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High Energy Physics – Phenomenology

Charged Higgs-boson decays into quarks

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Abstract

We consider the full genuine next-to-leading order SUSY–QCD corrections to the charged Higgs decays into quarks supplemented by the NNLO corrections to the effective top and bottom Yukawa couplings. The NNLO corrections to the effective top Yukawa coupling are a new ingredient of our analysis. We arrive at an approximate NNLO prediction for MSSM charged Higgs decays after including the N^4LO QCD corrections for large charged Higgs masses. The residual uncertainties are in the percent range or below, depending on the particular MSSM scenario.

1. Introduction

The Standard Model (SM) of particle physics has been very successful in describing scattering and decay processes from the high- to the low-energy frontiers [1]. A cornerstone of this model is the Higgs mechanism for electroweak symmetry breaking [2]. The discovery of the Higgs boson in 2012 [3,4] has confirmed this realization of spontaneous symmetry breaking and completed the required particle content of the SM [5]. Although the SM describes all processes at the high-energy frontier and shows the proper behavior of processes in the high-energy limit, it cannot

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describe all observations. In particular, the evidence for Dark Matter and the inability of the SM to generate the baryon asymmetry of the universe call for physics beyond the SM.

The Higgs sector allows the electroweak force to remain weakly interacting up to very high-energy scales [6,7]. One of the major lines of research beyond the SM is the formulation of grand unified theories (GUTs) which are broken down to the low-energy SM at an energy scale of the order of 10^{16} GeV and can be motivated by the approximate unification of the gauge couplings of the SM. This requires the SM to be weakly interacting up to these high-energy scales, which is only possible if the Higgs mass takes a value in the range between about 130 and 190 GeV [8]. This is compatible with the observed Higgs mass of (125.09 ± 0.24) GeV, if the universe is allowed to be metastable [9].

However, even if the Higgs mass is in this range, a further problem arises. If the SM is embedded in a GUT, quadratic divergences in higher-order corrections to the Higgs mass suggest that, if the Higgs boson couples directly or indirectly to particles with masses at the GUT scale, quantum fluctuations tend to raise the Higgs mass to this new mass scale [10]. To stabilize the Higgs mass at the electroweak scale, an extreme fine-tuning of the corresponding mass counterterm is necessary. This hierarchy problem can be avoided by the introduction of supersymmetry [11], a novel symmetry between the bosonic and fermionic degrees of freedom of the model. Supersymmetric models are free of quadratic divergences and solve the hierarchy problem if the superpartners of all SM particles acquire masses below about $1-10$ TeV. Supersymmetric GUTs predict a value of the Weinberg angle that is in striking agreement with the precision measurements at LEP and SLC [12]. The minimal supersymmetric extension of the SM (MSSM) [13,14] requires the existence of 5 elementary Higgs bosons. These consist of two neutral CP-even states h, H, one neutral CP-odd A state, and two charged Higgs states $H^{\pm}$. For real MSSM parameters, the Higgs sector can be described by two input parameters at leading order (LO): the pseudoscalar mass M_A and $\mathrm{tg}\beta$. The latter is the ratio of the two vacuum expectation values involved in the neutral components of the two Higgs doublets.

An immediate sign of an extended Higgs sector beyond the SM would be the discovery of a charged Higgs boson. The main search channel for heavy charged Higgs bosons is its decay into a $t\bar{b}$ final state. This work addresses charged Higgs-boson decays into heavy quarks. The decay $H^+ \to t\bar{b}$ provides the dominant charged Higgs decay channel for a large range of the charged Higgs mass, depending on the MSSM scenario. In our numerical analysis, we have adopted the M_h^{125} benchmark scenario of Ref. [15] as a representative case of the MSSM. This scenario is not excluded by the experimental searches. The input parameters defined in the on-shell scheme of Ref. [15] are

$$\begin{aligned} M_h^{125}\colon\ & M_{\tilde{Q}_3} = 1.5 \text{ TeV}, \quad M_{\tilde{\ell}_3} = 2 \text{ TeV}, \quad M_{\tilde{g}} = 2.5 \text{ TeV}, \\ & M_1 = M_2 = 1 \text{ TeV}, \quad A_b = A_\tau = A_t = 2.8 \text{ TeV} + \mu/\mathrm{tg}\beta, \quad \mu = 1 \text{ TeV}, \end{aligned} \tag{1}$$

where $M_{\tilde{Q}_3}$ ($M_{\tilde{\ell}_3}$) denotes the (common) left- and right-handed soft SUSY-breaking mass parameter of the third generation squarks (sleptons), $M_{\tilde{g}}$ is the gluino mass, M_1, M_2 are the soft SUSY-breaking gaugino mass parameters, μ is the higgsino mass parameter and A_b, A_t, A_τ are the soft SUSY-breaking trilinear coupling parameters of the third generation. We have determined all squark masses according to the procedure described in Ref. [16]. This leads to the following values of the stop and sbottom masses for two values of $\mathrm{tg}\beta = 10, 40$:

$\underline{\mathrm{tg}\beta = 10}$

$$m_{\tilde{t}_1} = 1340 \text{ GeV}, \quad m_{\tilde{t}_2} = 1662 \text{ GeV}, \quad m_{\tilde{b}_1} = 1496 \text{ GeV}, \quad m_{\tilde{b}_2} = 1508 \text{ GeV}$$

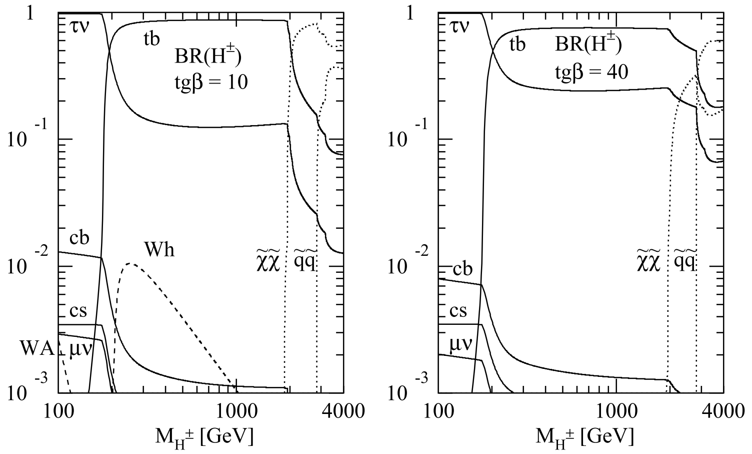


Fig. 1. Branching ratios of the charged Higgs boson within the M_h^{125} scenario as a function of the charged Higgs mass for two values of $\mathrm{tg}\beta = 10, 40$. The results shown here include the results of this paper. The branching ratios into charginos and neutralinos ($\tilde{\chi}\tilde{\chi}$) involve the sum over all gaugino final states, whereas those into squarks ($\tilde{q}\tilde{q}$) incorporate the sum over all squark final states. This figure has been obtained with `Hdecay` [19].

$$\underline{\mathrm{tg}\beta = 40}$$

$$m_{\tilde{t}_1} = 1340 \text{ GeV}, \quad m_{\tilde{t}_2} = 1662 \text{ GeV}, \quad m_{\tilde{b}_1} = 1479 \text{ GeV}, \quad m_{\tilde{b}_2} = 1525 \text{ GeV}. \tag{2}$$

We have used the renormalization-group-improved two-loop corrected charged Higgs mass and couplings of Ref. [17] in our analysis. The top pole mass has been chosen as $m_t = 172.5$ GeV, the bottom $\overline{\mathrm{MS}}$ mass as $\overline{m}_b(\overline{m}_b) = 4.18$ GeV and the strong coupling as $\alpha_s(M_Z) = 0.118$ in the $\overline{\mathrm{MS}}$ scheme. However, for the bottom mass involved in the derivation of the sbottom sector, the derived bottom mass has been employed to avoid large uncanceled $\mathrm{tg}\beta$-enhanced contributions (see discussion in Refs. [18]).

The branching ratios of the charged Higgs boson are shown in Fig. 1 as a function of the charged Higgs mass, for two values of $\mathrm{tg}\beta = 10, 40$. These plots already include the results of our work. The decay mode into $\tau^+\nu_\tau$ reaches branching ratios of more than 90% below the $t\bar{b}$ threshold and the muonic one reaches a few 10^{-3}. All other leptonic decay channels of the charged Higgs bosons are unimportant, while decays into a charm plus bottom or strange quark appear at the percent- or few permille-level. For large charged Higgs masses beyond the chargino, neutralino and squark thresholds, decays into these supersymmetric final states are significant, reaching branching ratios of up to $\sim 80\%$. Below the $t\bar{b}$ decay threshold, charged-Higgs decays into off-shell top quarks, $H^+ \to t^*\bar{b} \to b\bar{b}W^+$, are relevant [20]. Their branching ratio can reach the percent level for charged Higgs masses below the top-bottom threshold.

This article is structured as follows: In Section 2 we discuss the effective bottom and top Yukawa couplings at NNLO, in Section 3 we state our main results for the charged Higgs decays $H^+ \to t\bar{b}, c\bar{b}, c\bar{s}$ before concluding in Section 4.

