Received 19 July 2023; revised 7 November 2023; accepted 27 November 2023.
Date of publication 7 December 2023; date of current version 19 December 2023.

Digital Object Identifier 10.1109/OAJPE.2023.3340299

Modelling of 3-Phase p-q Theory-Based Dynamic Load for Real-Time Simulation

KARTHIK RAJASHEKARAIAH[1], COSIMO IURLARO[2] (Graduate Student Member, IEEE), SERGIO BRUNO[2] (Member, IEEE), AND GIOVANNI DE CARNE[1] (Senior Member, IEEE)

[1]Institute for Technical Physics, Karlsruhe Institute of Technology, 76344 Karlsruhe, Germany
[2]Department of Electrical and Information Engineering (DEI), Politecnico di Bari, 70125 Bari, Italy
CORRESPONDING AUTHOR: K. RAJASHEKARAIAH (karthik.rajashekaraiah@kit.edu)

This work was supported by the Helmholtz Association under the Program “Energy System Design” and under the Helmholtz Young Investigator Group “Hybrid Networks” under Grant VH-NG-1613. The work of Sergio Bruno was supported by Regione Puglia within the Framework of the Programme REFIN—Research for Innovation under Grant A1C03120.

ABSTRACT This article proposes a new method of modelling dynamic loads based on instantaneous p-q theory, to be employed in large powers system network simulations in a digital real-time environment. Due to the use of computationally heavy blocks such as phase-locked-loop (*PLL*), mean calculation,and coordinate transformation blocks (e.g., $abc - dq0$), real-time simulation of large networks with dynamic loads can be challenging. In order to decrease the computational burden associated to the dynamic load modelling, a p-q theory-based approach for load modelling is proposed in this paper. This approach is based on the well-known p-q instantaneous theory developed for power electronics converters, and it consists only of linear controllers and of a minimal usage of control loops, reducing the required computational power. This improves real-time performance and allows larger scale simulations. The introduced p-q theory-based load (PQL) model has been tested on standard networks implemented in a digital real time simulator, such as the SimBench semi-urban medium voltage network and the 118-bus Distribution System, showing significant improvement in terms of computational capability with respect to standard load models (e.g., MATLAB/Simulink dynamic load).

INDEX TERMS Load modelling, dynamic load, real time simulation, instantaneous p-q theory.

I. INTRODUCTION

LOAD modelling plays a vital role in understanding the power system dynamics and developing effective control actions [1]. Several studies, i.e. [1], [2], [3], and [4], show the critical aspects of load representation in power system, where over-simplified models can lead to inaccurate predictions of the system's response during a disturbance. As a consequence, the stability of the system may be over-estimated, causing unexpected events like cascading outages and blackouts.

Developing proper load models is not a simple task, as some modelling challenges are met during power system studies: 1) include loads with different nature (e.g., static or dynamic); 2) update the load composition and structure in continuous and accurate way; 3) validate the load model accuracy using single-event conditions (e.g., during natural disturbances like faults). These challenges can be solved by increasing the load complexity, and adding more details' layers. If, from one side, this solution brings higher simulation accuracy, on the other side, it requires higher computational effort, bringing the simulation time out of the boundaries that are suitable for digital real-time simulations [2].

However, creating tailored models for each single load is not realistic. Loads vary in nature, power level and external condition dependency (e.g., voltage or frequency, temperature, human behaviour), making it challenging to develop a general model fitting all. So, due to the lack of information about the load characteristics, the majority of the power system simulation software uses measurement-based approaches for load modelling [5], [6], where the loads' dynamics are included using induction motor models or exponential dynamic model [7]. Such models have been tested and accepted worldwide for performing transient studies of power systems. Furthermore, dynamic load models are essential to digital real time simulations and Power Hardware-in-the-Loop (PHIL) testing, reproducing the behaviour of complex grids [8] and emulating dynamic distributed active resources (prosumers, storage, dynamic loads, etc.) [9].

Several simulation software are available for power system studies. MATLAB is among the commonly used ones, having applications from offline (e.g., SimScape) to real-time simulations (e.g., OPAL-RT uses it as implementation platform). The MATLAB library has implemented an exponential dynamic load model that can track a time-varying power profile and represent the dynamic behaviour of a power component.

This equivalent model includes a Phase-Locked-Loop (PLL) for the current synchronization, a variable mean value computation for the voltage dependency, and synchronous frame transformation blocks ($dq0 - abc$). As a main drawback, the aforementioned components require either continued access to the local computer memory (e.g., the variable mean needs an array of data for the mean calculation) or have iterative non-linear loops (e.g., PLL and $dq0 - abc$ transformation block), impacting heavily on the load model required simulation time. If this load is implemented in digital real-time simulations, it may lead to computational overruns, meaning that the simulated grid cannot be solved within the specified time step.

This paper introduces a novel dynamic load model for digital real-time power system simulations based on the mathematical formulation of the instantaneous p-q theory [10]. This work aims to identify and eliminate well-known components in dynamic load models (e.g., Simulink ones), which may require large computational effort (e.g., PLL or mean calculations), and substitute them with lighter algebraic calculations. The instantaneous p-q theory-based load model (PQL) uses algebraic equations to generate the current references, eliminating PLL, mean, and transformation blocks. The goal is to create a dynamic load model with similar accuracy performance to the existing ones (e.g., Simulink-based) while making it computationally lighter and enabling the simulation of large power system grids with the available real-time resources. In this work, the p-q theory-based load model has been validated for steady-state and transient analysis with existing large network benchmarks, such as the SimBench semi-urban medium voltage network [11] and the 118-bus Distribution System [12].

This work's contributions can be summarized as follows:

- Introduce and mathematically explain the p-q theory-based load model and provide details on implementing it in digital real-time simulation.
- Compare the accuracy of the PQL model with existing well-accepted dynamic load models, such as the Simulink ones. This analysis was carried out in steady-state and transient conditions with large network benchmarks (SimBench and 118-bus Distribution Systems). The models of the test benchmark networks have been uploaded in the open-access IEEE DataPort environment and can be accessed [13].
- Compare the required computational time of the PQL model against passive and dynamic models in Simulink.

