European Journal of Operational Research 305 (2023) 1–20

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Invited Review

Spatial coverage in routing and path planning problems

Katharina Glock[a,1], Anne Meyer[b,2,*]

[a] FZI Research Center for Information Technology, Haid-und-Neu-Str. 10-14, Karlsruhe D-76131, Germany
[b] TU Dortmund University, Leonhard-Euler-Straße 5, Dortmund D-44227, Germany

ARTICLE INFO

Article history:
Received 15 July 2021
Accepted 13 February 2022
Available online 18 February 2022

Keywords:
Vehicle routing problem
Informative path planning
Orienteering problem
Covering tour problem
Close-enough vehicle routing

ABSTRACT

Routing and path planning problems that involve spatial coverage have received increasing attention in recent years in different application areas. Spatial coverage refers to the possibility of considering nodes that are not directly served by a vehicle as visited for the purpose of the objective function or constraints. Despite similarities between the underlying problems, solution approaches have been developed in different disciplines independently, leading to different terminologies and solution techniques. This paper proposes a unified view of the approaches: Based on a formal introduction of the concept of spatial coverage in vehicle routing, it presents a classification scheme for core problem features and summarizes problem variants and solution concepts developed in the domains of operations research and robotics. The connections between these related problem classes offer insights into common underlying structures and open possibilities for developing new applications and algorithms.

1. Introduction

In recent years, in different disciplines and application areas, vehicle routing problems (VRP) have been extended such that the demand of nodes can, in some form, be fulfilled by visiting nearby nodes: In the field of logistics, for example, models have been proposed in which customer demand can be fulfilled anywhere within walking distance of some target instead of requiring service at one specific location. These types of routing problems arise in emergencies such as earthquakes, floods, or in mobile health care scenarios (Allahyari, Salari, & Vigo, 2015; Hachicha, Hodgson, Laporte, & Semet, 2000). Similar ideas have been used for deciding on distribution structures for humanitarian relief and medical supplies (Naji-Azimi, Renaud, Ruiz, & Salari, 2012a; Veenstra, Roodbergen, Coelho, & Zhu, 2018). Other applications include the routing of vehicles through transshipment facilities from which the final customers are served (Current & Schilling, 1989), the planning of routes for mobile meter reading services (Gulczynski, Heath, & Price, 2006), and ATM replenishment (Orlis, Bianchessi, Roberti, & Dullaert, 2020). Problems arising in the context of planning postal services and telecommunication networks share similar characteristics (Gendreau, Laporte, & Semet, 1997; Vogt, Poojari, & Beasley, 2007; Xu, Chiu, & Glover, 1999).

A different application area for these types of routing problems originates from robotics: Unmanned aerial vehicles (UAVs) are deployed to detect toxic gases in the air. The goal is to build a gas distribution map for the affected area after chemical emergencies or large fires. The gas concentration at a location is not independent of nearby locations. UAV-based routing and mapping approaches take this spatial dependency into account to infer gas concentrations at unsurveyed locations by using the information from nearby visited locations in probabilistic process models (Glock & Meyer, 2020; Singh, Krause, Guestrin, & Kaiser, 2009a). Other applications in aerial vehicle routing address the problem of identifying targets within a given distance of a vehicle trajectory (Behdani & Smith, 2014) or obtaining data on different targets from within a given maximum distance (Mennell, 2009).

These problems are closely related in the sense that they treat nodes in close spatial proximity as related. In the following, we refer to this as the concept of "spatial coverage". However, due to the difference in application areas and even disciplines, solution approaches have been developed independently. This has led to the establishment of different terminology for very similar problems, which hinders the development of solution concepts that exploit the similarity to problem variants from other domains or disciplines.

* Corresponding author.
E-mail addresses: kglock@fzi.de (K. Glock), anne2.meyer@tu-dortmund.de (A. Meyer).
[1] ORCID: https://orcid.org/0000-0003-0670-6269
[2] ORCID: https://orcid.org/0000-0001-6380-1348

https://doi.org/10.1016/j.ejor.2022.02.031
0377-2217/

In this study, we seek to offer a unified view on variants in vehicle routing an path planning that consider some form of spatial coverage. Our contributions toward this goal are:

(1) the definition of the spatial coverage concept in routing and path planning and a scheme for categorizing problems involving aspects of spatial coverage,
(2) the introduction of unified mathematical models to highlight similarities and show differences between models developed in different domains and communities,
(3) the overview of problems, solution approaches, and benchmark instances derived in different disciplines, and
(4) the identification of connections between structurally related problems that can build the basis of new solution approaches.

This paper is organized as follows. In the next section, we introduce a general description of VRP with spatial coverage, propose a classification scheme, and derive six important classes. In Section 3, we detail the six classes and their application areas, formally introduce the problem variants, and discuss exact and heuristic solution methods proposed in literature. We furthermore list problem variants that are closely related but cannot be assigned to one of the classes. In Section 4, we discuss and summarize insights that emerged from the overall review of models and identify promising future research avenues.

2. Definition and classification of the VRP with spatial coverage

In this section, we give a general description of the VRP with spatial coverage (VRP-SCOV) and introduce the necessary notation for defining its variants. We then propose a classification scheme for the variants of the VRP-SCOV and derive six classes wherein problems share important characteristics.

2.1. Definition of the general VRP-SCOV

As common for VRP, the VRP-SCOV is defined on a graph $G = (\mathcal{V}, \mathcal{A})$ with a set of nodes $\mathcal{V}$ and a set of arcs $\mathcal{A}$. The set $\mathcal{V}$ is composed of a set of depot nodes $\mathcal{V}^d$ and customer nodes $\mathcal{V}^c$ to be visited or covered in vehicle tours. The nodes can be visited or covered by one vehicle or a fleet of vehicles denoted $\mathcal{M}$. The tour length of each vehicle is limited by a value T. Traveling along an arc $(i, j) \in \mathcal{A}$ is associated with a travel distance d_{ij} and requires a travel time t_{ij}. If applicable, t_{ij} contains the service time at node i.

The special feature of the VRP-SCOV is the spatial covering mechanism: By visiting a node $i \in \mathcal{V}^c$, other nodes $k \in \mathcal{V}^c$ within a maximum covering distance d can be “covered”, i.e., they can be considered as visited for the purpose of the objective function or constraints of the problem. We refer to the strength of the interdependency, i.e., the degree of coverage of node k if node i is visited, as the weight w_{ik} with $w_{ik} \in [0, 1]$. The nodes k that a visit at node i can cover belong to the covering neighborhood $\mathcal{C}_i$ of i with $\mathcal{C}_i = \{k \in \mathcal{V}^c : w_{ik} > 0\}$.

Fig. 1 gives an illustrative example for the covering mechanism. The image depicts a vehicle route traveling through four nodes, starting and ending at a central depot (black triangle). The covering neighborhoods are depicted as circles with radius d. All nodes within these circles are considered covered by the vehicle tour. Three nodes remain uncovered.

Depending on the application, nodes $i \in \mathcal{V}^c$ additionally may be associated with a profit $p_i \geq 0$ that is collected when a node is visited directly or which is collected partially (depending on the degree of coverage w_{ij}) if a nearby visit covers a node.

Based on the introduced notation, we define the general variant of the VRP-SCOV as follows: For nodes within $\mathcal{V}^c$, assign a vehicle m from vehicle set $\mathcal{M}$ and find a tour through the assigned nodes for each vehicle $m \in \mathcal{M}$ such that the travel time budget T is not exceeded and an objective function is optimal. The objective function maximizes either the profit associated with visited and covered nodes or minimizes the cost for visiting or covering all nodes.

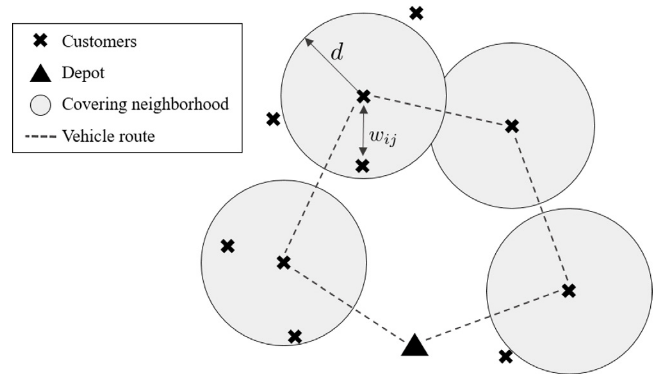


Fig. 1. Example for a vehicle routing problem with spatial coverage.

Table 1
Problem characteristics relating to spatial coverage.

Characteristic	Variants
Node types	mandatory-active (ma)
	mandatory-passive (mp)
	optional-active (oa)
	optional-passive (op)
Spatial coverage	complete
	partial
	probabilistic
Topological space model	discrete
	continuous
Planning objective	min routing
	min allocation
	max profit

So far, we discussed spatial coverage for node-based vehicle routing problems. If we come to the class of arc routing problems, in which all arcs or a given set of arcs must be visited, spatial coverage occurs in a very similar way: An arc provides coverage to a set of nodes if the distance between the arc and the nodes is below a certain threshold. As the majority of problems addresses node-based vehicle routing problems, we mainly discuss these types of problems indicating whenever the arc-based problem variant differs.

The particular challenges of all problem classes subsumed under VRP-SCOV arise from the spatial covering mechanism, which introduces interdependencies between nodes. For multi-vehicle problems, spatial coverage additionally means that routes are highly interdependent.

2.2. Classification scheme

The different problem variants can be distinguished by means of four major characteristics: the different types of nodes that are involved, the type of spatial coverage, the topological space model, and the planning objective. We give a summary of possible attributes in Table 1 and discuss them in more detail below.

2.2.1. Node types

The set of nodes $\mathcal{V}^c$ can be divided into active and passive nodes: **Active** nodes can be visited directly by a vehicle and can provide coverage for nodes close to them. It is important to note that they can also provide coverage to each other. **Passive** nodes, in contrast, cannot be visited directly but may be covered by visiting active nodes nearby. In case of arc-based routing problems, all nodes are passive and are covered if nearby arcs are part of the solution.

Within each group, nodes can be either mandatory or optional. **Mandatory** active nodes have to be included in a vehicle tour, while mandatory passive nodes have to be covered. **Optional** nodes can be covered or visited if this is beneficial with respect to the objective function or required by a constraint.

2.2.2. Spatial coverage

We can further distinguish between different forms of coverage that have been discussed in literature. Essentially, these forms describe how the weights w_{ij} that characterize the degree to which a node (or an arc) i can cover a node j are determined.

Complete coverage means that the full requirements of a node are met or its entire profit is collected as long as it is included in the covering neighborhood of at least one visited node, i.e., $w_{ij} = 1$ for all $i \in \mathcal{V}^c$, $j \in \mathcal{C}_i$.

Partial coverage indicates that any unvisited, but covered node only provide partial benefits compared to a solution where they are directly visited, i.e., $w_{ij} < 1$ for $i \in \mathcal{V}^c$, $j \in \mathcal{C}_i$.

Probabilistic coverage refers to approaches that use probabilistic models to determine the relationship between nodes. This means that weights w_{ij} are not set explicitly but may result from some formalized model for representing spatial relations. A more detailed discussion follows in Section 3.6.

In all cases, the maximum benefit of covering a node should never exceed the benefit provided if this node were visited by a vehicle.

2.2.3. Topological space models

Problems involving spatial coverage can be continuous or discrete. **Discrete** models are typically defined over a graph through which the vehicles are routed. **Continuous** models are defined over a plane in which vehicle routes can be freely determined.

2.2.4. Planning objectives

Generally speaking, the planning objectives relating to spatial coverage can be differentiated into profit maximization and cost minimization. The cost types must be further differentiated, so that the following three planning objective types can be derived:

Min routing objectives minimize the routing cost for visiting or covering all nodes. Routing cost depend on the distance or travel time of the resulting tour plan.

Min allocation objectives minimize (in addition to the routing cost) the allocation cost for visiting or covering all nodes. Allocation costs depend on the distance between the covered node j and the visited node i that is providing coverage, i.e., the node i that j it is “allocated” to.

Max profit objectives maximize the profits or contributions provided by the visited or covered nodes subject to route length restrictions.

We also consider some problem variants with more than one objective. In these cases, we classify the models based on the way how they treat the spatial coverage aspect.

2.3. Classes of VRP-SCOV

Based on the properties mentioned above, we derived six classes of VRP that include the spatial coverage mechanism. As far as possible, the proposed classification and terminology follows established understandings in literature where different problem classes have emerged independently from one another. In many cases, the problems are inspired by real-world applications. Hence, some of the models cannot be clearly assigned to one class or the other and the classes might not cover all possible model variants. However, our claim is not the perfect assignment of all possible model variations, but the elaboration of common features in addressing the aspect of spatial coverage. In this way, we want to simplify the search for similar problems, suitable modeling variants, and solution methods.

A short description of the six problem classes with respect to the characteristics as summarized in Table 1 follows below. The formal description is given in the next section.

Covering tour problems (CTP) (Section 3.1)
ma, oa, mp | complete | discrete | min routing
visit or cover all mandatory passive or active nodes respectively minimizing routing cost.

Close-enough vehicle routing problems (CEVRP) (Section 3.2)
mp | complete | continuous | min routing
cover all mandatory passive nodes by determining vehicle tours in a continuous plane that pass sufficiently close minimizing routing cost.

Close-enough arc routing problems (CEARP) (Section 3.3)
mp | complete | discrete | min routing
cover all mandatory passive nodes by visiting arcs that are sufficiently close minimizing routing cost.

Vehicle routing allocation problems (VRAP) (Section 3.4)
ma, mp, oa | complete | discrete | min routing, min allocation, (max profit)
visit or cover (allocate) all mandatory active and passive nodes minimizing routing and allocation cost. In some variants, a penalty for not visiting or covering optional active nodes is incurred. This can be interpreted as a max profit objective.

Orienteering problems with coverage (OPCov) (Section 3.5)
oa, op | complete or partial | discrete or continuous | max profit
visit or cover optional active or passive nodes respectively maximizing the profit. Only one of the introduced variants operates on a continuous plane.

Informative path planning (IPP) (Section 3.6)
oa | probabilistic | discrete or continuous | max profit
visit or cover optional active nodes maximizing the profit. The profit corresponds to the information gain about visited and unvisited nodes. Both discrete and continuous variants are subsumed under the term IPP in literature.

Table 2 describes all classes and notable corresponding problem variants based on the classification scheme introduced in the preceding subsections. Additionally, we indicate whether the problems have been studied for single vehicles or vehicle fleets.

3. Overview of classes of the VRP-SCOV

This section provides a detailed overview of the different problem classes incorporating coverage aspects in routing and path planning. The main objective is to present the different variants that have been proposed in literature in a unified way and to highlight the potential of solution approaches tailored to the VRP-SCOV. To this end, we present applications, problem variants, and representative problem formulations and summarize exact as well as heuristic solution methods for each problem class.

Note that, for some problem classes, the literature is already relatively “streamlined” in the sense that several authors agree on common problem features and terminologies. This is particularly true for CTP and CEVRP variants. In contrast, research related to the VRAP has led to similar problem variants published under different names. The same holds for the OPCov, where several problem

Table 2
Classification of problem variants introduced in literature.

