A Shapley value-based Distributed AC OPF Approach for Redispatch Congestion Cost Allocation

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ABSTRACT
The progressing energy transition induces a growing need for redispatch congestion management, and, thereby, a fair distribution of its respective costs among the different system operators. In this light, a very recent paper uses the Shapley value as such a fair allocation rule to assign redispatch congestion costs to system operators. However, this approach is based on DC optimal power flow (OPF) and requires the sharing of detailed grid models from all system operators. This is not preferred by them due to data privacy concerns. W.r.t. real-world implementation, the present paper extends the method by using AC OPF problem formulations for more realistic results, and solving them by a distributed optimization algorithm, i.e., Augmented Lagrangian based Alternating Direction Inexact Newton method (ALADIN), for preserving data privacy. Simulation results of an illustrative example show great potential of the proposed distributed approach in the aspects of both solution accuracy and computing time. This makes the presented approach generically feasible for real applications in the energy transition.

CCS CONCEPTS
• Mathematics of computing → Optimization; • Hardware → Power networks.

KEYWORDS
Shapley value, optimal power flow, game theory, ALADIN, distributed optimization algorithm

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1 INTRODUCTION
With a rising energy demand, a stronger focus on renewables, and more prosumers entering the market, power grids are working increasingly close to their limits. As a result, congestion management is becoming even more essential for operating electric power systems, and the respective redispatch cost increased dramatically recently.

E.g. Germany’s power grid is composed of 4 transmission system operators (TSOs) and more than 900 distribution system operators (DSOs). A centralized management is not preferred by system operators—or is even forbidden by the respective regulation. Furthermore, more grid operators prefer not to share their grid data. In 2021, the redispatch bill reaches 1.4 billion euros [1], and is expected to multiply in the following years. Therefore, digital platforms, e.g., PICASSO [2], are developed as an industry solution for coordinating redispatch among transmission and distribution system operators. However, under the current cost allocation rule, the TSOs pay the redispatch cost mostly and shift the full charge to consumers, while the DSOs profit from it without cost. Hence, the following unsolved question arises: How can we share the redispatch costs fairly without having to share grid model data?—This is a practical problem to which industry does not have off-the-shelf answers.

The Shapley value, an allocation rule from the field of collaborative game theory, could be such a fair solution, as it is the only solution that fulfills desirable axioms of fairness. Other allocation rules, such as the nucleolus [17] and core or Z-bus matrix [10] do not meet these properties. In fact, the Shapley value has been widely studied for allocating costs in operating power systems.

In the present paper, we focus on redispatch congestion cost allocation: Several works have dealt with congestion management already 20 years ago [8, 9]. In a very recent study [20], the congestion cost allocation problem is formulated as a cooperative game, and the Shapley value is introduced as a unique fair allocation rule to assign costs to all system operators. By exploiting the structure of the congestion cost allocation problem and the respective Shapley algorithm, the authors propose to obtain the Shapley value by solving several DC OPF problems sequentially and to reduce the amount of required computation by further simplification methods. Simulation results show that the proposed algorithm can save almost half of the computing time while preserving high accuracy. However, the algorithm has two main disadvantages w.r.t. real-world implementation. On the one hand, the respective Shapley value is obtained based on suboptimal solutions because solutions of DC OPF are never AC feasible [5]. On the other hand, the DC OPF problems are solved by a centralized approach, by which a centralized entity requires all detailed grid models from system operators.
As discussed above, this is not preferred or is even forbidden by the respective regulation due to data privacy concerns.

W.r.t. real-world implementation, solving AC OPF problems by distributed approach is the intuitive next step. The most well-known distributed algorithms for tackling the AC OPF problems are Optimality Condition Decomposition (ocd) [16], Auxiliary Problem Principle (APP) [6], and Alternating Direction Method of Multipliers (ADMM) [12]. However, AC OPF problems are generally NP-Hard [7], and all of these first-order optimization algorithms have no guaranteed convergence for the nonconvex AC OPF in general. They are hard to converge in practice. In [14], the Augmented Lagrangian based Alternating Direction Inexact Newton method (aladin) is first proposed as a second-order distributed optimization algorithm. In contrast to the mentioned existing approaches, aladin can provide a local convergence guarantee with a quadratic convergence rate for generic distributed nonconvex optimization problems, if suitable Hessian approximations are used. By applying aladin for solving AC OPF problems, exchanging the original grid data is not required, and thus the information privacy can be preserved. Some recent studies focus on AC OPF problems of transmission grids [3] and of AC/DC hybrid grids [22], in which medium-scale power systems are studied. They show high potential of scalability and numerical robustness for aladin in solving AC OPF problems.