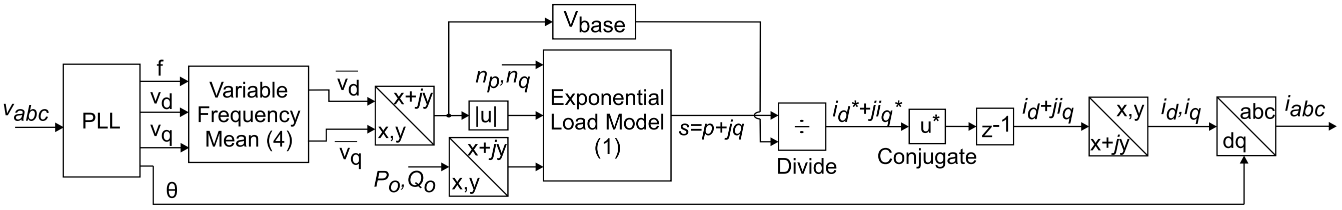
The paper is organized as follows: Section II reviews the current practices on load modelling for power system studies. Section III describes the Simulink dynamic load model and its implementation. Section IV introduces the concept of instantaneous p-q theory and a computational complexity analysis between the two models has been provided. A description of the test feeders is given in Section V. In Section VI, the p-q theory-based dynamic load model and Simulink load model are used to model the test feeders in real-time; results are obtained and compared with different scenarios. Section VII compares the test network using the proposed dynamic load model and Simulink load models in terms of real-time computation performance. Section VIII concludes the paper.

II. CURRENT PRACTICE ON LOAD MODELLING FOR POWER SYSTEM STUDIES

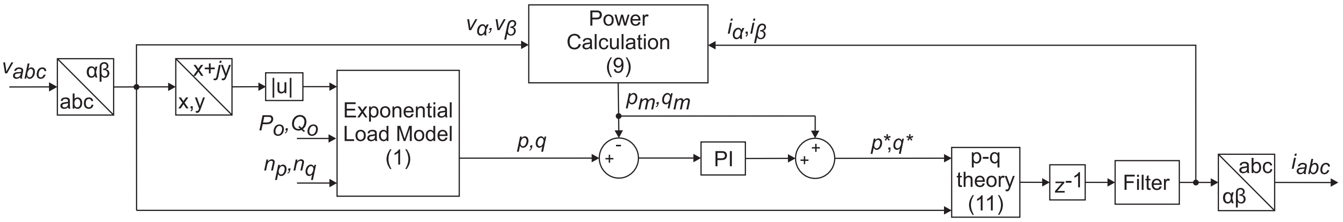
The measurement-based and component-based approaches are the existing methodologies for load modelling. The measurement-based load modelling, also known as the "top-down" approach, typically makes use of the measurements recorded during certain events or disturbances, which can naturally occur in the system, but can also be intentionally induced, in order to derive aggregate load characteristics. Component-based load modelling is instead also known as "physical" or "bottom-up" approach for modelling an individual type of load during its design or performance assessment. A detailed discussion of measurement and component-based modelling approaches have been made in [14].

Based on the above two approaches, the existing load models can generally be divided into static and dynamic load models. These load models provide relevant information on load characteristics like active and reactive power demand and responses to system frequency and voltage variations. The main difference lies in their transient behaviour capability. Static load models represent loads that respond instantaneously to a voltage and/or frequency change. Dynamic loads develop a time-dependent response to voltage and/or frequency variations based on their previous state and interaction with the system during the transition to the next state (e.g., loads generally found in the industrial load sector, or induction motors).

The most commonly used static load models are exponential, linear, and second-order polynomial models [14]. Among the existing dynamic load models, the most widely used ones are the exponential dynamic load model (for modelling predominately residential loads) and dynamic induction motor load models (for modelling consisting of induction motors). Other dynamic loads that can be included are bulk power bus load, Distributed Electric Storage System (DESS),



(a) Simulink dynamic load



(b) Three-phase p-q theory-based load

FIGURE 1. **Schematic of three-phase Simulink and p-q theory-based load.**

distribution load model, and load models of emerging devices.

In reality, although several software platforms, such as MATLAB/Simulink, PSCAD, EMTP-RV, PSSe, DigSILENT PowerFactory, Modelica, etc., can implement these kinds of loads and perform simulations and analysis, there are only few softwares which provide integration to perform real-time simulations. Among all, MATLAB/Simulink is the one that allows integration with the real-time environment such as OPAL-RT digital real-time simulator with limited changes. Matlab/Simulink software implements an exponential dynamic load model in their libraries, which can be used in power system studies and real-time simulations. This dynamic load model is characterized by the presence of several blocks characterized by heavy computational burden, such as Phase-Locked-Loop (PLL), discrete mean, and *abc*-to-*dq*0 transformation. The overall requirement of computing resources needed to solve this model can significantly limit the extent of real-time simulations, in both terms of time resolution and network extension or complexity.

In order to overcome these limitations, in this paper, a novel dynamic load model is proposed and implemented in a real-time simulation environment. The proposed model permits to reduce the computational burden through the elimination of the blocks characterized by heavier computational requirements. The proposed dynamic load model is tested and compared with the existing library load block of Simulink.

III. SIMULINK DYNAMIC LOAD MODEL

MATLAB/Simulink software uses an exponential dynamic load model to develop a three-phase dynamic load, which consumption is dependent on the voltage level as in (1). The active power P and reactive power Q are dependent on the measured voltage, and the exponential values allow it to work in different modes other than the constant power load, where, V_0 is the reference positive sequence voltage, n_p and n_q are the exponents controlling the nature of load (0 for constant power, 1 for constant current and 2 for constant impedance), P_0 and Q_0 are the initial active and reactive power at the voltage V_0, V represents the positive sequence voltage, $[T_{p1}, T_{p2}]$ are the time constants controlling the dynamics of the active power, and $[T_{q1}, T_{q2}]$ are the time constants controlling the dynamics of the reactive power. The Simulink implementation of reference currents to mimic the behaviour of the exponential load is explained in the upcoming section.

$$
\begin{aligned}
P &= P_0 \left(\frac{V}{V_0}\right)^{n_p} \left(\frac{1+T_{p1}}{1+T_{p2}}\right) \\
Q &= Q_0 \left(\frac{V}{V_0}\right)^{n_q} \left(\frac{1+T_{q1}}{1+T_{q2}}\right)
\end{aligned}
\tag{1}
$$

A. IMPLEMENTATION OF SIMULINK DYNAMIC LOAD MODEL (SDL)

In the Simulink dynamic load model, line voltages V_{ab} and V_{bc} are measured at the load connection point in per unit (pu). The measured line voltages are fed to a PLL. The structure of PLL in MATLAB/Simulink is the same as Synchronous Reference Frame-PLL (SRF-PLL) but with the addition of variable frequency mean block to reduce the oscillations caused by harmonics and to improve its performance [15] as shown in Figure 2. The measured voltages are transformed into *dq*0 synchronous reference frame using Park's transformation (2), which can be further optimized to (3) to reduce the measurements.

