Free-fall non-universality in quantum theory

Viacheslav A. Emelyanov*

Institute for Theoretical Physics
Karlsruhe Institute of Technology
76131 Karlsruhe, Germany

Abstract

We show by embodying the Einstein equivalence principle – local Poincaré invariance – and general covariance in quantum theory that wave-function spreading rules out the universality of free fall, i.e. the free-fall trajectory of a quantum (test) particle depends on its internal properties. We provide a quantitative estimate of the free-fall non-universality in terms of the Eötvös parameter, which turns out to be measurable in atom interferometry.

* viacheslav.emelyanov@partner.kit.edu

I. INTRODUCTION

According to Newton's gravitational law, any body having a non-zero gravitational mass is a source of gravity. It is a consequence of numerous experiments that gravitational mass M_g of a macroscopic body is equal with good accuracy to its inertial mass M_i. So, one might assume

$$\left(M_g/M_i\right)_{\rm classical} \; = \; 1 \, . \tag{1}$$

In Newton's theory, this means that small-enough test bodies fall down equally fast, provided same initial position and velocity. The general theory of relativity (GR) promotes this result to the weak equivalence principle which is also known in the literature as the universality of free fall [1]. This principle is a core argument for modelling gravitational interaction through space-time geometry [2], where particles' trajectories correspond to geodesic world lines.

In the framework of quantum theory, however, particles cannot be thought of as point-like objects which move along single world lines. Indeed, Heisenberg's uncertainty principle forces to abandon the idea that position and momentum can be simultaneously defined with perfect precision for quantum particles [3]. This quantum fuzziness originates from the fact that wave functions have finite localisation in space, resulting in the probability of finding a particle at a given space-time point, which is always less than unity. This suggests that quantum particles might not obey the weak equivalence principle, provided its potential breach does not involve tidal gravitational forces [1], which do modify free-fall trajectories of extended bodies.

In this article, we explore this conceptual conflict quantitatively. This is achieved, first, by working in the framework of quantum field theory over curved spacetime, where the latter is modelled by GR, and, second, by implementing Einstein's equivalence principle and general covariance by relating quantum fields to elementary particles.

II. FREE FALL OF CLASSICAL PARTICLES

According to Einstein's gravitational theory, matter is a source of a non-trivial spacetime curvature. The spacetime curvature is mathematically described by the Riemann tensor. This tensor has dimension of inverse length squared. In other words, we can characterise the space-time curvature by a length scale: The bigger this length scale, the weaker a gravitation field is. In particular, at the Earth's surface, it reads

$$L_\oplus \; \equiv \; R_\oplus \left(R_\oplus/R_{S,\oplus}\right)^{\frac{1}{2}} \; \approx \; 1.71\times 10^{11}\,{\rm m}\,, \tag{2}$$

where $R_\oplus \approx 6.37\times 10^6$ m denotes the Earth's radius, whereas $R_{S,\oplus} \approx 8.87\times 10^{-3}$ m stands for its Schwarzschild (S) radius. Thus, the Earth's curvature plays a little role in the dynamics of microscopic objects in quantum processes taking place over time intervals much smaller than $L_\oplus/c \approx 9.52$ min, where $c \approx 2.99\times 10^8$ m/s is the speed of light in vacuum. For this reason, we shall neglect the Earth's curvature in what follows until Sec. V, which is also needed not to go beyond the application domain of the weak equivalence principle.

This approximation means that the metric tensor at the Earth's surface can be replaced by the Minkowski metric $\eta \equiv \mathrm{diag}(+1,\,-1,\,-1,\,-1)$ iff one considers local inertial coordinates. To this end, we wish to introduce normal Riemann coordinates, y, defined at a given point at the Earth's surface, which corresponds to $y^a = 0$. In its vicinity, i.e. $|y| \ll L_\oplus$, we have

$$ds^2 \;=\; g_{ab}(y)\, dy^a dy^b \;\approx\; \eta_{ab}\, dy^a dy^b\,, \tag{3}$$

where the Latin indices lie in $\{0, 1, 2, 3\}$. We have neglected curvature-dependent terms on the right-hand side of (3), because of the weakness of the Earth's gravitational field. These terms can be found in [4]. The very fact that the metric tensor can always be locally brought to the Minkowski-metric form is a result of Einstein's equivalence principle – locally and at any non-singular point of the Universe, the special-relativity physics applies [2].

