



# On free fall of fermions and antifermions

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**Abstract** We propose a model describing spin-half quantum particles in curved spacetime in the framework of quantum field theory. Our model is based on embodying Einstein's equivalence principle and general covariance in the definition of quantum-particle states. With this model at hand, we compute several observables which characterise spin-half quantum particles in a gravitational field. In particular, we find that spin precesses in a normal Fermi frame, even in the absence of torsion. The effect appears to be complementary to free-fall non-universality we have recently reported about for spinless quantum particles. Furthermore, we find that quantum-particle gravitational-potential energy is insensitive to wave-packet spreading in the Earth's gravitational field, that is responsible for the non-universality of free fall in quantum theory. This theoretical result provides another channel for the experimental study of our quantum-particle model by using gravitational spectrometers. Finally, we also find that (elementary) fermions and antifermions are indistinguishable in gravity.

## 1 Introduction

There are several research fields in modern theoretical physics, focusing on various aspects of quantum field theory. This primarily finds its application in elementary particle physics. In fact, quantum theory of fields was originally developed to model electromagnetic interaction to naturally require a unification of quantum mechanics and special relativity. Nowadays, the Standard Model of elementary particle physics successfully involves the quantum-field-theory formalism for modelling high-energy scattering processes taking place in colliders.

In the framework of general relativity, however, special relativity is a special-case theory. It is in conflict with observations whenever gravity cannot be neglected with respect to the rest three fundamental interactions. In particular, the free-fall observation of neutrons [1–4] shows that the Standard Model has to be extended to comprise general relativity. This circumstance leads to a problem of field quantisation in curved spacetime.

This is problematic because it is not self-evident how to properly generalise quantum field theory over Minkowski spacetime to a non-Minkowski one. The basic reason of that is the role which the Poincaré group plays in the definition of quantum vacuum and, correspondingly, of Fock space in field quantisation in Minkowski spacetime [5–9]. The basic idea, which has been put forward in this regard, is to utilise isometry group of a given non-Minkowski spacetime for that purpose [10–12]. This particularly implies global field quantisation, because this approach demands the knowledge of metric tensor at all space-time points. Apart from this is unfeasible in practice, it is unknown if the observable Universe has to have any particular exact isometry group. Thus, global field quantisation assumes the usage of space-time geometries which have a non-trivial isometry group. These geometries must approximate some space-time regions of the observable Universe. In other words, their isometry groups can be thought of as local non-exact symmetries of the Universe. The Einstein equivalence principle states that the Poincaré group can be considered as its local non-exact symmetry too [13]. The question then arises as to whether global field quantisation locally reduces to that used in particle physics.

The Standard Model of particle physics has been in fact being tested in the presence of the Earth's gravitational field. It may be approximately described by Schwarzschild spacetime if the Earth's rotation is neglected. The Schwarzschild metric is invariant with respect to time translations and rotations. Although these form a subgroup of the Poincaré group, there is no guarantee that global field quantisation based

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on the isometry group of Schwarzschild spacetime locally agrees with global field quantisation in Minkowski spacetime. This is because the Schwarzschild-time and -space coordinates differ from local Minkowski-time and -space ones. Indeed, general coordinates  $x$  and normal Riemann coordinates  $y$  in the neighbourhood of  $X$  are related as follows [14]:

$$y^c(x) \approx Y^c + (x - X)^c + \frac{1}{2} \Gamma_{ab}^c(x - X)^a(x - X)^b, \quad (1)$$

where the Latin indices run over  $\{0, 1, 2, 3\}$ ,  $\Gamma_{ab}^c$  are Christoffel symbols computed at  $X$  and higher-order terms with respect to metric derivatives have been neglected on the right-hand side of (1). So, Schwarzschild spacetime is locally spherical symmetric at any  $X$ , which is due to Einstein’s equivalence principle, while globally spherical symmetric with respect to a single point which is known as the central singularity of the Schwarzschild geometry. Furthermore, the Schwarzschild time does asymptotically match a local Minkowski time at spatial infinity. However, this is non-existent in practice. That is, the Schwarzschild-time translations do not correspond to local-Minkowski-time translations if metric derivatives are non-zero.

The successful application of quantum field theory in elementary particle physics instructs us to consider plane-wave modes (used to expand a quantum field [5–9]), i.e.

$$\psi_P(y) = \exp(-i P_a y^a / \hbar), \quad (2)$$

in the Riemann inertial frame, where  $P$  is the on-mass-shell 4-momentum:  $\eta_{ab} P^a P^b = (Mc)^2$ , where  $c$  is the speed of light in vacuum in the absence of external fields [5–7]. We then obtain from (1) for  $\mathbf{X}$  being at the Earth’s surface that

$$\psi_P(y)|_{|\mathbf{P}| \ll Mc} \propto \exp(-i Mc^2(1 + g_{\oplus} z/c^2)t/\hbar), \quad (3)$$

where  $g_{\oplus} \approx 9.81 \text{ m/s}^2$  is the free-fall acceleration,  $z \equiv (x - X)^3$  is the vertical height above the Earth’s surface and  $t \equiv (x - X)^0/c$  is the Schwarzschild time. The  $g_{\oplus}$ -dependent term in (3) has been derived in [15] from the Schrödinger equation with Newton’s gravitational potential. This term gives rise to gravity-induced quantum interference which has been observed in the Colella–Overhauser–Werner experiment [16]. It is a non-inertial-frame effect, that also shows up in an accelerated frame, as that has been confirmed by the Bonse–Wroblewski experiment [17]. These experiments demonstrate that quantum interference cannot be used to distinguish between uniform gravity and acceleration, in accord with Einstein’s principle [18].

In the Schwarzschild frame, global field quantisation instructs us to consider modes which are eigenfunctions of the Killing vector generating the Schwarzschild-time translations [10–12]:

$$\psi_P(x)|_{|\mathbf{P}| \ll Mc} \propto \exp(-i Mc^2 t/\hbar), \quad (4)$$

where the coefficient of proportionality is independent of the Schwarzschild-time coordinate for any value of  $|\mathbf{P}|$ . The mode phase differs from that of (3). So, the physical meaning of Fock space based on these modes is obscure in light of the experiments mentioned above.

From other side, the plane-wave modes (2) cannot be exact solutions of a field equation in curved spacetime. Still, these can be treated as approximate solutions in local inertial frames. These frames can generically be introduced relative to either a point or a trajectory. However, a quantum-field operator, e.g.  $\hat{\Psi}(x)$ , depends on a single point. In other words,  $\hat{\Psi}(x)$  is a local operator, whereas the coordinate transformation  $x \rightarrow y(x)$  means that  $y$  depends on a pair of points –  $x$  and  $X$ . This might in turn mean that one has to consider  $\hat{\Psi}(x) \rightarrow \hat{\Psi}_X(y)$  under the coordinate transformation (1). Theoretical particle physics relies on the Minkowski-spacetime approximation, where one deals with  $\hat{\Psi}(y)$  which does not depend on  $X$  [5–9]. These all might mean that  $\hat{\Psi}(x)$  itself should appear in observables in such a way that  $X$  does not show up if the space-time curvature is neglected. It is in fact required for Einstein’s equivalence principle to hold in local quantum phenomena.

One of these phenomena is scattering of particles. At tree level of perturbation theory, the probability amplitude covariantly generalised to curved spacetime pictorially reads for a pair of charged particles in the framework of quantum electrodynamics as follows:

$$\propto \prod_{s \in \{1,2\}} \int d^4 x_s \sqrt{-g(x_s)} J_1^\mu(x_1) G_{\mu\nu}^F(x_1, x_2) J_2^\nu(x_2), \quad (5)$$

where  $G_{\mu\nu}^F(x_1, x_2)$  is a Feynman propagator of the photon field in curved spacetime, naturally depending on the geodesic distance between  $x_1$  and  $x_2$  and the space-time curvature [11], and

$$J_s^\mu(x) \equiv \langle \psi_{s,f} | \hat{J}_s^\mu(x) | \psi_{s,i} \rangle - \langle \psi_{s,f} | \psi_{s,i} \rangle \langle \Omega | \hat{J}_s^\mu(x) | \Omega \rangle, \quad (6)$$

where  $\hat{J}_s^\mu(x)$  is the current-density operator [6] and  $|\Omega\rangle$  stands for the quantum-vacuum state. This probability amplitude is diffeomorphism invariant if and only if the particle states  $|\psi_{s,i}\rangle$  and  $|\psi_{s,f}\rangle$  are independent of coordinate frames. Besides,  $\langle \psi_{s,f} | \psi_{s,i} \rangle$  turns identically to zero if the initial and final states of a particle are orthogonal to each other. In practice, this might not be the case for free particles if one deals with superpositions of plane waves in Minkowski spacetime. Alternatively, one may replace  $\hat{J}_s^\mu(x)$  by  $:\hat{J}_s^\mu(x):$  for which  $\langle \Omega | :\hat{J}_s^\mu(x): | \Omega \rangle = 0$  [5–9]. In any case, the current

density  $J_s^\mu(x)$  defined in (6) is localised in a spacetime region in which the states  $|\psi_{s,i}\rangle$  and  $|\psi_{s,f}\rangle$  overlap. It means that the probability amplitude (5) locally reduces to that computed by using the Minkowski-spacetime approximation if the in-coming particles are brought to a space-time region of size being much smaller than a local curvature length in that region. If otherwise, the probability amplitude tends to zero by increasing the distance between the localisation regions of  $J_1^\mu(x)$  and  $J_2^\mu(x)$ , following from the cluster decomposition principle (see Ch. 4 in [5] and [19] for concrete computations).

