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A hydraulic energy flow within the moving Earth

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Abstract

We consider the Earth moving through empty space at 30 km s^{-1} (in the Sun's frame of reference). Associated with this motion is a convective flow of kinetic and internal energy. Since there is high pressure inside the Earth, and since the Earth is moving, there is yet another 'hydraulic' energy flow. This latter is what this article is about. Although this energy flow is huge, it is not addressed in the textbooks. The reason is that for the explanation one needs a concept which is not introduced in traditional presentations of classical gravitation: the gravitomagnetic field. The corresponding theory, gravitoelectromagnetism, was formulated in 1893 by Heaviside in analogy to Maxwell's theory of electromagnetism. We discuss the question of what are the sources and sinks of this hydraulic, non-convective energy flow. To answer the question, we need to study the energy flow density distribution within the gravitational field. In doing so, we will make some interesting observations. The energy flow within the field is twice as large as it should be to transfer the field energy from one side of the Earth to the other. The excess flow goes back through the matter of the Earth. Since our readers may not be familiar with Heaviside's theory, we first treat the electromagnetic analogue of our problem and then translate the results to the gravitational situation.

Keywords: gravitational field, gravitoelectromagnetism, poynting vector, energy flow

(Some figures may appear in colour only in the online journal)

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1. Introduction

Our subject is an effect which can be treated by means of classical physics, but which nevertheless is not discussed in the textbooks. The effect is large and would certainly not be ignored in a different context.

Consider the Earth as it moves with 30 km s^{-1} through empty space (in the reference frame of the Sun). With this movement, an energy transport is associated. The kinetic and the internal energy of the Earth are taken along 'convectively'. What usually is not mentioned is that, in addition, there is a non-convective energy flow within the Earth in the direction of motion. It results from the fact that firstly, the interior of the Earth is under high pressure, and secondly, the Earth is moving.

The total energy flow density is [1]

$$j_E = (\rho_{E,\mathrm{kin}} + \rho_{E,\,\mathrm{int}} + p) \cdot \mathbf{v} \tag{1}$$

Here, $\rho_{E,\mathrm{kin}} \cdot \mathbf{v}$ describes the flow of the kinetic energy, $\rho_{E,\,\mathrm{int}} \cdot \mathbf{v}$ that of the internal energy, and $p \cdot \mathbf{v}$ the energy flow that we are concerned with in the following. This latter contribution to the energy transport is traditionally called 'realized work'. It is well known for technical applications as for instance the hydraulic lines of an excavator or of agricultural machinery. Since in the following, we will often refer to this energy flow, we shall give it a name of its own. Let us call it the *hydraulic energy* flow.

One might believe that in the case of the Earth, it is a small, insignificant effect. But actually, this is not the case. Let us estimate its magnitude. In the center of the Earth the pressure is about 3.6×10^{11} Pa. Together with the velocity $v = 30$ km s^{-1} we get an energy flow density of 10 000 TW m^{-2}. Towards the outside, i.e. towards the Earth's surface, the pressure and thus also the energy flow density decreases quadratically with the distance from the center.

Obviously, this hydraulic energy flow must have sources and sinks. In the following, we are concerned with the question of what these sources/sinks are all about. Even a qualitative answer would be valuable. While answering the question, we will make some interesting observations.

Of course, one could get rid of the problem by simply describing the situation in the reference frame of the Earth. In that case, there is no flow of kinetic energy and no flow of internal energy, and there is also no hydraulic energy flow. But that is not what we want. We want to describe the Earth in the reference frame of the Sun.

Our problem has to do with the gravitational field. To solve it, we need the theory of gravitoelectromagnetism (GEM). This theory was formulated by Heaviside in 1893 in complete analogy to Maxwell's theory of electromagnetism [2]. Since probably our readers are not familiar with GEM, we first translate our question into an electromagnetic one. As is well known, the energy flow in the electromagnetic field is described by the Poynting vector, which is introduced in every textbook on electromagnetism, and which is also a subject of numerous recent articles in the American Journal of Physics and the European Journal of Physics, see for example [3–9].