$$
\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \sin \omega t & \sin(\omega t - \frac{2\pi}{3}) & \sin(\omega t + \frac{2\pi}{3}) \\ \cos \omega t & \cos(\omega t - \frac{2\pi}{3}) & \cos(\omega t + \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}
\tag{2}
$$

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \frac{1}{3}\sin\omega t(2v_{ab}+v_{bc}) - \sqrt{3}v_{bc}\cos\omega t \\ \frac{1}{3}\cos\omega t(2v_{ab}+v_{bc}) + \sqrt{3}v_{bc}\sin\omega t \end{bmatrix}\begin{bmatrix} v_{ab} \\ v_{bc} \end{bmatrix} \tag{3}$$

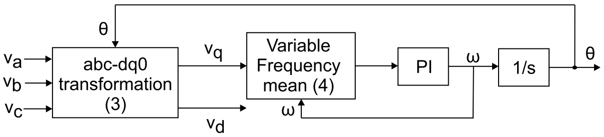


FIGURE 2. Structure of MATLAB/Simulink PLL [15].

The voltage V_q is fed to a variable frequency mean value block to reduce the oscillations caused by harmonics and to improve its performance. The variable frequency mean uses a running average window over a period of one cycle and is calculated using (4), where, $u(k)$ is the current input signal, $u(k-i)$ where $i \neq 0$ is the previous input, and N is the length of the average (one cycle of the input signal). The average output signal is held to its initial value for the first cycle.

$$\overline{u(k)} = \frac{1}{N}\sum_{i=0}^{N-1} u(k-i) \tag{4}$$

The output of the variable frequency mean block is fed through a PI controller to provide the angular frequency ω, which is intern fed through a integrator to generate the angular position θ. The angular position of the dq synchronous reference frame is controlled with a feedback loop that regulates the q-component to zero [16]. Thus, generating the voltage, frequency, and phase angle as the output of the PLL.

The generated V_d and V_q voltages from the PLL pass through variable frequency mean block to reduce the oscillations. The mean voltages are converted into a complex form, and its magnitude is used in the exponential load model block to have the characteristics of the load as mentioned in 1. The generated power is converted into complex form to obtain apparent power (s) and is used for calculating the positive sequence currents in dq synchronous frame ($i_d^* + ji_q^*$). The dq positive sequence currents are transformed into abc natural reference frame I_{abc} using $dq0 - abc$ transformation block.

The angular position is fed using the output of the PLL to ensure synchronization. The generated abc natural reference frame currents are fed to a controlled current source to depict the behaviour of the load as shown in Figure 1a.

Compiling this load model in a real-time environment may lead to higher usage of the CPU. This is due to the presence of computationally-heavy blocks such as PLL, mean, and transformation blocks. The PLL block and transformation block takes more CPU usage to calculate the trigonometric functions available in the equations. The digital or fixed-time solvers require more iterations to get accurate output, based on the approximation method used, such as the Taylor series algorithm according to (5), COordinate Rotation DIgital Computer (CORDIC), Lookup table, and so on [17], [18], [19]. The increase in the number of iterations can produce more accurate results, but on the other hand, it increases the latency and computation [20].

$$\begin{aligned} \sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots..; -\infty < x < \infty \\ \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots..; -\infty < x < \infty \end{aligned} \tag{5}$$

The increase in the computation and latency may lead to over-runs in a digital real-time environment, limiting the simulation of large models. To overcome this computational burden, an instantaneous p-q theory-based approach is proposed for load modelling and discussed in detail in the next section.

IV. INSTANTANEOUS p-q THEORY-BASED LOAD MODELLING

This section describes the mathematical approach of 3-phase p-q theory-based load model (PQL) proposed. The p-q theory is defined as a set of power equations [10], based on instantaneous values, expressed in the time domain, and it can be applied to any 3-phase system with or without a neutral conductor. This makes it valid for both steady-state and transient states. The p-q theory first transforms currents and voltages from abc to $\alpha\beta0$ coordinates and then defines instantaneous power on these coordinates. Hence, this theory considers the three-phase system as a unit, not a superposition or sum of three single-phase circuits.

In this work, as a matter of simplicity in explaining the concept, the p-q theory-based load model concept is discussed for a 3-phase, 3-wire system, and the time-dependency of main electric variables such as voltage, current, power, or frequency has been omitted in the equation notation. However, an extension to 4-wire systems is straightforward if the 0-component is included. The p-q theory for the 3-ϕ, 3-wire system uses the instantaneous voltage and current vectors, which are expressed in $\alpha\beta$ axes as (6):

$$\begin{aligned} v_{\alpha\beta} &= v_\alpha + \jmath v_\beta \\ i_{\alpha\beta} &= i_\alpha + \jmath i_\beta \end{aligned} \tag{6}$$

where v_α and v_β are given by power-variant transformation or simplified transformation for the balanced system in order to detect the amplitude of the input signal by,

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{7}$$

Therefore, the instantaneous complex power s can be represented as the product of voltage vector v and the conjugate of current vector i^*. Since instantaneous voltages and currents are considered, the complex power s definition is valid during both transients and steady states.

$$\begin{aligned} s &= v_{\alpha\beta} \cdot i_{\alpha\beta}^* = (v_\alpha + \jmath v_\beta)(i_\alpha - \jmath i_\beta) \\ &= (v_\alpha i_\alpha + v_\beta i_\beta) + \jmath(v_\beta i_\alpha - v_\alpha i_\beta) \end{aligned} \tag{8}$$

From (8), the instantaneous active power (p) and reactive power (q) can be expressed as,

$$\begin{aligned} p &= v_\alpha i_\alpha + v_\beta i_\beta \\ q &= v_\beta i_\alpha - v_\alpha i_\beta \end{aligned} \tag{9}$$

TABLE 1. Models characteristics comparison.