In theoretical particle physics, one commonly deals with in-coming states depending solely on initial momenta. In classical theory, it is additionally required to set their initial positions in order to ascertain particles' trajectories. This has to be the case in quantum theory as well. Clearly, this does not contradict to Heisenberg's uncertainty principle. It means that we need to replace  $|\psi_i\rangle$  by  $|\psi_{X,P}\rangle$ , where  $|\psi_{X,P}\rangle$  propagates over a trajectory passing through  $(X, P)$  in the phase space. In Minkowski spacetime, this trajectory must be  $x(\tau) = X + (P/M)\tau$ , where  $M$  is particle's mass and  $\tau$  denotes proper time. Therefore, in the coordinate representation,  $|\psi_{X,P}\rangle$  turns into the wave function  $\psi_{X,P}(x)$  to depend on  $x$  and  $X$ . Such wave functions enter the current densities (6). This line of reasoning shows, thereby, that all quantities entering the scattering amplitude depend not on absolute positions, like quantum-field operators,  $\hat{\Psi}_s(x)$ , but rather on relative positions.

To summarise, quantum-field operators are fundamental objects in quantum field theory. Still, quantum phenomena manifest themselves through the interaction of quantum particles. A quantum-particle model based on modes  $\psi_P(x)$  is not complete, because initial position  $X$  has to be part of the model, otherwise this is at odds with Bohr's correspondence principle. It means that the quantum-field expansion over modes  $\psi_P(x)$  should play no underlying role for the description of quantum particles (cf. [10–12]). Instead of that, one should look for  $\psi_{X,P}(x)$  to provide the wave-function description for quantum particles in the weak-gravity limit. This naturally allows us to fulfil the condition that  $\psi_{X,P}(x)$  reduces to a plane-wave superposition in a local Minkowski frame for  $x$  being close to  $X$  [20–22], in accordance with the application of quantum field theory in particle physics [5–9]. For this condition to be fulfilled in any local Minkowski frame,  $\psi_{X,P}(x)$  should, additionally, transform as a zero-rank tensor under general coordinate transformations. This explains how quantum field theory based on the Minkowski-spacetime approximation and high-energy experiments done in the Earth's gravitational field can locally reconcile with each other.

We intend here to study spin-half quantum particles in curved spacetime in the framework of quantum field theory. Our idea is here to build Einstein's equivalence prin-

ciple and general covariance [23] into the model of quantum particles in gravity, that we have lately put forward in [20–22]. Accordingly, we will construct and study a quantum spin-half state which is locally represented by a superposition of positive-frequency plane waves, as in Minkowski spacetime, and which is invariant under general coordinate transformations. In an arbitrary curved space-time, it is hard to construct such a state non-perturbatively in curvature. For this reason, we will do this in perturbation theory by only taking the leading-order curvature contribution to the quantum spin-half state into account. This approximation proves to be sufficient to reach the main goal of the article, that is to compare the kinematics of a quantum particle modelled by such a state with that of a classical particle of same mass and spin. This analysis allows us to determine possible experimental setups which are suitable for testing our model.

Throughout, we use natural units  $c = G = \hbar = 1$ , unless otherwise stated.

## 2 Covariant spin-half particle

### 2.1 Minkowski spacetime

Here we want to model a spin-half quantum particle in Minkowski spacetime. We consider that the particle is initially placed at  $Y$  in position space and at  $P$  in momentum space. So, it is modelled by the following state:

$$|\psi_{Y,P}\rangle \equiv \hat{a}^\dagger(\psi_{Y,P})|\Omega\rangle, \quad (7)$$

where  $|\Omega\rangle$  is the quantum vacuum and  $\hat{a}^\dagger(\psi_{Y,P})$  is an operator creating the spin-half quantum particle to be characterised by the wave function  $\psi_{Y,P}(y)$ .

First, theoretical particle physics defines a unique quantum vacuum which is known in the literature as the Minkowski vacuum [5–9]. It is unique with respect to the Poincaré isometry group of Minkowski spacetime. However, the observable Universe may have no exact isometry group. Yet, it does have approximate isometry groups emerging at different length scales. One of the physically relevant examples is a *local* Poincaré group. This emerges in a vicinity of any non-singular space-time point at length scales much smaller than a local curvature length at that point. This statement is a consequence of Einstein's equivalence principle which is so far in agreement with observations. At cosmological length scales, the Universe looks as de-Sitter spacetime [27]. There is no unique quantum vacuum in de-Sitter spacetime, which is invariant under the de-Sitter isometry group [11]. In general, there is no preferred procedure to choose a unique quantum vacuum in curved spacetime. In the absence of any exact universe isometry group,  $|\Omega\rangle$  is, at least, supposed to be unitarily equivalent to all local Minkowski vacua which can be defined in local inertial frames. This is required

for having a locally unique Fock space in the observable Universe. In other words, radio waves coming from Sagittarius A\* must have the same nature like those produced here on the Earth.

Second, the particle creation operator is defined as follows:

$$\hat{a}^\dagger(\psi_{Y,P}) \equiv \int d^3\mathbf{y} \hat{\Psi}^\dagger(\mathbf{y}) \psi_{Y,P}(\mathbf{y}), \quad (8)$$

where  $\hat{\Psi}(\mathbf{y})$  is a Dirac field satisfying Dirac's equation with the mass term  $M$ . It is common in theoretical particle physics to consider a definite momentum wave function, namely

$$(\psi_{Y,P}(\mathbf{y}))_{\text{non-normalisable}} = u(P) e^{-iP \cdot (\mathbf{y}-Y)}, \quad (9)$$

where  $u(P)$  is a 4-dimensional column vector (see Sec. 3.3 in [6]). Nevertheless, a single plane wave is not localised in space. As a result,  $|\psi_{Y,P}\rangle$  is non-normalisable, implying that  $|\psi_{Y,P}\rangle$  is physically obscure in this case. This problem can be re-solved by treating their (normalisable) superposition [7]:

$$(\psi_{Y,P}(\mathbf{y}))_{\text{non-spinorial}} = \int \frac{d^4K}{(2\pi)^3} \theta(K^0) \delta(K^2 - M^2) \times F_P(K) u(K) e^{-iK \cdot (\mathbf{y}-Y)}, \quad (10)$$

where  $F_P(K)$  has a narrow peak at  $K = P$ . Yet, unlike the plane wave (9), this wave function does not properly transform under the Lorentz transformations. In fact, it is not a spinor. For this reason, we instead consider in what follows that

$$\psi_{Y,P}(\mathbf{y}) \equiv \int \frac{d^4K}{(2\pi)^3} \theta(K^0) \delta(K^2 - M^2) \times F_P(K) \frac{\gamma \cdot K + M}{2M} u(P) e^{-iK \cdot (\mathbf{y}-Y)}, \quad (11)$$

where  $\gamma^a$  are the four Dirac matrices in Weyl's representation [6]. This wave function can be shown to be a solution of the Dirac equation and to transform as a spinor under the Lorentz transformations, assuming  $F_P(K)$  is a Lorentz scalar (see below).

As mentioned above, one normally treats on-mass-shell plane waves in theoretical particle physics, which are associated with in- and out-going particles in a given scattering amplitude. This amplitude depends on initial and final momenta of such particles, but not on their initial and final positions. Such amplitude does not disappear if the particles are infinitely separated away from each other. This is in contradiction to observations. In theory, this circumstance is taken into account via the cluster decomposition principle (see Ch. 4 in [5]), basically stating that distant scattering experiments yield uncorrelated results. For this principle to hold, one needs localised-in-space quantum states which correspond to wave packets.

Wave packets involve, at least, one additional parameter to determine the shape of  $F_P(K)$ . This parameter appears in the Heisenberg uncertainty relation and is known in the quantum-mechanics literature as momentum variance [24]. It ensures that

$$\begin{aligned} \langle \psi_{Y,P} | \psi_{Y,P} \rangle &= \int d^3\mathbf{y} (\psi_{Y,P}(\mathbf{y}))^\dagger \psi_{Y,P}(\mathbf{y}) \\ &= \frac{1}{2} \int \frac{d^3\mathbf{K}}{(2\pi)^3} \frac{|F_P(\mathbf{K})|^2}{\sqrt{\mathbf{K}^2 + M^2}} \frac{M^2 + P \cdot \mathbf{K}}{2M^2} \equiv 1. \end{aligned} \quad (12)$$

The right-hand side is Lorentz-invariant. It requires the momentum integral be also invariant under the Lorentz transformations. In other words, we must consider  $F_P(K) = F(P \cdot K)$ . We have dealt with Lorentz-invariant Gaussian wave functions in [20–22], which have been earlier studied in [25, 26]. We wish here to treat a modified Lorentz-invariant Gaussian wave packet:

$$F_P(K) \equiv \frac{2^{3/2} \pi M}{D \sqrt{K_1 \left( \frac{M^2}{D^2} \right)}} \frac{\exp\left(-\frac{P \cdot K}{2D^2}\right)}{\sqrt{M^2 + P \cdot K}}, \quad (13)$$

where  $D$  is the momentum variance and  $K_\nu(z)$  stands for the modified Bessel function of the second kind. The pre-factor in (13) has been chosen for the normalisation condition (12) to be fulfilled (cf. [20, 21]).