Therefore, in section 2, we establish the energy balance for a moving electrically charged sphere instead of the moving Earth, i.e. a massive sphere. In section 3 we translate the results into statements about gravitation.

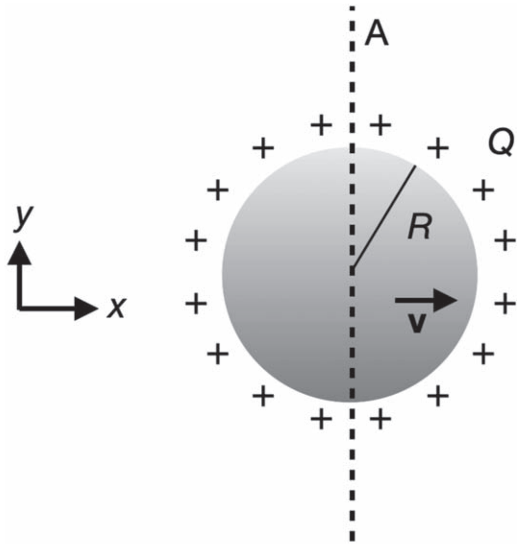

Figure 1. An electrically charged sphere (the charge is located at the surface) moves to the right (in the positive x-direction) thereby crossing the intersection plane A.

2. Energy flow in the case of a translational motion of a charge carrying sphere

2.1. Four contributions to the energy balance

We consider a sphere of radius R that carries the positive electric charge Q and is moving with the velocity $\mathbf{v}$ in the positive x-direction, thereby crossing the intersection surface A that lies in the $y - z$ plane, figure 1. The charge is uniformly distributed on the surface of the sphere and cannot be displaced. Let the velocity be small against c, so that we can consider the field as quasi-static. Thus, we do not need to take retardation into account.

We consider the situation at the moment when the sphere has just crossed the plane A halfway.

Together with the sphere, also the electric field must pass from the half-space on the left ($x < 0$) of A to that on the right ($x > 0$) without changing its shape. This means that the energy of the field must be transported from left to right as well.

In the following, we shall establish the energy balance for the right half-space, i.e. the region to the right of the plane A. For this purpose, we have to consider four contributions:

- the time rate of change of the energy content of the field $\mathrm{d}E_{\mathrm{vol}}/\mathrm{d}t$ that is due to the change of the energy density of the field;
- the time rate of change of the energy content of the field $\mathrm{d}E_{\mathrm{surf}}/\mathrm{d}t$, which is caused by the fact that the moving sphere is constantly cutting out or 'collecting' field at its front surface;
- the energy flow P_{field} through the plane A within the field, i.e. for $r > R$;
- the energy flow P_{matter} through the plane A within the matter of the sphere, i.e. for $r < R$.

For the energy balance to be correct, the following must apply:

$$\frac{\mathrm{d}E_{\mathrm{vol}}}{\mathrm{d}t} + \frac{\mathrm{d}E_{\mathrm{surf}}}{\mathrm{d}t} = P_{\mathrm{field}} + P_{\mathrm{matter}} \tag{2}$$

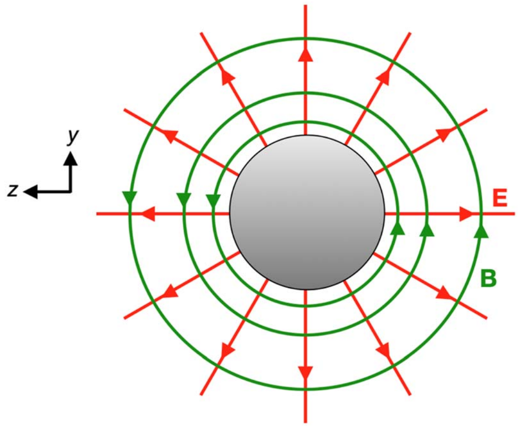

Figure 2. Electric and magnetic field lines. Notice that here the x-axis is perpendicular to the drawing plane; thus, the sphere moves out of the plane of the figure.