2. Effective Yukawa couplings

Within the MSSM, radiative corrections to the effective bottom and top Yukawa couplings are important for moderate to large values of $\mathrm{tg}\beta$. The dominant part of these corrections can be coped with by introducing effective Yukawa factors for the neutral Higgs bosons. For charged

Higgs bosons, the top and bottom Yukawa-coupling factors are identical to the pseudoscalar couplings at LO. In this work, we have also adopted the effective pseudoscalar Yukawa-coupling factors for the charged Higgs boson that include higher-order corrections. The expressions for the effective pseudoscalar Yukawa-coupling factors are

$$
\begin{aligned}
g_b^A \to \tilde{g}_b^A &= \frac{g_b^A}{1+\Delta_b}\left[1-\frac{\Delta_b}{\mathrm{tg}^2\beta}\right] \qquad && (g_b^A = \mathrm{tg}\beta) \\
g_t^A \to \tilde{g}_t^A &= \frac{g_t^A}{1+\Delta_t}\left[1-\Delta_t \mathrm{tg}^2\beta\right] \qquad && (g_t^A = 1/\mathrm{tg}\beta)\,.
\end{aligned} \tag{3}
$$

The QCD and electroweak corrections to the bottom Yukawa coupling [21,22] take the form ($C_F = 4/3$)

$$
\begin{aligned}
\Delta_b &= \Delta_b^{QCD} + \Delta_b^{elw,t} + \Delta_b^{elw,1} + \Delta_b^{elw,2} \\
\Delta_b^{QCD} &= \frac{C_F}{2}\frac{\alpha_s}{\pi}\, m_{\tilde{g}}\, \mu\, \mathrm{tg}\beta\, I(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{g}}) \\
\Delta_b^{elw,t} &= \frac{\lambda_t^2}{(4\pi)^2}\, A_t\, \mu\, \mathrm{tg}\beta\, I(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}, \mu^2) \\
\Delta_b^{elw,1} &= -\frac{\alpha_1}{12\pi}\, M_1\, \mu\, \mathrm{tg}\beta \left\{ \frac{1}{3} I(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, M_1^2) + \left(\frac{c_b^2}{2} + s_b^2\right) I(m^2_{\tilde{b}_1}, M_1^2, \mu^2) \right. \\
&\qquad \left. + \left(\frac{s_b^2}{2} + c_b^2\right) I(m^2_{\tilde{b}_2}, M_1^2, \mu^2) \right\} \\
\Delta_b^{elw,2} &= -\frac{\alpha_2}{4\pi}\, M_2\, \mu\, \mathrm{tg}\beta \left\{ c_t^2 I(m^2_{\tilde{t}_1}, M_2^2, \mu^2) + s_t^2 I(m^2_{\tilde{t}_2}, M_2^2, \mu^2) \right. \\
&\qquad \left. + \frac{c_b^2}{2} I(m^2_{\tilde{b}_1}, M_2^2, \mu^2) + \frac{s_b^2}{2} I(m^2_{\tilde{b}_2}, M_2^2, \mu^2) \right\},
\end{aligned} \tag{4}
$$

where $\lambda_t = \sqrt{2} m_t/(v \sin\beta)$ represents the top Yukawa coupling and $\alpha_1 = g'^2/4\pi$, $\alpha_2 = g^2/4\pi$ correspond to the electroweak gauge couplings. The masses $m_{\tilde{b}_{1,2}}$ and $m_{\tilde{t}_{1,2}}$ denote the sbottom and stop masses. The variables $s/c_{t,b} = \sin/\cos\theta_{t,b}$ are related to the stop/sbottom mixing angles $\theta_{t,b}$. The function I is defined as

$$
I(a,b,c) = \frac{ab\log\frac{a}{b} + bc\log\frac{b}{c} + ca\log\frac{c}{a}}{(a-b)(b-c)(a-c)} = \frac{1}{a-b}\left\{ \frac{a\log\frac{a}{c}}{a-c} - \frac{b\log\frac{b}{c}}{b-c} \right\}. \tag{5}
$$

As can be seen from Eq. (4), the Δ_b terms grow with tgβ in addition to the tgβ behavior of the bottom Yukawa coupling at LO. The effective coupling $\tilde{g}_b^A$ resums the Δ_b contributions to all orders. The two-loop SUSY–QCD corrections to all individual terms given in Eq. (4) are known [23–25]. They modify the effective Yukawa couplings by about 10% for the central renormalization-scale choice given by the average mass of the contributing SUSY particles at one-loop order. Potentially large terms growing with A_b can be incorporated as well by the simple replacement [22][1]

[1] Extensions of the treatment of A_q terms have also been discussed in Refs. [26].

$$\Delta_b \to \frac{\Delta_b}{1+\Delta_{b,1}}\,,$$
$$\text{where} \quad \Delta_{b,1} = -\frac{2}{3}\,\frac{\alpha_s}{\pi}\,m_{\tilde{g}}\,A_b\,I(m^2_{\tilde{b}_1}, m^2_{\tilde{b}_2}, m^2_{\tilde{g}})\,. \tag{6}$$

Here the two-loop SUSY–QCD corrections amount to $\sim 10\%$ [25]. These one- and two-loop calculations can be translated to the strange Yukawa coupling with the appropriate replacements of the bottom/top masses and couplings by their strange/charm counter parts [25].

For our calculation of the charged Higgs decays into quarks, the SUSY–QCD correction Δ_b^{QCD} is the relevant one for analyzing the different perturbative orders in the strong coupling α_s. The SUSY–QCD part Δ_b^{QCD} at one- and two-loop order is displayed in Fig. 2 as a function of the renormalization scale of the strong coupling α_s.

The Δ_b corrections to the bottom Yukawa coupling amount to up to $30\% - 40\%$, depending on the value of tgβ in the M_h^{125} benchmark scenario, and are thus sizable. The scale dependence is significantly reduced from one- to two-loop order so that the residual theoretical uncertainties remain at the few-percent level. For the central prediction of the Δ_b^{QCD} corrections to the bottom Yukawa coupling, we have chosen the average SUSY mass $\mu_R = (m_{\tilde{b}_1} + m_{\tilde{b}_2} + m_{\tilde{g}})/3$ as the renormalization scale. The corresponding effective Yukawa-coupling factor $\tilde{g}_b^A$ of Eq. (3) is displayed in Fig. 3 as a function of the renormalization scale μ_R. The radiative corrections included in Δ_b reduce the LO coupling factor g_b^A by about 10% (20%) for tg$\beta = 10(40)$, as can be inferred from the comparison of the effective $\tilde{g}_b^A$ factor with the LO coupling g_b^A for the central scale choice $\mu_R = \mu_0$.

2.1. *Effective top Yukawa couplings at NNLO*

The top Yukawa coupling is affected by analogous radiative corrections. The SUSY–QCD part is discussed at the NNLO level in this section.[2] At the one-loop level, it is given by

$$\Delta_t = \frac{C_F}{2}\,\frac{\alpha_s}{\pi}\,\frac{m_{\tilde{g}}\,\mu}{\mathrm{tg}\beta}\,I(m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}, m^2_{\tilde{g}})\,. \tag{7}$$

This term is the leading SUSY-QCD correction to the top Yukawa couplings of the *neutral* MSSM Higgs in the limit of heavy SUSY particles, analogously to the bottom case [22]. For the incorporation of the NNLO corrections, we follow the same line as in Refs. [22,23]. The one-loop corrections can be obtained by off-diagonal mass insertions as shown in Fig. 4 and by replacing the vacuum expectation value v_1 of the first Higgs doublet by the full Higgs field, $v_1 \to \sqrt{2}\phi_1^{0*}$. This method is based on the low-energy theorems for soft external Higgs fields [28] and results in the effective Lagrangian for the neutral Higgs fields

$$\begin{aligned}
\mathcal{L}_{eff} &= -\lambda_t \overline{t_R}\left[\phi_2^0 + \Delta_t \mathrm{tg}\beta\,\phi_1^{0*}\right] t_L + h.c. \\
&= -m_t \bar{t}\left[1 - i\gamma_5 \frac{G^0}{v}\right] t - \frac{m_t/v}{1+\Delta_t}\bar{t}\Big[g_t^h\,(1-\Delta_t\,\mathrm{tg}\alpha\,\mathrm{tg}\beta)\,h \\
&\qquad + g_t^H\left(1+\Delta_t\frac{\mathrm{tg}\beta}{\mathrm{tg}\alpha}\right) H - g_t^A\left(1-\Delta_t \mathrm{tg}^2\beta\right) i\gamma_5 A\Big] t\,,
\end{aligned} \tag{8}$$

[2] The two-loop corrections to the effective top Yukawa coupling of the light scalar Higgs boson h have first been calculated in Ref. [27].

