with the unit mol so that the conversion rate is expressed in the unit mol/s.

The conversion rates of nuclear reactions are so low that even the prefixes *nano*, *pico* or *femto* are not convenient. Therefore, the conversion is no longer measured in mol but by the number of the elementary reactions: the number of the nuclei that have decayed or that have been formed.

As we know, one mol corresponds to $6 \cdot 10^{23}$ particles or "elementary quantities":

$$1 \text{ mol} = 6 \cdot 10^{23} \text{ elementary quantities}$$

The conversion rate is measured accordingly by the number of elementary quantities per second, i.e.

$$1\frac{\text{mol}}{\text{s}} = 6 \cdot 10^{23} \frac{\text{elementary quantities}}{\text{s}} .$$

The unit "elementary quantity per second" was given its own name: *Becquerel*, abbreviated Bq. Hence:

$$1\frac{\text{mol}}{\text{s}} = 6 \cdot 10^{23} \text{Bq}.$$

Example:

Earlier we learned that a very small part of the hydrogen of common water is the hydrogen isotope tritium. This tritium decays according to the nuclear equation

$${}^{3}_{1}\mathrm{H}_{K} \rightarrow {}^{3}_{2}\mathrm{He}_{K} + e + \bar{\nu} .$$

The conversion rate in a cubic meter of normal water amounts to approximately

$$I_n = 750 \text{ Bq}.$$

In comparison, we look at a chemical reaction that is considered as being slow: corrosion of a car.

We make the realistic assumption that 1 mol of rust is formed in one year. Hence,

$$I_n = \frac{1 \text{ mol}}{1 \text{ year}} = \frac{1 \text{ mol}}{32 \cdot 10^6 \text{s}}$$

$$= \frac{1}{32 \cdot 10^6} 6 \cdot 10^{23} \text{Bq} = 1.9 \cdot 10^{16} \text{Bq}.$$

The comparison shows a typical difference between chemical and nuclear reactions: most nuclear reactions that we are dealing with have much lower conversion rates than typical chemical reactions. Therefore, the amount of the chemical elements on Earth only change to a very limited extent due to nuclear reactions.

> The conversion rates of typical nuclear reactions are much lower than those of typical chemical reactions.

We often seek to accelerate a chemical reaction, i.e. to increase the conversion rate. This can be achieved with two different methods:

- increase of the temperature
- addition of a catalyzer (a substance whose quantity will not change in the course of the reaction).

The same methods can be applied to nuclear reactions. The conversion rate can be increased by means of increasing the temperature or by means of a catalyzer.

However, temperatures of many million Kelvin are required to increase the conversion rate in a noticeable way through a temperature increase. Such conditions exist in the Sun and in other stars. In fact, the the nuclear energy production in the Sun and in stars is ensured by nuclear reactions. Another example for the increase of the conversion rate by means of high temperatures is the fusion reactor, i.e. a system that is in a development stage and that should be used for the production of electric energy one day.

The catalytic acceleration of a nuclear reaction occurs in the nuclear reactor. Here, neutrons work as a catalyzer.

We will later take a closer look at these processes: nuclear reactions in the nuclear reactor and in the Sun.

> Nuclear reactions can be accelerated in the same way as chemical reactions by
> - increasing the temperature;
> - using a catalyzer.

Exercise

1. Natural uranium contains 99.28 % of the isotope $^{238}_{92}\text{U}$ and 0.72 % of the isotope $^{235}_{92}\text{U}$. How many mol of the isotope $^{235}_{92}\text{U}$ are contained in one kg of natural uranium? The isotope $^{235}_{92}\text{U}$ is used in the nuclear reactor to produce nuclear energy. While it is still not in the reactor, it decays by itself according to the reaction equation .

   $$^{235}_{92}\text{U}_\text{K} \rightarrow {}^{231}_{90}\text{Th}_\text{K} + {}^{4}_{2}\text{He}_\text{K}\,.$$

   The conversion rate for 1 kg of the natural uranium isotope mix for this reaction is

   $$I_n = 5.76 \cdot 10^5 \text{ Bq}.$$

   Convert the conversion rate in mol/s. How much time is needed for the decay of 1% of the $^{235}_{92}\text{U}$?

10.11 The half-life

The phosphorus isotope $^{30}_{15}\text{P}_\text{K}$ is unstable. It decays according to

$$^{30}_{15}\text{P}_\text{K} \rightarrow {}^{30}_{14}\text{Si}_\text{K} + \bar{e} + \nu\,.$$

We look at an individual nucleus of this phosphor isotope. It can decay, and it will decay – but when? The answer to this question is the same as to a question that we have already formulated and even answered earlier. The time when the reaction takes place is indeterminate. But this does not mean that nothing can be said about this time. The *probability* of a nucleus to decay in a given time interval has a well-defined value. In case of our reaction, the probability that the nucleus will decay in 2.5 minutes is 50 %. It is as if the nucleus tossed a coin every 2.5 minutes. Depending on whether it has tossed head or tail, it will decay within the next 2.5 minutes or not.

If the nucleus has several possibilities of decay – which is the case for many nuclei – the result of "rolling a dice" will also determine in which way the nucleus is to decay.

We will now look at a very large number of $^{30}_{15}\text{P}_\text{K}$ nuclei, for example a million. Then we know that, after 2.5 minutes, only half of them will be left and the other half will have decayed. Hence, only approximately

500 000 phosphor nuclei will be left after 2.5 minutes. After further 2.5 minutes, there will be approximately 250 000 nuclei left; another 2.5 minutes later, the number will be 125 000, and so on, Fig. 10.7.

The time interval in which the reacting substance has decayed to half of its initial value is called *half-life* $T_{1/2}$. The decrease of the number N of the decaying nuclei in the course of time is described by an exponential function. We denominate the number at the time $t = 0$ with N_0. Then,

$$N = N_0 \cdot 2^{\frac{t}{T_{1/2}}}.$$

In our example we had:

$N_0 = 1\,000\,000$
$T_{1/2} = 2.5$ min.

Hence, the number of the phosphor nuclei as a function of time is

$$N(t) = 1000000 \cdot 2^{\frac{t}{2.5\,\text{min}}}.$$

If we insert for example 10 min for t, we will obtain

$$N(t) = 1000000 \cdot 2^{\frac{10\,\text{min}}{2.5\,\text{min}}}$$
$$= 1000000 \cdot 2^{-4} = 62500.$$

> The half-life is the interval of time in which half of the quantity of a substance has decayed.

Exercises

1. An unstable nuclide has a half-life of 5 minutes. How many of the originally existing 400 000 niclei will be left after 20 minutes?
2. The quantity of a nuclide whose half-life is 2 years has decayed from an original quantity of $8 \cdot 10^8$ to $1 \cdot 10^8$. How many years have passed?
3. The amount of a nuclide has decreased to 1/16 of the original amount in 4 months. What is the half-life?
4. A nuclide A decays into a nuclide B (plus two fast particles) with a half-life of 2 days and then into a nuclide C with a half-life of 120 years. At the time t_0, there is pure A. What is the proportional composition of the mix of the nuclei A, B and C at the times
   $t_0 = 0$ years,
   $t_1 = 2$ years,
   $t_2 = 1\,000\,000$ years?
5. As you know, the amount of a substance whose nuclei are unstable decays by half during the half-life. By which factor is the conversion rate reduced in this time interval?

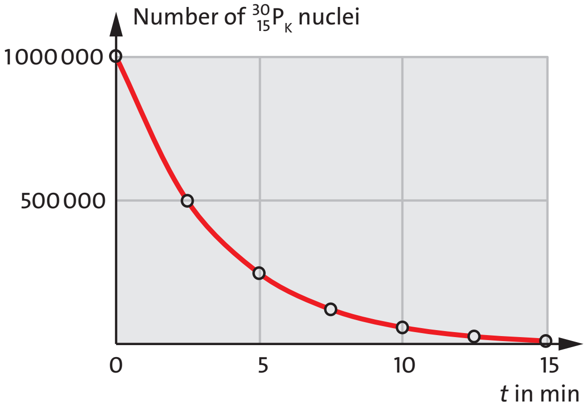

Fig. 10.7 The number of the nuclei decreases to half of its amount in each interval of 2.5 minutes.

10.12 Nuclear reactions in the Sun

The Sun "works" by means of nuclear reactions. With "working" we mean: the energy that comes with the light from the Sun is supplied by nuclear reactions.

The Sun is a star. There are countless other stars in the universe that have the same structure as the Sun and in which the same processes take place. The reason why the Sun appears larger to us than the other stars is only that we are much closer to the Sun.

To get an idea of the size, mass, temperature and composition of the Sun, we will compare it to the Earth. The most important data is summarized in Table 10.4. The density distribution of the Sun is particularly interesting.

	Earth	Sun
mass	$6 \cdot 10^{24}$ kg	$2.0 \cdot 10^{30}$
radius	6 000 km	700 000 km
density inside outside	17 g/cm^3 3 g/cm^3	100 g/cm^3 0.0001 g/cm^3
temperature inside outside	6000 K 300 K	15 000 000 K 6000 K
composition (The percentage values refer to the mass.)	35 % Fe 30 % O 15 % Si 13 % Mg 7 % others	75 % H 23 % He 2 % others

Table 10.4 Mass of Sun, planets and Moon

Fig. 10.8 shows that the density beyond half the Sun radius is very low. 90 % of the mass of the Sun is located within half of the radius. Hence, we can almost say that, in reality, the Sun is only half as large as it looks.

Please also consider what the Sun consists of: almost entirely of hydrogen and helium.

The structure of the Sun is not very complicated. It is simply a large gas ball. This is actually surprising as its density on the inside is higher than the density of all solid matter on Earth. In case of temperatures as high as they exist in the Sun, all matter behaves as a gas.

> The Sun consists almost exclusively of hydrogen and helium.

The Sun emits light and consequently energy. The energy current that leaves the Sun in this manner is very great. It amounts to

$$P = 4 \cdot 10^{26}\ \mathrm{W}.$$

Only the two billionth part of it hits the Earth. This is

$$P = 2 \cdot 10^{17}\ \mathrm{W}.$$

Where does this energy come from exactly? A nuclear reaction takes place inside the Sun. Hydrogen is transformed into helium according to the reaction equation.

$$4\,{}^{1}_{1}\mathrm{H}_{K} \rightarrow {}^{4}_{2}\mathrm{He}_{K} + 2\bar{e} + 2\nu\,.$$

We check the balance of the reaction:

	4 p		$p_2n_2 + 2\bar{e} + 2\nu$	
electric charge	4 · 1		2 + 2 · 1 + 0	
baryonic charge	4 · 1		4 + 0 + 0	
leptonic charge	0		0 + 2 · (–1) + 2 · 1	
rest energy in pJ	2 p	2 · 150.3277	2 n 2 $\bar{e}$	2 · 150.5349 2 · 0.0819
$-E_S$ in pJ	0		–4.5334	
sum in pJ	300.655		296.7	

Hence, an of amount of energy of ΔE = 3.955 pJ is left over. With this energy entropy is produced during the reaction.

