A Shapley value-based Distributed AC OPF Approach for Redispatch Congestion Cost Allocation

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ABSTRACT

The progressing energy transition induces a growing need for redispatch congestion management, and, thereby, a fair distribution of its respective costs among the different system operators. In this light, a very recent paper uses the Shapley value as such a fair allocation rule to assign redispatch congestion costs to system operators. However, this approach is based on DC optimal power flow (OPF) and requires the sharing of detailed grid models from all system operators. This is not preferred by them due to data privacy concerns. W.r.t. real-world implementation, the present paper extends the method by using AC OPF problem formulations for more realistic results, and solving them by using a distributed optimization algorithm, i.e., Augmented Lagrangian based Alternating Direction Inexact Newton method (ALADIN), for preserving data privacy. Simulation results of an illustrative example show great potential of the proposed distributed approach in the aspects of both solution accuracy and computing time. This makes the presented approach generically feasible for real applications in the energy transition.

CCS CONCEPTS

• **Mathematics of computing** → *Optimization*; • **Hardware** → **Power networks**.

KEYWORDS

Shapley value, optimal power flow, game theory, ALADIN, distributed optimization algorithm

ACM Reference Format:
Rebecca Bauer, Xinliang Dai, and Veit Hagenmeyer. 2023. A Shapley value-based Distributed AC OPF Approach for Redispatch Congestion Cost Allocation. In *The 14th ACM International Conference on Future Energy Systems (e-Energy '23), June 20–23, 2023, Orlando, FL, USA*. ACM, New York, NY, USA, 5 pages. https://doi.org/10.1145/3575813.3576881

e-Energy '23, June 20–23, 2023, Orlando, FL, USA

ACM ISBN 979-8-4007-0032-3/23/06…$15.00
https://doi.org/10.1145/3575813.3576881

1 INTRODUCTION

With a rising energy demand, a stronger focus on renewables, and more prosumers entering the market, power grids are working increasingly close to their limits. As a result, congestion management is becoming even more essential for operating electric power systems, and the respective redispatch cost increased dramatically recently.

E.g. Germanys power grid is composed of 4 transmission system operators (TSOs) and more than 900 distribution system operators (DSOs). A centralized management is not preferred by system operators—or is even forbidden by the respective regulation. Furthermore, more grid operators prefer not to share their grid data. In 2021, the redispatch bill reaches 1.4 billion euros [1], and is expected to multiply in the following years. Therefore, digital platforms, e.g. PICASSO [2], are developed as an industry solution for coordinating redispatch among transmission and distribution system operators. However, under the current cost allocation rule, the TSOs pay the redispatch cost mostly and shift the full charge to consumers, while the DSOs profit from it without cost. Hence, the following unsolved question arises: *How can we share the redispatch costs fairly without having to share grid model data?*—This is a practical problem to which industry does not have off-the-shelf answers.

The Shapley value, an allocation rule from the field of collaborative game theory, could be such a fair solution, as it is the only solution that fulfills desirable axioms of fairness. Other allocation rules, such as the nucleolus [17] and core or Z-bus matrix [10] do not meet these properties. In fact, the Shapley value has been widely studied for allocating costs in operating power systems.

In the present paper, we focus on redispatch congestion cost allocation: Several works have dealt with congestion management already 20 years ago [8, 9]. In a very recent study [20], the congestion cost allocation problem is formulated as a cooperative game, and the Shapley value is introduced as a unique fair allocation rule to assign costs to all system operators. By exploiting the structure of the congestion cost allocation problem and the respective Shapley algorithm, the authors propose to obtain the Shapley value by solving several DC OPF problems sequentially and to reduce the amount of required computation by further simplification methods. Simulation results show that the proposed algorithm can save almost half of the computing time while preserving high accuracy. However, the algorithm has two main disadvantages w.r.t. real-world implementation. On the one hand, the respective Shapley value is obtained based on suboptimal solutions because solutions of DC OPF are never AC feasible [5]. On the other hand, the DC OPF problems are solved by a centralized approach, by which a centralized entity requires all detailed grid models from system operators.

As discussed above, this is not preferred or is even forbidden by the respective regulation due to data privacy concerns.