Problem class	Problem variant	Objective: min routing	Objective: min allocation	Objective: max profit	Coverage: complete	Coverage: partial	Coverage: probabilistic	Nodes: mandatory active	Nodes: optional active	Nodes: mandatory passive	Nodes: optional passive	Model: discrete	Model: continuous	Vehicles
covering tour problems	covering salesman problem (CSP)	√			√				√			√		1
	generalized covering salesman problem (GCSP), *mm*-CTP	√			√				√			√		1, *m*
	covering tour problem (CTP), *m*-CTP, multi-depot CTP	√			√			√	√	√		√		1, *m*
close-enough vehicle routing problems	traveling salesman problem with neighborhoods (TSPN)	√			√					√			√	1
	close-enough traveling salesman problem (CETSP), CEVRP	√			√					√			√	1, *m*
close-enough arc routing problems	close-enough arc routing problem (CEARP)	√			√					√		√		1
	generalized directed rural postman problem (GDRP), DC-GDRP	√			√					√		√		1, *m*
vehicle routing allocation problem	median cycle problem (MCP), ring-star problem (RSP)	√	√		√				√			√		1
	Steiner ring-star problem (SRSP)	√	√		√				√	√		√		1
	m-ring-star problem (*m*-RSP)	√	√		√				√	√		√		*m*
	vehicle routing allocation problem (VRAP)	√	√	√	√			√	√	√		√		1,*m*
orienteering problems with coverage	time constrained maximal covering salesman problem (TCMCSP)			√		√			√			√		1
	set orienteering problem (SOP)			√	√				√			√		1
	correlated team orienteering problem (CorTOP), GCorTOP			√		√			√			√		*m*
	team orienteering problem with overlaps (TOPO)			√	√				√		√	√		*m*
	close-enough orienteering problem (CEOP)			√	√						√		√	1
informative path planning problems	informative path planning (IPP)			√			√		√			√[a]	√[a]	1, *m*

[a] Both variants exist under the same name.

Table 3
Overview of sets and parameters for modeling VRP-SCOV-variants.

Sets	
$\mathcal{V}$	Set of nodes with $\mathcal{V} = \mathcal{V}^d \cup \mathcal{V}^c$
$\mathcal{V}^d$	Set of depot nodes containing the single depot D
$\mathcal{V}^c$	Set of nodes to be visited or covered with $\mathcal{V}^c = \mathcal{V}^a \cup \mathcal{V}^p$
$\mathcal{V}^a$	Set of active nodes with $\mathcal{V}^a = \mathcal{V}^{ma} \cup \mathcal{V}^{oa}$
$\mathcal{V}^{ma}$	Set of mandatory active nodes
$\mathcal{V}^{oa}$	Set of optional active nodes
$\mathcal{V}^p$	Set of passive nodes with $\mathcal{V}^p = \mathcal{V}^{mp} \cup \mathcal{V}^{op}$
$\mathcal{V}^{mp}$	Set of mandatory passive nodes
$\mathcal{V}^{op}$	Set of optional passive nodes
$\mathcal{C}_i$	Set of nodes that can be covered by $i \in \mathcal{V}^a$
$\mathcal{A}$	Set of arcs
Parameters	
$\alpha \in (0, 1)$	Fixed factor used for discounting profit of covered nodes
$c_{ij} \geq 0$	Cost for covering node j from node i
$d \geq 0$	Maximum coverage radius
$d_{ij} \geq 0$	Distance between nodes i, j
$\epsilon > 0$	Minimum segment length
$f(d_{ij})$	Auxiliary function for determining weights w_{ij}
$\lambda_1, \lambda_2, \lambda_3 > 0$	Objective function weights
$(lon_i, lat_i) \in \mathbb{R}^2$	Position of node i
$p_i > 0$	Profit associated with visiting or covering i
$T > 0$	Maximum route budget
$w_{ij} \in [0, 1]$	Weight indicating the degree of coverage provided by i for j

variants with relatively minor differences and no consistent terminology have emerged. To highlight similarities and differences, we adapted formulations from literature such that all are presented as variants of the VRP-SCOV. An overview of the harmonized notation relevant for all models is given in Table 3. Since the focus in this work lies on the spatial coverage concept, we limit the problem formulations to the single-vehicle variants. This enables us to distinguish more clearly between the different models. For better readability, we define a set of depot nodes $\mathcal{V}^d$ containing one depot D with exactly one vehicle. Multi-vehicle cases will be discussed for those problem variants where they have been studied in literature.

The literature for each class is summarized in a table in Appendix B (Tables B1 to B6). The tables indicate the proposed solution approaches and characteristics of the instances used for evaluation purposes. However, it is important to note that few consistent benchmark instances have emerged in literature, and even these are used inconsistently.

3.1. Covering tour problem

Coverage aspects have been considered in VRP literature in form of the covering salesman problem (CSP), the covering tour problem (CTP), and the multi-vehicle covering tour problem (m-CTP). These problems deal with the determination of cost-minimal routes such that every node is either visited directly or is within a given maximum distance to a node directly visited by a vehicle. The CTP can be formulated as the generalized traveling salesman problem (GTSP) given some assumptions about the covering neighborhoods discussed at the end of Section 3.1.2.

3.1.1. Problem definition and applications

The CSP was first introduced and formulated by Current & Schilling (1989) as a variant of the traveling salesman problem (TSP) where all nodes have to be within a predetermined maximum distance of a node that is visited by the vehicle. Applications proposed by Current & Schilling (1989) include mobile health care, where it might be sufficient to provide service within reach of the population, and aircraft transport, where it is sufficient to deliver goods reasonably close to the customer, while a different mode of transport completes the final delivery. Furthermore, the authors introduce a bi-objective variant differentiating between the routing cost and the cost for opening facilities at the selected locations.

Afterward, the problem class did not receive much attention until Gendreau et al. (1997) proposed a more general model that distinguishes between optional and mandatory active nodes, which they refer to as the covering tour problem (CTP). Hodgson, Laporte, & Semet (1998) apply this problem for the provisioning of health care in Ghana. Golden, Naji-Azimi, Raghavan, Salari, & Toth (2012) propose a problem generalization where some nodes need to be covered or visited multiple times in order to fulfill the entire demand. The authors refer to this problem as the generalized covering salesman problem (GCSP).

The first multi-vehicle variant was discussed by Hachicha et al. (2000) as an extension of the CTP, denoted the m-CTP. Additional constraints restrict the number of visits per tour as well as the tour length. The authors apply their solution procedures for the same use case in mobile health care as Hodgson et al. (1998). Flores-Garza, Salazar-Aguilar, Ngueveu, & Laporte (2017) introduce an extension of the m-CTP where the objective does not lie in the determination of the shortest tour but in the minimization of the sum of arrival times at visited locations. The authors denote this problem the multi-vehicle cumulative covering tour problem (m-CCTP). An extension of the GCSP with multiple vehicles, referred to as the multi-vehicle multi-covering tour problem (mm-CTP) is discussed by Pham, Hà, & Nguyen (2017).

3.1.2. Problem formulation

The CTP can be formulated as follows (adapted from Gendreau et al., 1997): The set $\mathcal{V}^c$ is separated into three sets: the set of mandatory active nodes $\mathcal{V}^{ma}$, the set of optional active nodes $\mathcal{V}^{oa}$, and the set of mandatory passive nodes $\mathcal{V}^{mp}$. In the model formulation below, mandatory and optional active nodes are comprised in the set of active nodes $\mathcal{V}^a$.

With respect to the VRP-SCOV, the coverage of passive nodes is considered as complete and, hence, can be represented by weight parameters $w_{ij} = 1$ if $d_{ij} \leq d$ and $w_{ij} = 0$ otherwise for $i \in \mathcal{V}^{mp}, j \in \mathcal{V}^a$. Based on the weight w_{ij} we can determine the covering neighborhood $\mathcal{C}_i$ of each active node $i \in \mathcal{V}^{ma} \cup \mathcal{V}^{oa}$ as $\mathcal{C}_i = \{j \in \mathcal{V}^{mp} \mid w_{ij} = 1\}$.

Binary decision variables x_{ij} indicate whether node i is visited immediately before node j. Decision variables y_i indicate whether nodes are part of the tour or not. Note that, in contrast to Gendreau et al. (1997), we do not use an undirected graph in order to keep this problem formulation consistent with other problems discussed in this paper.

The CTP can be stated as follows:

$$\textbf{(CTP)} \qquad \min \sum_{i,j \in \mathcal{V}^a \cup \mathcal{V}^d \mid i \neq j} d_{ij} x_{ij} \tag{1}$$

s.t.

$$\sum_{j \mid i \in \mathcal{C}_j} y_j \geq 1 \qquad i \in \mathcal{V}^{mp} \tag{2}$$

$$\sum_{j \in \mathcal{V}^a} x_{ij} = 1 \qquad i \in \mathcal{V}^d \tag{3}$$

$$\sum_{j \in \mathcal{V}^a} x_{ji} = 1 \qquad i \in \mathcal{V}^d \tag{4}$$

$$\sum_{i \in \mathcal{V}^a \cup \mathcal{V}^d \mid i \neq k} x_{ik} + \sum_{j \in \mathcal{V}^a \cup \mathcal{V}^d \mid k \neq j} x_{kj} = 2 y_k \qquad k \in \mathcal{V}^a \cup \mathcal{V}^d \tag{5}$$

$$\sum_{\substack{i \in S, j \in \mathcal{V}^a \cup \mathcal{V}^d \setminus S \\ \text{or } j \in S, i \in \mathcal{V}^a \cup \mathcal{V}^d \setminus S}} x_{ij} \geq 2 y_k \qquad S \subset \mathcal{V}^a, \quad 2 \leq |S| \leq |\mathcal{V}^a \cup \mathcal{V}^d| - 2, \quad \mathcal{V}^{ma} \setminus S \neq \emptyset, k \in S \tag{6}$$

$$y_i = 1 \qquad i \in \mathcal{V}^{ma} \tag{7}$$

$$x_{ij} \in \{0, 1\} \qquad i, j \in \mathcal{V}^a \cup \mathcal{V}^d \mid i \neq j \tag{8}$$

$$y_i \in \{0, 1\} \qquad i \in \mathcal{V}^{oa} \cup \mathcal{V}^{ma} \cup \mathcal{V}^d \tag{9}$$

Objective (1) minimizes the length of the vehicle tour. Constraints (2) ensure that all mandatory passive nodes are covered by at least one visited active node. Constraints (3) and (4) ensure that tours start and end at the depot. Constraints (5) ensure that all visited customer nodes are entered and left exactly once. Constraint set (6) eliminates subtours. These constraints work as follows: For all nodes k that are included in a tour (i.e., for nodes with $y_k = 1$), the left-hand side ensures that a set S containing node k is connected by at least two arcs to the set $\mathcal{V}^{ma} \setminus S$, which by definition includes at least one node that also has to be included in the vehicle tour. Constraints (7) ensure that all mandatory active nodes are included in the vehicle route. Finally, constraints (8) and (9) define binary variable domains.

As stated before, the CTP can be formulated as GTSP given some assumptions. In the GTSP, the nodes are partitioned into clusters and a vehicle has to visit at least (or exactly) one node of each cluster. It was studied, among others, by Fischetti, Salazar González, & Toth (1997). The GTSP is generalized to the generalized vehicle routing problem (GVRP) and, due to its wide range of application areas, it has attracted a lot of attention (Bektaş, Erdoğan, & Røpke, 2011). A node is considered as covered if one node of the cluster is visited. Clusters can be – but do not need to be – built based on spatial or geographic conditions. As described by Gendreau et al. (1997), the CTP can be modeled as a GTSP by determining clusters of active nodes providing coverage for mandatory passive nodes depending on their distance and assuming that each mandatory active node builds a separate cluster. This modeling variant requires that clusters can have non-empty intersections. However, for example, Bektaş et al. (2011) explicitly assume that the clusters are non-empty and disjoint. Following this definition, the CTP cannot be modeled generally as GTSP. In the following, we do not further consider the GTSP and the GVRP, as solution techniques that explicitly consider spatial dependencies cannot be applied to this problem class. Nevertheless, we would like to point out that some of the solution techniques of this widely studied class are applicable to problems considered in this paper.

3.1.3. Solution approaches

All approaches introduced in this section are summarized in Table B1 of the Appendix B.

Exact solution approaches

Gendreau et al. (1997) propose the first exact solution approach for the single-vehicle CTP and adopt valid inequalities that have been proposed for the set covering problem (SCP). This is possible as the SCP is strongly related to the CTP: In the SCP, each set can be interpreted as a node together with its covering neighborhood. The objective is to select the minimum number of sets such that all nodes are covered, i.e., included in a selected set, without considering travel times. Moreover, the authors transfer the concept of "dominance" from the SCP, where one optional active node dominates another if it covers at least the same set of nodes. The exact solution procedure is based on a branch-and-cut scheme. The results highlight that (1) the problem difficulty largely depends on the number of active nodes, while passive nodes have a lower impact on the overall computation time, and (2) problems become easier when the size of the covering neighborhood increases, i.e. when passive nodes can be covered by a larger number of active ones.

Model formulations in the form of mixed integer problems are used to solve smaller instances to optimality (e.g., Naji-Azimi et al., 2012a), but are not refined further. Hà, Bostel, Langevin, & Rousseau (2013) derive an exact algorithm for the m-CTP based on a new problem formulation and several valid inequalities for the linear relaxation. Jozefowiez (2014) transfer a branch-and-price algorithm for the ring star problem (see Section 3.4) to the m-CTP. The multi-vehicle multi-covering variant mm-CTP is solved using a branch-and-cut approach by Pham et al. (2017), who strengthen the model using several new valid inequalities. Most recently, Glize, Roberti, Jozefowiez, & Ngueveu (2020) have proposed a column-generation based approach for solving the m-CTP as well as its bi-objective variant.

Heuristic approaches

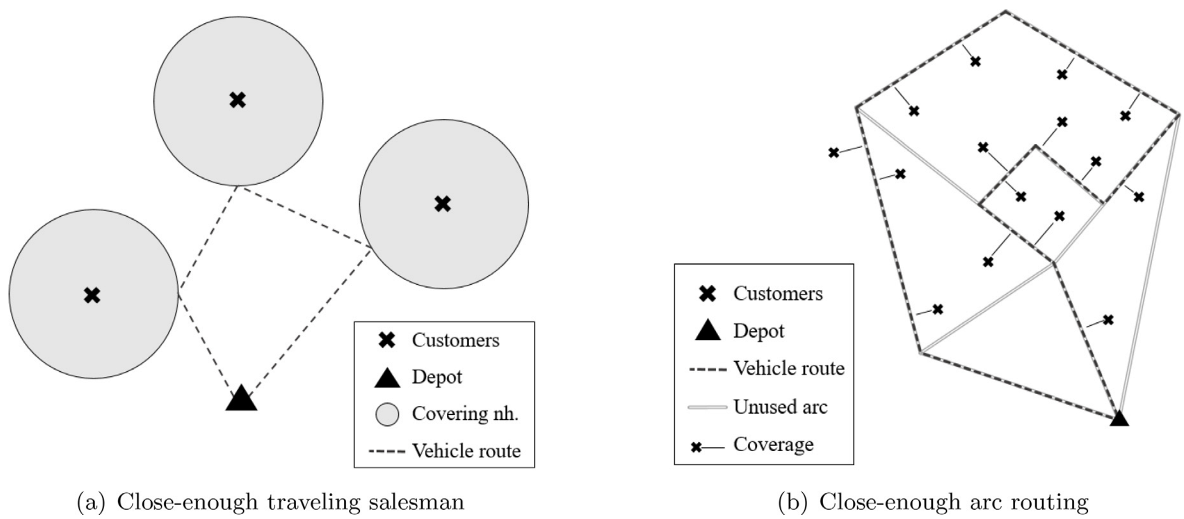
Current & Schilling (1989) apply a two-stage approach: In a first step, they optimally solve a SCP ignoring the distance traveled by the vehicle to determine the nodes that make up the tour. In a second step, they optimally solve a TSP for each of the (symmetric) candidate solutions found in the first step. However, the authors note that while they expect the approach to perform well, especially when there are costs incurred for opening facilities, it is not possible to find the optimal solution to the CSP in any case where opening more facilities reduces routing costs.

