MAGNETIC FIELD MEASUREMENTS AND RADIATION SIMULATION FOR A SUPERCONDUCTING TRANSVERSE-GRADIENT UNDULATOR*

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Abstract

The transverse gradient undulator (TGU) concept is a way to enable short-gain-length free electron lasers with laserplasma accelerated electron bunches, although their energy spread is typically in the percent range. In this contribution, we report on the magnetic field measurements on a 40-period superconducting TGU designed, manufactured and commissioned at KIT, Karlsruhe. As the figure of merit for the field quality, tracking and radiation field simulations, based on the measured fields, will be presented.

INTRODUCTION

A transverse gradient undulator (TGU) features an approximately linear dependence of the undulating magnetic field amplitude on one transverse coordinate, usually the horizontal coordinate x. If matched to the electron beam dispersion, this transverse field amplitude gradient can compensate for the energy spread of the electron beam, resulting in monochromatic radiation even for an energy spread on the percent level [1]. Using the TGU scheme makes it moreover possible to achieve an effective free-electron laser (FEL) amplification for such electron beams and could in this way enable compact and efficient laser-plasma accelerator (LPA) driven high-gain FELs [2] as well as storage ring driven low-gain FELs for the EUV to X-ray range [3, 4]. Aiming at these wavelength ranges, it is particularly attractive to make the TGU superconducting, which renders possible short period lengths (and thus short radiation wavelengths), sufficiently high field amplitudes and a large transverse field gradient at the same time [5, 6]. At the Karlsruhe Institute of Technology (KIT), a 40-period superconducting TGU has been designed, built and commissioned in a cryostat specially manufactured for its operation by the company Cryovac, Germany [7, 8]. The TGU consists of two superconducting coils, each wound with a single Nb-Ti wire and subdivided into sub-coils with alternating current direction. The transverse field gradient is achieved by a cylindrical shape of the coils and poles. The TGU is designed iron-free in order to allow for the generation of a correction field by two long racetrack coils situated inside the coil former. The combined dipole-sextupole field produced by these coils is required for suppressing the ponderomotive transverse drift of the particles in the TGU field [1, 7]. Table 1 summarizes the most relevant parameters of the superconducting TGU.

In order to understand how the properties of the real TGU affect the particle trajectories and thereby the generated ra-

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Figure 1: Left: Part of the brass slide with Hall probe array mounted. Right: cross-sectional view of the TGU with magnetic measurement system.

diation, two-dimensional field maps of the TGU's magnetic field were measured by scanning a Hall probe array through the TGU gap. The measured data should also provide a basis for further realistic extrapolations, for instance with regard to the FEL performance of a TGU of this design.

EXPERIMENT

The field mapping was done using a longitudinal sliding system with a carriage guide directly attached to the TGU and a carriage moved by a magnetically coupled linear translation unit mounted on one of the outer flanges of the cryostat, thus operating at ambient temperature [9]. The Hall probe array, manufactured by AREPOC s.r.o., Slovakia, a circuit board with seven Hall probes equidistantly placed in the transverse gradient direction, is fixed to the carriage and in this way moved through the TGU gap. The fixture consists of a thin MACOR[®] plate to which the circuit board is glued and which is bolted to the carriage (Fig. 1). The position of the Hall probes is a priory known only to limited accuracy and thus needs to be consistently reconstructed from the measured data, as discussed below.

A systematic uncertainty of the absolute measured field is caused by the fact that no calibration data for the Hall probes at 4.2 K were available for this experiment. Following the approach of Ref. [10], the Hall probe sensitivities were extrapolated from calibration data for 77 K using the assumption that their temperature dependence follows the

Table 1: Basic parameters of the superconducting TGU. The nominal values for an operating current of 750 A are quoted.

Parameter	Value
Period length $\lambda_{\rm u}$ at 4 K	10.46 mm
Full Periods $N_{\rm u}$	40
Transverse beam pos. $x_0(E_{e0})$	4.6 mm
Flux density $B_{y}(x_{0})$ (nominal)	1.05 T

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Figure 2: Magnetic flux density B_{\perp} at TGU operating current $I_{op} = 30$ A, comparison of measured data with Opera-3d simulation for the first periods as function of z (left) and amplitude average over all periods as function of x (right).

"typical" calibration of Hall probes from the same manufacturer for which the according data were available.

Further limitations emerged during the experiment: unlike in several test runs at 80 K, at 4.5 K mechanical instabilities did not permit to cover the entire scanning range, but limited the scans to 32 out of 40 periods of the TGU. The huge thermal load introduced by the linkage of the measurement system puts further limits on both the scanning range and the operating current of the TGU. The latter, however, is not a severe limitation since in the iron-free TGU the magnetic flux density scales strictly proportional to the current.

MEASUREMENT RESULTS

The magnetic flux density of the TGU main coils was mapped at operating currents 10 A, 20 A, 30 A, 40 A and 50 A, the latter two in a reduced scanning range. In the following, we discuss the results of the measurements at 30 A (4 % of the nominal operating current), which for the iron-free magnet can be regarded as fully representative. The field of the correction coils was mapped at 5 A in the correction coils and 0 A in the main coils (after the downramp).

