How Automated Market Makers Approach the Thin Market Problem in Cryptoeconomic Systems

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Abstract

The proper design of automated market makers (AMMs) is crucial to enable the continuous trading of assets represented as digital tokens on markets of cryptoeconomic systems. Improperly designed AMMs can make such markets suffer from the thin market problem (TMP), which can cause cryptoeconomic systems to fail their purposes. We developed an AMM taxonomy that showcases AMM design characteristics. Based on the AMM taxonomy, we devised AMM archetypes implementing principal solution approaches for the TMP. The main purpose of this article is to support practitioners and researchers in tackling the TMP through proper AMM designs.

Index Terms

automated market makers, cryptoeconomic systems, decentralized exchange, decentralized finance, blockchain

I. INTRODUCTION

Organizations aim to efficiently allocate resources through markets, often through initial public offerings of stocks [1], [2]. Resource allocation in markets requires transfers of asset ownership [3]. Asset ownership transfers are commonly processed by intermediaries like banks and notaries [4], [5], which can increase transaction costs (e.g., banking and clearing fees) [6], slow down transaction settlement (e.g., cross-border payments) [7], and decrease flexibility (e.g., regarding offering structures of stocks) [7].

By leveraging distributed ledger technology (DLT), in particular blockchain technology, cryptoeconomic systems offer an alternative approach for resource allocation that can reduce the reliance on intermediaries, decrease transaction costs, and enhance flexibility [5], [8]. Cryptoeconomic systems are sociotechnical systems wherein market participants (e.g., individuals, organizations, and software components) manage ownership of assets based on digital tokens that are secured by principles of cryptographic systems [5], [9], such as digital signatures. To offer and trade tokens in cryptoeconomic systems, market participants commonly use automated market makers (AMMs). AMMs are software agents that are used to provide liquidity to cryptoeconomic system markets by continuously offering trades of token pairs to investors based on mathematically specified price functions [10]. For example, the fictive company Token Comp issues TKC tokens to raise capital. It offers the TKC tokens to investors in exchange for ETH tokens (i.e., the native currency of the Ethereum system) via an AMM. The AMM holds the token pair TKC/ETH and continuously offers token trades to market participants. Alice buys transfers 1 ETH to the AMM in order to buy TKC tokens. The AMM then calculates the token price using its price function (i.e., the number of TKC tokens Alice will receive for 1 ETH) and transfers the corresponding amount of TKC tokens to Alice. Conversely, Alice can exchange TKC for ETH tokens against the AMM at any time. By continuously offering exchanges of ETH and TKC tokens, the AMM provides liquidity to the TKC/ETH market.

The proper design of AMMs is crucial for successful resource allocation in cryptoeconomic systems. Improperly designed AMMs can make cryptoeconomic system markets suffer from the thin market problem (TMP). The TMP refers to unreliable asset pricing in markets of low liquidity, which increases financial risks for investors and organizations that issue tokens for resource allocation [11]–[13]. For example, in markets subject to the TMP, selling large token amounts in a short time strongly decreases token prices [12]. Continuing with the previous example, Alice can (unintentionally) strongly impact TKC token prices. During the process of selling tokens, Alice continuously decreases the token price along the price function of the AMM. Thereby, Alice experiences financial losses by selling TKC tokens at lower prices than expected. Financial risks, such as financial losses caused by strong price changes, can shy away market participants from investing in cryptoeconomic systems. Without investors buying a sufficient volume of tokens (e.g., TKC tokens), organizations and even the entire cryptoeconomic system fail in their resource allocation purposes (e.g., raising capital and exchanging tokens). Properly designed AMMs should implement mechanisms to solve the TMP.

Various AMM designs were developed that implement different approaches to support organizations in their resource allocation. For example, constant function market makers (e.g., Uniswap v2) use mathematical conservation functions to discover adequate token prices [14], [15]. Proactive market makers (e.g., DODO), adopt token prices from external price oracles [14], [16]–[18]. However, external price oracles cannot provide adequate token prices because they are unknown thin markets [12]. Thus, proactive market makers appear not to be able to solve the TMP. In contrast, Uniswap v2 discovers token prices based on buy and sell transactions that change the token reserves but require market participants to provide liquidity [19]. Uniswap v2 seems to solve the TMP if sufficient liquidity is provided. Apparently, AMM designs can strongly influence the efficacy of solving the TMP.

Understanding AMM designs is of particular value for the identification and the targeted use of solution approaches for the TMP. However, publications presenting a wide variety of AMM designs are scattered across various sources (e.g., blogs, whitepapers, and scientific databases). This makes it difficult to deduce AMM design characteristics that account for solution approaches for the TMP. Therefore, the design of such approaches can hardly be identified, inhibiting their targeted use.

To understand AMM designs and their characteristics, extant research [14], [15], [20]–[22] presents conceptualizations of AMM designs, covering basic AMM design characteristics, such as liquidity sensitivity and path independence. Although being of great value for understanding AMM designs, extant AMM conceptualizations mainly focus on a few AMM designs, such as constant function market makers [14], [15], [21]. Additional characteristics of other AMM designs, such as the price adoption of proactive market makers [16], [18], translation invariance of LMSR market makers [23]–[25], and token supply-sovereignty continuous liquidity market makers [9], are neglected. This makes it hard to understand the key characteristics of AMM designs and how they account for tackling the TMP. A conceptualization of AMM designs is needed to understand AMM design characteristics and to identify and compare solution approaches for the TMP implemented in AMMs. We ask the following research questions:

RQ1: What are the key characteristics of AMM designs?

RQ2: What are principal solution approaches for the TMP implemented in AMMs?

We applied a three-step research approach. First, we developed an AMM taxonomy based on 122 scientific publications and 110 AMMs following the method of taxonomy development of Nickerson et al. [26]. Second, we utilized the AMM taxonomy to identify important design characteristics for tackling the TMP. Based on the identified design characteristics, we developed AMM archetypes that implement common solution approaches for the TMP. Third, we assessed the efficacy of the developed AMM archetypes to solve the TMP.

Our work has the following main contributions. First, we contribute to the understanding of AMM designs by presenting an AMM taxonomy. The AMM taxonomy can be used to guide AMM development as it points out dimensions that need to be considered by developers and offers options to implement the dimensions. Moreover, the AMM taxonomy is useful for the systematic comparison of AMM designs. Second, by presenting AMM archetypes (i.e., Price-discovering LP-based AMM, Price-adopting LP-based AMM, Price-discovering Supply-sovereign AMM), we support the understanding of the basic functioning of common AMM designs that are used for specific resource allocation purposes (e.g., token issuance). The AMM archetypes can be used as abstract blueprints that can be refined to develop AMMs that meet resource allocation purposes. Third, by explaining the commonly used solution approaches for the TMP implemented in AMMs and the efficacy of these approaches, we support the anticipation of the TMP by proper AMM designs.

The remainder of this work is organized into six sections. In Section II, we introduce cryptoeconomic systems, AMMs, and principal typical uses of AMMs to meet resource allocation purposes. Moreover, we explain the TMP in more detail. In Section III, we describe how we developed the AMM taxonomy and the AMM archetypes. In Section IV, we present the groups, dimensions, and characteristics of our AMM taxonomy and demonstrate its applicability based on 110 AMMs. Section V describes the developed AMM archetypes and their solution approaches for the TMP. In Section VI, we discuss our principal findings and describe the contributions and limitations of this work. Moreover, we outline future research directions on AMMs. In Section VII, we conclude with our personal takeaways and thoughts on the development of AMMs for cryptoeconomic systems.

II. BACKGROUND AND RELATED RESEARCH

A. Cryptoeconomic Systems and Distributed Ledger Technology

Cryptoeconomic systems (e.g., based on the Bitcoin system or the Ethereum system) are sociotechnical systems that enable agents (e.g., individuals, organizations, and software artifacts) to manage ownership of assets, including claims, rights, and securities, by using principles of cryptographic systems [9], [27]. In this section, we introduce the foundations of cryptographic and economic systems combined in cryptoeconomic systems. Building on those foundations, we introduce DLT and its role in the operation of cryptoeconomic systems.

a) Cryptoeconomic Systems: Cryptoeconomic systems combine principles of cryptographic systems and economic systems. Cryptographic systems are suits of cryptographic algorithms that are used to reach certain security levels, such as in terms of confidentiality [28], [29]. The basic functions to be fulfilled by cryptographic algorithms in cryptographic systems are

key generation, encryption, and decryption. Key generation algorithms produce secrets, also called keys, that can be used to encrypt and decrypt data. For the authentication of identities in computer systems, such as DLT systems, by digital signatures, asymmetric key techniques are commonly used [30]. Digital signatures allow for the authentication of identities in computer systems [30]–[32] to authorize asset transfers as required in economic systems. For example, market participants in stock markets must authenticate against brokers to initiate asset transfers.

Economic systems are social systems in which market participants allocate resources in order to enable the trading of products and services. A prevalent form of economic system in modern times is the market economy [33]. In market economies, prices and production are determined by the interaction of supply and demand from all market participants (e.g., producers, consumers, investors, and traders) [34]. In these markets, market participants come together to exchange assets, such as goods and services. To enable an asynchronous exchange of assets, exchanges operate order books that record buy and sell offers and fulfill them when a matching counterparty is found. Basically, there are two types of orders. First, limit orders which are instructions to buy or sell assets at a specified price but without the guarantee of immediate execution [35]. Limit orders are stored in order books. As the counterpart to limit orders, market orders are instructions to buy or sell assets immediately at a given price [35]. Exchanges match limit orders and market orders in markets to settle assets to be traded. The immediately available volume to settle market orders (e.g., through limit orders) is the available liquidity in a market [36].

Market makers are used to enable smooth asset trading by providing liquidity to markets. A market maker is a rational market participant who quotes bid (buy) and ask (sell) prices for trading pairs [37]. A trading pair refers to two different assets that can be traded against each other, for example, Bitcoin against USD or Wheat against USD. Market makers actively place limit orders in the order book committing their willingness to trade at bid/ask prices to market participants, consequently providing liquidity to the market [37].

Market makers can apply different approaches to determine bid/ask prices for their limit orders. A simple approach is to incorporate bid/ask prices stated by market participants into a pricing function. Such a pricing function can compute the market maker's asset prices by averaging all bid/ask quotes of all investors. The pricing function of market makers is usually private and not known to other market participants [37], [38]. Market makers leverage bid/ask spreads by continuously buying and selling assets with added surcharges [10], [38].

As a prerequisite for the exchange of assets, market makers must continuously hold balanced amounts of all assets in their inventory that are offered in trading pairs to market participants (e.g., arbitrageurs and investors) [37]. Market makers ideally sell a number of assets (e.g., USD) and simultaneously buy the equivalent number of assets of the same kind. When arbitrageurs and investors buy more underpriced assets of one kind, the market maker sells more of this asset than it buys. Thereby, the market maker becomes subject to inventory imbalances that can render the market maker unable to trade asset pairs [38], [39].

Cryptoeconomic systems combine principal techniques of cryptographic systems to enable the safe and secure operation of economic systems [5], [9].

b) Cryptoeconomic Systems based on Distributed Ledger Technology: DLT is often used to instantiate cryptoeconomic systems. DLT enables the operation of distributed ledgers, a kind of distributed database that stores records of transactions. Often, these transactions represent resource allocations of the economic system. DLT systems usually implement techniques and functionalities to operate the infrastructure of cryptoeconomic systems [5], [9], for example, asymmetric cryptography for the authentication of identities by digital signatures and a database management system for distributed ledgers. The DLT-based infrastructure is governed by an economic system in terms of the creation, allocation, and distribution of assets that are represented by digital tokens in digital economies. In DLT-based cryptoeconomic systems, tokens are typically specified and managed in smart contracts (e.g., ERC-20 standard) that map token balances to unique identifiers of the market participants (e.g., externally owned addresses in the Ethereum system) [40]. Smart contracts are software programs that allow for the automated execution of transaction logic [41], [42]. Transactions can manipulate the token mapping, enabling asset ownership transfers.