In a similar approach, Gendreau et al. (1997) combine methods for solving SCP and TSP problems. Instead of a strict two-phase approach, they incrementally expand a tour using SCP heuristics, where new vertices are included based on the cost of including them in the route and the additional coverage that they provide. The authors note that the resulting heuristic performs well, especially when the set of mandatory nodes is small.

Baldacci, Boschetti, Maniezzo, & Zamboni (2005) propose three scatter search heuristics that operate on populations of reference solutions that are improved using local search heuristics and recombined to generate new candidate solutions. Golden et al. (2012) propose two local search procedures based on the exchange of visited nodes for the GCSP. Search starts from a random initial ordering of visited nodes, as the authors note that neither a TSP solution on the active node set nor a selection based on the SCP yields better results. Moreover, the search incorporates classic TSP search moves that yield significant improvements. The results show that both approaches outperform the two-stage approach by Current & Schilling (1989) while requiring less computation time.

Similar to earlier approaches for the CSP and CTP, Hachicha et al. (2000) propose several heuristics for the multi-vehicle variant that extend known approaches for solving the SCP as well as popular VRP heuristics, notably a local improvement heuristic and the savings and sweep heuristics. Again, the main drivers of problem complexity and computation time are the number of active nodes and the size of the covering neighborhood, with larger neighborhoods leading to lower gaps and computation times as well as shorter tours.

For the m-CTP, Naji-Azimi et al. (2012a) propose a local search with added diversification steps that can significantly improve solution quality and runtime. It is embedded in a multi-start procedure used to decrease the dependency on the starting solution. In contrast to single-vehicle variants, additional moves are used to exchange nodes between routes. The evolutionary local search approach for the m-CTP by Hà et al. (2013) first determines covering subsets. Finding tours with minimum distance through these subsets can be considered as a VRP with unit demand. Solutions are improved by local search moves, both classical VRP moves and operators that replace nodes within the tours. Kammoun, Derbel, Ratli, & Jarboui (2017) propose a variable neighborhood search (VNS) algorithm that integrates a randomized construction and shaking heuristic together with insertion and swap moves that outperform earlier approaches by Hà et al. (2013). Pham et al.



(a) Close-enough traveling salesman (b) Close-enough arc routing

Fig. 2. Examples of close-enough vehicle and arc routing problems, respectively.

(2017) apply a genetic algorithm for the *mm*-CTP that is based on the unified hybrid genetic search (UHGS) framework proposed by Vidal, Crainic, Gendreau, & Prins (2014) for VRP variants. Flores-Garza et al. (2017) solve the *m*-CCTP is via a GRASP mechanism that seeks to construct routes based on widely spread initial nodes and to improve them further using local search moves.

3.2. Close-enough vehicle routing problems

The core idea of close-enough vehicle routing problems (CEVRP) is to build a tour of minimum length such that all nodes are covered by a vehicles' tour, or stated differently, such that at least one vehicle passes through any point within the covering neighborhood of each node. Problems in this class are typically defined in the two-dimensional plane where covering neighborhoods are most commonly defined as circles with diameter d. The corresponding arc-routing problem (CEARP) is summarized in Section 3.3. Both problem classes are illustrated in Fig. 2.

3.2.1. Problem definition and applications

The first single-vehicle variant relating to the CEVRP has been proposed by Arkin & Hassin (1994) under the name geometric covering salesman problem. The authors discuss it as a generalization of the TSP where the customer is willing to meet a salesperson at any point within a limited region. A similar application is discussed by Mata & Mitchell (1995), Gudmundsson & Levcopoulos (1999) and Dumitrescu & Mitchell (2003), who refer to this problem as the traveling salesman problem with neighborhoods (TSPN) and generalize the possible shapes of the covering regions. The goal of these problem variants is to determine a tour of minimum length that, at some point, passes through each region. This means that, unlike the CTP, the locations at which a vehicle tour changes direction are part of the decision variables and can be chosen freely within a two-dimensional plane. Gentilini, Margot, & Shimada (2013) suggest applying this model for automated visual inspection in robotics, where images may be taken anywhere within a limited region. Dong, Yang, & Chen (2007) and Gulczynski et al. (2006) apply similar models for planning tours of mobile meter reading services, where mobile sensors can collect readings from nearby RFID-equipped devices. The latter have introduced the term close-enough traveling salesman problem (CETSP) for this problem, which has been used in most of the later literature. Mennell (2009) has extended the problem to the multi-vehicle case, i.e., the CEVRP.

3.2.2. Problem formulation

As shown by Behdani & Smith (2014), any optimal solution to the CETSP can be represented by discrete points in the plane that cover at least one passive node each and that are connected by straight lines to form the vehicle tour. Earlier, Dong et al. (2007) proposed such a representation to formulate the problem as a mixed-integer nonlinear problem formulation (MINLP): The node set $\mathcal{V}$ is composed of only passive mandatory nodes $\mathcal{V}^{mp}$ and vehicle depot nodes $\mathcal{V}^{d}$ with positions $(lon_i, lat_i), i \in \mathcal{V}$. This means that $\mathcal{V}^{op} = \mathcal{V}^{a} = \emptyset$. Nodes $i \in \mathcal{V}^{mp}$ are covered if a vehicle passes through any point in space within distance d to i. This means that, for each node i, we can define its covering neighborhood as $\mathcal{C}_i = \{(a,b) \in \mathbb{R}^2 : \|(a,b) - (lon_i, lat_i)\| \leq d\}$ where $\|\cdot\|$ indicates the application of the Euclidean metric.

The points on the tours from which a node is covered are modeled as continuous decision variables $(a_i, b_i) \in \mathbb{R}^2$, which are typically referred to as representative points or turn(ing) points. Note that for the depot, these variables are fixed to the known depot location, i.e., $(a_i, b_i) = (lon_i, lat_i)$ for all $i \in \mathcal{V}^d$. The turning points are connected by straight line segments and thus define the tour. Finally, we use binary decision variables x_{ij} to indicate the sequence in which nodes are covered in a tour. Then, the CETSP is formulated as follows:

$$\textbf{(CETSP)} \qquad \min \sum_{i,j \in \mathcal{V}^{mp} \cup \mathcal{V}^d \mid i \neq j} x_{ij} \sqrt{(a_i - a_j)^2 + (b_i - b_j)^2} \tag{10}$$

s.t.

$$(a_i - lon_i)^2 + (b_i - lat_i)^2 \leq d^2 \qquad i \in \mathcal{V}^{mp} \tag{11}$$

$$\sum_{j \in \mathcal{V}^{mp} \cup \mathcal{V}^d \mid i \neq j} x_{ji} = 1 \qquad i \in \mathcal{V}^{mp} \cup \mathcal{V}^d \tag{12}$$

$$\sum_{j \in \mathcal{V}^{mp} \cup \mathcal{V}^d \mid i \neq j} x_{ij} = 1 \qquad i \in \mathcal{V}^{mp} \cup \mathcal{V}^d \tag{13}$$

$$\sum_{i,j \in S} x_{ij} \leq |S| - 1 \qquad S \subseteq \mathcal{V}^{mp}, 2 \leq |S| \leq |\mathcal{V}^{mp}| \tag{14}$$

$$x_{ij} \in \{0,1\} \qquad i,j \in \mathcal{V}^{mp} \cup \mathcal{V}^d \mid i \neq j \tag{15}$$

$$(a_i, b_i) \in \mathbb{R}^2 \qquad i \in \mathcal{V}^{mp} \cup \mathcal{V}^d \tag{16}$$

The objective function (10) minimizes total distance, while constraints (11) ensure that the turning point (a_i, b_i) is sufficiently close to the covered node i. Constraints (12) and (13) state that each node is included once in the tour. Subtours are prohibited by constraint set (14). Constraints (15) and (16) define the decision variables.

As noted by Mennell (2009), the formulation above might lead to numerical issues as several nodes may be covered from the exact same position. Hence, Mennell (2009) proposes an additional constraint

$$\sqrt{(a_i - a_j)^2 + (b_i - b_j)^2} \geq \epsilon \qquad i, j \in \mathcal{V}^{mp} \cup \mathcal{V}^d \mid i \neq j \tag{17}$$

to ensure that turning points are separated from one another by a small positive value ϵ. While this may lead to suboptimal solutions in some cases, the resulting formulation avoids numerical issues that would otherwise arise.

In the following section, we additionally consider approaches in which the covering neighborhoods cannot be described by circles but by arbitrary polygons. However, no mathematical models were proposed in literature for these variants.

3.2.3. Solution approaches

The approaches introduced for the CEVRP are summarized in Table B2 in Appendix B.

Approximation algorithms Several approximation algorithms exist for variants of the close-enough routing problem in the plane, all focusing on the single-vehicle case. The earliest publications on the TSPN focus on algorithms achieving provable worst-case bounds on the length of the obtained routes relative to the optimum. Arkin & Hassin (1994) have proposed methods that first identify turning points within the covering regions in a heuristic fashion. Then, a vehicle route is planned through these points. The algorithm chooses turning points by placing lines that intersect as many neighborhoods as possible (“covering lines”). The authors also highlight cases where simple myopic heuristics might yield arbitrarily bad solutions. For the more general TSPN where covering neighborhoods can be represented by arbitrary polygonal regions, an approximate algorithm has been proposed by Mata & Mitchell (1995) and developed further by Gudmundsson & Levcopoulos (1999) and Dumitrescu & Mitchell (2003). The broad idea of these schemes is first to find a bounding square that includes or at least touches all neighborhoods. This square is then subdivided until all neighborhoods are intersected or touched, at which point a tour can be planned that passes through all neighborhoods.

Exact solution approaches

There are few exact solution approaches for CEVRP variants defined in the plane. While the MINLP formulation above can be solved optimally, several authors note that this is impractical (Dong et al., 2007; Mennell, 2009). Consequently, several authors focus on finding bounds to the objective function that allow, e.g., to assess the quality of heuristic approaches. Mennell (2009) provide the first steps in this direction but note that the achieved bounds are weak. Behdani & Smith (2014) determine tighter lower and upper bounds based on discretization schemes. These schemes yield partitionings of the search space, e.g., in the form of cells that intersect at least one neighborhood; solutions can then be planned by determining routes through these partitions. Still, the approach quickly becomes impractical even in cases with fewer than 20 nodes to be covered. Carrabs, Cerrone, Cerulli, & Gaudioso (2017), Carrabs, Cerrone, Cerulli, & D’Ambrosio (2018) extend this work by providing new discretization schemes. A branch-and-bound approach for the TSPN is introduced by Gentilini et al. (2013) who exploit the fact that, once all binary variables are fixed, the continuous relaxation of the problem can be solved with reasonable efficiency. Still, the problem remains intractable for more than 15 neighborhoods. A branch-and-bound algorithm for the CETSP is proposed by Coutinho, Do Nascimento, Pessoa, & Subramanian (2016). The algorithm works on ordered lists of covered vertices which are expanded as one travels further down the search tree. The approach performs well on larger instances, especially if more neighborhoods overlap.

Heuristic approaches

Gulczynski et al. (2006) summarize several possible methods for the CETSP. All of these approaches are based on the determination of “supernode” sets, i.e., sets of candidate turning points in the two-dimensional plane such that all customer nodes $\mathcal{V}^{mp}$ are within a given distance to at least one node in the supernode set. The authors show that two heuristics are particularly successful: One based on decomposing the area into tiles covered by one supernode in the center, the other based on the determination of areas where the covering neighborhoods overlap, so-called Steiner zones. Assuming that it is beneficial to have as few supernodes as possible, the authors propose strategies for eliminating and replacing supernodes. Tours are then planned by solving a TSP through such a set. Dong et al. (2007) follow a very similar approach. Additionally, they introduce a method that seeks to find “compact” supernodes sets by iteratively computing the convex hull of all remaining uncovered nodes, computing its centroid, and selecting a new supernode as close as possible to the centroid. Yang et al. (2018) propose a hybrid evolutionary algorithm that combines a continuous procedure for determining turning points and a genetic algorithm for optimizing the visit sequence. Most recently, Carrabs, Cerrone, Cerulli, & Golden (2020) combine discretization concepts for computing lower and upper bounds for the CETSP with a Carousel greedy heuristic for incrementally constructing solutions, resulting in heuristic solutions with tight bounds in a relatively short computation time.

Solution approaches put forward by Mennell (2009) for solving the CEVRP are similarly based on Steiner zones. The authors propose an efficient heuristic to reduce the problem to a limited number of these zones. A vehicle route is then planned such that it connects turning points in each of the identified zones. The route is improved by modifying the turning points through which a vehicle passes to reduce overall tour length.

3.3. Close-enough arc routing problems

In contrast to the CEVRP, close-enough arc routing problems are defined on a graph, for example, a street network (see Fig. 2). Usually, a node is part of the covering neighborhood of a street segment if the distance to the arc is below a certain threshold. Similar to the CEVRP, this means that a node is covered as long as a vehicle passes through its neighborhood at any point during its tour.

3.3.1. Problem definition and applications

The arc-based problem variant has been first presented by Shuttleworth, Golden, Smith, & Wasil (2008), again in the context of mobile meter reading services. This problem has been referred to as the close-enough arc routing problem (CEARP) by Hà, Bostel, Langevin, & Rousseau (2014). Drexl (2007) and Drexl (2014) have studied the generalized directed rural postman problem (GDRP), which, given several subsets of arcs, seeks to determine a tour of minimum length that traverses at least one arc in each set. While customer nodes are not modeled explicitly, this corresponds to the CEARP if each of these groups is interpreted as the covering neighborhood of one customer node. It has been extended to the multi-vehicle case by Ávila, Corberán, Plana, & Sanchis (2017) in the form of the distance-constrained generalized directed rural postman problem (DC-GDRP). Renaud, Absi, & Feillet (2017) introduce the stochastic CEARP that accounts for the distance-dependent probability of failure when reading meters remotely.

3.3.2. Problem formulation

For modeling the CEARP, we follow the model proposed in Hà, Bostel, Langevin, & Rousseau (2012) for the sake of simplicity. More refined models are discussed in Hà et al. (2014). The model is defined on a directed graph $G = (\mathcal{V}^{\text{inter}}, \mathcal{A})$ representing, for example, a street network. In arc routing, nodes represent intersections as opposed to customers commonly represented in VRP variants. Street segments (arcs) are defined between these intersections. In the formulation below, the set $\mathcal{V}^{\text{inter}}$ represents nodes including the depot $\mathcal{V}^d$. The street segments connecting these nodes define the arc set $\mathcal{A}$. Each arc $(i, j) \in \mathcal{A}$ is associated with costs d_{ij}. The set of nodes to be covered is defined as $\mathcal{V}^{mp}$. Binary parameters w_{kij} are equal to one if and only if node $k \in \mathcal{V}^{mp}$ can be covered by arc $(i, j) \in \mathcal{A}$, i.e., if k is in the covering neighborhood of arc (i, j). Decision variables $x_{ij} \in \mathbb{N}^0$ indicate the number of times an arc $(i, j) \in \mathcal{A}$ is traversed by the vehicle.

$$\textbf{(CEARP)} \qquad \min \sum_{(i,j)\in\mathcal{A}} d_{ij}x_{ij} \tag{18}$$

s.t.

$$\sum_{i,j \mid i\in\mathcal{V}^d, j\in\mathcal{V}^{\text{inter}}, (i,j)\in\mathcal{A}} x_{ij} \geq 1 \tag{19}$$

$$\sum_{j \mid (i,j)\in\mathcal{A}} x_{ij} - \sum_{j \mid (j,i)\in\mathcal{A}} x_{ji} = 0 \qquad i \in \mathcal{V}^{\text{inter}} \tag{20}$$

$$\sum_{(i,j)\in\mathcal{A}} x_{ij}w_{kij} \geq 1 \qquad k \in \mathcal{V}^{mp} \tag{21}$$

$$M \textstyle\sum_{i,j \mid i\in S, j\in\mathcal{V}^{\text{inter}}\setminus S, (i,j)\in\mathcal{A}} x_{ij} - \textstyle\sum_{i,j \mid i\in S, j\in S, (i,j)\in\mathcal{A}} x_{ij} \geq 0 \qquad S \subset \mathcal{V}^{\text{inter}} \setminus \mathcal{V}^d,\; 2 \leq |S| \leq |\mathcal{V}^{\text{inter}}| - 2 \tag{22}$$

$$x_{ij} \in \mathbb{N}^0 \qquad (i, j) \in \mathcal{A} \tag{23}$$

Objective (18) minimizes travel distance. Constraint (19) ensures that the vehicle leaves the depot, while constraints (20) maintain flow conservation and ensure that the vehicle returns to the depot. Constraints (21) ensure coverage of passive nodes. Disjoint subtours, i.e., subtours that are disconnected from the tour containing the depot, are eliminated by constraint set (22). Note that an arc routing solution can generally contain subtours. Only subtours that are not connected to the depot tour need to be eliminated. Parameter M is a sufficiently large integer. Constraints (19) define the decision variables.