Following the main idea from [20], the present paper aims to propose a Shapley value-based distributed AC OPF approach for redispatch congestion cost allocation respecting data privacy to distribute the total cost to system operators fairly. The main contributions of the present paper further develop the approach in [20] as follows:

(1) Instead of the DC approximation in [20], the Shapley value in the present paper is computed based on AC OPF problems, which would provide more realistic results.

(2) By using the aladin algorithm, AC OPF problems within Shapley algorithm are solved by a distributed optimization algorithm. In this way, sharing detailed grid models is not required.

The rest of this paper is structured as follows. In Section 2 describes a distributed approach of Shapley value calculation for the redispatch congestion cost allocation problem. The numerical simulations are presented in Section 3 for validating the proposed approach using an illustrative example. Section 4 concludes this paper and gives an outlook on future work.

2 METHODOLOGY

In this section, a Shapley-based distributed approach is introduced for the redispatch congestion cost allocation problem, in which AC OPF is solved by a distributed optimization algorithm and thus the detailed grid data is preserved.

2.1 Nomenclature

In the present paper, we describe the network of a power system as $\mathcal{W} = (\mathcal{R}, \mathcal{N}, \mathcal{L})$, where $\mathcal{R}$ represents the set of all regions operated by different system operators, $\mathcal{N}$ denotes the set of all buses, $\mathcal{L}$ the set of all branches. For the purpose of simplifying the calculation of the Shapley value, following [20], for all congested lines connecting bus $i$ and bus $j$, are taken as players $\xi := (i, j) \in \mathcal{L}^p$, where $\mathcal{L}^p \subseteq \mathcal{L}$ denotes the set of all congested lines (players).

Besides, we use coalition $\Omega \in \mathcal{P}(\mathcal{L}^p)$ to represent a possible group of congested lines, where $\mathcal{P}(\mathcal{L}^p)$ is the power set of $\mathcal{L}^p$. As a result, the congestion cost allocation problem can be described as a cooperative game $G = (\mathcal{L}^p, \Phi)$, where $\Phi$ is a map that assigns the operation cost $\Phi(\Omega)$ to every possible coalition $\Omega$.

2.2 Shapley Value for Cost Allocation

The goal of congestion cost allocation is to find a fair distribution of the costs of a grand coalition $\Phi(\mathcal{L}^p)$, under which power systems operate in a safe mode and none of the line is overloaded. In the present work, the Shapley value is applied for redispatch congestion cost allocation following [20]. It yields the expected marginal contribution of a player over all collaborations, and is a unique allocation rule that satisfies fairness conditions for cost allocation problems [20].

Remark 1 (Fairness Condition). An allocation rule is called fair, if the following properties are satisfied

- **Null Agent**: for all players who do not invoke costs should have a Shapley value of 0, i.e.,
  $$\Psi_\xi(\Phi) = 0, \quad \forall \xi \in (\mathcal{L}^p) / \Phi(\xi) = 0,$$

- **Additivity**: a combined cost function $\Phi + \Lambda$ should yield the sum of both Shapley values, i.e.,
  $$\Psi(\Phi + \Lambda) = \Psi(\Phi) + \Psi(\Lambda),$$

- **Symmetry**: two players with the same costs yield the same Shapley values, i.e.,
  $$\Phi_\xi(\Phi) = \Phi_\zeta(\Phi), \quad \forall (\xi, \zeta) \in (\mathcal{L}^p)^2 / \Phi(\xi) = \Phi(\zeta).$$