Features Load model	Dynamic model	Constant Impedance Load	Constant Current Load	Constant Power Load	External Power Control	No Need for Synchronization	Low Computational Burden
SSL	X	✓	X	X	X	✓	✓
SDL	✓	✓	✓	✓	✓	X	X
PQL	✓	✓	✓	✓	✓	✓	✓

SSL - Static Simulink Load; SDL - Simulink Dynamic Load; PQL - $p-q$ theory-based Load

Using (9), the α-β currents can be expressed as functions of the real power p and imaginary power q in matrix form as

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{1}{v_\alpha^2 + v_\beta^2} \begin{bmatrix} v_\alpha & v_\beta \\ v_\beta & -v_\alpha \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} \tag{10}$$

Expanding the right-hand side of the equation (10) yields,

$$\begin{aligned} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} &= \frac{1}{v_\alpha^2 + v_\beta^2} \begin{bmatrix} v_\alpha & v_\beta \\ v_\beta & -v_\alpha \end{bmatrix} \begin{bmatrix} p \\ 0 \end{bmatrix} \\ &+ \frac{1}{v_\alpha^2 + v_\beta^2} \begin{bmatrix} v_\alpha & v_\beta \\ v_\beta & -v_\alpha \end{bmatrix} \begin{bmatrix} 0 \\ q \end{bmatrix} \\ &= \begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \end{bmatrix} + \begin{bmatrix} i_{\alpha q} \\ i_{\beta q} \end{bmatrix} \end{aligned} \tag{11}$$

the instantaneous active and reactive current on α axis can be obtained from (11) and can be expressed as,

$$i_{\alpha p} = \frac{v_\alpha}{v_\alpha^2 + v_\beta^2} p \tag{12}$$

$$i_{\alpha q} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2} q \tag{13}$$

Similarly, the instantaneous active and reactive current on β axis can be expressed as,

$$i_{\beta p} = \frac{v_\beta}{v_\alpha^2 + v_\beta^2} p \tag{14}$$

$$i_{\beta q} = \frac{-v_\alpha}{v_\alpha^2 + v_\beta^2} q \tag{15}$$

Using (10) and inverse Clarke-transformation, we obtain the reference currents i_{abc} as mentioned in (16)

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \tag{16}$$

The above equation can be used when the system is balanced. The positive, negative, and zero sequences should be considered if the system is unbalanced, as mentioned in [10].

A. IMPLEMENTATION of p-q THEORY-BASED LOAD MODEL (PQL)

In the p-q theory-based load modelling approach, the phase voltages v_{abc} are measured in per unit (pu) at the point of load connection. The measured voltages are transformed into a stationary frame to obtain v_α and v_β according to (7). The generated quadrature voltages are converted into a complex form, and the magnitude of this signal is fed as input to the exponential block to get the behaviour of the load as in (1).

The generated active and reactive powers are compared with the measured power (p_m and q_m) (9), and the error signal is passed through a PI controller to follow the reference powers. The feed-forward technique is used to speed up the response of the system. The corrected powers (p^* and q^*) are passed through the p-q theory block to obtain the quadrature currents i_α and i_β according to (10). The quadrature currents are filtered using a first-order Butterworth filter with a cut-off frequency of $400Hz$ to reduce the discontinuities in the currents and to have a smooth transition.

The filtered quadrature currents are transformed into reference currents i_{abc} by using (16). These generated reference currents i_{abc}, shown in Fig. 1b, are fed to the controlled-current sources to depict the behaviour of the load, thus eliminating the computational burdening blocks such as PLL, mean, $abc - dq0$ transformation (and vice-versa). Table 1 shows different features supported using Simulink load models (static and three-phase dynamic load models) and the proposed p-q theory-based load model. This model is tested for its characteristics and computational performance using a couple of case studies in the upcoming section.

TABLE 2. Mathematical operations comparison.

Load Models Operations	Simulink Dynamic Load (SDL)	p-q theory-based Load (PQL)
Multiplication	50	38
Integration	5	2
Unit Delay	8	2
Discrete variable transport Delay	3	0
Switch	3	1
Summation/Subtraction	24	18
Trigonometric Calculations	6	0

B. COMPARISON OF CALCULATION COMPLEXITY

Before the experimental validations presented in Section VII, it is possible to investigate on the calculation complexity introduced by the different dynamic load models. The number of mathematical operations required by each model has been compared in Table 2. As can be observed, the Simulink model requires a high number of multiplications, integrations, and delays, adding also discrete variable transport delays

TABLE 3. Mathematical operations for different modules.

Operations \ Modules	Phase-Locked-Loop (PLL)	Mean (Variable frequency)	$dq0$-to-abc transformation
Multiplication	22	8	10
Integration	3	1	0
Unit Delay	3	2	0
Variable Transport Delay	1	1	0
Switch	1	1	0
Summation/Subtraction	10	5	4
Trigonometric Calculations	2	0	4

and trigonometric calculations. In detail, Table 3 shows that the PLL requires the highest amount of calculation blocks, making the Simulink model relatively heavy to compute. In addition, the Mean and the synchronous frame transformations increase the overall computational burden of the model.

On the opposite, the proposed $p-q$-based theory load model is characterized by a reduced amount of multiplications, integrations, and delays, plus no discrete variable transport delays and trigonometric calculation are needed. The reduced number of required operations permits to lighten the overall computational burden, making the proposed model more suitable for digital real-time simulations.

V. TEST FEEDERS

In order to realistically validate the performance of the proposed p-q based load model, two standard benchmarks have been employed in this work: the SimBench semi-urban medium voltage (MV) network [21], available for studying German distribution networks, and the 118-bus Distribution System, derived from a reduced network of the 123-bus distribution system [12]. These two networks have been chosen considering their size and complexity as optimal benchmark for large grids performance analysis. The OPAL-RT ARTEMiS suite is used as a platform for the digital real-time testing, considering both accuracy and required computational effort of the model. In order to let the reader replicate this paper's results, the p-q theory-based dynamic load model and the two benchmark networks simulated in this work have been uploaded in an open-access IEEE DataPort directory [13].

A. SIMBENCH SEMI-URBAN MV NETWORK

As mentioned above, the SimBench Semi-Urban MV Network benchmark has all the required characteristics to represent with high fidelity the German distribution system. This benchmark has been already used to publish, test, compare algorithms and methods for different use cases [21], [22].

The SimBench semi-urban grid is a open-ring system of 117 nodes system [23]. The switches and open lines are neglected to simplify the testing, making only 115 nodes available in the grid in Fig. 3 with a rated voltage of 20 kV. The network consists of two in-line high voltage to medium voltage transformer Tf1, and Tf2 (HV/MV) of 40 MVA connected in parallel to supply 9 distribution feeders. The sums of the loads and Distributed Energy Resources (DERs) are approximately 40.4 MW and 17.4 MW respectively. It is composed of long feeders from 3.8 up to 12.1 km. The data of this network can be found in [24], and are not repeated in this work for the sake of brevity. The balanced equivalent loads (loads and DERs) have been connected directly at each node for simulation.