## 2.2 Minkowski-spacetime approximation

We have considered so far a spin-half wave packet in Minkowski spacetime. The Universe is a non-Minkowski spacetime [27]. Yet, it follows from Einstein's equivalence principle that the observable Universe can be locally approximated by Minkowski spacetime at any point  $Y$  for  $y$  satisfying

$$|y - Y| \ll l_c(Y), \quad (14)$$

where  $l_c(Y)$  is a local curvature length at  $Y$ , estimated by the inverse of the fourth root of the Kretschmann scalar at that point. At the Earth's surface, we obtain  $l_c(r_\oplus) \sim 10^{11}$  m, meaning that the Earth's curvature can be basically ignored in collider physics. We want to go beyond this approximation. In other words, (11) turns into a leading-order approximation of the wave function which does not involve metric derivatives:

$$\begin{aligned} \psi_{Y,P}^{(0)}(\mathbf{y}) &= \int \frac{d^4K}{(2\pi)^3} \theta(K^0) \delta(K^2 - M^2) \\ &\times F_P(K) \psi_{Y,P|K}^{(0)}(\mathbf{y}), \end{aligned} \quad (15a)$$

where we have on the mass shell that

$$\psi_{Y,P|K}^{(0)}(\mathbf{y}) = \frac{\gamma \cdot K + M}{2M} u(P) e^{-iK \cdot (\mathbf{y}-Y)}. \quad (15b)$$

It should be pointed out that the momentum integral can be exactly evaluated with the choice (13) for  $F_P(K)$ , which is given by a combination of elementary functions.

The wave function is given in terms of local Minkowski (Riemann) coordinates  $y$  defined at  $Y$ . It can be re-written in general coordinates  $x = x(y)$  with  $X = x(Y)$ . In these coordinates,  $\psi_{X,P}^{(0)}(x)$  depends on  $x$  via  $\sigma(x, X)$ ,  $P^M(X) \sigma_M(x, X)$  and  $\gamma^M(X) \sigma_M(x, X)$ , where  $\sigma(x, X)$  is a geodetic distance – Synge’s world function,  $-\sigma_M(x, X)$  is its derivative with respect to  $X^M$ ,  $P^M(X) \equiv e_a^M(X) P^a$  and  $\gamma^M(X) \equiv e_a^M(X) \gamma^a$ , where  $e_a^M(X)$  are vierbein fields at  $X$ , namely  $g_{MN}(X) e_a^M(X) e_b^N(X) = \eta_{ab}$ . These all mean that the wave packet transforms as a zero-rank tensor under general coordinate transformations, as required.

### 2.3 Beyond Minkowski-spacetime approximation

We now wish to obtain the leading-order curvature correction to  $\psi_{X,P}^{(0)}(x)$ . The Dirac-field equation generically reads

$$(i\gamma^\mu(x)D_\mu - M) \psi_{X,P}(x) = 0, \tag{16}$$

where  $D_\mu$  is the spinorial covariant derivative. By setting  $Y = 0$  in what follows for the sake of simplicity, we obtain in normal Riemann coordinates that

$$(i\gamma^a \partial_a - M) \psi_{Y,P}^{(0)}(y) = 0, \tag{17a}$$

$$(i\gamma^a \partial_a - M) \psi_{Y,P}^{(2)}(y) = -\frac{i}{6} R^a{}_{cbd} y^c y^d \gamma^b \partial_a \psi_{Y,P}^{(0)}(y) - \frac{i}{8} R_{abcd} \gamma^c \gamma^b \gamma^a y^d \psi_{Y,P}^{(0)}(y). \tag{17b}$$

Note,  $\psi_{Y,P}^{(2)}(y)$  depends apparently on the curvature tensor at  $Y$ , where the index “(2)” refers to the number of metric derivatives involved, while  $\psi_{Y,P}^{(1)}(y)$  does not exist in normal Riemann coordinates.

In vacuum, i.e.  $R_{ab} = 0$ , the second term on the right-hand side of (17b) is identically zero. In this case, we have

$$\psi_{Y,P}^{(2)}(y) = \int \frac{d^4 K}{(2\pi)^3} \theta(K^0) \delta(K^2 - M^2) \times F_P(K) \psi_{Y,P|K}^{(2)}(y), \tag{18a}$$

where we find on the mass shell that

$$\psi_{Y,P|K}^{(2)}(y) = \mathcal{O}^{(2)} \psi_{Y,P|K}^{(0)}(y), \tag{18b}$$

where by definition

$$\begin{aligned} \mathcal{O}^{(2)} \equiv & \frac{iK \cdot y}{6M^2} R_{abcd} K^a K^b y^c y^d + \frac{1}{12M} R_{abcd} K^a y^c y^d \gamma^b \\ & + \frac{K \cdot y + i}{12M^3} R_{abcd} K^a K^b y^d \gamma^c \\ & - \frac{2iK \cdot y + 1}{8M^2} R_{abcd} K^a y^c S^{bd}, \end{aligned} \tag{18c}$$

and

$$S^{ab} \equiv \frac{i}{4} [\gamma^a, \gamma^b]. \tag{18d}$$

This solution is non-unique, like in case of the scalar-field model [21]. For instance, we obtain others by adding multiples of  $R_{abcd} K^a y^c (K^b y^d - S^{bd})$  and  $R_{abcd} K^a (K^b y^d \gamma^c - i y^c S^{bd})$  to  $\mathcal{O}^{(2)}$ . In this article, however, we shall focus our study on  $\mathcal{O}^{(2)}$  as given in (18c).

Among of all terms entering  $\mathcal{O}^{(2)}$ , only the first term modifies the wave-function phase in a gravitational field. This term coincides with that we have found for spin-zero particles at the leading order in space-time curvature [21]. For this reason, spin should not influence quantum interference induced by space-time curvature. In fact, the relative phase shift of wave packets obtained via splitting an ultra-cold  $^{87}\text{Rb}$  atom cloud [28] is oblivious to atoms’ spin degree of freedom.

## 3 Observables

### 3.1 Quantum particle

The quantum-particle state  $|\psi_{Y,P}\rangle$  is defined in Minkowski spacetime through the creation operator (8). We covariantly generalise it to curved spacetime as follows:

$$|\psi_{X,P}\rangle \equiv \hat{a}^\dagger(\psi_{X,P})|\Omega\rangle, \tag{19}$$

where

$$\hat{a}^\dagger(\psi_{X,P}) \equiv \int_\Sigma d\Sigma_\mu(x) \hat{\Psi}(x) \gamma^\mu(x) \psi_{X,P}(x), \tag{20}$$

where  $\Sigma$  is a time-like Cauchy surface on which, however, the integral does not depend. This is because both the Dirac field and the wave packet are solutions of the Dirac-field equation (16) and  $\psi_{X,P}(x)$  is localised in space. Next, making use of the anti-commutation relation for  $\hat{\Psi}(x)$  and its canonical conjugate  $\hat{\Pi}(x)$  and taking into account  $\hat{a}(\psi_{X,P})|\Omega\rangle = 0$ , we obtain

$$\langle \psi_{X,P} | \psi_{X,P} \rangle = \int_\Sigma d\Sigma_\mu(x) J^\mu(x), \tag{21}$$

where

$$J^\mu(x) \equiv \overline{\psi}_{X,P}(x) \gamma^\mu(x) \psi_{X,P}(x) \tag{22}$$

is covariantly conserved, namely  $\nabla_\mu J^\mu(x) = 0$ . It is worth emphasising that  $J^\mu(x)$  is a vector, because  $\psi_{X,P}(x)$  is a scalar with respect to general coordinate transformations. This covariant conservation law means that  $\langle \psi_{X,P} | \psi_{X,P} \rangle$  is a constant which we set to unity:

$$\langle \psi_{X,P} | \psi_{X,P} \rangle \equiv 1. \tag{23}$$

This defines the normalisation condition for the wave function  $\psi_{X,P}(x)$  in curved spacetime. Note that both sides of

(23) are diffeomorphism invariant. This physically leads to the frame-independent existence of quantum particles modelled by  $|\psi_{X,P}\rangle$ , even in the presence of a non-stationary gravitational field (see [20] for the de-Sitter-universe case).

This circumstance is at odds with the idea that quantum particles may be created in non-stationary spacetimes [11, 12]. This relies on a few assumptions. For the sake of concreteness, we focus on the flat de-Sitter spacetime, i.e. on that patch of the de-Sitter hyperboloid, which can be parametrised by Friedmann–Robertson–Walker coordinates with no spatial curvature. In this example, one first expands  $\hat{\Psi}(x)$  over modes which are eigenfunctions of Killing vectors generating translations in space. In the absence of cosmic-time-translation symmetry, there is no unique choice of modes' dependence on cosmic time. For this reason, one second imposes a condition on how the modes have to depend on time at past cosmic infinity, defining  $\psi_{-\infty}(x)$ . One instead chooses  $\psi_{+\infty}(x)$  for that condition to be fulfilled at future cosmic infinity as well. Both  $\psi_{-\infty}(x)$  and  $\psi_{+\infty}(x)$  can be utilised to define wave functions to satisfy the normalisation condition (23) with  $\psi_{X,P}(x)$  replaced by  $\psi_{-\infty}(x)$  and  $\psi_{+\infty}(x)$ , respectively. The cosmological particle creation is owing to the replacement of  $\psi_{-\infty}(x)$  by  $\psi_{+\infty}(x)$  [29]. In contrast,  $\psi_{X,P}(x)$  locally reduces to the plane-wave superposition in the vicinity of any  $X$ , which is required for quantum particle theory to be consistent with Einstein's equivalence principle [20–22].