W.r.t. real-world implementation, solving AC OPF problems by distributed approach is the intuitive next step. The most well-known distributed algorithms for tackling the AC OPF problems are Optimality Condition Decomposition (OCD) [16], Auxiliary Problem Principle (APP) [6], and Alternating Direction Method of Multipliers (ADMM) [12]. However, AC OPF problems are generally NP-Hard [7], and all of these first-order optimization algorithms have no guaranteed convergence for the nonconvex AC OPF in general. They are hard to converge in practice. In [14], the Augmented Lagrangian based Alternating Direction Inexact Newton method (ALADIN) is first proposed as a second-order distributed optimization algorithm. In contrast to the mentioned existing approaches, ALADIN can provide a local convergence guarantee with a quadratic convergence rate for generic distributed nonconvex optimization problems, if suitable Hessian approximations are used. By applying ALADIN for solving AC OPF problems, exchanging the original grid data is not required, and thus the information privacy can be preserved. Some recent studies focus on AC OPF problems of transmission grids [3] and of AC/DC hybrid grids [22], in which medium-scale power systems are studied. They show high potential of scalability and numerical robustness for ALADIN in solving AC OPF problems.

Following the main idea from [20], the present paper aims to propose a Shapley value-based distributed AC OPF approach for redispatch congestion cost allocation respecting data privacy to distribute the total cost to system operators fairly. The main contributions of the present paper further develop the approach in [20] as follows:

(1) Instead of the DC approximation in [20], the Shapley value in the present paper is computed based on AC OPF problems, which would provide more realistic results.
(2) By using the ALADIN algorithm, AC OPF problems within Shapley algorithm are solved by a distributed optimization algorithm. In this way, sharing detailed grid models is not required.

The rest of this paper is structured as follows. In Section 2 describes a distributed approach of Shapley value calculation for the redispatch congestion cost allocation problem. The numerical simulations are presented in Section 3 for validating the proposed approach using an illustrative example. Section 4 concludes this paper and gives an outlook on future work.

2 METHODOLOGY

In this section, a Shapley-based distributed approach is introduced for the redispatch congestion cost allocation problem, in which AC OPF is solved by a distributed optimization algorithm and thus the detailed grid data is preserved.

2.1 Nomenclature

In the present paper, we describe the network of a power system as $\mathcal{W} = (\mathcal{R}, \mathcal{N}, \mathcal{L})$, where $\mathcal{R}$ represents the set of all regions operated by different system operators, $\mathcal{N}$ denotes the set of all buses, $\mathcal{L}$ the set of all branches. For the purpose of simplifying the calculation of the Shapely value, following [20], for all congested lines connecting bus i and bus j, are taken as players $\xi := (i, j) \in \mathcal{L}^p$, where $\mathcal{L}^p \subseteq \mathcal{L}$ denotes the set of all congested lines (players). Besides, we use coalition $\Omega \in \mathbb{P}(\mathcal{L}^p)$ to represent a possible group of congested lines, where $\mathbb{P}(\mathcal{L}^p)$ is the power set of $\mathcal{L}^p$. As a result, the congestion cost allocation problem can be described as a cooperative game $\mathcal{G} = (\mathcal{L}^p, \Phi)$, where Φ is a map that assigns the operation cost $\Phi(\Omega)$ to every possible coalition Ω.

2.2 Shapley Value for Cost Allocation

The goal of congestion cost allocation is to find a fair distribution of the costs of a grand coalition $\Phi(\mathcal{L}^p)$, under which power systems operate in a safe mode and none of the line is overloaded. In the present work, the Shapley value is applied for redispatch congestion cost allocation following [20]. It yields the expected marginal contribution of a player over all collaborations, and is a unique allocation rule that satisfies fairness conditions for cost allocation problems [20].

