Letter

Ultrafast dynamics of cold Fermi gas after a local quench

N. V. Gnezdilov,[1,*] A. I. Pavlov,[2,3] V. Ohanesjan,[4] Y. Cheipesh,[5] and K. Schalm[4]

[1]*Department of Physics, University of Florida, Gainesville, Florida 32601, USA*
[2]*The Abdus Salam International Centre for Theoretical Physics (ICTP), Strada Costiera 11, I-34151 Trieste, Italy*
[3]*IQMT, Karlsruhe Institute of Technology, 76344 Eggenstein-Leopoldshafen, Germany*
[4]*Instituut-Lorentz, Delta ITP, Universiteit Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands*
[5]*Instituut-Lorentz, Universiteit Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands*

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We consider nonequilibrium dynamics of two initially independent reservoirs *A* and *B* filled with a cold Fermi gas coupled and decoupled by two quantum quenches following one another. We find that the von Neumann entropy production induced by the quench is faster than thermal transport between the reservoirs and defines the short-time dynamics of the system. We analyze the energy change in the system which adds up the heat transferred between *A* and *B* and the work done by the quench to uncouple the reservoirs. In the case when *A* and *B* interact for a short time, we notice an energy increase in both reservoirs upon decoupling. This energy gain results from the quench's work and does not depend on the initial temperature imbalance between the reservoirs. We relate the quench's work to the mutual correlations of *A* and *B* expressed through their von Neumann entropies. Utilizing this relation, we show that once *A* and *B* become coupled, their entropies grow (on a timescale of the Fermi time) faster than the heat flow within the system. This result may provide a track of quantum correlations' generation at finite temperatures which one may probe in ultracold atoms, where we expect the characteristic timescale of correlations' growth to be ~0.1 ms.

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Introduction. Experimental techniques in ultracold quantum gases have greatly advanced in recent years, providing a vigorous control over transport phenomena [1–16]. In contrast to their electronic counterparts, the reservoirs formed out of trapped cold Fermi gas are well isolated from the outer environment and allow for highly tunable interaction strength and disorder. This level of adjustment makes the ultracold atomic systems particularly attractive to probe nonequilibrium dynamics of quantum many-body systems in transport observables.

Due to the atomic nature of carriers, the characteristic timescales of the tunneling phenomena differ by many orders from the electron transport. The shortest timescale relevant for transport in a Fermi system is the Fermi time $\tau_F \sim 1/\varepsilon_F$—the time a particle travels a distance comparable to the Fermi wavelength, where ε_F is the Fermi energy. In turn, the transport measurements are performed on a timescale much longer than the Fermi time [5]. Indeed, for a quantum point contact, it takes $\sim 10\,\tau_F$ to form a steady flow pattern after inducing the potential difference within a system [5,17]. Whereas in an electronic setup this falls into the category of ultrafast processes, being in the femtosecond range, in ultracold atoms the Fermi time is on the order of 0.1 ms. The magnitude of the timescale difference allows one to study the processes specific to ultrafast physics in a moderate millisecond time frame [18].

A natural way to study the early-time evolution of a many-body system is via quantum quench. A quantum quench drives the system out of equilibrium by an explicit change of a system's Hamiltonian parameters [19], e.g., turning on or off the interaction between the subparts of a composite system. Achievable in highly controlled cold atomic platforms, the postquench dynamics provide significant insights into the keystone concepts of many-body physics such as entanglement, ergodicity, and thermalization [19–26].

Conventionally, one considers the early-time dynamics of a many-body system after the quantum quench in a nearly adiabatic regime. In this case, the quench turns on slowly compared to the characteristic timescale of the problem [19]. Instead, interested in a system's dynamics on a timescale comparable to the Fermi time, we focus on a local quench that instantly changes the Hamiltonian of the system.

