Temporal Parity-Time-Symmetric Metasurfaces

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Abstract – **In this talk, we present the concept of temporal parity-time-symmetric metasurfaces. In contrast to spatial parity-time symmetric structure, here, the metasurface exhibits gain and loss in the temporal domain, and the gain and loss are virtually created by time-varying lossless reactive components. It is found that the alternation of gain and loss in the temporal domain results in the closing of the momentum bandgap. At the exceptional point where the size of the momentum bandgap becomes zero, the metasurface exhibits an exotic response for pulse illuminations, expressed in the localization and linear growing of the pulse energy.**

I. INTRODUCTION

Non-hermitian electromagnetic systems with gain and loss equally distributed in space can exhibit real eigenfrequencies. Such systems are known as parity-time (PT) symmetric systems [1]. Recent years have witnessed tremendous efforts on PT-symmetric systems due to their intriguing electromagnetic properties, such as optical isolation and nonreciprocity, band merging, and unidirectional invisibility, generated at the exceptional points where the eigenfrequency of the system undergoes a transition from a real to complex value [2]. While PT-symmetric structures were initially introduced in the spatial domain, in more recent works, the concept was extended to the temporal domain. For example, by introducing gain and loss media in time, exotic energy transformation has been observed [3, 4, 5]. In this talk, we introduce a PT-symmetric periodic system in photonic time crystals. Instead of switching the material properties from gain to loss, which is difficult to implement, we use time-varying inductance and capacitance to mimic the gain and loss. We found that such a system results in an exotic bandgap effect: when the gain and loss reach balance, the size of momentum bandgap become zero, resulting in localization of the continuous wave and linear growing of the pulse energy.

II. RESULTS

A. Virtual gain and loss elements

First, we introduce how to implement the gain and loss by using temporal switching of lossless reactive components. The gain or loss can be realized through temporal switching of an inductance or capacitance. Figure 1 qualitatively shows the energy variation of the two components under periodic temporal switching. For the capacitance, the stored energy is written as $W_C = C(t)V^2(t)/2 = Q^2(t)/2C(t)$, where $V(t)$ and $Q(t)$ are the electric voltage and charge of the capacitor, respectively. According to this relation, when the capacitance suddenly drops, the charge accumulation $Q(t)$ is not able to respond immediately. Therefore, the electric energy increases, which results in an increase of $V(t)$. However, at the moment in time when the capacitance increases $(t = T)$, $V(t) = W_C = 0$, the switching process does not increase or decrease the energy. Therefore, the time-varying capacitance accumulates energy when the capacitance drops. Similar logic can be applied to time-varying inductance. The magnetic energy of an inductor can be written as $W_L = L(t)I^2(t)/2 = \Phi^2(t)/2L(t)$, where $\Phi(t)$ is the magnetic flux, and $I(t)$ is the electric current flowing in the inductor. When the inductance increases abruptly, the magnetic flux keeps unchanged, and the electric current and magnetic energy decreases. Therefore, with proper phases between modulation and signal, the time-varying inductance can serve as a virtual lossy element.
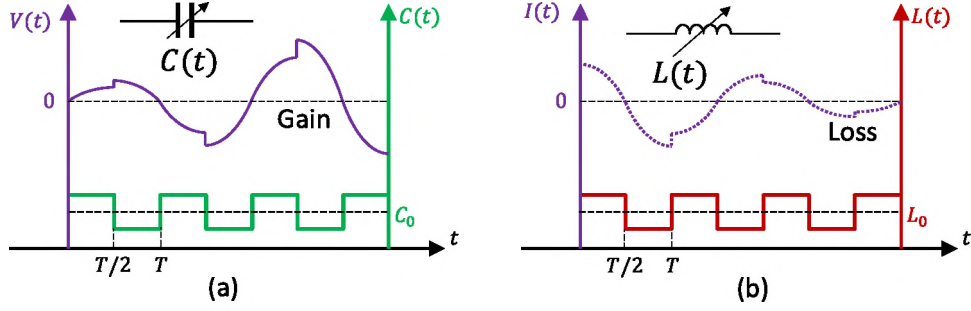
Fig. 1: Qualitative explanation of virtual gain and loss generated by time-varying (a) capacitance and (b) inductance.

