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A closer look at isodoublet vector leptoquark solution to the $R_{D^{(*)}}$ anomaly

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ABSTRACT: We discuss a model with a $SU(2)_L$ doublet vector leptoquark (LQ), motivated by the recent experimental results relating to the lepton universality of $\bar{B} \rightarrow D^{(*)}\tau\bar{\nu}_\tau$. We find that scalar operators predicted by the LQ are favored to explain the deviations, taking into account the recent LHCb result. We investigate the extensive phenomenology of the model and conclude that $B_s \rightarrow \tau\bar{\tau}$, $B \rightarrow K\tau\bar{\tau}$, $B_u \rightarrow \tau\bar{\nu}_\tau$ and high- p_T di- τ lepton signatures at the LHC will probe the interesting parameter region in the near future.

KEYWORDS: Bottom Quarks, Lepton Flavour Violation (charged), Semi-Leptonic Decays

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1 Introduction

The semi-tauonic B -meson decays, $\bar{B} \rightarrow D^{(*)}\tau\bar{\nu}$, have been interesting processes to measure the lepton flavor universality (LFU):

$$R_D \equiv \frac{\text{BR}(\bar{B} \rightarrow D\tau\bar{\nu}_\tau)}{\text{BR}(\bar{B} \rightarrow D\ell\bar{\nu}_\ell)}, \quad R_{D^*} \equiv \frac{\text{BR}(\bar{B} \rightarrow D^*\tau\bar{\nu}_\tau)}{\text{BR}(\bar{B} \rightarrow D^*\ell\bar{\nu}_\ell)}, \quad (1.1)$$

where ℓ denotes light charged leptons. Interestingly, deviations from the SM predictions [1–4]¹ have been reported by the BaBar [9, 10], Belle [11–15] and LHCb [16–20] collaborations.² Last and early this years, the LHCb collaboration reported the first result of R_{D^*} along with R_D [21] and another R_{D^*} data [22], respectively. These latest measurements are consistent with the previous world average within the uncertainty, but the resulting world average prefers larger (smaller) deviation in R_D (R_{D^*}). The current significance of the deviation is 3-4 σ [23] and the new physics (NP) interpretations are updated in refs. [23–26].³ One of the significant points, compared with the previous result, is the revival of the NP interpretation with scalar operators. The relevant interaction, in addition to the SM contribution, is

$$\mathcal{H}_{\text{eff}} = 2\sqrt{2}G_F V_{cb} \left[C_{S_L} O_{S_L} + C_{S_R} O_{S_R} \right], \quad (1.2)$$

¹Recently the dispersive matrix approach of the form factors found the larger R_{D^*} [5, 6] based on the Fermilab-MILC lattice result [7] while this method produce the 3 σ tension in the angular observable [8].

² $R_{D^{(*)}}$ are defined by $\ell = e, \mu$ for the BaBar/Belle and $\ell = \mu$ for the LHCb.

³See table 6 of ref. [23] for the recent summary of the situation.

with

$$O_{S_L} = (\bar{c}P_L b)(\bar{\tau}P_L\nu_\tau), \quad O_{S_R} = (\bar{c}P_R b)(\bar{\tau}P_L\nu_\tau), \quad (1.3)$$

where $P_L = (1 - \gamma_5)/2$ and $P_R = (1 + \gamma_5)/2$. The NP contribution is taken into account by the Wilson coefficients (WCs), C_X ($X = S_L, S_R$), normalized by the SM factor of $2\sqrt{2}G_F V_{cb}$.

It has been well known that the B_c lifetime constrains the scalar interpretation [27–32]. However, the recent result makes it possible to explain the deviations at the 1σ level using the scalar operators [25]. Furthermore, the only scalar contributions enhance the polarization observable, $F_L^{D^*}$,⁴ where the SM prediction is slightly lower than the measurement [34].

A famous mediator that induces sizable semileptonic scalar contribution is a charged Higgs in a generic two Higgs doublet model (2HDM). This possibility has been thoroughly surveyed [25, 35–50] and it is found that sizable contribution to WC is possible only in O_{S_L} . It is noted that the type-II 2HDM can contribute to O_{S_R} , but the contribution is not favored since the sign of C_{S_R} is always negative and does not comply with data.

A leptoquark (LQ) is considered to be one of the best candidates for the $R_{D^{(*)}}$ anomaly explanation. There are three kinds of LQs widely investigated so far [51]. In this paper, we focus on an isodoublet vector LQ (V_2) that significantly contributes to C_{S_R} . Recently, the LQ is studied motivated by $R_{K^{(*)}}$ anomaly [52–54] and $R_{D^{(*)}}$ anomaly [54, 55]. The contribution of the V_2 LQ to C_{S_R} could be positive and solves the anomalies. This LQ possibility is very interesting in view of the current status, but has not been well studied.⁵ In this work, we construct an effective model with V_2 from the phenomenological point of view. We study correlations between $R_{D^{(*)}}$ and other observables, and discuss how to test this LQ possibility.

Before the recent LHCb result, the V_2 LQ could not explain the R_{D^*} within 2σ [68]. The situation, however, changes: the current minimal χ^2 for R_{D^*} becomes 3.7 with O_{S_R} [23] which should be compared to $\chi_{\text{SM}}^2 = 13.6$. It would be timely to analyze the model with V_2 . Compared to the previous work [54], that appeared before the LHCb results, new parts are as follows. First, we assign a τ number to V_2 . This assignment forbids a proton decay and suppresses many flavor violating processes. The underlying theory is beyond our scope, but our setup would be a guiding principle to construct a concrete model. We study relevant flavor phenomenology in this effective model. We newly examine correlations between $R_{D^{(*)}}$ and other observables in this model, and find that $B_s \rightarrow \tau\bar{\tau}$ and $B \rightarrow K\tau\bar{\tau}$ are greatly enhanced.⁶ Second, we find that $B \rightarrow \tau\bar{\nu}_\tau$, that is not studied in ref. [54], excludes the simplest setup for the explanation of $R_{D^{(*)}}$. We rescue the possibility by

⁴See ref. [33] for the explicit definitions.

⁵A $SU(2)_L$ singlet leptoquark U_1 LQ, that is predicted by the Pati-Salam model, also induces C_{S_R} in general. The $R_{D^{(*)}}$ anomaly explanation, however, would suffer from the collider bound on extra gauge boson searches, if U_1 is originated from the massive gauge boson in the Pati-Salam model. $U(2)$ flavor symmetric models [24, 56–67] can evade the stringent collider bounds and predict also C_{S_R} accompanied by the contribution to the SM-like operator which substantially differentiates phenomenology.

⁶Similar correlations have been scrutinized within other LQ scenarios see refs. [24, 63–67, 69] for instance.

adding one more interaction. Third, we investigate the LHC implication of the model with the help of the public tool HighPT [70]. We conclude that signals with two oppositely charged τ leptons in the final states will also probe the interesting parameter region in the near future.

The outline of the paper is given as follows. In section 2 we introduce the working model for the V_2 LQ and summarize the model parameters. In section 3 we discuss the relevant flavor observables and investigate the phenomenology. Then we also consider the constraint from the LHC and discuss the future prospect. Section 4 is devoted to summary and discussion.

2 Model setup

In this section we introduce the working model and four-fermi interactions relevant to the phenomenology.

2.1 Simplified model with the V_2 LQ

We shall consider an extended SM model with a $SU(2)_L$ doublet vector LQ, V_2 . The charge assignment of V_2 is $(SU(3)_c, SU(2)_L, U(1)_Y) = (\bar{3}, 2, 5/6)$ and the field is described as

$$V_2 = \begin{pmatrix} V_2^{4/3} \\ V_2^{1/3} \end{pmatrix}, \quad (2.1)$$

where the electromagnetic charges of the upper and lower component are $4/3$ and $1/3$, respectively. This charge assignment is the same as that of a X boson in the $SU(5)$ grand unified theory (GUT). In this paper we do not specify the UV completion, and simply assign τ number and mass to this doublet. As a result, a disastrous rapid proton does not occur since the di-quark coupling is forbidden by the τ number conservation.