The general principle of relativity, saying that dynamical laws of nature are the same in all reference frames, ensures that physics does not depend on coordinates utilised. Nevertheless, the same physical process can look different in different coordinate frames. In particular, the local inertial coordinates y and general coordinates $x \equiv (ct, x, y, z)$ are related as follows [4]:

$$x^c \;\approx\; y^c - \frac{1}{2}\,\Gamma^c_{ab}\, y^a y^b\,, \tag{4}$$

where Γ^c_{ab} are Christoffel symbols computed at the Earth's surface and we have omitted terms which depend on higher-order derivatives of metric, in accord with the Minkwoski-spacetime approximation (3). Taking into account that the Earth's gravitational field is approximately described by the Schwarzschild geometry, we obtain

$$\Gamma^0_{ab}\, y^a y^b \;\approx\; +\frac{2g_\oplus}{c^2}\, y^3 y^0\,, \tag{5a}$$

$$\Gamma^1_{ab}\, y^a y^b \;\approx\; -\frac{2g_\oplus}{c^2}\, y^3 y^1\,, \tag{5b}$$

$$\Gamma^2_{ab}\, y^a y^b \;\approx\; -\frac{2g_\oplus}{c^2}\, y^3 y^2\,, \tag{5c}$$

$$\Gamma^3_{ab}\, y^a y^b \;\approx\; +\frac{g_\oplus}{c^2}\,\big((y^0)^2 + (y^1)^2 + (y^2)^2 - (y^3)^2\big)\,, \tag{5d}$$

where the free-fall acceleration points down in the negative z-direction with the magnitude at the Earth's surface reading

$$g_\oplus \;\equiv\; \frac{c^2 R_{S,\oplus}}{2(R_\oplus)^2} \;\approx\; 9.81\,\mathrm{m/s}^2\,. \tag{6}$$

Now, in the Riemann frame, all geodesics passing through $y^a = 0$ are straight world lines [4]. This is basically the condition which determines normal Riemann coordinates. Considering a classical (point-like) particle being initially at rest in the Riemann frame, we have

$$y^a(\tau) \;=\; c\tau\delta^a_0\,, \tag{7}$$

where τ is the proper time and $\delta \equiv \mathrm{diag}(+1,\,+1,\,+1,\,+1)$ is the Kronecker delta. It turns into

$$x^a(\tau) \;\approx\; c\tau\delta_0^a - \frac{1}{2}\,g_\oplus\tau^2\delta_3^a \tag{8}$$

in the non-inertial frame associated with the Earth's surface, where we have substituted (7) into (4) and (5) to get (8). This is the well-known result of Newton's gravitational theory, that explicitly demonstrates the universality of free fall in classical theory.

III. FREE FALL OF QUANTUM PARTICLES

Quantum field theory (QFT) is a mathematical formalism which enables us to successfully describe high-energy processes taking place between particles. This formalism is based on the unification of the underlying principles of quantum mechanics (QM) and the special theory of relativity (SR). The observable Universe cannot be described by Minkowski spacetime, which is a basic mathematical structure of SR. Hence, the application of QFT in theoretical particle physics relies on the Minkowski-spacetime approximation (3).

It is apparent that we need to go beyond this approximation in order to describe quantum particles in the presence of a gravitational field. We thereby wish to demand that the Einstein equivalence principle and the general principle of relativity be also implemented in quantum theory. The former principle implies then that quantum particles must locally be modelled by wave functions which, in local inertial frames, are given by plane-wave superpositions. In fact, it ensures that such quantum particles move along straight world lines in local inertial frames. The latter principle says in turn that wave functions must transform as tensors under general coordinate transformations. In particular, a spin-zero-particle wave function must correspond to a rank-zero tensor – scalar. This ensures that the semi-classical Einstein field equation is in accord with general covariance.

