Unleashing Infinitely Wide Momentum Bandgaps in Photonic Time Crystals

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Abstract – **The emergence of photonic time crystals has engendered considerable scientific curiosity, owing to their unique features, including the momentum bandgaps. However, the generation of experimentally detectable momentum bandgaps poses a formidable challenge, particularly at high frequencies, necessitating the use of high-power pumping that may lead to deleterious material overheating. To tackle this problem, we propose two routes toward theoretically unlimited enhancement of the momentum bandgap size.**

I. Introduction

In the first two decades of the current century, photonic crystal and metamaterial researchers attempted to manipulate light by engineering the geometry and size of inclusions (as well as unit cells) or the distance between them. However, for the next generation of these artificial materials, there is already a tremendous effort to use temporal variations of macroscopic effective parameters such as refractive index or surface impedance for controlling light and enhancing the functionalities or breaking limitations [1]. One of the seductive features of such time-varying structures is the generation of momentum bandgaps [2], which has important applications, for instance, for creating mirror-less lasers [3]. This is because, in contrast to the energy bandgap in time-invariant photonic crystals, in photonic *time* crystals, the field amplitude of electromagnetic waves excited within a momentum bandgap increases exponentially [4]. However, investigations of momentum bandgaps have been focused on non-resonant materials or structures (e.g. Ref. [5]), in which the produced bandgap is not wide. One approach to increasing the width of the bandgap is enhancing the modulation depth. In practice, this means that one needs to inject more pumping power into the system. For example, in an optical regime, such an amount of power injection may result in overheating problems. In Ref. [6], it was pointed out that even opening a moderate-width bandgap causes heating problems.

In this talk, we propose two different approaches to augment the bandgap size while keeping the modulation depth small. The first one is based on modulating material parameters simultaneously in space and time (constructing a photonic space-time crystal). Specifically, we indicate that if a spatially-varying structure is temporally modulated in a standing waveform and the modulation frequency corresponds to twice the energy-bandgap frequency, the momentum bandgap is significantly increased. This approach stands out from previous studies on space-time metamaterials [7] due to its distinct emphasis on enhancing the size of the momentum bandgap, which has not received significant attention in prior research. The second approach uses the notion of "resonance" in a time-varying structure. We demonstrate that temporally modulating a resonant photonic time crystal gives rise to a bandgap, ideally with an infinite width, which provides wave amplification for all the incident wave vectors (momenta). For demonstrating both methods, we choose a metasurface-based photonic time crystal platform [8]. However, our concept of expanding momentum bandgaps is general and can be applied also to bulk (three-dimensional) photonic time crystals.

II. Results

A. Space-time metasurfaces for expanding momentum bandgaps

Consider a spatially modulated metasurface whose surface impedance is capacitive and the surface capacitance is described by the modulation function $C(z) = C_0[1+m\cos(k_\mathrm{m}z)]$ (C_0 is the surface capacitance of the stationary

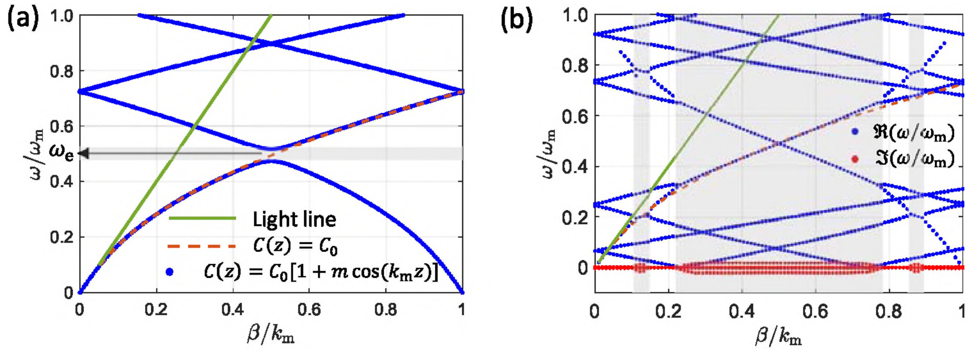

Fig. 1: Band structure of (a) spatially modulated and (b) space-time modulated (standing waveform) metasurface. The frequency and momentum axis are normalized with the temporal and spatial modulation frequencies respectively. The shaded region in Fig. 1(a) represents energy bandgap and the shaded region in Fig. 1(b) represents momentum bandgap where the eigen frequency has complex solutions.

metasurface, m represents the modulation strength, and k_m denotes the modulation spatial frequency). For such a metasurface equivalent to a two-dimensional photonic crystal, there is an energy bandgap with a finite width in the frequency domain. Figure 1(a) shows that the center of this energy bandgap is at $\omega = \omega_e$, which is, in fact, the eigenfrequency of a uniform metasurface, characterized by C_0, for the wavevector of $\beta = 0.5k_m$. Now, we suppose that this metasurface is also modulated in time at twice the central forbidden frequency (i.e., $\omega_m = 2\omega_e$). Therefore, this simultaneous modulation in time and space is represented by a standing-wave modulation function as $C(z,t) = C_0[1 + m\cos(k_m z)\cos(\omega_m t)]$, which results in creation of infinitely many frequency harmonics $\omega_n = \omega_0 + n\omega_m$ (ω_0 denotes the fundamental angular frequency). Note that in this case, each frequency harmonic corresponds to infinitely many Bloch modes $k_p = \beta + pk_m$ (β is the fundamental spatial frequency).