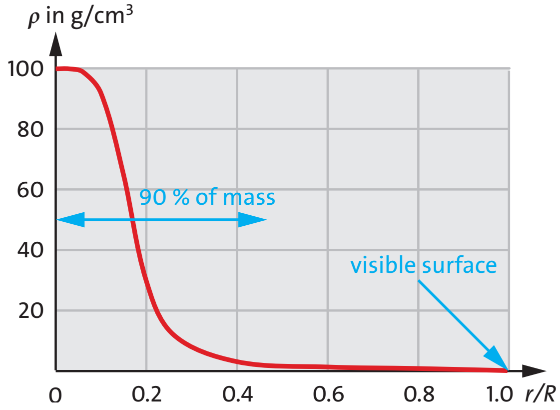

Fig. 10.8 The density of the Sun as a function of the distance from the center. The distance *r* is measured in units of the Sun radius *R*. We can see that the largest part of the Sun's mass is located within half of the radius, i.e. within *r*/*R* = 0.5

Under normal conditions, i.e. conditions that we as humans living on Earth consider as normal, the helium synthesis reaction does not occur, though: the reaction resistance is much too high. The reaction is possible in the Sun only because there is a very high temperature. In fact, the reaction takes place exclusively inside the Sun where there is a temperature of approximately 15 million Kelvin. For us as inhabitants of the Earth, this is an extreme temperature. For nuclear reactions, however, it is not very high. This is demonstrated by the fact that the reaction that we are just examining still runs quite slowly even at 15 000 000 K. The reaction conversion of 1 liter of Sun matter from inside the Sun, i.e. from where the temperature is highest, only amounts to approximately

$$\begin{aligned} I_n &= 2.5 \cdot 10^{9}\ \mathrm{Bq} \\ &= 2.5 \cdot 10^{9}\ \text{elementary conversions per second.} \end{aligned}$$

Let's compare once again with the corroding car for which we have calculated approximately $2 \cdot 10^{16}$ Bq. It is reassuring to know the following: a car will be completely rusty after several decades. The Sun, in contrast, has already "worked" for approximately 5 billion years and it has still enough "fuel", i.e. hydrogen, to shine for another similar period.

Due to the low conversion rate, also the amount of energy that is supplied by one liter of Sun per second is not very high either. We obtain the energy current by multiplying the conversion rate In (in the measurement unit Becquerel) with the energy that is released per elementary conversion, i.e. by 3.955 pJ:

$$\begin{aligned} P &= 2.5 \cdot 10^{9} \cdot 3.955\ \mathrm{pJ/s} = 2.5 \cdot 10^{9} \cdot 3.955 \cdot 10^{-12}\ \mathrm{J/s} \\ &= 0.01\ \mathrm{W}. \end{aligned}$$

Therefore, the energy current that comes out of each liter of Sun due to the nuclear reaction amounts to 0.01 W.

We compare it to other energy suppliers that we are familiar with a common terrestrial flame with a volume of 1 liter produces approximately 1000 W, i.e. 100 000 times more than 1 liter of Sun.

The fact that the Sun emits so much energy in total is due to the large volume of the Sun as a "reactor".

> In the Sun, helium nuclei are produced out of hydrogen nuclei in a nuclear reaction. In spite of the high temperature, the conversion rate is very low. The energy that is left from the reaction is transported to the outside, out of the reaction zone, with entropy.

Now we know what the energy source of the Sun consists of. However, this is not enough to explain the overall working principle of the Sun. We need to answer two more questions to understand it:

- How is it possible that the temperatures inside the Sun are so high although the reaction occurs at a very slow speed?
- The nuclear reaction in the Sun is the same as the one that takes place in a hydrogen bomb. But why does the Sun not explode like a huge hydrogen bomb?

Exercises

1. How much energy is released in the Sun during the formation of 1 mol of helium? Compare to the energy that is released during a common combustion process of 1 mol of hydrogen (e.g. reaction with oxygen).
2. All human beings produce heat: they release energy and entropy. Estimate how much energy is released by "1 liter of human being". Compare to the energy that is released by 1 liter of Sun.

10.13 Why it is so hot inside the Sun

The fact that very high temperatures can be achieved in spite of a very low conversion rate is also known to us from phenomena on Earth, e.g. from a haystack. We imagine that the hay has just been stacked up freshly. A very slow reaction takes place in fresh hay during which entropy is produced – albeit very little. The haystack warms up very slowly in the process. As the inside of the haystack is thermally insulated from the environment very well, the temperature slowly becomes higher and higher. Due to the higher temperature inside, entropy flows away from the inside to the outside. However, the temperature will only increase until the entropy that flows away is equal to the generated one: a flow equilibrium establishes. The temperature inside the haystack is the higher the larger the distance and hence the thermal resistance is that the entropy has to overcome on its way from the inside to the outside. In a large haystack, the temperature can become so high that the hay starts burning. Therefore, the temperature inside a larger haystack has to be monitored.

Precisely this "haystack effect" causes the high temperature inside the Sun and we can understand why the temperatures inside the Sun have reached such high levels: as the Sun is very large, the way between the reaction zone inside and the surface is very long: several 100 000 km. A temperature difference of several million Kelvin had to establish in order to enable the entropy to overcome this distance.

> The Sun must be much hotter inside than outside so that a flow equilibrium can be maintained for the entropy.

Exercise

1. Estimate the temperature gradient in the Sun from the inside to the outside. Estimate the temperature gradient in the Earth. Compare to the temperature gradient in the wall of a house.

10.14 The peculiar reaction of the Sun on an entropy supply

If a body is supplied with entropy, it will become warmer, i.e. its temperature will increase. At least, this is the normal behavior. We also know that there are cases in which the temperature does not change during entropy supply: for example when water is boiling. When the water has reached 100 °C and is left on the hotplate, i.e. entropy supply is going on, the temperature does not increase any further. Instead, the water evaporates.

Now we imagine that we could freely supply the Sun with entropy or extract entropy from it. We would observe a peculiar behavior. When supplying the Sun with entropy, it would not become warmer but colder, i.e. the temperature would decrease. When extracting entropy from it, it would not become colder but warmer; the temperature would increase.

This sounds paradox, but it is not. To understand this, we will make a few imaginary experiments with a gas, e.g. air, in a cylinder with a movable piston, Fig. 10.9.

First, we supply the gas with entropy while retaining the piston in its position. We observe that the temperature of the gas increases – very plausible, right?

In a second experiment, we move the piston so that the volume of the gas will increase. Hopefully you remember what will happen then: the temperature of the gas will decrease. (The effect is the same as if air flows out of a car tire. Its temperature decreases as well.)

Finally a third experiment: we supply the gas with entropy and simultaneously increase the volume. The entropy supply alone would cause a temperature increase, the volume increase would lead to a reduced temperature. Depending on the extent of the entropy supply and the volume increase, one or the other effect will be dominant. If the entropy supply is low and the volume increase high, the temperature will decrease, which leads us to what we are looking for: temperature reduction in spite of entropy supply.

You see what the trick is: the temperature does not only depend on the entropy but also on the volume. Hence, we can achieve a temperature decrease in spite of an entropy increase.

It is similar as in the following example: how can a rectangular piece of land be reduced in size by means of increasing its length? By means of reducing the width to a greater extent than the length, Fig. 10.10.

You could say that this is a cheap trick. Maybe, but this is the trick that is used in the Sun. In case of entropy supply, the Sun simultaneously increases its volume strongly enough as to achieve a temperature decrease. The fact that its volume is increased so significantly is due to the law of gravity. The more strongly a celestial body increases in size, the easier is the process of increasing its size even further. The matter is further away from the center and is therefore drawn towards the center less and less.

> If its entropy increases, the Sun will become larger and cooler; if its entropy is reduced, it will become smaller and hotter.

Exercise

1. Describe a process with a cylinder with a piston (according to Fig. 10.9) in which the temperature of the gas increases although it is deprived of entropy.

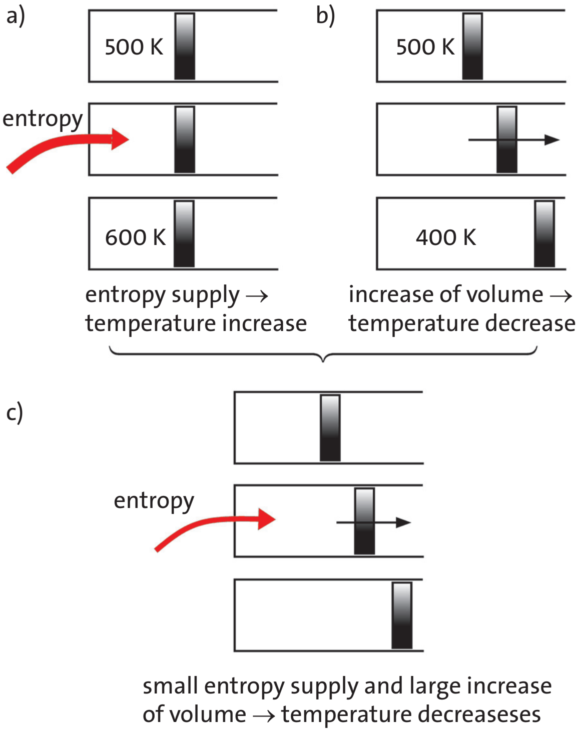

Fig. 10.9 (a) The piston is fixed. The entropy supply causes a temperature increase. (b) The volume increase causes a temperature decrease. (c) The temperature will decrease in case of low entropy supply and a strong increase of the volume.

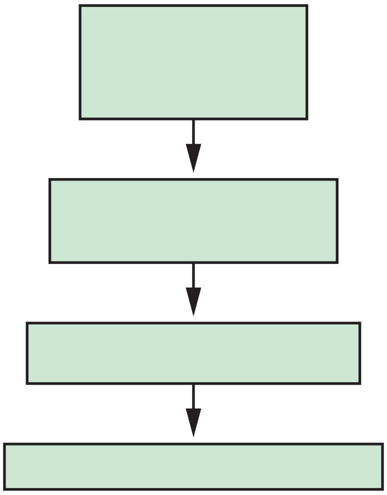

Fig. 10.10 Although the length of the rectangle increases, its surface area is reduced.

10.15 Why the Sun does not explode

The oxyhydrogen reaction

At first, we will look at an explosion that has not yet anything to do with the Sun.

Oxyhydrogen is a mix of hydrogen and oxygen with a ratio of 2 : 1. The hydrogen can react with the oxygen:

$$2\,H_2 + O_2 \rightarrow 2\,H_2O\,.$$

The energy balance of the reaction is positive, i.e. energy is left over. If no particular measures are taken, this remaining energy will simply be used to produce entropy so that the temperature will increase.

At a normal temperature, the reaction runs extremely slowly. The reaction resistance is very high; the reaction is practically inhibited. However, it will become faster if the temperature is increased: the higher the temperature, the higher the conversion rate.