In the following sections, we first consider the four contributions one by one and then relate them to each other.

For the calculation, we need the distribution of the two fields **E** and **B**.

The electric field **E** of our charged sphere is

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\frac{\mathbf{r}}{r} \tag{3}$$

The field lines run radially outward, figure 2.

We obtain the magnetic flux density **B** by a transformation of the reference frame. We know that in the reference frame, in which the Sphere is at rest **B** is zero everywhere. We now make a transformation into the reference frame in which it is moving with the velocity **v** and obtain:

$$\mathbf{B} = -\frac{\mathbf{v} \times \mathbf{E}}{c^2} \tag{4}$$

where we have again assumed $v \ll c$. The field lines are circles orthogonal to the direction of motion, figure 2, in which we have chosen the x-direction orthogonal to the drawing plane.

In addition, we need the energy density of the electric field

$$\rho_E(\mathbf{r}) = \frac{\varepsilon_0}{2}|\mathbf{E}(\mathbf{r})|^2 \tag{5}$$

(the energy density of the magnetic field is negligible in our context) and Poynting's formula for the energy flow density $\mathbf{j}_E$ within the electromagnetic field:

$$\mathbf{j}_E = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B} \tag{6}$$

Figure 3 shows the energy flow lines in a sectional surface in the $x - y$ plane passing through the center of the sphere. Since the sources and sinks are distributed all over the space, it is convenient to draw broad lines, and to express the magnitude of the current density not by the line density, but by a gray shading.

From equations (3), (4) and (6) it follows that the energy flow lines are circular.

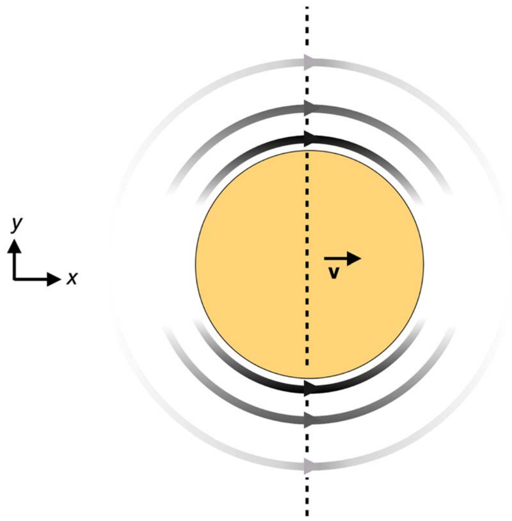

Figure 3. The energy flow lines form circular arcs. They have sources in the left half-space and sinks in the right. The magnitude of the energy flow density is suggested by the gray shading of the flow lines.

2.2. Energy change due to the change of the energy density within the field

From equations (3) and (5) we obtain the energy density of the electric field as a function of r:

$$\rho_E(r) = \frac{1}{32\pi^2\varepsilon_0}\frac{Q^2}{r^4} \tag{7}$$

Since the electric field strength changes with time, the energy density is also time-dependent. In the right half-space, which we are considering, it increases with time. Derivation with respect to time and integration over the right half-space from $r = R$ to $r = \infty$ yields the time rate of change of the energy 'within the volume'

$$\frac{\mathrm{d}E_{\mathrm{vol}}}{\mathrm{d}t} = \frac{1}{16\pi\varepsilon_0}\frac{Q^2}{R^2}v \tag{8}$$

Here v is the velocity of the moving sphere.

2.3. Energy change by collecting field energy at the surface of the sphere

The field strength at the surface of the sphere is

$$|\mathbf{E}(R)| = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} \tag{9}$$

from which the energy density of the field next to the surface is obtained. Because of the movement of the sphere we get for the 'collected' energy, i.e. the energy transferred from the field to the matter of the sphere

$$\frac{\mathrm{d}E_{\mathrm{surf}}}{\mathrm{d}t} = -\rho_E\pi R^2 v \tag{10}$$

and after inserting the energy density

Table 1. Four contributions to the energy balance.