The evolution of a quantum system after the local quench is known to pave the way towards measuring entanglement entropy [20–24]. The entanglement entropy is a measure of nonclassical correlations in composite quantum systems commonly defined as the von Neumann entropy of a subpart of a total system, which in turn is described by a pure state [27]. At zero temperature, the local quench connecting the two subspaces of a bipartite system generates entanglement entropy measurable in the particle density fluctuations in free fermion and fractional quantum Hall systems [20,21]. The generalization of Refs. [20,21] to finite temperature is not straightforward since entanglement and thermal contributions to the von Neumann entropy are nonadditive. However, if the temperature is low enough, one expects an instant coupling of two tanks of cold Fermi gas prepared at different temperatures to generate quantum correlations between them. Then, a question arises: What is the characteristic timescale of correlation generation initiated by the quench coupling, and how

*n.gnezdilov@ufl.edu

does this timescale compare to the one of the heat flow due to the initial thermal imbalance?

In this Letter, we show that correlation generation induced by a local quench is an ultrafast process. To track the correlation generation, we consider two reservoirs A and B filled with cold noninteracting Fermi gas sequentially coupled and decoupled by the two local quenches. We evaluate the von Neumann entropy production in the reservoirs by relating it to the energy change of A and B after decoupling. Then, we compare the resulting entropy dynamics to the heat current within the system. We find that the entropy production occurs on a timescale of τ_{F}, which is considerably faster than the thermal transport.

Thermal state driven out of equilibrium. We begin with two systems A and B, each initially prepared in a thermal state, that are instantaneously coupled by an interaction term V_{AB}. The generic Hamiltonian is

$$H(t) = H_A + H_B + g(t)V_{AB}, \tag{1}$$

where the function $g(t) = \theta(t) - \theta(t-\tau)$ defines a quench protocol that couples A to B at time $t = 0$ and disconnects them at $t = \tau$.

The initial state of the full system is given by the product of two thermal density matrices

$$\rho_0 = \rho_A \otimes \rho_B, \tag{2}$$

$$\rho_\alpha = Z_\alpha^{-1} \sum_{n_\alpha} e^{-\frac{E_{n_\alpha}}{T_\alpha}} |n_\alpha\rangle\langle n_\alpha| = e^{\frac{\mathcal{F}_\alpha - H_\alpha}{T_\alpha}}, \tag{3}$$

where $|n_\alpha\rangle$ is an eigenstate of the Hamiltonian H_α with energy E_{n_α}, T_α is the initial temperature, $\mathcal{F}_\alpha = -T_\alpha \ln Z_\alpha$ is the thermal free energy, and $Z_\alpha = \mathrm{Tr}_\alpha e^{-H_\alpha/T_\alpha}$ is the partition function for $\alpha = A, B$ [28].

Once the two systems are coupled they become correlated. A natural measure to study the correlations between A and B is the von Neumann entropy. The von Neumann entropy for system A is

$$S_{\mathrm{vN}}(t) = -\mathrm{Tr}_A\, \rho_A(t) \ln \rho_A(t), \tag{4}$$

where $\rho_A(t) = \mathrm{Tr}_B\, U(t)\rho_0 U^\dagger(t)$ is reduced density matrix and $U(t) = \hat{\mathrm{T}} \exp[-i\int_0^t dt' H(t')]$ is the time-ordered evolution operator.

Let us introduce the relative entropy, which is often used in both quantum information processing [27] and quantum thermodynamics [29] to distinguish between two quantum states and as a measure of irreversibility of a thermodynamic process [30]. For our purpose, we define the relative entropy between the evolved state $\rho_A(t)$ of the system A from its initial thermal state ρ_A:

$$S(\rho_A(t)||\rho_A) = \mathrm{Tr}_A\, \rho_A(t)[\ln \rho_A(t) - \ln \rho_A] \geqslant 0. \tag{5}$$

Using that the initial state of A is a thermal state at temperature T_A, we relate the expectation value of the Hamiltonian H_A to the combination of the von Neumann entropy (4) and the relative entropy (5) [31]: $\mathrm{Tr}_A\, \rho_A(t) H_A = \mathcal{F}_A - T_A \mathrm{Tr}_A\, \rho_A(t) \ln \rho_A = \mathcal{F}_A + T_A[S_{\mathrm{vN}}(t) + S(\rho_A(t)||\rho_A)]$. Subtracting the initial energy value $\mathrm{Tr}_A\, \rho_A H_A = \mathcal{F}_A + T_A S_{\mathrm{vN}}(0)$ from $\mathrm{Tr}_A\, \rho_A(t) H_A$, we get