Now we will slightly heat up the oxyhydrogen so that the reaction can start. The energy balance of the reaction is positive, i.e. energy is left over. Entropy is generated with this remaining energy. The temperature increases in the process and the reaction is accelerated. Even more energy will be left over and even more entropy will be created and the temperature will increase further – and so forth.

What we have described here is an explosion.

> Explosion: a reaction creates the preconditions for its increasingly fast progression.

An argument that supports the assumption that the Sun explodes

The Sun consists to the largest extent of hydrogen. Hydrogen can be transformed into helium in a nuclear reaction:

$$4\,{}^{1}_{1}\mathrm{H}_K \rightarrow {}^{4}_{2}\mathrm{He}_K + 2\,\bar{e} + 2\,\nu$$

Energy is left over when this reaction takes place. When we conserve hydrogen under terrestrial conditions, the reaction does practically not take place; it is inhibited. It would occur if the temperature were sufficiently high. Also in this case, we can now imagine that the reaction would only have to be "ignited" in order to start. It would be sufficient to bring the hydrogen to a sufficiently high temperature. It would react and entropy would be produced in the process; the temperature would increase further and the whole amount of hydrogen would react very fast and explosively.

The process described here does actually exist. But it is not equivalent to the process in the Sun but that in a hydrogen bomb.

We know that the Sun has not exploded so far and that it will certainly not explode in the nearer future. This means that there must be an error in our approach. We illustrate it once again, left side of Fig. 10.11.

The temperature increases; therefore, the conversion rate $I_n(R)$ increases; therefore, more entropy is produced, whereby the temperature is increased even further, etc. We can say that a *positive feedback* takes place here.

However, applying this conclusion to the Sun is a mistake. Please remember the message of the previous section: if the entropy of the Sun increases, the temperature will be reduced. The conclusion corrected accordingly is illustrated on the right in Fig. 10.11. The original temperature increase leads to a process that entails a reduction of the temperature. If the temperature has decreased due to some disturbance at the beginning, the process will make it increase once again. We can say that a *negative feedback* takes place during such a process. Hence, a positive feedback leads to an explosion, a negative feedback to the stabilization of a state.

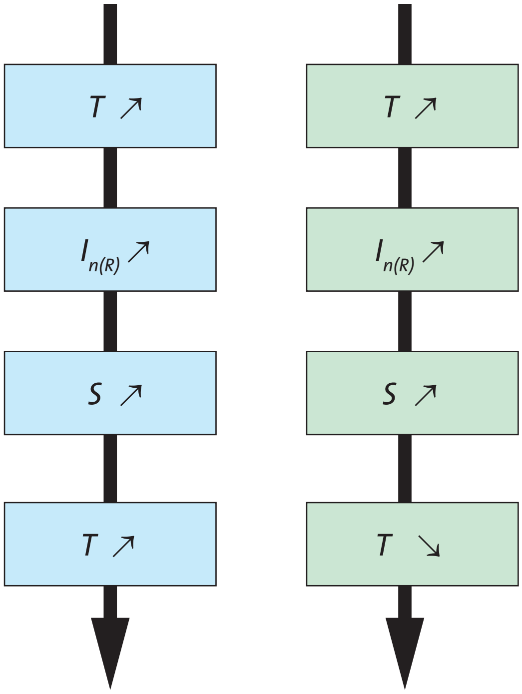

Fig. 10.11 Left: positive feedback; a temperature increase leads to a further temperature increase. Right: negative feedback; a temperature increase reverses itself.

In our case, the temperature of the Sun is stabilized and consequently also the conversion rate and the entropy production.

> Since the temperature of the Sun goes down when the entropy increases, the nuclear reaction is stabilized.

Exercise

1. Name and describe other processes with a positive and with a negative feedback.

10.16 The nuclear reactor

The nuclear power plant

Fig. 10.12 shows the structure of a nuclear power plant.

The way a nuclear power plant works is very similar to a coal-fired power plant. Steam is generated in both power plants, which has a high pressure and a high temperature. Therefore, entropy at a high temperature is needed. This entropy is generated in the boiler by means of burning coal in case of the coal-fired power plant. In nuclear power plants, the entropy creation takes place in the nuclear reactor. The required energy is supplied by a nuclear reaction.

The entropy is transported from the reactor convectively to the *steam generator* with the water of the primary loop. (The primary loop works like a central heating system: also in a central heating system, entropy is transported convectively from one place, i.e. the boiler, to another one, i.e. the radiators.)

In the steam generator, water steam with a high pressure (about 50 bar) and a high temperature (about 270 °C) is created out of the liquid water of the primary loop. A steam turbine is driven by the steam. At the outlet of the turbines, the steam only has a pressure of approximately 70 millibar and a temperature of 40 °C. In the *condenser*, the decompressed steam releases its entropy to the water of the cooling loop. The cooling water then brings the entropy convectively to some place in the environment: through a further heat exchanger into a river that flows past or into the air via a cooling tower.

The nuclear reactions

Let's talk about the nuclear reactions that take place in the nuclear reactor. The nuclide that is responsible for the nuclear energy production is the uranium isotope $^{235}_{92}\mathrm{U_K}$. Natural uranium contains only 0.7% of it. The remaining 99.3 % are made up by the isotope $^{238}_{92}\mathrm{U_K}$. For the use in nuclear reactors, the uranium is first enriched to a $^{235}_{92}\mathrm{U_K}$ content of 3 %.

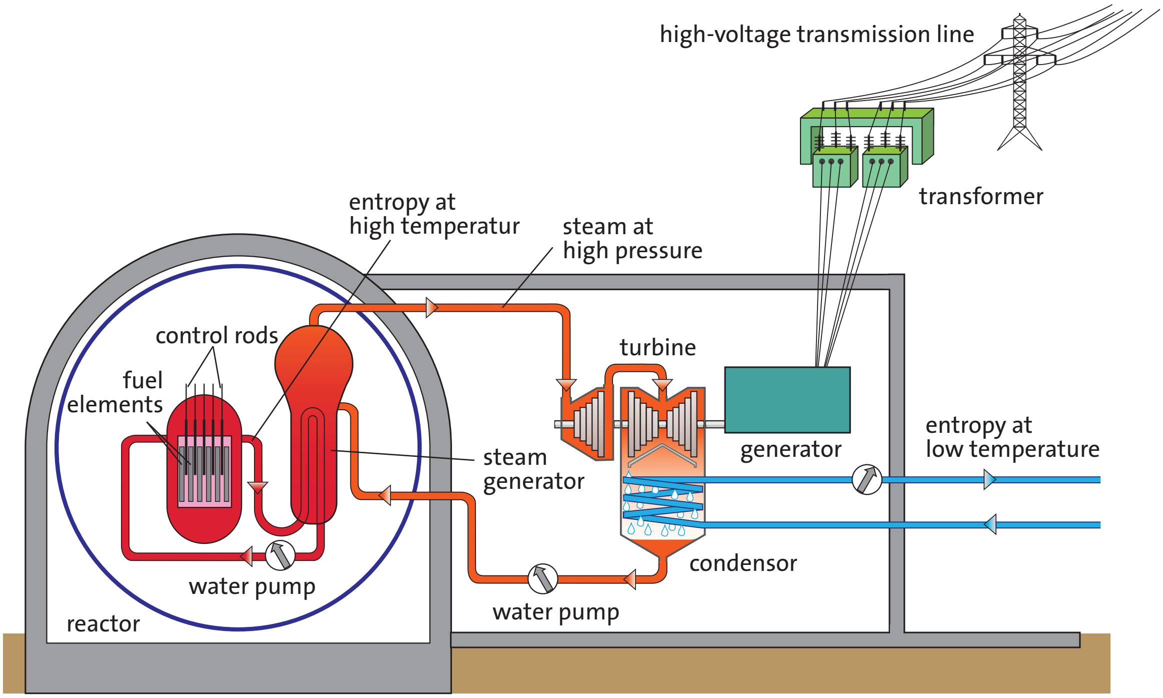

Fig. 10.12 Schematic of a nuclear power plant

The $^{235}_{92}\mathrm{U_K}$ is unstable and can decay into two other nuclides, for example like this:

$$^{235}_{92}\mathrm{U_K} \rightarrow {}^{141}_{56}\mathrm{Ba_K} + {}^{92}_{36}\mathrm{Kr_K} + 2\,\mathrm{n}$$

or:

$$^{235}_{92}\mathrm{U_K} \rightarrow {}^{139}_{56}\mathrm{Ba_K} + {}^{94}_{36}\mathrm{Kr_K} + 2\,\mathrm{n}$$

or:

$$^{235}_{92}\mathrm{U_K} \rightarrow {}^{144}_{55}\mathrm{Cs_K} + {}^{93}_{37}\mathrm{Rb_K} + 2\,\mathrm{n}$$

or in many other ways. Energy is left over in all these reactions.

You see that two neutrons are formed besides the two larger nuclides. We will see in the following that this is important.

All these decay reactions have a high reaction resistance at first. Their reaction yield is so low that they are actually not suitable for the production of nuclear energy at all.

However, the reaction can be easily accelerated by means of a catalyzer. Remarkably, neutrons are suitable as a catalyzer, i.e. precisely the substance that is itself formed during the reaction. Reactions in which the catalyzer that accelerates the reaction is produced itself are called *autocatalytic reactions*. To emphasize the role of the catalyzer, we could add a neutron to the left side of the reaction equation. This would, of course, lead to an additional neutron on the right side as well. Hence, we obtain for example:

$$^{235}_{92}\mathrm{U_K} + \mathrm{n} \rightarrow {}^{141}_{56}\mathrm{Ba_K} + {}^{92}_{36}\mathrm{Kr_K} + 3\,\mathrm{n}\,.$$

The uranium reacts with *one* neutron, and *three* neutrons are formed in this process.

This decay into two larger fragments plus a few neutrons is only the first reaction step though. This is because not only the newly formed neutrons react further, but also the larger fragments that have been formed are unstable. They decay in a series of secondary reactions in which, however, only small particles are detached from the respective nucleus: electrons or $^{4}_{2}\mathrm{He}$ nuclei. As an example, we look at the $^{141}_{56}\mathrm{Ba_K}$ nucleus that is formed during the decay of uranium.

$$\begin{array}{lll}
^{141}_{56}\mathrm{Ba_K} \rightarrow {}^{141}_{57}\mathrm{La_K} + \mathrm{e} + \bar{\nu} & & \\
\qquad\qquad\searrow & & \\
 & ^{141}_{57}\mathrm{La_K} \rightarrow {}^{141}_{58}\mathrm{Ce_K} + \mathrm{e} + \bar{\nu} & \\
 & \qquad\qquad\searrow & \\
 & & ^{141}_{58}\mathrm{Ce_K} \rightarrow {}^{141}_{59}\mathrm{Pr_K} + \mathrm{e} + \bar{\nu}\,.
\end{array}$$

An unstable $^{141}_{57}\mathrm{La_K}$ nucleus is formed at first out of the $^{141}_{56}\mathrm{Ba_K}$ nucleus; then, a $^{141}_{58}\mathrm{Ce_K}$ nucleus emerges, that is also unstabale. Only the $^{141}_{59}\mathrm{Pr_K}$ nucleus, that is formed in the last reaction step is stable.

