Sources Localization via Multi-Dimesional Scaling Based On Subspace Method

Hesham Ibrahim Ahmed [1, 2], Wan Qun [1], M. Ramadan[2]
1. University of electronic Science and Technology of China
School of communication and information engineering
Chengdu, Sichuan, China P.R.
2. Karary University
Omdurman, Khartoum, Sudan
Hesham21060@hotmail.com,wanqun@uestc.edu.cn

Ding Xue-ke, Zhou Zhi-ping
Tongfang Electronic Science and Technology co., ltd,
Jiujiang, Jiangxi, China P.R.

***Abstract*— There is no doubt that the issue of estimate locations for unknown sources has been mattering a lot of debate in recent years, in the field of communication, Radar, Navigation, wireless sensor networks, …etc. In this paper, the multidimensional scaling (MDS) based on subspace method was applied to solve the addressed issue in the presence of distance information that acquired from TOA measurements. The Simulation results show the significant consequences compared with Cramer-Rao lower bound (CRLB) as well as the modified version of classical Multi-dimensional scaling, it became close to the lower bound of variance when the signal to noise ratio increased.**

Keywords— **multi-dimensional scaling, subspace Method, time of arrival, distance measurements.**

I. INTRODUCTION

Estimating positions of emitted sources either are mobile or static has received a lot of attention in many aspects, particularly after the US Federal commission has adopted decision to develop the Emergency 911(E-911) services by increasing the accuracy for a precise location of clients [1]

Nowadays Multidimensional scaling (MDS) [2, 3] has become a powerful tool in exploratory data analysis [4]. It has been used to find the position of mobile and static emitting sources in the complex environment. The MDS is started by construct a similarity/dissimilarity distance matrix among all pairs of objects, and then the double centering procedure is applied to it. Finally, the eigenvalue decomposition is figured the relative coordinate positions out. In order to get the actual coordinates, we must apply the translation, rotation and scaling to them. This step is well-known as rigid transformation. In many articles, the projection distance matrix used to recover the actual coordinates. The classical MDS requires a fully connected distance matrix then, the target positions can be obtained by exploiting the eigenvalue decomposition. Subspace-based methods have the main advantage as the resolution of parameter estimation is increased.

Generally the source position can be estimated by exploiting time of arrivals (TOA), angle of arrival (AOA), time difference of arrival (TDOA), frequency difference of arrivals (FDOA), and received signal strength (RSS). TOAs TDOAs and RSSs provide distance measurements between source and sensors. FDOAs provide the rate of distance measurements, while the AOAs are the source bearings relative to sensors. Distances and bearing information are derived from the measurements and locations for the known position sensors [5].

Time difference of arrival can be remarked as the one way time of flight from the emitting source to the receiver. The source and corresponding sensors must be synchronized. To solve the localization problem by using TOA measurements, first it converted to range differences measurements then arrange it to a nonlinear equation involves the range information. Many iterative methods have been addressed such as Taylor method [6]. This method is computationally intensive and requires a good initial guess in order to achieve an accurate solution. The least squares based method attempts to reorganize the nonlinear equations into linear manner, then the position is estimated by using the least squares approach [7, 8]. Beside linearization the localization problem, subspace methods have been addressed using TOA measurements or range difference measurements. Wan Qun et al [9] introduced a noise subspace algorithm of multidimensional scaling matrix to localize a mobile station (MS) using three base stations (BS). K. W. Cheung and H. C. So [10] developed an algorithm of localize a MS by exploit the classical MDS. But both [9] and [10] do not consider the multi-target scenario. In our article we used subspace method to localize number of distributed targets in a plane. We assume the TOA information have been already acquired and converted to noisy distance measurements. The mean square position error MSPE compared with Cramer Rao lower bound CRLB applied to evaluate the performance of addressed method.

The rest of paper is organized as follows: the second section gives the Cramer-Rao lower bound (CRLB) Model and the model assumption that have been used in this experiment. The development of the proposed algorithm located in section three include the classical MDS review and the developing algorithm, while the simulation and results

This work was supported in part by the National Natural Science Foundation of China under grant NSFC 61172140 and 61401069.

discussion introduced in the fourth section and finally, the fifth section conclude the paper.