Field	Matter
$\frac{dE_{vol}}{dt} = \frac{1}{16\pi\varepsilon_0}\frac{Q^2}{R^2}v$	$\frac{dE_{surf}}{dt} = -\frac{1}{32\pi\varepsilon_0}\frac{Q^2}{R^2}v$
$P_{\mathbf{field}} = \frac{1}{16\pi\varepsilon_0}\frac{Q^2}{R^2}v$	$P_{matter} = -\frac{1}{32\pi\varepsilon_0}\frac{Q^2}{R^2}v$

$$\frac{dE_{surf}}{dt} = -\frac{1}{32\pi\varepsilon_0}\frac{Q^2}{R^2}v \tag{11}$$

The minus sign indicates that the energy in the right half-space decreases due to this contribution.

2.4. The energy flow within the field

For the energy flow crossing the intersection plane A within the field (i.e. for $r > R$) the energy flow density is

$$\mathbf{j}_{E,\text{field}} = \frac{1}{16\pi^2\varepsilon_0}\frac{Q^2}{r^4}\mathbf{v} \tag{12}$$

By integrating over the surface from $r = R$ to $r = \infty$ we obtain the total flow through A:

$$P_{\text{field}} = \frac{1}{16\pi\varepsilon_0}\frac{Q^2}{R^2}v \tag{13}$$

The fact that the energy flow is positive means that the flow is in the direction of the movement of the sphere.

2.5. The energy flow within the matter of the sphere

We calculate the force component parallel to the direction of motion, multiply by the velocity and obtain

$$P_{\text{matter}} = -\frac{1}{32\pi\varepsilon_0}\frac{Q^2}{R^2}v \tag{14}$$

The minus sign indicates that the energy flow is in the direction opposite to the movement. This is to be expected. The matter is moving and is under tensile stress, such as a drive belt. Here as there, the energy is transported in the opposite direction to the movement. So we have a return flow of the energy.

If the material of the sphere is homogeneous and isotropic, the energy flow density field is homogeneous. The flow density vectors are parallel to the direction of motion of the sphere. In the case of a hollow sphere, the entire energy would flow through the spherical shell.

2.6. The energy balance

What makes our considerations interesting can be seen when we compare the four contributions to the energy balance. We have compiled them in table 1.

We can make three observations:

1. The energy balance, equation (2), is satisfied.
2. The energy change $\mathrm{d}E_{\mathrm{vol}}/\mathrm{d}t$ within the field is equal to the energy flow P_{field} across the field. We could have made this statement without any calculation. The change of the energy density at any position within the field can only come about by an energy flow within the field itself, described by the Poynting vector. Likewise, the energy flow within the matter can only come about by energy being absorbed or 'collected' by the matter at the surface of the sphere.
3. Another observation is less evident: In the sectional plane A, the energy flow through the field is twice as large as the return flow through the matter. However, if one knows the corresponding energy balance for a moving capacitor [10], this result is not surprising either. When a charged capacitor is moved parallel to its plates, an energy current flows within the field parallel to the plates, i.e. in the direction of the movement. Also, this energy flow is twice as large as would be necessary to remove the field on one side and to build up field on the other one. Nevertheless, the energy balance is correct, because the excess energy flow, which is not needed to build up the field, flows back within the matter of the plates.
4. We have now the answer to our initial question: Which are the sources and sinks of the mechanical energy flow within the matter of the sphere? It is the collected field that acts as the source, and the deposited field that acts as the sink of the hydraulic energy flow.

2.7. A sphere whose charge is homogeneously distributed over the volume

Before comparing with the moving Earth—and this is our goal after all—we still have to investigate what changes in our results if the charge is not located on the surface of the sphere but is homogeneously distributed over its volume.

The field strengths **E** and **B** outside the sphere remain the same as when the total charge is located on the surface. As a consequence, nothing changes in the expressions $\mathrm{d}E_{\mathrm{vol}}/\mathrm{d}t$ and P_{field} as long as they refer to the region with $r > R$.