B. Automated Market Makers

In cryptoeconomic systems, AMMs are market makers implemented as software agents that commonly trade tokens with market participants at self-determined prices in an automated manner. In contrast to conventional market makers (e.g., trading organizations), AMMs execute the token settlement and use trading strategies that are based on mathematical formulas and are transparent to all market participants.

AMM designs commonly incorporate a price discovery component, price determination component, a parameter component, a token settlement component, token management components, and an liquidity provider (LP) token management component. An exemplary AMM component overview of Uniswap v2 is shown in Figure 1.

The *price discovery component* implements the logic to discover the token price. Typically, the price discovery component is part of the AMM and uses parameters of the parameter component for price discovery. The token price is passed to the price determination component.

The *price determination component* implements a price determination mechanism to calculate the bid/ask token prices. The price determination component adjusts the token price based on the parameters of the parameter component. Market participants



Fig. 1: Exemplary AMM component overview of Uniswap v2 [19]

can exchange tokens at the stated bid/ask token prices of the price determination component. When market participants execute transactions, the price determination component calculates the amount of bought tokens the market participant will receive in return for the amount of tokens sold to the AMM. Both token amounts are passed to the token settlement component.

The *parameter component* stores parameters the AMM uses to determine prices, settle transactions, and govern parameter changes of the AMM. Exemplary parameters of the parameter component are trading fees, amount of token reserves, token weights, and recent prices [19]. A set of parameters defines the state of an AMM. State transitions are carried out by trades of market participants with the AMM. The price discovery component and price determination component calculate the amount of tokens the market participant will receive in return for the provided amount of tokens. The calculation is based on the parameters in the AMM, the input parameters of the transaction, and parameters external from the AMM. The token price results from the amount of received tokens divided by the amount of provided tokens (from the market participant's perspective).

The token settlement component calls the token management component of the individual token keepers to initiate the actual token transfer through a token keeper. A token keeper is a software agent, often implemented as a smart contract, that controls the token management component. Token keepers are part of at least one cryptoeconomic system. They are often external to AMMs. AMMs can be token keepers themselves. Within each token keeper, a token management component manages the tokens of market participants, including AMMs in inventories. Token management components maintain account books that map token balances to unique identifiers (e.g., account addresses) of market participants. Token management components updates account books to *transfer, mint*, and *burn* tokens. AMMs have token inventories managed by at least one token keepers that manage tokens involved in the transactions to transfer tokens by updating their account books. For example, a market participant exchanges 1 WETH for 1000 USDC. To settle this transaction, the AMM instructs the ERC-20 smart contract of WETH, the market participant's token balance is decreased by one. The token balance of the AMM's liquidity pool is increased by one. Vice versa, the ERC-20 smart contract of USDC is instructed to increase the market participant's token balance by 1,000 and decrease the token balance of the AMM's liquidity pool by 1,000.

The *LP token management component* is an optional AMM-internal token management component that is used to handle LP tokens. Market participants can deposit tokens into the liquidity pools of AMMs. Such market participants are called liquidity providers (LPs) [19], [43]. When depositing tokens into the liquidity pools, LPs receive LP tokens. LP tokens represent a claim on a share of the liquidity pool that allows the liquidity providers to withdraw their share of the liquidity pool. LP tokens cannot be traded via the AMM [14], [19], [44]. The LP token management component stores and administrates the account book that maps LP tokens of the AMM to the unique identifiers of the LPs.

Building on the introduced components, different AMM designs were presented. Constant function market makers implement the predominant AMM design used by Uniswap v2 [19], PancakeSwap [45], and SushiSwap [46]. Constant function market makers implement mathematical conservation functions for price discovery. The price discovery mechanism adjusts token prices based on buy and sell transactions of market participants that change the AMM's token reserves [10], [15], [22], [43], [47], [48]. Proactive market makers, such as DODO [16] and WooFi [49], use external price oracles that incorporate price discovery

components. Proactive market makers do not discover token prices on their own. Instead, they adopt token prices from external price oracles that are often operated by third parties [14], [16]–[18].

C. Principal Purposes of Automated Market Makers

AMMs are used to meet two principal purposes: *decentralized exchange (DEX)* and *token issuance*. These purposes can be further nuanced into six subordinate ones, as described in the following.

1) Decentralized Token Exchange: Decentralized token exchanges allow market participants to swap tokens between market participants without the need for central authorities (e.g., brokers) [50], [51]. There are four different types of decentralized token exchanges: correlated tokens, uncorrelated tokens, non-fungible tokens (NFTs), and prediction tokens.

a) Correlated Tokens: Correlated tokens are tokens with linked prices. When the price of one token increases (or decreases), correlated token prices also increase (or decrease). Strongly correlated token pairs are supposed to be exchanged at a constant rate [52]. To enable cost-efficient exchanges at a constant rate, the market must be highly liquid at this exchange rate [53]. Exemplary correlated token pairs are Circle's USDC and Tether's USDT. Both tokens are paired with each other and exchangeable at a one-to-one ratio.

b) Uncorrelated Tokens: Uncorrelated token exchanges enable trades of tokens whose token prices are weakly or not correlated to each other. Uncorrelated token pairs require liquidity in wider price ranges because those tokens cannot be exchanged at a constant rate [21]. Instead, the exchange rate varies due to volatility and price fluctuations. An exemplary uncorrelated token pair is Bitcoin paired with a stable token, such as Circle's USDC.

c) Non-fungible Tokens: Via NFT exchanges, market participants can swap NFTs with fungible tokens. Each NFT has its individual value due to its inherent uniqueness. Therefore, NFTs are often non-interchangeable. However, to enable interchangeability, all NFTs of one collection are treated equally and assumed to be inter-changeable [54]. For example, NFTs of the Bored Ape Yacht Club collection could be paired with a rather stable token such as Ether. NFTs in the Bored Ape Yacht Club collection are treated equally and do not have a unique value assigned.

d) Prediction tokens: Prediction tokens are used in prediction markets that enable market participants to bet on outcomes of future events, for example, the outcomes of elections or company stock prices at a specific future point in time [55]. Market participants can place their bets into AMMs. When market participants place bets, they deposit tokens into liquidity pools of corresponding AMMs. The occurrence of events on which market participants have bet triggers the closure of the prediction market. When the prediction market is closed, AMMs evaluate the outcomes of triggered events against the bets of market participants. The AMM initiates payouts of tokens deposited by market participants depending on the event outcome [56]–[58]. For example, market participants who bet on the outcomes of events receive tokens; other market participants lose their deposits. Exemplary prediction markets are offered by Augur [57] and Zeitgeist [59]. Those prediction markets enabled market participants to bet on events such as the outcome of the 2020 U.S. presidential election.

2) Token Issuance: Token issuance refers to the process of minting and distributing tokens of cryptoeconomic systems to market participants. In token issuance, AMMs create a functional relationship between the price of a token and the corresponding token supply [13]. Practically speaking, the token price is mapped to the token supply. The use case of token issuance can be grouped into *curation tokens* and *initial token offerings*.

a) Curation Tokens: Curation tokens are used to curate market participant perceptions of the value of an asset, such as data sets, machine learning (ML) models, or artworks. AMMs are used to adjust token prices to update token prices according to market participants' perceptions continuously [60], [61]. For example, the Ocean Protocol offers an AMM for curation markets that issues tokens for data sets that are used to value the provided data sets in regard to their quality of training ML models. It curates the perceptions of market participants regarding the quality of the data set [62].

b) Initial Token Offering: In initial token offerings, token issuers (e.g., individuals and organizations) collect funding to finance endeavors by selling shares of the endeavor represented in the form of tokens [63]. Such endeavors include new infrastructure, new Dapps, or other projects (e.g., The Dao, Fetch.ai, Bancor) [13], [64], [65]. AMMs support initial token offerings by providing a thick market that enables market participants to buy and sell the cryptoeconomic system's tokens. Exemplary initial token offerings were conducted to finance the development of the Ethereum system in 2014 [66] and the Tezos system in 2017 [67].

D. Principal Challenges of Automated Market Makers

AMMs can become subject to the Liquidity Accumulation Problem (LAP) and the Price Determination Problem (PDP), which must be solved to tackle the TMP. The following briefly introduces the LAP, the PDP, and the TMP and their relationships.

a) Liquidity Accumulation Problem: The LAP refers to the accumulation of token reserves that can be used to settle transactions of market participants.

To settle transactions with market participants, AMMs draw on token reserves deposited in liquidity pools. When insufficient token reserves are available, AMMs are low-liquid and become oversensitive to transactions. For example, low-volume transactions can lead to large token price changes, and transactions cannot be settled because of insufficient token reserves [43]. Consequently, the AMM is rendered unattractive to trade with. Market participants shy away from using the AMM because of its ineffectiveness. Sufficient token reserves must be available for AMMs to tackle the TMP successfully.

b) Price Determination Problem: The PDP refers to the reliable determination of adequate token prices based on market information that may be hard to interpret in an automated manner.

AMMs must determine adequate token prices for which market participants buy or sell tokens. Adequate token prices represent the current cumulative perception of market participants. The adequate token price approximates the efficient token price but does not equal the efficient token price because it is unknown in inefficient markets according to the efficient market hypothesis [68], [69]. When AMMs set inadequate token prices (e.g., diverging from other markets), AMMs sell tokens at prices that are too low or buy tokens at prices that are too high, resulting in financial losses for the respective AMM. For example, a news report increases the actual value of tokens from the perspectives of market participants. The AMM may not be able to directly incorporate information from that news report into its pricing mechanism because the AMM is unable to interpret the market information [20], [70]. Thus, the AMM has incomplete market information and lags behind the market participants' perceptions of the adequate token price. This leads the AMM to sell tokens too cheaply.

AMMs that adjust token prices based on market information solve the PDP. Thus, these AMMs can offer tokens at adequate prices in cryptoeconomic system markets. Having market information incorporated, the AMM can state adequate token prices approaching the efficient token price that enables the AMM to tackle the pricing issue of the TMP successfully.

c) Thin Market Problem: The TMP refers to the unreliable asset pricing caused by the low liquidity of corresponding markets.

When buyers and sellers infrequently make bid-and-ask quotes, orders seldom match, which is a symptom of markets with low liquidity [12], [13]. Low liquidity can make markets very sensitive to transactions. Even transactions with little volume can strongly influence the token price. Increasing market sensitivity, which may even render markets over-sensitive, ultimately decreases the reliability of token prices [12], [71]. Large deviations from adequate token prices characterize unreliable token prices. The unreliability of token prices can be amplified by the exploitation of market over-sensitivity for market manipulations. Oversensitive thin markets can usually be successfully exploited at low cost [71], [72]. For example, in traditional financial markets, penny stocks that are typically prone to the TMP are often used for pump-and-dump schemes [72]. In a pump-and-dump schema, the market manipulator artificially increases the asset price (pump) to attract other buyers. Afterward, the bought assets are sold at the artificially pumped price to realize profits (dump) [73], [74] In summary, the TMP arises from low trading volume available to market participants, leading to high bid-ask spreads and unreliable token prices.

Various AMM designs were developed that implement different solution approaches for the LAP, PDP, and TMP. Such approaches have individual efficacy in solving the TMP. In this work, we describe the characteristics of AMM designs and corresponding principal solution approaches implemented in AMMs to tackle the LAP, the PDP, and the TMP. Moreover, we offer an explanation of their efficacy in solving the TMP.

III. METHODS

We applied a three-step method to understand the characteristics that constitute AMM designs and to develop AMM archetypes that implement principal solution approaches for the TMP. First, we developed an AMM taxonomy based on literature and AMM implementations following Nickerson et al. [26]. Second, we used the AMM taxonomy to develop AMM archetypes. Third, we analyzed scientific publications, gray literature, and AMM implementations to extract solution approaches implemented in AMM archetypes for the TMP.