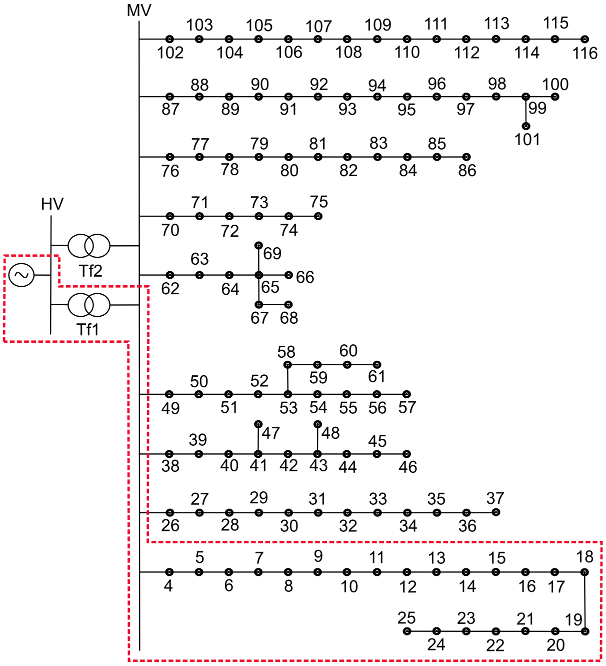


FIGURE 3. SimBench semi-urban MV network.

B. 118-BUS DISTRIBUTION SYSTEM

The 118-bus distribution system, shown in Fig. 4, has been introduced for the first time in [12] and represents a distribution network consisting of 118 load buses, 117 branches and a three-phase power source. The system operates at 11 kV rated voltage and the total active and reactive power amounting at 22.7 MW and 17.0 Mvar, respectively. In order to simplify the testing, in this work, the default network configuration has been considered, neglecting tie-lines and switches. The detailed parameters on loads and lines data can be found in [12], and are not repeated in this work for sake of brevity.

VI. MODEL VALIDATION

To examine the proposed p-q theory-based load model approach, a "SimBench semi-urban MV network" and "118-bus Distribution System" have been used for testing. These models were built using MATLAB/Simulink v2021b development platform. The models were decoupled using OPAL-RT's ARTEMiS SSN [25], compiled, and executed in real-time with RT-Lab software (RTLAB v2022.1). The performance of the systems was analyzed using the proposed p-q theory-based load (PQL) and Simulink dynamic load (SDL) models. The real-time simulation of the networks was

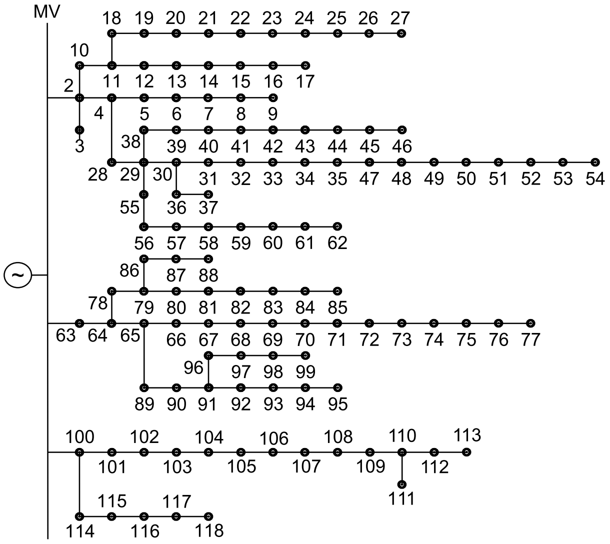


FIGURE 4. **118-bus Distribution System.**

carried out adopting a time step of 50μs and choosing the 4th order Runge-Kutta method as the fixed-step numerical solver.

A. STEADY-STATE ANALYSIS

The PQL model was tested for steady-state analysis by reducing the SimBench network to a single feeder network, as illustrated in the Figure. 3 (marked with a red dotted line). A nominal voltage of 20kV and a nominal power of 40MVA is used for per unit calculation. Each SDL model has been substituted with the PQL model for comparison purpose. The steady-state RMS voltages and currents of each phase were measured at each node in both the models and compared as shown in the Figure. 5.

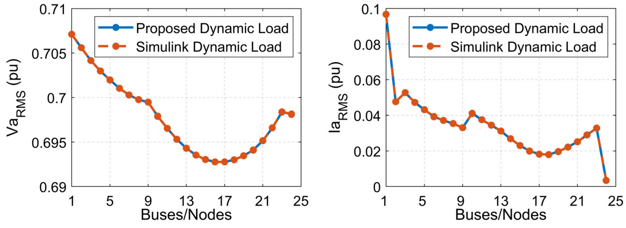
In the figure. 5, we observe that the RMS voltages and currents of each phase in the network with PQL model exhibits almost no deviation from the behaviour of the SDL model behaviour. The deviation between the RMS voltages and RMS currents of the compared load models are both below the 0.1% threshold, making their mismatch negligible.

B. TRANSIENT ANALYSIS

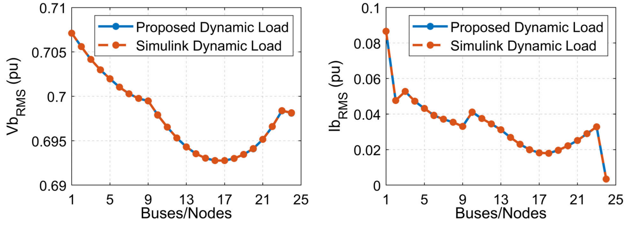
To validate the performance of the PQL model, transient analysis has been performed, considering active/reactive power and voltage changes in both benchmark grids. The transient study on SimBench network is done by a step change in active and reactive power on the load connected at bus 25, the farthest from the voltage source, in the sequence [1.0, 1.1, 0.9, 1.0] p.u. at the time instants [0, 0.4, 0.8, 1.2] s, respectively. The transient study on 118-bus Distribution System is done by applying a step change to the supply voltage in the sequence [1.0, 0.8, 1.2, 1.0] p.u. at the instants [0, 0.4, 0.8, 1.2] s, respectively.

1) VALIDATION OF EXTERNAL POWER CONTROL USING SIMBENCH SEMI-URBAN MV NETWORK

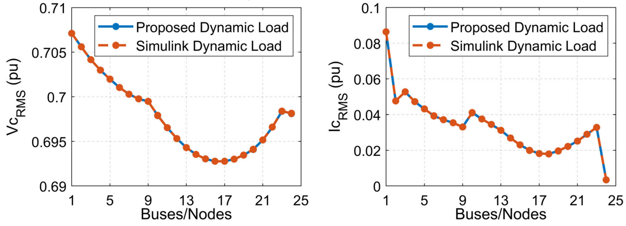
This test was performed to study the response of the PQL to a step change in reference power. The loads of the grid were set as constant power load ($n_p = 0$, $n_q = 0$) and the reference power of the load at bus 25 is subjected to a change in active and reactive power of 10% compared to its steady state.