II. CRLB AND THE MODEL ASSUMPTION

Suppose that we have $T=[x_i, y_i]^T$ $i=1,2,....,N$ where $N>=2$ be the unknown target positions in a particular field distributed in 2-D plane (or in 3-D straightforward). $(.)^T$ stands for transpose factor. Assume that a set of sensors $s=\left[x_j, y_j\right]$ $j=N+1, N+2,..M$ are in known position coordinates, M is the total number of sensors. Assume that the TOA information has already obtained and converted to noisy distance measurements. Also we assume line-of-sight propagation between the targets and all sensors. The distance pairs between targets and sensors given by converting the TOA measurement to range differences which are given as

$$r_{i,j}=d_{i,j}+n_{i,j} \tag{1}$$

Where $d_{i,j}=\sqrt{\left(x_i-x_j\right)^2+\left(y_i-y_j\right)^2}$ is the real distance and $n_{i,j}$ represents the range noise. The noise is sufficiently small, based on the assumption that it is white Gaussian noise with zero mean and variance equal to σ^2

The Cramer-Rao Lower bound (CRLB) of TOA measurements is introduced in [1] as

$$CRLB(\mathbf{x})=\left\{\mathbf{R}^H\mathbf{Q}_n^{-1}\mathbf{R}\right\}^{-1}, \tag{2}$$

where

$$\mathbf{R}=\begin{bmatrix}(x-x_1)/d_1 & (y-y_1)/d_1\\ \vdots & \\ (x-x_M)/d_M & (y-y_M)/d_M\end{bmatrix} \tag{3}$$

$$\mathbf{Q}_n=E\left\{\mathbf{nn}^T\right\}=diag\left\{\sigma_1^2,\sigma_2^2,......,\sigma_M^2\right\}$$

The corresponding CRLB of $\mathbf{x}$ is the summation of the diagonal elements of eq. (2).

III. THE ALGORITHM DEVELOPMENT

A. The MDS Analysis Review

The localization using TOA measurements is given as the following: the matrix $\mathbf{X}$ could be formed as

$$\mathbf{X}_i=\begin{vmatrix}x_1-x_i & y_1-y_i\\ x_2-x_i & y_2-y_i\\ & \\ x_M-x_i & y_M-y_i\end{vmatrix} \tag{4}$$

which is $M\times 2$ matrix.

After that, the multidimensional dissimilarity matrix which is a squared Euclidean distance matrix; denoted as $\mathbf{D}$, is constructed as

$$\mathbf{D}_i=\mathbf{X}_i\mathbf{X}_i^T \tag{5}$$

The entries inside the dissimilarity matrix are given by

$$[\mathbf{D}]_{m,n}=0.5(d_m^2+d_n^2-d_{mn}^2), \tag{6}$$

which its rank equal d+2 and it is semi-definite symmetric matrix. It is important to realize that the matrix $\mathbf{D}$ is a zero diagonal elements and only its upper/lower triangular part are informative. d is the noise-free version of r and $d_{m,n}=d_{n,m}$ .The distances among sensors will be free of error. In order to use the noisy measurements in the previous equation, so is rewritten as

$$\left[\hat{\mathbf{D}}\right]_{m,n}=0.5(r_m^2+r_n^2-d_{mn}^2) \tag{7}$$

The scalar product matrix $\mathbf{B}$ could be obtained by applying double centering procedure. The form of $\mathbf{B}$ is

$$\mathbf{B}_i=-\frac{1}{2}\mathbf{J}_{M+N}\hat{\mathbf{D}}_i\mathbf{J}_{M+N} \tag{8}$$

where $\mathbf{J}=\mathbf{I}-M^{-1}(1^T1)$ is a centering matrix, $\mathbf{I}$ is an $M\times M$ identity matrix, the ones vector has length equal to M. By applying the eigenvalue decomposition (SVD) for $\mathbf{B}$ yields

$$\mathbf{B}_i=\mathbf{U}_i\mathbf{\Lambda}_i\mathbf{U}_i^T, \tag{9}$$

where $\mathbf{\Lambda}=diag\left(\lambda_1,\lambda_2,.....,\lambda_m\right)$ is the diagonal elements of the eigenvalue matrix of $\mathbf{D}$ where $\lambda_1\geq\lambda_2\geq.....\geq\lambda_m\geq 0$, $\mathbf{U}$ is an unitary matrix i.e. $\mathbf{U}^T\mathbf{U}=\mathbf{I}$, and they are called the eigenvectors associated to eigenvalues.