Quantum fields are operator-valued distributions which form a quantum-field algebra [5]. To model a quantum particle in this framework, we need to select the operator $\hat{a}^\dagger(\psi)$ from this algebra, that gives the state $|\psi\rangle = \hat{a}^\dagger(\psi)|\Omega\rangle$ describing this particle, where $|\Omega\rangle$ is the quantum vacuum. In a local Minkowski frame, $|\psi\rangle$ must reduce to an asymptotically free state entering the definition of S-matrix elements in particle physics. To guarantee that, we define

$$\hat{a}^\dagger(\psi) \;\equiv\; -i\int_\Sigma d\Sigma^a(x)\Big(\psi(x)\partial_a\hat{\Phi}^\dagger(x) - \hat{\Phi}^\dagger(x)\partial_a\psi(x)\Big)\,, \tag{9}$$

where Σ is a Cauchy surface and $\hat{\Phi}(x)$ denotes a scalar field, because then we locally recover the Lehmann-Symanzik-Zimmermann reduction formula for the scalar field. In general, this formula relates S-matrix elements with time-ordered products of quantum fields [6, 7], or, in other words, it relates the mathematical formalism of QFT to physics.

Now, $\psi(x)$ in (9) corresponds to a wave function, at least in the weak-gravity limit, i.e. we assume that the characteristic linear size of $\psi(x)$ in space, L, is much smaller than $L_\oplus$. In the

Riemann frame, Einstein's equivalence principle tells us that the wave function of a spin-zero particle of mass $M > 0$ is a superposition of (positive-energy) plane waves, namely

$$\psi(y) \; = \; \frac{1}{(2\pi\hbar)^3}\int \frac{d^3\mathbf{K}}{2E_\mathbf{K}}\, F_\mathbf{P}(\mathbf{K}) \exp\left(-\frac{iK\!\cdot\! y}{\hbar}\right), \tag{10a}$$

where $\hbar \approx 1.05\times 10^{-34}$ J·s is the reduced Planck constant and

$$K \;\equiv\; (E_\mathbf{K}/c, \mathbf{K}) \;\equiv\; \left(\sqrt{(Mc)^2 + \mathbf{K}^2}, \mathbf{K}\right), \tag{10b}$$

$$K\!\cdot\! y \;\equiv\; \eta_{ab}K^a y^b\,. \tag{10c}$$

The function $F_\mathbf{P}(\mathbf{K})$ must have a narrow peak at $\mathbf{K} \sim \mathbf{P}$, where $P \equiv (E_\mathbf{P}/c, \mathbf{P})$ is an initial 4-momentum of the particle. This is in effect required for $\psi(y)$ to be a localised-in-space packet. Furthermore, the general principle of relativity forces us to deal with $F_\mathbf{P}(\mathbf{K}) = F(K\!\cdot\! P)$. For instance, a covariant Gaussian wave function [8, 9] is characterised by

$$F_\mathbf{P}(\mathbf{K}) \;\propto\; \exp\left(-\frac{K\!\cdot\! P}{2D^2}\right), \tag{11}$$

where $D > 0$ stands for momentum variance. The covariance principle leads, thereby, to $\psi(y)$ which is invariant under the (local) Lorentz transformations. This, in particular, ensures that $\psi(y)$ gains a phase shift in quantum-interference experiments [10–13], which is in agreement with the observations [14, 15].

