Mathematical Social Sciences 128 (2024) 16–17

Contents lists available at ScienceDirect

Mathematical Social Sciences

journal homepage: www.elsevier.com/locate/mss

Short communication

Note on "A classification of peak-pit maximal Condorcet domains" by Guanhao Li, Mathematical Social Sciences 125 (2023), 42–57

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ARTICLE INFO

Keywords:
Condorcet domains
Peak-pit domains

ABSTRACT

The note presents a counter example to Li (2023).

In this note we present a counterexample to Lemma 1 in Li (2023). Lemma 1 in Li (2023) states that any locally maximal weakly separated system of sets is an ideal. (All definitions are taken from the paper by Li (2023); see Definitions 18 and 19 for the notion of a locally maximal weakly separated set-system, and Definition 11 for the notion of an ideal.)

We will now exhibit a system of subsets on the set $[5] := \{1,2,3,4,5\}$ that is locally maximal weakly separated but not an ideal. Consider the graded system $S := \cup_{i=0}^{5} S_i$ where

$$S_0 = \{\emptyset\},$$

$$S_1 = \{\{1\},\{2\},\{5\}\},$$

$$S_2 = \{\{1,2\},\{2,4\},\{2,5\},\{4,5\}\},$$

$$S_3 = \{\{1,2,3\},\{1,2,4\},\{2,3,4\},\{2,4,5\},\{3,4,5\}\},$$

$$S_4 = \{\{1,2,3,4\}\},$$

$$S_5 = \{\{1,2,3,4,5\}\}.$$

This is not an ideal since the sets $\{2,4,5\}$ and $\{3,4,5\}$ do not have corresponding supersets in S of cardinality 4. Let us check that S is locally maximal weakly separated. We need to consider for every triple $\{i,j,k\}$ of pairwise different elements of [5] the reduced set system

$$S_{\{i,j,k\}} = \{X \cap \{i,j,k\} \mid X \in S, \text{ and } \{i,j,k\} \subseteq [5]\},$$

and show that it has cardinality 7 with either a singleton or a pair missing from $2^{\{i,j,k\}}$. We have:

$$S_{\{1,2,3\}} = \{\emptyset,\{1\},\{2\},\{3\},\{1,2\},\{2,3\},\{1,2,3\}\} \text{ and } \{1,3\} \text{ is absent},$$

$$S_{\{1,2,4\}} = \{\emptyset,\{1\},\{2\},\{4\},\{1,2\},\{2,4\},\{1,2,4\}\} \text{ and } \{1,4\} \text{ is absent},$$

$$S_{\{1,2,5\}} = \{\emptyset,\{1\},\{2\},\{5\},\{1,2\},\{2,5\},\{1,2,5\}\} \text{ and } \{1,5\} \text{ is absent},$$

$$S_{\{1,3,4\}} = \{\emptyset,\{1\},\{4\},\{1,3\},\{1,4\},\{3,4\},\{1,3,4\}\} \text{ and } \{3\} \text{ is absent},$$

$$S_{\{1,3,5\}} = \{\emptyset,\{1\},\{3\},\{5\},\{1,3\},\{3,5\},\{1,3,5\}\} \text{ and } \{1,5\} \text{ is absent},$$

$$S_{\{1,4,5\}} = \{\emptyset,\{1\},\{4\},\{5\},\{1,4\},\{4,5\},\{1,4,5\}\} \text{ and } \{1,5\} \text{ is absent},$$

$$S_{\{2,3,4\}} = \{\emptyset,\{2\},\{4\},\{2,3\},\{2,4\},\{3,4\},\{2,3,4\}\} \text{ and } \{3\} \text{ is absent},$$

$$S_{\{2,3,5\}} = \{\emptyset,\{2\},\{5\},\{2,3\},\{2,5\},\{3,5\},\{2,3,5\}\} \text{ and } \{3\} \text{ is absent},$$

$$S_{\{2,4,5\}} = \{\emptyset,\{2\},\{5\},\{2,4\},\{2,5\},\{4,5\},\{2,4,5\}\} \text{ and } \{4\} \text{ is absent},$$

$$S_{\{3,4,5\}} = \{\emptyset,\{3\},\{4\},\{5\},\{3,4\},\{4,5\},\{3,4,5\}\} \text{ and } \{3,5\} \text{ is absent}.$$

Hence S is locally maximal and weakly separated but not an ideal.

Thus, Lemma 1 in Li (2023) is not correct. Since Lemma 1 represents a pivotal step in the analysis of that paper, most of the proofs in Sections 3 and 4 of Li (2023) are therefore incomplete.

Indeed, in Section 3, the proof of Lemma 2 relies on Lemma 1; the proof of Theorem 1, in turn, relies on both Lemmas 1 and 2; the proof

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https://doi.org/10.1016/j.mathsocsci.2024.01.006
Received 15 December 2023; Received in revised form 12 January 2024; Accepted 16 January 2024
Available online 19 January 2024

of Lemma 4 relies on Theorem 1; the proof of Theorem 2 relies on Lemma 4; the proof of Lemma 5 relies on Lemma 1 (via Theorem 1); the proof of Corollary 1 relies on Lemmas 4 and 5, as well as Theorems 1 and 2; the proof of Lemma 6 relies on Corollary 1, and the proof of Corollary 2 relies on Corollary 1 and Lemma 6.

In Section 4, the pivotal statement is the aforementioned Corollary 1 from Section 3 which is used to prove both Lemmas 8 and 9. The proof of Theorem 3 relies on Lemma 9, and the proof of Theorem 4 uses both Theorems 2 and 3.

CRediT authorship contribution statement

Clemens Puppe: Writing – original draft. **Arkadii Slinko:** Writing – original draft.

Data availability

No data was used for the research described in the article.

References

Li, G., 2023. A classification of peak-pit maximal condorcet domains. Math. Social Sci. 125, 42–57.