In classical Multidimensional scaling, the coordinate matrix $\mathbf{X}$ can be recovered up to rigid motion from eq. (9) as.

$$\mathbf{X}=\mathbf{\Lambda}_{\{2,2\}}^{1/2}\mathbf{U}_{\{2,m\}}^T \tag{10}$$

The result is the standard MDS solution. Rigid motion includes scaling, rotation and transformation and it can be achieved by Procrustes transformation

B. The Developing Algorithm

In [9] , the least squares have been applied to estimate the rotated version of coordinates matrix, then the recovered coordinate matrix translated to the original coordinate matrix The estimated coordinates matrix could be recovered as the following

$$\hat{\mathbf{X}}_i^r = \arg\min_{\mathbf{x}} \left\| \mathbf{B}_i - \hat{\mathbf{X}}_i \hat{\mathbf{X}}_i^T \right\|_F^2 , \tag{11}$$

where $\| \,.\, \|_F$ denotes to Frobenius norm. $\hat{\mathbf{X}}_i^r$ is a rotated coordinates of estimated position this results .Noting that the rank of matrix is 2. We have $\lambda_3 = \lambda_4 = ... = \lambda_M = 0$. According to the rank property, eq. (9) could be also written as

$$\mathbf{B}_{si} = \mathbf{U}_{i\{2,m\}} \mathbf{\Lambda}_{i\{2\times2\}} \mathbf{U}_{i\{2,m\}}^T , \tag{12}$$

$\mathbf{U}_{\{2\times m\}} = [\mathbf{u}_1 \; \mathbf{u}_2]$ and $\mathbf{\Lambda}_{\{2\times2\}} = diag(\lambda_1, \lambda_2)$ while the closed form solution could be obtained from eq(9). is the same as the classical solution without rigid transformation given as

$$\hat{\mathbf{X}}_i^r = \mathbf{\Lambda}_{i\{2,2\}}^{1/2} \mathbf{U}_{i\{2,m\}}^T \tag{13}$$

The relationship between the centered coordinates and the actual coordinates introduced in given by

$$\mathbf{X}_i = \hat{\mathbf{X}}_i^{rT} \mathbf{\Omega}_i \tag{14}$$

where $\mathbf{\Omega}$ is the unknown transformation/rotation matrix; such that $\mathbf{\Omega}\mathbf{\Omega}^T = \mathbf{I}$, it can be determined via pseudo-inverse

$$\mathbf{\Omega}_i = \left(\mathbf{X}_i^{rT} \mathbf{X}_i^r \right)^{-1} \mathbf{X}_i^{rT} \mathbf{X}_i \tag{15}$$

Note that this matrix applied instead of the Procrustes transformation to recover the coordinates of estimated position matrix. The algorithm that used this kind of transformation matrix called the modified classical MDS

From equations (11) , (14) and(15), the orthogonal rotation matrix can be proved through the following steps

$$\begin{aligned} \mathbf{\Omega}_i &= (\Lambda_{i\{1:2,1:2\}}^{1/2} \mathbf{U}_{i\{1:2,1:m\}}^T \mathbf{U}_{i\{1:2,1:m\}} \mathbf{\Lambda}^{1/2})^{-1} \Lambda_{i\{1:2,1:2\}}^{1/2} \mathbf{U}_{i\{1:2,1:m\}}^T \mathbf{X} \\ &= (\mathbf{\Lambda})^{-1} \Lambda_{i\{1:2,1:2\}}^{1/2} \mathbf{U}_{i\{1:2,1:m\}}^T \mathbf{X} \\ &= \Lambda_{i\{1:2,1:2\}}^{-1/2} \mathbf{U}_{i\{1:2,1:m\}}^T \mathbf{X} \end{aligned} \tag{16}$$

From eq. (16) and eq.(14) we can get

$$\mathbf{X}_i = \mathbf{U}_{i\{1:2,1:m\}} \mathbf{U}_{i\{1:2,1:m\}}^T \mathbf{X}_i , \tag{17}$$

which indicates that could be obtained from the subspace eigenvector. Note that eq.(17) is an approximate relation. The term $\mathbf{U}_{\{1:2,1:m\}}$ is the signal subspace $\mathbf{U}_s$. To drive the position estimate, we could rewrite $\mathbf{X}$ as

$$\mathbf{X} = \mathbf{Y} - \mathbf{1Z} \tag{18}$$

$$\mathbf{Y} = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ . & . \\ . & . \\ x_M & y_M \end{bmatrix} \tag{19}$$