What changes is the interior of the sphere, i.e. the space with $r < R$, which was previously field-free.

The magnitude of the two field strengths decreases from the surface to the center linearly to zero. Correspondingly, the energy density of the electric field, as well as the magnitude of the energy current density decrease quadratically.

In the sectional plane A, the energy current density in the field results to be

$$\mathbf{j}_{E,\mathrm{field,inside}} = \frac{1}{16\pi^2\varepsilon_0} Q^2 \frac{r^2}{R^6}\mathbf{v} \tag{15}$$

By integration from $r = 0$ to $r = R$ we obtain the total energy flow through A inside the sphere:

$$P_{\mathrm{field,inside}} = \frac{1}{32\pi\varepsilon_0} \frac{Q^2}{R^2} v \tag{16}$$

From equations (13) and (16) we obtain the total energy flow through the plane A within the field:

$$P_{\mathrm{field}} = P_{\mathrm{field,inside}} + P_{\mathrm{field,outside}} = \frac{3}{32\pi\varepsilon_0} \frac{Q^2}{R^2} v \tag{17}$$

Now about the mechanical quantities. There is tensile stress within the matter of the sphere. The corresponding stress tensor has the same structure as that for hydrostatic pressure, i.e. it has a diagonal form, but it differs from the latter in the sign.

The magnitude of the tensile stress decreases quadratically with the distance from the center to zero at the surface:

$$p = -\frac{3}{32\pi^2\varepsilon_0}Q^2\frac{(R^2 - r^2)}{R^6} \tag{18}$$

Because of

$$\mathbf{j}_{E,\mathrm{matter}} = p \cdot \mathbf{v} \tag{19}$$

the energy flow density in plane A also decreases quadratically from the center to the outside:

$$\mathbf{j}_{E,\mathrm{matter}} = -\frac{3}{32\pi^2\varepsilon_0}Q^2\frac{(R^2 - r^2)}{R^6}\mathbf{v} \tag{20}$$

By integrating, we obtain the energy flow within the matter of the sphere through the cross-sectional area:

$$P_{\mathrm{matter}} = -\frac{3}{64\pi\varepsilon_0}Q^2\frac{1}{R^2}v \tag{21}$$

In figure 4, both energy flow densities, i.e. that of the field and that of the matter, are plotted on the y-axis. The figure would look the same for any other straight line lying in the A-plane and passing through the center of the sphere.

Figure 5 shows the shape of the energy flow lines. The magnitude of the current density is suggested by the gray shading of the flow lines.

Again, it is interesting to compare the forward flow through the field, equation (17), with the backward flow through the matter, equation (21). It turns out that also in the case of the homogeneously charged sphere, the energy flow crossing A in the forward direction (within the field) is twice as large as that in the backward direction (within the matter):

$$P_{\mathrm{field}} = -2 \cdot P_{\mathrm{matter}} \tag{22}$$

3. Translation of the results to gravitation

We recall our original question: How does the hydraulic energy flow within the Earth come about? Where does it originate and where does it end? In the previous section on the electromagnetic analogue, we found and calculated such a mechanical energy flow and we have determined its sources and sinks.

Now it is not necessary to repeat the whole calculation for the gravitational case. It is sufficient to simply translate the results of the electromagnetic analog.

However, there is still some clarification to be done for those readers who are not familiar with Heaviside's GEM.

3.1. General remark on Heaviside's theory

For the description of gravitational phenomena we usually learn and teach two theories: the Newtonian theory of gravitation and Einstein's general theory of relativity. Because of the tensor character of the physical quantities and because of the non-linearity of Einstein's

