

ONLINE FIT OF AN ANALYTICAL RESPONSE MATRIX MODEL FOR ORBIT CORRECTION AND OPTICAL FUNCTION MEASUREMENT

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Abstract

At the Karlsruhe Research Accelerator (KARA), an analytical online model of the orbit response matrix (ORM) has been implemented. The model, called the bilinear-exponential model with dispersion (BE+d model), is derived from the Mais-Ripken parameterization of coupled betatron motion. The online fit continuously adapts the model to changing beam optics without dedicated measurements using only orbit correction results as input. This gives access to an up-to-date ORM for orbit correction as well as estimates for the coupled beta function, betatron phase, and the tunes. After comparing such beta function fit results to an optics simulation and evaluating orbit correction with the model, problems of the approach are discussed.

INTRODUCTION

KARA is a 2.5 GeV synchrotron light source and accelerator test facility at the Institute for Beam Physics and Technology (IBPT) of the Karlsruhe Institute of Technology (KIT). Here, a new orbit correction software is under development. The goal is a program that performs well with different experimental operation modes, such as negative alpha optics [1], and to investigate novel orbit correction approaches in general. A first iteration was derived from a program used at the Dortmund Elektronenspeicherring-Anlage (DELTA) [2]. It relies on a conic solver for convex constrained optimization for calculating orbit corrections that allows orbit and steerer strength constraints, and can also correct the orbit length by modulating the frequency of the radio frequency (RF) accelerating cavity. At KARA, an analytical online model of the ORM based on the BE+d model that had been proposed in [3] was added to the software. Its ring buffer is loaded with tuples of orbit and steerer strength changes resulting from orbit corrections, and gives access to estimates for the coupled beta function, the betatron phase, and the tunes, as well as an analytical representation of the ORM. Similarly to the local optics from closed orbits (LOCO) approach [4], the method measures the linear optics of the storage ring without turn-by-turn capable beam position monitors (BPMs). An advantage of the BE+d model fit is that it requires no detailed lattice information to do so. Another usecase is orbit correction where it can be used as a replacement for a measured ORM. Compared to an online fit of the ORM itself, the fit of the analytical model is expected to require less measurements for the same signal-to-noise ratio. The reason is that the analytical model has only a fraction of the degrees of freedom of the naive matrix.

First tests of the online fit of the model were done. To minimize the required machine time, the program was trained with previously measured ORMs. Although these feature a better signal-to-noise ratio than a buffer of orbit corrections, they allow to give a basic assessment of the program's capabilities. Comparisons show that the linear-optics fit matches the OCELOT [5] optics model of the storage ring fairly well, and that the corresponding analytical ORM representations can be used for orbit correction. However, problems arise from a non-linear dependence of the orbit on the steerer strengths.

ONLINE MODEL

The online model relies on the response set fit algorithm (RSFM) [6] to fit the product of an analytical representation of the ORM R_{wjk} and the kick angle θ_{ks} to a ring-buffered set of measured orbit responses r_{wjks} by solving

$$\min_{\vec{\varphi}} \sum_{sjw} (R_{wjk}\theta_{ks} - r_{wjks})^2. \quad (1)$$

Here w indexes the horizontal or vertical plane, j references the BPM, k the steering magnet, and s is the number of the sample in the ring buffer. The analytical ORM representation is the BE+d model [7]

$$R_{wjk} = R_{wjk}(\beta_{mwj}, \Phi_{mwj}, q_m, D_{wj}\delta_k), \quad (2)$$

which is based on a complex variant [8] of the Mais-Ripken parameterization [9] that describes the trajectories in a linear storage ring as a superposition of two modes of betatron motion indexed by m . As such, the model depends on the coupled beta function β_{mwj} , the coupled betatron phase Φ_{mwj} and the mode tune q_m , as well as a dispersion term $D_{wj}\delta_k$. Together these quantities make up the vector of fit variables $\vec{\varphi}$.