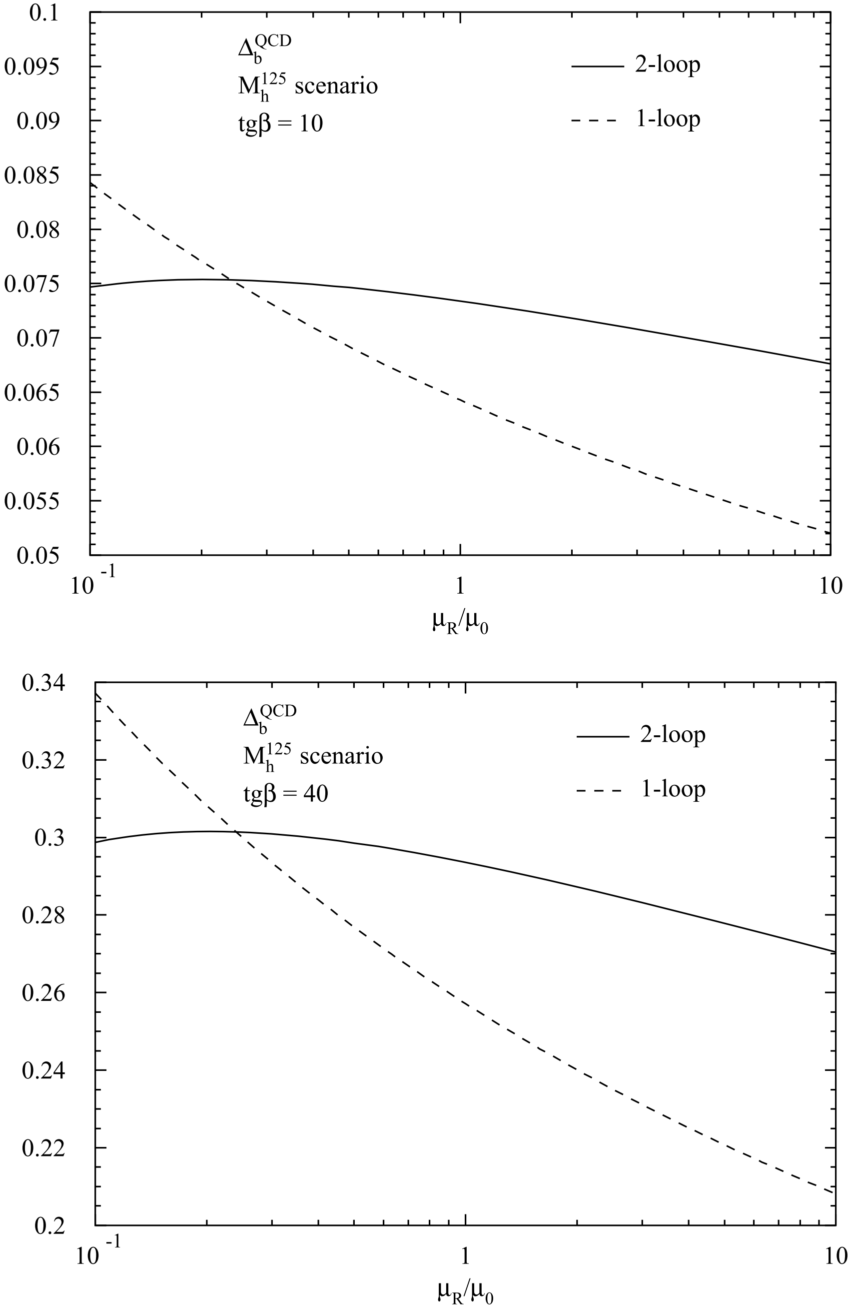


Fig. 2. Scale dependence of the SUSY–QCD correction Δ_b^{QCD} given in Eq. (4) at one-loop and two-loop order in the M_h^{125} scenario for $\mathrm{tg}\beta = 10, 40$ as a function of the renormalization scale μ_R and in units of the central scale choice $\mu_0 = (m_{\tilde{b}_1} + m_{\tilde{b}_2} + m_{\tilde{g}})/3$.

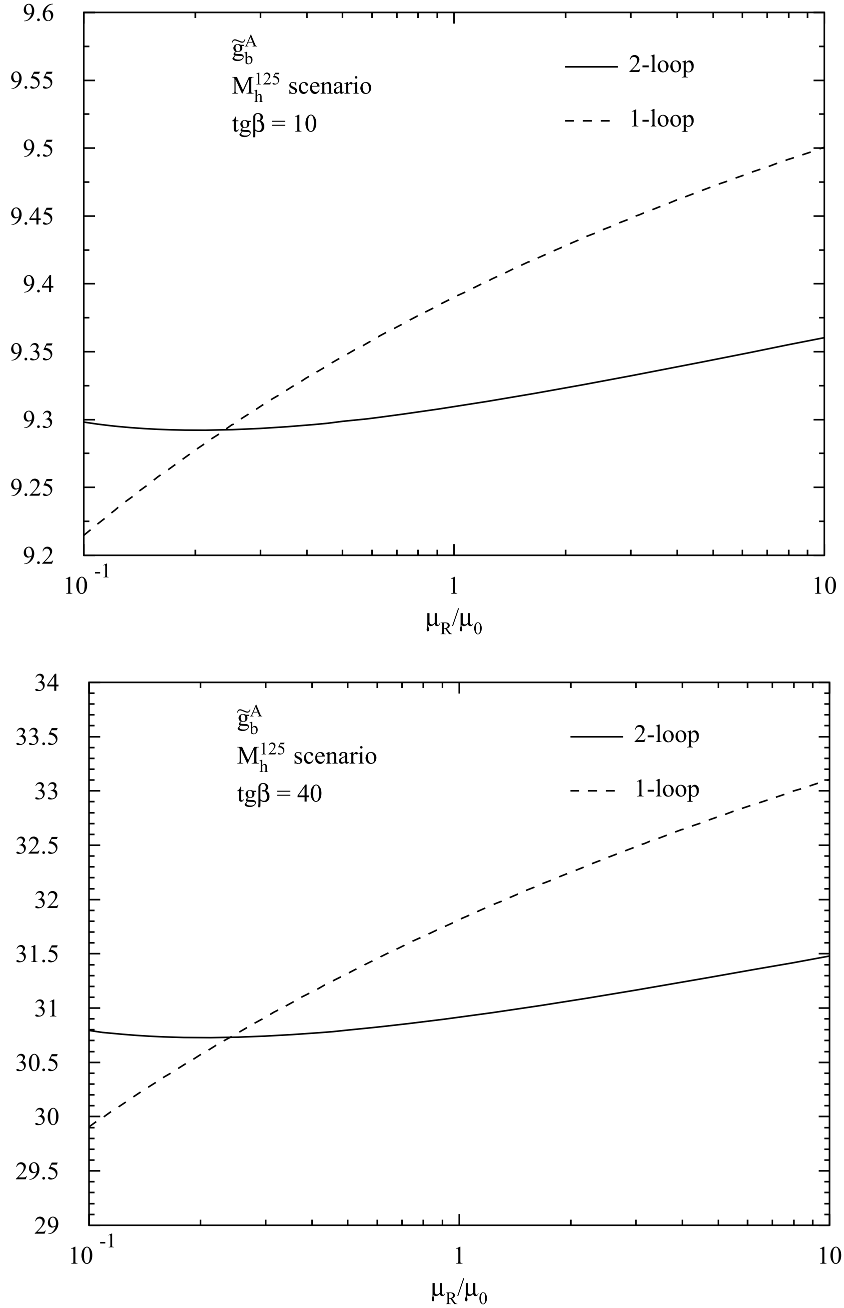


Fig. 3. Scale dependence of the SUSY–QCD corrected effective Yukawa coupling factors $\tilde{g}_b^A$ of Eq. (3) at one-loop and two-loop order in the M_h^{125} scenario as a function of the renormalization scale μ_R and in units of the central scale choice $\mu_0 = (m_{\tilde{b}_1} + m_{\tilde{b}_2} + m_{\tilde{g}})/3$. At leading order, $g_b^A = 10(40)$ for $\mathrm{tg}\beta = 10(40)$.

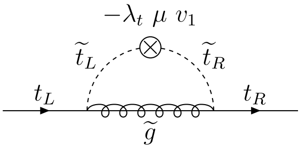


Fig. 4. One-loop diagrams of the SUSY–QCD contributions to the top self-energy with the off-diagonal mass insertions corresponding to the corrections Δ_t of the top Yukawa couplings. The contributing particles involve top quarks t, top squarks $\tilde{t}$ and gluinos $\tilde{g}$.

where $g^A_{t,b}$ are as defined in Eq. (3) and we used the explicit representation of the neutral components of the Higgs iso-doublets,

$$\phi_1^0 = \frac{1}{\sqrt{2}}\left[v_1 + H\cos\alpha - h\sin\alpha + iA\sin\beta - iG^0\cos\beta\right]$$
$$\phi_2^0 = \frac{1}{\sqrt{2}}\left[v_2 + H\sin\alpha + h\cos\alpha + iA\cos\beta + iG^0\sin\beta\right]. \quad (9)$$

In these expressions, α denotes the mixing angle of the neutral CP-even Higgs sector. The relation between the top Yukawa coupling λ_t and the top mass m_t is modified as

$$\lambda_t = \sqrt{2}\frac{m_t}{v\sin\beta(1+\Delta_t)}. \quad (10)$$

In this work, we assume the pseudoscalar coupling g_t^A to top quarks to be the relevant effective top-Yukawa coupling for the charged Higgs boson, also at higher perturbative order. Power counting allows us to show that the effective Lagrangian on the first line of Eq. (8), which is given in the current-eigenstate basis, is exact for the leading terms scaling with $1/\mathrm{tg}\beta$. Any additional insertion of the vertex Feynman rule of Fig. 4 yields another power of m_t/M_{SUSY}. The leading term of the full vertex in the expansion in m_t^2/M_{SUSY}^2 and M_Φ^2/M_{SUSY}^2 ($\Phi = h, H, A$) is given by the contribution Δ_t in the limit of vanishing top and Higgs masses.

