NUMERICAL INVESTIGATION OF LIQUID METAL FLOW IN SQUARE CHANNELS UNDER INCLINED MAGNETIC FIELDS FOR FUSION RELEVANT PARAMETERS

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Abstract: Laminar liquid metal flows in straight ducts of square cross-section under strong transverse magnetic fields have been investigated using a numerical code based on finite volume methods. The scope of the simulations is the liquid metal transport in nuclear fusion power reactors where orientations of duct walls do not ideally align with the direction of imposed magnetic field. The detailed knowledge about combined impact of field strength, inclination angle of walls with respect to magnetic field lines, wall thickness, and electric conductivity on the flow of the liquid metal breeder and coolant PbLi is essential for the design of future fusion breeding blankets. Simulations have been performed in the range of fusion relevant parameters and results have been confirmed by asymptotic results for some special cases.

Key words: MHD duct flow, inclined magnetic fields, numerical simulations

1. Introduction The assessment of magneto hydrodynamic (MHD) effects governing the flow of the liquid metal breeder and coolant PbLi in breeding blankets of fusion power reactors requires predictive tools that are capable of covering the relevant parametric spectrum. Numerical MHD codes used in fusion research must apply to realistic blanket geometries and complex magnetic field configuration for confining the fusion plasma [1]. The orientation of the magnetic field is determined by control fields superimposed on the main toroidal field component and by the reactor design causing toroidal filed ripples. As a matter of fact, in general, the magnetic field may have any orientation with respect to duct walls as shown in Figure 1a. While some preceding studies considered effects of field inclination and wall conductivity by applying analytical, asymptotic or numerical methods [2] [3] [4] [5] using the thin-wall approximation, the present work additionally investigates the impact of wall thickness and thereby supplements results published previously.

2. Formulation of the Problem Viscous incompressible flow of an electrically conducting fluid in a magnetic field **B** is governed by equations for conservation of momentum and mass:

$$Re\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla\right)\mathbf{v}=-\nabla p+\nabla^2\mathbf{v}+Ha^2\mathbf{j}\times\mathbf{B}, \qquad \nabla\cdot\mathbf{v}=0, \tag{1}$$

where **v**, p and **j** denote the non-dimensional velocity, pressure and electric current density. In the inductionless approximation, Ohm's law and continuity of current density

$$\mathbf{j}=-\nabla\phi+\mathbf{v}\times\mathbf{B}, \qquad \nabla\cdot\mathbf{j}=0, \tag{2}$$

determine current density and electric potential ϕ. The non-dimensional groups, the Reynolds number Re and the Hartmann number Ha, represent the system-dependent parameters:

$$Re = \frac{u_0 L}{\nu}, \quad Ha = B_0 L \sqrt{\frac{\sigma}{\rho \nu}}. \tag{3}$$

They specify the ratio of inertia or electromagnetic forces to viscous forces, respectively. Moreover, Ha is an indicator for the strength of the magnetic field. The variables L, B_0, u_0, ρ, ν and σ denote characteristic length, imposed magnetic field, mean velocity, mass density, kinematic viscosity and electric conductivity, respectively.

We investigate the impact of inclined magnetic fields on unidirectional laminar MHD flows in straight rectangular channels, as displayed in Figure 1a, where σ_w and t_w indicate electrical conductivity and thickness of the wall. While in several previous investigations, the wall conductance has been treated using the thin-wall approximation based on the wall conductance parameter

$$c = \frac{t_w \sigma_w}{L\sigma}, \tag{4}$$

the present work considers the influence of t_w/L and σ_w/σ separately. The magnetic field **B** transverse to the channel axis x may take any angle of inclination with respect to the duct walls. Parameters in the range Ha=1000, $0<c<0.1$ and $0.001<t_w/L<0.1$ constitute adequate configurations to study flows at fusion relevant conditions. The Reynolds number Re has been chosen small enough to avoid turbulence.

Equations (1)-(2) are solved by a numerical code [5] based on finite volume methods and a current conservative scheme [6]. The equations are solved simultaneously in the fluid and wall domains, coupled by conditions for continuity of potential and wall-normal currents at the fluid-wall interface. Cyclic conditions are applied in axial direction. Grid-independent results are obtained using a structured mesh as shown in Figure 1b, which contains 200×200 grid points in the cross-sectional plane of the fluid and additional 20 grid points to resolve the solid domain in wall-normal direction. A strong grading in wall-normal direction helps resolving thin viscous boundary layers at the walls.