3.3.3. Solution approaches

The approaches introduced for the CEARP are summarized in Table B3 in Appendix B.

Exact solution approaches

Compared to their counterparts in the CEVRP problem class, CEARP variants can, in general, be solved more efficiently as they are defined on a graph and do not represent continuous problems. Hà et al. (2014) propose several exact methods. Notably, they compare two existing problem formulations by Hà et al. (2012) and Drexl (2014) with a new one, which they strengthen using several valid inequalities. They furthermore propose a branch-and-cut procedure for these formulations. Ávila, Corberán, Plana, & Sanchis (2016) develop these approaches further by introducing new problem formulations together with valid inequalities. The authors solve instances with several hundred nodes and arcs, stating that instances with disjoint covering neighborhoods are more challenging than those where neighborhoods overlap. The same authors furthermore introduce a branch-and-cut approach based on similar concepts for the multi-vehicle variant (Ávila et al., 2017). Renaud et al. (2017) solve the stochastic version of the CEARP via a cutting-plane algorithm that is enhanced with preprocessing methods to decrease the problem size and heuristics to construct feasible solutions. Finally, Corberán, Plana, Reula, & Sanchis (2020) complement these approaches with a formulation and several valid inequalities. The resulting exact algorithm tends to be faster, especially as the fleet size increases.

Heuristic approaches

Solution approaches for the CEARP follow similar concepts as those for CEVRP variants. Shuttleworth et al. (2008) propose a two-stage approach, where the first step consists of greedily constructing subsets of arcs to be traversed in order to ensure full coverage, and the second one finds a complete cycle comprising these arcs. Drexl (2014) adapt a genetic algorithm for the GTSP for solving the DC-GDRP. Corberán, Plana, Reula, & Sanchis (2019) suggest a matheuristic for solving this problem. Routes are either constructed in parallel or sequentially and are improved using heuristic exchanges as well as exact procedures for the CEARP introduced by Ávila et al. (2016). The heuristic scales well but remains limited to instances with up to 5 vehicles. For solving the stochastic CEARP, Renaud et al. (2017) combine an approach adapted from Hà et al. (2013) and TSP-based heuristics.

3.4. Vehicle routing allocation problems

In this section, we apply the term “routing-allocation problem” for problems where neighborhood size is not fixed. Instead, covering a node incurs a cost that typically increases with the distance between the covering and the covered (also called “allocated”) node. The objective is to find a cost-minimal solution where all nodes are either visited or allocated.

3.4.1. Problem definition and applications

The first variant in this line of work has been proposed by Akinc & Srikanth (1992), who suggest a single-vehicle problem with mandatory coverage of all active nodes for selecting oil rig locations and planning maintenance and health care services. The more general problem variant that includes active and passive nodes has been proposed by Beasley & Nascimento (1996) for planning mobile medical care or postal collection routes. The problem seeks to minimize a weighted cost function that accounts for the routing costs of the vehicle. Additionally, allocation costs are incurred for covering a node i from node j Furthermore, high penalty costs for leaving nodes unvisited and uncovered are considered. This generalized problem is referred to as the vehicle routing allocation problem (VRAP), while the single-vehicle variant is abbreviated SVRAP. Note that the VRAP comprises both the CTP and the OPCov as a special case. Allahyari et al. (2015) introduced a problem referred to as the multi-depot CTP (MDCTP) for the provisioning of humanitarian aid after a disaster. The authors consider the cost of covering unvisited nodes, which increases with the distance between a covered node and the node providing coverage. Due to the this distinction, we subsume this problem under the VRAP class, where it represents the first multi-vehicle version.

Labbé, Laporte, Rodríquez Martín, & Salazar Gonzáles (1999) introduced the median cycle problem (MCP). The planning objective is to find a cycle in a graph such that a set of mandatory active nodes is either visited or allocated to a visited node. Concerning the objective function, two variants of the problem were proposed. In the first variant, the sum of routing and allocation costs is minimized. The second variant seeks to minimize routing costs subject to a maximum value constraint for the total allocation cost. The first variant of the MCP has also been published under the name ring-star problem (RSP) (Labbé, Laporte, Martín, & González, 2004). A suggested application for this problem is the design of telecommunication networks (Xu et al., 1999). The Steiner ring-star problem (SRSP) is a variant that distinguishes between optional active nodes which may be part of the cycle but do not need to be covered (so-called *Steiner* nodes) and active nodes that can be either visited directly or covered. Multi-vehicle applications of the RSP in which each cycle is limited in the number of assigned customers have also been introduced by Baldacci, Dell'Amico, & González (2007).

3.4.2. Problem formulation

Below, we indicate the formulation for the single-vehicle routing allocation problem (SVRAP) due to its versatility (Vogt et al., 2007). This problem comprises the introduced node categories except for optional passive nodes, i.e., $\mathcal{V}^c = \mathcal{V}^{ma} \cup \mathcal{V}^{oa} \cup \mathcal{V}^{mp}$, and depot nodes $\mathcal{V}^d$. Coverage is not limited in distance, and nodes may be covered by any active node. The cost of covering node j from node i is denoted c_{ij} (Vogt et al. (2007) refer to this as "allocating" node j to node i) and typically increases with distance between the two nodes. Leaving an optional active node i unvisited and uncovered incurs a penalty $p_i > 0$.

Binary decision variables x_{ij} indicate whether the vehicle travels from node i to node j. Binary variables y_i denote whether an active node $i \in \mathcal{V}^a$ is included in a vehicle tour or not. In addition, the SVRAP uses binary decision variables z_{ij} to indicate whether an active node $i \in \mathcal{V}^a$ covers an optional active or mandatory passive node $j \in \mathcal{V}^{oa} \cup \mathcal{V}^{mp}$. Decision variables z_i indicate whether an optional active or mandatory passive node $i \in \mathcal{V}^{oa} \cup \mathcal{V}^{mp}$ is covered by any node in the tour. Finally, $\lambda_1, \lambda_2, \lambda_3 > 0$ are objective function weights. Based on this notation, the problem can be formulated as follows:

$$\textbf{(SVRAP)} \quad \min \lambda_1 \sum_{i \in \mathcal{V}^{oa}} p_i(1 - y_i - z_i) + \lambda_2 \sum_{i \in \mathcal{V}^a, j \in \mathcal{V}^{oa} \cup \mathcal{V}^{mp} \mid i \neq j} c_{ij} z_{ij} + \lambda_3 \sum_{i,j \in \mathcal{V}^a \cup \mathcal{V}^d \mid i \neq j} d_{ij} x_{ij} \tag{24}$$

s.t.

$$\sum_{j \in \mathcal{V}^a} x_{ij} = 1 \qquad i \in \mathcal{V}^d \tag{25}$$

$$\sum_{j \in \mathcal{V}^a} x_{ji} = 1 \qquad i \in \mathcal{V}^d \tag{26}$$

$$\sum_{j \in \mathcal{V}^a \cup \mathcal{V}^d \mid i \neq j} x_{ji} = y_i \qquad i \in \mathcal{V}^a \tag{27}$$

$$\sum_{j \in \mathcal{V}^a \cup \mathcal{V}^d \mid i \neq j} x_{ij} = y_i \qquad i \in \mathcal{V}^a \tag{28}$$

$$\sum_{i \in \mathcal{V}^a} z_{ij} = z_j \qquad j \in \mathcal{V}^{oa} \cup \mathcal{V}^{mp} \tag{29}$$

$$z_{ij} \leq y_i \qquad i \in \mathcal{V}^a, j \in \mathcal{V}^{oa} \cup \mathcal{V}^{mp} \tag{30}$$

$$y_i + z_i \leq 1 \qquad i \in \mathcal{V}^{oa} \tag{31}$$

$$\sum_{j \in S, k \in \mathcal{V}^a \cup \mathcal{V}^d \mid k \notin S} x_{jk} \geq y_i \qquad i \in S \cap \mathcal{V}^a, S \subset \mathcal{V}^a, S \neq \emptyset \tag{32}$$

$$z_i = 1 \qquad i \in \mathcal{V}^{mp} \tag{33}$$

$$y_i = 1 \qquad i \in \mathcal{V}^{ma} \tag{34}$$

$$x_{ij} \in \{0, 1\} \qquad i, j \in \mathcal{V}^a \cup \mathcal{V}^d \mid i \neq j \tag{35}$$

$$y_i \in \{0, 1\} \qquad i \in \mathcal{V}^a \tag{36}$$

$$z_{ij} \in \{0, 1\} \qquad i \in \mathcal{V}^a, j \in \mathcal{V}^{oa} \cup \mathcal{V}^{mp} \mid i \neq j \tag{37}$$

$$z_i \in \{0, 1\} \qquad i \in \mathcal{V}^{oa} \cup \mathcal{V}^{mp} \tag{38}$$

Objective (24) minimizes the weighted sum of three types of cost: The first term represents the penalty for not visiting or covering optional active nodes. The second term sums up the distance-dependent cost for covering optional active or passive nodes, while the third term sums up the traveling cost of the tour. Constraints (25) and (26) ensure that the tour starts and ends at the depot, while sets (27) and (28) maintain flow conservation. Constraints (29) ensure that covered vertices are assigned to exactly one covering node. Meanwhile, constraint set (30) ensures that if an optional active node covers a node, the covering node is included in the tour. For all optional active nodes, constraints (31) ensure that they cannot be simultaneously visited and covered. Constraints (32) eliminate subtours. Constraints (33) ensure that all mandatory passive nodes are covered while constraints (34) ensure that mandatory active nodes are visited. Constraints (35) to (38) define the decision variables.

3.4.3. Solution approaches

The approaches introduced in the following paragraphs are given in Table B4 in Appendix B.

Exact solution approaches

Akinc & Srikanth (1992) solve the SVRAP with mandatory active nodes using a branch-and-bound procedure but note that further work is necessary to exploit problem-specific properties. Labbé et al. (1999) propose a branch-and-cut approach for MCP variants that is strengthened with several valid inequalities and integrates heuristic procedures for determining feasible solutions. The algorithm yields promising results with instances with up to 150 nodes. The authors note that the problems tend to be challenging when few nodes make up the optimal cycle. Exact approaches are discussed by the same authors in later works (Labbé et al., 2004; Labbé, Laporte, Martın, & González, 2005) and are shown to be able to solve larger instances as well as a real-world scenario. Simonetti, Frota, & de Souza (2011) propose an alternative formulation that is integrated in a branch-and-cut procedure. They integrate a GRASP for finding good initial solutions and thus reliable upper bounds.

The first approach for a multi-vehicle problem in this line of research has been proposed by Baldacci et al. (2007) for solving the m-RSP. The authors introduce two formulations together with valid inequalities that are integrated in a branch-and-cut procedure. Furthermore, they apply the algorithm to two large real-world instances with more than 2000 nodes. The instances are simplified in a preprocessing step. Baldacci, Hill, Hoshino, & Lim (2017) propose several problem relaxations that provide tighter lower bounds than previous approaches.

Heuristic approach

Xu et al. (1999) solve the SRSP using a tabu search approach. Search steps modify the selected active nodes while a short-term memory prevents the reversal of recent moves and a long-term memory emphasizes diversification by encouraging less frequently used moves. Pérez, Moreno-Vega, & Martın (2003) propose a variable neighborhood tabu search, which they apply to the MCP. Moves include node insertion, exchange, and deletion together with a shaking step that modifies the selected nodes as well as common VRP search moves. In contrast to observations made by authors of exact approaches, Pérez et al. (2003) note that the performance deteriorates when cycles include a larger percentage of nodes. Renaud, Boctor, & Laporte (2004) propose two heuristics for MCP variants: one combines a construction heuristic and local search approaches, the other is based on an evolutionary algorithm that seeks to improve solutions found by the first heuristic. Calvete, Galé, & Iranzo (2013) also propose an evolutionary algorithm for the RSP which achieves very low computation times. The authors demonstrate how the relationship between routing and assignment cost impacts solution structures.

For solving the SVRAP, Vogt et al. (2007) employ a tabu search variant. The initial solution is built based on the observations that nodes selected for visitation are rarely located at the border or the center of the considered area. This serves to select an initial set of nodes to be routed. Subsequently, the search iteratively changes the nodes included in the vehicle tour and optimally allocates all unvisited nodes. The solution is further strengthened by path-relinking and diversification steps that modify the set of selected nodes such that new options are explored.

Considering multi-vehicle variants, Naji-Azimi, Salari, & Toth (2010) propose a local search based heuristic for the m-RSP. Initial routes distributed as widely as possible across the graph are constructed and then improved using swap and deletion moves and strategies for improving the allocation of covered nodes. Naji-Azimi, Salari, & Toth (2012b) propose a VNS that comprises an improvement phase based on optimally solving restricted problems seeking to reallocate nodes. Zhang, Qin, & Lim (2014) propose a memetic algorithm for the problem class that combines a genetic algorithm with local search operations for modifying the visited nodes. For solving the multi-depot case, Allahyari et al. (2015) combine a greedy randomized adaptive search procedure (GRASP) and iterated local search (ILS). Initial routes are constructed by building clusters that assign nodes to depots and then building routes for each cluster.

3.5. Orienteering problems with spatial coverage

The goal of orienteering problems (OP) is to determine a subset of nodes to visit and the corresponding visit order so that the total collected profit is maximized and a time limit is not exceeded (Gunawan, Lau, & Vansteenwegen, 2016). For orienteering problems with spatial coverage (OPCov), no general model exists. Nonetheless, there has been an influx of publications in recent years that integrate aspects of spatial coverage and profit maximization. This section summarizes these approaches and highlights common principles and ideas. To this end, we formulate all models such that they follow the VRP-SCOV as closely as possible. All approaches are summarized in Table B5 in Appendix B.

3.5.1. Time constrained maximal covering salesman problem

Ozbaygin, Yaman, & Karasan (2016) have proposed an extension to the OP with additional consideration of coverage constraints. Thereby, nodes that are not included in a vehicle tour but are within a specified maximum distance of a visited node provide a positive contribution to the objective function. This contribution is less than the benefit yielded by directly including the node in a vehicle tour, i.e., it corresponds to a partial coverage in our terminology. This problem is denoted the time constrained maximal covering salesman problem (TCMCSP).