All the mentioned reactions contribute to the nuclear energy production of the reactor, albeit to a very different extent. Approximately 95 % of the overall released energy is formed by the decay of the uranium that is catalyzed by the neutrons and only 5 % in the secondary reactions.

In spite of the limited significance for the energy balance of the reactor, the secondary reactions in the reactor operation play a very important role: they are the reason why a reactor requires very costly safety features.

> In a nuclear reactor, $^{235}_{92}\mathrm{U}$ nuclei decay into two nuclei with approximately the same size as well as a few neutrons. The reaction is catalyzed by neutrons (autocatalysis). All emerging nuclides are unstable and continue to decay.

The control system of the reactor

First, let's remember how the boiler of a coal-fired power plant is controlled: in the same way as an oil-fired heating system, a common coal stove or an open fire. To make the reaction run faster, more fuel and more oxygen is introduced; to reduce the conversion rate, the fuel or air supply is reduced.

As an autocatalytic reaction takes place in the nuclear reactor, something different can be done here: the reactor is charged with the "fuel" once a year, i.e. with the amount of uranium that is required for the whole year.

However, this is a system with a positive feedback: neutrons are created during the reaction; the neutrons lead to a higher conversion rate and more neutrons are created, etc. What is described here is not a nuclear reactor but an atomic bomb. In order to transform it into a reactor, the reaction has to be controlled by influencing the amount of the catalyzer, i.e. the neutrons.

The neutron concentration in the reactor can be reduced by inserting control bars into the reaction zone. The *control bars* are bars made of a metal that absorbs neutrons, i.e. that reacts particularly well with the neutrons.

The safety devices

Many different nuclides are formed during the catalytic decomposition of the uranium; and all of them are unstable – they continue to react. However, the secondary reactions are not accelerated by just any catalyzer. Each of them takes place with its own characteristic speed: some of them are fast, others are slow.

We have seen that in these secondary reactions, a light particle is always formed besides a heavy nuclide: an electron or a $^{4}_{2}$He nucleus. We already know that, in this case, almost all of the energy that is left from the reaction is taken along by the light particle. This means that the decay products of the uranium are radioactive. Due to the high conversion rates in the reactor, the nuclear radiation that is emitted by the decay products is very intensive and dangerous.

This leads to one of the problems during the operation of a reactor. The decay products of the uranium may not escape to the environment. Avoiding this is quite complicated. However, there is yet another problem: the nuclear energy production of a reactor cannot be stopped quickly. Although it is possible to quickly stop the uranium decay reaction by means of the control bars, the secondary reactions cannot be influenced. The secondary reactions continue and only subside gradually. Table 10.5 shows how the nuclear energy production keeps going after the stop of the uranium fission.

Under normal operating conditions, a large power plant produces approximately 1000 MW. If it is shut down, it will still produce 0.7% of 1000 MW after one day, i.e. 7 MW. Therefore, the energy removal still needs to be ensured for several days after the switch-off. Otherwise, the reactor could heat up so strongly that the reactor container melts.

The safety systems of nuclear power plants have to be set up in a way that the removal of the heat is also ensured in emergency situations so that melting of the reactor container becomes very unlikely. Should it still occur, other safety barriers have to ensure that no radioactive substances are released to the environment.

Time after shutdown	Nuclear energy production (based upon normal operation of the plant)
5 seconds	5 %
1 minute	4 %
10 minutes	3 %
1 hour	1 %
1 day	0.7 %
2 days	0.5 %

Table 10.5

Due to the secondary reactions of the decay products of the uranium, a reactor needs sophisticated safety systems because

- the decay products of the uranium are radioactive and must not escape to the environment;
- the nuclear energy production during the secondary reactions cannot be switched off.

Radioactive waste

When the reactor is charged with new uranium once a year, the reaction products from the previous year have to be removed from the reactor first. These reaction products are a mix of a variety of nuclides. Many of these nuclides are unstable, and there are nuclides with a diversity of half-lives.

Of course, it is not possible to wait with the discharging process of the reaction products until all unstable nuclides have been transformed into stable ones as some nuclides have half-lives of several years, others of several decades and some even of centuries and millennia.

Hence, the reaction products that are removed from the reactor once a year are still radioactive and will remain radioactive for quite a long time. They consequently have to be stored in a place from which they cannot come in contact with the biological cycle of substances.

Plutonium

The decay processes of the uranium and the secondary products are the most important and most frequent reactions in the nuclear reactor. In addition, other reactions of a different type take place, too: synthesis reactions.

The $^{238}_{92}\mathrm{U_K}$ reacts with neutrons to form heavy nuclides. Among them, plutonium is particularly important. Plutonium is an element of which no isotope can be found in nature. In the reactor, the $^{239}_{94}$Pu nuclide is created out of $^{238}_{92}\mathrm{U_K}$ in the following reaction:

$$^{238}_{92}\mathrm{U_K} + \mathrm{n} \rightarrow {}^{239}_{94}\mathrm{Pu_K} + 2\,\mathrm{e} + 2\,\overline{\nu}.$$

Hence, plutonium is also contained in the radioactive waste. $^{239}_{94}$Pu is radioactive as well. In the process of its decay, it releases helium nuclei with a half-life of 24 000 years.

Just as $^{235}_{92}\mathrm{U_K}$, the decay of $^{239}_{94}$Pu into two parts with approximately the same size is catalyzed by neutrons. $^{239}_{94}$Pu could therefore be used as a reactor fuel. For this, it would have to be separated from the other radioactive waste products first. But this comes with another problem. An atomic bomb could be built in a relatively convenient way with such separated, pure plutonium.

10.16 The nuclear reactor

Exercises

1. Compare the energy produced by the uranium decay reaction

$$ {}^{235}_{92}\mathrm{U}_{\mathrm{K}} \rightarrow {}^{141}_{56}\mathrm{Ba}_{\mathrm{K}} + {}^{92}_{36}\mathrm{Kr}_{\mathrm{K}} + 2\,\mathrm{n} $$

with the nuclear energy produced by the subsequent reaction

$$ {}^{141}_{56}\mathrm{Ba}_{\mathrm{K}} \rightarrow {}^{141}_{57}\mathrm{La}_{\mathrm{K}} + \mathrm{e} + \bar{\nu}. $$

2. We consider the consecutive reaction where, at first, ${}^{141}_{56}\mathrm{Ba}_{\mathrm{K}}$ transforms in ${}^{141}_{57}\mathrm{La}_{\mathrm{K}}$, then in ${}^{141}_{57}\mathrm{La}_{\mathrm{K}}$ and finally in ${}^{141}_{59}\mathrm{Pr}_{\mathrm{K}}$:

$$\begin{array}{lll}
{}^{141}_{56}\mathrm{Ba}_{\mathrm{K}} \rightarrow {}^{141}_{57}\mathrm{La}_{\mathrm{K}} + \mathrm{e} + \bar{\nu} & & \\
\qquad\quad\searrow & & \\
 & {}^{141}_{57}\mathrm{La}_{\mathrm{K}} \rightarrow {}^{141}_{58}\mathrm{Ce}_{\mathrm{K}} + \mathrm{e} + \bar{\nu} & \\
 & \qquad\quad\searrow & \\
 & & {}^{141}_{58}\mathrm{Ce}_{\mathrm{K}} \rightarrow {}^{141}_{59}\mathrm{Pr}_{\mathrm{K}} + \mathrm{e} + \bar{\nu}
\end{array}$$

The half-life values of the three nuclides are: $T_{1/2}({}^{141}_{56}\mathrm{Ba}) = 18$ minutes, $T_{1/2}({}^{141}_{57}\mathrm{La}) = 4$ hours, $T_{1/2}({}^{141}_{58}\mathrm{Ce}) = 32$ days. We will begin at time t_0 with a certain amount of ${}^{141}_{56}\mathrm{Ba}$. Give a rough estimate of how much Ba, La, Ce, and Pr are present ("a lot", "medium", "little", "very little"):

a) after 30 seconds,
b) after 18 minutes,
c) after 5 days,
d) after 1 year.

11 PARTICLE PHYSICS

11.1 Radiation – elastic and inelastic scattering

If any type of substance moves in a straight direction without being disturbed, it will form a beam. Examples are a water jet or a light beam. Many other types of beams are created for scientific and technical purposes: radar and X-rays, electron, proton and atomic beams. We also say that the electrons, the protons, etc., constitute *radiation*.

When radiation hits an obstacle, there will generally be some sort of occurrence: energy and momentum of the radiation particles can change, the radiation can disappear and new particles can be created. In any case, we say that the radiation has been *scattered*.

The many different processes that take place can be roughly categorized into two classes.

Elastic scattering

The particles of the radiation remain what they are – electrons remain electrons, photons remain photons. They are merely diverted. This means that their momentum changes. (The momentum is a vector whose direction changes.) Their kinetic energy can be redistributed in the process.

Inelastic scattering

The radiation particles disappear and new, other particles are created.

If white light falls onto a sheet of white paper, the light will be scattered back. Its energy does not change in the process; only its direction changes. Elastic scattering takes place.

If X-rays fall onto a "crystal lattice", an interference pattern that allows for conclusions about the structure of the crystal will be formed. The energy of the X-rays does not change in the process – only the direction and hence the momentum. Elastic scattering takes place.

If UV light falls upon a white material and if then the white material emits visible light, the energy of the light changes. The scattering is inelastic. (We talk about *fluorescence* in this context.)

If a proton of a proton beam collides with a beam that runs in the opposite direction (for example in the LHC in Geneva), hundreds of new particles will be produced. The scattering is inelastic.

11.2 Elastic scattering – "seeing" with different types of radiation

To be able to see an object, it has to emit light. Some objects emit light by themselves, others emit light when they are illuminated. At the moment we are only interested in the second case. Hence, the object is illuminated and the light is partially absorbed and partially scattered elastically.

Objects can be seen in the figurative sense by taking advantage not of the "visible light" but of radiation of other wavelengths that we do not perceive with our eyes. Then, we have to use an appropriate detector, i.e. a suitable detection system or meter.

For example, we can "see" a plane by means of radar waves, i.e. electromagnetic waves with wavelengths from 1 to 10 cm. X-rays, i.e. electromagnetic waves with wavelengths from 10 to 1000 pm (1 picometer = 10^{-12} m), can be used to see the inside of the human body.

The radiation does not even have to be an electromagnetic radiation. It also works with any other radiation. For example, something can be "seen" with sound waves. The echo sounder is used to examine the ocean floor from a ship. The ultrasound waves used have a wavelength of approximately 10 cm. Ultrasound can also be used to see the inside of the human body. Thereby, the sound waves have a wavelength of approximately 0.5 mm. Bats orient themselves by means of ultrasound waves. Similar to the radar, they "illuminate" the world with sound waves with a wavelength of a few millimeters and receive the sound signal that is scattered back by the objects, e.g. by insects.