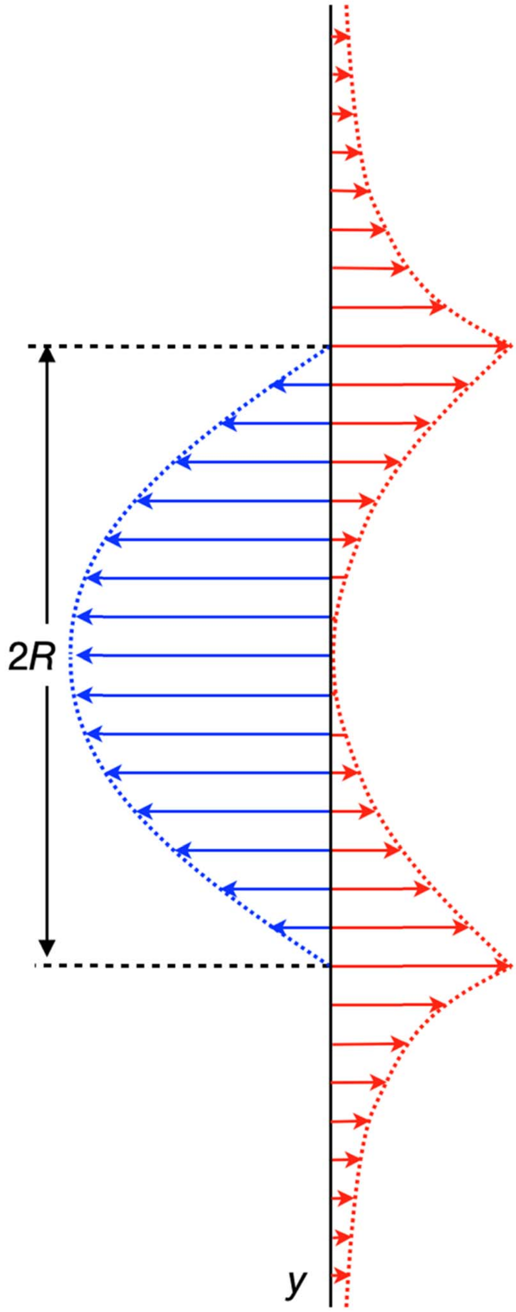

Figure 4. Energy flow density in the field (red) and in the matter (blue) on a straight line lying in the sectional plane and passing through the center of the sphere.

equations, general relativity is mathematically demanding. The much simpler Newtonian theory is sufficient for many purposes as an approximation, and it is therefore taught at schools and universities.

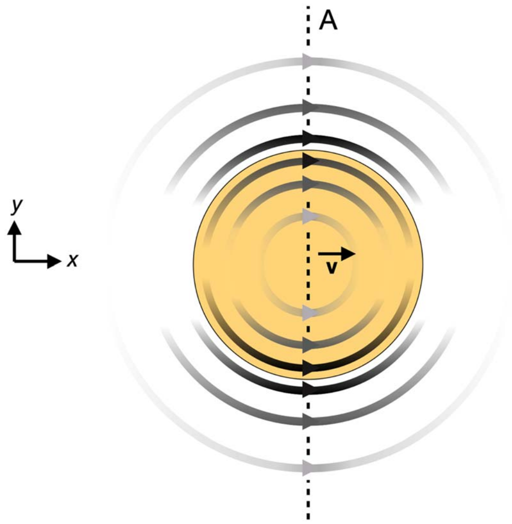

Figure 5. Inside as well as outside of the sphere, the energy flow lines form circular arcs.

Actually, Newton's theory of gravitation has a conceptual flaw: it operates with actions at a distance. This becomes manifest in the handling of the energy. It does not allow to formulate local energy balance. In Newton's time, the physical quantity energy did not yet exist and the field concept was not yet known. The deficiency could be remedied 150 years later by taking the first field theory, Maxwell's electrodynamics, as a model. In 1893, Heaviside presented a theory of gravitation largely analogous to Maxwell's theory, which could be used to establish a local energy balance [2, 11].

Of course, also Heaviside's theory cannot keep up with general relativity. The limits of its validity are discussed in [12]. It is not Lorentz invariant and it can only describe quasi-static processes. But this is also true for Newton's theory, which we are using every day. In any case, Heaviside's GEM is more powerful than the Newtonian theory, and we can safely say that it fills the gap between the Newtonian and the Einsteinian theory.