The Shapley value of a specific player $\xi$ can be written as following, cf.[19]

$$\Psi_\xi(\Phi) = \sum_{\Omega \in \mathcal{P}(\mathcal{L}^p) / \xi} |\Omega|!(|\mathcal{L}^p| - |\Omega|)! / |\mathcal{L}^p|! \{\Phi(\Omega \cup \xi) - \Phi(\Omega)\}.$$ 

where $\Phi(\Omega \cup \xi) - \Phi(\Omega)$ can be interpreted as congestion cost with respect to the player $\xi$, and its weight $|\Omega|!(|\mathcal{L}^p| - |\Omega| - 1)! / |\mathcal{L}^p|!$ can be interpreted as the probability of occurrence of the coalition $\Omega$. Consequently, the Shapley value can be viewed as the average marginal costs added to all coalition $\Omega$ by player $\xi$ [20], and satisfied the efficiency criterion.

Remark 2 (Efficiency Criterion). The total cost of all players must be shared precisely among the players, i.e.,

$$\sum_{\xi \in \mathcal{L}^p} \Psi_\xi(\Phi) = \Phi(\mathcal{L}^p).$$

The Shapley algorithm is depicted in Figure 1. It is executed sequentially and can be divided in two parts. Firstly, the operation cost $\Phi$ is determined by the corresponding OPF problem. Based on those operation costs of different coalitions, the Shapley values are computed in the second part. Different from [20], we use AC model for OPF problems for a more realistic and fair allocation, and solve them by using the distributed optimization approach to preserve data privacy.
2.3 Optimization Problems within Shapley

Within the Shapely algorithm, AC OPF problems are solved sequentially. We formulate the AC OPF problems following [13]. The complex voltages are formulated in polar coordinates, i.e., \( V_i = |V_i| e^{j\theta_i} \), where \( V_i \) is voltage magnitude, \( \theta_i \) is voltage angle, \( p_i \) and \( q_i \) denote active and reactive power at the bus \( i \), while \( p_{ij} \) and \( q_{ij} \) denote active and reactive power flow along the line \((i, j) \in L\).

W.r.t. a specific coalition \( \Omega \), only the power limits for all lines \((i, j) \in L = L \setminus \Omega^c \) are considered in the corresponding AC OPF problem, while the limits for all lines \((i, j) \in \Omega^c \) are ignored. Thereby, \( \Omega^c \) denotes the complement of the coalition \( \Omega \) in the set of all congested lines, i.e., \( \Omega^c = L \setminus \Omega \). Hence, the resulting AC OPF problem can be written as:

\[
\min_{x} f(x) = \sum_{k \in \mathbb{N}} \left[ a_k \left(p_k^a\right)^2 + b_k p_k^a + c_k \right] 
\quad \text{s.t.} \quad p_k^q - p_k^a = \sum_{k \in \mathbb{N}} a_k (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik}), \quad \forall (i, j) \in L \text{ (6a)}
\]

\[
q_k^q - q_k^a = \sum_{k \in \mathbb{N}} a_k (G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik}), \quad \forall (i, j) \in L \text{ (6b)}
\]

\[
p_{ij} = -\frac{\nu_{ij}}{2} G_{ij} + i \omega_{ij} B_{ij} \sin \theta_{ij}, \quad \forall (i, j) \in L \text{ (6c)}
\]

\[
q_{ij} = -\frac{\nu_{ij}}{2} G_{ij} + i \omega_{ij} B_{ij} \cos \theta_{ij}, \quad \forall (i, j) \in L \text{ (6d)}
\]

\[
p_{ij}^* + q_{ij}^* \leq \nu_{ij}^*, \quad \forall (i, j) \in L \text{ (6e)}
\]

\[
\omega_{ij} \leq \omega_{ij}^*, \quad p_{ij}^* \leq p_{ij}^* \leq p_{ij}^*, \quad q_{ij}^* \leq q_{ij}^* \leq q_{ij}^*, \quad \forall (i, j) \in L \text{ (6f)}
\]

with \( x = (\theta, u, p^q, q^q) \), where \( \theta_{ij} \) denotes the phase angle difference between buses \( i \) and \( j \), \( p_k^q, q_k^q \) the power of generator at bus \( i, p_i^q \) and \( q_i^q \) denote the load at the bus \( i \), \( G_{ij} \) and \( B_{ij} \) are the real and reactive components of the bus admittance matrix. Thereby, (6a) denotes a quadratic function for total generation cost; (6b)-(6c) denote nodal power balance at each bus; (6d)-(6e) represent line limits with respect to apparent power flow; (6g) denotes the corresponding upper and lower bounds on state variables. A solution to the problem (6) can be written as:

\[
x^* = \arg \min_{x} f(x), \quad \text{s.t.} \quad x \in \mathcal{X}(\Omega), \quad (7)
\]

where \( \mathcal{X}(\Omega) \) denotes the feasible set of \( x \) determined by the constraints (6b)-(6g) w.r.t. the coalition \( \Omega \). Thereby, the operation cost \( \Phi \) can be determined by optimal objective value of the problem (6):

\[
\Phi(\Omega) = f(x^*). \quad (8)
\]

2.4 Distributed Approach

In order to prevent exchanging detailed grid data, AC OPF problems in Shapely algorithm are solved by using a distributed approach in the present paper, i.e., the Augmented Lagrangian based Alternating Direction Inexact Newton method (ALADIN).

2.4.1 Distributed Problem Formulation. Before further description of the distributed optimization algorithm, the distributed formulation of OPF needs to be discussed first. Instead of cutting the tie-lines [3], we reformulate the OPF by sharing components to ensure physical consistence. As a result, OPF problems can be written in the affinely coupled distributed form:

\[
\min_{x} f(x) := \sum_{\ell \in \mathcal{R}} f_\ell(x_\ell) \quad (9a)
\]

\[
s.t. \quad \sum_{\ell \in \mathcal{R}} A_\ell x_\ell = b \quad | \lambda \quad (9b)
\]

\[
h_\ell(x_\ell) \leq 0 \quad | x_\ell, \ell \in \mathcal{R} \quad (9c)
\]

where (9a) is the separable objective, (9b) includes all the consensus constraints introduced by sharing components, (9c) summarizes all local constraints for local systems, and \( \lambda, \ell \) denote the dual variables (Lagrangian multipliers) of constraints (9b) and (9c), respectively. We refer to [11, 18] for more detailed information about the problem reformulation.

Algorithm 1 ALADIN

Input: \( z, \lambda, \rho > 0, \mu > 0 \) and scaling symmetric matrices \( \Sigma_x > 0 \)

Repeat:

leftm1rg1n=10pt solve the following decoupled nlp\('s for all \( \ell \in \mathcal{R} \):

\[
\min_{x_\ell} f_\ell(x_\ell) + \lambda^T A_\ell x_\ell + \frac{\rho}{2} \|x_\ell - z_\ell\|^2_{\Sigma_x} \quad (10a)
\]

s.t. \( h_\ell(x_\ell) \leq 0 \quad | \kappa_\ell \quad (10b) \)

leftm2rg2n=20pt compute the gradient \( q_\ell \), the Jacobian matrix \( f_\ell \) of active constraints \( h_\ell \) and the approximated Hessian \( H_\ell \) at the local solution \( x_\ell \) by:

\[
g_\ell = \nabla f_\ell(x_\ell), \quad J_\ell = \nabla h_\ell^T(x_\ell), \quad H_\ell = \nabla^2 \left[ f_\ell(x_\ell) + \kappa_\ell h_\ell(x_\ell) \right] > 0 \quad (11)
\]

leftm3rg3n=30pt terminate if \( \|x_\ell - b_\ell\|_2 \leq \varepsilon \) and \( \|z(\ell) - z_\ell\|_2 \leq \varepsilon \) are satisfied.

leftm4rg4n=40pt obtain \((p^q, \lambda^q)\) by solving coupled \( q^p \):

\[
\min_{p^{\mathcal{R}^2}, \lambda^p} \sum_{\ell \in \mathcal{R}} \left[ \frac{1}{2} \left(p^{\ell q}_\ell \right)^T H_\ell p^{\ell q}_\ell + 2 \nu_\ell \nu_\ell + \lambda^\ell \right] + \frac{\rho}{2} \|x_\ell - z_\ell\|^2_{\Sigma_x} \quad (12a)
\]

s.t. \( \sum_{\ell \in \mathcal{R}} A_\ell (x_\ell + p^{\ell q}_\ell) = b + s \quad | \lambda^p \quad (12b) \)

\[
J_\ell p^{\ell q}_\ell = 0, \quad \ell \in \mathcal{R} \quad (12c)
\]

leftm5rg5n=50pt update the primal and the dual variables with full step:

\[
z^{\ell} = x + p^q \quad \text{and} \quad \lambda^{\ell} = \lambda^q \quad (13)
\]

