Morphology-Based Correlation Parameter for Estimating the Effective Permittivity of Anisotropic Media Composed of Non-Spherical Inclusions

J. N. Camacho Hernandez[1], G. Link[1]

[1]*Institute for Pulsed Power and Microwave Technology, Karlsruhe Institute of Technology, Eggenstein-Leopoldshafen, 76344, Germany*
jesus.hernandez@kit.edu

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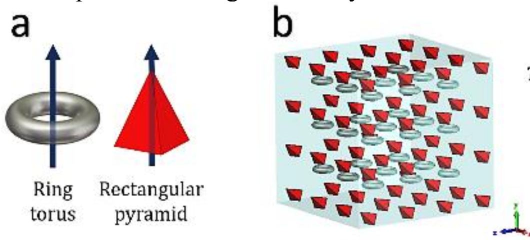
The effective permittivity characterizes the response of a composite material to an electric field. The effective permittivity is a complex quantity ($\varepsilon_{\text{eff}} = \varepsilon'_{\text{eff}} - j\varepsilon''_{\text{eff}}$) in which the real part $\varepsilon'_{\text{eff}}$ is known as the effective dielectric constant and describes the ability of a dielectric medium to store electric energy in an electric field. The imaginary part $\varepsilon''_{\text{eff}}$ is the effective dielectric loss, which accounts for the energy dissipation when exposed to an electric field (i.e. microwave radiation). The effective medium approximation (EMA) is a widely used technique to estimate the effective permittivity of composite media at the macroscopic scale [1]. The EMA approach assumes that the composite material can be treated as a homogeneous medium, also known as effective media, with inclusions distributed uniformly throughout a continuum medium. Describing the effective permittivity of effective media is accomplished by using mixing relations based on the EMA approach, i.e. the Maxwell-Garnett and Bruggeman relations. However, these mixture relations are limited to microstructures consisting of spherical and quasi-spherical (i.e. ellipsoidal) inclusions [2]. Consequently, the EMA relations are inadequate for estimating the effective permittivity of media with microstructures that deviate significantly from spherical or ellipsoidal shapes. Overcoming these limitations requires an appropriate consideration of the morphology and spatial orientation of inclusions to ensure an accurate estimation of the effective permittivity. Figure 1 provides an example of a material with complex microstructure, (a) including non-spherical or non-ellipsoidal inclusions, (b) anisotropic-oriented inclusions, (c) and the corresponding general effective permittivity.

In this contribution, we present the results on an approach for estimating the effective permittivity of anisotropic-oriented inclusions by considering their morphology. The approach employs the orientation tensor [3] and a correlation parameter tensor that contains geometrical details of the inclusions to estimate the effective permittivity tensor. We have used electromagnetic wave propagation calculations on media with various geometrical-microstructures to investigate the effect of the geometrical correlation parameter on the effective permittivity. The results demonstrate that the approach can produce reliable estimates of the effective permittivity tensor of media by considering their microstructure morphology. Moreover, the approach can be used to improve the design and analysis of electromagnetic devices and systems.



$$T_{\text{pyramid}} = \begin{pmatrix} 0.19 & 0.19 & 0.33 \\ 0.19 & 0.20 & 0.34 \\ 0.33 & 0.34 & 0.61 \end{pmatrix} \quad T_{\text{torus}} = \begin{pmatrix} 0.00 & 0.00 & 0.00 \\ 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 0.00 \end{pmatrix}$$

$$\varepsilon_{\text{eff}} = \begin{pmatrix} \varepsilon'_{\text{eff,xx}} - j\varepsilon''_{\text{eff,xx}} & \varepsilon'_{\text{eff,xy}} - j\varepsilon''_{\text{eff,xy}} & \varepsilon'_{\text{eff,xz}} - j\varepsilon''_{\text{eff,xz}} \\ \varepsilon'_{\text{eff,yx}} - j\varepsilon''_{\text{eff,yx}} & \varepsilon'_{\text{eff,yy}} - j\varepsilon''_{\text{eff,yy}} & \varepsilon'_{\text{eff,yz}} - j\varepsilon''_{\text{eff,yz}} \\ \varepsilon'_{\text{eff,zx}} - j\varepsilon''_{\text{eff,zx}} & \varepsilon'_{\text{eff,zy}} - j\varepsilon''_{\text{eff,zy}} & \varepsilon'_{\text{eff,zz}} - j\varepsilon''_{\text{eff,zz}} \end{pmatrix}$$

Fig. 1. (a) Examples of inclusions with non-spherical or non-ellipsoidal shapes and their basis orientation direction, (b) material with anisotropic-oriented inclusions according to their second-order orientation tensor T, (c) permittivity tensor, which describes the anisotropic behaviour of a material in response to an electric field.

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