According to Born's statistical interpretation, the wave packet $\psi(y)$ yields the probability amplitude of measuring the particle at a given place [3]. Thus, the probability to find it somewhere in space must be unity:

$$\int d^3\mathbf{y}\, \psi^*(y)\, \psi(y) \;=\; 1\,. \tag{12}$$

This is a normalisation condition for the wave function $\psi(y)$ in QM. It is evident though that this normalisation condition is at odds with special covariance, since the integration measure in (12) is variant under the (local) Lorentz transformations. Therefore, it must be replaced in QFT by the Klein-Gordon product of $\psi(y)$ with itself, cf. [16]:

$$i\int d^3\mathbf{y}\left(\psi^*(y)\,\partial_0\psi(y) - \psi(y)\,\partial_0\psi^*(y)\right) \;=\; 1\,. \tag{13}$$

In fact, this equation corresponds to $\langle\psi|\psi\rangle = 1$. This directly follows from the definition of $|\psi\rangle$ and the canonical commutation relation of $\hat{\Phi}(x)$ and its canonical conjugate. Note that (13) is independent on (local) inertial frames, provided $\psi(y)$ is a scalar. This physically implies that quantum particles are reference-frame-independent objects, i.e. their very existence does not depend on coordinates utilised [14, 15].

We wish to derive a free-fall trajectory of the quantum particle. In real-world experiments, quantum-particle trajectories are determined with the help of detectors. Any detector has a finite extent in space (and time). One can describe this by the scalar $W(y)$ which is essentially unity inside a particle detector and tends to zero outside that. This is a window-function-like scalar which can be understood in QFT as being due to the spontaneous breakdown of spatial translation symmetry. So, $W(y)$ is an order parameter. The device clicks if the particle passes through it, omitting details of how those interact with each other for the sake of simplicity. In this case, the particle position matches the detector location at the time moment of click. The covariant probability to find the particle in an infinitesimal spatial volume at y follows from (9) and reads

$$dP(y) \;\equiv\; -id\Sigma^a(y)\Big(\psi(y)\partial_a\psi^*(y) - \psi^*(y)\partial_a\psi(y)\Big)\,. \tag{14}$$

It is non-negative and drops substantially to zero away from the wave-function support. Thus,

$$P_W(\Sigma) \;\equiv\; \int_\Sigma dP(y)\, W(y) \tag{15}$$

gives the probability to observe the particle by this device. Note, $W(y)$ is to covariantly limit the integration volume to that which the detector occupies, cf. [17]. If there is an array of such small-enough detectors, then the particle position can be determined with some accuracy. On the other hand, we have from probability theory that

$$\langle y^a(\Sigma)\rangle \;\equiv\; \int_\Sigma dP(y)\, y^a \tag{16}$$

gives the expected value of y^a, which, in physics, corresponds to the center-of-mass position of the wave function $\psi(y)$. In terms of this quantity, the device clicks if the wave-function center of mass is localised within the support of $W(y)$.

These observations suggest that the quantum-particle position corresponds to

$$\langle y^a(\tau)\rangle \;\equiv\; i\int_\tau d^3\mathbf{y}\, y^a\Big(\psi^*(y)\,\partial_0\psi(y) - \psi(y)\,\partial_0\psi^*(y)\Big)\,, \tag{17}$$

which turns into the quantum-mechanics definition of position expectation value in the non-relativistic limit $|\mathbf{P}| \ll Mc$ [15]. Note that the position expectation value $\langle y^c(\tau)\rangle$ depends on the proper time τ. This is a physical hypothesis, meaning that quantum particles measure τ. This, however, can be justified by recalling that a lifetime of cosmic-ray (relativistic) muons is bigger than that of muons at rest. This discrepancy arises due to the time-dilation effect in SR [18]: The laboratory lifetime of the cosmic-ray muons is by a Lorentz factor bigger than their proper lifetime. This experimental result validates our hypothesis.