REMARK 1 (FAIRNESS CONDITION). *An allocation rule is called fair, if the following properties are satisfied*

– *NULL AGENT: for all players who do not invoke costs should have a Shapley value of 0, i.e.,*

$$\Psi_\xi(\Phi) = 0, \quad \forall \xi \in \{\xi \in \mathcal{L}^p \mid \Phi(\xi) = 0\}, \tag{1}$$

– *ADDITIVITY: a combined cost function $\Phi + \Lambda$ should yield the sum of both Shapley values, i.e.,*

$$\Psi(\Phi + \Lambda) = \Psi(\Phi) + \Psi(\Lambda), \tag{2}$$

– *SYMMETRY: two players with the same costs yield the same Shapley values, i.e.,*

$$\Phi_\xi(\Phi) = \Phi_\zeta(\Phi), \quad \forall (\xi, \zeta) \in \{(\xi, \zeta) \mid \Phi(\xi) = \Phi(\zeta)\}. \tag{3}$$

The Shapley value of a specific player ξ can be written as following, cf.[19]

$$\Psi_\xi(\Phi) = \sum_{\Omega \in \mathcal{L}^p \setminus \{\xi\}} \frac{|\Omega|! \, (|\mathcal{L}^p| - |\Omega| - 1)!}{|\mathcal{L}^p|!} \{\Phi(\Omega \cup \xi) - \Phi(\Omega)\}, \tag{4}$$

where $\Phi(\Omega \cup \xi) - \Phi(\Omega)$ can be interpreted as congestion cost with respect to the player ξ, and its weight $\frac{|\Omega|! \, (|\mathcal{L}^p| - |\Omega| - 1)!}{|\mathcal{L}^p|!}$ can be interpreted as the probability of occurrence of the coalition Ω. Consequently, the Shapley value can be viewed as the average marginal costs added to all coalition Ω by player ξ [20], and satisfied the efficiency criterion.

REMARK 2 (EFFICIENCY CRITERION). *The total cost of all players must be shared precisely among the players, i.e.,*

$$\sum_{\xi \in \mathcal{L}^p} \Psi_\xi(\Phi) = \Phi(\mathcal{L}^p). \tag{5}$$

The Shapley algorithm is depicted in Figure 1. It is executed sequentially and can be divided in two parts. Firstly, the operation cost Φ is determined by the corresponding OPF problem. Based on those operation costs of different coalitions, the Shapley values are computed in the second part. Different from [20], we use AC model for OPF problems for a more realistic and fair allocation, and solve them by using the distributed optimization approach to preserve data privacy.

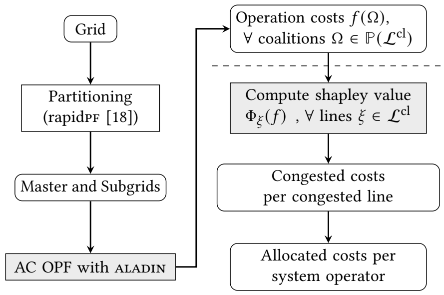

Figure 1: Shapley algorithm with ALADIN for distributed optimal power flow calculations.

2.3 Optimization Problems within Shapley

Within the Shapely algorithm, AC OPF problems are solved sequentially. We formulate the AC OPF problems following [13].The complex voltages are formulated in polar coordinates, i.e., $V_i = v_i e^{\mathrm{j}\theta_i}$, where v_i is voltage magnitude, θ_i is voltage angle, p_i and q_i denote active and reactive power at the bus i, while p_{ij} and q_{ij} denote active and reactive power flow along the line $(i, j) \in \mathcal{L}$.

W.r.t. a specific coalition Ω, only the power limits for all lines $(i, j) \in \tilde{\mathcal{L}} = \mathcal{L} \setminus \Omega^c$ are considered in the corresponding AC OPF problem, while the limits for all lines $(i, j) \in \Omega^c$ are ignored. Thereby, Ω^c denotes the complement of the coalition Ω in the set of all congested lines, i.e., $\Omega^c = \mathcal{L}^p \setminus \Omega$. Hence, the resulting AC OPF problem can be written as

$$\min_x f(x) = \sum_{i \in \mathcal{N}} \left\{ a_i \left(p_i^g\right)^2 + b_i p_i^g + c_i \right\} \tag{6a}$$

$$\text{s.t. } p_i^g - p_i^l = v_i \sum_{k \in \mathcal{N}} v_k \left(G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik}\right), \quad \forall i \in \mathcal{N} \tag{6b}$$

$$q_i^g - q_i^l = v_i \sum_{k \in \mathcal{N}} v_k \left(G_{ik} \sin\theta_{ik} - B_{ij} \cos\theta_{ik}\right), \quad \forall i \in \mathcal{N} \tag{6c}$$