A. AMM Taxonomy Development

To understand the differences between AMM designs, we developed an AMM taxonomy following the method proposed by Nickerson et al. [26]. First, we determined our meta-characteristic as AMM design characteristics to get an exhaustive overview of different AMM designs. Second, we specified six ending conditions (see Table I) that helped us recognize when

Туре	Name	Description
	Exhaustiveness	The characteristics and dimensions collectively exhaustively describe AMM designs
	Mutual Exclusiveness	Characteristics (and dimensions) do not semantically overlap
	Relevance	Each characteristic of each dimension is required for the classification of at least one AMM design in the taxonomy
Ohissting	Representativeness	A selection of publications and AMMs representative of AMM designs were incorporated into the taxonomy
Objective	Robustness	No changes were made to the taxonomy in the last iteration
Subjective	Conciseness	The taxonomy includes a limited number of relevant dimensions and characteristics to describe AMM designs

TABLE I: Ending conditions for the taxonomy development

TABLE II: Overview of the development of the AMM tax	onomy
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Approach	Туре	Iter. 0	Iter. 1	Iter. 2	Iter. 3	Iter. 4	Summary
Concentual	Confirmatory publications	5	25	n.a.	63	n.a.	93
to-empirical	Conflicting publications		8	n.a.	7	n.a.	29
to empirical	Overall publications	19	33	n.a.	70	n.a.	122
Empirical to	Confirmatory AMMs	n.a.	n.a.	46	n.a.	47	93
conceptual	Conflicting AMMs	n.a.	n.a.	14	n.a.	3	17
F	Overall AMMs	n.a.	n.a.	60	n.a.	50	110

n.a.: not applicable Iter.: Iteration

the taxonomy reached a sufficient quality level to terminate the taxonomy development. Third, we applied the conceptual-toempirical approach and the empirical-to-conceptual approach (see Table II) in five iterations. In the conceptual-to-empirical approach, we conceptualized the dimensions of the AMM taxonomy without examining implementations of AMMs. In this deductive process, we used our knowledge of and experience with AMMs and cryptoeconomic systems and judgment to create relevant dimensions by analyzing literature. We analyzed scientific and non-scientific publications on cryptoeconomic systems and AMMs to extract AMM dimensions and corresponding characteristics for our taxonomy. In the empirical-toconceptual approach, we analyzed implementations AMMs to extract relevant characteristics for the classification of AMM designs. Table II presents an overview of our taxonomy development process. The taxonomy development process comprises five iterations. In total, we analyzed 122 publications in the conceptual-to-empirical approach and 110 implementations of AMMs in the empirical-to-conceptual approach. We describe each iteration in the taxonomy development process in more detail in the following.

a) Conceptual-to-empirical (Iteration 0): To develop our initial version of the AMM taxonomy, we applied a conceptual-to-empirical approach based on the analysis of literature on AMMs. We started the conceptual-to-empirical approach with a search for literature that is relevant to the development of the AMM taxonomy. To assess the relevance of publications on AMMs, we defined inclusion and exclusion criteria (see Table III). Then, we compiled a set of potentially relevant publications for the development of the AMM taxonomy. We selected publications we deemed particularly relevant for the development of the AMM taxonomy. This selection resulted in an initial set of 31 publications, including peer-reviewed and gray literature, potentially relevant for the AMM taxonomy development. After applying our inclusion and exclusion criteria, we excluded twelve publications because those did not include AMM characteristics or did not describe concrete AMM designs. Our final set of literature to be analyzed to develop an initial AMM taxonomy comprised 19 publications.

After the literature search, we read the full texts of the 19 publications. Then, we analyzed them by applying open coding [75], [76] to extract dimensions describing AMM designs and corresponding characteristics. We recorded a name, description, original source, and corresponding characteristics for each dimension. Our initial coding resulted in 41 preliminary characteristics associated with 31 preliminary dimensions. We resolved ambiguities and inconsistencies between the preliminary characteristics and dimensions in three refinement rounds. For example, we merged the characteristics *sufficient funds, path deficiency*, and *non-depletion* into *path deficiency*. After refining the preliminary characteristics and associated dimensions, our initial version of the AMM taxonomy comprised 15 AMM dimensions and 34 AMM characteristics.

Туре	Name	Description			
	AMM Design Description	The publication describes at least one AMM design			
Inclusion	English Language The publication must be in English language				
Inclusion	Topic Fit The publication must describe at least one AMM design				
	Uniqueness	The publication must not be already included in the set of literature			
	Books	The publication is a book			
	Duplicate	The publication is already included in the set of relevant literature			
Exclusion	Not English	The publication is in a non-English language			
	Off-topic	The publication does not deal with AMMs			

TABLE III: Inclusion and exclusion criteria for literature

b) Conceptual-to-empirical (Iteration 1): To gather a set of AMMs to be classified into the initial version of the AMM taxonomy, we conducted a backward- and forward search based on the previously analyzed 19 publications. The first round of backward and forward searches yielded 1,086 additional publications (i.e., 243 publications by backward search and 843 publications by forward search). We applied our inclusion and exclusion criteria (see Table III) to the meta-information (e.g., title, keywords, abstract) to the potentially relevant publications. We excluded 291 duplicate publications and 692 publications because they were off-topic or lacked AMM design descriptions. Finally, we added 103 relevant publications to the set of relevant literature on AMMs as objects for taxonomy development. In Iteration 1, we randomly selected 33 of 103 publications for analysis. In our coding in iteration 1, we resolved ambiguities and inconsistencies in four refinement rounds. For example, we merged the dimensions *liquidity reversibility* and *liquidity invariance* into *liquidity changeability*. We ended our analysis when no refinements of the AMM taxonomy were required for the last ten analyzed publications. This refined version of the AMM taxonomy resulted in a total of 29 AMM dimensions and 68 characteristics.

c) Empirical-to-conceptual (Iteration 2): After the analysis of 52 relevant publications, we applied an empirical-toconceptual approach to test the AMM taxonomy using implementations of AMMs. We selected a sample of the 100 largest existing AMMs based on their 24-hour trading volume reported on www.coinmarketcap.com. We treated AMMs with identical designs but deployments to different DLT systems (e.g., Uniswap v3 based in the Ethereum system, Uniswap v3 based in the Polygon system) as a single AMM and selected only one implementation of such AMMs for analysis. We treated different versions of AMMs as different AMMs (e.g., Uniswap v2 and Uniswap v3). We excluded order-book protocols, derivative protocols, and aggregator protocols that do not implement AMMs. Moreover, we added ten AMMs whose designs strongly differ from those of the 100 AMMs selected from www.coinmarketcap.com to increase the exhaustiveness of the AMM taxonomy. To classify the selected AMMs, we used corresponding official documentation, whitepapers, yellowpapers, and code repositories. If we could not extract all the necessary information from such official sources, we extended our search to gray literature and contacted the developers to gather the necessary information. We selected the largest 60 AMMs from the set of 110 AMMs to have two batches for AMM classification and used the second batch to test the robustness of the AMM taxonomy. We classified the selected AMMs into the AMM taxonomy. During our classification of the 60 AMMs, we decided to drop pure mathematical descriptions of (parts of) AMMs because they did not match our meta-characteristics. For example, we removed the curve monotony, curve scaling, and curve differentiability dimensions because we decided that those detailed mathematical descriptions are not concise and implicitly covered by other dimensions such as translation invariance. After having classified 60 AMMs into our taxonomy, no refinements of the AMM taxonomy were required for the last 10 analyzed AMMs. After iteration 2, the AMM taxonomy included 18 dimensions and 45 characteristics.

d) Conceptual-to-empirical (Iteration 3): We applied the conceptual-to-empirical approach to incorporate the remaining 70 publications into the AMM taxonomy. We analyzed the selected publications by applying open coding. We refined the AMM taxonomy when we recognized the need to add dimensions and characteristics to the AMM taxonomy or to redefine existing ones. For example, we merged the dimensions *dynamic trading fees* into *asset risk management* because of the latest publications providing new insights on Loss-Versus-Rebalancing [77]. We did not recognize the need to refine the AMM taxonomy in the analysis of the last 34 of the 70 publications. Overall, 63 of the 70 publications confirmed the preliminary AMM taxonomy. Eventually, the AMM taxonomy was comprised of 19 AMM dimensions and 50 AMM characteristics.

e) Empirical-to-conceptual (Iteration 4): To test the robustness of the AMM taxonomy, we analyzed the remaining 50 AMMs and classified them into the AMM taxonomy. For the analysis and classification, we proceeded as described in Iteration 2. Among the 50 AMMs, 3 AMMs required minor refinements of the AMM taxonomy. For example, we added the AMM dimension token price source and added constant-power-sum to the price discovery mechanism dimension. Our final AMM taxonomy and its usableness, we assigned the 21 AMM dimensions to four groups (i.e., governance, liquidity, pricing, and trading). We inductively developed these groups from the dimensions included in the AMM taxonomy. After the fourth iteration, we met our ending conditions (see Table I) and, thus, decided that our AMM taxonomy is final.

B. AMM Archetype Development

To develop AMM archetypes, we discussed the AMMs classified in the taxonomy in terms of their different solution approaches for the LAP, PDP, and TMP. Based on our discussion, we rated the influence the dimensions have on solving the LAP, PDP, and TMP as high, medium, and negligible. We rated the influence of a dimension on the solution of the LAP, PDP, and TMP as high if there is a direct relation to the provision of liquidity, to the price determination, or to the interpretation and acquisition of market information. We rated the influence of a dimension on the solution of the LAP, PDP, and TMP as medium if indirectly related to the provision of liquidity order to price order to interpret and acquire market information. We rated the influence of a dimension on the solution to price determination. We rated the influence of a dimension on the solution of the LAP, PDP, and TMP as medium if indirectly related to the provision of liquidity order to price order to interpret and acquire market information. We rated the influence of a dimension on the solution of the LAP, PDP, and TMP as negligible if no relation to price determination or to the interpretation and acquisition of market information was evident.

We made unanimous decisions on the relevance of all dimensions for solving the LAP, PDP, and TMP. We eventually evaluated the influence of four dimensions on the solution of the LAP, PDP, and TMP as high, that of six dimensions as medium, and that of eleven dimensions as negligible. For example, we rated the influence of the *source of liquidity* dimensions as very high on solving the LAP, PDP, and TMP because its characteristics could solve the LAP by design. We rated the influence of the *liquidity changeability* dimension as a medium because the characteristics influence the available liquidity. We rated the influence of the *path independence* dimension as negligible because the transaction sequencing was assessed as negligible for the LAP, PDP, and TMP.

Next, we removed dimensions presented in the taxonomies from the set of important dimensions for solving the LAP, PDP, and TMP if they have a negligible or medium influence on the LAP, PDP, and TMP. This gave us a set of four dimensions with a total of 15 characteristics.

Last, we grouped the AMMs based on the four dimensions with high influence on the solution of the LAP, PDP, and TMP. We identified 22 combinations of AMM characteristics that formed our preliminary AMM archetypes. We recognized that those 22 preliminary AMM archetypes implement similar solution approaches for the LAP, PDP, and TMP. To ensure the discriminatory power of the AMM archetypes, we decided to use the dimensions *token price source* and *source of liquidity* to separate the AMM archetypes clearly. The selected dimensions have a particularly strong influence on the solutions to the LAP, PDP, and TMP and define the ownership of the AMM components. Based on those two dimensions and four characteristics, we identified four AMM archetypes by building the cross-product. We dropped one AMM archetype because we were not able to identify a *Price-adopting Supply-sovereign AMM* among the 110 analyzed AMMs. We concluded with three AMM archetypes that implement individual solutions for solving the LAP, PDP, and TMP and reflect AMMs used in actual cryptoeconomic systems.

IV. OUR TAXONOMY OF AUTOMATED MARKET MAKERS IN CRYPTOECONOMIC SYSTEMS

This section presents the AMM dimensions and AMM characteristics of the AMM taxonomy. A shortened overview is given in Supplementary Material VIII. We classified 14 exemplary AMMs to demonstrate the applicability of the AMM taxonomy in Table IV.

A. Groups, Dimensions, and Corresponding Characteristics of Automated Market Makers

The AMM taxonomy comprises four groups of AMM dimensions: governance, liquidity, pricing, and trading. These groups include 21 dimensions which cover 53 characteristics of AMM designs. We describe each group, its associated dimensions, and corresponding characteristics in the following.

1) Governance: The rules and processes for adjusting the AMM parameters, including trading fees, token inventory weights, and bid/ask spreads.

a) Governance Model: The distribution of accountabilities and decision rights and the resources involved to decide on adjustments of AMM parameters.