Now we could ask why one radiation is used in one case and the other radiation in another case. There are two answers to this question:

1. The radiation has to freely enter the way between the observer and the observed object. It may not be absorbed on the way. This is why visible light cannot be used to see the inside of the human body because the light does not penetrate the body up to the organs that we wish to see. It is absorbed. X-rays or ultrasound, in contrast, can be used to see the inside of the body.

However, this is not yet enough. If the radiation is not sufficiently absorbed or scattered by the object to be observed, it is not useful either because it would pass through the object in an undisturbed way without providing us with information about it. X-rays penetrate the body but are partially absorbed there. This way, we obtain information about the structure of the absorbing body parts, for example the bones, by means of the escaping radiation. Radar waves pass through the air without being hampered but are scattered on metal objects. This is why planes or rain clouds can be "seen" by means of radar waves. This applies accordingly for ultrasound waves.

> An object can be imaged by means of radiation. The radiation may not be absorbed or scattered on the way to the object.
>
> The radiation has to be absorbed or scattered by the object.

2. An important decision in favor of or against a particular radiation depends on the *resolution* that we seek to obtain: how small are the details that we want to distinguish? A high resolution means that we can detect small details of the object. The resolution of a radiation can be evaluated easily. It is only possible to detect structures that are not smaller than the wavelength of the radiation used.

For example, a flight radar with $\lambda = 10$ cm can only be used to detect objects or structures on an object that are not smaller than 10 cm.

Atoms cannot be seen with visible light ($\lambda \approx 600$ nm) as their diameter is approximately 0.1 nm, i.e. roughly 10 000 times smaller than the wavelength. X-rays, in contrast, can be used to "see" atoms.

> Only structures that are not smaller than the wavelength of the radiation used can be detected.

Apart from the electromagnetic waves and the sound waves, there are matter waves. We have learned that electrons can form a sine wave and that interferences can be produced with such waves. What is their wavelength? What we usually know about the radiation is the energy of its particles. We would like to calculate the wavelength from the particle energy.

Let’s remember the relationship between momentum and wavelength

$$p = \frac{h}{\lambda}.$$

If the particles are very fast, i.e. if they almost have the limit speed c, their rest energy E_0 is only a small part of their overall energy E. Hence,

$$E_0 \ll E.$$

We had seen that the relationship between energy and momentum is very simple in this case:

$$E = c \cdot p.$$

This allows us to calculate the wavelength:

$$\lambda = \frac{h}{p} = \frac{h \cdot c}{E}.$$

Example: SLAC (Stanford Linear Accelerator Center)

The linear accelerator at the SLAC creates electrons with an energy of 30 GeV (= $30 \cdot 10^9$ eV). For their wavelengths, we obtain

$$\lambda = \frac{h \cdot c}{E} = \frac{6.6 \cdot 10^{-34}\,\mathrm{Js} \cdot 3 \cdot 10^{8}\,\mathrm{m/s}}{30 \cdot 10^{9} \cdot 1.6 \cdot 10^{-19}\,\mathrm{J}}$$
$$= 4.1 \cdot 10^{-17}\,\mathrm{m}.$$

This is much less than the diameter of a proton ($1.7 \cdot 10^{-15}$ m). In fact, it was discovered at the SLAC that the proton has an internal structure. It consists of smaller parts, i.e. the quarks.

Example: LHC (Large Hadron Collider)

In the LHC, protons orbit with an energy of 7000 GeV (7000 giga electron volts)

$$\lambda = \frac{h \cdot c}{E} = \frac{6.6 \cdot 10^{-34}\,\mathrm{Js} \cdot 3 \cdot 10^{8}\,\mathrm{m/s}}{7 \cdot 10^{12} \cdot 1.6 \cdot 10^{-19}\,\mathrm{J}}$$
$$= 1.8 \cdot 10^{-19}\,\mathrm{m}.$$

Hence, the wavelength is 200 times smaller than that of the electrons at the SLAC.

You see that a very high resolution can be achieved with matter waves.

11.3 The size of the particles

If we would like to know what an electron looks like, we could try this: we send two highly energetic electron beams in opposite directions towards each other and examine how one group of electrons is scattered by the other one. This can be used to calculate the size of the electrons. However, we have seen earlier that an electron, i.e. a portion of electronium, can take on a diversity of shapes. The electron can have a variety of different sizes. To be able to make it very small, it has to be compressed. Therefore, energy is needed. The more strongly it is compressed, i.e. the more energy it is provided with, the smaller it becomes. When we analyze one electron with a high-energy radiation, it is provided with energy. The higher the energy of the radiation, the shorter its wavelength and the better the resolution, but the smaller becomes also the examined electron. Hence, the question is not necessarily "how big is the electron?" but "how small can the electron be made?"

So far, one has not been able to discover a lower limit for the diameter of the electron. As the electrons have been examined with a radiation of a wavelength of 10^{-19} m, we can only conclude that the minimum size of the electron must be smaller than this value.

The situation is different for the proton. Findings have shown that the proton has a size of $1.7 \cdot 10^{-15}$ m. Hence, an finite extension has been discovered. This means that the proton has an internal structure, which has also been found: it consists of three smaller particles, the so-called quarks, for which, however, no minimum size has been detected yet.

Sometimes, the electrons and the quarks are said to be pointlike. This statement is a bit thoughtless for two reasons:

- If we know that the maximum size of a particle is 10^{-19} m, this shall not mean that it is not possible to find a diameter and a structure that are smaller than 10^{-19} m, e.g. 10^{-20} m or 10^{-200} m or 10^{-1000} m, one day.
- To reduce the size of a particle to one point, an infinite amount of energy would be needed. Hence, the point shape of something cannot be detected in principle.

11.4 Inelastic scattering – particle reactions

Particles with a high energy are not only created to "resolve" (i.e. to see) small structures, but especially to create new particles. How can this be done? Any particles that are available in a sufficient quantity are charged with as much energy as possible. In case of the LHC, these particles are usually protons. Two proton beams are generated and are made to intersect at a very small angle, i.e. that they hit each other almost frontally. Most protons fly through this intersection area in a straight direction. Only a few of them are scattered inelastically at each other. We could also say that they collide with each other. When this happens, a large number of new particles is formed. Among them, there are many that do not exist in our natural environment. You know that the matter that surrounds us consists of only three different particles: protons, neutrons and electrons.

Every time a new accelerator reached a higher energy, new particles with a greater rest mass were generated – more than a hundred different types up to present.

But why do these artificially created particles not exist in nature? Because they decay very quickly. This decay is a statistic process that we already know from nuclear physics. For a given particle, we cannot indicate when it will decay. However, if we look at a large number of similar particles, we can indicate an interval of time after which only a well-defined percentage of them will be left. When we choose 50% for this percentage, the corresponding time is called *half-life*. It is common to observe the time after which only 37% will be left. It is called the *lifetime* of the particles.

Due to the short lifetime of the particles, the detection and determination of their characteristics with so-called detectors is complicated. We would like to try to get an overview of the different particles.

11.5 The old and the new particles

Classification

All particles can be divided into tow classes: the *leptons* and the *hadrons*, Fig. 11.1. So far, no internal structure could be detected for the leptons. The hadrons, in contrast, are composed particles. They consist of so-called quarks. Also for the quarks, no internal structure has been found.

The hadrons, in turn, can be categorized into mesons and baryons. The mesons consist of two, the baryons of three quarks.

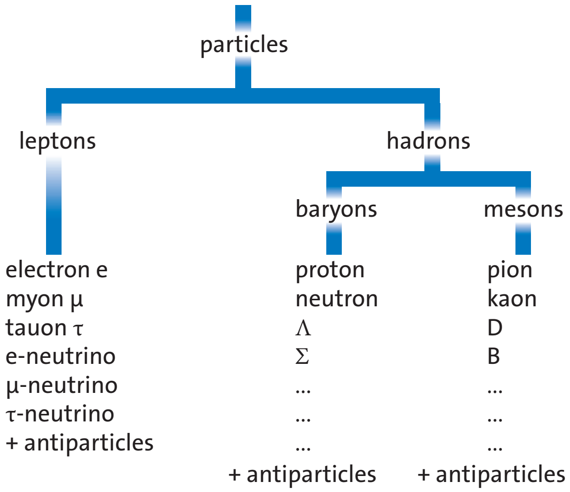

Fig. 11.1 Classification of the particles. Leptons have (according to today's knowledge) no internal structure. Hadrons consist of quarks. Quarks are structureless as well.

Leptons

There are 12 different leptons, Table 11.1, first two columns. Each of them belongs to a particle-antiparticle pair. The baryonic charge of all leptons is equal to zero. Among the leptons, there are a few particles we already know: the electron and the e-neutrino that we have simply called neutrino before, and their antiparticles. The myon has only a short lifetime, the tauon yet a much shorter one.

In spite of their short lifetime, myons can be found on Earth. They are created by the cosmic radiation at an altitude of approximately 10 km. The radiation from space largely consists of highly energetic protons. If these protons hit the atomic nuclei of the atmosphere, myons, whose velocity is close to the limit velocity *c*, are formed through a series of intermediate steps. The fact that they make it to the surface of the Earth is surprising because one would expect them to make a distance of only

$$s = v \cdot t = 3 \cdot 10^8\ \text{m/s} \cdot 2.2 \cdot 10^{-6}\ \text{s} \approx 600\ \text{m}$$

during their lifetime. But $2.2 \cdot 10^{-6}$ s is the lifetime in their own reference system. In the reference system of the Earth – and this is what matters – it is much longer.

lepton-antilepton pair		**electric charge**		**leptonic charge**		**rest energy (= rest mass/c^2)**	**lifetime in s**
e (electron)	$\bar{e}$	–1	+1	+1	–1	0.511 MeV	∞
μ (myon)	$\bar{\mu}$	–1	+1	+1	–1	105.7 MeV	$2.2 \cdot 10^{-6}$ s
τ (tauon)	$\bar{\tau}$	–1	+1	+1	–1	1777 MeV	$3 \cdot 10^{-13}$ s
ν_e (electron neutrino)	$\bar{\nu}_e$	0	0	+1	–1	< 2.2 eV	∞
ν_μ (myon neutrino)	$\bar{\nu}_\mu$	0	0	+1	–1	< 0.17 eV	∞
ν_τ (tauon neutrino)	$\bar{\nu}_\tau$	0	0	+1	–1	< 15.5 eV	∞