Problem definition

The set of nodes $\mathcal{V}$ contains optional active nodes $\mathcal{V}^{oa}$ and a depot node in $\mathcal{V}^d$. Visiting $i \in \mathcal{V}^{oa}$ directly yields a profit p_i; covering it yields only a fraction of p_i. The profit fraction is denoted α and is the same for all nodes within a covering neighborhood. With respect to the VRP-SCOV, this means that $w_{ij} = \alpha$ for $j \in \mathcal{C}_i$ with $\alpha \in (0, 1)$.

For modeling the problem, we use binary decision variables x_{ij} to indicate the sequence of nodes and variables y_i to represent which nodes are visited. Moreover, we use a binary variable z_i representing coverage of node i. Auxiliary variables u_i are used for eliminating subtours. Then, the problem can be formulated as follows:

$$\textbf{(TCMCSP)} \qquad \max \sum_{i \in \mathcal{V}^{oa}} (p_i y_i + \alpha p_i z_i) \tag{39}$$

s.t.

$$\sum_{j \in \mathcal{V}^{oa}} x_{ij} = 1 \qquad i \in \mathcal{V}^d \tag{40}$$

$$\sum_{j \in \mathcal{V}^{oa}} x_{ji} = 1 \qquad i \in \mathcal{V}^d \tag{41}$$

$$\sum_{j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j} x_{ji} = y_i \qquad i \in \mathcal{V}^{oa} \tag{42}$$

$$\sum_{j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j} x_{ij} = y_i \qquad i \in \mathcal{V}^{oa} \tag{43}$$

$$u_i - u_j + 1 \leq (|\mathcal{V}^{oa}| - 1)(1 - x_{ij}) \qquad i, j \in \mathcal{V}^{oa} \mid i \neq j \tag{44}$$

$$\sum_{i, j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j} x_{ij} d_{ij} \leq T \tag{45}$$

$$y_i + z_i \leq 1 \qquad i \in \mathcal{V}^{oa} \tag{46}$$

$$z_i \leq \sum_{j \mid i \in \mathcal{C}_j} y_j \qquad i \in \mathcal{V}^{oa} \tag{47}$$

$$x_{ij} \in \{0, 1\} \qquad i, j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j \tag{48}$$

$$y_i \in \{0, 1\} \qquad i \in \mathcal{V}^{oa} \tag{49}$$

$$z_i \in \{0, 1\} \qquad i \in \mathcal{V}^{oa} \tag{50}$$

$$u_i \in \{1, \ldots, |\mathcal{V}^{oa}|\} \qquad i \in \mathcal{V}^{oa} \tag{51}$$

Objective (39) maximizes the sum of profits of all directly visited nodes plus the partially considered profits of those that are covered. Constraints (40) and (41) ensure that the vehicle leaves and enters the depot exactly once, while constraints (42) and (43) state the same for all visited nodes. Subtours are eliminated by constraint set (44). Note that, in this model, we use the subtour elimination constraint commonly used for orienteering problems, see also Gunawan et al. (2016). The total route length is limited by constraint (45). Constraints (46) ensure that nodes that are visited directly are not considered covered. Constraints (47) state that all nodes are covered if they are included in the covering neighborhood of at least one visited node, while constraints (48) to (51) define the decision variables.

Exact approaches

Ozbaygin et al. (2016) propose several branch-and-cut schemes for solving the TCMCSP formulated above and strengthened with additional valid inequalities. The algorithms are tested on benchmark instances with up to 200 nodes. Additionally, the authors discuss the impact of problem parameters, notably, maximum tour length, the fraction α for indirect coverage, and properties of the underlying graph. The results highlight that the tour length restriction determines, in general, the achievable profitability. The value α changes the trade-off between the number of visited and unvisited nodes. The impact of increasing the maximum coverage distance d depends on the underlying graph structure: If the graph is clustered, the impact on the route can be relatively small. In other cases, the resulting tours are more narrow: This allows visiting more customers directly within the limited routing distance while maintaining the overall level of coverage of unvisited nodes.

3.5.2. Set orienteering problem

Another variant of the OPCov is the set orienteering problem (SOP) introduced by Archetti, Carrabs, & Cerulli (2018), who also introduced a set of benchmark instances. In this problem variant, customers are grouped into disjoint clusters. Visiting one customer within a cluster allows collecting the entire profit associated with this cluster. The SOP shares this interpretation of a neighborhood with the GTSP which we introduced in Section 3.1. In fact, the two problem classes only differ in the objective function. Applications addressed by Archetti et al. (2018) are supply chains or areas in which goods can be distributed further from a visited location. Hence, clusters can represent spatial relations but do not have to. Additional visits within one cluster do not provide additional benefits. Clusters are disjoint, i.e., visiting a customer from within one cluster does not provide benefits for other clusters. Note that only the single-vehicle case has been considered to this date.

Problem formulation

The set $\mathcal{V}^{oa}$ is composed of g clusters $\{C_1, \dots, C_g\}$ such that $C_k \cap C_m = \emptyset$ for all $k, m \in \{1, \dots, g\}$ with $k \neq m$. Visiting a node i or covering it by visiting another node in the same cluster yields a profit p_i. With respect to our notation for VRP-SCOV, we can define weights $w_{ij} = 1$ if i, j are in the same cluster and 0 otherwise. This means that clusters and covering neighborhoods are equivalent. As a consequence, the SOP can be formulated similarly to the TCMCSP with $\alpha = 1$. The problem uses the same decision variables as the TCMCSP (Section 3.5.1), with decision variables x_{ij}, y_i and z_i indicating the node sequence, visited nodes and covered nodes, respectively. Note that, to highlight the similarity between the models, the following formulation deviates from the one proposed in literature:

$$\textbf{(SOP)} \qquad \max \sum_{i \in \mathcal{V}^{oa}} z_i p_i \tag{52}$$

s.t.

$$\sum_{j \in \mathcal{V}^{oa}} x_{ij} = 1 \qquad i \in \mathcal{V}^d \tag{53}$$

$$\sum_{j \in \mathcal{V}^{oa}} x_{ji} = 1 \qquad i \in \mathcal{V}^d \tag{54}$$

$$\sum_{j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j} x_{ji} = y_i \qquad i \in \mathcal{V}^{oa} \tag{55}$$

$$\sum_{j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j} x_{ij} = y_i \qquad i \in \mathcal{V}^{oa} \tag{56}$$

$$u_i - u_j + 1 \leq (|\mathcal{V}^{oa}| - 1)(1 - x_{ij}) \qquad i, j \in \mathcal{V}^{oa} \mid i \neq j \tag{57}$$

$$\sum_{i, j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j} x_{ij} d_{ij} \leq T \tag{58}$$

$$z_i \leq \sum_{j \in C_i} y_j \qquad i \in \mathcal{V}^{oa} \tag{59}$$

$$x_{ij} \in \{0, 1\} \qquad i, j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j \tag{60}$$

$$y_i \in \{0, 1\} \qquad i \in \mathcal{V}^{oa} \tag{61}$$

$$z_i \in \{0, 1\} \qquad i \in \{1, \dots, g\} \tag{62}$$

$$u_i \in \{1, \dots, |\mathcal{V}^{oa}|\} \qquad i \in \mathcal{V}^{oa} \tag{63}$$

Objective (52) maximizes the sum of all covered nodes. Again, constraints (53) to (56) are vehicle flow constraints, constraints (57) eliminate subtours and constraints (58) ensure that the maximum route budget is not exceeded. Constraints (59) state that a node is covered if at least one other node belonging to the same neighborhood is included in a vehicle tour. Note that, for SOP, it is not necessary to distinguish between explicitly covered and visited nodes, as the objective function does not differentiate between these two groups of nodes. Finally, constraints (60) to (63) define the decision variables.

Heuristic approaches

Solution approaches for the SOP explicitly make use of the fact that clusters do not overlap. In the paper by Archetti et al. (2018), the SOP is solved using a matheuristic. An initial tour is constructed greedily. To reduce the initial routing cost, the search alternates between changing the nodes selected for visitation in each cluster and improving the sequence of the selected nodes. The main tabu search procedure seeks to improve the selection of clusters. If this fails, a MILP is applied to solve a simplified model in which clusters are partially fixed to take larger steps in the search space. Pěnička, Faigl, & Saska (2019) propose a VNS that operates on the sequence of clusters, while nodes to be visited in each cluster are selected using a shortest path search. Carrabs (2020) present a biased random-key genetic algorithm that similarly searches on the visiting sequence of clusters. Unused nodes and arcs are eliminated in a preprocessing step.

3.5.3. Correlated orienteering problem

Yu, Schwager, & Rus (2014) propose the correlated team orienteering problem (CorTOP) to model the planning problem for drones that execute surveillance and monitoring tasks of an environment with spatial correlations (see also informative path planning in the next section). They model the CorTOP as a variant of the team orienteering problem (TOP) to integrate information about spatial correlations in the model. In this model, covering a node only provides a limited fraction of its reward p_i. The unique feature of the CorTOP is that additional stops within covering distance provide an additional benefit such that the full reward for an unvisited target node is achieved when all covering nodes are included in a vehicle tour. In Glock & Meyer (2020), the mission planning problem for drones aims at providing a quick overview of the distribution of airborne substances after the release of hazardous substances and is modeled as a variant of the CorTOP. This application is referred to as rapid mapping.

Problem formulation

This problem only contains optional active nodes $\mathcal{V}^{oa}$ and a depot node contained in $\mathcal{V}^d$. The weights w_{ij} depend on the distance d_{ij} between two nodes, i.e., $w_{ij} = f^{weight}(d_{ij})$ for covering distances $d_{ij} \leq d$ with $f^{weight}(d_{ij}) \in [0, 1]$ for $i, j \in \mathcal{V}^a$. A possible representation for f^{weight} is an inverse distance weighting function with $f^{weight}(d_{ij}) = \frac{1}{d_{ij}}$. Additionally, we use the same decision variables x_{ij}, y_i and u_i to indicate node sequence, visited nodes and subtour elimination as in the previous orienteering variants (see Section 3.5.1). Consistent with the remainder of the formulations in this work, the single-vehicle variant of the CorTOP can be defined as mixed integer quadratic program (MIQP):

$$\textbf{(CorTOP)} \qquad \max \sum_{i \in \mathcal{V}^{oa}} y_i p_i + \sum_{i \in \mathcal{V}^{oa}} \sum_{j \in \mathcal{C}_i} y_i (1 - y_j) w_{ij} p_j \tag{64}$$

s.t.

$$\sum_{j \in \mathcal{V}^{oa}} x_{ij} = 1 \qquad i \in \mathcal{V}^d \tag{65}$$

$$\sum_{j \in \mathcal{V}^{oa}} x_{ji} = 1 \qquad i \in \mathcal{V}^d \tag{66}$$

$$\sum_{j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j} x_{ji} = y_i \qquad i \in \mathcal{V}^{oa} \tag{67}$$

$$\sum_{j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j} x_{ij} = y_i \qquad i \in \mathcal{V}^{oa} \tag{68}$$

$$u_i - u_j + 1 \leq (|\mathcal{V}^{oa}| - 1)(1 - x_{ij}) \qquad i, j \in \mathcal{V}^{oa} \mid i \neq j \tag{69}$$

$$\sum_{i, j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j} x_{ij} d_{ij} \leq T \tag{70}$$

$$x_{ij} \in \{0, 1\} \qquad i, j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j \tag{71}$$

$$y_i \in \{0, 1\} \qquad i \in \mathcal{V}^{oa} \tag{72}$$

$$u_i \in \{1, \ldots, |\mathcal{V}^{oa}|\} \qquad i \in \mathcal{V}^{oa} \tag{73}$$

The objective function (64) maximizes the sum of the priorities of all directly visited and covered nodes. Constraints (65) to (68) state that all visited nodes, included the depot, are entered and left once. Subtours are eliminated in constraint set (69). The route budget is limited by constraint (70). Constraints (71) to (73) define the decision variables.

In order to ensure that covering a visit can never be more beneficial than visiting it directly, Yu et al. (2014) enforce that $\sum_{i \in \mathcal{V}^{oa} \mid j \in \mathcal{C}_i} w_{ij} \leq 1, j \in \mathcal{V}^{oa}$, while Glock & Meyer (2020) introduce a more general nonlinear objective function

$$\textbf{(GCorTOP)} \qquad \max \sum_{i \in \mathcal{V}^{oa}} y_i p_i + \sum_{j \in \mathcal{V}^{oa}} \min\{ \sum_{i \in \mathcal{V}^{oa} \mid j \in \mathcal{C}_i} y_i (1 - y_j) w_{ij}, 1\} \; p_j \tag{74}$$

We refer to the corresponding multi-vehicle problem with objective function (74) as the generalized correlated team orienteering problem (GCorTOP).

Solution approaches Yu et al. (2014) solve small problem instances using the problem formulation above but do not seek to refine it further. Glock & Meyer (2020) propose a dynamic programming approach yielding optimal solutions for small instances of the GCorTOP. To solve larger instances, Glock & Meyer (2020) also propose a two-stage heuristic approach for solving real-world instances of the mission planning for emergency rapid mapping. In the first step, knowledge about "good" properties of solutions is exploited to find initial solutions that traverse large parts of the underlying graph. In a second step, these tours are improved further through an adaptive large neighborhood search (ALNS) procedure.

3.5.4. Team orienteering problem with overlaps

Orlis et al. (2020) develop a variant of the OPCov in the context of cash logistics. The objective is to select ATMs for replenishment via armored vehicles such that account holders nearby have access to serviced ATMs. The authors refer to this problem as the team orienteering problem with overlaps (TOPO).

Problem formulation

Locations providing service can be modeled as optional active nodes $\mathcal{V}^{oa}$ and customers as optional passive nodes $\mathcal{V}^{op}$. Each location $i \in \mathcal{V}^{oa}$ can service account holders within its covering neighborhood $\mathcal{C}_i$. As customers can obtain service from any service point nearby, these neighborhoods can overlap.

In Orlis et al. (2020), the TOPO is introduced as a multi-vehicle optimization problem. To maintain consistency with the remainder of this paper, we focus on the corresponding formulation for the single-vehicle case. To this end, binary decision variables x_{ij} again indicate the vehicle traveling from node i to node j. Whether a node is covered is indicated by variables z_i, while u_i are introduced for subtour elimination.

$$\textbf{(TOPO)} \qquad \max \sum_{i \in \mathcal{V}^{op}} z_i \tag{75}$$

s.t.

$$\sum_{j \in \mathcal{V}^{oa}} x_{ij} = 1 \qquad i \in \mathcal{V}^d \tag{76}$$

$$\sum_{j \in \mathcal{V}^{oa}} x_{ji} = 1 \qquad i \in \mathcal{V}^d \tag{77}$$

$$\sum_{j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j} x_{ji} \leq 1 \qquad i \in \mathcal{V}^{oa} \tag{78}$$

$$\sum_{j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j} x_{ij} - \sum_{j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j} x_{ji} = 0 \qquad i \in \mathcal{V}^{oa} \tag{79}$$

$$u_i + (T + d_{ij}) x_{ij} \leq u_j + T \qquad i, j \in \mathcal{V}^{oa} \mid i \neq j \tag{80}$$

$$u_i \leq (T - t_{ik}) \sum_{j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j} x_{ij} \qquad i \in \mathcal{V}^{oa}, k \in \mathcal{V}^d \tag{81}$$

$$\sum_{i, j \in \mathcal{V}^{a} \cup \mathcal{V}^d \mid i \neq j, k \in \mathcal{C}_i} x_{ij} \geq z_k \qquad k \in \mathcal{V}^{op} \tag{82}$$

$$x_{ij} \in \{0, 1\} \qquad i, j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j \tag{83}$$

$$z_i \in \{0, 1\} \qquad i \in \mathcal{V}^{op} \tag{84}$$

$$u_i \in \mathbb{R}^+ \qquad i \in \mathcal{V}^{oa} \tag{85}$$

In the formulation above, objective (75) maximizes the number of covered passive nodes, i.e., the number of served customers. Constraints (76) to (79) ensure that the depot is left and entered once and maintain flow conservation. Constraint set (80) eliminates subtours and sets the arrival time at each customer. With line (81), the maximum tour length is restricted. Constraints (82) ensure that a customer can only be covered if it is within reach of a visited location. Finally, decision variables are defined in constraints (83) to (85).