Under this assumption the couplings between V_2 and SM fermions relevant to our phenomenology are given by

$$\mathcal{L}_{V_2} = h_1^{ij} (\bar{d}_i^C \gamma_\mu P_L L_j^b) \epsilon^{ab} V_2^{\mu,a} + h_2^{ij} (\bar{Q}_i^{C,a} \gamma_\mu P_R e_j) \epsilon^{ab} V_2^{\mu,b} + h_3^{ij} (\bar{Q}_i^C \gamma_\mu P_R u_j) V_2^{\mu,*} + \text{h.c.} \quad (2.2)$$

where indices i, j and a, b are labels of flavor and $SU(2)_L$. We work on the down-quark basis. This choice is phenomenologically conservative since flavor changing neutral currents involving light down-type are suppressed at tree level. It is noted that within a $\mathcal{O}(1)$ TeV LQ scenarios 1-loop induced processes, e.g. meson mixing, is important although an UV completion is required to evaluate the correction. One possible underlying theory will be discussed in section 4. It is noted that those interactions change the fermion number by 2 units: $|\Delta F| = 2$ where $F = 3B + L$ and, B and L are baryon and lepton numbers, respectively. By assigning the τ number to V_2 we can eliminate h_3 that triggers a dangerous proton decay [71–73].⁷ Thanks to the τ charge assignment, the structure of the interaction

⁷ τ -flavored U_1 LQ is discussed in ref. [74].

is described as

$$h_1^{ij} = \begin{pmatrix} 0 & 0 & h_1^{13} \\ 0 & 0 & h_1^{23} \\ 0 & 0 & h_1^{33} \end{pmatrix}, \quad h_2^{ij} = \begin{pmatrix} 0 & 0 & h_2^{13} \\ 0 & 0 & h_2^{23} \\ 0 & 0 & h_2^{33} \end{pmatrix}, \quad h_3 = 0. \quad (2.3)$$

Assuming that those elements are real, we consider flavor and collider phenomenology in the next section. Now, the terms in \mathcal{L}_{V_2} are decomposed as

$$\begin{aligned} \mathcal{L}_{V_2} = & + h_1^{i3} (\bar{d}_i^C \gamma_\mu P_L \tau) V_2^{4/3, \mu} - h_1^{i3} (\bar{d}_i^C \gamma_\mu P_L \nu_\tau) V_2^{1/3, \mu} \\ & - h_2^{i3} (\bar{d}_i^C \gamma_\mu P_R \tau) V_2^{4/3, \mu} + h_2^{i3} (\bar{u}_i^C \gamma_\mu P_R \tau) V_2^{1/3, \mu} + \text{h.c.} \end{aligned} \quad (2.4)$$

The mass eigenstates are given by replacing as $(u_L, d_L) \rightarrow (V_Q^\dagger u_L, d_L)$

$$\begin{aligned} \mathcal{L}_{V_2} = & + h_1^{i3} (\bar{d}_i^C \gamma_\mu P_L \tau) V_2^{4/3, \mu} - h_1^{i3} (\bar{d}_i^C \gamma_\mu P_L \nu_\tau) V_2^{1/3, \mu} \\ & - h_2^{i3} (\bar{d}_i^C \gamma_\mu P_R \tau) V_2^{4/3, \mu} + (V_Q^* h_2)^{i3} (\bar{u}_i^C \gamma_\mu P_R \tau) V_2^{1/3, \mu} + \text{h.c.}, \end{aligned} \quad (2.5)$$

where V_Q denote Cabibbo-Kobayashi-Maskawa matrix [75, 76].

2.2 Four-fermi couplings

The interactions in eq. (2.5) contribute to the semileptonic operators through $V^{4/3}$ and $V^{1/3}$ exchanges:

$$\begin{aligned} \mathcal{L}_{NDSL} = & - \frac{h_1^{i3} h_1^{k3*}}{m_{V_2}^2} (\bar{d}_k \gamma_\mu P_R d_i) (\bar{\tau} \gamma^\mu P_L \tau) - \frac{h_1^{i3} h_1^{k3*}}{m_{V_2}^2} (\bar{d}_k \gamma_\mu P_R d_i) (\bar{\nu}_\tau \gamma^\mu P_L \nu_\tau) \\ & - \frac{h_2^{i3} h_2^{k3*}}{m_{V_2}^2} (\bar{d}_k \gamma_\mu P_L d_i) (\bar{\tau} \gamma^\mu P_R \tau) - \frac{(V_Q^* h_2)^{i3} (V_Q^* h_2)^{k3*}}{m_{V_2}^2} (\bar{u}_k \gamma_\mu P_L u_i) (\bar{\tau} \gamma^\mu P_R \tau) \\ & + \frac{2h_1^{i3} h_2^{k3*}}{m_{V_2}^2} (\bar{d}_k P_R d_i) (\bar{\tau} P_L \tau) + \frac{2h_1^{i3} (V_Q^* h_2)^{k3*}}{m_{V_2}^2} (\bar{u}_k P_R d_i) (\bar{\tau} P_L \nu_\tau) \\ & + \frac{2h_2^{i3} h_1^{k3*}}{m_{V_2}^2} (\bar{d}_k P_L d_i) (\bar{\tau} P_R \tau) + \frac{2(V_Q^* h_2)^{i3} h_1^{k3*}}{m_{V_2}^2} (\bar{d}_k P_L u_i) (\bar{\nu}_\tau P_R \tau), \end{aligned} \quad (2.6)$$

where the masses of $V_2^{4/3}$ and $V_2^{1/3}$ are assumed to be degenerate and m_{V_2} denotes the LQ mass. We categorize these four-fermi interactions, based on the induced processes:

- (i) down type neutral current (τ),
- (ii) down type neutral current (ν_τ),
- (iii) up type neutral current,
- (iv) charged current.

Our main goal of this paper is to find the correlation between $R_{D^{(*)}}$, to which $h_1^{33} \times h_2^{23*}$ dominantly contributes, and other observables. We introduce the following hierarchical coupling structure,

$$h_1^{ij} = \begin{pmatrix} 0 & 0 & \epsilon \\ 0 & 0 & \epsilon \\ 0 & 0 & \mathcal{O}(1) \end{pmatrix}, \quad h_2^{ij} = \begin{pmatrix} 0 & 0 & \epsilon \\ 0 & 0 & \mathcal{O}(1) \\ 0 & 0 & \epsilon \end{pmatrix}, \quad (2.7)$$

where ϵ is a small dimensionless parameter.

It is noted that $|h_{1,2}^{i3}|^2$ does not trigger lepton flavor violating processes, although it is important in collider phenomenology as we will see later. At $\mathcal{O}(\epsilon^0)$, we focus on the combination of $h_1^{33}h_2^{23*}$, that contributes to categories (i) and (iv). At $\mathcal{O}(\epsilon^1)$, we have 8 combinations that involve all of four categories. Those 9 combinations and the relevant flavor processes are summarised in table 1. Below, we summarize the parameterizations in the four categories.

(i) Down type neutral current (τ). In the categories (i), the induced operators are

$$\mathcal{H}_{\text{eff}}^\tau = -\frac{\alpha G_F V_{td_i} V_{td_k}^*}{\sqrt{2}\pi} \left(C_S^{ki} O_S^{ki} + C_S^{ki'} O_S^{ki'} + C_P^{ki} O_P^{ki} + C_P^{ki'} O_P^{ki'} \right. \\ \left. + C_9^{ki} O_9^{ki} + C_9^{ki'} O_9^{ki'} + C_{10}^{ki} O_{10}^{ki} + C_{10}^{ki'} O_{10}^{ki'} \right) + \text{h.c.}, \quad (2.8)$$

where

$$O_S^{ki} = (\bar{d}_k P_R d_i)(\bar{\tau}\tau), \quad O_P^{ki} = (\bar{d}_k P_R d_i)(\bar{\tau}\gamma_5\tau), \\ O_9^{ki} = (\bar{d}_k \gamma_\mu P_L d_i)(\bar{\tau}\gamma^\mu\tau), \quad O_{10}^{ki} = (\bar{d}_k \gamma_\mu P_L d_i)(\bar{\tau}\gamma^\mu\gamma_5\tau), \quad (2.9)$$

and the primed operators are obtained by exchanging $P_L \leftrightarrow P_R$. Matching onto the WCs at the LQ scale is

$$C_S^{ki} = -C_P^{ki} = \frac{\sqrt{2}\pi}{\alpha G_F V_{td_i} V_{td_k}^*} \frac{h_1^{i3} h_2^{k3*}}{m_{V_2}^2}, \quad C_S^{ki'} = C_P^{ki'} = \frac{\sqrt{2}\pi}{\alpha G_F V_{td_i} V_{td_k}^*} \frac{h_2^{i3} h_1^{k3*}}{m_{V_2}^2}, \\ C_9^{ki} = C_{10}^{ki} = -\frac{\pi}{\sqrt{2}\alpha G_F V_{td_i} V_{td_k}^*} \frac{h_2^{i3} h_2^{k3*}}{m_{V_2}^2}, \quad C_9^{ki'} = -C_{10}^{ki'} = -\frac{\pi}{\sqrt{2}\alpha G_F V_{td_i} V_{td_k}^*} \frac{h_1^{i3} h_1^{k3*}}{m_{V_2}^2}. \quad (2.10)$$

The relative factor of 2 and sign difference between scalar and vector operators come from the Fierz identities. It is noted that $h_1 \times h_1$ and $h_2 \times h_2$ contribute to vector operators while $h_1 \times h_2$ contributes to scalar operators. As we will see below, we find that the scalar operators are correlated with the charged current, while vector operators are independent of $R_{D^{(*)}}$ because of the structure. We note that there is a chirality enhancement in purely leptonic meson decays with the contribution of the scalar operators.