AMMs can implement centralized or decentralized governance models. Centralized governance models restrict the selection of market participants that are allowed to participate in the decision process (e.g., vote on parameter adjustments), for example, the developers that maintain the software of an AMM. Decentralized governance models allow various market participants to participate in the decision process.

b) Parameter Adjustment: The functionality to change AMM parameters.

AMMs can have fixed parameters (e.g., token weights), allow for manual parameter adjustments, or allow for automatic parameter adjustments. Fixed parameters are held constant over time [14], [78]. Manual adjustment allows a selection of market participants to change the parameters. AMMs with automatic adjustment have mechanisms implemented that automatically adjust parameters.

Price discovery and the available liquidity can depend on AMM parameters. For example, the token weights influence the amounts of individual tokens held in liquidity pools of AMMs. This can influence the available liquidity for the individual tokens. Thereby, parameter adjustments can eventually influence the efficacy of AMMs to solve the LAP and PDP.

c) Trading Fee Adjustment: The adjustment of trading fees charged by an AMM.

AMMs charge trading fees for transaction processing [14], [79]. AMMs can charge an adjustable trading fee or a static trading fee [80], [81].

The trading fee adjustment of an AMM can influence the bid/ask spread of token prices. The trading fee is subtracted/added to the token price leading to the bid/ask spread. Furthermore, trading fee adjustments can be used to mitigate potential attacks on AMMs. For example, front-running attacks during high token price volatility can be reduced if the trading fee can be increased in relation to the token price's volatility [14].

2) Liquidity: The availability, constraints, and source of liquidity in the AMM.

a) Liquidity Changeability: Liquidity changeability indicates whether the available liquidity for an AMM can vary.

The liquidity changeability can be constant or variable. For example, AMMs that rely on external liquidity providers often have variable liquidity because liquidity providers can withdraw and deposit liquidity over time. Supply-sovereign AMMs typically have constant liquidity because liquidity is sourced internally.

The liquidity changeability of an AMM influences the stability of the liquidity provided by the AMM in cryptoeconomic system markets. For example, if the liquidity available to AMMs is low, the AMMs can provide less liquidity to the cryptoeconomic system market. AMMs with prescribed liquidity can provide a guaranteed amount of liquidity to cryptoeconomic system markets leading to more stable liquidity.

b) Liquidity Provider Permission: The specification of market participants allowed to deposit tokens into liquidity pools. AMMs can have open liquidity pools that allow any market participant to provide liquidity. On the other side, AMMs can have permissioned liquidity pools that are exclusive for certain market participants to provide liquidity or that are exclusive to internal sources of liquidity.

Liquidity provider permissions can influence the amount of liquidity that can be provided to cryptoeconomic system markets. AMMs that restrict liquidity providers are less likely to accumulate sufficient liquidity and can consequently fail to solve the LAP.

c) Number of Tokens per Liquidity Pool: The variety of tokens that can be deposited in one liquidity pool. The number of tokens per liquidity pool can be exactly two or more.

The number of tokens per liquidity pool influences the flexibility to exchange tokens. For example, market participants can exchange a token A for a token B and a token B for a token A in a liquidity pool with two tokens. In a liquidity pool with three tokens, market participants can exchange A for B, A for C, B for C, B for A, C for A, and C for A. This increases the amount of possible transactions.

d) Risk Management: The mechanisms to manage the risk of holding eventually volatile tokens.

AMMs are exposed to the risk of inventory imbalance (see Section II-B). AMMs can have no risk management, imbalance surcharges, or loss insurance. AMMs with no risk management do not reduce the risk of inventory imbalance for liquidity providers and entirely pass the risk to the liquidity providers. AMMs with imbalance surcharges add surcharges to stated bid/ask token prices based on the current inventory imbalance. Market participants are disincentivized to execute transactions that further increase the imbalance. In addition, bid/ask spreads increase, increasing market-making profits. AMMs that implement loss insurance compensate losses of liquidity providers with external tokens (e.g., tokens of the AMM's cryptoeconomic system).

The risk management of AMMs influences the attractiveness of liquidity providers to provide liquidity to AMMs. This indirectly influences how AMMs can solve the LAP and how much liquidity AMMs can provide to the cryptoeconomic system market.

e) Source of Liquidity: The source of available liquidity for an AMM can be external from liquidity providers or internal by token supply sovereignty.

In external liquidity provision, external liquidity providers deposit tokens into liquidity pools of the AMM. In return, liquidity providers are commonly rewarded with a share of the trading fee. In internal liquidity provision, AMMs have supply sovereignty over tokens and do not depend on external liquidity providers. AMMs with internal liquidity provisions mint new tokens when market participants buy and burn tokens when market participants sell the tokens to the AMM.

An AMM's liquidity source influences how the AMM can solve the LAP. AMMs that rely on external liquidity provision need to solve the LAP because the AMM must attract sufficient liquidity. A token supply-sovereign AMM solves the LAP by design because liquidity is sourced through mint/burn actions of the tokens with token supply sovereignty.

f) Supported Token Pairs: The token pairs that can be traded against each other using an AMM.

AMMs can be open to all tokens and allow arbitrary token trading pairs. AMMs can have restricted token pairs that enforce any token to be paired with one certain token (e.g., USDC paired with any other token) or restrict the tokens that can be paired (e.g., USDC, BTC, and WETH but arbitrary token pairs of the tokens' cross-product).

The supported token pairs of an AMM influence the flexibility with which an AMM is used. If the token pairs are restricted, this eventually affects the demand for individual tokens. For example, suppose a cryptoeconomic system has its own token. All token pairs in the AMM must be paired with the proprietary token. In that case, this increases the demand for this token because liquidity providers must provide all tokens of the liquidity pool.

3) Pricing: The functionalities and properties that are required for asset pricing.

a) Information Expressiveness: The degree to which market information is incorporated into prices.

The price discovery mechanism can either be expressive or inexpressive. Market information can be incorporated based on liquidity that requires the traders to purchase tokens to adjust adequate token prices to approximate the efficient token price [25], [48], [82]. The token price adjustment is correlated with the transaction volume. An inexpressive price discovery mechanism does not adjust token prices based on the transactions of market participants. Thus, their perception of efficient token prices is not included [25].

The information expressiveness of AMMs influences the ability of the AMM to adjust token prices. Information-inexpressive AMMs do not adjust token prices based on the market participants' transactions. The token prices must be adjusted from an external token price source or the AMM must implement some other price discovery mechanism. During our literature and AMM analysis, no price discovery mechanism was found that adjusts token prices in other ways than based on market participants' transactions or price adoption. Information-expressive AMMs adjust token prices based on market participants' transactions.

b) Liquidity Sensitivity: The strength of the influence of liquidity in the AMM on the magnitude of token price changes. If an AMM is liquidity-sensitive, high liquidity mitigates the token price change, while low liquidity amplifies the price changes at a constant volume. For liquidity-insensitive AMMs, token price changes are not influenced by the available liquidity in the AMM [22], [80], [83].

The liquidity sensitivity of AMMs can influence the token price changes caused by transactions. Liquidity-sensitive AMMs adjust token prices in correlation to the available liquidity. Therefore, liquidity-sensitive AMMs can adapt the token price changes to the available liquidity. Liquidity-insensitive AMMs cannot adapt token price changes to the available liquidity. Thereby, liquidity-insensitive AMMs adjust token prices independently of the available liquidity. Token price changes must then be configured based on other parameters of the transaction, such as transaction volume.

c) Liquidity Concentration: The distribution of liquidity at different price levels.

The liquidity concentration in an AMM can be function-based liquidity concentration, LP-based liquidity concentration, or autonomous liquidity concentration [14], [80], [84]. AMMs with function-based liquidity concentration concentrate the liquidity at a certain token price. A mathematical function gives the token price with the highest liquidity and does not change over time. AMMs with LP-based liquidity concentration concentrate the liquidity at price ranges that are given by their liquidity providers [85]. The liquidity provider can decide in which price region its liquidity should be concentrated. AMMs with autonomous liquidity concentration automatically concentrate the liquidity at a certain token price [86]. The token price with the highest liquidity is chosen by some automatism. Typically, time-weighted average prices are used to determine the token price that the liquidity is concentrated on [22], [80].

The liquidity concentration of AMMs influences the efficacy of how AMMs solve the PDP because it influences the available liquidity at different token price levels. Therefore, the liquidity concentration can influence the token price evolution. For example, when liquidity for a correlated token pair (e.g., USDC/USDT) is concentrated at a token price of 1 USDC per 1 USDT, the AMM can settle large transaction volumes with small token price changes at this level. If the token price diverges to 10 USDC per 1 USDT, there is low liquidity. Token prices become more sensitive to changes. The token price can be more or less pinned to a token price of 1 USDC per 1 USDT. The way liquidity is concentrated thereby also determines how the token price is pinned. Function-based liquidity concentration has a static token price pinning. LP-based liquidity concentration allows users to decide where the token price is pinned. Autonomous liquidity concentration implements a mechanism determining where the token price is pinned.

d) Path Deficiency: The feasible transactions in relation to the token reserve of an AMM.

Path deficiency guarantees that the token reserves are always bounded from below (e.g., token reserves cannot shrink) for any set of transactions. Optimal transactions transition to the minimal reserve set that satisfies the lower bound. Transactions above the lower bound either receive less output or add more input as the optimal transaction. This guarantees that an AMM's token reserves cannot be depleted [20]. Strict path deficiency indicates that all transactions must be sub-optimal for the market participant. The market participant must receive less output or add more input than the optimal transaction. This ensures that the AMM's token reserves are increasing at any trade. Path-independent AMMs are path deficient by definition [20]. Strictly path-deficient AMMs are not path independent [20] because market participants must overpay the AMM to increase its token reserves. The AMM transitions to different states when a buy transaction and a sell transaction with equal volume are executed.

e) Price Bounding: The functionality to limit token prices of the AMM.

The price bounding of a pricing mechanism can be bounded from above, bounded from below, and bounded from above and below. Bounded token prices can move in certain price ranges. These price ranges are: bounded from above $[-\inf ...j]$, bounded from below [i...inf], and bounded from above and below [i...j] [17], [87].

The price bounding influences the possible token prices of an AMM. Price bounding limits the available liquidity to a certain token price range. This can cause market liquidity to decrease sharply when the adequate token price exits this price range. Furthermore, price bounding can pin token prices in prescribed ranges if there are no alternative markets to which market participants can switch.

f) Price Discovery: The definition of the process of discovering adequate token prices.

Pricing algorithms implemented in AMMs can be the constant product algorithm, geometric mean algorithm, constant sum algorithm, constant product-sum algorithm, constant power sum algorithm, logarithmic market scoring algorithm, exponential function, and price adoption algorithm.

A constant product algorithm uses a conservation function based on a constant product $c = \prod_{i=1}^{n} r_i$, with r being the amount of token i in reserve and n being the number of tokens that are considered for pricing [14], [19], [20].

Geometric mean algorithms add a weight w to the reserves of tokens and can be expressed as $c = \prod_{i=1}^{n} r_i^{w_i}$. Token reserves can be weighted to have a constant imbalance [44], [88].

Constant sum algorithms use a conservation function that is based on a constant sum $c = \sum_{i=1}^{n} r_i$ [20]. The constant sum algorithm allows for constant exchange rates that are not adjusted over time [14], [21], [88].

Constant product-sum algorithms use a conservation function based on a constant sum of products. The conservation function can be expressed as $\chi D^{x_i} + \prod x_i = \chi D^n + (\frac{D}{n})^n$ with D being the total amount of coins and χ being a leverage factor which is defined as $\chi \in \mathbb{R} | 0 \le \chi$. For $\chi = 0$, the constant product-sum algorithm is a constant product. For $\chi = \infty$, the constant-sum algorithm is a constant sum [52], [53].

Constant power-sum algorithms use a conservation function based on a constant sum of multiple powers. The conservation function can be expressed as: $c = \sum_{i=1}^{n} r_i^{1-t}$ with t being a parameter to change the curvature of the conservation function [89].