Solution approaches Orlis et al. (2020) introduce a branch-and-cut-and-price algorithm for which they describe several acceleration techniques and methods for tightening bounds. Moreover, they propose a large neighborhood search in which routes are partially destroyed and repaired by modifying the selected locations and the routes through these locations in two distinct steps. The improvement of the selected active nodes specifically takes into account that covering neighborhoods may overlap. In this case, their combined contribution to the objective can be less than if they were considered separately (i.e., $f(A \cup B) \leq f(A) + f(B)$ for a given objective function $f : S \subset \mathcal{V} \to \mathbb{R}_0^+$ and $A, B \subset \mathcal{V}$).

3.5.5. Close-enough orienteering problem

The only problem variant subsumed under OPCov defined on the plane is the close-enough orienteering problem (CEOP). Since this is the only variant, we have refrained from introducing a separate class. The CEOP combines properties of the orienteering problem with the CEVRP (also defined on the plane) in the sense that the profit associated with a node is collected as long as a vehicle travels sufficiently close to this node at any point during its tour. First referred to as the orienteering problem with neighborhoods (Faigl, Pěnička, & Best, 2016), the CEOP was developed for mobile data collection in the field of robotics. Due to its application in this domain, the authors extend the problem to the Dubins orienteering problem with neighborhoods (DOPN), which adds additional restrictions on the maneuverability of the mobile robot (Faigl & Pěnička, 2017). Similar to the CEVRP, a solution to the CEOP is typically represented by turning points within sufficient distance to the nodes that are selected to be covered. However, no mixed-integer formulation or exact solution approach has been proposed for this problem. Therefore, this section summarizes the heuristic methods developed in literature.

Heuristic approaches Faigl et al. (2016) develop a self-organizing map (SOM) based procedure based on a neural network that works on a discretization of the plane. However, the algorithm does not outperform other approaches used for benchmarking on the orienteering problem without spatial coverage. The DOPN is solved using VNS as well as a SOM-based approach by Faigl & Pěnička (2017). Štefaníková, Váňa, & Faigl (2020) propose a GRASP for the CEOP. In the construction phase, new turning points are iteratively inserted into emerging tours. If the tour budget is exceeded, the route is repaired by removing segments until the constraint is satisfied. The core distinction of the novel approach is the determination of turning points such that the detour for each insertion is minimal.

3.6. Informative path planning

The literature related to the VRP-SCOV in the domain of robotics has its origins in sensor placement problems without consideration of routing decisions. These problems address the question of designing sensor networks for monitoring environmental phenomena (e.g., Krause, Singh, & Guestrin, 2008). Examples include water contamination (Krause et al., 2008), temperature and salinity in bodies of water (Binney, Krause, & Sukhatme, 2013; Binney & Sukhatme, 2012) or wireless signal strength (Hollinger & Sukhatme, 2014). In these applications, each sample provides additional information about the overall monitored distribution. These distributions are usually spatially correlated, which means that similar values can be observed at locations close to one another. Hence, each sensor also provides data about its surrounding area. The overall information about the surveyed phenomenon is diminished if sensors are placed too closely together.

The sensor placement problem has been extended to mobile sensor systems. The resulting planning problem is referred to as informative path planning (IPP). Both discrete variants, where vehicles move through predetermined candidate sampling locations, as well as continuous models in which samples can be taken freely at any point in the plane have been proposed in literature. In consistency to literature, we subsume both variants under the term IPP.

3.6.1. Problem definition and applications

The objective of the IPP is to determine vehicle trajectories that provide as much information as possible about a spatially correlated phenomenon while respecting the vehicles' maximum mission duration. Most of the work addresses environmental monitoring applications, e.g., oceanic monitoring using autonomous underwater vehicles, where large areas have to be surveyed.

Similarly to the orienteering problem, IPP models seek to maximize some measure indicating the benefit of the vehicle tours. As opposed to orienteering problem methods, IPP approaches do not account for profits associated with specific target locations. Instead, they use probabilistic models for determining the information gain achieved by the vehicles with respect to the observed phenomenon, notably Gaussian process (GP) models. An overview of these models is provided in Appendix A. An illustrative example for such a GP is provided in Fig. 3: A UAV takes samples, e.g., of hazardous gases, across the target area, which yields the predicted distribution indicated on the left-hand side. The right-hand side presents the remaining uncertainty after accounting for these samples: While the prediction is likely to be accurate close to the to the UAV's route, the uncertainty increases with increasing distance from the closest sampled location.

This information about the uncertainty of predictions is addressed in objective functions of the IPP. This means that these models seek to determine sets of sensing locations such that the overall uncertainty about the interpolated phenomenon is minimal. Models furthermore ensure that additional measurements remain beneficial while reducing the marginal contribution of additional nearby samples to the objective function. They share this property with several orienteering problems with spatial coverage, e.g., TOPO. A main IPP challenge lies in the a priori assessment of the benefit of additional samples. This is because an accurate assessment requires full knowledge of the visited points of the mission. Variants such as CorTOP and GCorTOP have been introduced as approximative representations of these models in the form of mixed integer models in order to address this disadvantage.

3.6.2. Problem formulation

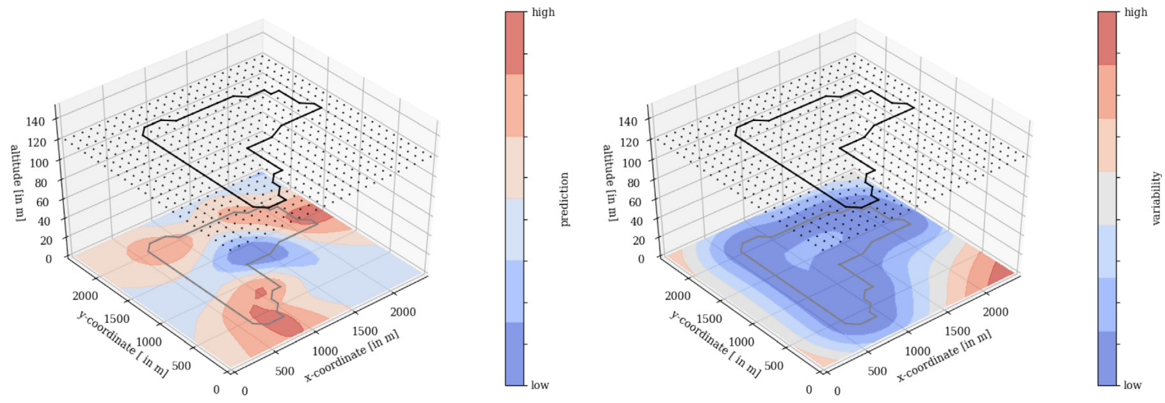
As IPP models differ most from the other lines of research summarized in this work, we need to adjust our mathematical formulation: Most importantly, we cannot define a closed-form expression for the total profit of a solution. Instead, the value of a solution is usually defined using an informativeness measure $\mathcal{I}(S)$, i.e., some error measure based on predictive model conditioned on the included sampling locations $S \subseteq \mathcal{V}^{oa}$. Possible candidate locations at which samples may be obtained are included in the set of optional active nodes $\mathcal{V}^{oa}$. For consistency with the models provided for the OPCov in Section 3.5, variables x_{ij} and y_i again indicate the sequence of sampling locations, while y_i is used to indicate locations at which samples are taken during the tour. Auxiliary variables u_i are used for subtour elimination. The discrete version of IPP distinguishes itself from orienteering models by means of a modified objective function:

$$\textbf{(IPP)} \qquad \max \quad \mathcal{I}(\{i \in \mathcal{V}^{oa} \mid y_i = 1\}) \tag{86}$$

s.t.

$$\sum_{j \in \mathcal{V}^{oa}} x_{ij} = 1 \qquad i \in \mathcal{V}^d \tag{87}$$

$$\sum_{j \in \mathcal{V}^{oa}} x_{ji} = 1 \qquad i \in \mathcal{V}^d \tag{88}$$



(a) Predicted values (b) Remaining uncertainty

Fig. 3. Example of a surveyed phenomenon and remaining uncertainty.

$$\sum_{j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j} x_{ji} = y_i \qquad i \in \mathcal{V}^{oa} \tag{89}$$

$$\sum_{j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j} x_{ij} = y_i \qquad i \in \mathcal{V}^{oa} \tag{90}$$

$$u_i - u_j + 1 \leq (|\mathcal{V}^{oa}| - 1)(1 - x_{ij}) \qquad i, j \in \mathcal{V}^{oa} \mid i \neq j \tag{91}$$

$$\sum_{i,j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j} x_{ij} d_{ij} \leq T \tag{92}$$

$$x_{ij} \in \{0, 1\} \qquad i, j \in \mathcal{V}^{oa} \cup \mathcal{V}^d \mid i \neq j \tag{93}$$

$$y_i \in \{0, 1\} \qquad i \in \mathcal{V}^{oa} \tag{94}$$

$$u_i \in \{1, \dots, |\mathcal{V}^{oa}|\} \qquad i \in \mathcal{V}^{oa} \tag{95}$$

Objective (86) maximizes the informativeness function for the set of selected samples. Constraints (87) to (95) follow the CorTOP formulation (see Section 3.5.3).

3.6.3. Solution approaches

Table B6 in Appendix B summarizes the solution approaches for IPP. In contrast to research in the domain of Operations Research, which emphasize heuristics for solving large-scale problems, literature on the IPP has focused on approximation algorithms with bounded performance. This offers certain guarantees when faced with high costs for sensing and monitoring.

Several approaches have been put forward that are based on the recursive greedy heuristic proposed by Chekuri & Pal (2005) for solving discrete single-vehicle problems with submodular objective functions. This approach operates by splitting the problem into two subproblems, solving the first one recursively, and then solving the other while keeping the solution obtained for the first one fixed. Singh, Kaiser, Batalin, Krause, & Guestrin (2007) improve the running time of the recursive greedy approach by decomposing the target area into independent cells. Furthermore, the authors address the multi-vehicle case by applying the recursive greedy algorithm to a series of single-vehicle problems sequentially, in each step taking into account the information obtained using all previously planned vehicle routes. These concepts are also discussed and evaluated in detail in a later publication (Singh et al., 2009a). Meliou, Krause, Guestrin, & Hellerstein (2007) embed the recursive greedy approach in a framework that seeks to minimize sensing cost for a given threshold value for $\mathcal{I}$. Singh, Krause, & Kaiser (2009b) propose an alternative to the recursive greedy approach in which node clusters are precomputed first, and a route is then obtained by solving an orienteering problem on the approximated graph obtained in this preprocessing step. A version of the recursive greedy algorithm is also used by Binney, Krause, & Sukhatme (2010), who solve an IPP variant with time windows that limit the accessibility of certain areas. The authors also demonstrate how available information, for example, the information obtained using previous missions, can be incorporated to improve subsequent tours. Stranders, De Cote, Rogers, & Jennings (2013) propose a near-optimal heuristic that first decomposes the graph into distinct clusters, for which routes are planned greedily. For multiple vehicles, routes are planned sequentially. Binney et al. (2013) extend the recursive greedy approach to a case with time-varying fields. Similar to previous approaches, the algorithm does not scale well for instances with more than a few dozen candidate locations. Jawaid & Smith (2015) propose approximate solution techniques for a generalized problem seeking to maximize arbitrary submodular objectives. One is based on a greedy approach; the other relaxes the subtour elimination constraint and then repairs a solution.

A branch-and-bound algorithm for the single-vehicle discrete IPP is proposed by Binney & Sukhatme (2012). Due to the high runtime required to solve even small instances to optimality, the authors limit the search space. This significantly improves runtime, but problems remain computationally intractable for vehicle routes comprising more than 15 locations.

The continuous IPP has received less attention in literature. Hollinger & Sukhatme (2013, 2014) propose a rapidly-exploring information gathering algorithm, which iteratively assigns random sampling locations to vehicle routes and expands vehicle paths towards these nodes. This approach applies to both discrete and continuous planning problems. However, its performance is highly dependent on the maximum route length. Note that many algorithms in this domain, especially those developed more recently, focus on variants where routes are planned adaptively based on previous measurements (e.g., Bottarelli, Bicego, Blum, & Farinelli, 2019; Lim, Hsu, & Lee, 2016) and are out of the scope of this review.

3.7. Related and combined models

Due to the heterogeneous nature of the research up to this point, a variety of models has been proposed that can be seen as variants of the VRP-SCOV but do not exclusively fit into one problem category. In this section, we review selected models to illustrate this variety. Please note that we do not seek to provide a complete overview but to summarize variants that offer new insights how different aspects may be combined.

3.7.1. Bi-objective models

An early version of a bi-objective model variant is the maximal covering tour problem (MCTP) introduced by Current & Schilling (1994). The authors address application areas such as the design of service delivery systems, bi-modal transportation systems, or distributed computer networks. In this model, only a given number p of the optional active nodes needs to be visited directly, while the other nodes are considered as covered if a visited node is closer than the maximal covering distance d. The two objectives are the minimization of the total tour length and the minimization of the total demand that is not covered. In turn, that means that the total covered demand is maximized. Following our introduced logic, on the one hand, the problem minimizes cost for visiting p nodes resembling a CTP. On the other hand, the covered demand is maximized as in OPCov. The authors propose a local search heuristic to approximate the efficient frontier among the two objectives.

Jozefowiez, Semet, & Talbi (2007) introduced the bi-objective covering tour problem (BOCTP) as a generalization of the CTP. They had the same application areas as for the CTP in mind. As in the CTP, there exist three sets of customers ($\mathcal{V}^{ma}, \mathcal{V}^{oa}, \mathcal{V}^{oa}$) and the first objective seeks to find a tour of minimum length visiting or covering all mandatory passive and active nodes. The second objective strives to minimize the greatest distance between covered nodes and the nearest visited nodes. The authors note that a strength of this approach is the fact that it avoids the need to specify a covering distance d a-priori. As the BOCTP is a generalization of the CTP the close relationship is obvious. However, the problem also resembles the VRAP, as the second objective considers the allocation decision of the covered nodes to the visited nodes. Jozefowiez et al. (2007) solve the problem using a combination of an evolutionary algorithm and the exact approach for the CTP proposed by Gendreau et al. (1997). The evolutionary algorithm focuses on the selection of nodes to be visited, i.e., on solutions to the CSP subproblem. Its objective is to construct candidate pareto-optimal solutions, which are improved by the exact approach.

Another bi-objective variant of the CTP is the bi-objective stochastic covering tour problem proposed by Tricoire, Graf, & Gutjahr (2012) as an extension of the bi-objective MCTP. Even if the authors see other application areas, the model is introduced having disaster relief operations in mind delivering goods to the population in the affected area. In contrast to CTP variants, the authors assume that the number of people – and, hence, the demand – can be expressed as a function of the distance between the place they come from and the opened delivery center. Furthermore, they assume the demand is uncertain. They model the demand as random variable in a two-stage stochastic optimization problem. All nodes need to be either visited or covered. The first objective minimizes routing cost and cost for opening a delivery station. The second objective minimizes the expected uncovered demand: it comprises the demand of clients, who do not go to the closest delivery center because it is too far, the demand not satisfied as the capacity of the delivery center is too small, and the demand that cannot be satisfied as the vehicle capacity is exceeded. The actual uncovered demand is determined by solving the second stage of the model. For solving the problem the authors propose both an exact algorithm and a heuristic. This model combines aspects of the CTP and the OPCov.