It is noteworthy that the linear equation could be solved by exploit the least squares (LS) method to estimate the positions of targets, but here we carry out the positions of targets using noise subspace method. From the relationship between the signal subspace and noise subspace we have

$$\mathbf{I}_M - \mathbf{U}_s \mathbf{U}_s^T = \mathbf{U}_n \mathbf{U}_n^T , \tag{20}$$

where $\mathbf{U}_n$ corresponds to the noise subspace, for multi-targets estimation we can use the following equation

$$\mathbf{U}_n \mathbf{U}_n^T \mathbf{1}_M \mathbf{z}_i^T \approx \mathbf{U}_n \mathbf{U}_n^T \mathbf{Y} , \tag{21}$$

where $\mathbf{z}_i = [x_i \quad y_i]$; $i = 1, 2, ..., N$ and is the number of targets, $\mathbf{1}_M$ stands for $M \times 1$ vector which has all entries equal to one.

In order to estimate positions of targets we use the linear least square approach so,

$$\hat{\mathbf{z}}_i = \left(\left(\mathbf{U}_{n,i} \mathbf{U}_{n,i}^T \mathbf{1}_M \right)^{\dagger} \mathbf{U}_{n,i} \mathbf{U}_{n,i}^T \mathbf{Y} \right), \tag{22}$$

while the subscript notation $(.)^{\dagger}$ stands for the pseudo-inverse operation.

IV. . SIMULATION AND RESULTS

Computer simulation has been done through Matlab platform. Our proposed scheme is executed to evaluate the performance. the minimum square position errors MSPEs have been applied to evaluate the proposed method and modified multidimensional scaling (modified MDS) that introduced in [3]. In addition, the average of Cramer-Rao Lower bound CRLB has been considered too. All the results are averaged from 1000 independent trials.

The model of noise that used in our work is white Gaussian noise with zero mean and variance equals to σ^2 , the range measurements acquired after converting the TOA measurements. The SNR in each measurement were specified as $SNR = d_i / \sigma_i^2$.

In the first scenario; the impact of increasing SNR versus minimum square position errors has been shown in the second plotted figure. The targets located at $[-4000 \ \ 4000]$, $[4000 \ \ -4000]$, $[-4000 \ \ -4000]$ and $[4000 \ 4000]$ longitude units, the sensors located at coordinates $[0 \ 0]$,$[0 \ 6000]$,$[0 \ \ -6000]$,$[6000 \ 0]$,$[-6000 \ \ 0]$ $[-6000 \ \ -6000]$, $[6000 \ \ -6000]$, $[-6000 \ \ 6000]$ and $[6000 \ 6000]$. Figure (1) shows the configuration of sensors and targets in 2-dimensional space. Figure (2) shows the improvement of mean square of position errors (MSPE) with different value of SNR for each target and taking the average of MSPEs. The SNR is changed from -5dB to 60dB. The mean square position errors is defined as

$$MSPEs = \frac{\sum_{i=1}^{N}\sum_{j=N+1}^{M}\left\{(\hat{x}_i - x_i)^2 + (\hat{y}_i - y_i)^2\right\}}{M - N}$$

The performance is better than modified MDS method when SNR increases. Subspace methods are considered as suboptimal estimators that could not achieve the CRLB bound but it become close to the lower bound with increasing SNR.

The second scenario includes the effect of increasing number of sensors with fixed SNR; we set the SNR equal to 10 dB. The number of sensor varying from four to nine. Figure (3) shows that the performance becomes better when the number of sensor increases. Comparing to the modified classical MDS, the subspace approach remain close to lower bound while the modified MDS looks far from the lower bound, overall speaking the proposed method has better performance than the modified version of classical MDS.

V. CONCLUSION

In this paper the distributed sources localization were investigated by introducing MDS analysis supported by subspace method, the estimation of positions of targets have been localized, moreover with increasing SNR and number of sensors a better estimation of position could be achieved which it improves the performance, the results are verified by computer simulations