B. Basic principles of temporal PT-symmetric metasurfaces

Next, we integrate the virtual gain and loss elements into the metasurface and create a temporal PT-symmetric metasurface. Let us first consider a spatially homogeneous metasurface, the electromagnetic properties of the metasurface are periodically varying in time, as shown in Fig. 2(a). Within one period ($0 < t < T$), the metasurface is inductive L_0 when $0 < t < T/2$, it becomes capacitive C_0 in the next half cycle. Such a temporal metasurface can be implemented as an LC-circuit with its time-varying unit-cell capacitance $C(t)$ and inductance $L(t)$ connected in series [see Fig. 2(b)]. Figures 2(c)–(d) show the variations of $C(t)$ and $L(t)$. When $0 < t < T/2$, the capacitance is infinite, the capacitor is shorted, and the LC-circuit can be regarded as a purely L-circuit. In the next half cycle ($T/2 < t < T$), $L = 0$, meaning that the inductor is shorted, and the metasurfaces can be represented by a pure capacitance $C = C_0$. According to the analysis in Section II-A, at the time moment of $t = T/2$, the gain is generated due to the sudden decrease of surface capacitance. At $t = T$, the loss is generated due to the sudden increase in inductance. Properly choosing the values of L_0 and C_0, the gain and loss can reach a balance. Such inductive-capacitive alternating metasurfaces can be implemented by periodical metallic patches inserted with electronic switches (see Fig. 6 in [6]).
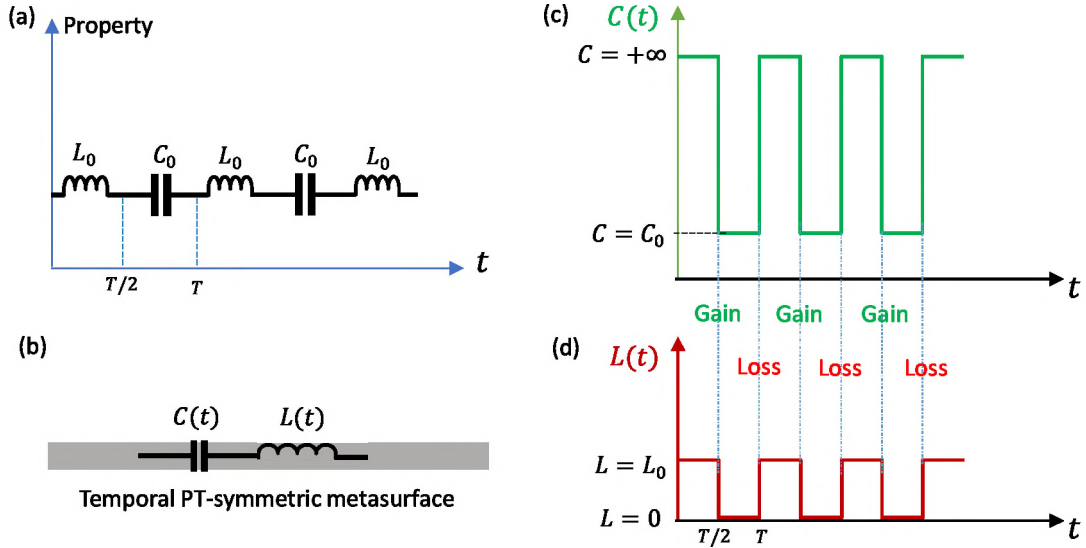