The two-loop corrections to Δ_t can be derived from the related two-loop corrections to the top-quark self-energy (see Fig. 5) in the limit of vanishing quark masses (including the top mass) for the first term of a large SUSY-mass expansion. Applying all possible off-diagonal mass insertions in the stop propagators of Fig. 5, we have obtained the two-loop corrections to the Δ_t term. The stop and gluino masses have been renormalized on-shell. For the strong coupling α_s we have applied the $\overline{\mathrm{MS}}$ scheme with five active flavors, where the top quark and SUSY particles have been decoupled. The explicit expressions of the renormalization constants have been obtained by replacing the sbottom masses by the stop masses in the results given in Ref. [23]. In order to restore the supersymmetric relations between the SM couplings and their supersymmetric counterparts, we have included the anomalous counterterms within dimensional regularization which we used in the calculation of the NNLO corrections to the top Yukawa coupling. In this way, we have translated the calculation of the NNLO corrections presented for the bottom-quark case in Ref. [23] to the top-quark case. The results for Δ_t at one- and two-loop order are shown in Fig. 6 as a function of the renormalization scale of the strong coupling α_s. The Δ_t terms turn out to be at the permille level in the M_h^{125} benchmark scenario. The scale dependence is significantly reduced from the one- to two-loop level so that the residual relative theoretical uncertainties are at the few-percent level. For the central prediction of the Δ_t contributions to the effective top Yukawa coupling we have chosen the average SUSY mass $\mu_R = (m_{\tilde{t}_1} + m_{\tilde{t}_2} + m_{\tilde{g}})/3$ as the renormalization scale. The corresponding effective top Yukawa-coupling factor $\tilde{g}_t^A$ of Eq. (3) is

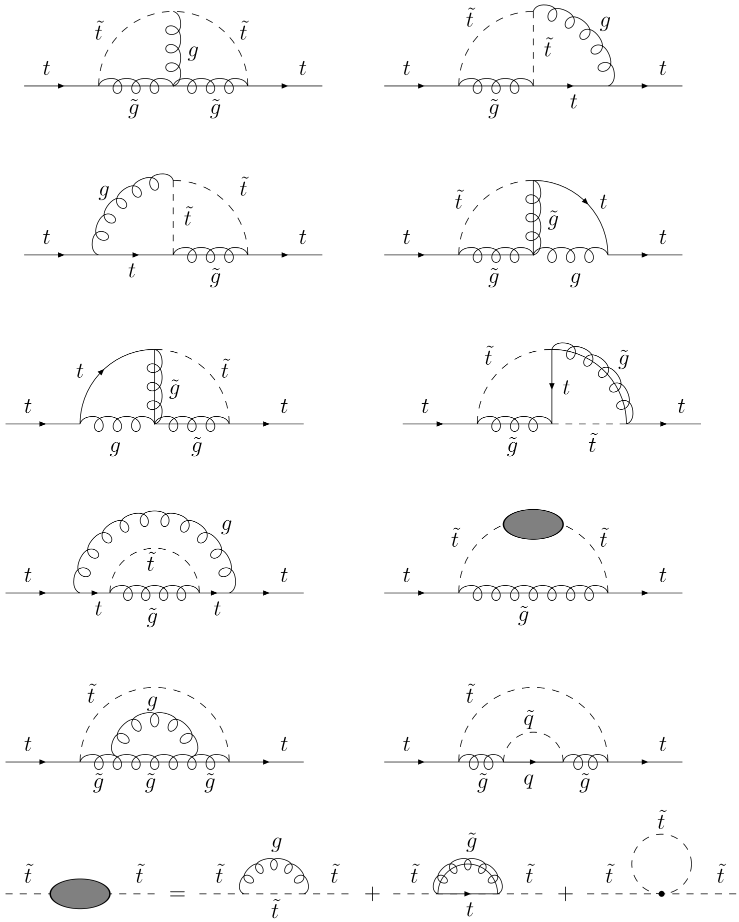


Fig. 5. Two-loop diagrams of the SUSY–QCD contributions to the top self-energy involving top quarks t, top squarks $\tilde{t}$, gluons g and gluinos $\tilde{g}$. The squark-quark contributions to the gluino propagator have to be summed over all quark/squark flavors $q/\tilde{q}$, including both directions of the flavor flow due to the Majorana nature of the gluino.

presented in Fig. 7 at one- and two-loop order as a function of the renormalization scale μ_R. Due to the additional $\text{tg}^2\beta$ enhancement of $\tilde{g}_t^A$, the SUSY–QCD corrections to the effective Yukawa-coupling factor turn out to be sizable, i.e. about 5%-10% (30%) for $\text{tg}\beta = 10(40)$, and thus of similar size as the bottom Yukawa-coupling factor $\tilde{g}_b^A$ discussed in the previous subsection.

3. Charged Higgs decays into quarks

In this section, we consider the genuine NLO SUSY–QCD corrections to the charged Higgs decays into quarks. These consist of vertex-correction terms and self-energy contributions to the external legs, see Fig. 8, which have been combined with their respective counterterms.

For the calculation of the vertex corrections, the charged Higgs couplings to up- and down-type squarks are required. The generic Feynman rule is depicted in Fig. 9. In the chiral basis, the corresponding couplings read as ($g_D^A = 1/g_U^A = \text{tg}\beta$)

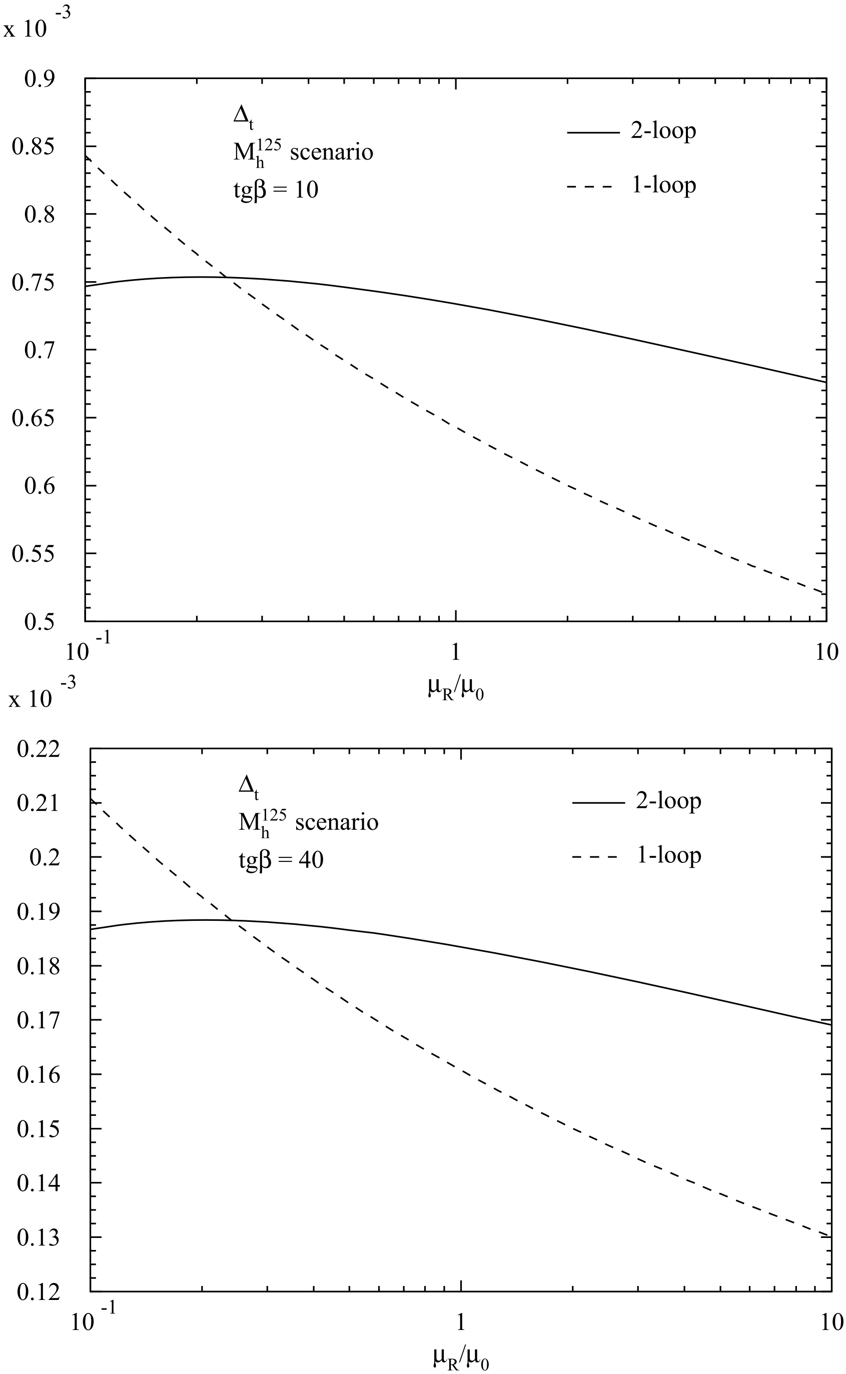


Fig. 6. Scale dependence of the SUSY–QCD correction Δ_t at one-loop and two-loop order in the M_h^{125} scenario for tg$\beta = 10, 40$ as a function of the renormalization scale μ_R and in units of the central scale choice $\mu_0 = (m_{\tilde{t}_1} + m_{\tilde{t}_2} + m_{\tilde{g}})/3$.