$$\Delta E_A(t) = T_A[\Delta S_{\mathrm{vN}}(t) + S(\rho_A(t)||\rho_A)], \tag{6}$$

where $\Delta E_A(t) = \mathrm{Tr}_A\, \rho_A(t) H_A - \mathrm{Tr}_A\, \rho_A H_A$ and $\Delta S_{\mathrm{vN}}(t) = S_{\mathrm{vN}}(t) - S_{\mathrm{vN}}(0)$. The relation (6) is the first law of thermodynamics for a subpart of a composite quantum system driven from its initial thermal state [32]. A thermodynamic standpoint on the evolution of a quantum system enables one to characterize the irreversibility of a dynamical process [33] and the emergence of decoherence [34] and to relate multipartite quantum correlations to extractable work [35–37].

Equation (6) is most appropriately seen as a thermodynamic statement. It establishes the energy-to-entropy balance after the process is over, i.e., the two systems are decoupled. Indeed, one cannot completely isolate the two systems from each other when they are coupled and determine the actual energy shift in A or B. As such, we shall understand $\Delta E_{A/B}$ as the energy change in the system after decoupling at $t = \tau$.

At zero temperature, turning on the interaction between the subparts of a composite quantum system may induce quantum correlations that increase the von Neumann entropy of each subpart [20,21]. At finite temperature, when A and B are decoupled the energies of both systems will change by $\Delta E_{A/B} \geqslant T_{A/B} \Delta S_{\mathrm{vN}}$, where we used that the relative entropy is nonnegative [27]. So if the change of the von Neumann entropy is positive, the energy change in the system is also inevitably positive. In particular, such an energy increase may be relevant for quantum technology applications, e.g., quantum digital cooling, where a quantum system is brought to the low-energy state by a coupling/decoupling protocol with a cool bath [38]. Utilizing the free fermions example, we demonstrate that at low temperatures the von Neumann entropy increases under the fast decoupling condition $\tau \lesssim \tau_{\mathrm{F}}$.

The case study: free fermions. In free fermion systems, quantum correlations in the Fermi sea are well studied [39–41], including the generation of quantum correlations after a local quench where the entanglement entropy is related to the particle number fluctuations [20]. At the same time, free-particle motion defines transport properties in ultracold Fermi gas [5]. Hence, we proceed with a free fermions model to compare the characteristic timescales for thermal transport and the entropy production induced by a local quench.

Consider for systems A and B two two-dimensional reservoirs with spinless free fermions. The Hamiltonian (1) reads

$$H_A = \sum_{\mathbf{p}} \xi_{\mathbf{p}} a_{\mathbf{p}}^\dagger a_{\mathbf{p}}, \quad H_B = \sum_{\mathbf{p}} \xi_{\mathbf{p}} b_{\mathbf{p}}^\dagger b_{\mathbf{p}}, \tag{7}$$

$$V_{AB} = \lambda a^\dagger(\mathbf{r} = 0) b(\mathbf{r} = 0) + \mathrm{H.c.}, \tag{8}$$

where A and B are coupled locally in space at $\mathbf{r} = 0$. Here $a, a^\dagger$ and $b, b^\dagger$ are the fermionic operators in reservoirs A and B, $\mathbf{p}$ is the momentum, $\xi_{\mathbf{p}}$ is the corresponding dispersion, and λ is the coupling constant. The size of each reservoir is V. Both reservoirs are at equal chemical potential $\mu \simeq \varepsilon_{\mathrm{F}}$. Note that $[H_\alpha, V_{AB}] \neq 0$.

We begin our analysis with the energy transfer in the system. To determine the overall energy shift in reservoir A, we compute the energy flux $\frac{d\langle H_A\rangle}{dt} = ig(t)\langle [V_{AB}, H_A]\rangle = -ig(t)V^{-1}\sum_{\mathbf{p}\mathbf{p}'} \xi_{\mathbf{p}}(\lambda\langle a_{\mathbf{p}}^\dagger b_{\mathbf{p}'}\rangle - \mathrm{H.c.})$ within time-dependent perturbation theory in λ [42]. Consequently, in the lowest order, we obtain the Fermi golden rule formula for the energy shift