(ii) Down type neutral current (ν_τ). The induced operators involving ν_τ are

$$\mathcal{H}_{\text{eff}}^\nu = -\frac{\sqrt{2}G_F\alpha}{\pi} V_{td_i} V_{td_k}^* C_R^{ki} \left(\bar{d}_i \gamma^\mu P_R d_k \right) (\bar{\nu}_\tau \gamma_\mu P_L \nu_\tau), \quad (2.11)$$

Coupling product	$V_2^{4/3}$	$V_2^{1/3}$
$h_1^{33} \times h_2^{23*}$	(i) $b \rightarrow s\tau\bar{\tau}$ $B_s \rightarrow \tau\bar{\tau}, B \rightarrow K\tau\bar{\tau}$	(iv) $b \rightarrow c\tau\bar{\nu}_\tau$ $\bar{B} \rightarrow D^{(*)}\tau\bar{\nu}_\tau$ $B_c \rightarrow \tau\nu_\tau, B_u \rightarrow \tau\bar{\nu}_\tau$
$h_1^{33} \times h_2^{13*}$	(i) $b \rightarrow d\tau\bar{\tau}$ $B_d \rightarrow \tau\bar{\tau}, B \rightarrow \pi\tau\bar{\tau}$	(iv) $b \rightarrow u\tau\bar{\nu}_\tau$ $\bar{B} \rightarrow D^{(*)}\tau\bar{\nu}_\tau$ $B_u \rightarrow \tau\nu_\tau, B \rightarrow \pi\tau\bar{\nu}_\tau$
$h_1^{33} \times h_2^{33*}$	(i) $b\bar{b} \rightarrow \tau\bar{\tau}$ $\Upsilon(nS) \rightarrow \tau\bar{\tau}$	(iv) $t \rightarrow b\tau\bar{\nu}_\tau$ —
$h_1^{33} \times h_1^{13*}$	(i) $b \rightarrow d\tau\bar{\tau}$ $B_d \rightarrow \tau\bar{\tau}, B \rightarrow \pi\tau\bar{\tau}$	(ii) $b \rightarrow d\nu_\tau\bar{\nu}_\tau$ $B_d \rightarrow \nu_\tau\bar{\nu}_\tau, B \rightarrow \pi\nu_\tau\bar{\nu}_\tau$
$h_1^{33} \times h_1^{23*}$	(i) $b \rightarrow s\tau\bar{\tau}$ $B_s \rightarrow \tau\bar{\tau}, B \rightarrow K\tau\bar{\tau}$	(ii) $b \rightarrow s\nu_\tau\bar{\nu}_\tau$ $B_s \rightarrow \nu_\tau\bar{\nu}_\tau, B \rightarrow K\nu_\tau\bar{\nu}_\tau$
$h_1^{13} \times h_2^{23*}$	(i) $s \rightarrow d\tau\bar{\tau}$ —	(iv) $c \rightarrow d\tau\bar{\nu}_\tau$ $D_d \rightarrow \tau\bar{\nu}_\tau$
$h_1^{23} \times h_2^{23*}$	(i) $s\bar{s} \rightarrow \tau\bar{\tau}$ —	(iv) $c \rightarrow s\tau\bar{\nu}_\tau$ $D_s \rightarrow \tau\bar{\nu}_\tau$
$h_2^{13} \times h_2^{23*}$	(i) $s \rightarrow d\tau\bar{\tau}$ —	(iii) $c \rightarrow u\tau\bar{\tau}$ —
$h_2^{33} \times h_2^{23*}$	(i) $b \rightarrow s\tau\bar{\tau}$ $B_s \rightarrow \tau\bar{\tau}, B \rightarrow K\tau\bar{\tau}$	(iii) $t \rightarrow c\tau\bar{\tau}$ —

Table 1. Summary table for the relevant flavor processes. In the first row we list up the category and parton level processes and if it exists mesonic in the second row. Processes with the strikethrough are prohibited by the symmetry argument or suppressed by the neutrino mass.

where

$$C_R^{ki} = -\frac{h_1^{i3}h_1^{k3*}}{m_{\sqrt{2}}^2} \frac{\pi}{\sqrt{2}G_F\alpha V_{td_i}V_{td_k}^*}. \quad (2.12)$$

The combination of $h_1^{i3}h_1^{k3*}$ contributes to this category mediated by the $V^{1/3}$ LQ and only vector operators are generated. As a result $M_1 \rightarrow \nu_\tau\bar{\nu}_\tau$ process is suppressed by the neutrino mass and negligible in our setup where M denotes a meson. Therefore we focus on $M_1 \rightarrow M_2\nu_\tau\bar{\nu}_\tau$.

(iii) Up type neutral current. The $h_2 \times h_2$ combination only gives the operators involving τ and up-type quarks. $h_2^{23} \times h_2^{33*}$ and $h_2^{23} \times h_2^{13*}$ induce $t\bar{c}\tau\bar{\tau}$ and $c\bar{u}\tau\bar{\tau}$ vector operators respectively. Regarding the latter interaction, it is difficult to obtain the constraint at the tree level in flavor physics because of the heavy τ mass with respect to the charm

mass. Although $t \rightarrow c\tau\bar{\tau}$ transition is kinematically allowed, the experimental sensitivity to $\text{BR}(t \rightarrow c\tau\bar{\tau})$ is several orders away from the prediction even at the high luminosity (HL) LHC [77]. Therefore we will not discuss the physics induced by those terms below.

(iv) Charged current. Finally, we discuss the charged current involving τ . This interaction contributes to $R_{D^{(*)}}$, and described by the $h_1 \times h_2^*$ combination. The resulting semitauonic scalar operator is

$$\mathcal{H}_{\text{CSR}} = 2\sqrt{2}G_F V_{ki} C_{S_R}^{u_k d_i} (\bar{u}_k P_R d_i) (\bar{\tau} P_L \nu_\tau). \quad (2.13)$$

where the coefficient at the LQ scale, μ_{LQ} , is evaluated as

$$C_{S_R}^{u_k d_i}(\mu_{\text{LQ}}) = - \sum_n \frac{V_Q^{kn}}{\sqrt{2}G_F V_Q^{ki}} \frac{h_1^{i3} h_2^{n3*}}{m_{V_2}^2}. \quad (2.14)$$

We note that $i = 3, k = 2$ corresponds to eq. (1.2). The operator triggers $M_1^- \rightarrow \tau\bar{\nu}_\tau$ and $M_1 \rightarrow M_2\tau\bar{\nu}_\tau$ decays. It is noted that the former again receives the chirality enhancement while the enhancement in the case with $M_1 = D_{(s)}$ is moderate due to $m_{D_{(s)}} \simeq m_\tau$.

3 Phenomenology

In this section, we discuss the phenomenology in this model, assuming the LQ couplings are aligned as in eq. (2.7). In section 3.1, we study the processes where our predictions are not suppressed by ϵ nor CKM λ in the Wolfenstein parameterization [78]. The LHC phenomenology of $\mathcal{O}(\epsilon^0)$ will be given in section 3.2. In section 3.3, we discuss our predictions at $\mathcal{O}(\epsilon^0)$ and $\mathcal{O}(\lambda)$. Finally the $\mathcal{O}(\epsilon)$ phenomenology is given in section 3.4.

3.1 Flavor phenomenology at $\mathcal{O}(\epsilon^0)$ and $\mathcal{O}(\lambda^0)$

First of all, we consider the $b \rightarrow c\tau\bar{\nu}_\tau$ transition corresponding to the category (iv). As discussed above, the semileptonic charged current is generated by the $V_2^{1/3}$ exchange, and it is proportional to $h_1^{33} \times h_2^{23*}$. The induced operator, O_{S_R} , at the LQ scale is evaluated as

$$C_{S_R}^{cb}(\mu_{\text{LQ}}) \simeq 0.18 \left(\frac{2 \text{ TeV}}{m_{V_2}} \right)^2 \left(\frac{h_1^{33} h_2^{23*}}{-0.5} \right). \quad (3.1)$$

We adopt the generic formula given in ref. [23] for the prediction of $R_{D^{(*)}}$. It is known that the imaginary part of C_{S_R} is not helpful to fit the current $R_{D^{(*)}}$ result and hence we assume those couplings to be real. The constraint on WC from high- p_T di- τ search, which we see later in this section, is almost independent of the LQ scale. As a benchmark, we set the LQ mass to $m_{V_2} = 2 \text{ TeV}$. To connect the coefficient to the B meson scale, $\mu_b = 4.2 \text{ GeV}$, we use the renormalization group evolution (RGE) for the dimension-six operators at the QCD next-to-leading and the electroweak leading orders including the top-quark threshold corrections [79–82]. We also include the QCD one-loop matching corrections [83]. As a result we approximately obtain

$$C_{S_R}(\mu_b) = 2.0 C_{S_R}(\mu_{\text{LQ}}). \quad (3.2)$$

Thanks to the LHCb downward (upward) shift of R_{D^*} (R_D), we find that C_{S_R} can explain the anomaly within 2σ .