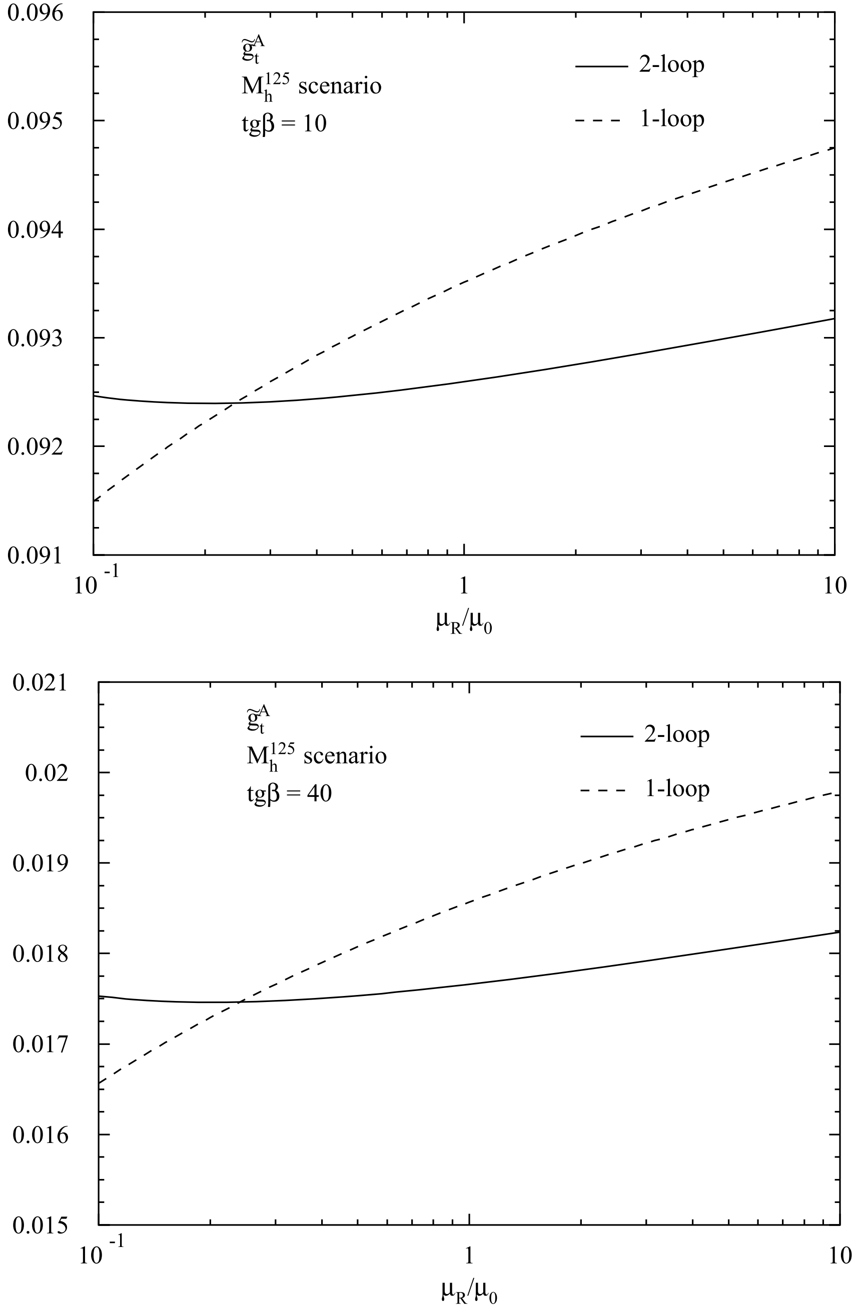


Fig. 7. Scale dependence of the SUSY–QCD corrected effective Yukawa-coupling factors $\tilde{g}_t^A$ at one-loop and two-loop order in the M_h^{125} scenario as a function of the renormalization scale μ_R and in units of the central scale choice $\mu_0 = (m_{\tilde{t}_1} + m_{\tilde{t}_2} + m_{\tilde{g}})/3$. At leading order, $g_b^A = 0.1(0.025)$ for $\mathrm{tg}\beta = 10(40)$.

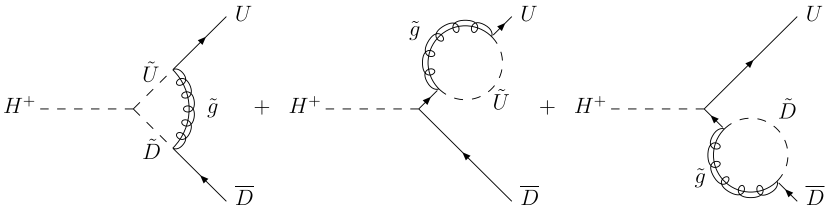


Fig. 8. Virtual SUSY–QCD corrections to charged Higgs boson decays into $U\overline{D}$ quark pairs at next-to-leading order. U denotes up-type and D down-type quarks.

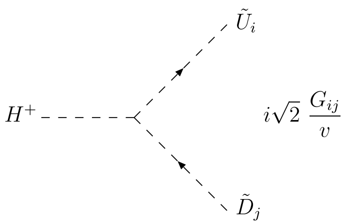


Fig. 9. Feynman rule for the charged Higgs coupling to up- and down-type squarks, $\tilde{U}_i$ and $\tilde{D}_j$ ($ij = L, R$ or $1, 2$), respectively.

$$
\begin{aligned}
G_{LL} &= m_D^2 g_D^A + m_U^2 g_U^A - M_W^2 s_{2\beta} \\
G_{LR} &= m_D (A_D g_D^A + \mu) \\
G_{RL} &= m_U (A_U g_U^A + \mu) \\
G_{RR} &= m_D m_U (g_D^A + g_U^A)\,.
\end{aligned}
\tag{11}
$$

Rotating these couplings to the sfermion mass eigenstate basis results in

$$
\begin{aligned}
G_{11} &= G_{LL} c_U c_D + G_{LR} c_U s_D + G_{RL} s_U c_D + G_{RR} s_U s_D \\
G_{12} &= -G_{LL} c_U s_D + G_{LR} c_U c_D - G_{RL} s_U s_D + G_{RR} s_U c_D \\
G_{21} &= -G_{LL} s_U c_D - G_{LR} s_U s_D + G_{RL} c_U c_D + G_{RR} c_U s_D \\
G_{22} &= G_{LL} s_U s_D - G_{LR} s_U c_D - G_{RL} c_U s_D + G_{RR} c_U c_D\,,
\end{aligned}
\tag{12}
$$

where $s_i = \sin\theta_i, c_i = \cos\theta_i$ denote the mixing-angle contributions. Since the sfermion mixing angles are proportional to the SM fermion masses, we have kept the mixing angles for the stop and sbottom states, but have neglected them for the first two generations.

For the SUSY–QCD part of the NLO calculation, we have renormalized the quark masses on-shell. The contribution of the SUSY masses to the QCD-running of the quark masses is decoupled in this way. We have assumed that the pure QCD corrections factorize from the genuine SUSY–QCD corrections, since the QCD corrections are dominated by light-particle contributions. In the QCD part, the up-type (down-type) quark masses have been implemented as the $\overline{\text{MS}}$ masses $\overline{m}_{U,D} = \overline{m}_{U,D}(M_{H^\pm})$ at the scale of the charged Higgs mass in the limit of large charged Higgs masses, while closer to the decay threshold we adopted the pole masses[3] $M_{U,D}$. Close to threshold, the partial decay width at NLO reads

[3] Both regions have been combined by a quartic interpolation w.r.t. the masses.

$$
\begin{aligned}
\Gamma[H^+ \to U\bar{D}] = \frac{3G_F M_{H^\pm}}{4\sqrt{2}\pi} |V_{UD}|^2 \lambda^{1/2} \Bigg\{ \\
&(1-\mu_U-\mu_D)\left[M_U^2 (g_U^A)^2 \left(1+\frac{4}{3}\frac{\alpha_s}{\pi}\delta^+_{UD}\right)\left(1+\tilde{\delta}^+_{UD}\right) \right. \\
&\left. + M_D^2 (g_D^A)^2 \left(1+\frac{4}{3}\frac{\alpha_s}{\pi}\delta^+_{DU}\right)\left(1+\tilde{\delta}^+_{DU}\right) \right] \\
&-4M_U M_D \sqrt{\mu_U\mu_D}\, g_U^A g_D^A \left(1+\frac{4}{3}\frac{\alpha_s}{\pi}\delta^-_{UD}\right)\left[1+\tilde{\delta}^-_{UD}\right]\Bigg\}, \qquad (13)
\end{aligned}
$$

with $\mu_i = M_i^2/M_{H^\pm}^2$ and $\lambda = (1-\mu_U-\mu_D)^2 - 4\mu_U\mu_D$. The $\tilde{\delta}^\pm_{ij}$ denote the individual parts of the SUSY–QCD corrections. We have assumed the CKM matrix element V_{UD} to be the same in the quark and squark sector so that it factorizes globally.[4] The coupling factors g_Q^A are given in Eq. (3) at LO. The QCD-correction factors $\delta^\pm_{ij}$ $(i, j = U, D)$ read as [30,31]

$$
\begin{aligned}
\delta^+_{ij} &= \frac{9}{4} + \frac{3-2\mu_i+2\mu_j}{4}\log\frac{\mu_i}{\mu_j} + \frac{(\frac{3}{2}-\mu_i-\mu_j)\lambda + 5\mu_i\mu_j}{2\lambda^{1/2}(1-\mu_i-\mu_j)}\log x_i x_j + B_{ij} \\
\delta^-_{ij} &= 3 + \frac{\mu_j-\mu_i}{2}\log\frac{\mu_i}{\mu_j} + \frac{\lambda+2(1-\mu_i-\mu_j)}{2\lambda^{1/2}}\log x_i x_j + B_{ij}\,, \qquad (14)
\end{aligned}
$$

with $x_i = 2\mu_i/[1-\mu_i-\mu_j+\lambda^{1/2}]$ and the terms

$$
\begin{aligned}
B_{ij} = \frac{1-\mu_i-\mu_j}{\lambda^{1/2}} &\left[4\mathrm{Li}_2(x_i x_j) - 2\mathrm{Li}_2(-x_i) - 2\mathrm{Li}_2(-x_j) + 2\log x_i x_j \log(1-x_i x_j)\right. \\
&\left. - \log x_i \log(1+x_i) - \log x_j \log(1+x_j)\right] \\
&-4\left[\log(1-x_i x_j) + \frac{x_i x_j}{1-x_i x_j}\log x_i x_j\right] \\
&+\frac{\lambda^{1/2}+\mu_i-\mu_j}{\lambda^{1/2}}\left[\log(1+x_i) - \frac{x_i}{1+x_i}\log x_i\right] \\
&+\frac{\lambda^{1/2}-\mu_i+\mu_j}{\lambda^{1/2}}\left[\log(1+x_j) - \frac{x_j}{1+x_j}\log x_j\right].
\end{aligned}
$$