$$\Delta E_A = -\frac{\mathcal{T}}{(2\pi)^2}\int_{-\varepsilon_F}^{\varepsilon_F} d\omega d\omega'\,\omega\,\frac{\sin^2(\delta\omega\tau/2)}{(\delta\omega/2)^2} \times \left[n_A^{(0)}(\omega) - n_B^{(0)}(\omega')\right], \tag{9}$$

where $\mathcal{T} = (2\pi)^2\nu_A\nu_B|\lambda|^2$ is the transmission coefficient [43], $\delta\omega = \omega - \omega'$, and $n_\alpha^{(0)}(\omega) = (e^{\omega/T_\alpha} + 1)^{-1}$ are the initial occupation numbers. In the above, we introduced the density of states $\nu_\alpha = V^{-1}\sum_{\mathbf{p}}\delta(\omega - \xi_{\mathbf{p}}) = p_F^2/(4\pi\varepsilon_F)$ and replaced sums over momenta with integrals over energy [44]. Here ε_F is the Fermi energy in the reservoir that we use as the UV cutoff for the energy integrals and p_F is the Fermi momentum.

Let us consider A and B at zero temperature. Turning on the coupling entangles the states in the reservoirs and thus generates entanglement entropy between previously disconnected systems [20]. Alongside, energy measurements are known to exhibit entanglement properties in a quantum system [45]. Hence, we investigate whether the energy of the reservoirs remains unchanged after decoupling.

At zero temperature the distribution function is $n_\alpha^{(0)}(\omega) = \theta(-\omega)$ for both reservoirs. Substituting the unit-step distribution functions into Eq. (9) and evaluating the energy integrals, we derive the energy shift in reservoir A:

$$\Delta E_A^{q} = \frac{\mathcal{T}}{2\pi}\,\varepsilon_F\int_0^{\varepsilon_F\tau}\frac{d\zeta}{\pi}\,\sin\zeta\,\frac{\sin^2(\zeta/2)}{(\zeta/2)^2}. \tag{10}$$

The second reservoir acquires equal energy increment $\Delta E_B^{q} = \Delta E_A^{q}$. As shown in Fig. 1 (top) (solid blue curve), the energy of the reservoir increases in the absence of temperature or particle imbalances. For times $\tau \ll 1/\varepsilon_F$, the energy grows quadratic in time: $\Delta E_A^{q} \simeq \mathcal{T}/(2\pi)^2\varepsilon_F^3\tau^2$. The superscript "q" punctuates a quantum origin of the effect obtained within zero-temperature quantum-mechanical perturbation theory.

Now suppose that reservoirs A and B are prepared at low temperatures ($T_A, T_B \ll \varepsilon_F$) and consider a cooling protocol for reservoir A: $T_A > T_B$. The temperature imbalance between the reservoirs inevitably leads to heat transport. The heat current across the tunneling contact is $\mathcal{I}_T = -\frac{d}{dt}\frac{1}{2}\sum_{\mathbf{p}}\xi_{\mathbf{p}}(\langle a_{\mathbf{p}}^\dagger a_{\mathbf{p}}\rangle - \langle b_{\mathbf{p}}^\dagger b_{\mathbf{p}}\rangle)$. We evaluate the overall heat transmitted from A to B by the moment of decoupling as $\Delta Q = \int dt\,\mathcal{I}_T$, leading to

$$\Delta Q = \tfrac{1}{2}(\Delta E_B - \Delta E_A), \tag{11}$$

and focus on a short-time limit $\tau \sim 1/\varepsilon_F \ll 1/\max(T_A, T_B)$. The heat transfer in Eq. (11) accounts for the relative energy flux between the reservoirs to exclude the external contribution due to the explicit time dependence of the Hamiltonian (1).