2.4.2 Distributed Optimization Algorithm. ALADIN was first introduced in [14] to handle distributed nonlinear programming. ALADIN for AC OPF problems is outlined in Algorithm 1. The algorithm has two main steps, i.e., a decoupled nlp step 1 and a coupled q step 4. Following the idea of augmented Lagrangian, the local problem is formulated as (10) in step 1, where \( \rho \) is the penalty parameter and \( \Sigma_x \) is the positive-definite weighted matrix for the region \( \ell \). Based on the result from local nlp\('s, the ALADIN algorithm terminates...
The costs allocated to the congested lines by the Shapley value are 70 MW each. They are overloaded by 382% and 123%, corresponding on busses 5, 7 and 9. Minus losses, all three sum up to the load of 315 MW, distributed based on their average contribution to any combination of lines.

3.2.1 Shapley values.

The Shapley value is heavily dependent on the solution accuracy of the corresponding AC OPF problem, cf. (4). In order to validate the distributed approach proposed in the present paper, i.e., ALADIN, we introduce two quantities:

- deviation of optimization variables: \( \|x - x^*\|_2 \)
- solution gap: \( \frac{f(x^*) - f(x)}{f(x^*)} \)

where \( x^* \) and \( f(x^*) \) are provided by the centralized MATPOWER default solver.

As shown in Table 1, solving AC OPF problems by using ALADIN can obtain a very high accuracy solution in terms of the optimal value. The deviation of the resulting Shapley value is less than \( 1 \times 10^{-6} \), compared with a centralized reference value.

Table 1: Comparisons

<table>
<thead>
<tr>
<th>Coalition ( \Omega )</th>
<th>Iter</th>
<th>Time [s]</th>
<th>( |x - x^*|_2 )</th>
<th>Solution Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>{}</td>
<td>7</td>
<td>0.129</td>
<td>4.54 \times 10^{-8}</td>
<td>2.19 \times 10^{-6}</td>
</tr>
<tr>
<td>{(1, 4)}</td>
<td>7</td>
<td>0.128</td>
<td>7.82 \times 10^{-8}</td>
<td>4.69 \times 10^{-7}</td>
</tr>
<tr>
<td>{(3, 6)}</td>
<td>7</td>
<td>0.131</td>
<td>9.90 \times 10^{-8}</td>
<td>6.04 \times 10^{-8}</td>
</tr>
<tr>
<td>{(1, 4), (3, 6)}</td>
<td>7</td>
<td>0.129</td>
<td>1.135 \times 10^{-7}</td>
<td>9.54 \times 10^{-8}</td>
</tr>
</tbody>
</table>

Figure 3 displays the convergence behavior of ALADIN for AC OPF problems w.r.t. the grand coalition \( \Omega = L^P = \{(1, 4), (3, 6)\} \), i.e., all congested lines are taken into consideration. It can be concluded that ALADIN can converge to a high accuracy solution rapidly for the optimization problems in Shapley value, in terms of the deviation of state variables, the solution gap, primal residual \( \|Ax - b\|_2 \) and dual residual \( \|x - z\|_2 \).

4 CONCLUSIONS AND FUTURE WORK

In the present paper, a distributed approach to the calculation of the Shapley value for redispach congestion cost allocation is introduced. Different from the recently introduced method in [20], the...
proposed method uses AC models for OPF problems for a more realistic and fairer allocation. Besides, by using the ALADIN algorithm for solving the AC OPF, data privacy can be preserved while the high-quality solution can be maintained. Simulation results of an illustrative example show great potential of the proposed distributed approach in the aspects of both solution accuracy and computing time. This makes the presented approach generically feasible for real applications in the energy transition. Future directions of the work include introducing approximation methods of both the Shapley value, the OPF and the grid. Another important direction is scaling up the method to larger systems that also differentiate between transmission and distribution systems.

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