3.7.2. Combining several aspects of spatial coverage

The two-echelon routing problem with truck and drones (2ERTD) was introduced by Vu, Vu, Hà, & Nguyen (2021). It seeks to find a truck route through dedicated nodes that can be interpreted as optional active nodes. UAVs then deliver parcels to final customers that can be modeled as mandatory passive nodes and that are within flying time to visited truck nodes. The truck has to wait for the UAVs to return, and the optimization objective is to minimize the total duration of the truck tour. The problem combines the CTP with aspects of VRAP models as the objective function accounts for the time required for the UAV deliveries, which depends on the allocation of passive to active nodes. The number of passive nodes that can be covered by an active one are limited. The authors solve the problem using a GRASP based on dedicated local search moves and demonstrate that the coverage via drones can yield improvements especially in regions with lower customer density.

The clustered coverage orienteering problem (CCOP) is a problem introduced for planning information-maximizing tours for autonomous vehicles (Zhang, Wang, Wang, & Laporte, 2020). In this problem, an area of interest is separated into distinct regions, each of which comprises several candidate sampling sites that provide information within a given covering range. A minimum number of samples has to be taken per cluster in order to provide sufficient information. The planning objective is to maximize the total area covered by at least one sensor while ensuring that the required number of samples is taken in all regions. This problem has characteristics of the VRP-SCOV in two aspects: First, similar to the CSP, it is mandatory to visit all regions. Second, the model encourages taking disperse samples, as overlapping coverage areas are penalized in the objective. This mirrors concepts of the CorTOP and IPP, even though the proposed model differs considerably from these variants.

Oruc & Kara (2018) study a post-disaster assessment routing problem in which UAVs and motorcycles are deployed to assess critical infrastructure. Both the survey of roads (modeled as arcs) and points of interest (modeled as optional active nodes) in a network provide information. The planning problem maximizes the importance of the nodes and arcs that are accessed. Coverage aspects are introduced for drones as their larger field of view allows covering nearby nodes and arcs. The resulting model combines aspects of the CEARP, which is extended by the coverage of arcs as well as nodes, with the OPCov. Both an exact model as well as construction and improvement heuristics are proposed.

3.7.3. Specialized covering mechanisms

Veenstra et al. (2018) present a case study in which patients need to be supplied with medicine. If patients do not have access to a suitable pharmacy within reasonable range, they are supplied either by direct delivery or from pickup lockers close to their homes. The planning objective is to minimize the total routing and locker installation cost. Planning this delivery network closely resembles the CTP, with lockers and customers represented as optional active nodes, but only locker nodes being able to provide coverage.

A stochastic variant of the m-CTP is introduced by Karaoğlan, Erdoğan, & Koç (2018). In this model, the demand of a customer (i.e., passive node) is covered with a given probability from a visited optional active node, with multiple visits within covering distance increasing the overall probability that a customer is served. The planning objective is to maximize expected demand served. Additional visits within covering distance increase the share of demand that is covered. The model is similar to the CorTOP but differs in how the coverage is modeled. The authors propose two solution approaches: a branch-and-cut algorithm and a VNS.

Margolis, Song, & Mason (2021) propose a multi-vehicle variant of the VRP-SCOV for surveillance where targets (mandatory passive nodes) have to be monitored for a given time by vehicles traveling between waypoints (optional active nodes). A vehicle may adjust its speed in order to increase the time it covers a

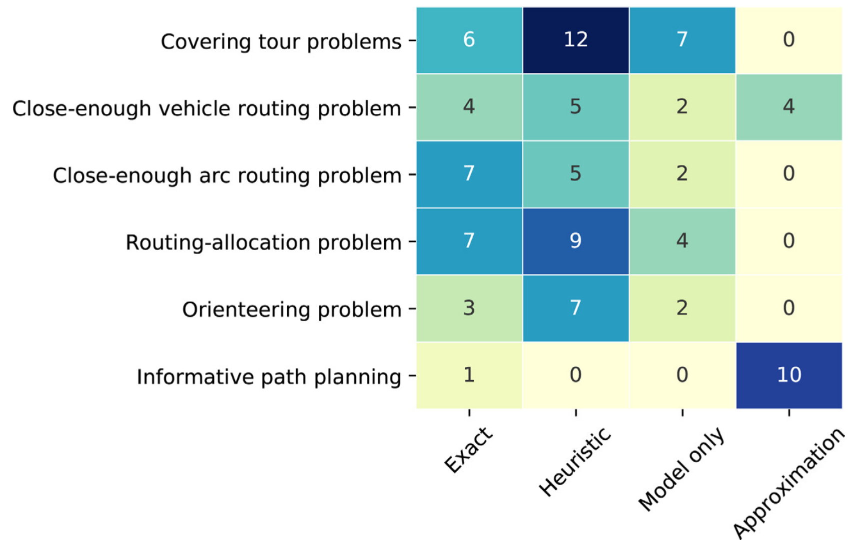


Fig. 4. Overview of approaches by VRP-SCOV class.

node, i.e., remains within covering distance to this node. While all passive nodes have to be covered, the maximization objective increases with increasing coverage time for nodes that are more important instead of minimizing total tour length, which means that the problem shares aspects of the OPCov. The problem is solved using a branch-and-price framework that accounts for speed adjustments and propose a heuristic construction strategy.

4. Discussion and prospective research directions

4.1. Discussion and insights

In this section, we summarize insights that go beyond specific problem instances and can be applied to a range of problems related to the VRP-SCOV. We start with a short overview and discuss important aspects in more detail.

Figure 4 gives a condensed view of the category of approaches proposed for each VRP-SCOV class. This heatmap shows that there are quite some publications (17) which introduced only a model formulation without proposing dedicated exact solution methods ("Model only"). As it could have been expected, most solution methods are heuristic. 38 of the surveyed publications provide heuristic approaches, while 28 introduce exact solution formulations. Earlier heuristic approaches often apply a two-stage concept, where nodes are selected in the first step, and a tour is planned in the second one. Later works are more inspired by heuristics developed for classical VRP problems, notably local search and metaheuristic concepts. Furthermore, we can see that decomposition-based exact approaches which are successful for solving VRP variants are frequently applied for the problems discussed in this work. Approximation algorithms have been mainly developed independently from this body of work, originating in the domains of mathematics and robotics for the TSPN and IPP. It is worth mentioning that these approaches often address continuous model variants, whereas other heuristics and exact approaches heavily rely on the discretization of the search space.

Below, we list insights and observations that are relevant for all classes of the VRP-SCOV:

Similarity of models The unified definition of the mathematical models in Section 3 reveals the common covering mechanisms considered in different vehicle routing and path-planning approaches. The review shows that especially the models subsumed under orienteering problems with coverage, which have been recently introduced independently and under different names, are strongly related.

Impact of node types In the case of discrete models, the main drivers of problem difficulty are the number of active nodes and the length of the tours. This effect holds for all surveyed problem classes. Meanwhile, mandatory active nodes provide structure, which can be used, e.g., for constructing initial vehicle routes or for applying dominance concepts (e.g., Gendreau et al., 1997) to eliminate optional candidates. Passive nodes are only a minor driver of problem difficulty and have a far less severe impact on solution times for exact and heuristic approaches alike. Approaches put forward in recent literature solve instances with several tens of thousands of passive nodes (e.g., Orlis et al., 2020).

Size of covering neighborhoods The size of the covering neighborhood has a major impact on computation times and the structure of the obtained solutions. In general, larger covering neighborhoods (relative to the distance between nodes) mean that it is easier to decrease routing costs in CTP and CEVRP problems and find high objective values for IPP and OPCov variants. Increasing covering distance has been shown to yield substantial improvements in practical applications (see, e.g., Ozbaygin et al., 2016; Shuttleworth et al., 2008).

Exploiting spatial coverage - Passive nodes and covering distance Coverage aspects can be exploited to decrease problem size. For example, mandatory passive nodes might determine which active nodes need to be visited or which are dominated by others. Moreover, they determine desirable solution properties: For example, visited nodes tend to be evenly distributed across the considered area, with few visits near the border or at the center. Increasing the coverage distance means that visits are sparser and vehicles travel farther in between them.

Exploiting spatial coverage - Algorithm design Many approaches exploit spatial coverage concepts to guide the search process. Several early solution concepts apply a two-stage approach where the first step lies in determining nodes to be visited, and the second step seeks to find a good route through these nodes. This allows major speed-ups as well as the usage of existing tools such as set covering techniques. Despite their potential to increase search performance, these ideas need to be considered with care: Selecting the smallest possible number of nodes to be visited does not necessarily yield the shortest tours in discrete problems. In continuous problems such as the CEVRP class, determining Steiner zones where the largest number of covering neighborhoods overlap provides a useful structure that can be used during the search. However, basing the search on these zones restricts possible shapes of tours and may not always yield the best solutions. Furthermore, coverage can be used to decompose a problem into distinct, non-overlapping areas or clusters, which, in turn, can be used during search instead of the individual nodes. While this leads to substantially less complex problems, some options to improve the final tours become unavailable. Later approaches are typically more flexible and combine the consideration of clusters and individual nodes, e.g., by using local search moves that alternate between the selection of nodes and route improvement strategies.

4.2. Prospective research directions

Several research directions merit further consideration:

Flexible models for spatial coverage From a modeling perspective, it is often assumed that essential parameters are known in advance. This concerns the covering distance in particular. As this assumption does not always hold in practice, variants such as bi-objective models that explicitly seek to minimize maximum covering distance seem promising. Examples come from applications where increasing the covering distance at the expense of routing costs or customer satisfaction is a feasible option, e.g., when designing delivery networks or postal services.

Furthermore, models developed in robotics and vehicle routing differ considerably in the level of detail that they provide and the computational effort required. Some models, such as the CorTOP, bridge the gap between the disciplines by modeling spatial dependencies in more detail while they can still be efficiently evaluated. This is promising for both domains: On the one hand, these models make VRP heuristics accessible for path planning problems from robotics. On the other hand, the more detailed coverage models might be promising for service provisioning where customers benefit from having access to services at multiple locations. An example would be a more detailed model variant of the ATM replenishment problem considered in Orlis et al. (2020).

Transfer of solution concepts Despite the fundamental similarities between the problems, different disciplines have focused on different solution strategies. In general, research has emphasized heuristic approaches over exact solution schemes. Approximation algorithms have been put forward for single-vehicle variants of IPP and the CETSP but are largely missing for the other problem classes as well as for multi-vehicle applications. Thus, transferring approaches developed for one problem variant to other types of the VRP-SCOV offers the potential to strengthen solution concepts further.

Benchmark instances and practical applications Aside from the modeling and solving aspects, a topic rarely addressed is the study of practical problem applications. Authors have proposed applying the VRP-SCOV to a wide range of use cases in transportation, health care, and information gathering. Despite this, only a few real-world problems and datasets have been studied to this date, notably in health care (Hachicha et al., 2000; Jozefowiez et al., 2007), meter reading (Shuttleworth et al., 2008) and environmental mapping (Popović, Vidal-Calleja, Chung, Nieto, & Siegwart, 2019; Popović et al., 2020; Smith et al., 2011). Unfortunately, most datasets are not publicly available, hindering the development of new solution approaches and making the comparability of approaches impossible. Hence, it can be assumed that future research will benefit from a larger range of applications and published benchmark instances.

Dynamic and stochastic variants Finally, little attention has been directed toward dynamic and stochastic problem variants, especially in vehicle routing. Some authors have introduced stochastic or probabilistic problem variants (Karaoğlan et al., 2018; Renaud et al., 2017; Tricoire et al., 2012), but few models and solution concepts exist. Adaptive mechanisms have been studied in the domain of environmental mapping that can adjust to dynamically evolving environments. However, other settings where the information evolves over time have not yet been studied but are highly relevant in practical applications. In particular, future research efforts should be devoted to robust optimization methods for critical applications, for example, in health care and emergency logistics.

In summary, considering spatial coverage in routing and path planning problems is a powerful concept for many different application areas. While research in this domain has been fragmented in the past, we believe that the different disciplines can benefit from one another, thereby developing powerful methods for practical problems. This classification and overview is a first step toward this goal.

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2022.02.031.

References

Akinc, U., & Srikanth, K. (1992). Optimal routing and process scheduling for a mobile service facility. *Networks, 22*(2), 163–183.
Allahyari, S., Salari, M., & Vigo, D. (2015). A hybrid metaheuristic algorithm for the multi-depot covering tour vehicle routing problem. *European Journal of Operational Research, 242*(3), 756–768.
Archetti, C., Carrabs, F., & Cerulli, R. (2018). The set orienteering problem. *European Journal of Operational Research, 267*(1), 264–272.
Arkin, E. M., & Hassin, R. (1994). Approximation algorithms for the geometric covering salesman problem. *Discrete Applied Mathematics, 55*(3), 197–218.
Ávila, T., Corberán, A., Plana, I., & Sanchis, J. M. (2016). A new branch-and-cut algorithm for the generalized directed rural postman problem. *Transportation Science, 50*(2), 750–761.
Ávila, T., Corberán, Á., Plana, I., & Sanchis, J. M. (2017). Formulations and exact algorithms for the distance-constrained generalized directed rural postman problem. *EURO Journal on Computational Optimization, 5*(3), 339–365.
Baldacci, R., Boschetti, M. A., Maniezzo, V., & Zamboni, M. (2005). Scatter search methods for the covering tour problem. In R. Sharda, S. Voß, C. Rego, & B. Alidaee (Eds.), *Metaheuristic optimization via memory and evolution* (pp. 59–91). Springer.
Baldacci, R., Dell'Amico, M., & González, J. S. (2007). The capacitated m-ring-star problem. *Operations Research, 55*(6), 1147–1162.
Baldacci, R., Hill, A., Hoshino, E. A., & Lim, A. (2017). Pricing strategies for capacitated ring-star problems based on dynamic programming algorithms. *European Journal of Operational Research, 262*(3), 879–893.
Beasley, J., & Nascimento, E. (1996). The vehicle routing-allocation problem: A unifying framework. *Top, 4*(1), 65–86.
Behdani, B., & Smith, J. C. (2014). An integer-programming-based approach to the close-enough traveling salesman problem. *INFORMS Journal on Computing, 26*(3), 415–432.
Bektaş, T., Erdoğan, G., & Røpke, S. (2011). Formulations and branch-and-cut algorithms for the generalized vehicle routing problem. *Transportation Science, 45*(3), 299–316.
Binney, J., Krause, A., & Sukhatme, G. S. (2010). Informative path planning for an autonomous underwater vehicle. In *2010 IEEE International Conference on Robotics and Automation* (pp. 4791–4796). IEEE.
Binney, J., Krause, A., & Sukhatme, G. S. (2013). Optimizing waypoints for monitoring spatiotemporal phenomena. *The International Journal of Robotics Research, 32*(8), 873–888.
Binney, J., & Sukhatme, G. S. (2012). Branch and bound for informative path planning. In *IEEE International Conference on Robotics and Automation* (pp. 2147–2154). IEEE.
Bottarelli, L., Bicego, M., Blum, J., & Farinelli, A. (2019). Orienteering-based informative path planning for environmental monitoring. *Engineering Applications of Artificial Intelligence, 77*, 46–58.
Calvete, H. I., Galé, C., & Iranzo, J. A. (2013). An efficient evolutionary algorithm for the ring star problem. *European Journal of Operational Research, 231*(1), 22–33.
Carrabs, F. (2020). A biased random-key genetic algorithm for the set orienteering problem. *European Journal of Operational Research, 292*(3), 830–854.
Carrabs, F., Cerrone, C., Cerulli, R., & D'Ambrosio, C. (2018). Improved upper and lower bounds for the close enough traveling salesman problem. In *International conference on green, pervasive, and cloud computing* (pp. 165–177). Springer.
Carrabs, F., Cerrone, C., Cerulli, R., & Gaudioso, M. (2017). A novel discretization scheme for the close enough traveling salesman problem. *Computers & Operations Research, 78*, 163–171.
Carrabs, F., Cerrone, C., Cerulli, R., & Golden, B. (2020). An adaptive heuristic approach to compute upper and lower bounds for the close-enough traveling salesman problem. *INFORMS Journal on Computing, 32*(4), 1030–1048.