We plot ΔQ computed from Eqs. (11) and (9) in Fig. 1 (bottom) (dotted black curve). Comparing the heat to the energy curves in Fig. 1 (bottom), one notices that for short τ, the heat transfer is considerably slower than the energy increment. Pushing the short-time limit to the extreme, $\varepsilon_F\tau \ll 1$, we find that the heat transfer is suppressed by the temperature-dependent coefficient if compared to the energy change in the same regime. Estimating the finite temperature corrections to Eq. (10) using the Sommerfeld expansion [42], we find

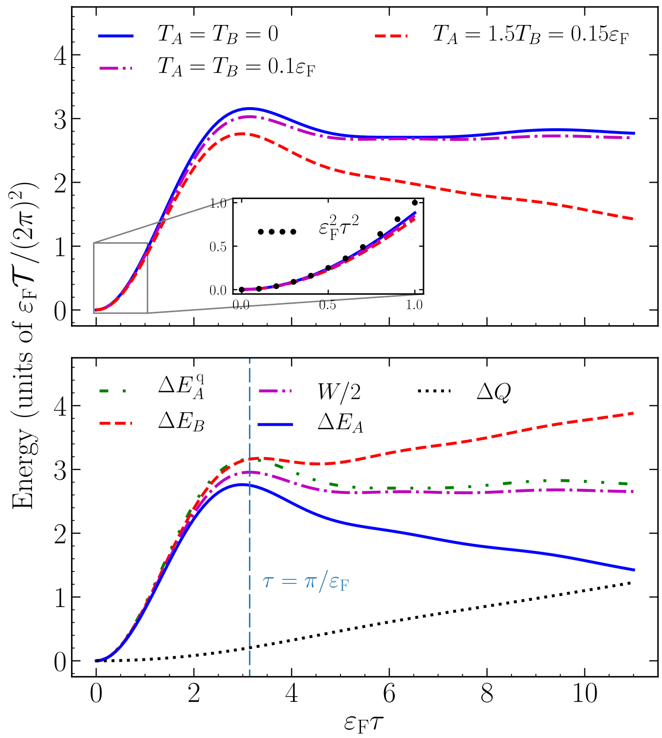

FIG. 1. (Top) Energy increment in reservoir A, ΔE_A, due to quench-coupling with B as a function of time calculated from Eq. (9). The inset demonstrates that the approximation of $\tau \ll 1/\varepsilon_F$ in Eq. (10) accurately describes the energy increment up to $\tau \sim 1/\varepsilon_F$ for a given initial temperature imbalance. (Bottom) Energy balance in the system upon decoupling at $t = \tau$ for $T_A = 0.15\varepsilon_F$ and $T_B = 0.1\varepsilon_F$. ΔE_A^q is the energy change at zero temperature (10), ΔE_B is the energy change in the initially colder system, ΔE_A is the energy change in the initially hotter system, W is the work done by the quench upon decoupling, and ΔQ is the heat transferred from A to B. The vertical line marks the maximum of ΔE_A^q. The figures for a different set of initial temperatures are presented in the Supplemental Material [42].

$\Delta Q \propto \pi^2(T_A^2 - T_B^2)\varepsilon_F\tau^2/6$, so that the ratio $\Delta Q/\Delta E_A^{q} = \pi^2(T_A^2 - T_B^2)/(6\varepsilon_F^2)$ vanishes in the low-temperature limit.

As shown in Fig. 1, the energy increment in both reservoirs does not depend on temperature up to $\tau \sim 1/\varepsilon_F$ and is well described by the quantum contribution (10) ($T_A = T_B = 0$). The refrigerated system starts showing energy decrease (cooling down) near $\tau \sim \pi/\varepsilon_F$, which corresponds to the maximum of the zero-temperature energy curve (10). Accordingly, the heat contribution to energy increases as τ approaches the inverse temperature [46].

The heat transfer lowers the energy in reservoir A. Thus, the energy gain originates from external work. Indeed, the generic form of the Hamiltonian (1) implies that energy can be added to or subtracted from the total system when turning on or off the interaction between A and B: $d\langle H\rangle/dt = \delta(t)\langle V_{AB}(0)\rangle - \delta(t-\tau)\langle V_{AB}(\tau)\rangle$. Evaluating the expectation value of the coupling term as we did for the energy (9), we find $\langle V_{AB}(\tau)\rangle = -\Delta E_A - \Delta E_B$ and consequently $\langle V_{AB}(0)\rangle = 0$, meaning that the first quench does not transfer energy in or out of the system. Hence, the total energy of $\Delta E_A + \Delta E_B$ is added to the system at $t = \tau$,

$$\frac{d\langle H\rangle}{dt} = \delta(t-\tau)W, \tag{12}$$

as a work $W = \Delta E_A + \Delta E_B$ done by the quench to decouple the two reservoirs. Alternatively, one can think of the energy $\Delta E_A + \Delta E_B$ as the binding energy of A and B—the energy required to decouple the reservoirs.