Logarithmic market scoring algorithms use a cost function C of the total of assets in the market that can be expressed as follows: $C(q) = b \cdot log(\sum_{j=1}^{n} exp(q_j/b))$ with q being the vector of quantities, b being a strictly positive parameter to control liquidity in the market and n the number of assets [23], [58], [90], [91].

Price adoption algorithms use external price oracles to adopt token prices, which are then adjusted by a mathematical function [16]. Typically, token prices are adjusted based on the token reserve imbalance. The token reserve imbalance can be expressed as: $\Delta R_i = \frac{r_{0,target} - r_{0,current}}{r_{0,target}}$ with $r_{0,target}$ being the targeted token reserves of r_0 and $r_{0,current}$ being the current token reserves of r_0 . The offered token price can be expressed as follows: $P_i = p_{adopted} + p_{adopted}k(\Delta R_i)$ with k being a parameter to configure the magnitude of token price adjustments, which is defined as $k \in \mathbb{R} | 0 \le k \le 1$ [14], [16].

Exponential function pricing algorithms use an exponential conservation function that is based on a constant exponent $c = S_a^{\kappa}/r_b$, with S_a being the total supply of token a, r_b being the amount of token b in reserve and κ being a parameter for curvature [9], [92], [93].

g) Path Independence: The independence of AMMs' state transitions from the order of buy and sell transactions with identical cumulative volume.

Pricing mechanisms can be path-dependent or path-independent. Rearranging the sequential order of buy and sell transactions with identical cumulative volumes leads path-dependent pricing mechanisms to transition to different AMM states. Because AMM states often form the foundation of AMMs to self-determine token prices, path-dependent pricing mechanisms can state different token prices for the same transaction depending on its position in the transaction sequence [15], [22], [83]. For example, executing one buy transaction with 100 USDT volume increases the token price of the trading pair by 1, while executing ten transactions with each 10 USDT volume increases the token price of the pair only by 0.5. Path-independent pricing mechanisms transition to the same state for different sequential orders of transactions with identical cumulative volumes. For example, following the previous example, buy transactions with a cumulative volume of 100 USDT always result in a token price increment of 1, no matter how the transactions are split.

The path independence of an AMM can influence transaction volumes. For path-independent AMMs, transactions are likely to be executed with their full volume in one transaction to save execution costs of the underlying infrastructure (e.g., gas fees in the Ethereum system). For path-dependent AMMs, market participants are likely to split up transactions if transactions with little volume are beneficial (e.g., decrease overall transaction cost). For example, if the AMM charges a transaction fee that is quadratic to the volume of the transactions, it is beneficial for market participants to split up their transactions into smaller-volume transactions to save on transaction costs. In practice, there would be some equilibrium between optimized transaction costs and the execution costs of the infrastructure.

h) Token Price Source: The source of adequate token prices that the AMM uses to quote bid/ask token prices.

The token price source can be internal or external. AMMs with internal token price sources incorporate a price discovery component that discovers the adequate token price [20], [21]. AMMs with external token price sources outsource the token price discovery to a price oracle that provides adequate token prices to the AMMs [16].

The token price source can influence the cost-efficiency of AMMs to adjust token prices to adequate token prices. External token price sources are often more cost-efficient than internal token price sources because the price discovery is outsourced.

The AMM adopts adequate token prices in the absence of transactions. AMMs with internal token price sources are often less cost-efficient because token prices are updated based on buy/sell transactions that indicate an increase/decrease of the adequate token price. Therefore, AMMs sell undervalued or buy overvalued tokens to adjust the token prices. This decreases the cost-efficiency of AMMs.

i) Translation Invariance: The payoff from a portfolio consisting of equal amounts of each asset.

Translation invariance is primarily used in the context of prediction markets. Translation invariant AMMs always charge the same cost for the same amount of each asset at each AMM state [47], [83], [91], [94]. For example, a translation invariant AMM charges 0.5 USD for token A and 0.5 USD for token B. The cost of one token A and one token B (same amount) is 1 USD. Later, AMM's token prices changed to 0.8 USD for token A and 0.2 USD for token B. The cost of one token A and one token A and one token A and one token B at any state of the AMM. Non-translation invariant AMMs charge different costs for the same amount of each asset at different states of the AMM [47]. A non-translation invariant AMM charges, for example, 0.5 USD for token A and 0.5 USD for token B. The cost of one token A and 0.5 USD for token B. The cost of one token A and 0.5 USD for token B. The cost of one token A and 0.5 USD for token B. The cost of one token A and 0.5 USD for token B. The cost of one token A and 0.5 USD for token B. The cost of one token A and 0.5 USD for token B. The cost of one token B and 0.5 USD for token B. The cost of one token A and 0.5 USD for token B. The cost of one token A and 0.5 USD for token B. The cost of one token A and 0.5 USD for token B. The cost of one token A and 0.5 USD for token B. The cost of one token A and 0.5 USD for token B. The cost of one token A and 0.5 USD for token B. The cost of one token A and 0.5 USD for token B. The cost of one token A and 0.3 USD for token B. The cost of one token A and one token B at different states of the AMM charges different costs for one token A and one token B at different states of the AMM.

4) Trading: The types, functionalities, and properties of trade execution in the AMM.

a) Interoperability: The capability of an AMM to execute transactions across multiple infrastructures (e.g., multiple DLT systems) of cryptoeconomic systems.

AMMs can be interoperable or non-interoperable. For example, interoperable AMMs enable market participants to transact across DLT systems [5]. Non-interoperable AMMs settle transactions in cryptoeconomic systems built on a single DLT system. Senders and recipients of tokens must be part of the same DLT systems. The interoperability of an AMM influences the ease of transaction execution for market participants, as they can switch infrastructures simultaneously through one transaction.

b) Limit Order Functionality: The functionality to create limit orders.

Limit orders are instructions to buy or sell tokens at specified token prices but without the guarantee of immediate execution [35]. The instruction is triggered when the AMM's token price strikes the specified token price of the limit order. The AMM can either provide limit order functionality or not. The limit order functionality of AMMs can influence market participants' usage of AMMs. Limit order functionality can be essential for larger organizations to have conditional transaction execution.

c) Price Guarantee: The process that determines token prices in cryptoeconomic system markets.

Token prices may change in the time between issuing transactions to a cryptoeconomic system and the settlement of those transactions. Market participants may thus sell or buy tokens at a different price than actually intended. AMMs can be designed to deal with changing token prices in three ways: give no price guarantee, guarantee price ranges, or guarantee exact prices. First, AMMs without a token price guarantee do not guarantee any price for transaction settlement. The settlement price is unknown at transaction issuance. Second, AMMs can implement ranged price determination that guarantees transaction settlement only in a specified price range, known as slippage. The slippage of token prices can usually be set per transaction. Third, AMMs can guarantee exact price determination. The token price at transaction issuance equals the token price at transaction settlement.

The price guarantee of an AMM influences the ease of transaction execution for market participants. Configurable price ranges and exact price guarantees reduce the risk of transaction execution for market participants. Little price guarantees increase financial risks for market participants because transactions could be settled at unprofitable prices.

d) Volume Dependency: The dependency of token prices on the transaction volume in a cryptoeconomic system market. The volume dependency of a pricing mechanism can be dependent and independent. Volume-dependent pricing mechanisms differ in the mean price of transactions based on their transaction volume. Volume-independent pricing mechanisms state the equivalent mean prices for different transaction volumes. Volume dependency leads AMMs to have different token price changes, while volume-independent AMMs have equal or no token price changes resulting from transactions.

B. Applicability Demonstration of the AMM Taxonomy

We demonstrate the applicability of the AMM taxonomy by illustrating the classification of exemplary AMMs in the AMM taxonomy in Table IV. Each AMM has exactly one characteristic of each dimension. The demonstration of the AMM taxonomy shows its mutual exclusiveness because all AMMs have exactly one characteristic per dimension. The relevance and representativeness are shown because all characteristics are occupied by at least one AMM design. The limited number of dimensions and characteristics shows the conciseness and exhaustiveness of the AMM taxonomy.

			[7]	WP [95]	MP [96]	5T [13]	[52]	16]	2021 [21], [97]	nce [98]	vap Sb [99]	v2 [19]	v3 [85]	[6]	ce [89]
dno.			ıgur [5	lancer	lancer	ncor S	tve v1] 000	Stable	f.Finaı	IORSv	uiswap	uiswap	ooFi [∠	eldSpa
5	Dimension	Characteristic	Au	Ba	Ba	Ba	Cn	μă	n d	Re	Ē	Un	Un	M	Ϋ́,
	Governance Model	Centralized			х				x						
nce		Decentralized	X	X		X	X	X		X	X	х	х	х	X
nai		Automatic						x			X				X
ver	Parameter Adjustment	Fixed	X	X		X	X		X			X	X		
8		Dynamic			X			v		X	x			X	
	Trading Fee Adjustment	Static	x	x	•	x	x		x	x		x	x	x	x
		Prescribed				x									
	Liquidity Changeability	Variable	x	x	х		x	x	x	x	X	х	х	х	x
	Liquidity Provider	Open	x	x			х	x	x	x	X	х		х	X
	Permission	Permissioned			Х	X									
ty	Number of Tokens per	Three or More				X	X	x	X	X	X	Х	X	х	X
idi	Liquidity Pool	Two	X	X	X										
iqu	Pick Management	Imbalance Surcharges						X			v			X	
	KISK Management	No Risk Management	v	v	v	v	v		v	v	А	v	v	-	v
		External	x	X	X		X	x	x	x	x	X	X	x	X
	Source of Liquidity	Internal				x									
	Summented Tradine Dains	Open	x	x			x	x		x	x	х	x	x	
	Supported Trading Pairs	Restricted			х	x			x						X
	Interoperability Limit Order Functionality	Interoperable								X				х	
		Non-interoperable	X	X	Х	X	x	x	x		X	х	х		X
ng		Included								X					
adi		Not Included	X	X	X	X	X	X	X		X	X	X	X	X
Ë	Price Guarantee	Price Pange	v	v	v	v	v	v	v	v	v	v	v	v	X
		Volume-dependent	x	x	x	x	x	x		x	x	x	x	x	x
	Volume Dependency	Volume-independent							x						
	Information	Expressive	x	x	х	x	x	x		x	X	х	х	х	x
	Expressiveness Pool	Inexpressive							x						
	Liquidity Concentration	Automatic						x						х	
		Function-based	X	X	Х	X	x		x		X	Х			X
		LP-based								X			X		
	Liquidity Sensitivity	Songitive							X						v
		Deficient	A Y	A	л	A Y	Λ	<u> </u>		<u> </u>	Λ	Λ	А.	А.	
	Path Deficiency	Strictly Deficient		x	x		x	x	x	x	x	x	x	x	x
	Dath Indonen I	Path Dependent						x			X				
	Pain Independence	Path Independent	x	x	х	x	x		x	x		х	х	х	X
<u>5</u> 0	Price Bounding	Bounded from Above													X
icir		Bounded from Above and Below		X	Х		x	X		X	X	Х	х	х	
Pr		Bounded from Below							X						
		Constant-sum							X						
	Price Discovery	Constant-product					v			X	X				
		Constant-product-sum					<u>л</u>								x
		Exponential Function				x									
		Geometric Mean		x	x										
		Logarithmic Market Scoring	x												
		Price Adoption						X						X	
	Token Price Source	External	X	X	Х	X	х		Х	X	Х	Х	Х		Х
	Token Thee Source	Internal						x						Х	
	Translation Invariance	Non-translation Invariant		X	X	X	X	X		X	X	X	X	X	X
		Translation Invariant	X						X						

TABLE IV: Demonstration of the AMM Taxonomy

V. AMM ARCHETYPES AND THEIR SOLUTION APPROACHES FOR THE THIN MARKET PROBLEM

We developed three AMM archetypes (i.e., Price-discovering LP-based AMM, Price-adopting LP-based AMM, and Pricediscovering Supply-sovereign AMM) that can be distinguished by two dimensions of the AMM taxonomy (i.e., price discovery and source of liquidity). Figure 2 illustrates the designs of the three AMM archetypes. Corresponding to their different characteristics associated with the dimensions *price discovery* and *source of liquidity*, the three archetypes implement different solution approaches to tackle the LAP, PDP, and TMP. In this section, we describe the three AMM archetypes in more detail.