The expressions for the SUSY–QCD corrections $\tilde{\delta}^\pm_{ij}$ are quite involved [32]. However, in the limit of large SUSY parameters $A_{U/D}, \mu, M_{\tilde{g}}$ and $m_{\tilde{U}_i/\tilde{D}_i}$ $(i = 1, 2)$, they approach simple expressions,

$$
\begin{aligned}
\tilde{\delta}^+_{UD} = -2\delta_U + C_F\frac{\alpha_s}{\pi}\Bigg\{ c_{2U}\frac{m^2_{\tilde{U}_1}-m^2_{\tilde{U}_2}}{8}&\left[M_{\tilde{g}}^2 J(m^2_{\tilde{U}_1}, m^2_{\tilde{U}_2}, M^2_{\tilde{g}}) - I(m^2_{\tilde{U}_1}, m^2_{\tilde{U}_2}, M^2_{\tilde{g}})\right] \\
-c_{2D}\frac{m^2_{\tilde{D}_1}-m^2_{\tilde{D}_2}}{8}&\left[M_{\tilde{g}}^2 J(m^2_{\tilde{D}_1}, m^2_{\tilde{D}_2}, M^2_{\tilde{g}}) - I(m^2_{\tilde{D}_1}, m^2_{\tilde{D}_2}, M^2_{\tilde{g}})\right] \\
-\frac{m^2_{\tilde{D}_1}-m^2_{\tilde{U}_1}}{8}&\left[M_{\tilde{g}}^2 J(m^2_{\tilde{D}_1}, m^2_{\tilde{U}_1}, M^2_{\tilde{g}}) - I(m^2_{\tilde{D}_1}, m^2_{\tilde{U}_1}, M^2_{\tilde{g}})\right]
\end{aligned}
$$

[4] A discussion of flavor constraints in charged Higgs-boson production can be found in Ref. [29].

$$
\begin{aligned}
&-\frac{m^2_{\tilde{D}_2}-m^2_{\tilde{U}_2}}{8}\left[M^2_{\tilde{g}}J(m^2_{\tilde{D}_2},m^2_{\tilde{U}_2},M^2_{\tilde{g}})-I(m^2_{\tilde{D}_2},m^2_{\tilde{U}_2},M^2_{\tilde{g}})\right]\\
&-M_{\tilde{g}}\left(A_U+\frac{\mu}{g^A_U}\right)\Big[s^2_U c^2_D I(m^2_{\tilde{U}_1},m^2_{\tilde{D}_1},M^2_{\tilde{g}})\\
&\qquad +c^2_U c^2_D I(m^2_{\tilde{U}_2},m^2_{\tilde{D}_1},M^2_{\tilde{g}})\\
&\qquad +s^2_U s^2_D I(m^2_{\tilde{U}_1},m^2_{\tilde{D}_2},M^2_{\tilde{g}})\\
&\qquad +c^2_U s^2_D I(m^2_{\tilde{U}_2},m^2_{\tilde{D}_2},M^2_{\tilde{g}})\\
&\qquad -I(m^2_{\tilde{U}_1},m^2_{\tilde{U}_2},M^2_{\tilde{g}})\Big]\Big\}\\
\tilde{\delta}^+_{DU}=&-2\delta_D+C_F\frac{\alpha_s}{\pi}\Big\{c_{2D}\frac{m^2_{\tilde{D}_1}-m^2_{\tilde{D}_2}}{8}\left[M^2_{\tilde{g}}J(m^2_{\tilde{D}_1},m^2_{\tilde{D}_2},M^2_{\tilde{g}})-I(m^2_{\tilde{D}_1},m^2_{\tilde{D}_2},M^2_{\tilde{g}})\right]\\
&-c_{2U}\frac{m^2_{\tilde{U}_1}-m^2_{\tilde{U}_2}}{8}\left[M^2_{\tilde{g}}J(m^2_{\tilde{U}_1},m^2_{\tilde{U}_2},M^2_{\tilde{g}})-I(m^2_{\tilde{U}_1},m^2_{\tilde{U}_2},M^2_{\tilde{g}})\right]\\
&-\frac{m^2_{\tilde{U}_1}-m^2_{\tilde{D}_1}}{8}\left[M^2_{\tilde{g}}J(m^2_{\tilde{D}_1},m^2_{\tilde{U}_1},M^2_{\tilde{g}})-I(m^2_{\tilde{D}_1},m^2_{\tilde{U}_1},M^2_{\tilde{g}})\right]\\
&-\frac{m^2_{\tilde{U}_2}-m^2_{\tilde{D}_2}}{8}\left[M^2_{\tilde{g}}J(m^2_{\tilde{D}_2},m^2_{\tilde{U}_2},M^2_{\tilde{g}})-I(m^2_{\tilde{D}_2},m^2_{\tilde{U}_2},M^2_{\tilde{g}})\right]\\
&-M_{\tilde{g}}\left(A_D+\frac{\mu}{g^A_D}\right)\Big[s^2_D c^2_U I(m^2_{\tilde{D}_1},m^2_{\tilde{U}_1},M^2_{\tilde{g}})\\
&\qquad +c^2_D c^2_U I(m^2_{\tilde{D}_2},m^2_{\tilde{U}_1},M^2_{\tilde{g}})\\
&\qquad +s^2_D s^2_U I(m^2_{\tilde{D}_1},m^2_{\tilde{U}_2},M^2_{\tilde{g}})\\
&\qquad +c^2_D s^2_U I(m^2_{\tilde{D}_2},m^2_{\tilde{U}_2},M^2_{\tilde{g}})\\
&\qquad -I(m^2_{\tilde{D}_1},m^2_{\tilde{D}_2},M^2_{\tilde{g}})\Big]\Big\}\\
\tilde{\delta}^-_{UD}=&-\delta_U-\delta_D\\
&-\frac{C_F}{2}\frac{\alpha_s}{\pi}M_{\tilde{g}}\Big\{\left(A_D+\frac{\mu}{g^A_D}\right)\Big[c^2_U s^2_D I(m^2_{\tilde{D}_1},m^2_{\tilde{U}_1},M^2_{\tilde{g}})+s^2_U s^2_D I(m^2_{\tilde{D}_1},m^2_{\tilde{U}_2},M^2_{\tilde{g}})\\
&\qquad +c^2_U c^2_D I(m^2_{\tilde{D}_2},m^2_{\tilde{U}_1},M^2_{\tilde{g}})+s^2_U c^2_D I(m^2_{\tilde{D}_2},m^2_{\tilde{U}_2},M^2_{\tilde{g}})\\
&\qquad -I(m^2_{\tilde{D}_1},m^2_{\tilde{D}_2},M^2_{\tilde{g}})\Big]\\
&+\left(A_U+\frac{\mu}{g^A_U}\right)\Big[s^2_U c^2_D I(m^2_{\tilde{D}_1},m^2_{\tilde{U}_1},M^2_{\tilde{g}})+c^2_U c^2_D I(m^2_{\tilde{D}_1},m^2_{\tilde{U}_2},M^2_{\tilde{g}})\\
&\qquad +s^2_U s^2_D I(m^2_{\tilde{D}_2},m^2_{\tilde{U}_1},M^2_{\tilde{g}})+c^2_U s^2_D I(m^2_{\tilde{D}_2},m^2_{\tilde{U}_2},M^2_{\tilde{g}})\\
&\qquad -I(m^2_{\tilde{U}_1},m^2_{\tilde{U}_2},M^2_{\tilde{g}})\Big]\Big\}\,, \qquad (15)
\end{aligned}
$$

with the function I of Eq. (5) and

$$J(a,b,c) = \frac{\partial I(a,b,c)}{\partial c} = \frac{1}{a-b}\left\{ \frac{a\log\frac{a}{c}}{(a-c)^2} - \frac{b\log\frac{b}{c}}{(b-c)^2} \right\} + \frac{1}{(a-c)(b-c)}. \quad (16)$$

The terms δ_Q $(Q = U, D)$ are related to the $\Delta_{U/D}$ terms, respectively,

$$\delta_Q = \Delta_Q \left[1 + \frac{1}{\left(g_Q^A\right)^2} \right] \qquad (Q = U, D). \quad (17)$$

For the third generation, Δ_U is given by Δ_t of Eq. (7) and Δ_D by Δ_b^{QCD} of Eq. (4).

The use of the effective up- and down-type Yukawa couplings $\tilde{g}_U^A, \tilde{g}_D^A$ at LO leads to the subtraction of the δ_U and δ_D terms in Eq. (15), leaving the SUSY-remainders

$$\begin{aligned} \tilde{\delta}_{UD}^{+} &\to \tilde{\delta}_{UD,rem}^{+} = \tilde{\delta}_{UD}^{+} + 2\delta_U \\ \tilde{\delta}_{DU}^{+} &\to \tilde{\delta}_{DU,rem}^{+} = \tilde{\delta}_{DU}^{+} + 2\delta_D \\ \tilde{\delta}_{UD}^{-} &\to \tilde{\delta}_{UD,rem}^{-} = \tilde{\delta}_{UD}^{-} + \delta_U + \delta_D \end{aligned} \quad (18)$$

in the SUSY–QCD corrections to the charged Higgs decay of Eq. (13). This yields the improved expression of the partial decay width

$$\begin{aligned} \Gamma[H^+ \to U\bar{D}] = \frac{3G_F M_{H^\pm}}{4\sqrt{2}\pi} |V_{UD}|^2 \lambda^{1/2} \Big\{ \\ (1-\mu_U-\mu_D)\Big[M_U^2 \tilde{g}_U^A \left(1 + \frac{4}{3}\frac{\alpha_s}{\pi}\delta_{UD}^{+}\right)\left(\tilde{g}_U^A + g_U^A \tilde{\delta}_{UD,rem}^{+}\right) \\ + M_D^2 \tilde{g}_D^A \left(1 + \frac{4}{3}\frac{\alpha_s}{\pi}\delta_{DU}^{+}\right)\left(\tilde{g}_D^A + g_D^A \tilde{\delta}_{DU,rem}^{+}\right)\Big] \quad (19) \\ -4M_U M_D \sqrt{\mu_U \mu_D}\left(1 + \frac{4}{3}\frac{\alpha_s}{\pi}\delta_{UD}^{-}\right)\left[\tilde{g}_U^A \tilde{g}_D^A + \frac{\tilde{g}_U^A g_D^A + g_U^A \tilde{g}_D^A}{2}\tilde{\delta}_{UD,rem}^{-}\right]\Big\} \end{aligned}$$

for all charged Higgs decays into quarks. We extended our NNLO calculation of the Δ_Q $(Q = t, b)$ contributions to the charm quarks and included Δ_s terms in the effective strange Yukawa coupling [25]. We have implemented the effective Yukawa-coupling factors $\tilde{g}_U^A, \tilde{g}_D^A$ in every term emerging from the LO amplitude during the contraction with the NLO amplitude, in complete analogy to the computation of neutral Higgs-boson decays into bottom quarks, where this prescription avoids artificial singularities [22].