The standing-wave modulation scheme is different from travelling-wave modulations, in which one spatial mode corresponds to one frequency mode. Here, we extend the conventional travelling-wave space-time modulation function to a general and arbitrary periodic space-time modulation, and, subsequently, we calculate the band diagram of the standing-wave modulated structure, as shown in Fig. 1(b). We observe that the width of the momentum bandgap (the middle one) is significantly increased. The maximum width occurs when the modulation frequency is twice ω_e. Intriguingly, unlike momentum bandgaps of photonic time crystals where the real part of the eigenfrequency is constant, here, the real part of eigenfrequency is seen as a tilted line. In the oral presentation, we will also show other interesting features of space-time metasurfaces. For example, we will illustrate overlapping of energy and momentum bandgaps resulting in a linear growth of electromagnetic wave amplitude.

B. Time-varying resonant metasurfaces for infinitely wide momentum bandgaps

The second alluring approach to enhance the momentum bandgap width is to significantly modify the initial dispersion relation (corresponding to the time-invariant scenario) by using a *resonant* structure. The purpose of using a resonant structure is to obtain a nearly flat dispersion curve ($d\omega/d\beta \approx 0$) when there is no modulation. In such structures, upon modulation at twice the resonant frequency, the folding of bands along the frequency axis will result in nearly parallel bands crossing. Such crossing yields an ultra-wide momentum bandgap. In the metasurface setting, we can ponder mushroom structures that are theoretically modeled as parallel resonant circuits (parallel connection of inductance and capacitance). Below the resonance ($\omega < \omega_{res}$), the surface is inductive and supports TM-polarized surface waves. As the phase constant increases from zero value, the dispersion curve associated with this polarization tends to become flat (similarly to moving toward saturation). At the resonance, the curve is approximately flat and the phase constant is practically infinite. Note that the dispersion curve is symmetric with respect to the β-axis (horizontal axis), giving information about negative angular frequencies. Regarding TM-polarized surface wave excitations, we temporally modulate the metasurface at twice the resonant frequency $\omega_m = 2\omega_{res}$. Accordingly, the complete dispersion curve (including the negative angular frequency part) will repeat itself consecutively along the vertical axis (the angular-frequency axis). As a result, two flat curves approach each other near the resonance ($\pm\omega_{res}$), and, therefore, an ultra-wide bandgap is generated.

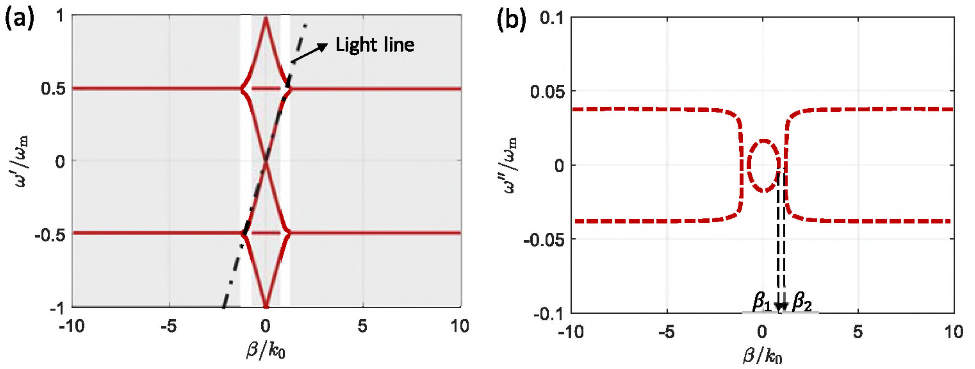

Fig. 2: (a) Real part of the eigenfrequency. The shaded region represents the bandgap. (b) Imaginary part of the eigenfrequency for $\omega' = 0.5\omega_{\rm m}$. In the plots, k_0 is the reference wavevector.

Figure 2 illustrates the real and imaginary parts of the eigenfrequency. We observe that within $|\beta| = [0, \beta_1] \cup [\beta_2, +\infty]$, the eigenfrequency is a complex value confirming the creation of such an ultra-wide bandgap. Here, the quality factor plays a significant role (for the parallel LC model, $Q = R\sqrt{C/L}$, where R represents the loss resistance). Indeed, further increasing the quality factor – increasing C and decreasing L – gives rise to even wider bandgaps so that β_1 and β_2 become identical. This means that the bandgap covers the whole momentum space, and, intriguingly, there is an amplification of free-space waves from all the incident angles as well as surface waves of all momenta.

III. Conclusion

In this study, we have proposed two methods for broadening the bandgap of metasurface-based photonic time crystals. One is based on space-time standing-wave modulation, and the other is based on temporal modulations of resonant structures. The results show that both approaches can effectively enhance the width of momentum bandgaps with only a moderate modulation strength. Moreover, the results reveal new physical phenomena, such as simultaneous parametric amplification of waves with any wavenumber.

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