Next we consider the $B_s \rightarrow \tau\bar{\tau}$ decay, that is predicted by the operators in the category (i). This process is correlated with $R_{D^{(*)}}$ in our model. Using the operators in eq. (2.9) the branching ratio is given as

$$\text{BR}(B_s \rightarrow \tau\bar{\tau}) = \frac{\tau_{B_s} f_{B_s}^2 m_{B_s} G_F^2 m_\tau^2 \alpha^2 |V_{tb} V_{ts}|^2}{16\pi^3} \sqrt{1 - \frac{4m_\tau^2}{m_{B_s}^2}} \times \left\{ \left| (C_{10}^{sb} - C_{10}^{sb'}) + \frac{m_{B_s}^2}{2m_b m_\tau} (C_P^{sb} - C_P^{sb'}) \right|^2 + \frac{m_{B_s}^4}{4m_b^2 m_\tau^2} \left(1 - \frac{4m_\tau^2}{m_{B_s}^2} \right) |C_S^{sb} - C_S^{sb'}|^2 \right\}. \quad (3.3)$$

We note that the coefficient in the SM is estimated as $C_{10}^{sb, \text{SM}}(\mu_b) = -4.3$ [84, 85]. In our model, the scalar semileptonic operator is induced at the tree level, so that the leptonic meson decay has the chirality enhancement. Currently the LHCb with Run 1 data sets the leading upper limit on the decay as [86]

$$\text{BR}(B_s \rightarrow \tau\bar{\tau}) \leq 6.8 \times 10^{-3}. \quad (3.4)$$

The future prospects of the Run 3 and the HL LHC are estimated in ref. [87]: compared to the current bound the sensitivities will be improved by factor of 5 and 13, respectively.

The coupling product also contributes to $B \rightarrow K\tau\bar{\tau}$. The current limit announced by the BaBar collaboration is $\text{BR}(B \rightarrow K\tau\bar{\tau}) \leq 2.25 \times 10^{-3}$ [88] where the SM prediction is 1.4×10^{-7} [69]. The relevant formula is given as [24]

$$\frac{\text{BR}(B \rightarrow K\tau\bar{\tau})}{\text{BR}(B \rightarrow K\tau\bar{\tau})_{\text{SM}}} \simeq \left(1 + 0.17\text{Re}[C_P^{sb}] + 0.14\text{Re}[C_S^{sb}] + 0.06|C_S^{sb}|^2 + 0.06|C_P^{sb}|^2 \right). \quad (3.5)$$

The Belle II experiment with 5ab^{-1} of data [89] will be sensitive to $\text{BR}(B \rightarrow K\tau\bar{\tau}) = 6.5 \times 10^{-5}$.

It is noted that other LQ, S_1 , R_2 and $U(2)$ flavored U_1 do not contribute to a single scalar operator. Given that $R_{D^{(*)}}$ anomaly is explained by O_{S_R} in the V_2 LQ scenario, the coupling product, $h_1^{33} \times h_2^{23}$, should be sizable. As shown in eqs. (2.10), (3.3) and (3.5), the sizable $h_1^{33} \times h_2^{23}$ enhances $\text{BR}(B_s \rightarrow \tau\bar{\tau})$ and $\text{BR}(B \rightarrow K\tau\bar{\tau})$ so that they are key predictions to test this model. This correlation has not been pointed out in the previous works to our best knowledge. In figure 1, we show the correlation among $R_{D^{(*)}}$, $\text{BR}(B_s \rightarrow \tau\bar{\tau})$ and $\text{BR}(B \rightarrow K\tau\bar{\tau})$. The colored horizontal line is the model prediction of $R_{D^{(*)}}$. Blue dashed region is excluded by the current $B_s \rightarrow \tau\bar{\tau}$ constraint. Blue and red solid lines are expected to be probed at the Run 3 and the HL LHC. The green would not be uncovered. We see that the Run 3 and the HL LHC data will be the interesting probe of the 2σ region. For the SM prediction that is depicted by a star symbol, we adopt the latest HFLAV2023 average of $R_D = 0.298$ and $R_{D^*} = 0.254$.⁸ We also see the current constraint of $B \rightarrow K\tau\bar{\tau}$

⁸It is noted if we rely on the Lattice predictions of $R_D = 0.299$ and $R_{D^*} = 0.265$ [7, 25, 90], where R_{D^*} is shifted by 0.01, the model prediction goes through the 1σ region. On the other hand, if we adopt $R_D = 0.290$ and $R_{D^*} = 0.248$ [2] where the form factor is also fitted also with full angular data from the Belle [91, 92], the V_2 prediction contour goes through the 2σ region.

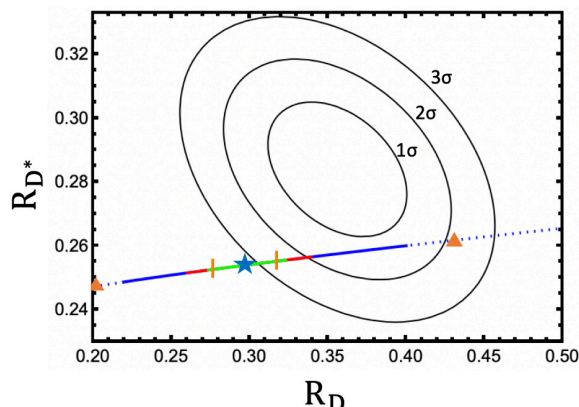


Figure 1. Correlation among $R_{D^{(*)}}$, $\text{BR}(B_s \rightarrow \tau\bar{\tau})$ and $\text{BR}(B \rightarrow K\tau\bar{\tau})$ is shown. Colored lines are prediction of V_2 LQ model. The star mark corresponds to the SM prediction. Blue dotted lines are excluded by the $B_s \rightarrow \tau\bar{\tau}$ measurement. Blue solid lines (red solid lines) will be probed with $B_s \rightarrow \tau\bar{\tau}$ at Run 3 (HL LHC). The current constraint and near future prospect of the $B \rightarrow K\tau\bar{\tau}$ measurement are shown in orange. Region between triangles are currently allowed and the gap between the triangle and vertical bar will be probed with Belle II early data of 5 ab^{-1} . 1, 2, 3 σ contours for $R_{D^{(*)}}$ are shown in black.

in figure 1. The region between the two orange triangles satisfies the $B \rightarrow K\tau\bar{\tau}$ constraint. It is found that the constraint is weaker than that from $B_s \rightarrow \tau\bar{\tau}$. The Belle II with 5 ab^{-1} of data will probe the region between triangle and vertical orange line. Therefore $B \rightarrow K\tau\bar{\tau}$ will probe the interesting parameter region in near future.

The same semitauonic scalar operator which again corresponds to the category (iv) largely enhances $B_c \rightarrow \tau\bar{\nu}_\tau$ branching ratio. Although R_{D^*} is not largely deviated from the SM prediction, R_{D^*} and $\text{BR}(B_c \rightarrow \tau\bar{\nu}_\tau)$ has a correlation:

$$\text{BR}(B_c \rightarrow \tau\bar{\nu}_\tau) = \text{BR}(B_c \rightarrow \tau\bar{\nu}_\tau)_{\text{SM}} \left| 1 + \frac{m_{B_c}^2}{m_b + m_c} C_{SR}^{cb} \right|^2, \quad (3.6)$$

where $\text{BR}(B_c \rightarrow \tau\bar{\nu}_\tau)_{\text{SM}} \simeq 0.022$ [23]. In the numerical evaluation, $m_b = 4.18 \text{ GeV}$ and $m_c(m_b) = 0.92 \text{ GeV}$ are used. The upper bound of the coupling product from $\text{BR}(B_s \rightarrow \tau\bar{\tau})$ indirectly set that of $\text{BR}(B_c \rightarrow \tau\bar{\nu}_\tau)$ as

$$\text{BR}(B_c \rightarrow \tau\bar{\nu}_\tau) \leq 8\%. \quad (3.7)$$

This satisfies the current conservative limit, $\text{BR}(B_c \rightarrow \tau\bar{\nu}_\tau) \lesssim 60\%$ [31] while future lepton colliders can test the SM prediction at $\mathcal{O}(1)\%$ accuracy [93–95].