For large charged Higgs masses, the above expressions for the partial widths simplify. Due to the chiral structure of the charged Higgs vertex, the QCD corrections approach the QCD corrections to the scalar current correlator, which are known up to N^{4}LO [33]. In the large Higgs mass regime $M_{H^\pm} \gg M_U + M_D$, the improved partial decay width is given by [30,31,34]

$$\begin{aligned} \Gamma[H^+ \to U\overline{D}] = \frac{3G_F M_{H^\pm}}{4\sqrt{2}\pi} |V_{UD}|^2 \Big[\overline{m}_U^2 \tilde{g}_U^A \left(\tilde{g}_U^A + g_U^A \tilde{\delta}_{UD,rem}^{+}\right) \quad (20) \\ + \overline{m}_D^2 \tilde{g}_D^A \left(\tilde{g}_D^A + g_D^A \tilde{\delta}_{DU,rem}^{+}\right)\Big](1 + \delta_{\rm QCD}), \end{aligned}$$

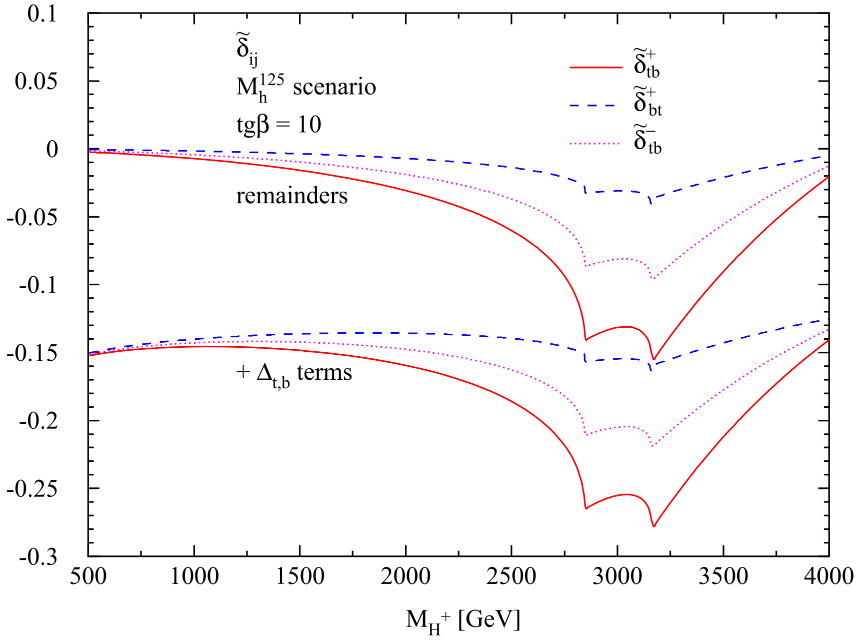
where $\overline{m}_{U,D}$ denote the $\overline{\rm MS}$ masses evaluated at the scale of the charged Higgs-mass $M_{H^\pm}$. In this expression, we have used the effective Yukawa coupling $\tilde{g}_{U/D}^A$ in every contribution that emerges from the LO amplitude, as in Eq. (19), but we use the LO Yukawa-coupling factors for the SUSY-remainder itself at the amplitude level. The QCD corrections δ_{QCD} are given by [33]

$$\begin{aligned}\delta_{\text{QCD}} = {} & 5.67\frac{\alpha_s(M_{H^\pm})}{\pi} + (35.94 - 1.36N_F)\left[\frac{\alpha_s(M_{H^\pm})}{\pi}\right]^2 \\ & +(164.14 - 25.77N_F + 0.259N_F^2)\left[\frac{\alpha_s(M_{H^\pm})}{\pi}\right]^3 \\ & +(39.34 - 220.9N_F + 9.685N_F^2 - 0.0205N_F^3)\left[\frac{\alpha_s(M_{H^\pm})}{\pi}\right]^4 ,\end{aligned} \tag{21}$$

where large logarithms are absorbed in the $\overline{\text{MS}}$ masses $\overline{m}_{U,D}$ at the charged-Higgs mass scale and we have used $N_F = 5$.

We have implemented these improved results for the partial charged Higgs decay widths into heavy quarks, $H^+ \to t\bar{b}, c\bar{b}, c\bar{s}$, using the corresponding expressions for Δ_t, Δ_b, Δ_c and Δ_s up to NNLO in the code `Hdecay` [19]. This allows us to predict these partial charged Higgs decay widths with approximate NNLO SUSY–QCD precision within the MSSM. In our numerical analysis, for the SUSY-remainders $\tilde{\delta}^{\pm}_{ij}$ themselves we have consistently used the top pole mass and the derived bottom mass, in accordance with their treatment in the stop and sbottom sectors [16]. Fig. 10 shows the size of the individual NLO SUSY coefficients $\tilde{\delta}^{\pm}_{ij}$ $(i, j = t, b)$ as a function of the charged Higgs mass within the M_h^{125} benchmark scenario for two values of $\text{tg}\beta = 10, 40$. The upper curves correspond to the SUSY-remainders $\tilde{\delta}^{\pm}_{ij,rem}$ after absorbing the dominant $\Delta_{t/b}$ terms in the effective Yukawa coupling factors $\tilde{g}^A_{t/b}$, while the lower curves exhibit the full NLO SUSY–QCD corrections without using the effective Yukawa couplings, i.e. the $\Delta_{b,t}$ terms are part of the fixed-order NLO correction. It is clearly visible that for charged Higgs masses up to about 2 TeV the SUSY-remainders are in the percent range and below so that the effective Yukawa-coupling factors $\tilde{g}_{t,b}$ alone yield a reliable approximation of the full NLO results. It should be noted that also the Δ_t terms contained in the effective top Yukawa coupling factor $\tilde{g}^A_t$ provide an excellent approximation of the NLO SUSY–QCD corrections emerging from the top-Yukawa coupling. This can be understood from the leading terms in the large SUSY-mass expansion of Eq. (15): The remainder terms are either not enhanced for large values of $\text{tg}\beta$ or the $\text{tg}\beta$-enhanced contributions are proportional to differences of the function I of Eq. (5) that are small for nearly degenerate soft SUSY-breaking squark-mass parameters. Only for larger Higgs masses close to and above the virtual stop and sbottom thresholds the remainders turn out to be more sizable, as expected for kinematical reasons. For a charged Higgs mass below about 2 TeV, this result can also be used for an NNLO approximation of the SUSY–QCD corrections, since the SUSY remainder of the NNLO corrections is expected to be even further suppressed in the limit of large SUSY-masses.

In Fig. 11 we show the partial decay width $\Gamma(H^+ \to t\bar{b})$ with pure QCD corrections and NLO/approximate NNLO (in terms of the effective Yukawa couplings) SUSY–QCD corrections as a function of the charged Higgs mass. We show the results for two values of $\text{tg}\beta = 10, 40$ and for the central scale choices. While the NLO SUSY–QCD corrections modify the partial decay width substantially, the approximate NNLO corrections are still relevant. They are at the level of a few percent over the full charged Higgs-mass range and reduce the theoretical uncertainties related to the scale choice to the percent level. The analogous picture emerges for the subleading decay modes $H^+ \to c\bar{b}$ and $H^+ \to c\bar{s}$, as can be inferred from Fig. 12 that includes the analogous SUSY–QCD corrections to the charm and strange Yukawa couplings.



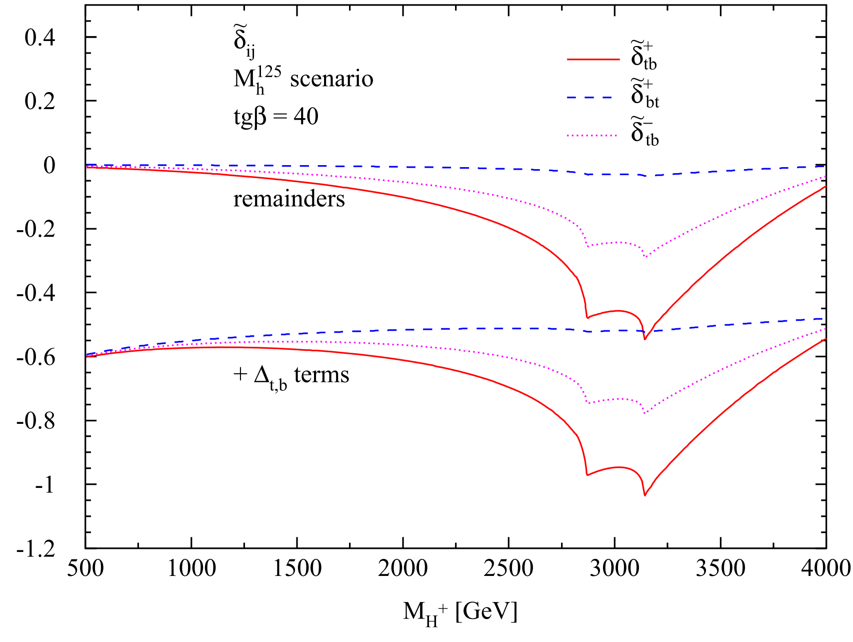
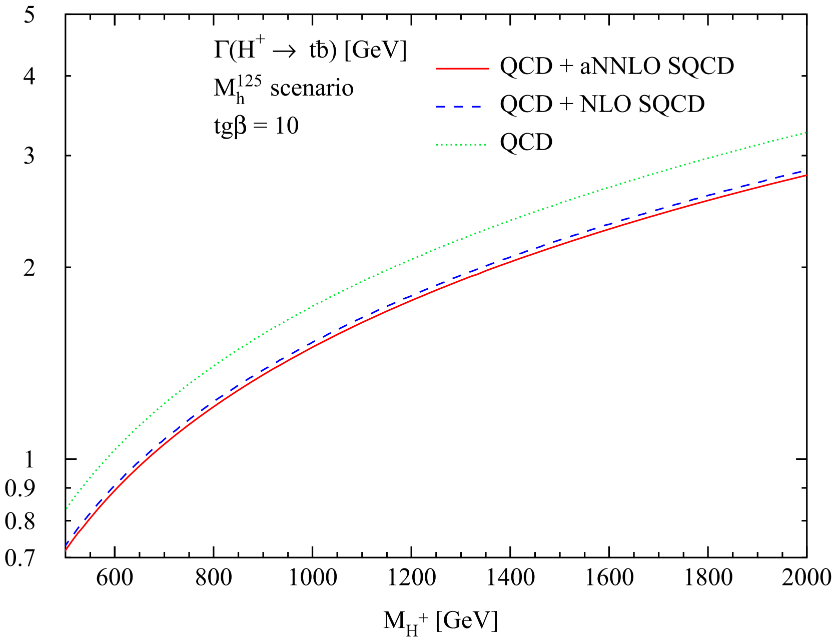