Table 11.1 Leptons and antileptons with their electric charge (the unit is the elementary charge), leptonic charge, rest energy and lifespan

quark-antiquark pair		electric charge		baryonic charge		rest energy (= rest mass/c^2)
u (up)	$\bar{u}$	+2/3	−2/3	+1/3	−1/3	~ 3 MeV
d (down)	$\bar{d}$	−1/3	+1/3	+1/3	−1/3	~ 5 MeV
s (strange)	$\bar{s}$	−1/3	+1/3	+1/3	−1/3	~ 100 MeV
c (charm)	$\bar{c}$	+2/3	−2/3	+1/3	−1/3	~ 1.25 GeV
b (bottom)	$\bar{b}$	−1/3	+1/3	+1/3	−1/3	~ 4.2 GeV
t (top)	$\bar{t}$	+2/3	−2/3	+1/3	−1/3	~ 171 GeV

Table 11.2 Quarks and antiquarks with their electric charge (the unit is the elementary charge), baryonic charge and energy at rest

hadron	rest energy (= rest mass/c^2)	electric charge	baryonic charge	leptonic charge	lifetime in s	composition
proton	938.3 MeV	+1	+1	0	∞	uud
neutron	939.6 MeV	0	+1	0	885.7	udd
Λ^0	1115.6 MeV	0	+1	0	$2.6 \cdot 10^{-10}$	uds
Σ^0	1189.4 MeV	+1	+1	0	$0.8 \cdot 10^{-10}$	uus
π^+	139.6 MeV	+1	0	0	$2.6 \cdot 10^{-8}$	$u\bar{d}$
K^0	493.7 MeV	−1	0	0	$1.2 \cdot 10^{-8}$	$u\bar{s}$
η_e	2980 MeV	0	0	0	$2.5 \cdot 10^{-23}$	$c\bar{c}$
B^+	5270.8 MeV	+1	0	0	$1.4 \cdot 10^{-12}$	$u\bar{b}$

Table 11.3 Data of several hadrons

Quarks

There are 12 different quarks – just as many as there are leptons. Each quark belongs to a quark-antiquark pair, Table 11.2. The leptonic charge of all quarks is equal to zero.

To describe the new particles that are created in a collider, it is useful to introduce new physical quantities. There are no corresponding characteristics for these physical quantities that we know from our normal macroscopic experience. Hence, they only appear to us as mathematical aids. You have already come across two of these quantities in connection with the nuclear reactions: the baryonic charge and the leptonic charge. For particle physics, several other such quantities have been introduced. As the values of these quantities are always integer multiples of a specific value, just as in the case of the electric charge or the angular momentum, they are always indicated in multiples of this unit. Hence, the values that are indicated are integers. These numbers are called *quantum numbers*.

Hadrons

By combining the six quarks and their antiquarks, we obtain the different hadrons of which more than 100 are known. Two of them, i.e. the proton and the neutron, together with the electron virtually form all the matter that exists in nature, also the matter of celestial bodies. All the others have been created artificially. The baryons consist of three, the mesons of two quarks. Table 11.3 contains the data of four baryons and four mesons as an example. In the last column, there are the three or two quarks the baryon consists of.

The quarks are held together in a baryon or meson by a field. It is the field of the *strong interaction*. As it is located exclusively inside the hadrons, we do not know it from the macroscopic world.

A comparison of the values of the rest energy from Table 11.2 and 11.3 shows that the rest energy of the hadrons is much higher than the sum of the rest energies of the quarks they consist of. We look at the proton as an example:

Proton 938 MeV

$$\left.\begin{array}{l}\text{u-quark 3 MeV}\\ \text{u-quark 3 MeV}\\ \text{d-quark 5 MeV}\end{array}\right\}\ \text{11 MeV}$$

The biggest part of the rest energy of the proton does not come from the quarks but from the field of the strong interaction that holds the quarks together.

One peculiar feature of the quarks has not yet been addressed here: quarks only exist inside the hadrons; there are no free quarks. We could also express this differently: quarks only exist in groups of two or three. But why shouldn't it be possible to kick a quark out of a proton? We would like to try to understand by means of a comparison why this is not possible.

Two dogs are attached to each other by a spring, Fig. 11.2. As long as the spring is not stretched, each dog has the feeling of being able to move freely. Only when they try to come loose from each other, they notice that they are attached. They can move away from each other a little bit, but only with an energy input. The further they wish to move away from each other, the more energy they need to put into the spring. The same is true for the quarks in a baryon or in a meson.

If the dogs pump more and more energy into the spring, the spring will finally break and the dogs will be truly free. In case of the quarks, something different will happen if more and more energy is shot into the meson or the baryon: when the energy reaches the amount that is necessary to create a new quark-antiquark pair, the two original quarks will actually be detached but each of them will immediately form a new pair with one of the newly created quarks, Fig. 11.3. Hence, we have failed to obtain a free quark once again.

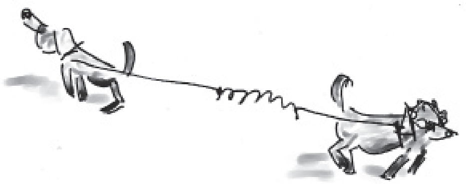

Fig. 11.2 As long as the spring is loose, the two dogs feel free. To move away from each other, they have to put energy into the spring. If they pull too strongly, i.e. if they pump too much energy into the spring, the spring will break and the two dogs will be free again.

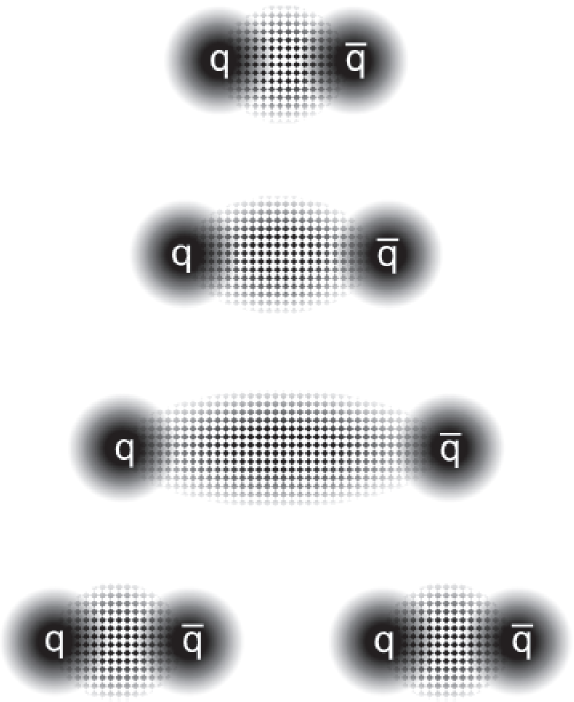

Fig. 11.3 As long as the quark and the antiquark (above) are close to each other, they are "free". In order to move them away from each other, energy needs to be put into the field of the strong interaction. If the energy is sufficient to create a new quark and a new antiquark, the original quark-antiquark pair will split and two new pairs will be formed.

Exercises

1. Which regularities are contained table 11.1?
2. Which regularities are contained in table 11.2?
3. Check whether the entries of the columns with the electric and the baryonic charge of table 11.3 are correct.

12 ORGANZING PRINCIPLES IN PHYSICS

Why are we interested in the creation and the properties of artificially created particles so that billions of euros are invested in devices that can be used to create them?

The world looks puzzling and complex to us. Understanding the world means discovering organizing principles in the apparent chaos – if there are any such organizing principles. The experience made during the last centuries, i.e. since the emergence of sound natural sciences, has shown that such organizing principles indeed exist. There are two types of them:

- The diversity of phenomena is explained with reference to the interaction of many particles of which there is only a limited number of types. All particles of one type are equal among each other. We would like to call this method the "Lego principle".
- The world is described mathematically, i.e. with theories.

12.1 The "Lego principle"

The first method to obtain a simplified idea of the world can be understood best by means of a comparison. We look at Lego bricks. For the sake of simplicity, we assume that Lego bricks exist in ten different shapes, each of them in six different colors. Hence, there is a total of 60 brick types. They can be used to build a variety of different things: houses, vehicles, people, animals, robots.... None of these things is identical to another one. However, the chaotic diversity of the Lego works is based on a simple principle. The variety only emerges through different combinations of just a few elementary components.

This method can also be applied beyond the world of toys.

Such an experiment to explain the world has already been made by *Leukippos* and *Demokrit* in the 5th century before Christ. From today’s perspective, these ideas might be regarded as conjectures, but it was discovered later that the explanation method actually works. Chemistry is the science that deals with it. The existence of the countless different (pure) substances can be understood with the "Lego principle": a pure substance consists of molecules that are equal among each other. A molecule is composed of atoms, and there are only around 100 of them – as many as there are chemical elements. Hence, the existence of the millions of different substances that are known to us can be traced back to only a hundred fundamental components.

In our example, the different atoms correspond to the different types of Lego bricks. A molecule corresponds to a structure that was built of Lego bricks.

We have not yet considered in this context that the atoms of each chemical element exist with different masses. There is a series of isotopes of each element. If we look at the different isotopes of all elements as fundamental components, we will have to deal with around 3000 fundamental substances: the nuclides. The atoms of a nuclide are equal among each other; the atoms of different nuclides are different. By "equal among each other" we mean: they have the same chemical characteristics and they have the same mass, Fig. 12.1.

Hence, the Lego principle can explain the diversity of substances in a genial way. Millions of different molecules are traced back to around 3000 different atoms. But this leads us directly to the next question: what do the individual atoms consist of? We have found the reason for the diversity of the pure substances. But how can we explain the diversity of the different atoms? You already know the answer. We apply the Lego principle once again: as before, the atoms are built in a Lego-like way of components that only exist as three different types: protons, neutrons and electrons.

So, have we come to an end? You have just learned that the answer to this question must be "no". Although only three components of the deeper Lego level are involved in the composition of the natural matter that surrounds us, they are not the only components that exist on this level. We know that, besides the proton and the neutron, there are many other hadrons, and there are other leptons apart from the electrons. And the Lego principle can be applied once again to the hadrons. The diversity of the hadrons can be traced back to the quarks of which there are only 12. Therefore, there would only be 12 leptons and 12 quarks as components of matter.

You can certainly imagine how it continues. The assumption that this is not the final state seems logical. In fact, there are recent theories that suggest that there are more particles on the quark-lepton level and that a new organizing principle has to be found for them. These theories are called *supersymmetric theories.*

Exercise

1. Continue Figure 12.1 in an upward direction. Several continuations are possible: one towards an increasing complexity and one towards an increasing size.

12.2 Theories

The second method to put the apparent chaos of the world into an order is to apply mathematics to nature. It involves describing the different phenomena at the different places and times with an identical formula. The relationship $P = v \cdot F$ applies for a car, an electron and a star. It is valid in Germany, in China, on Mars and in the Andromeda galaxy, it is valid now and it was valid 10 000 years ago and 10 billion years ago, and it will still be valid in 10 billion years. Hence, a theory or a formula embraces common features of many different processes.

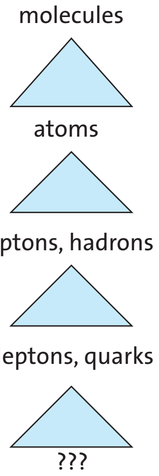

Fig. 12.1 Hierarchy of components: molecules consist of atoms, atoms consist of hadrons and leptons, hadrons consist of quarks.