A. Price-discovering LP-based AMM

The Price-discovering LP-based AMM (see Figure 2a) incorporates a price discovery component, a price determination component, a token settlement component, a parameter component, and an LP token management component. The Price-discovering LP-based AMM trades tokens of token management components that at least one external token keeper operates. Following the AMM taxonomy, the Price-discovering LP-based AMM is characterized by an *internal price discovery* and *LP-based source of liquidity*.

When the Price-discovering LP-based AMM is set up, its price discovery component is initialized with a token price determined by the first liquidity provider that initializes the AMM. After the token price initialization, the price discovery component determines token prices based on a prescribed mathematical function that incorporates market participant perceptions of the token price as follows. Market participants only buy undervalued tokens and sell overvalued tokens to take profits. When a market participant buys/sells tokens at a specific token price, it is assumed that the rational market participant knows information by which the new adequate token price diverges from the stated token price. The Price-discovering LP-based AMM increases the adequate token price as higher than the token price stated by the Price-discovering LP-based AMM. Conversely, the Price-discovering LP-based AMM decreases the adequate token price when market participants sell tokens because it is assumed that market participants because it is assumed token price [20], [44].

The magnitude of price changes depends on the liquidity available to the Price-discovering LP-based AMM. High liquidity leads to smaller token price changes. Low liquidity leads to larger token price changes (see Section II-D0c). Because the price discovery mechanism of the Price-discovering LP-based AMM is typically deterministic, price changes of transactions with a given volume are predictable. The price determination component of the Price-discovering LP-based AMM calls the price discovery component to fetch the current token price, adjusts it by adding trading fees provided by the parameter component, and passes the determined token amounts to the token settlement component.

The token settlement component calls external token management components to transfer tokens between the Price-discovering LP-based AMM and market participants. Token management components record the token balances of the AMM and other market participants. The balances of the Price-discovering LP-based AMM in the token management components form the token reserves of its liquidity pool. The token management components are controlled by token keepers that can be part of multiple cryptoeconomic systems at the same time. Thus, tokens managed by token keepers may be accessed by multiple AMMs and other market participants.

Third-party liquidity providers deposit tokens into liquidity pools in the token management components of external token keepers. In exchange for depositing tokens, liquidity providers receive LP tokens in return for their token deposits. LP tokens are issued by the LP token keeper component of the Price-discovering LP-based AMM and represent a claim on a share of tokens in the Price-discovering LP-based AMM's liquidity pool. Using the LP tokens, liquidity providers can withdraw their share of tokens from liquidity pools. By depositing and withdrawing tokens, the liquidity offered by the Price-discovering LP-based AMM can change over time.

The Price-discovering LP-based AMM implements an incentive mechanism to accumulate sufficient liquidity and keep liquidity providers motivated not to withdraw deposited tokens. Common incentive mechanisms distribute revenues for token deposits to liquidity providers. Such revenues correspond to shares of the transaction fees charged by the AMM for transaction settlement. The following describes how the Price-adopting LP-based AMM can solve the PDP, LAP, and TMP.

a) Price Determination Problem: The Price-discovering LP-based AMM incorporates available market information via trade indications into the token prices to solve the PDP by discovering adequate token prices. Transactions of market participants with the Price-discovering LP-based AMM convey such market information. When new market information (e.g., fundamentals, regulatory news, new products) is available, the token price stated by the Price-discovering LP-based AMM can diverge from the adequate token price. This divergence incentivizes rational market participants to collect and analyze market information to get the chance to buy temporarily underpriced tokens and sell them in the future to realize profits. Rational market participants sell temporarily overpriced tokens. Buying and selling tokens feeds the Price-discovering LP-based AMM with market information required to adjust its stated token price to the adequate token price. The buy and sell transactions of the market participants function as indicators for the Price-discovering LP-based AMM. The Price-discovering LP-based AMM can solve the PDP depending on the market information provided by the market participants.



Fig. 2: Overview of the designs of the developed AMM archetypes

b) Liquidity Accumulation Problem: To solve the LAP, the Price-discovering LP-based AMM must accumulate sufficient liquidity. The Price-discovering LP-based AMM incentivizes liquidity providers to deposit tokens in its liquidity pools by distributing token rewards. Liquidity providers receive a share of the collected trading fee that is charged when market participants trade with the AMM. Rational liquidity providers are likely to deposit tokens into the liquidity providers will not deposit tokens, which can render the Price-discovering LP-based AMM unable to settle transactions or oversensitive to low-volume transactions. The Price-discovering LP-based AMM can solve the LAP depending on its ability to incentivize liquidity providers to deposit tokens.

c) Thin Market Problem: To solve the TMP, the Price-discovering LP-based AMM must solve the LAP to provide sufficient liquidity to the cryptoeconomic system market and solve the PDP to state reliable token prices. It must be distinguished between two cases of the efficacy of how the LAP is solved by the Price-discovering LP-based AMM to evaluate if the TMP can be solved.

In the first case, the Price-discovering LP-based AMM implements suitable incentives for accumulating sufficient liquidity to solve the LAP. High liquidity leads to less sensitive token prices. Thus, market participants can buy/sell larger amounts of tokens when the stated token price diverges from the adequate token price. This increases the possible profit for market participants. Market participants are more likely to collect and analyze all available market information to profit from the token price divergence. The increased number of settled transactions provides more market information to the Price-discovering LP-based AMM to adjust its stated token price to the new adequate token price. This increases the reliability and timeliness of token price adjustments by the Price-discovering LP-based AMM.

In the second case, the Price-discovering LP-based AMM offers too few incentives to solve the LAP and accumulates insufficient liquidity. Low liquidity increases the sensitivity of token price adjustments. Thus, small amounts of tokens are required to adjust the stated token price to the new adequate token price. Consequently, market participants can only buy/sell smaller amounts of tokens when the token price stated by the Price-discovering LP-based AMM diverges from the adequate token price. This decreases the possible profit for market participants. Market participants become unlikely to collect and analyze market information because their costs to collect and analyze market information may not exceed the possible profit. Market participants trade less frequently against the Price-discovering LP-based AMM, which decreases its reliability and timeliness of token price adjustments. Thus, the Price-discovering LP-based AMM cannot solve the TMP if the incentives for liquidity providers to solve the LAP are insufficient.

In summary, if the Price-discovering LP-based AMM solves the LAP, it can solve the TMP. To solve the LAP, the Pricediscovering LP-based AMM must implement incentives that motivate market participants to provide sufficient liquidity.

d) Purposes of the Price-discovering LP-based AMM: The Price-discovering LP-based AMM is used for the purposes of *decentralized token exchange* and *token issuance*. Common purposes for decentralized token exchange are correlated token markets, uncorrelated token markets, non-fungible token markets, and prediction markets. Depending on the tokens traded by the Price-discovering LP-based AMM, different designs of price discovery mechanisms and liquidity sensitivity are implemented. For example, constant product price discovery mechanisms are often used with user-based liquidity concentration suitable for exchange correlated tokens because liquidity can be concentrated around the current exchange rate [45], [46], [52]. To exchange correlated tokens, constant product-sum price discovery mechanisms are often used in combination with function-based liquidity concentration to concentrate liquidity at a fixed exchange rate [46], [100]. For prediction tokens, the token prices of Price-discovering LP-based AMM are typically bounded in the price range of 0 to 1 to reflect the occurrence probability of the event that is adjusted by the price discovery mechanism.

Price-discovering LP-based AMM can be used for token issuance, more specifically for initial token offerings and curation tokens. In both purposes, the token issuer (e.g., developers of a cryptoeconomic system) is the first liquidity provider that deposits the tokens to issue in the liquidity pool. The Price-discovering LP-based AMM trades these tokens with market participants to bring them into circulation.

B. Price-adopting LP-based AMM

The Price-adopting LP-based AMM (see Figure 2b) incorporates a price determination component, a token settlement component, a parameter component, and an LP token management component. The Price-adopting LP-based AMM uses at least one external price discovery component that is part of the token keeper of eventually other cryptoeconomic systems. The traded tokens are managed by external token management components. Following the AMM taxonomy, the Price-adopting LP-based AMM is characterized by an *external price discovery* and *LP-based source of liquidity*.

The price determination component fetches token prices from at least one external price discovery component that is operated by an external *price oracle* (e.g., Chainlink). The price oracle is configured when the AMM is set up and discovers adequate token prices in a proprietary way (e.g., retrieving token prices from Binance or Coinbase). Retrieved token prices can be modified by the price determination component based on parameters of the parameter component. For example, the Priceadopting LP-based AMM commonly adds a surcharge to the token price when its token reserves become imbalanced. Given equal-sized transaction volumes, the token reserves are getting less imbalanced in percentage terms when having high liquidity compared to low liquidity. Thus, in the case of high liquidity, the surcharge remains smaller. In consequence, high liquidity decreases the token price sensitivity. Low liquidity increases the token price sensitivity. The adjusted token price is subsequently passed from the price determination component to the token settlement component. The Price-adopting LP-based AMM's token settlement component interacts with external token management components that manage tokens and record balances of market participants. The LP token management component issues LP tokens to the liquidity providers, presenting a claim on a share of tokens in the liquidity pool of the Price-adopting LP-based AMM. Thus, the offered liquidity can change over time. The Price-adopting LP-based AMM implements an incentive mechanism that distributes revenues for token deposits to incentivize market participants to deposit tokens into the liquidity pools of the Price-adopting LP-based AMM. The following describes how the Price-adopting LP-based AMM can solve the PDP, LAP, and TMP.

a) Price Determination Problem: The Price-adopting LP-based AMM outsources the price discovery by using a price discovery component operated by an external price oracle. The external price oracle must solve the PDP and employ a price discovery mechanism. For example, external price oracles provide token prices that are typically derived from thick markets such as Coinbase or Binance. However, these thick markets are assumed to manifest the adequate token price but are mainly centralized. Consequently, the Price-adopting LP-based AMM does not solve the PDP itself but, in practice, relies on a lead market to resolve the PDP on its behalf. Thus, the Price-adopting LP-based AMM can never become the lead market itself and fails to replace centralized markets.

b) Liquidity Accumulation Problem: To solve the LAP, the Price-discovering LP-based AMM must accumulate sufficient liquidity. As it is for the Price-discovering LP-based AMM, the Price-adopting LP-based AMM incentivizes liquidity providers to deposit tokens in its liquidity pools and rewards them with a share of the collected trading fee. Thus, rational liquidity providers are likely to deposit tokens into the liquidity pools if they perceive the token rewards as profitable. Consequently, the Price-adopting LP-based AMM can solve the LAP depending on its ability to incentivize liquidity providers to deposit tokens.

c) Thin Market Problem: The Price-adopting LP-based AMM does not incorporate a price discovery component. Thus, it depends on external price oracles to discover adequate token prices. Consequently, the Price-adopting LP-based AMM cannot create a cryptoeconomic system lead market. In most cases, price oracles only provide token prices for lead markets. The Price-adopting LP-based AMM cannot provide liquidity to a cryptoeconomic system market that is subject to the TMP. Only if an omniscient price oracle provides adequate token prices in thin markets, the Price-adopting LP-based AMM can solve the TMP. The efficacy of the Price-adopting LP-based AMM to solve the TMP depends on the reliability of external liquidity providers in providing tokens to solve the LAP.

In summary, the Price-adopting LP-based AMM can only solve the TMP and become a lead market if an omniscient price oracle is available. If an omniscient price oracle is available, the Price-adopting LP-based AMM must implement suitable incentives for liquidity providers to deposit sufficient tokens to solve the LAP.

d) Purposes of the Price-adopting LP-based AMM: The Price-adopting LP-based AMM is used for the purpose of decentralized token exchange, in particular, correlated tokens, uncorrelated tokens, non-fungible tokens, and prediction markets [16], [49]. The principal design of the components in the Price-adopting LP-based AMM is equal for all types of tokens that can be traded. However, the parameters in the parameter component vary based on the token price correlation of the traded tokens. For example, correlated tokens are likely to have smaller surcharges when having a token reserve imbalance because correlated tokens do not diverge in their price development. There is a low risk of quantitatively holding more of the less valuable tokens. Vice versa, uncorrelated tokens are likely to have higher surcharges when having token reserve imbalance because there is a higher risk of holding more of the less valuable tokens because uncorrelated tokens are likely to diverge in their price development.