The TGU coil has inter-turn and ground shorts, which lead to current redistribution after each current ramp. These processes are visible in the measured on-axis magnetic flux density as an approximately exponential asymptotic drift with a time constant $\tau = 1400$ s. The field maps were measured after a waiting time of 5τ after the current ramp.

In order to compare the measured flux densities with theoretical expectations, a least square fit to a SIMULIA Opera- $3D^{\textcircled{8}}$ model was performed, using the *x*-, *y*-, *z*-displacement, roll and yaw angle of the Hall probe array as free parameters, in the following referred to as alignment parameters. For the fit, the Opera model was evaluated for each data point at the calculated Hall probe positions, assuming one set of alignment parameters globally valid for the complete scan. Since





Figure 3: Magnetic flux density generated by the correction coils, averaged over the scanning range, as a function of reconstructed transverse position. The measured data are compared to an Opera-3d simulation.



Figure 4: Magnetic flux density generated by the TGU, averaged over a rolling window spanning over one period length, corresponding residual net kicks to the beam along its trajectory. The data are shown for all full TGU periods in the scanning range (excluding the matching periods).

Hall probe #7 (the Hall probe closest to the TGU center) showed non-physical offsets, it was excluded from further evaluation. Figure 2 shows a comparison of the measured data to the model calculation resulting from the fit of the Hall probe array alignment, for Hall probes #1 to #6 and the first six periods of the TGU including the matching period. The agreement is excellent.

For a cross-check the measured correction coil field, averaged over the scanning range, is compared to the according model calculation for the same set of alignment parameters. The comparison here is somewhat complicated by the fact that the correction field amplitude is so low that the Hall probe signals are in the same order of magnitude as the variation of the offset voltages over the set of probes. Because no independent calibration measurement at true zero-Gauss conditions was available, in this case, a least square fit with the offset voltages as free parameters was performed. As shown in Fig. 3, the measured data and reconstructed Hall probe positions are consistent with the model calculation.



Figure 5: Photon flux density over photon energy calculated using the measured field maps, scaled to nominal operating current (750 A). The simulation assumes zero-emittance electron beamlets with a beam current of 10 pA, traveling through the TGU at the reconstructed Hall probe positions. The bottom panel shows the according spectral dispersion, resulting from an optimization of the electron energies for a monochromatic photon spectrum.

A first direct measure of the local field quality can be extracted from the measured data by taking the average over a rolling window spanning over one period length, as shown and compared to the Opera model data for selected Hall probes in Fig. 4. The average over one period length should be zero (except for the undulator terminals). The periodic structure of the theoretical curve is due to a sampling effect. The deviations seen in the plot for the measured data correspond to the local residual field integral, which can be translated into a kick map for the beam along the undulator. Such a representation of the local field errors will in the upcoming evaluation steps be used to include the properties of the real magnet in the particle tracking and radiation field simulations.

RADIATION FIELD SIMULATIONS

The main goal of the presented field measurements is to determine the local field errors and their distribution along the particle trajectories through the TGU in order to investigate their impact on these trajectories, on the resulting radiation field and on the interaction between particle motion and radiation field. Our first, simplified approach to that investigation is to directly track through the field map measured by each Hall probe assuming it represents an undulator field constant in both transverse directions. In other words, the tracking simulation disregards the transverse field gradient present in the TGU and treats the measured fields as field maps of six independent planar undulators. For better comparability with our model-based simulations [11], the field maps were scaled to the nominal operating current of 750 A. For each field map an individual field integral correction was applied. The longitudinal starting position and track angle were chosen such that the net transverse drift of the zeroemittance particle beam was minimized. The particle energy was adjusted to achieve the same undulator radiation wavelength for each of the six field maps [12]. Figure 5 shows the resulting undulator radiation spectra and the according particle energies plotted as a function of the transverse Hall probe positions obtained from the least-square fit described above. The tracking and radiation field simulations were done using the WAVE code by Michael Scheer [13].

The following conclusions can be drawn: (1) The calculated line width of the undulator radiation is consistent with the natural line width for the number of periods considered here, i.e. within the given boundaries no significant impact of the local field errors on the quality of the radiation is visible. (2) Within the limitations of the chosen approach, the measurements confirm the theoretical expectations on the TGU scheme in terms of dispersion matching and monochromaticity of the TGU radiation.

STRATEGY FOR REFINED RADIATION SIMULATIONS AND EXTRAPOLATIONS

For a more realistic study on particle trajectories and radiation fields generated inside the real TGU it is necessary to construct a true 3D field map from the measured data, including also the properly set correction field. This study is still to be done and will follow two approaches. The first approach will be to superimpose the local field integral error map extracted from the measurements to the 3D magnetic field map generated from the Opera model of the ideal TGU. The second is to apply errors to the coil positions and radii in the Opera model and to fit these errors to the measured data. Both techniques can be used to extrapolate with a view to longer undulators and FEL performance.

SUMMARY

The magnetic flux density in the gap of KIT's superconducting TGU was mapped on a 2d-patch by scanning an array of Hall probes through the TGU gap. The measurement was performed in the TGU's own cryostat. By comparison to the Opera-3D model of the TGU, it was possible to reconstruct the path of the Hall probes through the gap and to quantify local field errors. The agreement between simulation and experiment is excellent. Radiation field simulations based on the measured data show that the field quality of the TGU is sufficient to enable high-quality undulator radiation.

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