$$p_{ij} = \ \ v_i^2 G_{ij} - v_i v_j \left(G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}\right), \quad \forall (i,j) \in \tilde{\mathcal{L}} \tag{6d}$$

$$q_{ij} = -v_i^2 G_{ij} - v_i v_j \left(G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}\right), \quad \forall (i,j) \in \tilde{\mathcal{L}} \tag{6e}$$

$$p_{ij}^2 + q_{ij}^2 \le \bar{s}_{ij}^2, \quad \forall (i,j) \in \tilde{\mathcal{L}} \tag{6f}$$

$$\underline{u}_i \le u_i \le \bar{u}_i, \ \ \underline{p}_i^g \le p_i^g \le \bar{q}_i^g, \ \ \underline{q}_i^g \le q_i^g \le \bar{q}_i^g, \quad \forall i \in \mathcal{N} \tag{6g}$$

with $x = (\theta, v, p^g, q^g)$, where θ_{ij} denotes the phase angle difference between buses i and j, p_i^g q_i^g the power of generator at bus i, p_i^l and q_i^l denote the load at the bus i, G_{ij} and B_{ij} are the real and reactive components of the bus admittance matrix. Thereby, (6a) denotes a quadratic function for total generation cost; (6b)-(6c) denotes nodal power balance at each bus; (6d)-(6c) represent line limits with respect to apparent power flow; (6g) denotes the corresponding upper and lower bounds on state variables. A solution to the problem (6) can be written as

$$x^* = \arg\min_x f(x), \quad \text{s.t.} \quad x \in \mathcal{X}(\Omega), \tag{7}$$

where $\mathcal{X}(\Omega)$ denotes the feasible set of x determined by the constraints (6b)-(6g) w.r.t. the coalition Ω. Thereby, the operation cost Φ can be determined by optimal objective value of the problem (6)

$$\Phi(\Omega) := f(x^*). \tag{8}$$

2.4 Distributed Approach

In order to prevent exchanging detailed grid data, AC OPF problems in Shapely algorithm are solved by using a distributed approach in the present paper, i.e., the Augmented Lagrangian based Alternating Direction Inexact Newton method (ALADIN).

2.4.1 Distributed Problem Formulation. Before further description of the distributed optimization algorithm, the distributed formulation of OPF needs to be discussed first. Instead of cutting the tie-lines [3], we reformulate the OPF by sharing components to ensure physical consistence. As a result, OPF problems can be written in the affinely coupled distributed form

$$\min_x \quad f(x) := \sum_{\ell \in \mathcal{R}} f_\ell(x_\ell) \tag{9a}$$

$$\text{s.t.} \quad \sum_{\ell \in \mathcal{R}} A_\ell x_\ell = b \qquad | \ \lambda \tag{9b}$$

$$h_\ell(x_\ell) \le 0 \qquad | \ \kappa_\ell, \ \ell \in \mathcal{R} \tag{9c}$$

where (9a) is the separable objective, (9b) includes all the consensus constraints introduced by sharing components, (9c) summarizes all local constraints for local systems, and λ, κ_ℓ denote the dual variables (Lagrangian multipliers) of constraints (9b) and (9c), respectively. We refer to [11, 18] for more detailed information about the problem reformulation.

Algorithm 1 ALADIN

Input: z, λ, $\rho > 0$, $\mu > 0$ and scaling symmetric matrices $\Sigma_\ell > 0$
Repeat:

leftm1rg1n=10pt solve the following decoupled NLPs for all $\ell \in \mathcal{R}$

$$\min_{x_\ell} \quad f_\ell(x_\ell) + \lambda^\top A_\ell x_\ell + \frac{\rho}{2} \|x_\ell - z_\ell\|^2_{\Sigma_\ell} \tag{10a}$$

$$\text{s.t.} \quad h_\ell(x_\ell) \le 0 \qquad | \ \kappa_\ell \tag{10b}$$

leftm2rg2n=20pt compute the gradient g_ℓ, the Jacobian matrix J_ℓ of active constraints h_ℓ^{act} and the approximated Hessian H_ℓ at the local solution x_ℓ by

$$g_\ell = \nabla f_\ell(x_\ell), \ J_\ell = \nabla h_\ell^{\text{act}}(x_\ell), \ H_\ell \approx \nabla^2 \left\{ f_\ell(x_\ell) + \kappa_\ell^\top h_\ell(x_\ell) \right\} > 0 \tag{11}$$

leftm3rg3n=30pt terminate if $\|Ax - b\|_2 \le \epsilon$ and $\|\Sigma(x - z)\|_2 \le \epsilon$ are satisfied.