C. Price-discovering Supply-sovereign AMM

The Price-discovering Supply-sovereign AMM (see Figure 2c) incorporates a price discovery component, a price determination component, a token settlement component, and a token management component. The Price-discovering Supply-sovereign AMM controls at least one internal token management component and, thus, can transfer, mint, and burn tokens independent from external token keepers. In contrast to the previous two AMM archetypes, the Price-discovering Supply-sovereign AMM does not incorporate an LP token management component. Following the AMM taxonomy, the Price-discovering supplysovereign AMM is characterized by *internal price discovery* and *supply-sovereign source of liquidity*.

When the Price-discovering Supply-sovereign AMM is set up, the creator (e.g., AMM developer) defines a supply curve for the issued token within the price discovery component. The supply curve is a mathematical function that maps the circulating token supply to a token price. Although arbitrary supply curve shapes would be possible, it is best practice to use monotonically increasing supply curve shapes. As for the Price-discovering LP-based AMM, the market participant perceptions of the token price are incorporated. Market participants can buy/sell undervalued/overvalued tokens to profit from price divergences. The price discovery component changes token prices based on the prescribed supply curve. However, when market participants buy/sell tokens, the circulating token supply increases/decreases because tokens are minted/burned by the internal token management component. The magnitude of price changes depends on the slope of the supply curve. A steep slope for a given transaction volume leads to larger token price changes. A flat slope leads to smaller token price changes.

Established cryptoeconomic systems are operated with a fixed token supply or slowly increasing/decreasing token supply via inflation/deflation. Perceived value changes of the entity (e.g., the cryptoeconomic system, organization, product, right, or claim) that is represented by the token result in token price adaption. For the Price-discovering Supply-sovereign AMM, token prices and the amount of circulating tokens change in a concerted way, defined by the supply curve. Therefore, perceived value changes of the entity lead to the adaption of token price and circulating token supply. For example, if the perceived value increases, the supply curve of the Price-discovering Supply-sovereign AMM increases the token price and the token supply. An increasing token supply decreases the value of each individual token. The increasing token supply attenuates token price changes. In consequence, the token price changes can be decoupled from value changes.

Because the supply curve is prescribed, the price discovery is typically deterministic. Thus, price changes of transactions with a given volume are predictable. The price discovery component of the Price-discovering Supply-sovereign AMM passes the adequate token price to the price determination component. The price determination component adjusts the token price based on parameters of the parameter component (e.g., trading fees). The adjusted token price is subsequently passed to the token settlement component.

The token settlement component calls the internal token management component and external ones. The internal token management component is instructed to burn/mint tokens of/for market participants. The external token management components transfer tokens between the Price-discovering Supply-sovereign AMM and market participants.

The Price-discovering Supply-sovereign AMM sources liquidity by burning/minting tokens in the internal token management component. In the case that zero tokens have been issued, there is no market participant that can sell a token. Thus, no token reserves on external token management components are required. When market participants buy the first tokens, these tokens are minted by the internal token management component. Market participants pay with tokens of external token management components. The Price-discovering Supply-sovereign AMM accumulates token reserves on the external token management components that back the issued tokens and are later used when market participants sell tokens back to the AMM. If there is no supply limit for the token of the internal token management component, the Price-discovering Supply-sovereign AMM is always liquid. In the case of a token supply limit, the Price-discovering Supply-sovereign AMM could get illiquid when the upper supply limit is reached. No new tokens can be minted.

In contrast to the Price-discovering LP-based AMM and the Price-adopting LP-based AMM, the Price-discovering Supplysovereign AMM does not depend on liquidity providers to deposit tokens. Instead, the liquidity of the Price-discovering Supply-sovereign AMM is prescribed by the supply curve that gives the price changes and the token supply limit. Because the supply curve is prescribed and the price discovery is deterministic, the available liquidity is also prescribed at any point in time. In the following, we describe how the Price-adopting LP-based AMM can solve the PDP, LAP, and TMP.

a) Price Determination Problem: To solve the PDP by discovering adequate token prices, the Price-discovering Supplysovereign AMM incorporates available market information via trade indications into the stated token prices. Correspondingly to the Price-discovering LP-based AMM, stated token prices can diverge from the adequate token price. Thus, rational market participants collect and analyze market information to profit from token price divergences and induce token price changes through buy/sell transactions. The Price-discovering Supply-sovereign AMM can solve the PDP depending on the market information provided by the market participants.

b) Liquidity Accumulation Problem: To solve the LAP, the Price-discovering Supply-sovereign AMM creates its own liquidity by burning/minting the tokens of its internal token management component. As described above, the Price-discovering Supply-sovereign AMM is always liquid as long as an eventual token supply limit is not reached. The available liquidity is prescribed by the supply curve of the price discovery component. In consequence, the Price-discovering Supply-sovereign AMM solves the LAP by design.

c) Thin Market Problem: The efficacy with which the Price-discovering Supply-sovereign AMM can solve the TMP depends on the supply curve chosen by the issuer. The supply curve indicates the direction of future price evolution based on the value of the underlying entity. Consequently, choosing a suitable supply curve for the Price-discovering Supply-sovereign AMM is difficult. In most cases, issuers are interested in rising token prices. We assume that issuers would choose a monotonously increasing supply curve to issue tokens. To evaluate whether the TMP can be solved, we distinguish between three cases of supply curves.

In the first case, the issuer chooses a supply curve for the Price-discovering Supply-sovereign AMM that is too steep. The steep supply curve leads to a steep price evolution. Early investors (e.g., developer team and seed-investors) have high profit margins. They can buy tokens very cheaply and profit from the steep price evolution. The steep supply curve results

in low liquidity because token prices change rapidly for small-volume transactions that burn/mint tokens. Consequently, the Price-discovering Supply-sovereign AMM cannot solve the TMP because of high token price fluctuation and low liquidity but allows for high profit margins.

In the second case, the issuer chooses a supply curve that is too flat. Early investors have little profits because of the flat price evolution. The flat supply curve results in high liquidity because token prices change very little, even for large-volume transactions. Consequently, the Price-discovering Supply-sovereign AMM can solve the TMP by providing stable token prices and high liquidity but attenuating profit margins.

In the third case, the issuer chooses a suitable slope for the supply curve and suitable profits for early investors, suitable liquidity, and suitable price evolution. Consequently, the Price-discovering Supply-sovereign AMM can solve the TMP with reasonable profit margins.

In summary, there is a dependency between the supply curve slope and the efficacy with which the Price-discovering Supplysovereign AMM can solve the TMP. Steep supply curve slopes attract more investors because of eventually high profits, but the TMP cannot be solved. A flat slope attracts fewer investors because of low profits, but the TMP can be solved. Consequently, issuers must choose a suitable supply curve within this trade-off to attract sufficient investors while solving the TMP.

d) Purposes of the Price-discovering Supply-sovereign AMM: The Price-discovering Supply-sovereign AMM is exclusively used for *token issuance* because supply-sovereignty is needed by design. Common purposes for token issuance are initial token offerings [13] and curation tokens [61], [62]. In both purposes, issuers must define the supply curves implemented in the Price-discovering Supply-sovereign AMM.

VI. DISCUSSION

A. Principal Findings

In this work, we present an AMM taxonomy including four groups (i.e., pricing, liquidity, trading, and governance) comprising 21 dimensions with a total of 53 characteristics. Building on the design specifications of AMMs presented in the AMM taxonomy, we introduced three AMM archetypes (i.e., the Price-discovering LP-based AMM, the Price-adopting LP-based AMM, and the Price-discovering Supply-sovereign AMM) implementing distinct solution approaches for the TMP by tackling the LAP and PDP.

In our literature analysis, we recognized the dominance of gray literature, technical documentation, and blog articles in the field of AMMs. The development of AMMs seems to be mainly driven by practice and only gradually gaining ground in science. In the AMM taxonomy development, we recognized that boundaries between AMMs and other parts of cryptoeconomic systems are often not clearly defined. For example, Uniswap is described as a peer-to-peer protocol for decentralized token exchange that utilizes an AMM [100]. In contrast, the official Uniswap v3 whitepaper says that Uniswap is a non-custodial AMM [85]. Such inconsistent wording makes drawing clear system boundaries for AMMs and their uses in cryptoeconomic systems difficult. We argue that AMMs are market makers implemented as software agents that trade tokens with market participants at self-determined prices in an automated manner. The purposes of AMMs include decentralized token exchanges and token issuance. A decentralized exchange (DEX) is a decentralized marketplace that is the decentralized equivalent of traditional exchanges. DEXs allow market participants to trade tokens with each other. Consequently, AMMs are market participants in DEXs that continuously offer token trades at stated bid/ask token prices with other market participants. DEXs can contain several types and instances of AMMs. Each instance of AMM is responsible for exactly one liquidity pool that allows other market participants to exchange the tokens included in its liquidity pool.

In the classification of AMMs into the AMM taxonomy, we recognized that the price discovery mechanisms implemented in AMMs strongly differ depending on the purposes of the AMMs. There is a strong influence of the price discovery mechanism on the token price evolution. Some price discovery mechanisms are better suited for certain purposes (e.g., correlated or uncorrelated token exchange) than others. For example, constant-product price discovery mechanisms are frequently used in exchanges of uncorrelated tokens because token prices are changed almost equally at all token prices [14]. Constant-product-sum price discovery mechanisms are used in exchanges of correlated tokens because token prices are less changed at a specified exchange rate than constant-produce price discovery mechanisms [14]. Such dynamics in the price evolution of different price discovery mechanisms on token price evolution are needed to enable AMMs to meet their individual purposes.

Many AMM designs are customized forks of code repositories of established AMMs, such as Uniswap v2 and Uniswap v3. Such forks of AMM code bases are partially modified (e.g., SushiSwap adding staking and liquidity mining to Uniswap v2 or PancakeSwap running on Binance Smart Chain instead of the Ethereum system). Such modifications lead to a large variance between AMMs classified into the AMM taxonomy (see Table IV). For example, the dimensions *allowed trading pairs, trading fee adjustment*, and *parameter adjustment* are only slightly modified in most forks of the AMM designs (e.g., Sushiswap, PancakeSwap). The solution approaches for the LAP, PDP, and TMP remain largely unchanged. Since AMM operators usually issue their own tokens (e.g., UNI, SUSHI), the operators can profit by selling these issued tokens. We suspect that this monetary interest has created a large number of new AMMs that have modified existing AMMs with little effort to create their own AMMs to profit from selling custom tokens.

Despite the arbitrary variance of characteristics that are easy to modify in AMM designs, the applicability demonstration of the AMM taxonomy (see Table IV) shows that single characteristics per dimension have become dominant in AMM designs. For example, most AMM designs are non-translation invariant and information expressive based on liquidity [19], [45], [101]. Only a few AMM designs with constant-sum price discovery are translation invariant and information inexpressive [14], [21], [97]. The dimensions *information expressiveness, translation invariance,* and *volume dependency* appear to depend on the dimension *price discovery*. For example, AMMs with constant-sum price discovery are *information inexpressive, translation invariant*, and *volume independent*. Such AMMs have known weaknesses for the purpose of decentralized token exchange. For example, due to *information in-expressiveness* and *volume independence*, they cannot adjust token prices [21], [47]. In addition, AMMs with constant-sum price discovery until 2021 when they switched to constant-product-sum because of the weaknesses of constant-sum price discovery [21]. It is unclear whether this characteristic will still be relevant in the future.

We identified the Price-discovering LP-based AMM as the predominant archetype that is widespread with 98 occurrences in our analysis of 110 AMMs (see Table IV). Extant literature also focuses on Price-discovering LP-based AMM. The Pricediscovering Supply-sovereign AMM and Price-adopting LP-based AMM are much less represented in extant literature. The Price-discovering LP-based AMM appears frequently due to forks of well-known AMMs, such as Uniswap v2, Uniswap v3, and Curve, which are attributed to the Price-discovering LP-based AMM. Because those forks change a few characteristics with minor influences on the solution of the LAP, PDP, and TMP, there is a large variance in the analyzed AMM designs.