Fig. 2: (a) Temporal variations of metasurface response. (b) Equivalent circuit of the time-varying metasurface. Temporal variations of (c) $C(t)$ and (d) $L(t)$ functions.
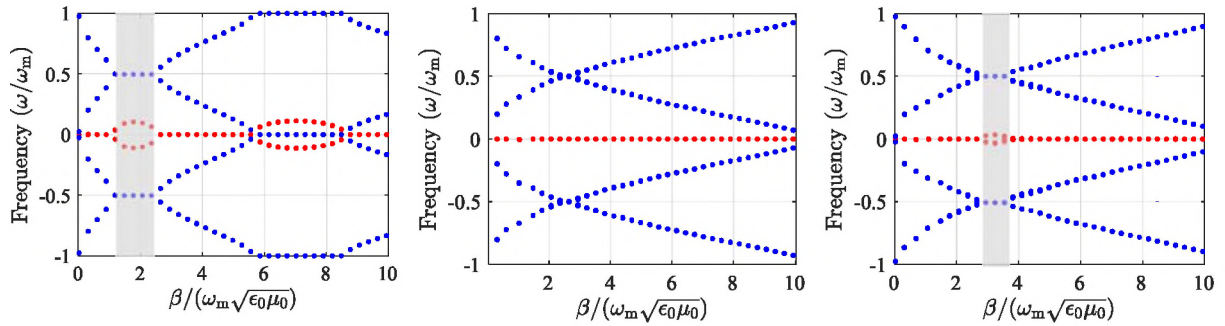
Fig. 3: Band structure of metasurface for (a) $L_0 = 0$ nH, (b) $L_0 = 33$ nH, and (c) $L_0 = 100$ nH. In all three cases, $C(t) = C_0 = 0.73$ pF and $L(t) = 0$ when $T/2 < t < T$, and $C(t) = 5C_0$, $L(t) = L_0$ when $0 < t < T/2$. The modulation frequency is $\omega_m = 2\pi \times 2$ GHz. The blue and red dots represent real and imaginary parts of eigenfrequency, respectively. The shaded region represents the first momentum bandgap.

C. Closed momentum bandgap and field localization

Here, we focus on the effect of the temporal gain-loss system on the momentum bandgap. The band structure is calculated with the mode-matching method [7]. When the inductance is $L_0 = 0$ nH, the metasurface is purely capacitive, supporting TE-polarized surface wave. Temporal modulation of capacitance induces a maximum momentum bandgap, as shown in Fig. 3(a), where the surface wave exponentially grows in time. Next, we introduce the modulation of the inductive part, gradually increasing the inductance L_0. The variation of inductance results in losses that compensate for the gain induced by the capacitance. Therefore, the momentum bandgap becomes narrower. When $L_0 = 33$ nH, the bandgap is completely closed, meaning that, the metasurface reaches a balance of gain and loss at each point. It is important to note that the zero bandgap does not imply that waves inside the bandgap propagate regularly as in an unmodulated material. Instead, here, the wave at the zero bandgap point generates time reflection and transmission and forms a standing waveform. Since the momentum bandgap is closed, the imaginary part of eigenfrequency is zero, the system does not provide or absorb energy to the wave. In other words, the wave is localized in the material without growing or decaying. Further increasing the inductance will break the energy balance, and the momentum bandgap will be reopened. It is interesting to note that purely inductive metasurfaces cannot support TE-polarized surface mode. However, for temporal PT metasurfaces with hybrid inductive and capacitive properties with a fast alternating speed, TE-polarized waves can propagate on such metasurfaces.

III. CONCLUSION

In this study, we introduce the concept of temporal PT-symmetric metasurfaces with balanced gain and loss distributed in the temporal dimension. The gain and loss are virtually generated by time-varying reactive components. It was found that by properly choosing the values of alternating surface inductance and capacitance, one can close the momentum bandgap, resulting in a zero-width bandgap, which is an exceptional point. The eigenfrequency at the exceptional point becomes purely real, and the wave is localized in the structure without growing and decaying.

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