Consequently, we obtain from (10), (11), (13) and (17) for the spin-zero quantum particle being initially at rest ($|\mathbf{P}| = 0$) that

$$\langle y^a(\tau)\rangle \;=\; c\tau\delta_0^a \tag{18}$$

in the Riemann or, in other words, local inertial frame, while, by bearing in mind (4) and (5),

$$\langle x^a(\tau)\rangle \;\approx\; c\tau\delta_0^a - \frac{1}{2}\,g_\oplus\left(\left(1 + \frac{D^2}{(Mc)^2}\right)\tau^2 + \frac{\hbar^2}{4(Dc)^2}\right)\delta_3^a \tag{19}$$

in the non-inertial frame associated with the Earth's surface. The quantum result (19) differs from the classical one (8) by terms to depend on internal quantum-particle properties. Note, the deviation from the geodesic depends on the characteristic quantum-particle extent $\hbar/D$, following from Heisenberg's uncertainty relation, but is not due to tidal gravitational forces. In fact, the tidal-force impact on free fall diminishes with decreasing extent of a freely falling body, unlike the *time-dependent* correction to (8) in (19). Our result (19) means thus that the weak equivalence principle does not hold in quantum theory [14, 15].

The origin of the free-fall non-universality in quantum theory is wave-function spreading. Indeed, this universal phenomenon follows from the circumstance that the wave function $\psi(y)$ obeys the Heisenberg uncertainty principle. This manifests itself through

$$\langle y^i y^j(\tau)\rangle \;\approx\; \left(\frac{\hbar^2}{4D^2} + \tau^2\,\frac{D^2}{M^2}\right)\delta^{ij}\,, \tag{20}$$

where $i, j \in \{1, 2, 3\}$, meaning that $\psi(y)$ expands in space. The combination of this quantum-mechanical result with (5d) explains the quantum corrections to (8) in (19).

Our result (19) may be interpreted in Newton's gravitational theory as (1) cannot hold in quantum theory, namely we instead have

$$\big(M_g/M_i\big)_{\text{quantum}} \;\approx\; 1 + \frac{D^2}{(Mc)^2}\,, \tag{21}$$

because, owing to the time-dependent term in (20), it follows from (19) that

$$\frac{d^2}{d\tau^2}\,\langle z(\tau)\rangle \;\approx\; -g_{\oplus}\,\big(M_g/M_i\big)_{\text{quantum}}\,. \tag{22}$$

It is worth pointing out that (21) is a relativistic result, because the quantum correction to (1) disappears in the quantum-mechanics limit, in accordance with [19]. It originates from going beyond Newton's theory by taking into account gravitational-length contraction, as this gives rise to terms in (5), depending quadratically on y^i. We intend next to study whether (19) is at least approximately consistent with other observables.

IV. FOUR-MOMENTUM OF QUANTUM PARTICLES

The stress-energy-tensor operator for the Klein-Gordon quantum field $\hat{\Phi}(y)$ reads

$$\hat{T}_{ab}(y) \;=\; \partial_a\hat{\Phi}(y)\partial_b\hat{\Phi}(y) - \frac{1}{2}\,\eta_{ab}\big(\partial_c\hat{\Phi}(y)\partial^c\hat{\Phi}(y) - (Mc/\hbar)^2\hat{\Phi}^2(y)\big)\,, \tag{23}$$

Making use of the canonical commutation relation for $\hat{\Phi}(y)$ and its canonical conjugate $\hat{\Pi}(y)$, we obtain for the single-particle state $|\psi\rangle$ that

$$\langle\psi|\hat{T}_{ab}(y)|\psi\rangle \;=\; \langle\Omega|\hat{T}_{ab}(y)|\Omega\rangle + T^{ab}(\psi(y))\,, \tag{24}$$

where $\langle\Omega|\hat{T}_{ab}(y)|\Omega\rangle$ stands for the quantum-vacuum stress tensor [20–22] and

$$T_{ab}(\psi(y)) \;\equiv\; 2\partial_{(a}\psi^*(y)\,\partial_{b)}\psi(y) - \eta_{ab}\big(|\partial\psi(y)|^2 - (Mc/\hbar)^2|\psi(y)|^2\big)\,. \tag{25}$$

Apparently, the quantum vacuum $|\Omega\rangle$ does not carry information about the quantum particle modelled by $|\psi\rangle$. That is a no-particle state by its very definition. This means that we need to renormalise $\langle\psi|\hat{T}_{ab}(y)|\psi\rangle$ by subtracting $\langle\Omega|\hat{T}_{ab}(y)|\Omega\rangle$ from it. This gives rise to $\langle\psi|{:}\hat{T}_{ab}(y){:}|\psi\rangle$, where the colons mean the normal ordering, being equal to $T_{ab}(\psi(y))$.