For $W = 2$ planes, $M = 2$ betatron modes, $J = 40 - 1 = 39$ BPMs [10] (one BPM is not in use), and $K = 44$ horizontal and vertical steering magnets [11] that KARA is equipped with, the BE+d model has [12]

$$2MWJ + 2MK + M + WJ + K - (2M + 1) = 607 \quad (3)$$

degrees of freedom. The measured ORM has about five times more $2JK = 3432$. Recording an ORM at KARA therefore requires at least 44 distinct orbit measurements with all steering magnets perturbed sequentially, while the RSFM-based online model only requires $607/(2J) < 8$ measurements with all steerers perturbed simultaneously. The latter should be seen as a bare minimum though. A previous study at DELTA recommends twice this number [6].

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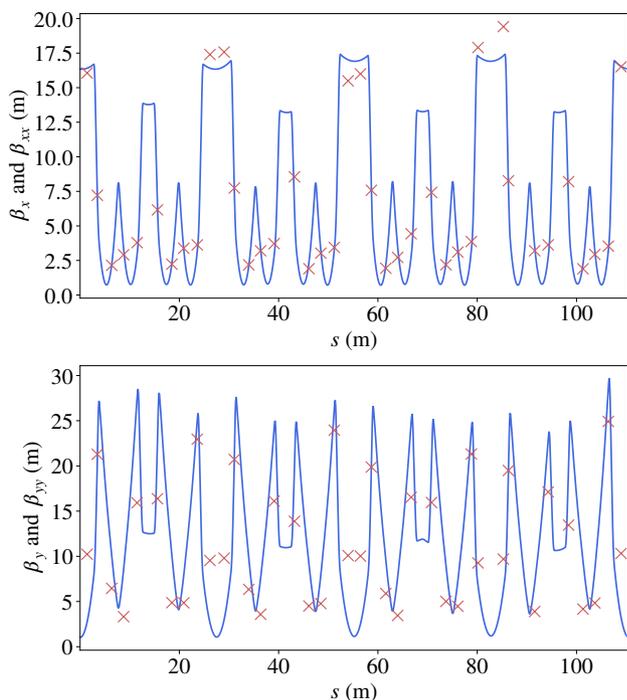


Figure 1: Comparison of the horizontal (top) and vertical beta function (bottom) from OCELOT (blue) and the RSFM (red). The standard deviations of the measurement were so small that they are not shown (see text).

BETA FUNCTION COMPARISON

The coupled beta function estimates resulting from a BE+d model fit of an ORM recorded after injection at 0.5 GeV with open insertion devices β_{xxj} and β_{yyj} are compared to the beta function values calculated with an optics model of KARA in OCELOT [5] $\beta_x(s)$ and $\beta_y(s)$ in Fig. 1. Here, β_{xxj} is the the horizontal projection of the primarily horizontal betatron mode at BPM j and β_{yyj} the vertical projection of the primarily vertical mode. The fitted tunes are $Q_x \approx 6.767$ as well as $Q_y \approx 2.785$ and the fractional tunes measured by the bunch-by-bunch feedback are $q_x = 0.771$ as well as $q_y = 0.791$. While the fitted tunes and the measured ones are very similar, the RSFM beta function estimates and the optics calculation only match fairly well. The fit was therefore repeated 100 times in a Monte-Carlo analysis with a uniformly distributed noise of 5 mm/mrad on the input data to give an estimate of the statistical uncertainties of the tune and beta function estimates. Of these fits, one diverged and was, on the condition of $\max(\beta_{xy}) > 0.1 \cdot \max(\beta_{yy})$ and $\max(\beta_{yx}) > 0.1 \cdot \max(\beta_{xx})$ (KARA has practically no coupling), excluded from the analysis. The standard deviations across the remaining 99 fits were so small that they are not shown here. The deviation of the fitted peak beta function values from the optics calculation therefore must be a systematic error (for example due to non-linearities in the ring buffer measurements).