Fig. 10. Individual NLO coefficients $\tilde{\delta}^{\pm}_{ij}$ $(i, j = t, b)$ of the decay width $\Gamma(H^+ \to t\bar{b})$ as a function of the charged Higgs mass with and without the $\Delta_{t/b}$ contributions in the M_h^{125} scenario. The upper curves show the sizes of the SUSY-remainders, while the lower curves correspond to the full corrections including the $\Delta_{t/b}$ contributions, without using the effective Yukawa couplings.



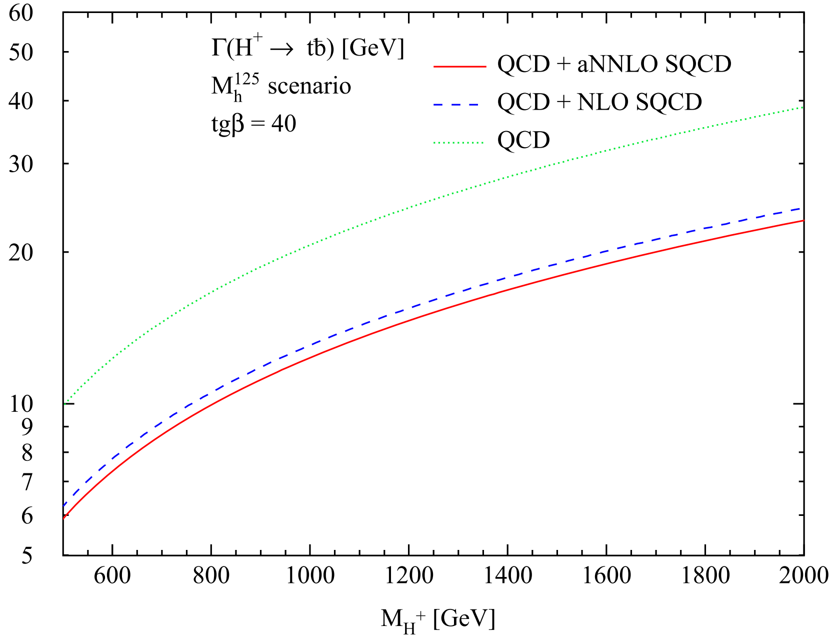
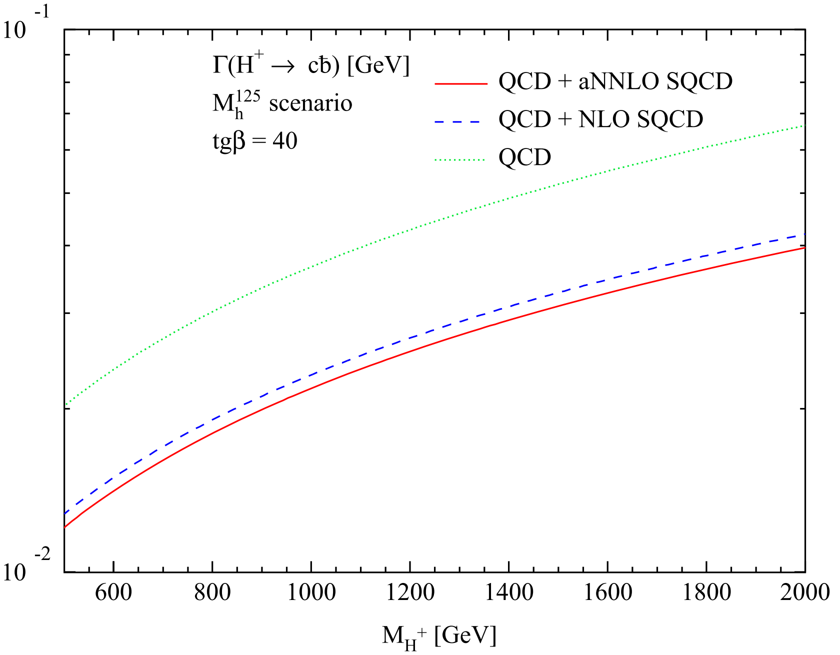


Fig. 11. Partial decay width of $H^+ \to t\bar{b}$ at different orders of the SUSY–QCD corrections. The dotted (green) lines include the usual QCD corrections involving the $\overline{\mathrm{MS}}$ Yukawa couplings, but no genuine SUSY–QCD corrections. The dashed (blue) lines involve the full NLO SUSY–QCD remainders in addition to the effective Yukawa couplings that have been used at the 1-loop level in SUSY–QCD. The full (red) lines display the approximate results (aNNLO) including the 2-loop SUSY–QCD corrections to the effective Yukawa couplings.



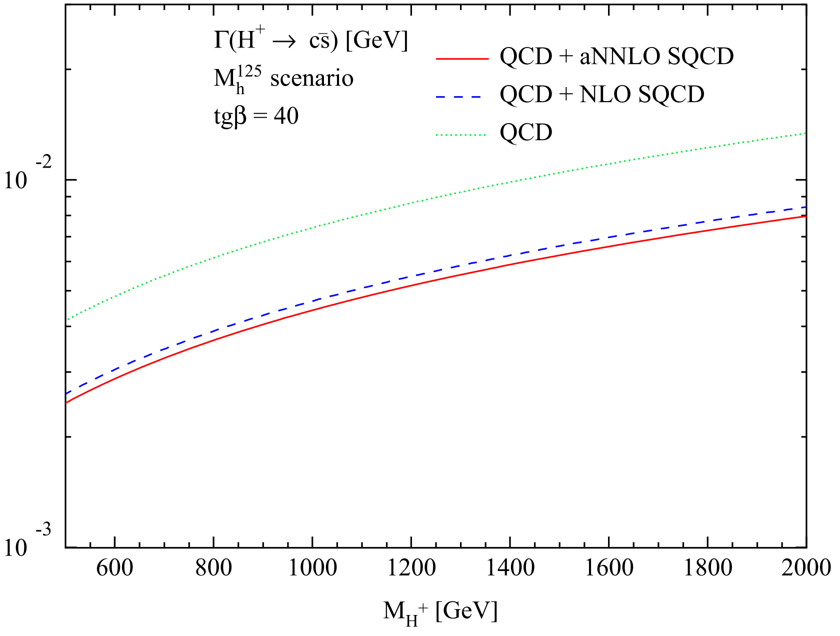


Fig. 12. Same as Fig. 11 but for $H^+ \to c\bar{b}$ (top) and $H^+ \to c\bar{s}$ (bottom) for tg$\beta = 40$.

4. Conclusions

In this work we have re-examined the charged Higgs-boson decays into heavy quarks within the MSSM. We rederived the genuine NLO SUSY–QCD corrections and combined them with the usual QCD corrections in a factorized form. We discussed the role of introducing effective down-type and up-type Yukawa couplings that constitute the dominant contributions to the genuine SUSY–QCD corrections and analyzed the accuracy of this approximation to the full contributions. Since the SUSY-remainder beyond the effective Yukawa couplings turned out to be small at NLO and we expect the same to be valid beyond NLO, we can obtain approximate NNLO results for the genuine SUSY–QCD corrections by evaluating these corrections to the effective Yukawa couplings. This amounts to calculating the down-type Δ_D and up-type Δ_U corrections at NNLO. The NNLO calculation to Δ_U is new in our work. Combining these approximate NNLO SUSY–QCD corrections with the usual QCD corrections that are known up to N^{4}LO for large Higgs masses, we have obtained an approximate NNLO result for charged Higgs-boson decays into heavy quarks. We have applied our calculation to the charged Higgs-boson decays $H^+ \to t\bar{b}, c\bar{b}, c\bar{s}$ in particular, but have implemented these corrections in the public code `Hdecay` [19] for all charged Higgs decays into quarks. This makes the theoretical predictions of the charged Higgs decays more precise than previously. The new implementations in `Hdecay` will be made public soon.

Our calculation can easily be extended to non-minimal supersymmetric models as e.g. the next-to-MSSM (NMSSM). This requires taking into account an extended Higgs sector involving more mixing angles than in the MSSM. The translation is straightforward, as demonstrated for the public code `NMSSMCALC` [35] that computes the radiatively corrected Higgs masses and decay width including all available higher-order corrections.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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