The Price-adopting LP-based AMM is less used in practice, presumably because of its dependency on external price oracles and its incapability of solving the TMP. However, our results indicate that the Price-adopting LP-based AMM can enable a cost-efficient decentralized token exchange if they are used in thick markets with reliable external price oracles. AMM archetypes with internal price discovery mechanisms suffer from financial losses if token prices diverge from adequate token prices because tokens are sold under price or bought over price. In contrast, the Price-adopting LP-based AMM does not sell tokens under price or buy tokens over price if the price oracle reliably provides adequate token prices. In this case, token prices are adjusted without selling tokens under price or buying tokens over price.

The Price-discovering Supply-sovereign AMM is exclusively used for the purpose of token issuance because it requires supply sovereignty over at least one token. Our findings indicate that the Price-discovering Supply-sovereign AMM is most suitable for the purpose of token issuance because issuers can prescribe the supply curve of the token. Thereby, token issuers can guide price evolution to increase its predictability for the issuers and facilitate the assessment of financial risks for investors. However, the Price-discovering Supply-sovereign AMM is scarcely researched. Specific effects on price evolution still remain unclear. We suppose that Price-discovering Supply-sovereign AMMs offer a new way of token issuance. To the best of our knowledge, no equivalent concept in economics considers the issuance of tokens or stocks with unlimited supply. The mechanism to burn/mint tokens based on a defined supply curve allows issuers to control or even dictate the price evolution based on the total value of the entity the token represents. Because the Price-discovering Supply-sovereign AMM backs issued tokens according to the supply curve, the tokens can be sold at any time using the token prices calculated from the supply curve. This creates a guaranteed minimum token price for investors. The guaranteed value of the underlying entity is given by the tokens that back the issued tokens.

B. Contributions to Practice and Research

Our study bridges the gap between practice and research and informs about the dimensions and characteristics of AMM designs and commonly implemented solution approaches for the TMP. We contribute to research in three ways. First, we contribute to the understanding of AMM designs by presenting an AMM taxonomy. The AMM taxonomy can be used to guide AMM development as it points out dimensions that need to be considered by developers and offers options to implement the dimensions in the form of characteristics. Moreover, the AMM taxonomy is useful for the systematic comparison of AMM designs, for example, to identify design differences.

Second, by presenting AMM archetypes (i.e., Price-discovering LP-based AMM, Price-adopting LP-based AMM, Pricediscovering Supply-sovereign AMM), we support the understanding of the basic functioning of common AMM designs that are used for specific resource allocation purposes (e.g., token issuance). The AMM archetypes can be used as abstract blueprints that can be refined to develop AMMs that meet resource allocation purposes. For example, AMM developers can fine-tune AMM archetypes by selecting characteristics of the AMM taxonomy.

Third, by explaining the principal solution approaches implemented in AMMs to tackle the TMP and the efficacy of these solution approaches, we contribute to understanding how the TMP can be addressed through AMM designs. Moreover, we explain how the individual solution approaches to tackling the TMP depend on market dynamics, such as the frequency of buy and sell orders. This understanding is useful to support practitioners in predicting financial risks arising from the TMP and taking corresponding actions to avert such risks. For example, investors can precisely predict the price impact caused by buying and selling larger amounts of tokens.

C. Limitations and Future Research

Extant research is mainly concerned with Price-discovering LP-based AMMs (e.g., constant-function market makers [14], [21], [102]) and Price-adopting LP-based AMMs (e.g., proactive market makers [16], [21], [72]). Research on Price-discovering Supply-sovereign AMM is still in its infancy. Only a few publications on Price-discovering Supply-sovereign AMMs are available [9], [103], [104]. Influences on cryptoeconomic system markets emerging from an eventually unlimited token supply are not addressed yet and are still unclear. In the description of the Price-discovering Supply-sovereign AMM (see Section V-C), we addressed the influences of the unlimited token supply to the best of our knowledge. Such assumptions should be evaluated in a future study to provide evidence on the influence of unlimited token supply in cryptoeconomic system markets.

We analyzed AMMs based on related official whitepapers (e.g., [62], [65], [85]), blog articles (e.g., [60], [93], [103]), and official documentation (e.g., [16], [100]). Most of those publications are not peer-reviewed. Moreover, many publications do not present sufficiently detailed information to classify AMMs into the AMM taxonomy. To still classify all AMMs into our taxonomy, we made informed guesses based on source code. For example, if the AMM builds on a fork of established AMMs, we used publications on the original AMM design to complete the classification. We discussed such assumptions to complete AMM design classifications but cannot guarantee the validity of all assumptions.

We identified three AMM archetypes with individual solution approaches to tackle the LAP, PDP, and TMP. The AMM archetypes are differentiated by two AMM dimensions (*token price source* and *source of liquidity*), which most influence the solution approaches of AMM designs to tackle the LAP, PDP, and TMP. Among the 21 AMM dimensions included in the AMM taxonomy, we identified *price discovery mechanism* and *source of liquidity* as particularly relevant to differentiate AMM designs with respect to their efficacy in solving the LAP, PDP, and TMP. The cross-product of all characteristics per dimension resulted in four archetypes. The Price-adopting Supply-sovereign AMM archetype could not be identified in our analysis of 110 AMMs. We dropped this AMM archetype, but it may become relevant in the future.

Our taxonomy includes additional characteristics, such as liquidity concentration and price discovery, that can also influence the efficacy of solution approaches for the LAP, PDP, and TMP, including *liquidity concentration*, *price discovery mechanism*, *token price source*, and *source of liquidity*. We neglected these characteristics to increase the discriminatory power of the AMM archetypes. In future research, more granular AMM archetypes could be elaborated.

We extracted the solution approaches implemented in AMM archetypes for the LAP, PDP, and TMP based on literature and AMMs used in cryptoeconomic systems. In Section V, we explain how the AMM archetypes solve the LAP, PDP, and TMP. Quantitative analyses could be initiated that eventually yield further evidence supporting our qualitative results. Quantitative analyses may be based on simulations to investigate how using the three AMM archetypes presented in this work influences cryptoeconomic system markets regarding metrics like token price volatility, price elasticity, and market depth.

VII. CONCLUSION

Cryptoeconomic systems can offer a decentralized option for organizations to allocate resources based on digital tokens, for example, by issuing tokens representing assets and trading such tokens. In cryptoeconomic systems, AMMs are often used to issue and trade tokens and to solve the TMP. The efficacy with which AMMs solve the TMP is strongly influenced by their designs.

The proper design of AMMs for cryptoeconomic systems is difficult because the characteristics of AMM designs are unclear. Improperly designed AMMs can lead to the failure of cryptoeconomic systems due to the TMP. Since AMM design characteristics are unclear, the designs of solution approaches for the TMP that are implemented in AMM designs remain unclear. This eventually hinders the development of proper AMM designs that can solve the TMP. To support the development of proper AMM designs, this manuscript presents an AMM taxonomy with 21 dimensions and 53 characteristics. The AMM taxonomy includes dimensions shared by any AMM design and characteristics that inform how the dimensions can implemented in AMM designs. Thereby, the AMM taxonomy can guide the development of AMM designs. Leveraging the AMM taxonomy, we developed three AMM archetypes (i.e., Price-discovering LP-based AMM, Price-adopting LP-based AMM, and Price-discovering Supply-sovereign AMM) that reflect common AMM designs with different solution approaches for the TMP. We explain the implemented solution approaches and their efficacy in solving the LAP, PDP, and TMP for each AMM archetype. The three AMM archetypes can support the design and selection of AMM designs to meet individual purposes of cryptoeconomic systems, such as decentralized token exchanges and token issuance.

Research on AMM designs with internal token supply sovereignty is still in its infancy. Our work indicates that AMM designs with internal token supply sovereignty can enable organizations to decouple value changes from token price changes to reduce volatility and prescribe token price evolution, liquidity, and price elasticity. We believe that AMMs with internal token supply sovereignty can establish independent and continuous lead markets. The prescribed supply curve allows for a transparent representation of value and increases predictability in terms of price evolution and price elasticity. The dependency of price evolution and price elasticity on the supply curve allows developers to specify the price development of the token already during the development of AMMs. This allows AMM developers to actively define the token price developments when issuing tokens and eventually avoid severe token price fluctuations. We believe AMMs with internal token supply sovereignty will greatly support the viable operation of future cryptoeconomic systems.

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SUPPLEMENTARY MATERIAL

VIII. OVERVIEW OF THE DIMENSIONS AND CHARACTERISTICS IN THE AMM TAXONOMY

đ							
Lot							
U	Dimension	Description	Characteristic				
	Commence Madal	The desision model for nonemator adjustments of an AMM	Centralized				
ance	Governance Model	The decision model for parameter adjustments of an AMM.	Decentralized				
		The functionality to change parameters of the AMM over	Automatic				
l III	Parameter Adjustment	time.	Fixed				
Ň	-		Manual				
0	Trading Eas Adjustment	The adjustment of trading fees sharged by an AMM	Dynamic				
	Trading Fee Adjustment	The adjustment of trading lees charged by an Alvini.	Static				
	Liquidity Changaability	Liquidity changeability indicates whether the amount of avail-	Prescribed				
	Elquidity Changeability	able liquidity in the AMM can vary.	Variable				
	Liquidity Provider	The specification of what market participants are allowed to	Open				
	Permission	deposit tokens into liquidity pools.	Permissioned				
N	Number of Tokens per	The variety of tokens that can be deposited in one liquidity	Two				
dit	Liquidity Pool	pool.	Three or More				
ji ji		The mechanisms to manage the risk of holding eventually	Imbalance Surcharges				
Fi	Risk Management	volatile tokens.	Loss Insurance				
			No Risk Management				
	Source of Liquidity	The source of liquidity for an AMM can be external from	External				
	Source of Equality	liquidity providers or internal by token supply sovereignty.	Internal				
	Supported Trading Pairs	The token pairs that can be traded against each other using	Open				
	Supported Trueing Funs	an AMM.	Restricted				
	Interoperability	The capability of an AMM to execute transactions across	Interoperable				
	Interoperatinty	multiple infrastructures of cryptoeconomic systems.	Non-interoperable				
l g	Limit Order Functionality	The functionality to create limit orders	Included				
ip		The functionality to create mint orders.	Not Included				
12	Price Guarantee	The process that determines token prices in cryptoeconomic	No Guarantee				
		systems.	Price Range				
	Volume Dependency	The dependency of token prices on the transaction volume in	Volume-dependent				
		a cryptoeconomic system market.	volume-independent				
	Information	The degree to which market information is incorporated into	Expressive				
	Expressiveness Pool	prices.	Inexpressive				
	Liquidity Concentration	The distribution of liquidity at different price levels.	Automatic				
			Function-based				
			LP-based				
	Liquidity Sensitivity	The influence strength of liquidity in the AMM on the	Insensitive				
		magnitude of token price changes.	Sensitive				
	Path Deficiency	The feasible transactions in relation to the token reserve of	Dencient				
		an AMM.	Strictly Deficient				
	Path Independence	The independence of AMM state transitions from the order	Path Dependent				
	-	of transactions with identical cumulative volume.	Path Independent				
gu	Drice Bounding	The functionality to limit token prices of the AMM.	Bounded from Palew				
ici	Frice Bounding		Bounded from Above and Below				
- A		The deficition of the process of discounting foils to be provided	Constant sum				
		The definition of the process of discovering fair token prices	Constant power sum				
		based on at least one mathematical function.	Constant-power-sum				
			Constant-product sum				
	Price Discovery		Exponential Europion				
			Geometric Mean				
			Logarithmic Market Scoring				
			Price Adoption				
		The source of fair token prices that are used by the AMM to	External				
	Token Price Source	auote hid/ask token prices	Internal				
		The payoff from a portfolio consisting of aqual amounts of	Non-translation Invariant				
	Translation invariance	each asset	Translation Invariant				

TABLE I: Overview of the AMM taxonomy