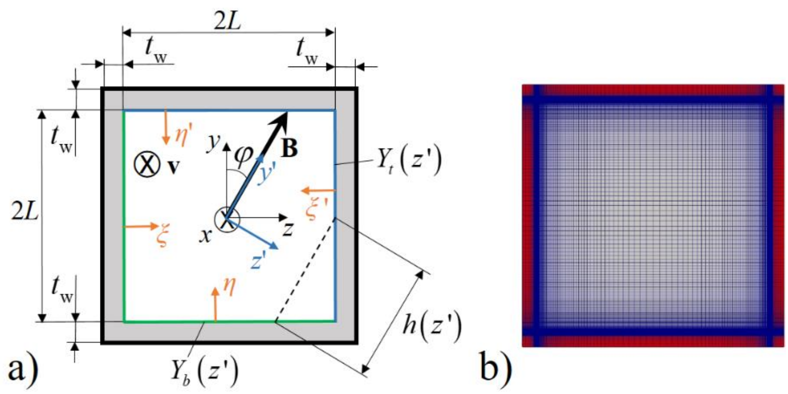

Figure 1: a) Sketch of geometry and coordinate systems; b) Structured discretization for fluid (grey) and wall (red).

For validation purposes at high Hartmann number and for insulating duct walls (c = 0), an asymptotic solution has been used, based on the fact that in insulating ducts of arbitrary shape, the core velocity is constant along field lines and proportional to the duct height, measured along magnetic field lines, i.e. $u_c=u_c(z') \sim h(z')=Y_t - Y_b$ [7]. Here, Y_t and Y_b are the y' coordinates of the top and bottom duct contour (Figure 1a). As a result, the velocity becomes constant in the central core where $h(z')=$ *constant* and it decays linearly towards the external corners. The asymptotic analysis [7], valid for $Ha >> 1$, has been extended by viscous corrections in the Hartmann layers, where η and ξ denote the local wall-normal coordinates, scaled by the corresponding thickness of the layers $\delta_\eta = Ha^{-1}/\cos\varphi$ and $\delta_\xi = Ha^{-1}/\sin\varphi$, respectively (see Figure 1a). This leads to viscous corrections of velocity near the walls as

$$u_{\text{asym}} = u_c - \left(u_t e^{-\eta'} + u_b e^{-\eta} + u_b e^{-\xi} + u_t e^{-\xi'} \right), \tag{5}$$

where u_t and u_b denote core velocity, evaluated at Y_t and Y_b. We find the flow rate and mean velocity $\bar{u}$ by integration over the cross-section. The fact that $\bar{u} = 1$ determines the axial pressure gradient as

$$\left(\frac{dp}{dx}\right)_{asym} = -Ha\cos\varphi\left(1 - \frac{1}{3}\tan\varphi + \frac{1}{2Ha}\left(\frac{3\tan\varphi - 2}{\cos\varphi} - \frac{1}{\sin\varphi}\right)\right)^{-1}. \tag{6}$$

3. Results

Figure 2 displays three cases of MHD flow in square channels for different wall conductivities under a strong magnetic field, inclined by φ=30°.