ORBIT AND RF FREQUENCY CORRECTION EVALUATION

The analytical ORM representation for an ORM recorded in user operation at 2.5 GeV with closed insertion devices was evaluated for orbit correction. In general, this worked as well as any measured ORM but it also inherited a problem. In user operation at KARA, a method for RF frequency correction [13] is usually used to maintain a small orbit RMS even if the orbit lengthens or shortens due to thermal effects. The correction of the RF frequency is calculated by adding the corresponding orbit response as a column to the ORM and solving the orbit correction problem for the modified matrix [14]. While using a simulated ORM and dispersion function works fairly well, utilizing measured data leads to oscillations such as shown in Fig. 2. Although these can be suppressed via regularization, selecting a suitable singular-value cut-off is neither straight forward nor necessarily always possible. This problem persisted when using the analytical ORM.

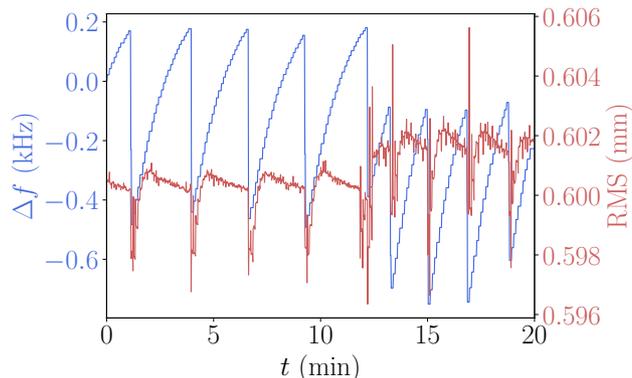


Figure 2: Correction of RF frequency (blue) and horizontal orbit RMS over all BPMs (red) vs time. After about 12 min, the singular value cut-off is changed. The oscillations shift slightly but persist anyways.

NON-LINEAR BPM READINGS

The orbit response of a storage ring is only approximately linear in a small parameter range, outside which the effects of sextupoles, BPM geometry, and hysteresis introduce a non-linear behavior. At KARA, this was now investigated by ramping-up the currents of the horizontal steering magnets (HSMs) in steps of $\Delta I = 4$ mA until a maximum orbit deviation of ± 6 mm was reached at a particular BPM (shown in the top plot of Fig. 3). Each power source was then ramped down until the orbit reached the maximum orbit deviation for a second time before it was ramped up again to return to its initial setting. Each data series was then fitted with a third-order polynomial

$$f(I) = aI^3 + bI^2 + cI + d, \quad (4)$$

where I is the steerer current. Examples of this measurement and the fitted polynomials for two BPM-HSM pairs are given in Fig. 3. While the bottom plot presumably shows the effect of a sextupole magnet where the linear increase in steerer strength is firstly off-set and then reversed by the sextupole field strength that quadratically depends on the orbit deviation (large b), the top measurement displays the typical third-order symmetry of the non-linear behavior introduced by the BPM geometry (large a). Both data series also show the effect of hysteresis.