Taking into account that $T_{ab}(\psi(y))$ is a tensor, we find in the frame resting on the Earth's surface for the particle with the initial momentum $|\mathbf{P}| = 0$ that

$$\begin{aligned} \langle p^a(\tau)\rangle \;&\equiv\; \int_\tau d\Sigma^c(y)\,\frac{\partial x^a}{\partial y^b}\,T^b_c(\psi(y)) \\ &\approx\; Mc\left(1 + \frac{3}{2}\frac{D^2}{(Mc)^2}\right)\delta^a_0 - Mg_\oplus\tau\left(1 + \frac{5}{2}\frac{D^2}{(Mc)^2}\right)\delta^a_3\,. \end{aligned} \tag{26}$$

This result can be immediately obtained from $M_i\langle\dot{x}^a(\tau)\rangle$ with (19), which, in classical theory, gives particle's 4-momentum, where the inertial mass M_i has been defined via the Lagrangian mass M at the leading order of the approximation as follows:

$$M_i \;\equiv\; M\left(1 + \frac{3}{2}\frac{D^2}{(Mc)^2}\right). \tag{27}$$

These computations give an independent support for the result (22), as we have from (26) with (27) that

$$\frac{d}{d\tau}\,\frac{\langle p^z(\tau)\rangle}{M_i} \;\approx\; -g_\oplus\left(M_g/M_i\right)_{\text{quantum}}. \tag{28}$$

It should be mentioned that this derivation makes no use of Born's statistical interpretation we have utilised above to link the quantum-particle trajectory with the wave-function center-of-mass position.

V. GEODESIC DEVIATION FOR QUANTUM PARTICLES

The free-fall acceleration is a non-inertial-frame effect which is, accordingly, absent in local inertial frames. In contrast, the spacetime curvature is non-vanishing in all reference frames. In particular, it shows itself as a relative acceleration between geodesics. This starts to play an important role in satellite-borne experiments.

Considering a detector at rest at the origin of a Riemann frame parametrised by χ, we find in terms of the normal Riemann coordinates y that

$$\chi^c \;\approx\; X^c + y^c - \frac{1}{3}\,R^c{}_{adb}\left(y^a X^d y^b - X^a y^d X^b\right) \tag{29}$$

where $y^a = 0$ corresponds to X^a in the satellite's rest frame, and $R^c{}_{adb}$ is the Riemann tensor at that point. Taking into account that $\langle y^c(\tau)\rangle$ gets no contribution linearly depending on the curvature tensor in vacuum, we find

$$\frac{d^2}{d\tau^2}\,\langle\chi^c(\tau)\rangle \;\approx\; -\frac{2}{3}\,R^c{}_{adb}\,U^a X^d U^b\left(1 + \frac{D^2}{(Mc)^2}\right), \tag{30}$$

where $U^a \equiv P^a/M$ is the initial 4-velocity of the quantum particle.

This result is in accord with the geodesic deviation equation up to the factor depending on the internal quantum-particle properties. This factor fully agrees with that in (22), suggesting that the (passive) gravitational mass of the quantum particle is by that factor bigger than its inertial mass. This is, apparently, in agreement with (21).

VI. QUANTITATIVE ESTIMATE

The result (22) implies quantum particles fall down faster than classical ones. This effect is negligibly small for macroscopic objects. In particular, one gram of iron has the size of about 6.24×10^{-3} m, which may be equated to $\hbar/D$, according to Heisenberg's uncertainty relation, giving $D/Mc \approx 5.63\times10^{-38}$. However, a rubidium atom, ^{85}Rb, has the radius of 220×10^{-12} m and, thus, we get the estimate $D/Mc \approx 5.63\times10^{-9}$.