Chekuri, C., & Pal, M. (2005). A recursive greedy algorithm for walks in directed graphs. In *IEEE Symposium on foundations of computer science (FOCS)* (pp. 245–253). IEEE.
Corberán, Á., Plana, I., Reula, M., & Sanchis, J. M. (2019). A matheuristic for the distance-constrained close-enough arc routing problem. *Top, 27*(2), 312–326.
Corberán, A., Plana, I., Reula, M., & Sanchis, J. M. (2020). On the distance-constrained close enough arc routing problem. *European Journal of Operational Research, 291*(1), 32–51.
Coutinho, W., Do Nascimento, R., Pessoa, A., & Subramanian, A. (2016). A branch-and-bound algorithm for the close-enough traveling salesman problem. *INFORMS Journal on Computing, 28*(4), 603–799.
Current, J. R., & Schilling, D. A. (1989). The covering salesman problem. *Transportation Science, 23*(3), 208–213.
Current, J. R., & Schilling, D. A. (1994). The median tour and maximal covering tour problems: Formulations and heuristics. *European Journal of Operational Research, 73*(1), 114–126.
Dong, J., Yang, N., & Chen, M. (2007). Heuristic approaches for a TSP variant: The automatic meter reading shortest tour problem. In E. K. Baker, A. Joseph, A. Mehrotra, & M. A. Trick (Eds.), *Extending the horizons: Advances in computing, optimization, and decision technologies* (pp. 145–163). Springer.
Drexl, M. (2007). *On some generalized routing problems*. Deutsche Post Lehrstuhl für Optimierung von Distributionsnetzwerken, RWTH Aachen Ph.D. thesis..
Drexl, M. (2014). On the generalized directed rural postman problem. *Journal of the Operational Research Society, 65*(8), 1143–1154.
Dumitrescu, A., & Mitchell, J. S. (2003). Approximation algorithms for TSP with neighborhoods in the plane. *Journal of Algorithms, 48*(1), 135–159.
Faigl, J., & Pěnička, R. (2017). On close enough orienteering problem with Dubins vehicle. In *2017 IEEE/RSJ International conference on intelligent robots and systems (IROS)* (pp. 5646–5652). IEEE.
Faigl, J., Pěnička, R., & Best, G. (2016). Self-organizing map-based solution for the orienteering problem with neighborhoods. In *2016 IEEE International conference on systems, man, and cybernetics (SMC)* (pp. 1315–1321). IEEE.
Fischetti, M., Salazar González, J. J., & Toth, P. (1997). A branch-and-cut algorithm for the symmetric generalized traveling salesman problem. *Operations Research, 45*(3), 378–394.
Flores-Garza, D. A., Salazar-Aguilar, M. A., Ngueveu, S. U., & Laporte, G. (2017). The multi-vehicle cumulative covering tour problem. *Annals of Operations Research, 258*(2), 761–780.
Gendreau, M., Laporte, G., & Semet, F. (1997). The covering tour problem. *Operations Research, 45*(4), 568–576.
Gentilini, I., Margot, F., & Shimada, K. (2013). The travelling salesman problem with neighbourhoods: MINLP solution. *Optimization Methods and Software, 28*(2), 364–378.
Glize, E., Roberti, R., Jozefowiez, N., & Ngueveu, S. U. (2020). Exact methods for mono-objective and bi-objective multi-vehicle covering tour problems. *European Journal of Operational Research, 283*(3), 812–824.
Glock, K., & Meyer, A. (2020). Mission planning for emergency rapid mapping with drones. *Transportation Science, 54*(2), 534–560.
Golden, B., Naji-Azimi, Z., Raghavan, S., Salari, M., & Toth, P. (2012). The generalized covering salesman problem. *INFORMS Journal on Computing, 24*(4), 534–553.
Gudmundsson, J., & Levcopoulos, C. (1999). A fast approximation algorithm for TSP with neighborhoods and red-blue separation. In *International computing and combinatorics conference* (pp. 473–482). Springer.
Gulczynski, D. J., Heath, J. W., & Price, C. C. (2006). The close enough traveling salesman problem: A discussion of several heuristics. In F. B. Alt, M. C. Fu, & B. L. Golden (Eds.), *Perspectives in operations research* (pp. 271–283). Springer.
Gunawan, A., Lau, H. C., & Vansteenwegen, P. (2016). Orienteering problem: A survey of recent variants, solution approaches and applications. *European Journal of Operational Research, 255*(2), 315–332.
Hà, M. H., Bostel, N., Langevin, A., & Rousseau, L.-M. (2012). An exact algorithm for the close enough traveling salesman problem with arc covering constraints. In *Proceedings of the 1st international conference on operations research and enterprise systems (ICORES)* (pp. 233–238).
Hà, M. H., Bostel, N., Langevin, A., & Rousseau, L. M. (2013). An exact algorithm and a metaheuristic for the multi-vehicle covering tour problem with a constraint on the number of vertices. *European Journal of Operational Research, 226*(2), 211–220.
Hà, M. H., Bostel, N., Langevin, A., & Rousseau, L.-M. (2014). Solving the close-enough arc routing problem. *Networks, 63*(1), 107–118.
Hachicha, M., Hodgson, M. J., Laporte, G., & Semet, F. (2000). Heuristics for the multi-vehicle covering tour problem. *Computers & Operations Research, 27*(1), 29–42.
Hodgson, M. J., Laporte, G., & Semet, F. (1998). A covering tour model for planning mobile health care facilities in Suhum District, Ghana. *Journal of Regional Science, 38*(4), 621–638.
Hollinger, G. A., & Sukhatme, G. S. (2013). Sampling-based motion planning for robotic information gathering. *Robotics: Science and Systems, 3*(5), 1–8.
Hollinger, G. A., & Sukhatme, G. S. (2014). Sampling-based robotic information gathering algorithms. *The International Journal of Robotics Research, 33*(9), 1271–1287.
Jawaid, S. T., & Smith, S. L. (2015). Informative path planning as a maximum traveling salesman problem with submodular rewards. *Discrete Applied Mathematics, 186*, 112–127.
Jozefowiez, N. (2014). A branch-and-price algorithm for the multivehicle covering tour problem. *Networks, 64*(3), 160–168.
Jozefowiez, N., Semet, F., & Talbi, E.-G. (2007). The bi-objective covering tour problem. *Computers & Operations Research, 34*(7), 1929–1942.
Kammoun, M., Derbel, H., Ratli, M., & Jarboui, B. (2017). An integration of mixed VND and VNS: The case of the multivehicle covering tour problem. *International Transactions in Operational Research, 24*(3), 663–679.
Karaoğlan, İ., Erdoğan, G., & Koç, Ç. (2018). The multi-vehicle probabilistic covering tour problem. *European Journal of Operational Research, 271*(1), 278–287.
Krause, A., Singh, A., & Guestrin, C. (2008). Near-optimal sensor placements in Gaussian processes: Theory, efficient algorithms and empirical studies. *Journal of Machine Learning Research, 9*, 235–284.
Labbé, M., Laporte, G., Martín, I. R., & González, J. J. S. (2004). The ring star problem: Polyhedral analysis and exact algorithm. *Networks: An International Journal, 43*(3), 177–189.
Labbé, M., Laporte, G., Martın, I. R., & González, J. J. S. (2005). Locating median cycles in networks. *European Journal of Operational Research, 160*(2), 457–470.
Labbé, M., Laporte, G., Rodríquez Martín, I., & Salazar Gonzáles, J. (1999). The median cycle problem. *Technical Report*. Department of Operations Research and Multicriteria Decision Aid at Université Libre de Bruxelles.
Lim, Z. W., Hsu, D., & Lee, W. S. (2016). Adaptive informative path planning in metric spaces. *The International Journal of Robotics Research, 35*, 585–598.
Margolis, J. T., Song, Y., & Mason, S. J. (2021). A multi-vehicle covering tour problem with speed optimization. *Networks*, 1–24. Advance online publication
Mata, C. S., & Mitchell, J. S. (1995). Approximation algorithms for geometric tour and network design problems. In *Proceedings of the eleventh annual symposium on computational geometry* (pp. 360–369). ACM.
Meliou, A., Krause, A., Guestrin, C., & Hellerstein, J. M. (2007). Nonmyopic informative path planning in spatio-temporal models. *AAAI, 10*(4), 602–607.
Mennell, W. (2009). *Heuristics for solving three routing problems: Close-enough traveling salesman problem, close-enough vehicle routing problem, sequence-dependent team orienteering problem*. University of Maryland Ph.D. thesis..
Naji-Azimi, Z., Renaud, J., Ruiz, A., & Salari, M. (2012a). A covering tour approach to the location of satellite distribution centers to supply humanitarian aid. *European Journal of Operational Research, 222*(3), 596–605.
Naji-Azimi, Z., Salari, M., & Toth, P. (2010). A heuristic procedure for the capacitated m-ring-star problem. *European Journal of Operational Research, 207*(3), 1227–1234.
Naji-Azimi, Z., Salari, M., & Toth, P. (2012b). An integer linear programming based heuristic for the capacitated m-ring-star problem. *European Journal of Operational Research, 217*(1), 17–25.
Orlis, C., Bianchessi, N., Roberti, R., & Dullaert, W. (2020). The team orienteering problem with overlaps: An application in cash logistics. *Transportation Science, 54*(2), 470–487.
Oruc, B. E., & Kara, B. Y. (2018). Post-disaster assessment routing problem. *Transportation Research Part B: Methodological, 116*, 76–102.
Ozbaygin, G., Yaman, H., & Karasan, O. E. (2016). Time constrained maximal covering salesman problem with weighted demands and partial coverage. *Computers & Operations Research, 76*, 226–237.
Pěnička, R., Faigl, J., & Saska, M. (2019). Variable neighborhood search for the set orienteering problem and its application to other orienteering problem variants. *European Journal of Operational Research, 276*(3), 816–825.
Pérez, J. A. M., Moreno-Vega, J. M., & Martın, I. R. (2003). Variable neighborhood tabu search and its application to the median cycle problem. *European Journal of Operational Research, 151*(2), 365–378.
Pham, T. A., Hà, M. H., & Nguyen, X. H. (2017). Solving the multi-vehicle multi-covering tour problem. *Computers & Operations Research, 88*, 258–278.
Popović, M., Vidal-Calleja, T., Chung, J. J., Nieto, J., & Siegwart, R. (2019). Informative path planning and mapping for active sensing under localization uncertainty. In *2020 IEEE International Conference on Robotics and Automation (ICRA)* (pp. 10751–10757). IEEE.
Popović, M., Vidal-Calleja, T., Hitz, G., Chung, J. J., Sa, I., Siegwart, R., & Nieto, J. (2020). An informative path planning framework for UAV-based terrain monitoring. *Autonomous Robots, 44*(6), 889–911.
Renaud, A., Absi, N., & Feillet, D. (2017). The stochastic close-enough arc routing problem. *Networks, 69*(2), 205–221.
Renaud, J., Boctor, F. F., & Laporte, G. (2004). Efficient heuristics for median cycle problems. *Journal of the Operational Research Society, 55*(2), 179–186.
Shuttleworth, R., Golden, B. L., Smith, S., & Wasil, E. (2008). Advances in meter reading: Heuristic solution of the close enough traveling salesman problem over a street network. In B. Golden, S. Raghavan, & E. Wasil (Eds.), *The vehicle routing problem: Latest advances and new challenges* (pp. 487–501). Springer.
Simonetti, L., Frota, Y., & de Souza, C. C. (2011). The ring-star problem: A new integer programming formulation and a branch-and-cut algorithm. *Discrete applied mathematics, 159*(16), 1901–1914.
Singh, A., Kaiser, W., Batalin, M., Krause, A., & Guestrin, C. (2007). Efficient planning of informative paths for multiple robots. In *International joint conference on artificial intelligence (IJCAI)* (pp. 2204–2211).
Singh, A., Krause, A., Guestrin, C., & Kaiser, W. J. (2009a). Efficient informative sensing using multiple robots. *Journal of Artificial Intelligence Research, 34*, 707–755.
Singh, A., Krause, A., & Kaiser, W. J. (2009b). Nonmyopic adaptive informative path planning for multiple robots. In *International joint conference on artificial intelligence (IJCAI)* (pp. 1843–1850).
Smith, R. N., Schwager, M., Smith, S. L., Jones, B. H., Rus, D., & Sukhatme, G. S. (2011). Persistent ocean monitoring with underwater gliders: Adapting sampling resolution. *Journal of Field Robotics, 28*(5), 714–741.
Štefaníková, P., Váňa, P., & Faigl, J. (2020). Greedy randomized adaptive search procedure for close enough orienteering problem. In *Proceedings of the 35th annual ACM symposium on applied computing* (pp. 808–814). ACM.
Stranders, R., De Cote, E. M., Rogers, A., & Jennings, N. R. (2013). Near-optimal con-

tinuous patrolling with teams of mobile information gathering agents. *Artificial Intelligence, 195*, 63–105.
Tricoire, F., Graf, A., & Gutjahr, W. J. (2012). The bi-objective stochastic covering tour problem. *Computers & Operations Research, 39*(7), 1582–1592.
Veenstra, M., Roodbergen, K. J., Coelho, L. C., & Zhu, S. X. (2018). A simultaneous facility location and vehicle routing problem arising in health care logistics in the Netherlands. *European Journal of Operational Research, 268*(2), 703–715.
Vidal, T., Crainic, T. G., Gendreau, M., & Prins, C. (2014). A unified solution framework for multi-attribute vehicle routing problems. *European Journal of Operational Research, 234*(3), 658–673.
Vogt, L., Poojari, C. A., & Beasley, J. E. (2007). A tabu search algorithm for the single vehicle routing allocation problem. *Journal of the Operational Research Society, 58*(4), 467–480.
Vu, L., Vu, D. M., Hà, M. H., & Nguyen, V.-P. (2021). The two-echelon routing problem with truck and drones. *International Transactions in Operational Research*, 1–17. Advance online publication
Xu, J., Chiu, S. Y., & Glover, F. (1999). Optimizing a ring-based private line telecommunication network using tabu search. *Management Science, 45*(3), 330–345.
Yang, Z., Xiao, M.-Q., Ge, Y.-W., Feng, D.-L., Zhang, L., Song, H.-F., & Tang, X.-L. (2018). A double-loop hybrid algorithm for the traveling salesman problem with arbitrary neighbourhoods. *European Journal of Operational Research, 265*(1), 65–80.
Yu, J., Schwager, M., & Rus, D. (2014). Correlated orienteering problem and its application to informative path planning for persistent monitoring tasks. In *IEEE International Conference on Intelligent Robots and Systems* (pp. 342–349). IEEE.
Zhang, W., Wang, K., Wang, S., & Laporte, G. (2020). Clustered coverage orienteering problem of unmanned surface vehicles for water sampling. *Naval Research Logistics (NRL), 67*(5), 353–367.
Zhang, Z., Qin, H., & Lim, A. (2014). A memetic algorithm for the capacitated m-ring-star problem. *Applied intelligence, 40*(2), 305–321.