Another novel aspect is that  $|\psi_{X,P}\rangle$  is diffeomorphism invariant. For example, a part of the flat de-Sitter spacetime can be parametrised by static coordinates. In these coordinates, there is a time-like Killing vector which generates translations in time. This basically eliminates the ambiguity in the mode choice by selecting those which are its eigenfunctions. This property is correspondingly fulfilled for any moment of static-time coordinate. In fact, such modes define a quantum-particle state being unitarily equivalent to  $|\psi_{-\infty}\rangle$ . At future cosmic infinity,  $|\psi_{+\infty}\rangle$  is replaced by  $|\psi_{-\infty}\rangle \otimes \dots \otimes |\psi_{-\infty}\rangle$  under the coordinate transformation from the flat to static patch of the de-Sitter hyperboloid. This replacement is apparently in tension with the general principle of relativity.

In the absence of experimental data favouring the idea of quantum particles being created in a non-stationary spacetime geometry, it is unclear whether this model accurately describes particle physics in curved spacetime. From other side, the Einstein equivalence principle is by now well tested in various experiments [13], while general covariance is a guiding principle for formulating physical laws. These laws manifest themselves through interaction of (quantum) particles. Thus, general covariance makes physical sense if and only if (quantum) particles are independent on coordinate reference frames.

In interacting quantum field theories,  $|\psi_{X,P}\rangle$  should be interpreted as an asymptotic state to model either an in- or out-coming particle. In the former instance,  $|\psi_{X,P}\rangle$  does change with time. It is because  $\psi_{X,P}(x)$  is a solution of the linear Dirac-field equation, while  $\hat{\Psi}(x)$  satisfies a non-linear Dirac-field equation. In fact, we obtain from (20) that

$$\hat{a}^\dagger(\psi_{X,P})|_{\text{out}} = \hat{a}^\dagger(\psi_{X,P})|_{\text{in}} - i \int d^4x \sqrt{-g(x)} \times (i D_\mu \hat{\Psi}(x) \gamma^\mu(x) + M \hat{\Psi}(x)) \psi_{X,P}(x), \quad (24)$$

where we have taken into account that  $\psi_{X,P}(x)$  obeys (16) and turns to zero at spatial infinity, i.e.  $\psi_{X,P}(x)$  is localised in space. This diffeomorphism-invariant integral vanishes if and only if  $\hat{\Psi}(x)$  satisfies the linear Dirac-field equation. The result (24) can be utilised to generalise the Lehmann–Symanzik–Zimmermann reduction formula [9, 30] to curved spacetime. It is worth emphasising that this generalisation logically follows from the general principle of relativity. Hence, the quantum state  $|\psi_{X,P}\rangle$  may depend on time in a gravitational field if we go beyond classical gravity, by working, for instance, in the framework of the effective quantum-gravity theory [31].

Here we should digress to briefly discuss the quantum-vacuum decay in a constant electric-field background to compare it with the vacuum decay in a non-stationary gravitational field. The former is known in the literature as Schwinger's effect which was deduced in a manifestly gauge-invariant way by employing the proper-time method [32]. This has no explicit reference to quantum particles. A (gauge-dependent) derivation has been later proposed, making use of the quantum-particle notion based on the Feynman prescription distinguishing positive- and negative-frequency modes as, respectively, particles and antiparticles [33, 34], similar to [29]. In the electric-field presence,  $\hat{\Psi}(x)$  cannot generically satisfy the linear Dirac equation, meaning that the integral in (24) is non-trivial, unless  $\psi_{X,P}(x)$  propagates away from the electric field. Furthermore, to our knowledge, a gauge-invariant wave-function solution to model a charged test particle placed in a constant electric field is not yet known in the literature.

Working in normal Riemann coordinates, we have

$$\langle \psi_{Y,P} | \psi_{Y,P} \rangle \equiv \langle \psi_{Y,P} | \psi_{Y,P} \rangle_{(0)} + \langle \psi_{Y,P} | \psi_{Y,P} \rangle_{(2)} + \dots, \quad (25)$$

where the first term is generically independent on metric derivatives, whereas the second one depends on the curvature tensor at the point  $Y$ , i.e. on no more than two metric derivatives. We find in case of (15b) and (18b) that

$$\langle \psi_{Y,P} | \psi_{Y,P} \rangle_{(0)} = 1, \quad (26a)$$

$$\langle \psi_{Y,P} | \psi_{Y,P} \rangle_{(2)} = 0. \quad (26b)$$

These mean that the wave function  $\psi_{X,P}(x) \approx \psi_{X,P}^{(0)}(x) + \psi_{X,P}^{(2)}(x)$  is properly normalised up to the leading-order approximation in the curvature tensor.

### 3.2 Quantum-particle propagation

We define particle’s position through the first moment of the current density (22) [20, 21]:

$$\langle x^\mu(\Sigma) \rangle \equiv \int_\Sigma d\Sigma_\nu(x) x^\mu J^\nu(x), \tag{27}$$

where  $\Sigma$  is a time-like Cauchy surface. Apparently, the position expectation value depends on the choice of  $\Sigma$ . In quantum mechanics, this choice determines the notion of time, which is invariant under the Galilei transformations. In special relativity, this choice is ambiguous, as the Lorentz transformations mix temporal and spatial coordinates. This means that we need here a physical hypothesis which specifies preferred Cauchy surfaces. We intend to presume in this regard that quantum massive particles measure proper time which is commonly denoted by  $\tau$ . Despite of the fact that this assumption logically follows from the geodesic equation, it can also be justified by utilising experimental data. Specifically, muons are unstable quantum particles whose mean lifetime is about  $2.19 \times 10^{-6}$  s. It implies that a cosmic-ray muon cannot reach the Earth’s surface if its mean lifetime is measured by a clock to rest with respect to the Earth. Since this is at odds with observations, the clock and muon time differ from each other. This discrepancy comes from the time-dilation effect [35]: The laboratory lifetime of a cosmic-ray muon is by a Lorentz factor bigger than its proper lifetime.

Apart from (27) reduces in the quantum-mechanics regime to the well-known definition of position expectation value, we obtain from it in the particle’s rest frame  $\chi = (\tau, \boldsymbol{\chi})$  that

$$\langle \dot{\chi}^a(\tau) \rangle = \int d^3\boldsymbol{\chi} \sqrt{-g(\tau, \boldsymbol{\chi})} J^a(\tau, \boldsymbol{\chi}). \tag{28}$$

In Minkowski spacetime,  $\langle \dot{\chi}^i(\tau) \rangle$  identically vanishes. Under the Lorentz transformation from the rest frame  $\chi$  to  $y$ , we then find

$$\langle \dot{y}^a(\tau) \rangle = \Lambda^a_b \langle \dot{\chi}^a(\tau) \rangle = U^a \quad \text{with} \quad U^a \equiv P^a/M, \tag{29}$$

as in classical theory. Thus, our definition of quantum-particle position makes sense, at least in the absence of gravity (cf. equation (23) of Sec. III.B in [20]).

Working in Riemann normal coordinates, we have

$$\langle y^a(\tau) \rangle \equiv \langle y^a(\tau) \rangle_{(0)} + \langle y^a(\tau) \rangle_{(2)} + \dots \tag{30}$$

where we find

$$\langle y^a(\tau) \rangle_{(0)} = U^a \tau, \tag{31a}$$

$$\begin{aligned} \langle y^a(\tau) \rangle_{(2)} = & -\frac{1}{8M} \left( \frac{1}{4D^2} f_1 \left( \frac{M^2}{D^2} \right) + \tau^2 f_2 \left( \frac{M^2}{D^2} \right) \right) \\ & \times R^a_{bcd} U^b \langle S^{cd} \rangle, \end{aligned} \tag{31b}$$

where by definition

$$\langle S^{ab} \rangle \equiv \bar{u}(U) S^{ab} u(U), \tag{32}$$

where  $S^{ab}$  has been defined in (18d), and

$$f_1(z) \equiv 7 + \frac{11}{z} - \frac{55}{4z^2} + \mathcal{O}\left(\frac{1}{z^3}\right), \tag{33a}$$

$$f_2(z) \equiv 1 - \frac{5}{2z} + \frac{75}{8z^2} + \mathcal{O}\left(\frac{1}{z^3}\right). \tag{33b}$$

The result  $\langle \dot{y}^a(\tau) \rangle \neq 0$  implies that Dirac particles propagate along non-geodesic trajectories, because, in classical theory, geodesics passing through  $Y = 0$  are given by straight world lines in the Riemann frame [14]. Still, this effect comes from the spin degree of freedom. Classical spinning particles are known to be subject to Mathisson’s force, leading to their non-geodesic motion in curved spacetime [36, 37] (see [38] for a short review). To study this effect further in the framework of quantum field theory, we next intend to compute 4-momentum of spin-half quantum particles.