The work W does not depend on the thermal gradient in the system, as shown in Fig. 1 (bottom) (dash-dotted magenta curve compared to the solid blue and dashed red curves). Furthermore, on a timescale of $\tau \lesssim 1/\varepsilon_{\rm F}$, W defines the energy increment in both reservoirs, whereas the latter is given by the zero-temperature result (10). Combining Eqs. (11) and (12), we find for $\tau \lesssim 1/\varepsilon_{\rm F}$ and $T_A, T_B \ll \varepsilon_{\rm F}$ that

$$\Delta E_{A/B} = \mp\Delta Q + \frac{W}{2} \simeq \frac{W}{2}, \tag{13}$$

where we neglect the heat transfer compared to the work contribution to the reservoir's energy. In the above, the "−" sign is taken for reservoir A and the "+" sign for reservoir B.

Having the energy dynamics set, we proceed to the entropy analysis to account for the correlations' generation. From here on, we consider the reservoirs at equal temperatures $T_A = T_B = T \ll \varepsilon_{\rm F}$ since the short-time dynamics ($\tau \lesssim 1/\varepsilon_{\rm F}$) is not affected by temperature imbalance.

Once we decouple the system at $t = \tau$, while it remains well isolated from the outer environment, each reservoir pursues unitary evolution. We assume that the reservoirs are independent if observed much later after decoupling [47]. In this case, their von Neumann entropies equal the entropy of a Fermi gas expressed in their occupation numbers [48] and coincide with the diagonal entropy known for contributing to the energy change in an out-of-equilibrium process [49]. For reservoir A, the entropy is $S_{\rm vN}(t) = -\sum_{\bf p}[\langle a^\dagger_{\bf p} a_{\bf p}\rangle \ln\langle a^\dagger_{\bf p} a_{\bf p}\rangle + (1-\langle a^\dagger_{\bf p} a_{\bf p}\rangle)\ln(1-\langle a^\dagger_{\bf p} a_{\bf p}\rangle)]$. Since the postdecoupling unitary evolution of each reservoir implies the von Neumann entropy conservation, we have $S_{\rm vN}(t=\tau) = S_{\rm vN}(t \gg \tau)$.

To evaluate the entropy we begin with formally expanding it in occupation numbers:

$$S_{\rm vN}(\tau) = S^{(0)}_{\rm vN} - \sum_{\bf p}\left[n^{(1)}_{\bf p}(\tau) + n^{(2)}_{\bf p}(\tau) + \cdots\right]\ln\frac{n^{(0)}_{\bf p}}{1-n^{(0)}_{\bf p}} - \sum_{\bf p}\frac{n^{(1)}_{\bf p}(\tau)^2}{2n^{(0)}_{\bf p}(1-n^{(0)}_{\bf p})} + \cdots, \tag{14}$$

where $n^{(m)}_{\bf p}(\tau)$ are perturbative corrections to equilibrium occupation numbers $n^{(0)}_{\bf p} = (e^{\xi_{\bf p}/T}+1)^{-1}$, with the superscript m marking the order in $\mathcal{T}$ [50]. The first term in Eq. (14) is the initial entropy of the reservoir $S^{(0)}_{\rm vN} = N(\pi^2/3)(T/\varepsilon_{\rm F})$, where $N = Vp^2_{\rm F}/(4\pi)$ is the number of particles [51]. The second term in Eq. (14) is the overall energy increment ΔE_A divided by the initial temperature T, where we used $n^{(0)}_{\bf p}/(1-n^{(0)}_{\bf p}) = e^{-\xi_{\bf p}/T}$. We compare Eq. (14) to Eq. (6) and combine the remaining terms in Eq. (14) into the relative entropy taken with a minus sign:

$$S(\rho_A(\tau)||\rho_A) = \sum_{\bf p}\frac{n^{(1)}_{\bf p}(\tau)^2}{2n^{(0)}_{\bf p}(1-n^{(0)}_{\bf p})} - \cdots. \tag{15}$$

In contrast to the energy increment obtained within quantum-mechanical perturbation theory and most prominent at zero temperature, the entropy computation explicitly requires $T \neq 0$. Indeed, taking the lower energy bound in Eq. (6) leads to the entropy divergence $\Delta S_{\rm vN} = \Delta E_A/T$ at $T \to 0$ since the primary contribution to energy (10) does not depend on temperature. Furthermore, the relative entropy may also diverge at low temperatures due to the Fermi functions in the denominator in Eq. (15). Thus, combining quantum mechanical perturbation theory with nonequilibrium thermodynamics requires a lower bound on temperature T^*.