(a) Phase-A RMS voltage (b) Phase-A RMS current



(c) Phase-B RMS voltage (d) Phase-B RMS current



(e) Phase-C RMS (f) Phase-C RMS current

FIGURE 5. **RMS voltages and currents at different nodes.**

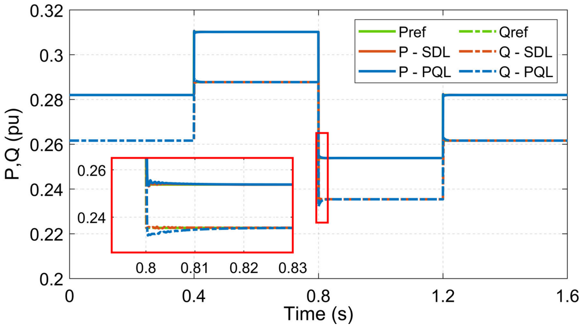
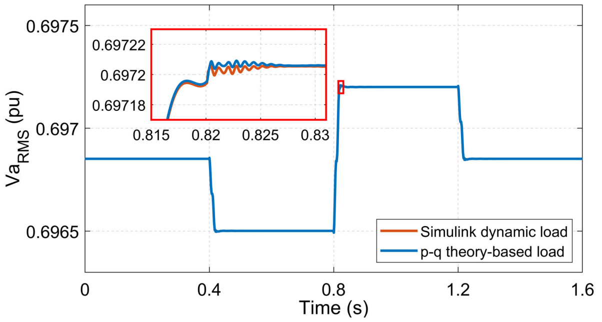


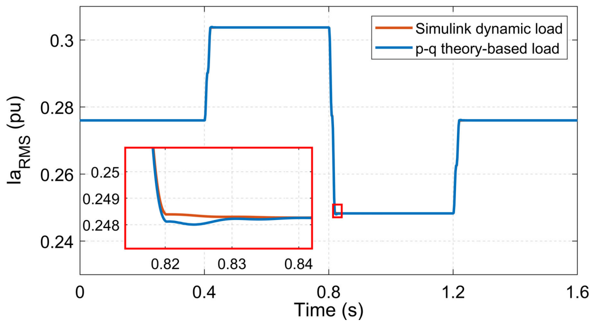
FIGURE 6. **Active and Reactive power variation of the load connected at Bus 25.**

A similar test was conducted by replacing the PQL model with SDL model in the SimBench network, and the comparison study was performed as shown in Fig. 6. We can observe from Figure. 6 that the PQL model and the SDL model follow the reference values with similar dynamics. The maximum deviation of both the models with respect to reference power is nearly 0.4%, which is negligible.

The change in the reference power leads to a change in voltage and current profiles to satisfy the power demand. For a constant power load scenario, voltage sags should increase the current and voltage swells in voltage should decrease the



(a) Phase-A RMS voltage at Bus 25 due to change in power



(b) Phase-A RMS current Bus 25 due to change in power

FIGURE 7. Phase-A RMS voltage and RMS current at Bus 25 due to change in power.

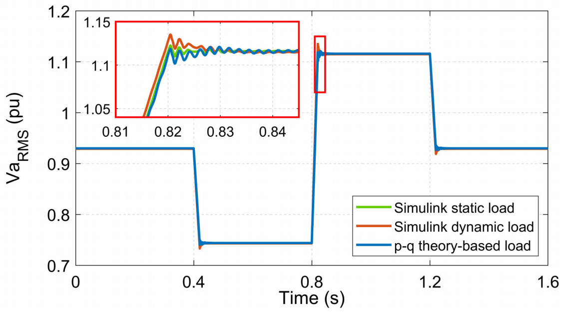
current to maintain the power constant. The same can be observed in the RMS voltage and RMS current behaviour as depicted in the Figure. 7. The maximum deviation of RMS voltage between PQL and SDL model is approximately 0.01% and, the maximum deviation of RMS current is approximately 0.04%. Concluding, the maximum deviation between the two models can be considered negligible.

2) VALIDATION OF CONSTANT IMPEDANCE LOAD MODE OF THE MODEL ON THE 118-BUS DISTRIBUTION SYSTEM

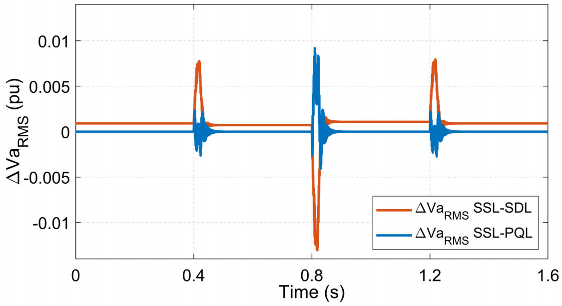
In this network, a sequence of step voltage variations has been imposed in order to test the behaviour of the PQL model and compare it to the performances of the commonly adopted SDL model. For this test, a step change was applied every 0.4 s to the voltage magnitude of the slack bus, which represents the secondary windings of the transformer in the primary substation. The step changes were applied following the sequence [1, 0.8, 1.2, 1] p.u. In this case, the PQL model and the SDL have been set as a constant impedance load ($n_p = 2$, $n_q = 2$) and were both compared to the Simulink Static Load (SSL) that can be considered as an ideal benchmark load. Please note that in the previous test case the static model (SSL) could not be employed as benchmark because simply it does not allow to change the power reference during the simulation. The Bus 54, which is farthest from the supply, has been chosen as an observatory point for this study. On this bus, indeed, the greatest voltage deviations can be observed as it is influenced by the greatest number of intermediate buses from the voltage source.

In Figure. 8, the behaviour of voltage at bus 54 has been shown for the three different load models. Figure. 8a shows that the voltage trend using the PQL model closely follows the voltage trend using the SSL model, with a negligible steady-state error. Using the SDL model, on the other hand, a steady-state error of 0.1% is present.The PQL model tracks the SSL model better than the SDL model, even during the transients. The percentage overshoot of the voltage trajectory using the PQL model is 0.45%, which is very similar to the overshoot using the SSL model (0.58%). Whereas, using the SDL model results in an overshoot of 1.8%, which is approximately 4 times greater.