3.2 LHC phenomenology at $\mathcal{O}(\epsilon^0)$

Since the LQ has a TeV-scale mass, the direct search at the LHC is a powerful tool to probe the scenario. V_2 is pair-produced by the strong interaction at the hadron collider. Depending on the subsequent decays, we can set the lower limit on the V_2 mass. The sizable h_1^{33} and h_1^{23} respectively lead the following decays: $V_2^{1/3} \rightarrow b\nu_\tau$, $V_2^{4/3} \rightarrow b\tau$, $V_2^{1/3} \rightarrow c\tau$ and

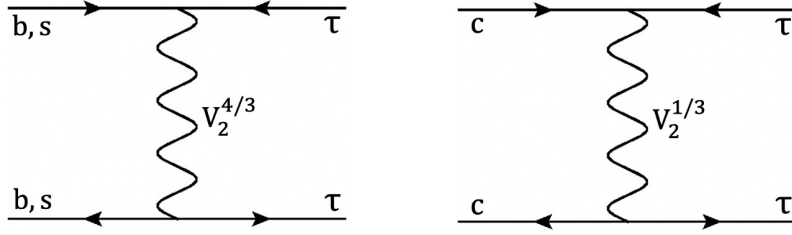


Figure 2. The contributing Feynman diagrams for $\tau\bar{\tau}$ final state at the LHC. Both $V_2^{4/3}$ (left) and $V_2^{1/3}$ (right) contribute to the high- p_T signature.

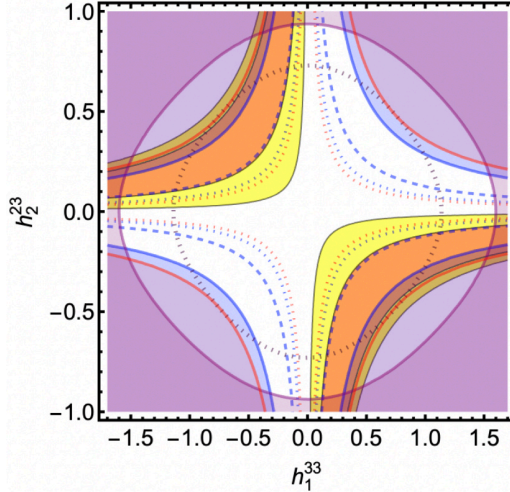


Figure 3. $R_{D^{(*)}}$ favored region, constraint from $B_s \rightarrow \tau\bar{\tau}$ and di- τ searches at the LHC are shown on h_1^{33} vs. h_2^{23} . We fixed the V_2 LQ mass to 2 TeV. Orange and yellow regions correspond to $\chi^2 \leq 6.18$ (orange) and $\chi^2 \leq 11.83$ (yellow) for the $R_{D^{(*)}}$ data, respectively. Blue shaded region is excluded by the current $B_s \rightarrow \tau\bar{\tau}$ and dashed and dotted contours denote the future prospect for the Run 3 and HL LHC. Similarly red shaded region is excluded by the current $B \rightarrow K\tau\bar{\tau}$ and dotted contours denote the future prospect for the early Belle II of 5 ab^{-1} . Purple shaded region is also excluded by the high- p_T di- τ searches at the LHC. The future projection is shown in the dotted contour.

$V_2^{4/3} \rightarrow s\tau$. The LQ mass has been directly constrained as $M_{LQ} \gtrsim 1.5 \text{ TeV}$ from the searches for the LQ pair-production [96–99]. Furthermore, it is known that the high- p_T region is important to prove the new physics scenario that explains the $R_{D^{(*)}}$ anomaly [41, 100–109]. In our model, $V_2^{4/3}$ also contributes to the di- τ final state, so that the searches for di- τ with high- p_T [110, 111] provide the better probe than the $\tau\nu$ searches studied in refs. [112–115]. See figure 2 for the contributing Feynman diagrams. We study the bounds from the di- τ and mono- τ signatures at the LHC. We constructed the χ^2 function based on the high- p_T bins of refs. [111, 115] using HighPT [70] as a function of couplings and draw the upper bounds on the LQ couplings where $m_{V_2} = 2 \text{ TeV}$ is fixed. It is found that the mediator mass dependence in di- τ final state is mild in terms of the four-fermi interactions. We note that the study of the interplay between $R_{D^{(*)}}$ and collider physics in this model has not been done before.

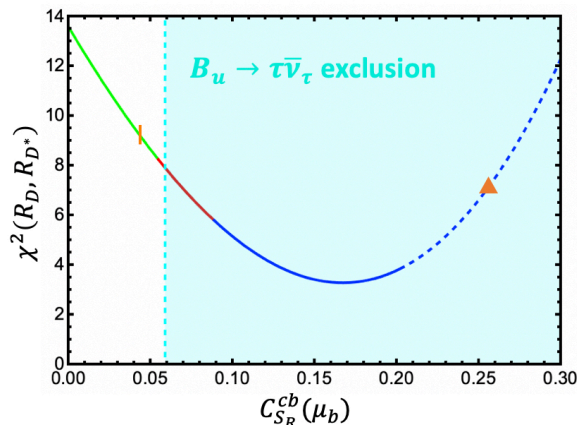


Figure 4. $\chi^2(R_D, R_{D^*})$ and R_{B_u} as a function of $C_{S_R}^{cb}(\mu_b)$. $C_{S_R}^{cb} = (0.88 + 2.45i)C_{S_R}^{ub}$ is fixed for R_{B_u} . The meaning of the color is the same as in figure 1. The blue dashed line is excluded by the current bound from $B_s \rightarrow \tau\bar{\tau}$. The shaded cyan region is excluded by the current $B_u \rightarrow \tau\bar{\nu}_\tau$.

In figure 3 solid and dotted purple lines show the current bound and future prospect of the LHC experiment, respectively. The shaded region is excluded. We overlaid the current constraint and future sensitivity of the Run 3 and the HL LHC from $B_s \rightarrow \tau\bar{\tau}$ with solid, dashed and dotted blue lines. We also show the current constraint and early Belle II sensitivity of $B \rightarrow K\tau\bar{\tau}$ with solid and dotted red lines. The regions favored by $R_{D^{(*)}}$ are shown in orange and yellow: $\chi^2 \leq 6.18$ (orange) and $\chi^2 \leq 11.83$ (yellow). We see that the Run 3 $B_s \rightarrow \tau\bar{\tau}$, early Belle II $B \rightarrow K\tau\bar{\tau}$ and high- p_T tail at the HL LHC will test the whole orange region and hence probe the remaining interesting parameter region.

We briefly summarize the difference in the prediction of the other LQ scenarios:

- $\text{BR}(B_s \rightarrow \tau\bar{\tau})$ and $\text{BR}(B \rightarrow K\tau\bar{\tau})$ are largely enhanced, while, for instance, it is not in the S_1 LQ case. Although R_2 and $U(2)$ flavored U_1 LQ enhance $\text{BR}(B_s \rightarrow \tau\bar{\tau})$ and $\text{BR}(B \rightarrow K\tau\bar{\tau})$, the former (latter) has C_T (C_{V_L}) contribution in $R_{D^{(*)}}$ too. Therefore the degree of the enhancement is milder for the other LQs. Since the coupling strength to explain the deviation is larger than the U_1 LQ model, we can test this scenario with smaller amount of the data.
- Furthermore, as is shown in ref. [68], polarization observables in $\bar{B} \rightarrow D^{(*)}\tau\bar{\nu}_\tau$ are helpful to distinguish those scenarios. Especially τ polarization will be a key observable.
- The larger signal rate in di- τ is predicted at the LHC, compared to the U_1 LQ. This is because that the larger couplings are necessary to explain $R_{D^{(*)}}$, and both $V_2^{4/3}$ and $V_2^{1/3}$ contribute to the processes. Therefore the LHC data in high- p_T di- τ channel will be very important to probe the model.

3.3 Flavor phenomenology at $\mathcal{O}(\epsilon^0)$ and $\mathcal{O}(\lambda^1)$

The leptonic B meson decay, $B_u \rightarrow \tau \bar{\nu}_\tau$, also constrain our model.⁹ This decay is enhanced by the scalar operator, although it is suppressed by an off-diagonal CKM element. Similar to the B_c decay, we can derive the numerical formula as

$$\text{BR}(B_u \rightarrow \tau \bar{\nu}_\tau) = \text{BR}(B_u \rightarrow \tau \bar{\nu}_\tau)_{\text{SM}} \left| 1 + 3.75 C_{S_R}^{ub} \right|^2. \quad (3.8)$$

The SM prediction is estimated as $\text{BR}(B_u \rightarrow \tau \bar{\nu}_\tau)_{\text{SM}} \simeq 0.95 \times 10^{-4}$ with $|V_{ub}| = 0.409 \times 10^{-2}$.¹⁰ $C_{S_R}^{ub}$ at the LQ scale is estimated as

$$C_{S_R}^{ub}(\mu_{\text{LQ}}) \simeq (0.15 + 0.44i) \left(\frac{2 \text{ TeV}}{m_{V_2}} \right)^2 \begin{pmatrix} h_1^{33} h_2^{23*} \\ -0.5 \end{pmatrix}. \quad (3.9)$$

It is noted that the coefficient in eq. (3.9) is bigger than that of eq. (3.1) because of the factor of $(V_{cb}V_{us})/V_{ub} = 0.84 + 2.46i$.