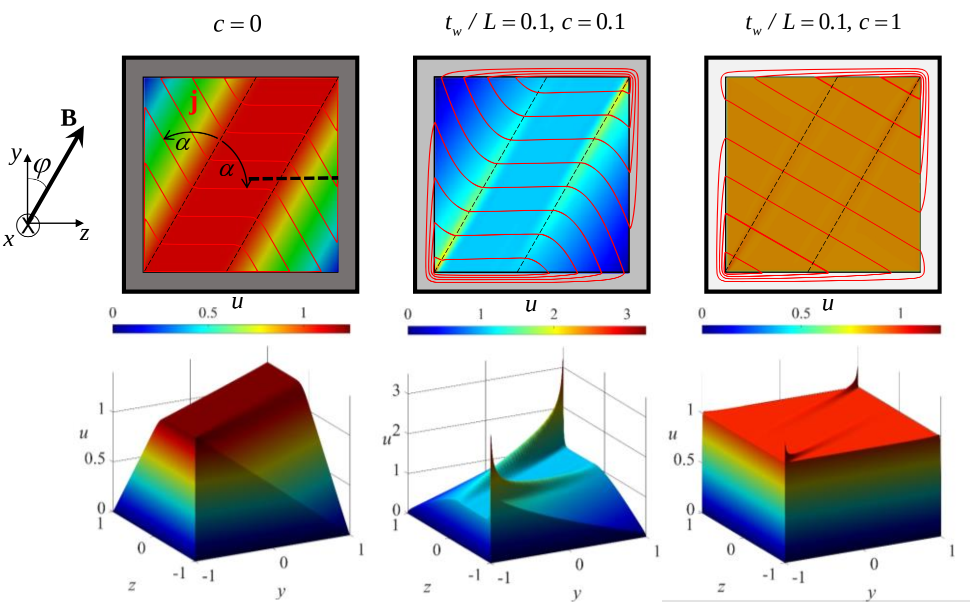

Figure 2: Velocity profiles for Ha = 1000, φ = 30°. Red solid lines represent current streamlines and black dashed lines indicate characteristic surfaces that form along magnetic field lines. The bold horizontal dashed line shown for c = 0 indicates positions where samples have been taken for code validation shown in Figure 3.

As anticipated from asymptotic analyses, e.g. [7] [8] [3], internal layers emerge from geometric singularities of the geometry (corners) and spread across the fluid along magnetic field lines. These layers separate the flow domain into three distinct core regions. One core is located in the center for which $h(z')$ and $u_c(z')$ are constant. The other cores connect the internal layers with the outer duct corners. For poorly conducting walls with $c<Ha^{-1}$, the velocity in the lateral cores decays monotonically towards the outer corners in accordance with [7]. Moreover, the velocity drops rapidly across thin Hartmann layers to satisfy the no-slip condition at all walls. For moderate wall conductance, currents in the central core flow parallel to the duct walls and their angle of inclination α with respect to the internal layers is the same as for currents in the outer cores (see Figure 2). With increasing c, more and more currents enter the wall domain. This raises the overall current density, leading to higher pressure gradients and jet-like peaks of velocity in internal layers. For highly conducting walls with $c>>Ha^{-1}$, currents in all cores flow perpendicular to the magnetic field and the velocity becomes constant, except for small jets in the corners and thin boundary layers.

Velocity profiles obtained by numerical simulations along the sample line y=0 in Figure 2 are compared with the asymptotic solution and presented in Figure 3 for c = 0 and Ha = 1000.

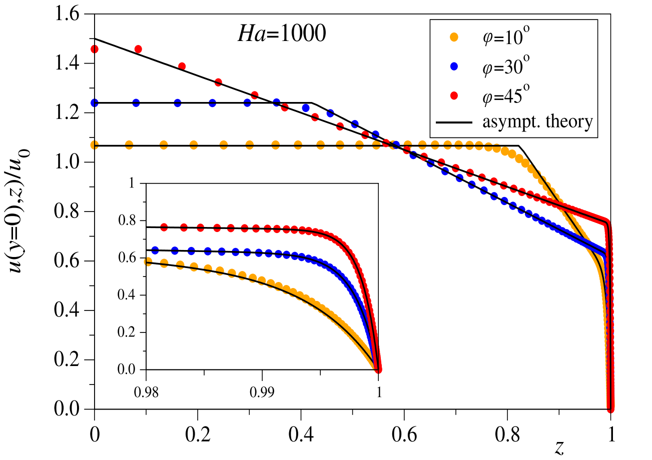

Figure 3: Validation of the flow profile at c = 0 with the asymptotic solution along the sample line in Figure 2.

We observe a very good agreement in almost the entire cross section and in particular inside the viscous Hartmann layers, as shown e.g. by the enlarged view in the subplot. The sharp change of slope visible in the asymptotic velocity profiles at the characteristic surface between the inner and outer cores, i.e. along the field lines that pass the duct corners, is caused by neglecting core viscosity in the asymptotic analysis. The numerical solutions, however, correctly account for viscous effects at the edges of the cores and smooth the profiles across the thin internal layers. Similar results have been obtained for flows up to Ha = 50000 and deviations for pressure gradients compared with asymptotic results are smaller than 0.3%.