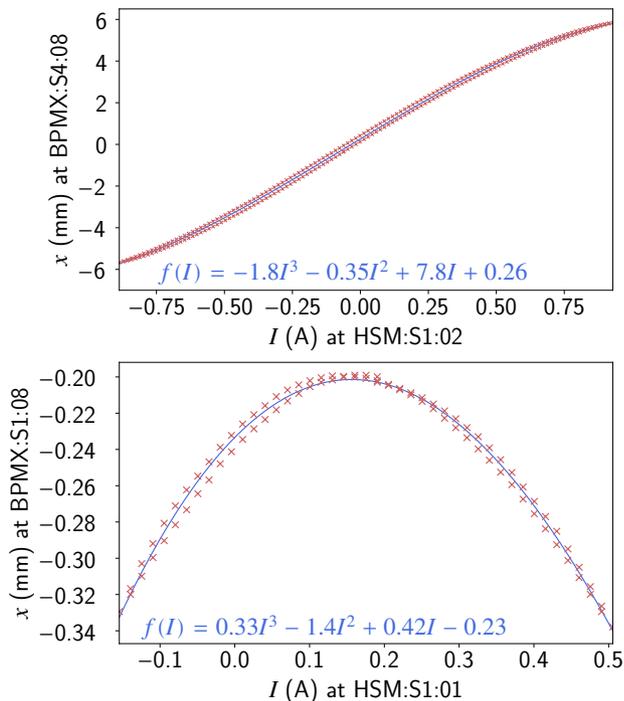


Figure 3: Orbit measurement (red) and polynomial fit (blue) for different steerer strengths at different BPM-HSM pairs.

The polynomial fit of the data was repeated for each BPM-HSM pair after transforming the orbit readings and changes of the steerer currents to a uniform interval $[-1, 1]$. A comparison of the second- and third-order coefficients of all pairs is given in Fig. 4. Unsurprisingly, many show some form of third-order distortion (bottom plot) as the BPM geometry affects all BPM readings. Large quadratic coefficients are considerably less prevalent (top plot) and mostly appear when the amplitude of betatron motion is small.

CONCLUSION

The RSFM analysis built into the online model produces reliable beta function and tune estimates and gives access to an analytical ORM representation that can be used for orbit correction. The deviation of the fitted beta function estimates from an OCELOT optics model in the peaks and oscillations appearing while correcting the RF frequency can probably be attributed to a non-linear dependence of the transverse orbit measurement on the steerer strengths.

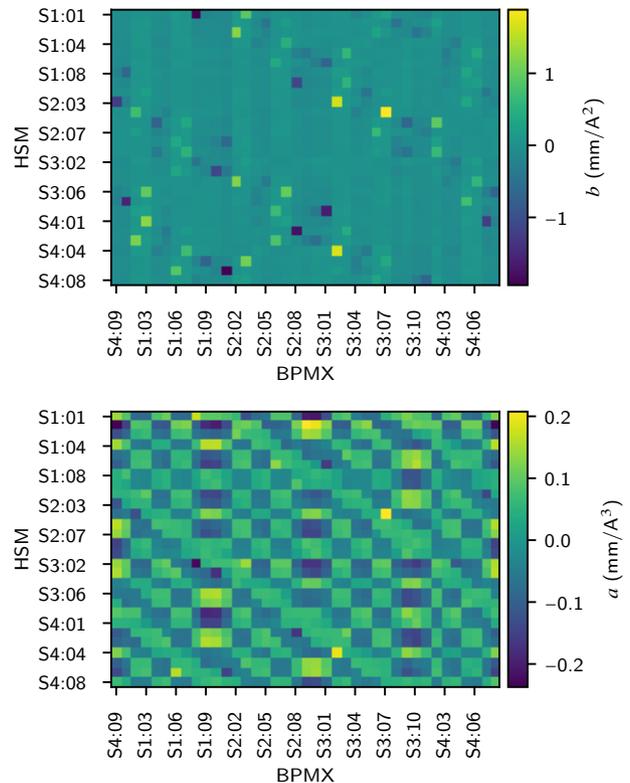


Figure 4: Second-order coefficients b (top) and third-order coefficients a (bottom) of the polynomial fit according to Eq. (4) for orbit and steerer-currents normalized to uniform intervals for all BPM-HSM pairs.

Most BPM-HSM pairs show either a second- or third-order dependence.

OUTLOOK

The problems arising from the linear assumption inherent to matrix-based orbit correction approaches are usually countered with regularization. As cutting of singular values does not work sufficiently well in our case, Thikonov regularization could be tried. However, it might be advisable to switch to a non-linear orbit response model instead as was shown in Refs. [15] and [16]. Such an approach would probably not only remove the problem of the oscillations during RF frequency correction but would also work better with non-linear and experimental optics such as negative alpha optics [1].

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