The current experimental world average is $\text{BR}(B_u \rightarrow \tau \bar{\nu}_\tau) = (1.09 \pm 0.24) \times 10^{-4}$ [116]. There is a notorious discrepancy between the inclusive and the exclusive determinations: $|V_{ub}|_{\text{inc}} = 4.25 (1 \pm 0.07) \times 10^{-3}$ and $|V_{ub}|_{\text{exc}} = 3.70 (1 \pm 0.04) \times 10^{-3}$. Therefore, we assign 14% uncertainty to the SM amplitude. On the other hand, the experimental result has 22% uncertainty. Combining those uncertainties at 2σ , we allow 70% uncertainty and set the following criteria:

$$R_{B_u} = \frac{\text{BR}(B_u \rightarrow \tau \bar{\nu}_\tau)}{\text{BR}(B_u \rightarrow \tau \bar{\nu}_\tau)_{\text{SM}}} \leq 1.7. \quad (3.10)$$

It is noted that the following observables,

$$R_{pl} = \frac{\text{BR}(B_u \rightarrow \tau \bar{\nu}_\tau)}{\text{BR}(B_u \rightarrow \mu \bar{\nu}_\mu)}, \quad R_{ps} = \frac{\Gamma(B_u \rightarrow \tau \bar{\nu}_\tau)}{\Gamma(B_d \rightarrow \pi^+ l \bar{\nu}_l)} \quad (3.11)$$

are free from V_{ub} and useful to test the NP [117]. The corresponding SM predictions are $R_{pl}^{\text{SM}} = 222$ and $R_{ps}^{\text{SM}} = 0.54 \pm 0.04$.¹¹ The current experimental constraint is given as $R_{ps}^{\text{exp}} = 0.73 \pm 0.14$ while R_{ps} is not measured due to the large uncertainty in $\text{BR}(B_u \rightarrow \mu \bar{\nu}_\mu)$ [89]. At 2σ level, this leads to $R_{B_u} \lesssim 2$, that is weaker than the constraint in eq. (3.10).

The Belle II with 50 ab^{-1} of the data will measure R_{pl} and R_{ps} at 12% and 7% at 1σ . Even if we adopt the current theoretical uncertainty for R_{ps} to be conservative, we obtain the similar uncertainty. It is noted that in our model, the modification of the denominator mode is negligible and hence the uncertainty of the ratio corresponds to the sensitivity to $\text{BR}(B_u \rightarrow \tau \bar{\nu}_\tau)$.

In figure 4 we show χ^2 for R_D and R_{D^*} . The meaning of color and style of lines are the same as in figure 1. The current conservative exclusion of eq. (3.10) is shown in cyan region. It is seen that currently $B_u \rightarrow \tau \bar{\nu}_\tau$ is more sensitive to $h_1^{33} \times h_2^{23*}$ than $B_s \rightarrow \tau \bar{\nu}_\tau$.

⁹This constraint has not been pointed out in the previous work [54].

¹⁰This is the average of inclusive and exclusive V_{ub} .

¹¹The large part of the uncertainty comes from $B \rightarrow \pi$ transition form factor [117].

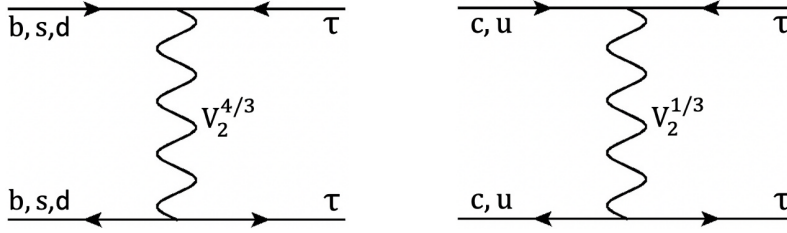


Figure 5. The contributing Feynman diagrams for $\tau\bar{\tau}$ final state at the LHC in the presence of non-vanishing h_2^{13} , h_2^{23} , h_1^{33} . In addition the one in figure 2, h_2^{13} induces $d\bar{d} \rightarrow \tau\bar{\tau}$ and $u\bar{u} \rightarrow \tau\bar{\tau}$ processes mediated by $V_2^{4/3}$ and $V_2^{1/3}$.

and $B \rightarrow K\tau\bar{\tau}$. This figure clearly shows that the interesting parameter space is already excluded by the current result of R_{B_u} . This bound, however, can be relaxed by sizable other LQ couplings, as discussed in section 3.4.

3.4 $\mathcal{O}(\epsilon)$ phenomenology

In this section, we investigate $\mathcal{O}(\epsilon)$ contributions and derive the upper limit on the coupling products. As is summarized in table 1, there are several processes to be discussed.

First, we consider h_2^{13} contribution. If h_2^{13} is sizable, the coupling contributes to $B_u \rightarrow \tau\bar{\nu}_\tau$. The contribution to C_{SR}^{ub} is expressed as

$$C_{SR}^{ub}(\mu_{\text{LQ}}) = -\frac{1}{\sqrt{2}G_F V_Q^{cb}} \frac{h_1^{33} (V_Q^{11} h_2^{13*} + V_Q^{12} h_2^{23*})}{m_{V_2}^2}. \quad (3.12)$$

If $h_2^{13*} = -V_Q^{12}/V_Q^{11} h_2^{23*} \simeq -0.23 h_2^{23*}$ is satisfied, C_{SR}^{ub} can be enough small to evade the bound from R_{B_u} mentioned above and rescue the solution.

The tree-level exchange of the LQ induces $B_d \rightarrow \tau\bar{\tau}$, and the amplitude is proportional to $h_1^{33} \times h_2^{13*}$. The $B_d \rightarrow \tau\bar{\tau}$ contribution has a chirality enhancement, so that it gives a strong bound. The current experimental upper limit is $\text{BR}(B_d \rightarrow \tau\bar{\tau}) \leq 2.1 \times 10^{-3}$ at 95% CL. [116] and the Belle II experiment is expected to probe $\text{BR}(B_d \rightarrow \tau\bar{\tau}) \simeq 9.6 \times 10^{-5}$ [89]. We note that $B \rightarrow \pi\tau\bar{\nu}_\tau$ is also induced by the same operator but the bound is weak. In addition, $c\bar{u}\tau\bar{\tau}$ and $s\bar{d}\tau\bar{\tau}$ four-fermi interactions are lead by the tree-level LQ exchange and the coefficients are proportional to $h_2^{13} \times h_2^{23*}$. The couplings, however, do not predict rare meson decays since τ mass does not allow decay processes such as $D \rightarrow \tau\bar{\tau}$ nor $D \rightarrow \pi\tau\bar{\tau}$.¹²

Our numerical analysis shows that the $R_{D^{(*)}}$ anomaly can be explained within 2σ , if $h_1^{33} \times h_2^{23*}$ is fixed within $[-0.29, -0.12]$ when $m_{V_2} = 2 \text{ TeV}$. Let us define the ratio of h_2^{13} to h_2^{23} as $h_2^{13} = -\lambda^{uc} h_2^{23}$. This ratio is limited by $B_u \rightarrow \tau\bar{\nu}_\tau$. When $h_1^{33} \times h_2^{23*}$ is around -0.29 (-0.12), λ^{uc} should satisfy $0.07 \lesssim \lambda^{uc} \lesssim 0.57$ ($0.16 \lesssim \lambda^{uc} \lesssim 0.37$) to evade the $B_u \rightarrow \tau\bar{\nu}_\tau$ bound. In the future, the Belle II experiment could improve the bound and this

¹²The one-loop contribution contributes to $K - \bar{K}$ mixing, but the bound is not so tight since there is no chirality enhancement as long as $h_1^{13} \times h_1^{23*}$ is small. Furthermore for the correct loop calculation, we need the UV model and hence we limit ourselves to focus on the tree level phenomenology in this paper. We will come back to this point in section 4.