But scientists continuously have to invent new theories for different reasons:

- A theory always describes a phenomenon in a merely approximate way. The validity of a theory is confirmed through measurements. Measurements only have a limited accuracy in principle and will become more precise over time. At some point, scientists discover that the current theory does not describe the world precisely enough and consequently set up a new theory. An example is "classical" mechanics. It was useful as long as it dealt with velocities that were small compared to the limit velocity *c*. With an increasing measurement accuracy, but also with an increasing velocity of the analyzed objects, it turned out that traditional mechanics was no longer sufficient. This is why the relativity theory was invented.
- Theories that are used to describe different areas of phenomena are usually developed in isolation from each other. At some point, it is discovered that two areas can be combined, i.e. that they can be described with a single and more comprehensive theory. Examples are electricity and magnetism. At first, the electric and the magnetic phenomena seemed to have nothing to do with each other. Then, it turned out that they were interrelated very closely and the theory of electrodynamics that covers both the electric as well as the magnetic phenomena was developed.
- When the examination methods are improved, areas of phenomena that were still unknown before can be explored. A new theory has to be invented for this purpose. An example is particle physics. As

long as the internal structure of the hadrons was not yet known, quantum theory was a valid approach. A completely new theory was needed with the discovery of the quarks: quantum chromodynamics.

Researchers are still faced with many problems to be solved. This means that other new theories need to be conceived. There are already candidates awaiting their conclusion and confirmation: the supersummetric theories mentioned before, the "string theories" and the "Grand Unified Theory".

12.3 The very small and the very large

Particle physics has enabled us to look at even the smallest structures that have been explored up to present. To get a better understanding of the size of the particles, Figure 12.2 shows the size of different objects on a logarithmic scale. The distance between two adjacent small lines respectively corresponds to a factor 100.

In the Figure, the quark and the lepton are listed at 10^{-19} m, however with a question mark as everything we know so far is that the maximum size of these particles is 10^{-19} m. We do not know whether there are structures that are larger than 10^{26} m or smaller than 10^{-19} m.

12.4 The elementary particles and the Theory of Everything

Sometimes, science is said to be looking for the latest, final, structureless, elementary components of matter. And we hear that theorists work on searching a theory of everything, i.e. a theory that describes everything in a conclusive way.

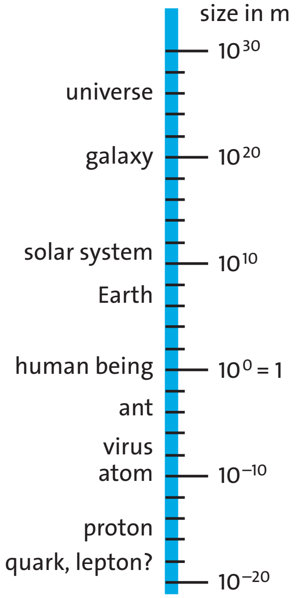

Fig. 12.2 Sizes of different objects, from the smallest to the largest one, on a logarithmic scale: the distance between two adjacent short lines corresponds to a factor 100.

In fact, some natural scientists may have the secret desire to arrive at a final point at some time, i.e. that the definite elementary particles and the theory that comprises everything will be found.

The quest for such final answers is human. It is the same search that led our ancestors to invent gods and demons. This way, there was the final explanation for any adversity such as strokes of lightning, poor harvests and lost wars – at least for those who were not very demanding in their reasoning. There was clarity about what was good and bad, and there were final answers to the question about the beginning and the end of the world. For those who seek knowledge, however, the perspective that everything has already been explored at some time is rather depressing. Continuous, successful research is much more satisfactory and exciting.

Table of separation energies

Z = Number of protons
N = Number of neutrons
E_S = Separation energy in pJ

	Z	N	E_S
H	1	0	0.00
		1	0.356
		2	1.359
He	2	1	1.2364
		2	4.5334
		3	4.39
		4	4.69
Li	3	2	4.22
		3	5.13
		4	6.29
		5	6.61
		6	7.26
Be	4	3	6.02
		4	9.05
		5	9.32
		6	10.41
		7	10.49
B	5	3	6.05
		4	9.02
		5	10.37
		6	12.21
		7	12.75
		8	13.53
C	6	3	6.25
		4	9.71
		5	11.77
		6	14.77
		7	15.56
		8	16.87
		9	17.06
N	7	5	11.86
		6	15.08
		7	16.77
		8	18.50
		9	18.90
		10	19.84
		11	20.30
O	8	6	15.82
		7	17.94
		8	20.45
		9	21.11
		10	22.40
		11	23.03
		12	24.25
F	9	8	20.54
		9	22.01
		10	23.68
		11	24.74
		12	26.04
		13	26.87
		14	28.08
Ne	10	7	18.09
		8	21.17
		9	23.04
		10	25.74
		11	26.82
		12	28.48
		13	29.31
		14	30.74
		15	31.42
Na	11	9	23.39
		10	26.13
		11	27.90
		12	29.89
		13	31.01
		14	32.45
		15	33.35
		16	34.44
Mg	12	9	23.90
		10	27.01
		11	29.12
		12	31.76
		13	32.94
		14	34.72
		15	35.75
		16	37.11
		17	37.72
Al	13	11	29.41
		12	32.13
		13	33.95
		14	36.04
		15	37.28
		16	38.79
		17	39.71
Si	14	12	33.01
		13	35.14
		14	37.90
		15	39.25
		16	40.95
		17	42.01
		18	43.48
		19	41.23
P	15	13	35.48
		14	38.34
		15	40.15
		16	42.12
		17	43.39
		18	45.01
		19	46.02
S	16	14	39.04
		15	41.13
		16	43.54
		17	44.93
		18	46.76
		19	47.88
		20	49.46
		21	50.15
		22	51.44
Cl	17	16	43.91
		17	45.75
		18	47.78
		19	49.15
		20	50.80
		21	51.78
		22	53.08
		23	54.01
		24	55.28
Ar	18	16	44.66
		17	46.70
		18	49.14
		19	50.55
		20	52.45
		21	53.50
		22	55.08
		23	56.06
		24	57.57
		25	58.47
		26	59.81

	Z	*N*	E_S
K	19	18	49.44
		19	51.37
		20	53.47
		21	54.72
		22	56.33
		23	57.54
		24	59.09
		25	60.25
		26	61.68
		27	62.78
		28	64.12
Ca	20	18	50.17
		19	52.30
		20	54.80
		21	56.14
		22	57.98
		23	59.25
		24	61.04
		25	62.22
		26	63.89
		27	65.05
		28	66.65
		29	67.47
		30	68.49
Sc	21	21	56.83
		22	58.77
		23	60.32
		24	62.14
		25	63.54
		26	65.25
		27	66.57
		28	68.19
		29	69.16

	Z	*N*	E_S
Ti	22	21	57.55
		22	60.16
		23	61.68
		24	63.80
		25	65.22
		26	67.08
		27	68.39
		28	70.14
		29	71.16
		30	72.41
		31	73.28
V	23	23	62.54
		24	64.62
		25	66.31
		26	68.16
		27	69.66
		28	71.43
		29	72.60
		30	73.96
		31	74.94
Cr	24	22	61.20
		23	63.31
		24	65.92
		25	67.62
		26	69.70
		27	71.18
		28	73.11
		29	74.39
		30	75.94
		31	76.94
		32	78.27
Mn	25	25	68.35
		26	70.55
		27	72.23
		28	74.16
		29	75.60
		30	77.24
		31	78.40
		32	79.79
		33	80.81

	Z	*N*	E_S
Fe	26	25	69.13
		26	71.73
		27	73.44
		28	75.58
		29	77.07
		30	78.87
		31	80.09
		32	81.70
		33	82.76
		34	84.17
		35	85.06
		36	86.35
Co	27	27	74.14
		28	76.39
		29	78.01
		30	79.83
		31	81.21
		32	82.88
		33	84.08
		34	85.57
		35	86.63
		36	87.99
Ni	28	27	74.88
		28	77.54
		29	79.18
		30	81.14
		31	82.58
		32	84.41
		33	85.66
		34	87.36
		35	88.45
		36	90.00
		37	90.98
		38	92.42
		39	93.34

	Z	*N*	E_S
Cu	29	30	81.69
		31	83.30
		32	85.18
		33	86.60
		34	88.34
		35	89.61
		36	91.20
		37	92.33
		38	93.79
		39	94.80
Zn	30	31	84.15
		32	86.21
		33	87.67
		34	89.58
		35	90.85
		36	92.63
		37	93.76
		38	95.39
		39	96.43
		40	97.90
		41	98.84
		42	100.26
		43	101.12
Ga	31	33	88.30
		34	90.21
		35	91.67
		36	93.47
		37	94.80
		38	96.45
		39	97.67
		40	99.17
		41	100.21
		42	101.68
		43	102.71
		44	104.07

	Z	N	E_S
Ge	32	34	91.21
		35	92.67
		36	94.65
		37	95.97
		38	97.81
		39	99.00
		40	100.72
		41	101.81
		42	103.45
		43	104.49
		44	106.00
		45	106.97
		46	108.37
		47	109.29
As	33	37	96.69
		38	98.55
		39	99.90
		40	101.63
		41	102.91
		42	104.55
		43	105.72
		44	107.28
		45	108.39
		46	109.82
Se	34	37	97.66
		38	99.72
		39	101.07
		40	103.00
		41	104.29
		42	106.07
		43	107.26
		44	108.94
		45	110.06
		46	111.65
		47	112.72
		48	114.21
		49	115.14
		50	116.53

	Z	N	E_S
Br	35	41	105.16
		42	106.92
		43	108.25
		44	109.96
		45	111.22
		46	112.85
		47	114.07
		48	115.60
		49	**116.70**
		50	118.12
Kr	36	39	102.77
		40	104.82
		41	106.31
		42	108.23
		43	109.57
		44	111.42
		45	112.68
		46	114.44
		47	115.63
		48	117.32
		49	118.46
		50	120.04
		51	120.92
		52	122.05
		53	122.87
		56	129.78
Rb	37	45	113.61
		46	115.36
		47	116.76
		48	118.44
		49	119.83
		50	121.42
		51	122.39
		52	123.54
		53	124.46

	Z	N	E_S
Sr	38	43	111.43
		44	113.45
		45	114.87
		46	116.78
		47	118.15
		48	119.99
		49	121.34
		50	123.12
		51	124.14
		52	125.39
		53	126.32
		54	127.50
		55	128.34
Y	39	45	115.63
		46	117.50
		47	119.02
		48	120.92
		49	122.41
		50	124.25
		51	125.35
		52	126.62
		53	127.67
		54	128.87
		55	129.86
Zr	40	47	120.20
		48	122.18
		49	123.67
		50	125.59
		51	126.74
		52	128.13
		53	129.21
		54	130.52
		55	131.56
		56	132.82
		57	133.71
		58	134.74