Figure 4 displays numerical results of pressure gradients depending on magnetic field inclination, wall conductance parameter and wall thickness. They are shown in dimensionless form (scaled by $\sigma u_0 B_0^2$) and compared to those of Hartmann flow $(dp/dx)_{Ha} = -(c+Ha^{-1})/(c+1)$.

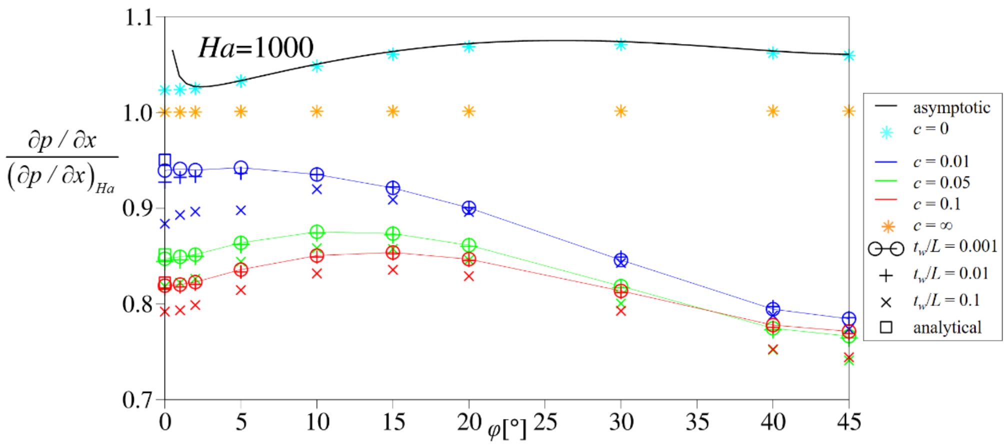

Figure 4: Numerical and analytical results of the pressure gradient dp/dx in relation to that of corresponding Hartmann flow $(dp/dx)_{Ha}$ for Ha = 1000 as a function of φ, for different t_w/L and c.

For insulating walls, we notice a very good agreement with the asymptotic solution. When the inclination angle approaches zero, numerical results reveal behavior, which the asymptotic theory cannot predict since local Hartmann layers δ_ξ at vertical walls would grow indefinitely. High velocity jets interact with the wall boundary layers resulting in viscous effects.

The solution for $c > 1$ shows a robust behavior of pressure gradient, apparently unaffected by field inclination and in perfect agreement with the solution of Hartmann flow. In such a flow regime, the pressure gradient is highest and MHD effects do not depend on inclination or wall thickness.

For arbitrary wall conductivities, pressure gradients vary significantly in the investigated range of parameters. In order to compare all results in a single diagram in Figure 3, pressure gradients have been normalized by the ones of corresponding Hartmann flow. For all investigated cases $c \leq 0.1$ we observe a clear dependence of pressure drop on the angle of inclination φ. Moreover, the results for a given wall conductance parameter c (same color in the plot) demonstrate clearly that the thin wall approximation applies only for very thin walls. For applications with finite wall thickness, results may deviate from those obtained for very thin walls, especially for $\varphi < 20°$. For $c = 0.1$ and $t_w/L = 0.1$ (relatively thick walls) results differ from those for thin walls over the entire range of φ. Nevertheless, we can conclude that the thin-wall approximation yields excellent results if $t_w/L \leq 0.05$.

4. Conclusions MHD flows in rectangular ducts have been investigated for different angles of inclination of the duct walls with respect to the applied strong magnetic field, for different wall conductance parameters and for various thicknesses of duct walls. For inclined ducts, internal layers emerging from duct corners spread along magnetic field lines across the entire fluid domain, giving rise to strong gradients of velocity in these layers. Results for pressure drop support the validity of the thin-wall approximation that appears to apply when walls are sufficiently thin ($t_w/L \leq 0.05$). The present numerical simulations for fully developed flow have been performed using a high-resolution structured grid.

Finding proper spatial discretization has been a particular challenge throughout this study, since high velocity gradients on small length scales occur along walls as well as across internal layers, which spread into the duct. While structured grids as shown in Figure 1b meet such requirements only under great computational expense, unstructured meshes employing local refinement overcome such issues. This topic will be discussed in an extended separate paper.

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