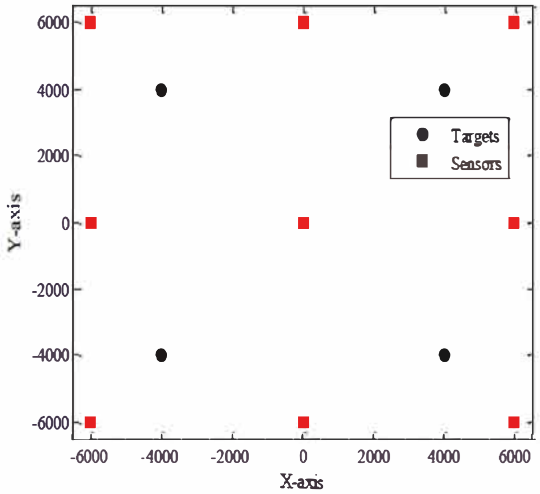

Figure (1) the configuration of the experiment

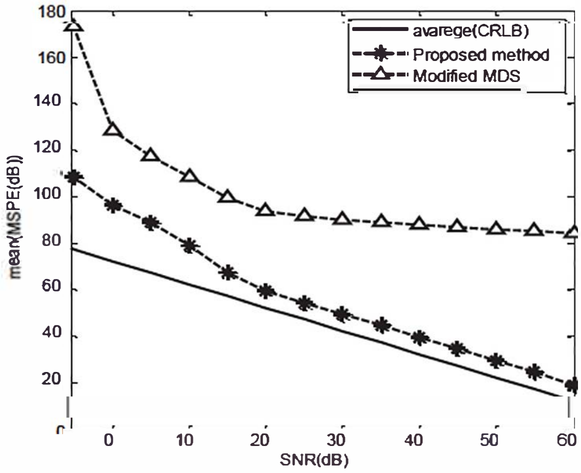

Figure (2) the MSPE versus the signal-to-noise ratio

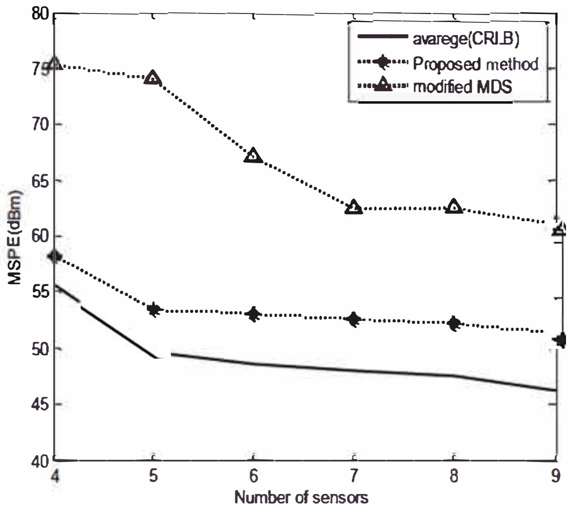

Figure (3) the MSPE versus the number of sensors.

REFERENCES

[1] F. C. Commission, "Revision of the commission's rules to ensure compatibility with enhanced 911 emergency calling systems," Report and Order and Further Notice of Proposed Rulemaking, Tech. Rep. CC Docket, 1996.

[2] I. Borg and P. J. Groenen, Modern multidimensional scaling: Theory and applications: Springer, 2005.

[3] T. F. Cox and M. A. Cox, Multidimensional scaling: CRC Press, 2000.

[4] W. L. Martinez, A. Martinez, and J. Solka, Exploratory data analysis with MATLAB: CRC Press, 2004.

[5] H. So. (2012). Source Localization: Algorithms and Analysis.

[6] W. H. Foy, "Position-Location Solutions by Taylor-Series Estimation," Aerospace and Electronic Systems, IEEE Transactions on, vol. AES-12, pp. 187-194, 1976.

[7] K. W. Cheung, H. C. So, W. K. Ma, and Y. T. Chan, "Least squares algorithms for time-of-arrival-based mobile location," Signal Processing, IEEE Transactions on, vol. 52, pp. 1121-1130, 2004.

[8] Ahmed H I, Wei P, et al. Estimation of Time Difference of Arrival (TDOA) for the Source Radiates BPSK Signal[J]. IJCSI International Journal of Computer Science Issues, 2013, 10(3).

[9] Q. Wan, Y.-J. Luo, W.-L. Yang, J. Xu, J. Tang, and Y.-N. Peng, "Mobile localization method based on multidimensional similarity analysis [cellular radio applications]," in Acoustics, Speech, and Signal Processing, 2005. Proceedings.(ICASSP'05). IEEE International Conference on, 2005, pp. iv/1081-iv/1084 Vol. 4.

[10] H.-C. So and F. Chan, "A generalized subspace approach for mobile positioning with time-of-arrival measurements," Signal Processing, IEEE Transactions on, vol. 55, pp. 5103-5107, 2007.

[11] H. So, "Source localization: algorithms and analysis," Handbook of Position Location: Theory, Practice, and Advances, pp. 25-66, 2011.

[12] K. W. Cheung and H. So, "A multidimensional scaling framework for mobile location using time-of-arrival measurements," Signal Processing, IEEE Transactions on, vol. 53, pp. 460-470, 2005.