	Z	N	E_S
Nb	41	48	122.87
		49	124.49
		50	126.42
		51	127.68
		52	129.10
		53	130.25
		54	131.61
		55	132.72
		56	134.01
Mo	42	48	123.96
		49	125.58
		50	127.61
		51	128.91
		52	130.46
		53	131.64
		54	133.10
		55	134.20
		56	135.58
		57	136.53
		58	137.86
		59	138.72
Tc	43	51	129.65
		52	131.24
		53	132.50
		54	134.02
		55	135.19
		56	136.62
		57	137.71
Ru	44	50	129.27
		51	130.70
		52	132.42
		53	133.72
		54	135.35
		55	136.54
		56	138.09
		57	139.18
		58	140.66
		59	141.66
		60	143.09
		61	144.03
		62	145.39
		63	146.26

	Z	*N*	E_S
Rh	45	53	134.41
		54	136.08
		55	137.39
		56	138.97
		57	140.17
		58	141.66
		59	142.78
		60	144.22
		61	145.27
Pd	46	53	135.42
		54	137.20
		55	138.53
		56	140.22
		57	141.44
		58	143.04
		59	144.18
		60	145.71
		61	146.76
		62	148.24
		63	149.22
		64	150.63
		65	151.55
		66	152.90
Ag	47	56	140.89
		57	142.23
		58	143.84
		59	145.11
		60	146.64
		61	147.80
		62	149.28
		63	150.37
		64	151.78
		65	152.82
Cd	48	56	141.92
		57	143.28
		58	145.02
		59	146.29
		60	147.94
		61	149.12
		62	150.70
		63	151.82
		64	153.33
		65	154.38
		66	155.82
		67	156.81
		68	158.20
		69	159.13
		70	159.75
In	49	61	149.95
		62	151.56
		63	152.79
		64	154.30
		65	155.47
		66	156.91
		67	158.00
		68	159.41
Sn	50	59	147.92
		60	149.73
		61	151.04
		62	152.77
		63	154.01
		64	155.66
		65	156.87
		66	158.40
		67	159.51
		68	161.01
		69	162.05
		70	163.51
		71	164.49
		72	165.91
		73	166.86
		74	168.22
		75	169.14
		76	170.45
		77	171.34
Sb	51	67	160.30
		68	161.83
		69	162.95
		70	164.43
		71	165.52
		72	166.96
		73	168.00
		74	169.39
		75	170.39
		76	171.73
Te	52	65	158.42
		66	160.12
		67	161.33
		68	162.98
		69	164.14
		70	165.71
		71	166.83
		72	168.34
		73	169.39
		74	170.85
		75	171.86
		76	173.26
		77	174.24
		78	175.59
		79	176.54
		80	177.83
		81	178.77
I	53	70	166.50
		71	167.70
		72	169.23
		73	170.38
		74	171.84
		75	172.94
		76	174.35
		77	175.39
		78	176.77
		79	177.78
		80	179.22

	Z	N	E_S
Xe	54	67	162.92
		68	164.70
		69	165.95
		70	167.63
		71	168.84
		72	170.45
		73	171.61
		74	173.15
		75	174.26
		76	175.74
		77	176.80
		78	178.23
		79	179.26
		80	180.63
		81	181.67
		82	182.95
		83	183.59
Cs	55	75	175.13
		76	176.62
		77	177.77
		78	179.21
		79	180.31
		80	181.73
		81	182.81
		82	184.14
		83	184.84
Ba	56	71	170.47
		72	172.20
		73	173.43
		74	175.08
		75	176.28
		76	177.85
		77	179.00
		78	180.52
		79	181.63
		80	183.09
		81	184.20
		82	185.58
		83	186.33
		84	187.37
		85	188.09

	Z	N	E_S
La	57	78	181.32
		79	182.51
		80	183.98
		81	185.17
		82	186.58
		83	187.41
		84	188.49
		85	189.30
Ce	58	75	177.95
		76	179.61
		77	180.87
		78	182.46
		79	183.65
		80	185.21
		81	186.41
		82	187.88
		83	188.75
		84	189.90
		85	190.73
		86	191.83
		87	192.59
Pr	59	79	184.38
		80	185.95
		81	187.22
		82	188.72
		83	189.66
		84	190.83
		85	191.76

	Z	N	E_S
Nd	60	79	185.37
		80	187.05
		81	188.30
		82	189.88
		83	190.86
		84	192.11
		85	193.03
		86	194.25
		87	195.09
		88	196.27
		89	197.08
		90	198.26
		91	199.11
		92	200.28
Pm	61	81	188.97
		82	190.57
		83	191.61
		84	192.88
		85	193.88
		86	195.11
		87	196.06
		88	197.22
		89	198.12
Sm	62	80	188.51
		81	189.89
		82	191.57
		83	192.66
		84	194.00
		85	195.02
		86	196.33
		87	197.27
		88	198.55
		89	199.44
		90	200.77
		91	201.71
		92	202.98
		93	203.91

	Z	N	E_S
Eu	63	85	195.71
		86	197.03
		87	198.06
		88	199.33
		89	200.34
		90	201.71
		91	202.74
		92	204.05
		93	205.06
		94	206.26
Gd	64	85	196.69
		86	198.09
		87	199.13
		88	200.51
		89	201.51
		90	202.93
		91	203.96
		92	205.33
		93	206.35
		94	207.62
		95	208.57
		96	209.77
		97	210.67
Tb	65	91	204.81
		92	206.22
		93	207.30
		94	208.60
		95	209.63
		96	210.86
		97	211.86
Dy	66	87	200.66
		88	202.15
		89	203.25
		90	204.76
		91	205.88
		92	207.33
		93	208.42
		94	209.79
		95	210.83
		96	212.14
		97	213.15
		98	214.37
		99	215.29
		100	216.42

	Z	N	E_S
Ho	67	95	211.67
		96	213.02
		97	214.08
		98	215.37
		99	216.37
		100	217.54
Er	68	92	208.96
		93	210.12
		94	211.60
		95	212.70
		96	214.12
		97	215.18
		98	216.54
		99	217.57
		100	218.82
		101	219.78
		102	220.94
		103	221.85
		104	222.95
Tm	69	97	215.93
		98	217.33
		99	218.42
		100	219.71
		101	220.77
		102	221.97
		103	222.97
Yb	70	96	215.76
		97	216.89
		98	218.34
		99	219.44
		100	220.80
		101	221.86
		102	223.14
		103	224.16
		104	225.36
		105	226.29
		106	227.39
		107	228.28
		108	229.37

	Z	N	E_S
Lu	71	101	222.61
		102	223.93
		103	225.01
		104	226.24
		105	227.25
		106	228.38
		107	229.34
Hf	72	99	220.98
		100	222.43
		101	223.55
		102	224.93
		103	226.01
		104	227.31
		105	228.34
		106	229.56
		107	230.54
		108	231.72
		109	232.63
		110	233.71
		111	234.56
Ta	73	105	229.13
		106	230.39
		107	231.46
		108	232.67
		109	233.64
		110	234.75
W	74	104	228.99
		105	230.10
		106	231.44
		107	232.52
		108	233.81
		109	234.80
		110	235.99
		111	236.91
		112	238.06
		113	238.94
		114	240.03

	Z	N	E_S
Re	75	107	233.23
		108	234.58
		109	235.62
		110	236.85
		111	237.84
		112	239.02
		113	239.96
		114	241.09
Os	76	106	232.97
		107	234.09
		108	235.50
		109	236.56
		110	237.89
		111	238.90
		112	240.18
		113	241.13
		114	242.37
		115	243.30
		116	244.51
		117	245.40
		118	246.54
Ir	77	111	239.60
		112	240.91
		113	241.93
		114	243.22
		115	244.22
		116	245.46
		117	246.43
		118	247.59
Pt	78	109	237.94
		110	239.40
		111	240.48
		112	241.90
		113	242.93
		114	244.32
		115	245.33
		116	246.67
		117	247.64
		118	248.91
		119	249.85
		120	251.06
		121	251.95
		122	253.12

	Z	N	E_S
Au	79	115	246.14
		116	247.48
		117	248.55
		118	249.84
		119	250.88
		120	252.10
		121	253.10
Hg	80	113	244.54
		114	246.01
		115	247.11
		116	248.53
		117	249.62
		118	250.98
		119	252.05
		120	253.33
		121	254.33
		122	255.57
		123	256.53
		124	257.73
		125	258.64
Tl	81	119	252.81
		120	254.13
		121	255.23
		122	256.49
		123	257.55
		124	258.76
		125	259.80
		127	261.51
Pb	82	119	253.70
		120	255.10
		121	256.21
		122	257.55
		123	258.63
		124	259.92
		125	261.00
		126	262.18
		127	262.81
		128	263.65
		129	264.26
		130	265.08

	Z	N	E_S
Bi	83	123	259.20
		124	260.49
		125	261.60
		126	262.79
		127	263.53
		128	264.35
		129	265.05
Po	84	122	258.78
		123	259.90
		124	261.25
		125	262.36
		126	263.59
		127	264.32
At	85	123	260.33
		124	261.68
		125	262.83
		126	264.07
		127	264.88
		128	265.84
		130	267.58
Rn	86	121	258.29
		124	262.32
		125	263.48
		126	264.76
		132	270.29
		136	273.68
Fr	87	122	259.98
		125	263.82
		128	267.07
		133	271.75
		136	274.52
Ra	88	134	273.76
		135	274.58
		136	275.62
		137	276.40
		138	277.43
		139	278.16
		140	279.18

	Z	N	E_S
Ac	89	135	275.27
		136	276.34
		137	277.20
		138	278.25
		139	279.05
Th	90	138	279.27
		139	280.11
		140	281.20
		141	282.02
		142	283.05
		143	283.82
		144	284.81
Pa	91	138	279.94
		139	280.86
		140	281.96
		141	282.85
		142	283.89
U	92	141	283.86
		142	284.95
		143	285.80
		144	286.85
		145	287.67
		146	288.66
		147	289.43
Np	93	143	286.58
		144	287.63
		145	288.51
		146	289.50
Pu	94	143	287.47
		144	288.59
		145	289.50
		146	290.54
		147	291.38
		148	292.39
		149	293.20
Am	95	145	290.20
		146	291.26
		147	292.15
		148	293.17
		149	294.03

	Z	N	E_S
Cm	96	150	296.05
		151	296.88
		152	297.87
		153	298.63
Bk	97	149	295.70
		150	296.76
Cf	98	153	300.42
		154	301.41
Es	99	153	301.08
		154	302.10
Fm	100	156	304.82
		157	305.61
Md	101	156	305.41
		157	306.26
No	102	156	306.20
		157	307.08
Lr	103	153	303.51
		154	304.65
		157	307.59
Rf	104	157	308.25
Db	105	156	307.66
		157	308.67
Sg	106	157	309.17
Bh	107	155	307.05