In Figure. 9, the current RMS trajectories have been plotted for the three load models under test. It can be observed in Fig. 9a, that the currents of the PQL model reach the steady-state value faster than the SDL model, matching the dynamics of the SSL model. This means that, in terms of RMS values and with respect to the SDL, the PQL model allows to obtain a better approximation of the SSL model transient response, here assumed as the ideal benchmark. This result is also confirmed by the plots in Figure. 9b, where it can be observed that the current deviation between PQL and SSL is always well below a 0.4% error, whereas the current deviation between the SDL and SSL is always more pronounced, reaching a peak of about 0.8% error.



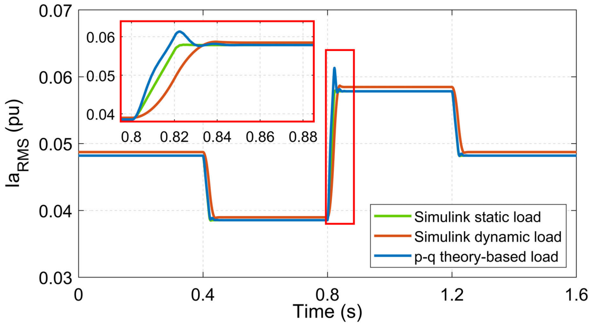
(a) Phase-A RMS voltage at bus 54



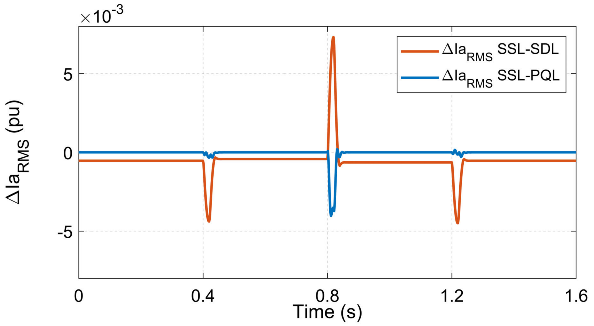
(b) Deviation of Phase-A RMS voltage at bus 54

FIGURE 8. Phase-A RMS voltage at Bus54 due to change in supply voltage.

Similarly, in Fig. 10, the comparison results between the active and reactive power waveforms have been shown. The



(a) Phase-A RMS current at bus 54



(b) Phase-A RMS current deviation at bus 54

FIGURE 9. **Phase-A RMS current at Bus54 due to change in supply voltage.**

PQL model is able to follow the SSL model power behaviour with minimum error and a relatively small overshoot (5% maximum error for active power and 9.4% maximum error for reactive power). In contrast, the SDL model is slower in reaching the steady-state power value, due to the delays introduced in the dynamic model by the PLL and the moving average block.

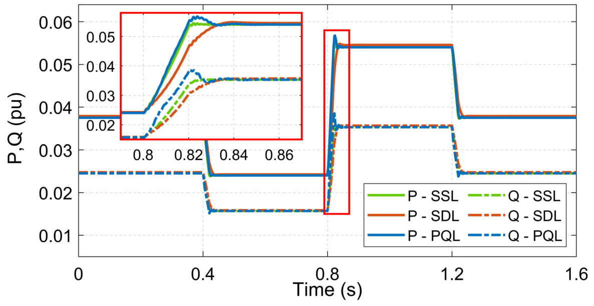


FIGURE 10. **Active and Reactive power variation of the load connected at bus 54 due to change in supply voltage.**

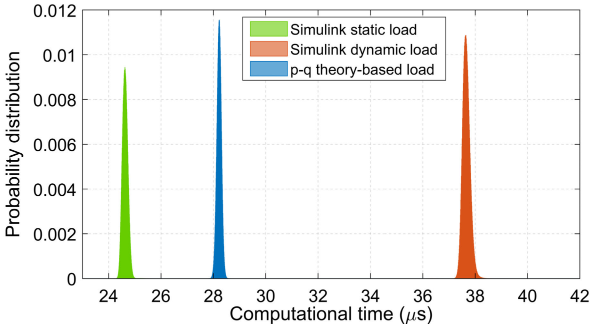
VII. EXECUTION TIME REQUIREMENTS

A comparison study was made between the proposed dynamic load (PQL) model and the Simulink load blocks (SDL and SSL) to assess the computational burden of the models. The computational analysis was carried out on the two large networks mentioned in Section V.

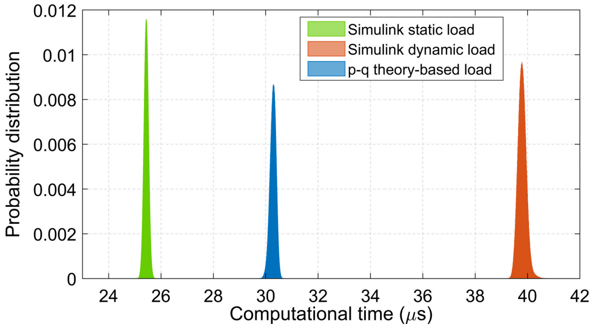
The computational study of the models was performed using an OPAL-RT's OP5700 real-time simulator. The system was configured with Red Hat Enterprise Linux Server release 5.2 OS version and 2.6.29.6-opalrt-6.2.1 Kernel version. The RT-LAB version used was v2022.1.

Since the main goal is to assess the impact in terms of computation time of the different load models, the real-time network models have been programmed to be run in real-time in software-synchronized mode and neglecting all measurements or data exchange blocks which could affect the overall computational performance. These tests are aimed at finding, for each network model, the minimum time-step which can be adopted without experiencing overruns.

Starting from a minimum value of 50μs, each network model was tested for a couple of hours in order to verify whether overruns would occur. In the case of overruns, the time step was increased by 50μs, and the network model was tested again for overruns, until no overruns were experienced in the entire test window. The tests are performed considering also a different number of active cores. When multiple cores are adopted (i.e two or three) the models are decoupled using the OPAL-RTs ARTEMiS stubline and the SSN block in the OPAL-RT's ARTEMiS library.



(a) Probability distribution of required computational time for the SimBench semi-urban MV network for 5 minutes of simulation using 3 cores



(b) Probability distribution of required computational time for the 118-bus Distribution System for 5 minutes of simulation using 3 cores

FIGURE 11. **Probability distribution of computational time.**

Please note that the SimBench semi-urban MV network is characterized by the presence of some nodes where, due to the presence of distributed generation, the equivalent load

TABLE 4. Overall computation time comparison.