leftm4rg4n=40pt obtain $(p^{\text{QP}}, \lambda^{\text{QP}})$ by solving coupled QP

$$\min_{p^{\text{QP}}, s} \quad \sum_{\ell \in \mathcal{R}} \left\{ \frac{1}{2} \left(p_\ell^{\text{QP}}\right)^\top H_\ell \, p_\ell^{\text{QP}} + g_\ell^\top p_\ell^{\text{QP}} \right\} + \lambda^\top s + \frac{\mu}{2} \|s\|_2^2 \tag{12a}$$

$$\text{s.t.} \quad \sum_{\ell \in \mathcal{R}} A_\ell (x_\ell + p_\ell^{\text{QP}}) = b + s \quad | \ \lambda^{\text{QP}} \tag{12b}$$

$$J_\ell \, p_\ell^{\text{QP}} = 0, \quad \ell \in \mathcal{R} \tag{12c}$$

leftm5rg5n=50pt update the primal and the dual variables with full step

$$z^+ = x + p^{\text{QP}} \quad \text{and} \quad \lambda^+ = \lambda^{\text{QP}} \tag{13}$$

2.4.2 Distributed Optimization Algorithm. ALADIN was first introduced in [14] to handle distributed nonlinear programming. ALADIN for AC OPF problems is outlined in Algorithm 1. The algorithm has two main steps, i.e., a decoupled NLP step 1 and a coupled QP step 4. Following the idea of augmented Lagrangian, the local problem is formulated as (10) in step 1, where ρ is the penalty parameter and Σ_ℓ is the positive-definite weighted matrix for the region ℓ. Based on the result from local NLPs (10), the ALADIN algorithm terminates

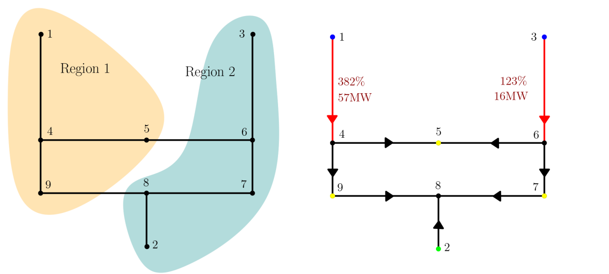

Figure 2: Regions of system operators and simulation results of the IEEE 9-bus grid. Red = congested lines, blue/green = generator ramping up/down, yellow = load.

if both the primal and the dual residuals are smaller than tolerance ϵ.

Based on curvature information (11), ALADIN builds a coupled quadratic program (QP) (12) to coordinate the results of the decoupled step from all regions. Additionally, a slack variable s and a corresponding penalty parameter μ is introduced in the consensus step to ensure feasibility of the coupled QP. Based on the second-order information, ALADIN achieves a locally quadratic convergence rate. A detailed proof of local convergence can be found in [3] [15].

3 ILLUSTRATIVE EXAMPLE

3.1 Setting

The standard IEEE 9-bus system is used as an illustrative example, cf. Figure 2. Using the open-source toolbox rapidPF [18][1], the grid is partitioned into two regions, $\mathcal{R}_1 = \{1, 4, 5, 9\}$ and $\mathcal{R}_2 = \{2, 3, 6, 7, 8\}$, representing different transmission grids. The framework is built on MATLAB-R2021a, the case studies are carried out on a standard desktop computer with `Intel® i5-6600K CPU @ 3.50GHz` and 32.0 GB installed RAM. The casADi toolbox [4] is used in MATLAB, and IPOPT [21] is used as nonlinear solver. The centralized reference solution is obtained by using the default solver in MATPOWER.

3.2 Shapley value-based Cost Allocation

The costs allocated to the congested lines by the Shapley value are based on their average contribution to any combination of lines being congested at the same time. They are usually not proportional to the overloads, but redistribute the costs depending on the lines overall influence.