### 3.3 Quantum-particle 4-momentum

Making use of the anti-commutation relation for  $\hat{\Psi}(x)$  and its canonical conjugate  $\hat{\Pi}(x)$ , we obtain

$$\begin{aligned} \langle p^\mu(\Sigma) \rangle & \equiv \int_\Sigma d\Sigma^\nu(x) \left( \langle \psi_{X,P} | \hat{T}_\nu^\mu(x) | \psi_{X,P} \rangle - \langle \Omega | \hat{T}_\nu^\mu(x) | \Omega \rangle \right) \\ & = \int_\Sigma d\Sigma_\nu(x) \frac{i}{2} \bar{\psi}_{X,P}(x) \gamma^{(\mu}(x) D^{\nu)} \psi_{X,P}(x) + \text{c.c.}, \end{aligned} \tag{34}$$

where we have explicitly subtracted the stress-tensor vacuum expectation value  $\langle \Omega | \hat{T}_\nu^\mu(x) | \Omega \rangle$  from  $\langle \psi_{X,P} | \hat{T}_\nu^\mu(x) | \psi_{X,P} \rangle$ , because that does not depend on the wave packet  $\psi_{X,P}(x)$  and, thus, cannot provide a physical contribution to the particle energy-momentum tensor. It should be, however, mentioned that  $\langle \Omega | \hat{T}_\nu^\mu(x) | \Omega \rangle$  is commonly supposed to have a physical meaning [11]. Apparently, this vacuum expectation value is ill-defined because of the mathematical nature of quantum-field operators. A properly regularised and then renormalised  $\langle \Omega | \hat{T}_\nu^\mu(x) | \Omega \rangle$  turns out to depend on the spacetime-curvature length  $l_c(x)$  [11], which is finite due to the presence of (quantum) matter (unless one treats a purely de-Sitter spacetime with  $l_c(x) = \text{const}$ ). If so,  $\langle \Omega | \hat{T}_\nu^\mu(x) | \Omega \rangle$  has to depend on wave packets of particles constituting (quantum) matter. This appears to point a logical flaw in the statement that the quantum vacuum  $|\Omega\rangle$  is a no-particle state and

$\langle \Omega | \hat{T}_\nu^\mu(x) | \Omega \rangle$  makes a non-zero contribution to the energy budget of the observable Universe. For this reason, we suppose that  $\langle \psi_{X,P} | \hat{T}_\nu^\mu(x) | \psi_{X,P} \rangle - \langle \Omega | \hat{T}_\nu^\mu(x) | \Omega \rangle$  might enter the semi-classical Einstein equation, while a curvature length arising from that does not source a non-zero stress-tensor vacuum expectation value for other quantum fields from the Standard Model.

Working in normal Riemann coordinates, we have

$$\langle p^a(\tau) \rangle \equiv \langle p^a(\tau) \rangle_{(0)} + \langle p^a(\tau) \rangle_{(2)} + \dots, \tag{35}$$

where we find<sup>1</sup>

$$\langle p^a(\tau) \rangle_{(0)} = f_3 \left( \frac{M^2}{D^2} \right) P^a, \tag{36a}$$

$$\langle p^a(\tau) \rangle_{(2)} = -\frac{\tau}{4} R^a{}_{bcd} U^b \langle S^{cd} \rangle, \tag{36b}$$

where by definition (see also Sec. III.B.3 in [20])

$$f_3(z) \equiv K_2(z)/K_1(z). \tag{37}$$

As noted above, point-like spinning bodies are subject to Mathisson’s spin-curvature force [36,37]. We find from the first Mathisson–Papapetrou equation (see (18) in [38]) by solving it in normal Riemann coordinates that

$$p^a(\tau) \approx M U^a - \frac{\tau}{4} R^a{}_{bcd} U^b \langle S^{cd} \rangle, \tag{38}$$

where higher-order curvature terms have been omitted. Hence,  $\langle p^a(\tau) \rangle$  reduces to the classical result in the limit  $D/M \rightarrow 0$ . To our knowledge, this was first derived in quantum theory in the framework of relativistic quantum mechanics in [39,40]. Later, it was obtained by making use of the WKB approximation in [41] (see also [42,43]). Recent quantum-mechanics results for spin-half particles in curved spacetime can be found in [44]. Recent results gained in the framework of the effective quantum theory of gravity show that the gravitational deflection depends on quantum-particle spin [45–47].

In classical theory, 4-momentum is proportional to 4-velocity for point-like particles. Still, in quantum theory over curved spacetime, we have from (31) that

$$\langle p^a(\tau) \rangle \approx M_i \langle \dot{y}^a(\tau) \rangle - \frac{\tau D^2}{4M^2} R^a{}_{bcd} U^b \langle S^{cd} \rangle, \tag{39}$$

where the inertial mass  $M_i$  has been defined through the Lagrangian mass  $M$  as follows [22]:

$$M_i \equiv M f_3 \left( \frac{M^2}{D^2} \right). \tag{40}$$

<sup>1</sup> It turns out that  $\langle p^a(\tau) \rangle_{(2)}$  is independent on  $D/M$ . First, it follows from direct computations by assuming  $D/M$  is small. We obtain no contributions up to the sixth order in perturbation theory. Second, numerical computations show that  $\langle p^a(\tau) \rangle_{(2)}$  is insensitive to various values of  $D/M \geq 1$ .

We recover the classical result for point-like particles in the limit  $D/M \rightarrow 0$ , which means that the wave function has a definite value of the momentum, but no definite position in space. The latter can also be seen in (31b), which makes no longer physical sense in the limit of vanishing momentum variance.

To study this limit in more detail, we want to consider (15b) and (18b) with  $K \rightarrow P$ . This corresponds to setting the momentum variance  $D$  to zero. In this instance, the wave function is non-normalisable. It particularly means that we must re-consider our method of computing observables. We have learned above that  $J^a(y)$  may be interpreted as 4-velocity density of the quantum particle. Therefore, we assume for the moment that

$$v^a(y) \propto \bar{\psi}_{Y,P}(y) \gamma^a(y) \psi_{Y,P}(y), \tag{41}$$

where the normalisation factor needs to be determined. We then find

$$v^a_{(0)}(y) \Big|_{y=U\tau} \propto U^a \bar{u}(P) u(P), \tag{42a}$$

$$v^a_{(2)}(y) \Big|_{y=U\tau} = 0. \tag{42b}$$

Apparently, the last result cannot be consistent with ours for  $\langle \dot{y}(\tau) \rangle_{(2)}$ . It was proposed in the references [41,51], however, to define 4-velocity in the WKB approximation as follows:

$$v^a(y) \propto \frac{1}{2Mi} \left( D^a \bar{\psi}(y)_{Y,P} \psi_{Y,P}(y) - \bar{\psi}_{Y,P}(y) D^a \psi_{Y,P}(y) \right), \tag{43}$$

giving

$$v^a_{(0)}(y) \Big|_{y=U\tau} \propto U^a \bar{u}(P) u(P), \tag{44a}$$

$$v^a_{(2)}(y) \Big|_{y=U\tau} \propto -\frac{\tau}{2} M R^a{}_{bcd} U^b \langle S^{cd} \rangle. \tag{44b}$$

If the normalisation factor is  $1/2M$ , then we obtain  $\langle \dot{y}(\tau) \rangle_{(0)}$  and  $\langle \dot{y}(\tau) \rangle_{(2)}$  with  $D$  set to zero. According to Gordon’s decomposition, (41) differs from (43) by a term which is proportional to  $\nabla_b (\bar{\psi}(y) S^{ab}(y) \psi(y))$ . This contributes neither to  $\langle \dot{y}(\tau) \rangle_{(0)}$  nor to  $\langle \dot{y}(\tau) \rangle_{(2)}$  if  $D/M \rightarrow 0$ , but is non-vanishing if computed on  $y = U\tau$ . Furthermore, we find

$$(v^b(y) \nabla_b v^a(y))_{(0)} \Big|_{y=U\tau} = 0, \tag{45a}$$

$$(v^b(y) \nabla_b v^a(y))_{(2)} \Big|_{y=U\tau} \propto -\frac{1}{2} M R^a{}_{bcd} U^b \langle S^{cd} \rangle, \tag{45b}$$

which are consistent with  $\langle \dot{y}(\tau) \rangle_{(0)}$  and  $\langle \ddot{y}(\tau) \rangle_{(2)}$ , respectively, if  $D/M \rightarrow 0$  is considered.

### 3.4 Quantum-particle spin

We define the spin matrix as follows:

$$\begin{aligned} \langle s^{\mu\nu}(\Sigma) \rangle &\equiv \int_{\Sigma} d\Sigma_{\lambda}(x) \left( \langle \psi_{X,P} | \hat{S}^{\lambda\mu\nu}(x) | \psi_{X,P} \rangle - \langle \Omega | \hat{S}^{\lambda\mu\nu}(x) | \Omega \rangle \right) \\ &= \int_{\Sigma} d\Sigma_{\lambda}(x) \frac{1}{2} \bar{\psi}_{X,P}(x) \{ \gamma^{\lambda}(x), S^{\mu\nu}(x) \} \psi_{X,P}(x), \end{aligned} \tag{46}$$

where  $\hat{S}^{\lambda\mu\nu}(x)$  denotes the spin-matrix operator [7,8].