Dynamic Load Model used	Min. Ts required [μs]	Comp. time required [μs]	Comp. time increase [%]	Min. Ts required [μs]	Comp. time required [μs]	Comp. time increase [%]	Min. Ts required [μs]	Comp. time required [μs]	Comp. time increase [%]
Simbench semi-urban MV network									
	1 core			*2 cores*			*3 cores*		
SDL	150	118.92	63.78%	100	56.06	47.02%	50	37.66	52.90%
PQL	100	82.35	13.41%	50	44.05	15.53%	50	28.22	14.58%
SSL	100	72.61	0%	50	38.13	0%	50	24.63	0%
118-bus Distribution System									
	1 core			*2 cores*			*3 cores*		
SDL	150	130.75	61.92%	100	62.21	58.86%	50	39.74	56.21%
PQL	100	93.31	15.55%	50	46.39	18.46%	50	30.28	19.03%
SSL	100	80.75	0%	50	39.16	0%	50	25.44	0%

is negative. The SSL model, being based on a fixed RLC load, is not able to represent this kind of equivalent. For such reason, in the implementation of the SSL network model, these particular nodes were treated adding a SDL block to simulated the fixed power injection. The number of nodes which required this modification is very limited (about 15 in the entire network) and therefore the influence in the overall computation time can be considered negligible.

Table 4 shows the minimum possible simulation time-step for each model and considering the simulation to be running on 1, 2, and 3 cores of the real-time simulator. This minimum time step is the one that ensures that no over-runs were experienced during the simulation. For simplicity, the average computation time is calculated over a simulation time of 5 minutes. The increase of computation time is calculated by using the SSL model as a reference.

Fig. 11a and Fig. 11b show the probability distribution of the required computation time of the two large networks under study. These timing are referred to the case of 3 cores and 50μs fixed time step. It can be seen that the SDL model shows a larger deviation of the computation time to its mean value. This may result in overruns during real-time simulations.

The adoption of the PQL model allows to achieve a significative reduction in computation time, compared the SDL model. The average computation time of the PQL model is 16% more, with a maximum of 19%, compared to the SSL model. On the contrary, the test networks with the SDL model require an average additional computation time of 55% higher than the SSL model, with a peak of 63.8% higher in the worst case. The significant reduction in computational burden using the PQL model allows the two large networks under study to be simulated safely with a time-step of 50μs, even using only two cores. This means that, thanks to the SDL, a real-time simulator can deal with larger networks without the need of additional cores. Moreover, if the number of available cores is not an issue, the load model reduction permits to reduce the number of cores dedicated to the network simulation and exploit the remaining idle resources for other complex functions which might also include the interaction with power devices, control equipment and communication systems.

VIII. CONCLUSION

The instantaneous p-q theory-based dynamic load modelling for the digital simulation of power systems is presented. The 3-phase p-q theory-based load model is proposed as an alternative to the existing three-phase dynamic load model in MATLAB/Simulink with significant improvement. The improvement is made by eliminating the heavy computational blocks such as PLL, mean, and $dq0 - abc$ transformation block for real-time simulation. By the testing shown in this work, the 3-phase p-q theory-based dynamic load was found to have a lower computation time without degradation of accuracy, compared to the Simulink three-phase dynamic load model. Applying the instantaneous theory, it is possible to simulate large and complex networks, such as the SimBench network and the 118-bus distribution system, with considerably reduced computation time (in average 30%) than the load model proposed by Simulink. The tests performed in this work demonstrated how the instantaneous p-q theory-based load modelling approach can allow larger and more complex networks to be simulated in real-time under the same available computing capacity.

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KARTHIK RAJASHEKARAIAH received the B.Tech. degree in electrical and electronics and the M.Tech. degree in power electronics from the Visvesvaraya Institute of Technology, Belgaum, Karnataka, India, in 2012, and 2016, respectively. He is currently pursuing the Ph.D. degree with the Institute of Technical Physics (Energy Laboratory 2.0), Karlsruhe Institute of Technology, Karlsruhe, Germany, under the supervision of Prof. Giovanni De Carne. Previously, he was a Field Application Engineer with OPAL-RT Technologies India (P) Ltd., Bengaluru, India, from November 2016 to August 2021. Also, he was an Engineer Trainee with TeleDNA Communications India (P) Ltd., Bengaluru, from September 2013 to September 2014. His research interests include modelling, simulation and analysis of power electronics in power system applications, and power hardware-in-the-loop studies in real-time applications.

COSIMO IURLARO (Graduate Student Member, IEEE) was born in Taranto, Italy, in 1995. He received the degree (Hons.) in electrical engineering with specialization in power system from the Polytechnic University of Bari, Italy, in 2019. He is currently pursuing the Ph.D. degree in electrical and information engineering on the topic of power systems. His research interests include technologies and methodologies to provide energy services to the power systems, microgrids, and real-time simulation.

SERGIO BRUNO (Member, IEEE) received the master's and Ph.D. degrees in electrical engineering from Politecnico di Bari in 2000 and 2004, respectively. He has been involved as a Senior Scientist in several research and development project in the field of smart grids and energy systems. Currently, he is a tenure-track Associate Professor with Politecnico di Bari, where he holds the courses in power system automation and power quality and manages the operational activities of the laboratory LabZERO. Since 2023, he has been the Vice-Chair of the IEEE Italy Section Chapter PE31. His recent studies deal with power system automation, smart distribution systems, microgrids, energy services, and real-time simulation.

GIOVANNI DE CARNE (Senior Member, IEEE) received the B.Sc. and M.Sc. degrees in electrical engineering from the Polytechnic University of Bari, Italy, in 2011 and 2013, respectively, and the Ph.D. degree from the Chair of Power Electronics, Kiel University, Germany, in 2018. He is currently a tenure-track Professor with the Institute for Technical Physics, Karlsruhe Institute of Technology, Karlsruhe, Germany, where he leads the "Real Time System Integration" Group and the "Power Hardware In the Loop Laboratory." In 2020, he has been awarded with the highly-competitive Helmholtz "Young Investigator Group" for the Project "Hybrid Networks: A Multi-Modal Design for the Future Energy System." He has authored/coauthored more than 90 peer-reviewed scientific papers. His research interests includes power electronics integration in power systems, solid state transformers, real time modelling, and power hardware in the loop. He is the Chairperson of the IEEE PES Task Force on Solid State Transformer Integration in Distribution Grids. He is an Associate Editor of *IEEE Industrial Electronics Magazine* and IEEE OPEN JOURNAL OF POWER ELECTRONICS.

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