Minding the low-temperature divergences, we aim to compute both the entropy production $\Delta S_{\rm vN} = S_{\rm vN}(\tau) - S^{(0)}_{\rm vN}$ and the relative entropy $S(\rho_A(\tau)||\rho_A)$ in the leading order in $\mathcal{T}$. We begin with evaluating the first correction to the occupation numbers analogously to the energy shift (9): $n^{(1)}_{\bf p}(\tau) = -\frac{|\lambda|^2}{V^2}\sum_{\bf p'}\frac{\sin^2(\delta\xi_{\bf pp'}\tau/2)}{(\delta\xi_{\bf pp'}/2)^2}(n^{(0)}_{\bf p} - n^{(0)}_{\bf p'})$, where $\delta\xi_{\bf pp'} = \xi_{\bf p} - \xi_{\bf p'}$. Then we substitute $n^{(1)}_{\bf p}(\tau)$ into the first entropy contributions in Eqs. (14) and (15). For temperatures in the range

$$T^* \lesssim T \ll \varepsilon_{\rm F}, \tag{16}$$

where $T^* \sim \varepsilon_{\rm F}/\ln N$, we find the entropy production

$$\Delta S_{\rm vN} \simeq \Delta E_A/T, \tag{17}$$

where ΔE_A is the energy increment computed in the leading order in $\mathcal{T}$ in Eq. (9). In turn, the relative entropy is subleading in $\mathcal{T}$ compared to Eq. (17):

$$S(\rho_A(\tau)||\rho_A) = \frac{2\mathcal{T}^2}{(2\pi)^4}\frac{\varepsilon_{\rm F}}{N}\int_{-\varepsilon_{\rm F}}^{\varepsilon_{\rm F}} d\omega J(\omega,\tau)^2\cosh^2\frac{\omega}{2T}, \tag{18}$$

where $J(\omega,\tau) = \int_{-\varepsilon_{\rm F}}^{\varepsilon_{\rm F}} d\omega' \frac{\sin^2(\delta\omega\tau/2)}{(\delta\omega/2)^2}[n^{(0)}(\omega) - n^{(0)}(\omega')]$.

The lower temperature bound T^* in Eq. (16) extensively depends on the particle number in the reservoir and originates from regularizing the perturbative series for entropy that we discuss in the Supplemental Material [42]. Consequently, within the settled temperature range (16), the perturbative expressions for the von Neumann entropy production (17) and the relative entropy (18) are well defined. Though the lower temperature bound is suppressed only logarithmically with N, for a trapped atomic cloud, e.g., see Ref. [5], $N \sim 10^5$ atoms giving $T^* \sim 0.1\varepsilon_{\rm F}$—well within the experimental reach.

The relative entropy (18) is a measure of state separation indicating how far the evolved state of the reservoir is from its initial thermal state. In Fig. 2 we observe that the state separation occurs on a timescale of $\tau_{\rm F} \sim 1/\varepsilon_{\rm F}$. For the initial temperatures of the reservoirs equal to T^*, this regime is well described by the $\varepsilon_{\rm F}\tau \ll 1$ approximation of Eq. (18), $S(\rho_A(\tau)||\rho_A) \simeq 2[\mathcal{T}^2/(2\pi)^4/N]\varepsilon^4_{\rm F}\tau^4[T\sinh(T/\varepsilon_{\rm F})/\varepsilon_{\rm F} - 1]$, illustrated in the inset in Fig. 2. Alongside, the relative entropy is subleading to the energy contribution to the von Neumann entropy, which leaves us with the expression (17) in the leading order in tunneling coefficient. Combining the perturbative result for the von Neumann entropy production (17) and the energy balance in the reservoir upon decoupling (13), we deduce that the von Neumann entropy of reservoir A accumulated during its mutual evolution with B defines the work required