3.2.1 Shapley values. The Shapley values of the 9-bus example represent the congestion management costs of congested lines.

Figure 2 shows the two regions of system operators and the simulation results. Lines $(1, 4)$ and $(3, 6)$ have a capacity of 20MW and 70MW each. They are overloaded by 382% and 123%, corresponding to 57MW and 16MW. The two generators at busses 1 and 3 produce 72MW and 85MW each, the generator at bus 2 produces 163MW. Minus losses, all three sum up to the load of 315MW, distributed on busses 5, 7 and 9.

[1]The code is available on https://github.com/xinliang-dai/rapidPF

The Shapley values of the overloaded lines are 424 and 135. This is roughly proportional to the overloads of the lines, although the system operator of region $\mathcal{R}_1$ pays slightly more in comparison.

3.2.2 Distributed vs. Centralized. The Shapley value is heavily dependent on the solution accuracy of the corresponding AC OPF problem, cf. (4). In order to validate the distributed approach proposed in the present paper, i.e., ALADIN, we introduce two quantities:

- deviation of optimization variables: $\|x - x^*\|_2$,
- solution gap: $|\frac{f(x^*)-f(x)}{f(x^*)}|$,

where x^* and $f(x^*)$ are provided by the centralized MATPOWER default solver.

As shown in Table 1, solving AC OPF problems by using ALADIN can obtain a very high accuracy solution in terms of the optimal objective value. The deviation of the resulting Shapley value is less than 1×10^{-6}, compared with a centralized reference value.

Table 1: Comparisons

Coalition Ω	Iter	Time [s]	$\|x - x^*\|_2$	Solution Gap
$\{\emptyset\}$	7	0.129	4.54×10^{-4}	2.19×10^{-6}
$\{(1,4)\}$	7	0.128	7.82×10^{-6}	4.60×10^{-7}
$\{(3,6)\}$	7	0.131	9.90×10^{-4}	6.04×10^{-8}
$\{(1,4),\ (3,6)\}$	7	0.129	1.135×10^{-6}	9.54×10^{-8}

Figure 3 displays the convergence behavior of ALADIN for AC OPF problems w.r.t. the grand coalition $\Omega = \mathcal{L}^p = \{(1, 4), (3, 6)\}$, i.e., all congested lines are taken into consideration. It can be concluded that ALADIN can converge to a high accuracy solution rapidly for the optimization problems in Shapley value, in terms of the deviation of state variables, the solution gap, primal residual $\|Ax - b\|_2$ and dual residual $\|x - z\|_2$.

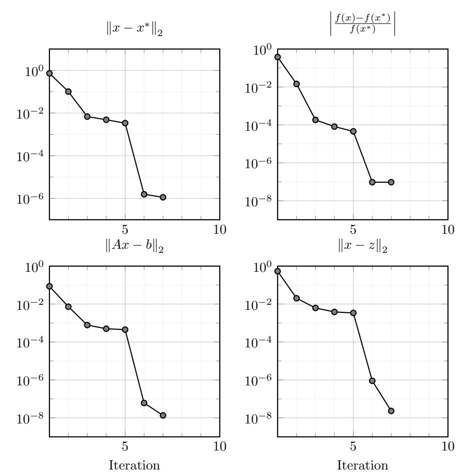

Figure 3: Convergence behaviours of ALADIN for the grand coalition $\Omega = \mathcal{L}^p$

4 CONCLUSIONS AND FUTURE WORK

In the present paper, a distributed approach to the calculation of the Shapley value for redispatch congestion cost allocation is introduced. Different from the recently introduced method in [20], the proposed method uses AC models for OPF problems for a more realistic and fairer allocation. Besides, by using the ALADIN algorithm for solving the AC OPF, data privacy can be preserved while the high-quality solution can be maintained. Simulation results of an illustrative example show great potential of the proposed distributed approach in the aspects of both solution accuracy and computing time. This makes the presented approach generically feasible for real applications in the energy transition. Future directions of the work include introducing approximation methods of both the Shapley value, the OPF and the grid. Another important direction is scaling up the method to larger systems that also differentiate between transmission and distribution systems.

ACKNOWLEDGMENTS

Rebecca Bauer and Xinliang Dai acknowledge funding by the BMBF-project ENSURE II with grant number 03SFK1F0-2.

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