A dimensionless parameter, which quantifies relative free-fall acceleration of a pair of test bodies of different composition, is known as the Eötvös parameter η. We find from (22) that

$$\eta(A,B) \approx \frac{D_A^2}{(M_A c)^2} - \frac{D_B^2}{(M_B c)^2}\,. \tag{31}$$

It approximately reads 3.16×10^{-17} in case of ^{85}Rb and a heavier atom. This is by five orders of magnitude smaller than the atom-interferometer sensitivity recently achieved in [23] (see also [24, 25]) by experimental tests of the universality of free fall, where the heavier atom was the rubidium isotope ^{87}Rb. Yet, the Eötvös parameter increases by use of lighter atoms:

	$(D/Mc)^2$	$(L/R_\oplus)^2$
One gram of iron	3.17×10^{-75}	9.59×10^{-19}
Rubidium atom (^{85}Rb)	3.16×10^{-17}	4.77×10^{-33}
Potassium atom (^{39}K)	1.78×10^{-16}	1.02×10^{-33}
Hydrogen atom (H)	1.14×10^{-11}	2.37×10^{-35}

TABLE I. The first column shows that the tinier a quantum particle is, the bigger the effect of wave-function spreading influences the particle's free-fall trajectory. In particular, one might expect that the effect is suppressed for Bose-Einstein condensates in free fall, since these have a relatively slowly expanding wave function, see [26, 27]. The second column illustrates the effect of tidal gravitational forces on free-fall trajectories of extended objects, estimated within Newton's theory (see also [15]).

Satellite-borne experiments have much better sensitivity with respect to the Earth-based ones by quantum tests of the free-fall universality – at the 10^{-17} level or better, – where their main advantage consists in the fact that these tests can potentially be made over infinite free-fall times [28]. Their sensitivity will thus be sufficient to empirically discover if wave-function spreading is more fundamental than the weak equivalence principle.

VII. CONCLUDING REMARKS

Here we have treated the free-fall propagation of spinless quantum particles from different standpoints. From the perspective of a particle detector being at rest on the Earth's surface, e.g. of that installed in the Bremen Drop Tower, a quantum particle falls down faster than its classical counterpart:

$$\frac{d^2}{d\tau^2}\big(\langle z(\tau)\rangle - z(\tau)\big) \;\approx\; -\frac{g_\oplus D^2}{(Mc)^2}\,. \tag{32}$$

This effect is due to wave-function spreading and gravitational-length contraction, both well known in quantum theory and general relativity, respectively. Moreover, from the perspective of a detector that freely falls down in the vicinity of the Earth's surface, the quantum particle approaches this detector in the horizontal direction faster than its classical counterpart, while moves faster away from it in the vertical one:

$$\frac{d^2}{d\tau^2}\big(\langle \chi^c(\tau)\rangle - \chi^c(\tau)\big) \;\approx\; -\,\frac{D^2}{3(L_\oplus M)^2}\,\big(X\delta_1^c + Y\delta_2^c - 2Z\delta_3^c\big)\,, \tag{33}$$

as it follows from (30) with $X = (0, X, Y, Z)$ and $U = (c, 0, 0, 0)$. This is a consequence of the interplay of wave-function spreading and the Earth's curvature.

It is a result of lots of experiments that QFT over Minkowski space locally makes physical sense, although the observable Universe is actually curved. This observation implies that both Einstein's equivalence principle and general covariance must be built into quantum theory for that to be in accordance with observations in particle colliders. This line of reasoning leads to our model of quantum particles in the presence of a gravitational field. It gives the results (32) and (33), which might be experimentally testable in the near future.

Still, there are two possible outcomes of these tests. If it will be experimentally discovered that the free-fall trajectory of a quantum (test) particle depends on its internal properties in accord with our results, then the weak equivalence principle – one of the underlying ideas of GR – should be re-thought in quantum theory. It is worth pointing out that this circumstance does not imply any modifications of the coupling of gravity to matter fields, since our results are based on the gravity theory described by a single space-time geometry. If otherwise, the wave-function description of quantum particles should be refined in GR. In either case, these will improve our insight of both quantum theory and gravity.

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