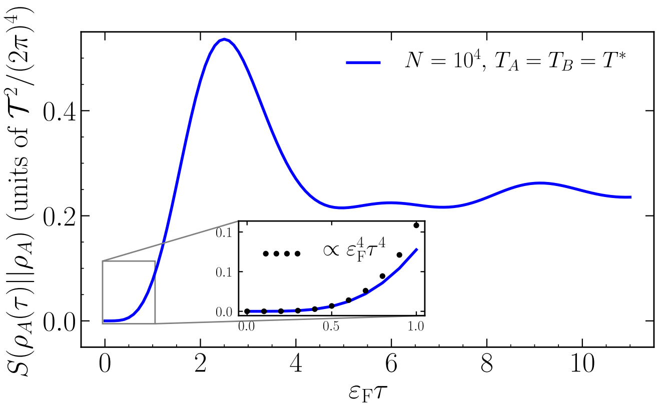

FIG. 2. Relative entropy computed in the second order in transmission $\mathcal{T}$ at $T = T^*$. The black dots in the inset show the relative entropy within $\varepsilon_F\tau \ll 1$ approximation.

to uncouple A from B:

$$W \simeq 2T\Delta S_{\rm vN}. \tag{19}$$

The same is valid for reservoir B, whose entropy equals the entropy of reservoir A.

The work-to-entropy relation (19) is derived for $T_A = T_B$, and therefore there is no heat flow. However, as seen from Eq. (13) and Fig. 1, even implying the initial temperature imbalance, the heat transfer is negligible compared to the work contribution to energy on a timescale of the Fermi time. Hence, on this timescale, one neglects the initial temperature imbalance and considers the reservoirs at equal temperatures with no loss of generality. Combining Eqs. (13) and (19) with quantum-mechanical energy increment (10), which defines the primary contribution to energy at low temperatures, we conclude that the von Neumann entropy is increasing on a timescale of τ_F, which is faster than the heat flow.

Determination of the regime where the influence of the heat flow on the entropy dynamics can be discarded may allow one to characterize the quantum correlations' generation after the local quench at finite temperatures, in contrast with the previous studies strictly implying zero temperature [20,21]. Furthermore, despite being focused on the free homogeneous Fermi gas in the tunneling regime, we anticipate our results to hold beyond these limitations. Indeed, relation (6) is nonperturbative and does not depend on the microscopic details of the system. A predicted energy change associated with quench-induced entropy requires (a) the initial state of the reservoir described by a thermal state and (b) the quench potential that does not commute with the initial Hamiltonian of the system. Accordingly, in Ref. [52], we study a setup with finite-size reservoirs with strongly interacting fermions using exact diagonalization and confirm Eq. (19) on a timescale set by the inverse largest energy scale in the problem.

Conclusion. Inspired by recent advances in ultracold Fermi gases, showing a drastic difference between the characteristic transport timescales in atomic systems compared to their electronic counterparts, we investigate the dynamics of Fermi gas on a timescale of τ_F after driving the system out of equilibrium.

We consider a two-terminal geometry confining two reservoirs filled with a cold Fermi gas and coupled by a tunneling contact. The reservoirs are initially at different temperatures, while the tunneling contact instantly opens at time $t = 0$ and closes at $t = \tau$ by a series of two local quenches following one another. The energy change in either reservoir consists of the heat transferred from the hotter system to the colder one through the open tunneling contact and the work done by the second quench to decouple the reservoirs. For $\tau \sim \tau_F$, we find that both reservoirs gain energy independent of their initial temperatures. This energy increment arises from work done by the second quench, whereas the heat contribution to the resulting energy change is negligible. We relate the work to the von Neumann entropy accumulated by the reservoir from $t = 0$ to $t = \tau$. This relation grants a dynamic track of the von Neumann entropy production. On a timescale of τ_F, the quench-induced entropy production is positive and grows sufficiently faster than the heat flow. This provides a thermodynamic insight into the ultrafast out-of-equilibrium dynamics of Fermi gas and a possibility to characterize quantum correlations' generation at finite temperature. In turn, the τ_F timeframe and the required temperature regime are experimentally accessible in ultracold atoms.

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