Working in normal Riemann coordinates, we have

$$\langle s^{ab}(\tau) \rangle \equiv \langle s^{ab}(\tau) \rangle_{(0)} + \langle s^{ab}(\tau) \rangle_{(2)} + \dots, \tag{47}$$

where we find

$$\langle s^{ab}(\tau) \rangle_{(0)} = \frac{1}{2} f_4 \left( \frac{M^2}{D^2} \right) \langle S^{ab} \rangle, \tag{48a}$$

$$\begin{aligned} \langle s^{ab}(\tau) \rangle_{(2)} &= \frac{1}{12} \left( \frac{1}{4D^2} f_5 \left( \frac{M^2}{D^2} \right) + \tau^2 f_6 \left( \frac{M^2}{D^2} \right) \right) \\ &\quad \times \left( R^{[a}_{\text{ecd}} U^{b]} U^e - \frac{1}{2} R^{ab}_{\text{cd}} \right) \langle S^{cd} \rangle, \end{aligned} \tag{48b}$$

where by definition

$$f_4(z) \equiv 1 - \frac{1}{z} + \frac{9}{4z^2} + O\left(\frac{1}{z^3}\right), \tag{49a}$$

$$f_5(z) \equiv 1 - \frac{14}{z} + \frac{87}{2z^2} + O\left(\frac{1}{z^3}\right), \tag{49b}$$

$$f_6(z) \equiv 1 - \frac{1}{z} - \frac{1}{4z^2} + O\left(\frac{1}{z^3}\right). \tag{49c}$$

First, note that both  $\langle s^{ab}(\tau) \rangle_{(0)}$  and  $\langle s^{ab}(\tau) \rangle_{(2)}$  separately satisfy the Pirani condition [48] in the following form:

$$U_b \langle s^{ab}(\tau) \rangle_{(0)} = 0, \tag{50a}$$

$$U_b \langle s^{ab}(\tau) \rangle_{(2)} = 0. \tag{50b}$$

Nevertheless, it seems that the condition might need to be re-defined in quantum theory, such that it is formulated as a single expectation value. This issue goes beyond our purpose in this article and, therefore, we leave it aside.

Second, we have from (48) that

$$\langle s^{ab}(\tau) \rangle \approx \frac{\tau}{6} \left( 1 - \frac{D^2}{M^2} \right) \left( R^{[a}_{\text{ecd}} U^{b]} U^e - \frac{1}{2} R^{ab}_{\text{cd}} \right) \langle S^{cd} \rangle. \tag{51}$$

If we assume for the moment that there is no spin precession in the sense of [49], namely in a local inertial frame with the origin at the particle’s centre of mass, which falls with it freely – a Fermi frame [50], – then we obtain from the second Mathisson–Papapetrou equation (see (19) in [38]) by solving it in normal Riemann coordinates that

$$\dot{s}^{ab}(\tau) \approx \frac{\tau}{6} \left( R^{[a}_{\text{ecd}} U^{b]} U^e - \frac{1}{2} R^{ab}_{\text{cd}} \right) \langle S^{cd} \rangle. \tag{52}$$

Note, the spin precession occurs in the Riemann frame, as the right-hand side of this equation does not vanish. It is due to the fact that the Riemann frame represents a local inertial frame at the given initial point ( $\tau = 0$ ), while the Fermi frame along the free-fall trajectory ( $\tau \geq 0$ ). The comparison of  $\langle \dot{s}^{ab}(\tau) \rangle$  with  $\dot{s}^{ab}(\tau)$  shows that spin precesses in quantum theory in curved spacetime in the Fermi frame, even in the absence of torsion [51], as the right-hand side of

$$\begin{aligned} &\frac{d}{d\tau} (\langle s^{ab}(\tau) \rangle - s^{ab}(\tau)) \\ &\approx -\frac{\tau D^2}{6M^2} \left( R^{[a}_{\text{ecd}} U^{b]} U^e - \frac{1}{2} R^{ab}_{\text{cd}} \right) \langle S^{cd} \rangle \end{aligned} \tag{53}$$

cannot generically vanish, unless  $D/M \rightarrow 0$ , as in classical theory. This circumstance might be of use to determine the ratio  $D/M$  for a given quantum spin-half particle in satellite-borne experiments, if the quantum-particle model proposed above adequately describes fermions in gravity.

## 4 Covariant spin-half antiparticle

### 4.1 Quantum antiparticle

Generalising the antiparticle creation operator (see Sec. 4.1 in [9]) to curved spacetime, we define

$$\hat{b}^{\dagger}(\varphi_{X,P}) \equiv \int_{\Sigma} d\Sigma_{\mu}(x) \bar{\varphi}_{X,P}(x) \gamma^{\mu}(x) \hat{\Psi}(x), \tag{54}$$

where the antiparticle wave function reads

$$\varphi_{X,P}(x) \equiv \hat{C} \psi_{X,P}(x) = -i\gamma^2 (\psi_{X,P}(x))^*, \tag{55}$$

where  $\hat{C}$  is the charge conjugation operator [6] and the star means complex conjugation.

### 4.2 Particle–antiparticle symmetry in gravity

Our computations of the quantum-antiparticle observables show that antiparticles cannot be distinguished from particles in a gravitational field. Specifically, we obtain the same results for  $\langle y^a(\tau) \rangle$ ,  $\langle p^a(\tau) \rangle$  and  $\langle s^{ab}(\tau) \rangle$  with  $u(U)$  replaced in (32) by  $v(U) = \hat{C}u(U)$ . This means the particle–antiparticle symmetry in gravity, because the charge conjugation operator preserves spin orientation.

## 5 Non-inertial-frame observables

### 5.1 Quantum corrections to free-fall trajectory

We have so far computed the quantum-particle trajectory in normal Riemann coordinates. From an experimental viewpoint, it is of interest to obtain its trajectory relative to a

detector being at rest with respect to a given reference frame. We assume that this frame is described by general coordinates,  $x = (t, \mathbf{x})$ . We then obtain from (1) (with higher-order corrections in metric derivatives included [14]) and (27) that

$$\langle x^\lambda(\tau) \rangle_{(0)} = X^\lambda + U^\lambda \tau, \tag{56a}$$

$$\langle x^\lambda(\tau) \rangle_{(1)} = -\frac{1}{2} \Gamma_{\mu\nu}^\lambda (U^\mu U^\nu \tau^2 + \frac{1}{3} \langle \chi^2(\tau) \rangle (U^\mu U^\nu - g^{\mu\nu})), \tag{56b}$$

$$\begin{aligned} \langle x^\lambda(\tau) \rangle_{(2)} = & -\frac{1}{6} (\Gamma_{\mu\nu,\rho}^\lambda - 2\Gamma_{\mu\sigma}^\lambda \Gamma_{\nu\rho}^\sigma) (U^\mu U^\nu U^\rho \tau^3 \\ & + \tau \langle \chi^2(\tau) \rangle (U^\mu U^\nu U^\rho - U^{(\mu} g^{\nu\rho)}) \\ & - \frac{1}{8M} \left( \frac{1}{4D^2} f_1 \left( \frac{M^2}{D^2} \right) + \tau^2 f_2 \left( \frac{M^2}{D^2} \right) \right) \\ & \times R^\lambda_{\mu\nu\rho} U^\mu \langle S^{\nu\rho} \rangle, \end{aligned} \tag{56c}$$

where  $U^\mu = e_a^\mu U^a$  and  $\langle \chi^2(\tau) \rangle$  describes wave-packet spreading [24], namely

$$\frac{1}{3} \langle \chi^2(\tau) \rangle \equiv \frac{1}{4D^2} f_7 \left( \frac{M^2}{D^2} \right) + f_8 \left( \frac{M^2}{D^2} \right) \frac{D^2 \tau^2}{M^2}, \tag{57}$$

where by definition

$$f_7(z) \equiv 1 - \frac{1}{2z} + \frac{7}{8z^2} + O\left(\frac{1}{z^3}\right), \tag{58a}$$

$$f_8(z) \equiv 1 - \frac{7}{2z} + \frac{123}{8z^2} + O\left(\frac{1}{z^3}\right). \tag{58b}$$

The free-fall trajectory is thus modified also due to the wave-packet spreading [21, 22]. It is a universal phenomenon in quantum theory. The momentum variance  $D$  enters the Heisenberg uncertainty relation, meaning that  $D > 0$ . As a result, the universality of free fall or, in other words, the weak equivalence principle is at odds with Heisenberg’s uncertainty principle.