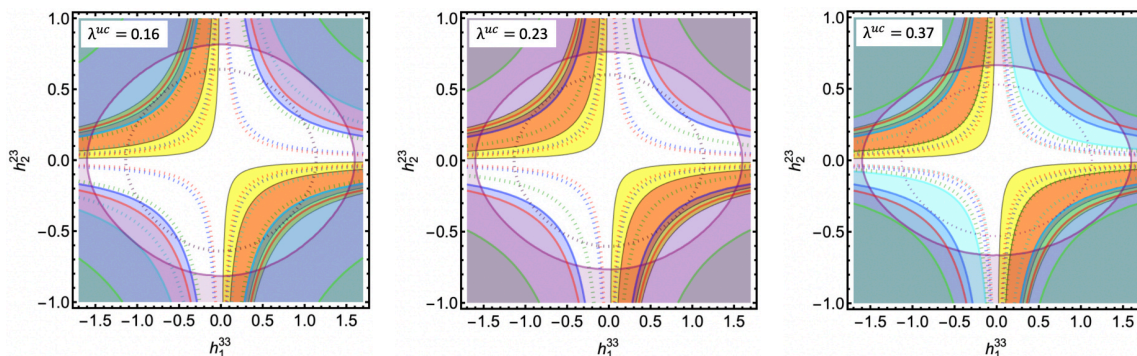


Figure 6. The color code is the same as in figure 3. Additionally, cyan and light green regions show the exclusion from $B_u \rightarrow \tau \bar{\nu}_\tau$ and $B_d \rightarrow \tau \bar{\tau}$. Their future prospects are also shown in dotted lines.

range could be reduced to be $0.20 \lesssim \lambda^{uc} \lesssim 0.34$ ($0.16 \lesssim \lambda^{uc} \lesssim 0.48$) if the experimental central value does not change [89].

h_2^{13} also contributes to $C_{S_R}^{cb}$ as

$$C_{S_R}^{cb}(\mu_{LQ}) = -\frac{1}{\sqrt{2}G_F V_Q^{ub}} \frac{h_1^{33} (V_Q^{22} h_2^{23*} + V_Q^{21} h_2^{13*})}{m_{V_2}^2}, \quad (3.13)$$

although it is suppressed by V_Q^{21} . Besides, h_2^{13} also contributes to $d\bar{d} \rightarrow \tau\bar{\tau}$ and $u\bar{u} \rightarrow \tau\bar{\tau}$ processes at the LHC as shown in figure 5. It is noted that h_2^{33} contributes to $C_{S_R}^{cb}$ with the CKM factor, V_Q^{23} . Due to the off-diagonal CKM suppression, both h_1^{33} and h_2^{33} should be sizable to enhance $C_{S_R}^{cb}$. This additional entry does not affect $B_s \rightarrow \tau\bar{\tau}$ and $B \rightarrow K\tau\bar{\tau}$ while contributes to high- p_T observables via $b\bar{b} \rightarrow \tau\bar{\tau}$. As a result, the impact on $R_{D^{(*)}}$ is small compared to that from h_2^{23} .

In figure 6, λ^{uc} is fixed at $\lambda^{uc} = 0.16, 0.23, 0.37$ on the left, middle and right panels, respectively. To see the prediction to motivate future experiments, the correlation among $\chi^2(R_D, R_{D^*})$, R_{B_u} , $\text{BR}(B_s \rightarrow \tau\bar{\tau}) \times 10^3$, $\text{BR}(B \rightarrow K\tau\bar{\tau}) \times 10^4$ and $\text{BR}(B_d \rightarrow \tau\bar{\tau}) \times 10^4$ is shown in figure 7. We see that in addition to $B_s \rightarrow \tau\bar{\tau}$ and $B \rightarrow K\tau\bar{\tau}$, $B_d \rightarrow \tau\bar{\tau}$ plays an important role in the probe when $\lambda^{uc} = 0.23$ and 0.37 . When $\lambda^{uc} = 0.16$ and 0.37 , the future R_{B_u} measurement can probe the best fit point of the model. On the other hand, R_{B_u} is suppressed when $\lambda^{uc} = 0.23$, because of the cancellation mentioned above. We note that real $h_1^{33} \times h_2^{23*}$ is favored by the current $R_{D^{(*)}}$ measurement. If $h_1^{33} \times h_2^{13*}$ is also real, we obtain the prediction for R_{B_u} as $0.89 \lesssim R_{B_u}$, due to the relative phase between the SM amplitude and V_2 amplitude. We note that the collider reach is also mildly extended with the inclusion of non-vanishing h_2^{13} .

We investigate the phenomenological impact of other couplings. As shown in table 1, sizable h_1^{13} and h_2^{23*} couplings predict the contributions to the leptonic D meson decays: $D_{d,s} \rightarrow \tau \bar{\nu}_\tau$, corresponding to the category (iv). To study the bounds numerically, we define

$$R_{D_d} = \frac{\text{BR}(D_d \rightarrow \tau \bar{\nu}_\tau)}{\text{BR}(D_d \rightarrow \tau \bar{\nu}_\tau)_{\text{SM}}} = \left| 1 + 1.47 C_{S_R}^{cd} \right|^2, \quad (3.14)$$

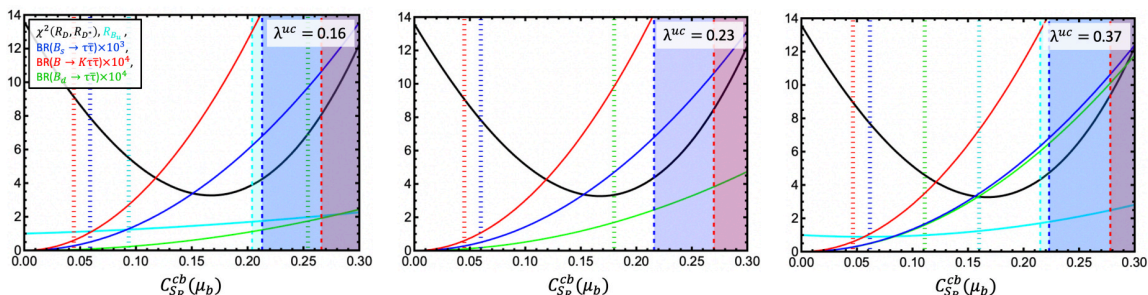


Figure 7. The correlation of prediction is shown as a function of $C_{S_R}^{cb}(\mu_b)$ fixing $\lambda^{uc} = 0.16$ (left), 0.23 (middle) and 0.37 (right). $\chi^2(R_D, R_{D^*}), R_{B_u}$ is shown in black and cyan. $\text{BR}(B_s \rightarrow \tau\bar{\tau}) \times 10^3$, $\text{BR}(B \rightarrow K\tau\bar{\tau}) \times 10^4$ and $\text{BR}(B_d \rightarrow \tau\bar{\tau}) \times 10^4$ are shown in blue, red and light green lines. The current exclusion is shown in each color while the future prospect is shown in dotted line.

and

$$R_{D_s} = \frac{\text{BR}(D_s \rightarrow \tau\bar{\nu}_\tau)}{\text{BR}(D_s \rightarrow \tau\bar{\nu}_\tau)_{\text{SM}}} = \left| 1 + 1.72C_{S_R}^{cs} \right|^2, \quad (3.15)$$

where $\text{BR}(D_d \rightarrow \tau\bar{\nu}_\tau)_{\text{SM}} \simeq 1.06 \times 10^{-3}$ and $\text{BR}(D_s \rightarrow \tau\bar{\nu}_\tau)_{\text{SM}} \simeq 5.2 \times 10^{-2}$. The numerical estimations are obtained using $|V_{cd}| = 0.221 \pm 0.004$, $|V_{cs}| = 0.975 \pm 0.006$ [116], $f_D = 0.2120 \pm 0.007 \text{ GeV}$ and $f_{D_s} = 0.2499 \pm 0.005 \text{ GeV}$ [3]. Compared to B_u and B_c decays, those enhancement factors are small where $m_c(m_c) = 1.27 \text{ GeV}$ is used. $C_{S_R}^{cs}$ is given by eq. (2.14), and proportional to $h_1^{13} \times h_2^{23*}$.

There are experimental results of those decays: $\text{BR}(D_d \rightarrow \tau\bar{\nu}_\tau)_{\text{exp}} = (1.20 \pm 0.27) \times 10^{-3}$ and $\text{BR}(D_s \rightarrow \tau\bar{\nu}_\tau)_{\text{exp}} = 5.32 \pm 0.11 \%$ [116]. Furthermore, the BES III experiment reports the recent result [118]: $\text{BR}(D_d \rightarrow \tau\bar{\nu}_\tau)/\text{BR}(D_d \rightarrow \mu\bar{\nu}_\mu) = 3.21(1 \pm 0.24)$ where the SM prediction is 2.67. When we require $C_{S_R}^{cs}$ to satisfy $0.5 \leq \text{BR}(D_d \rightarrow \tau\bar{\nu}_\tau)/\text{BR}(D_d \rightarrow \tau\nu_\tau)_{\text{SM}} \leq 1.5$ and $0.95 \leq \text{BR}(D_s \rightarrow \tau\bar{\nu}_\tau)/\text{BR}(D_s \rightarrow \tau\nu_\tau)_{\text{SM}} \leq 1.05$, we obtain

$$-1.3 \lesssim h_1^{13} \times h_2^{23*} \lesssim 1.1, \quad -0.46 \lesssim h_1^{23} \times h_2^{23*} \lesssim 0.48. \quad (3.16)$$