The idea behind Heaviside's theory is simple: take Maxwell's theory and replace the electromagnetic quantities with quantities, some known and some to be redefined, that relate to the gravitational field.

It should be noted that apart from Heaviside's theory there is a second theory, also called gravitoelectromagnetism, which differs from Heaviside's theory by a factor of 4 in some equation, see for instance Thirring [13]. In the following, we always refer to Heaviside's version. Thirring's theory arose after general relativity and represents a linear approximation of Einstein's theory. It allows to describe some general relativistic effects locally.

3.2. The gravinetic field

When discussing the electromagnetic case, we used Poynting's formula. It contains the electric field strength and the magnetic flux density.

To understand the gravitational analogue, we have to take note that besides the gravitational field strength $\mathbf{g}$, which is the analogue to the electric field strength $\mathbf{E}$, we have to introduce an analogue to $\mathbf{B}$, the so-called gravitomagnetic, or more briefly gravinetic field $\mathbf{b}$ [14]. This quantity is hardly known among physicists. The reason is that the associated force

Table 2. Corresponding physical quantities.

EM	GEM
Q	m
$\mathbf{E}$	$\mathbf{g}$
$\mathrm{B} = -\frac{\mathrm{v} \times \mathrm{E}}{c^2}$	$\mathrm{b} = \frac{\mathrm{v} \times \mathrm{g}}{c^2}$
ε_0	$\varepsilon_\mathrm{g} = \frac{1}{4\pi G}$
μ_0	$\mu_\mathrm{g} = \frac{4\pi G}{c^2}$

(the analogue of the magnetic Lorentz force) is extremely small in typical situations and plays practically no role anywhere. However, the gravinetic field manifests itself clearly in the energy flow [11, 14], and an equation analogous to the Poynting formula also applies to the gravitational field:

$$\mathbf{j}_E = -\frac{1}{\mu_\mathrm{g}}\mathbf{g} \times \mathbf{b} \tag{23}$$

3.3. The sign

Since bodies attract each other due to their mass, while similarly electrically charged objects repel each other, several signs in the GEM equations are opposite to those in the electromagnetic equations. Therefore, if we translate our electromagnetic results to gravitation, we have to reverse signs. This means in particular, that the energy density of the field is negative, and that current density vectors have the opposite direction. Table 2 shows the translation of those physical quantities that are needed for our translation.

3.4. The forward flow in the field and the backward flow in the matter

Everything said about the electrically charged sphere can now –*mutatis mutandis*– be transferred to the moving Earth. Here, we are interested in particular in the translation of equations (17) and (21). Equation (17) becomes

$$P_\mathrm{field} = -\frac{3G}{8}\frac{m^2}{R^2}v \tag{24}$$

and equation (21)

$$P_\mathrm{matter} = -\frac{3G}{16}\frac{m^2}{R^2}v \tag{25}$$

As we might have expected, we find

$$P_\mathrm{field} = -2 \cdot P_\mathrm{matter} \tag{26}$$

In this case, however, the energy flow within the field is in the direction opposite to the velocity, while the energy flow in the matter is in the same direction as the velocity. The deposited field acts as a source and the collected one as a sink for the ‘hydraulic’ energy flow in the matter of the moving Earth.

4. Conclusion

In the introduction, we had stated that a qualitative answer to our question would be valuable. We are now in a position to give such an answer. It could be somewhat like this:

As the Earth is moving, it is constantly picking up field in the forward direction, while depositing new field at its backside. The corresponding energy must be transported by the matter of the Earth from the one to the other side. Since the energy density in the gravitational field is negative, this energy flow runs from the back to the front side, i.e. in the direction of the movement.

This process has yet another consequence: the energy flow that passes through the field from front to back, is twice as large as would be necessary to remove the field on one side and to build up field on the other one. The energy balance is correct, because the excess energy flow, which is not needed to build up the field, flows back within the matter of the Earth.

Data availability statement

No new data were created or analysed in this study.

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