### 5.1.1 Earth’s gravitational field

The Earth’s gravitational field may be approximately modelled by the line element

$$\begin{aligned} ds_\oplus^2 \approx & \left( 1 - \frac{r_{S,\oplus}}{|\mathbf{x}|} \right) dt^2 - 4 \frac{\mathbf{J}_\oplus \times \mathbf{x}}{|\mathbf{x}|^3} \cdot d\mathbf{x} dt \\ & - \left( 1 + \frac{r_{S,\oplus}}{|\mathbf{x}|} \right) d\mathbf{x}^2, \end{aligned} \tag{59}$$

where  $r_{S,\oplus}$  is the Schwarzschild radius of Earth and  $\mathbf{J}_\oplus$  is its angular momentum. We obtain from (56a) and (56b) in the non-relativistic limit at the Earth’s surface that

$$\begin{aligned} M_i \langle \ddot{\mathbf{x}}(\tau) \rangle \approx & -M_g g_\oplus \mathbf{n} - 2M_g (\boldsymbol{\omega}_\oplus \times \mathbf{V}) \\ & - 3M_g ((\mathbf{n} \cdot \boldsymbol{\omega}_\oplus) \mathbf{n} \times \mathbf{V} - \boldsymbol{\omega}_\oplus \times \mathbf{V}), \end{aligned} \tag{60}$$

where the gravitational mass  $M_g$  has been defined through the Lagrangian mass  $M$  as follows:

$$M_g \equiv M_i \left( 1 + f_8 \left( \frac{M^2}{D^2} \right) \frac{D^2}{M^2} \right). \tag{61}$$

Furthermore,  $\mathbf{n}$  stands for the three-dimensional unit vector radially pointing outwards, and

$$\boldsymbol{\omega}_\oplus \equiv \frac{2\mathbf{J}_\oplus}{r_\oplus^3}, \tag{62}$$

where  $r_\oplus$  is the Earth’s radius and

$$V^i \equiv P^i / P^0. \tag{63}$$

It should be noted that the third term on the right-hand side of (60) arises from the gradient of the Earth’s angular velocity and vanishes if  $\boldsymbol{\omega}_\oplus \propto \mathbf{n}$ .

### 5.1.2 Uniformly accelerated and rotating frame

The result (60) should next be compared with the Dirac-particle trajectory in a uniformly accelerated and rotating frame:

$$\begin{aligned} ds^2 = & ((1 + \mathbf{a} \cdot \mathbf{x})^2 - (\boldsymbol{\omega} \times \mathbf{x})^2) dt^2 \\ & - 2(\boldsymbol{\omega} \times \mathbf{x}) \cdot d\mathbf{x} dt - d\mathbf{x}^2, \end{aligned} \tag{64}$$

where  $\mathbf{a}$  and  $\boldsymbol{\omega}$  are, respectively, constant acceleration and angular velocity. We find from (56) in the non-relativistic limit that

$$M_i \langle \ddot{\mathbf{x}}(\tau) \rangle \approx -M_i \mathbf{a} - 2M_g (\boldsymbol{\omega} \times \mathbf{V}), \tag{65}$$

where we have only taken into account terms linearly depending on the acceleration and the angular velocity.

If we treat the acceleration-dependent part of the results (60) and (65), then the difference between these is owing to gravitational-length contraction which is, apparently, non-existent in the uniformly accelerated frame. In other words,  $M_g \neq M_i$  cannot be gained by considering the gravitational time dilation only. The latter might be interpreted as causing free fall in its standard form [52].

If we take into account the angular velocity, then we find that the Coriolis force enters the equation of motion with the gravitational mass  $M_g$  instead of the inertial mass  $M_i$ . This also holds in a uniformly accelerated and rotating frame. This may be of use to determine the ratio  $M_g/M_i$  for a quantum particle in such a frame.

Note, in general, the difference between  $M_g$  and  $M_i$  disappears in the quantum-mechanics limit,  $c \rightarrow \infty$ , as this results in  $D/Mc \rightarrow 0$ . The free-fall non-universality in quantum theory is in this sense a relativistic effect, which is in agreement with [53].

### 5.2 Quantum corrections to gravitational-potential energy

We compute next the quantum-particle energy in the general coordinate frame  $x = (t, \mathbf{x})$ . Its energy is given by  $\langle p_t(\tau) \rangle$  which, in general, may depend on the proper time. In particular, assuming that  $\partial_t$  is a Killing vector, we have from (1) (with higher-order corrections in metric derivatives included [14]) and (34) that

$$\langle p_t(\tau) \rangle_{(0)} = e_t^a \langle p_a(\tau) \rangle_{(0)}, \tag{66a}$$

$$\langle p_t(\tau) \rangle_{(1)} = \frac{1}{2} g_{t[\nu, \mu]} e_a^\mu e_b^\nu \langle l^{ab}(\tau) \rangle_{(0)}, \tag{66b}$$

$$\langle p_t(\tau) \rangle_{(2)} = 0, \tag{66c}$$

where  $\langle l^{ab}(\tau) \rangle$  is the angular-momentum matrix [7,8], defined as the skew-symmetric part of the first moment of the energy-momentum tensor. We find

$$\langle l^{ab}(\tau) \rangle_{(0)} = \frac{1}{2} \langle S^{ab} \rangle, \tag{67}$$

which should be compared with the result for  $\langle s^{ab}(\tau) \rangle_{(0)}$  derived above. It immediately shows that  $M_g$  cannot enter the quantum-particle energy in the leading order of the approximation. The physical consequence of this result will be discussed shortly.

It should be emphasised that the curvature tensor does not contribute to the gravitational-potential energy (at the leading order of perturbation theory). This is a consequence of a non-trivial cancellation of  $\langle p^a(\tau) \rangle_{(2)}$  in (36b) by a term arising from the coordinate transformation  $x = x(y)$  at the corresponding order of perturbation theory. This cancellation is in agreement with the observation we have made in the end of Sect. 2.3 that spin does not affect phase shift induced by the curvature tensor.

#### 5.2.1 Earth’s gravitational field

We find in the Earth’s gravitational field at the Earth’s surface that

$$\langle p_t(\tau) \rangle_{(0)} \approx \gamma M_i (1 - g_\oplus r_\oplus) - \gamma \boldsymbol{\omega}_\oplus \cdot \mathbf{L}, \tag{68a}$$

$$\begin{aligned} \langle p_t(\tau) \rangle_{(1)} \approx & -\boldsymbol{\omega}_\oplus \cdot \mathbf{S} + \mathbf{S} \cdot (g_\oplus \mathbf{n} \times \mathbf{V}) \\ & - \frac{3}{2} ((\mathbf{n} \cdot \boldsymbol{\omega}_\oplus) \mathbf{n} \cdot \mathbf{S} - \boldsymbol{\omega}_\oplus \cdot \mathbf{S}), \end{aligned} \tag{68b}$$

where we have taken into account terms depending linearly on  $\mathbf{g}$  and  $\boldsymbol{\omega}$  only, which explains the approximation sign used in both equations, and, by definition,  $\gamma$  is the Lorentz factor and

$$L^i \equiv \epsilon^{ijk} X^j (M_i V^k), \tag{69a}$$

$$S^i \equiv \frac{1}{4} \epsilon^{ijk} \langle S^{jk} \rangle. \tag{69b}$$

Let us now suppose that a quantum neutron is initially placed at the altitude  $Z$ . We then obtain from (68) in the

non-relativistic limit that

$$\langle E \rangle \approx \frac{1}{2} M_i \mathbf{V}^2 + M_i g_\oplus Z. \tag{70}$$

If we now assume that this quantum particle is observed at a later moment of time ( $\tau > 0$ ) at

$$\langle z \rangle \approx Z + V_Z \tau - \frac{1}{2} \frac{M_g}{M_i} g_\oplus \tau^2, \tag{71}$$

according to (60), then its energy after the measurement must be bigger than its initial value. It is owing to  $M_g/M_i > 1$ , namely the total-energy gain equals  $(M_g - M_i) g_\oplus Z$ . We then obtain for neutrons from [1,2] and (60) that

$$\left. \frac{D^2}{(Mc)^2} \right|_{\text{neutron}} \lesssim 10^{-3}. \tag{72}$$

Therefore, a gravitational-spectrometer resolution must be better than  $10^{-10}$  eV to probe the universality of free fall at quantum level with a better accuracy than in [1,2], if the free-fall altitude,  $Z$ , is about 1 m. A gravitational spectrometer of the type treated in [54] might thus be of use to provide an independent experimental result testing the quantum-particle model proposed here.

#### 5.2.2 Uniformly accelerated and rotating frame

We find in the uniformly accelerated and rotating frame that

$$\langle p_t(\tau) \rangle_{(0)} \approx \gamma M_i (1 + \mathbf{a} \cdot \mathbf{X}) - \gamma \boldsymbol{\omega} \cdot \mathbf{L}, \tag{73a}$$

$$\langle p_t(\tau) \rangle_{(1)} \approx -\boldsymbol{\omega} \cdot \mathbf{S} + \mathbf{S} \cdot (\mathbf{a} \times \mathbf{V}). \tag{73b}$$

This result is to compare with that obtained in [55], wherein the Hamilton operator has been derived in the uniformly accelerated and rotating frame in the non-relativistic limit,  $|\mathbf{V}| \ll 1$ . The spin-rotation effect, which is owing to  $-\boldsymbol{\omega} \cdot \mathbf{S}$ , has been first proposed in [56] for a neutron interferometer. We also have the so-called inertial spin-orbit coupling, i.e.  $\mathbf{S} \cdot (\mathbf{a} \times \mathbf{V})$ , which is seemingly by a factor of 2 bigger than that found in [55]. This can be readily accounted for the Foldy–Wouthuysen transformation used in [55] to derive a non-relativistic approximation for the Hamilton operator in the framework of relativistic quantum mechanics. In quantum field theory, we obtain the expectation values (73) in a single-particle state, which are independent on any unitary transformation.