Sizable h_1^{13} and h_1^{23} enhances or suppresses $B \rightarrow K^{(*)}\nu_\tau\bar{\nu}_\tau$ and $B \rightarrow \pi\nu_\tau\bar{\nu}_\tau$ corresponding to the category (ii). The contributions of the LQ exchange are proportional to $h_1^{33} \times h_1^{23*}$ and $h_1^{33} \times h_1^{13*}$, respectively.¹³ The LQ contributions correspond to the vector operators. It is conventional to define the ratio as $\mathcal{R}_{M_1}^\nu = \text{BR}(B \rightarrow M_1\nu\bar{\nu})/\text{BR}(B \rightarrow M_1\nu\bar{\nu})_{\text{SM}}$. The Belle collaboration has provided an upper bound as $\mathcal{R}_{K^*}^\nu \leq 2.7$ and $\mathcal{R}_K^\nu \leq 3.9$ at the 90% CL. [119]. The Belle II could measure the SM prediction with 10% accuracy [89]. Following ref. [120], those ratios are expressed as

$$\mathcal{R}_{K^*}^\nu = (1 - 2\eta_\nu)\epsilon_\nu^2, \quad \mathcal{R}_K^\nu = (1 + \kappa_\nu\eta_{\nu\nu})\epsilon_\nu^2, \quad (3.17)$$

where

$$\epsilon_\nu = \frac{\sqrt{|C_L^{\text{SM}}|^2 + |C_R^{\text{bs}}|^2}}{|C_L^{\text{SM}}|}, \quad \eta_\nu = \frac{-\text{Re}(C_L^{\text{SM}}C_R^{\text{bs}*})}{|C_L^{\text{SM}}|^2 + |C_R^{\text{bs}}|^2}, \quad (3.18)$$

¹³We find that $B \rightarrow \pi\tau\bar{\tau}$ is less constraining compared to $B_s \rightarrow \tau\tau$ [69]. In fact, the bound on $B \rightarrow \pi\tau\bar{\tau}$ is not available in PDG [116].

and $\kappa_\nu = 1.34 \pm 0.04$ with $C_L^{\text{SM}} \simeq -6.35$. Similarly the current upper limit on $B \rightarrow \pi\nu\bar{\nu}$ is approximately given as

$$\mathcal{R}_\pi^\nu \lesssim 6000, \quad (3.19)$$

using the SM prediction of $\text{BR}(B^0 \rightarrow \pi^0\nu\bar{\nu})_{\text{SM}} = (5.4 \pm 0.6) \times 10^{-8}$ [121] and the current experimental constraint, $\text{BR}(B^0 \rightarrow \pi^0\nu\bar{\nu}) \leq 0.9 \times 10^{-5}$ at 90% C.L. [116]. Those upper bounds lead to

$$-0.27 \lesssim h_1^{33} \times h_1^{23*} \lesssim 0.21, \quad -3.7 \lesssim h_1^{33} \times h_1^{13*} \lesssim 3.7. \quad (3.20)$$

We find that the constraints from $B_d \rightarrow \tau\bar{\tau}$ and $B_s \rightarrow \tau\bar{\tau}$ lead to $-1.8 \lesssim h_1^{33} \times h_1^{13*} \lesssim 1.8$ and $-2.65 \lesssim h_2^{33} \times h_2^{23*} \lesssim 2.65$, respectively.

The bounds on the coupling products are summarized in table 2. We see that there are several combinations that are allowed to be $\mathcal{O}(1)$. Especially, it is not easy to constrain $h_1^{33} \times h_1^{13*}$, $h_2^{13} \times h_2^{23*}$ and $h_2^{33} \times h_2^{23*}$ using tree level processes. In other words h_2^{33} and h_1^{13} can be of order 1 while h_2^{13} and h_1^{23} should be somewhat smaller. We may obtain strong bounds considering one-loop contributions. Such a higher-order contribution usually involves extra fields in the loop, so a concrete setup needs to be taken into account.

Finally, we discuss the bound from the collider experiments. When h_1^{13} and h_2^{13} , that correspond to the couplings involving light quarks, are sizable, our model can be constrained by the collider searches further. We study the bounds from the di- τ and mono- τ signatures at the LHC by repeating the procedure explained in section 3.2. As shown in figures 5, the t-channel diagrams given by the exchange of the LQ induce di- τ signatures. Analogously we have t-channel diagrams contributing to mono- τ signature. Based on Run 2 full data we derive the upper limit on the coupling at 2σ as follows:

$$\begin{aligned} |h_1^{13}| &\leq 0.55, & |h_2^{13}| &\leq 0.51, & |h_1^{23}| &\leq 1.01, & |h_2^{23}| &\leq 0.93, \\ |h_1^{33}| &\leq 1.60, & |h_2^{33}| &\leq 1.66. \end{aligned} \quad (3.21)$$

In this analysis, we turn on the only one coupling assuming that the other couplings are vanishing. We see that $h_{1,2}^{13}$ coupling can be at most 0.5. Also we can set the upper limit on the least constrained h_2^{33} with LHC data. This collider constraint is complementary to the flavor constraint. We introduce a Yukawa texture, that respects the τ number, as an illustration:¹⁴

$$h_1^{ij} \simeq \begin{pmatrix} 0 & 0 & \mathcal{O}(10^{-3}) \\ 0 & 0 & \mathcal{O}(10^{-2}) \\ 0 & 0 & \mathcal{O}(1) \end{pmatrix}, \quad h_2^{ij} \simeq \begin{pmatrix} 0 & 0 & -0.23h_2^{23} \\ 0 & 0 & \mathcal{O}(1) \\ 0 & 0 & \mathcal{O}(0.1) \end{pmatrix}. \quad (3.22)$$

4 Summary and discussion

In this paper we studied phenomenology of the model with isodoublet vector LQ, V_2 . In light of the recent result of $R_{D^{(*)}}$, the LQ becomes very interesting. $\chi^2(R_D, R_{D^*})$ can be

¹⁴We note that h_2^{33} could be sizable as shown in table 2.

Coupling product	bound
$h_1^{33} \times h_2^{23*}$	$[-0.28, -0.12]$
$h_1^{33} \times h_2^{13*}$	$[-0.02, 0.04]$
$h_1^{33} \times h_2^{33*}$	—
$h_1^{33} \times h_1^{13*}$	$[-1.8, 1.8]$
$h_1^{33} \times h_1^{23*}$	$[-0.27, 0.21]$
$h_1^{13} \times h_2^{23*}$	$[-1.3, 1.1]$
$h_1^{23} \times h_2^{23*}$	$[-0.46, 0.48]$
$h_2^{13} \times h_2^{23*}$	—
$h_2^{33} \times h_2^{23*}$	$[-2.65, 2.65]$

Table 2. Summary table for the non-LHC bound on the coupling product assuming $m_{V_2} = 2 \text{ TeV}$.

as small as 3.7 in this model and the minimal coupling scenario predicts that $B_s \rightarrow \tau\bar{\tau}$ and $B \rightarrow K\tau\bar{\tau}$ within the reach of the Run 3 LHCb measurement and early Belle II with 5 ab^{-1} , respectively. In the minimal setup, $B_u \rightarrow \tau\bar{\nu}_\tau$ is deviated from the SM prediction, so that the setup is excluded. This bound can be evaded by introducing another flavor violating coupling to the large contribution to $B_u \rightarrow \tau\bar{\nu}_\tau$. We conclude that there are setups that are consistent with the experimental results related to the flavor physics as well as the high- p_T signals.

We only discussed the tree-level contributions induced by the V_2 exchange. The LQ mass is not large, so it may be necessary to take into account the one-loop corrections involving V_2 . The study, however, would require a complete model since the loop diagrams involve extra fields e.g. extra fermions and scalars in general and the contributions would not be negligible [122]. It would be challenging to construct a complete model with V_2 , since the constraint from the lifetime of proton is very strong and a specific parameter setup is required to explain the $R_{D^{(*)}}$ anomaly. The quantum number of V_2 is the same as X boson in the SU(5) GUT. We could, for instance, consider the model where the SU(5) unification is realized in only one generation: the fields in the other generations are charged under $G'_{\text{SM}} = \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$. The SM gauge symmetry is given by the linear combination of G'_{SM} and the subgroups of SU(5). If $\text{SU}(5) \times G'_{\text{SM}}$ breaks down to the SM gauge symmetry at the low scale, V_2 would arise as a massive gauge boson with a light mass. In this setup, V_2 could approximately have a quantum number like the τ number. The couplings of V_2 with light fermions may be suppressed and may be able to suppress the dangerous couplings that cause proton decay, at the tree level. The fields and couplings to realize the realistic fermion mass matrices, however, may cause additional contributions to flavor physics at the tree and the one-loop levels, as discussed in ref. [122]. The constraints from searches for the particles predicted by the underlying theory may disturb the $R_{D^{(*)}}$ anomaly explanation [123]. We need further detailed study [124].

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