## 6 Concluding discussion

The Standard Model of elementary particle physics employs quantum field theory in order to describe high-energy scattering processes generically involving a non-conserved number of particle species [5–9]. The primary object in quantum field theory is a quantum field. This is a local

distribution-valued operator defined over a certain spacetime. The Poincaré isometry of Minkowski spacetime plays a key role in relating quantum fields to elementary and composite particles to acquire physical meaning well before and after scattering processes. Specifically, the Lehmann–Symanzik–Zimmermann reduction formula links free particles with asymptotic quantum states defining  $S$ -matrix elements [9, 30].

Gravity is still treated as physics beyond the Standard Model of elementary particles, even though the gravitational interaction is known by now for several centuries. Indeed, there are no data which would necessitate quantum gravity for their explanation – gravitation is far too weak with respect to the rest fundamental interactions to be noticeable in colliders. Classical gravitational phenomena are, however, successfully described by general relativity, providing a geometrical description for the gravitational interaction, based on a number of ideas among of which local Poincaré invariance of the laws of nature and general covariance. Matter curves spacetime in general relativity, meaning theoretical particle physics relies on the Minkowski-spacetime approximation.

Direct observations of free fall of neutrons at the Earth’s surface [1, 2] show these particles fall down over classical geodesics, or, at least, these experiments were not sensitive enough to notice any deviation from those. From a fundamental point of view, these experiments require that quantum field theory in Minkowski spacetime be replaced by that in curved spacetime. A serious obstacle arises here from the circumstance that it is unclear how to model elementary and composite particles in the absence of the global Poincaré symmetry.

Algebra of quantum fields, rather than their particular realisation in terms of creation and annihilation operators, should be of underlying relevance [57]. We have proposed in [20–22] to pick up operators from the quantum-field algebra over curved spacetime, which locally reduce to those which create free particles in Minkowski spacetime. These operators are employed in quantum field theory in Minkowski space by deducing the Lehmann–Symanzik–Zimmermann reduction formula [9, 30]. The selection is achieved by means of a bi-scalar that, at least in the weak-field limit, provides a wave-function description for quantum particles. Mathematically, this is accomplished by constructing a wave packet depending on the space-time point  $x$  via the geodesic distance  $\sigma(x, X)$  – Synge’s world function [58], – and their covariant derivatives at  $X$ , which are contracted with curvature tensors at that point. This naturally allows to get a wave function which is locally given by a superposition of positive-frequency plane waves as in Minkowski spacetime, and which is a zero-rank tensor with respect to general coordinate transformations. This basic idea has been applied so far to model spin-zero quantum particles in gravity [20–22].

We have proposed herein a model to describe spin-half quantum particles in curved space-time in the framework of quantum field theory. Its novelty consists again in assuming that the Einstein equivalence principle and general covariance hold for spin-half quantum particles. It is not a self-evident assumption, because the mainstream approach in quantum field theory in curved spacetime is based instead on exploiting global isometry group of a given spacetime for expressing quantum-field operators through creation and annihilation operators [10–12]. Since local Poincaré group and global isometry group are, generically, not isomorphic to each other, it is unclear if global isometry group should be favoured with respect to local Poincaré group, taking into account that it is the latter which plays a fundamental role in high-energy particle physics [5–9]. Furthermore, general covariance is abandoned in this approach. It follows from favouring different mode functions for the definition of creation and annihilation operators in different patches of same spacetime. From an experimental standpoint, this is in tension with the Bonse–Wroblewski experiment [17]. Indeed, the observed interference pattern is induced there by acceleration which, in the leading-order of approximation, enters wave-packet phase in the form  $-iM\mathbf{a}\cdot\mathbf{x}_R t_R$ , where  $x_R = (t_R, \mathbf{x}_R)$  are Rindler coordinates. Wave functions being eigenfunctions of the Killing vector  $\partial_{t_R}$  of Minkowski spacetime cannot have this acceleration-dependent correction to  $-iMc^2 t_R$ , which are, however, normally used to do quantum particle physics in Rindler spacetime [11]. In accord with general covariance, plane waves re-written in Rindler coordinates do have that observed correction [21].

The first ever experiment that has shown that quantum physics is affected by gravity is the Collela–Overhauser–Werner experiment [16]. The observed interference pattern produced via overlapping two beams of thermal neutrons, propagating at different altitudes with respect to the Earth’s surface, is due to the free-fall acceleration. It was shown in the Bonse–Wroblewski experiment, mentioned above, that an analogous effect takes place in an accelerated reference frame. So, uniform gravity and acceleration cannot be distinguished in quantum-interference experiments [18]. The quantum-particle model proposed here is consistent with this empirical result. It is also consistent with the observed phase shift due to space-time curvature [28], see [21], as well as with the gravitational Aharonov–Bohm effect recently experimentally probed in [59]. This suggests that our model deserves further scrutiny.

Quantum optical communication makes use of photons as elementary information carriers [60]. It is required here to take into account gravitational-field background to keep track of its systematic influence on the information distortion by long-distance quantum communication. Due to rapid developments of satellite-based quantum communication [61–63], it is necessary to acquire a comprehensive insight into how

photons are affected by the gravitational field of Earth. A model describing massless quantum particles needs to be established, going beyond the semi-classical approximation by relying on such ideas as Born's statistical interpretation of quantum measurements, general covariance and local Poincaré invariance.

The Earth's gravitational field is weak – the local curvature length at the Earth's surface is of the order of the astronomical unit. It implies that the Minkowski-spacetime approximation used in elementary particle physics is adequate for the description of microscopic processes, whereas gravity must be taken into account over macroscopic time intervals. Still, it has been argued in [64,65] that black holes of a microscopic size might have been created in the high-density state of the early Universe. The local curvature length nearby such black holes can be much smaller than the hydrogen-atom size. Thus, the question arises as to whether the wave-packet description of elementary and composite particles is adequate once their quantum size is comparable to a local curvature length. If affirmative, it must be clarified what the physical impacts of a strong gravitational field on quantum matter are.

We have partially studied these questions in de-Sitter spacetime [20]: A coordinate-frame-independent non-perturbative (in curvature) wave-packet solution deduced there reveals that the kinematic properties of this wave solution noticeably differ from geodesic motion, unless the inverse Hubble constant (curvature length) is much bigger than the wave-function extent (position variance) which in turn must be much bigger than the Compton or de-Broglie wave-length of spin-zero quantum particles. Preliminary computations show that there might exist a non-perturbative wave-packet solution in Einstein static universes. Notably, the space-time geometry inside an extremely slowly collapsing dust star [66] can be approximated by a closed Einstein universe. The difficulty consists in the circumstance that the wave function depends, generically, on four scalars there, while, in de-Sitter spacetime, on two scalars [20].

The Newton equivalence principle states that gravitational and inertial mass of a body are equal in the non-relativistic and weak-gravity limit [67]. This principle is a result of numerous empirical tests and appears to be entirely accidental from a theoretical standpoint. Still, any experiment has a limited degree of accuracy. Furthermore, quantum field theory and general relativity are the most fundamental theories utilised nowadays for the description of matter and gravitation. Quantum theory of both matter and gravity should be capable of addressing the question how underlying the Newtonian principle actually is. Even though we have found  $M_g \neq M_i$ , the definition of  $M_g$  does not follow from the computation of Newton's gravitation potential sourced by the quantum particle. This computation requires to go beyond the test-particle approximation. In this sense, our model is incomplete, because the

quantum-particle state  $|\psi_{X,P}\rangle$  is oblivious to gravity-field operators, e.g.  $\langle\psi_{X,P}|\hat{h}_{\mu\nu}(x)|\psi_{X,P}\rangle = 0$ , where  $\hat{h}_{\mu\nu}(x)$  is the graviton-field operator defined in the framework of the effective field theory of quantum gravity [31]. Appropriately dressing  $\hat{a}^\dagger(\psi_{X,P})$  by an operator depending on  $\hat{h}_{\mu\nu}(x)$  in the sense of [68] should give a way to determine active gravitational mass of a quantum particle. There is *a priori* no guarantee that it matches passive gravitational mass,  $M_g$ .

A closely related issue is the (main) cosmological constant problem [69–71] following from the *assumption* that the quantum vacuum  $|\Omega\rangle$  gravitates. Indeed, the semi-classical Einstein field equation in vacuum reads

$$R_{\mu\nu}(x) - \frac{1}{2}R(x)g_{\mu\nu}(x) = \frac{8\pi G}{c^4}\langle\Omega|\hat{T}_{\mu\nu}(x)|\Omega\rangle, \quad (74)$$

where, strictly speaking,  $\langle\Omega|\hat{T}_{\mu\nu}(x)|\Omega\rangle$  is divergent [70,71], giving thus rise to a non-physical space-time geometry. However, this problem is a result of the semi-classical approximation in the sense that the metric tensor  $g_{\mu\nu}(x)$  is obtained by solving (74), rather than by computing  $\langle\Omega|\hat{g}_{\mu\nu}(x)|\Omega\rangle$ . For the semi-classical Einstein field equation (74) to hold, the quantum vacuum  $|\Omega\rangle$  must, at least, depend on  $\hat{g}_{\mu\nu}(x)$  and  $\hat{T}_{\mu\nu}(x)$ . This can be achieved by dressing the vacuum state by an operator depending on both  $\hat{g}_{\mu\nu}(x)$  and  $\hat{T}_{\mu\nu}(x)$ . At the moment, it is unclear if this is logically and physically admissible.

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