Nonfactorizable corrections to Higgs boson production in weak boson fusion

Konstantin Asteriadis,[1,*] Christian Brønnum-Hansen,[2,†] and Kirill Melnikov[2,‡]

[1]*High Energy Theory Group, Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*

[2]*Institute for Theoretical Particle Physics, Karlsruhe Institute of Technology, 76128 Karlsruhe, Germany*

(Received 26 May 2023; accepted 24 October 2023; published 25 January 2024)

We discuss the nonfactorizable corrections to Higgs boson production in weak boson fusion at the Large Hadron Collider. Such corrections depend on the finite part of the two-loop virtual amplitude $qQ \to q'Q' + H$ which, up to now, has only been computed in the eikonal approximation. We combine this contribution with real-virtual and double-real nonfactorizable QCD corrections and study their impact on the various observables in weak boson fusion. We find that the nonfactorizable corrections are strongly dominated by the two-loop virtual contributions, while all other contributions play a very minor role. This striking imbalance between real and virtual contributions is caused by a process-specific kinematic suppression of the former and a particular enhancement of the virtual corrections related to a Glauber phase.

DOI: 10.1103/PhysRevD.109.014031

I. INTRODUCTION

Weak boson fusion (WBF) is an important Higgs boson production channel; it has the second-largest cross section at the Large Hadron Collider (LHC). In addition, it is directly sensitive to the couplings of the Higgs boson to W and Z bosons allowing for a detailed exploration of their strengths and Lorentz structures.

Theoretical predictions for Higgs boson production in weak boson fusion are very advanced. They include next-to-leading order (NLO) QCD [1] and electroweak [2] corrections as well as next-to-next-to-leading order (NNLO) QCD [3–5] and next-to-next-to-next-to-leading order ($\mathrm{N^3LO}$) QCD [6] corrections. In addition, effects of multijet merging and an interplay between fixed order perturbative computations and parton showers in weak boson fusion were studied in Ref. [7]. However, available QCD corrections are computed in the so-called factorization approximation where strong interactions between the incoming quark lines are systematically ignored. The only exception is the identical-flavor contributions where the interference of t- and u-channels appears. It is well-understood that this interference is very strongly suppressed both kinematically and by color at NLO QCD when event selection criteria are applied [1,2]. Note that at percent-level precision these effects should be taken into account. However, in the present work we do not include such interferences and focus solely on remaining contributions to nonfactorizable corrections, which are so far only partially known.

Historically, remaining nonfactorizable corrections were neglected because they are color-suppressed [3], and, moreover, they appear at NNLO QCD for the first time. However, it was pointed out in Ref. [8] that these corrections receive a peculiar π^2-enhancement associated with a Glauber phase. In Refs. [8,9] the numerical impact of nonfactorizable corrections on various observables in WBF was investigated. It was found that these corrections are somewhat smaller than the factorizable corrections at NNLO QCD but that they certainly exceed the magnitude of $\mathrm{N^3LO}$ QCD corrections.

To make further progress in understanding the nonfactorizable effects in weak boson fusion, there are two directions to take. First, one can extend the calculation of the nonfactorizable two-loop amplitude for the WBF process $qQ \to q'Q' + H$ beyond the eikonal approximation. This is a formidable task since it requires the computation of two-loop five-point amplitudes with two massive propagators and an additional external massive particle which is beyond the current state of the art. Second, one can study the effects of all the other contributions relevant for computing the nonfactorizable correction through NNLO in perturbative QCD while accounting for the double-virtual contribution in the eikonal approximation. This is what we do in this paper.

Computation of NNLO QCD corrections to WBF requires double-real and real-virtual contributions, in addition to the two-loop virtual corrections. Individually, each

*kasteriad@bnl.gov
†christian.broennum-hansen@partner.kit.edu
‡kirill.melnikov@kit.edu

of these contributions is infrared divergent; to properly define them a subtraction procedure is needed. Since in the past decade remarkable progress in the development of NNLO QCD subtraction schemes for collider processes has been made, and since certain features of the nonfactorizable correction to Higgs boson fusion in WBF make the infrared structure of this process simple, construction of the subtraction scheme for computing the nonfactorizable corrections to WBF becomes straightforward. In fact, the relevant computation can be borrowed, almost verbatim, from a similar computation of the nonfactorizable corrections to single-top production reported recently in Ref. [10].

It is worth pointing out that the situation with real-virtual contributions is somewhat peculiar. Although the relevant one-loop amplitudes can be extracted from an existing computation of NLO QCD corrections to $H + j$ production in weak boson fusion [11], the fact that the corresponding *six-point* amplitude needs to be evaluated close to singular limits makes its use in the computation of NNLO QCD corrections nontrivial.

The remaining part of the paper is organized as follows. In the next section we recapitulate the construction of the infrared-finite fully differential cross section suitable for numerical computation. We discuss the numerical implementation and address difficulties with evaluating subtracted real-virtual contributions in Sec. III. We then present the results of our computation and show that the nonfactorizable corrections are strongly dominated by two-loop virtual corrections. We conclude in Sec. V.

II. CONSTRUCTION OF AN INFRARED FINITE CROSS SECTION

A NNLO QCD computation requires the construction of an infrared-finite cross section which can be integrated over phase space of final-state particles in four dimensions. This requires the use of a subtraction scheme since contributions with different numbers of final-state partons are not separately finite.

The construction of such a subtraction scheme for the case of nonfactorizable contributions to single-top production was recently presented in Ref. [10]. The discussion in that reference applies almost verbatim to the computation of nonfactorizable corrections to Higgs boson production in weak boson fusion. Because of that, we confine ourselves to reviewing the major building blocks of such a construction in this section, and note that further details can be found in Ref. [10].

Nonfactorizable corrections involve exchanges of real and virtual gluons between the two quark lines of the partonic process $qQ \to q'Q' + H$, where q and Q are arbitrary quarks or antiquarks (see Fig. 1). Such corrections do not contribute at next-to-leading order due to color conservation. Indeed, both real and virtual nonfactorizable corrections at NLO QCD contain just one single color generator T^a on each fermion line. When one computes the interference of the one-loop virtual amplitude with the leading-order amplitude or the square of the real-emission amplitude, the corrections vanish since the color generators are traceless.

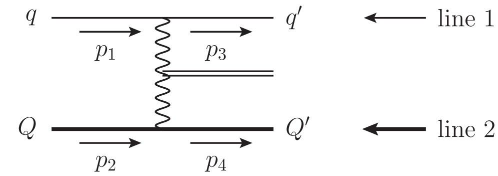

FIG. 1. Momentum, parton, and line conventions at Born level used throughout the discussion. We do not show fermion flow because q and Q each represent any (light) quark or antiquark.

Despite being absent at lower orders, nonfactorizable contributions do appear at NNLO in perturbative QCD. For example, virtual contributions with two gluons connecting the upper and lower quark lines lead to a color factor $\mathrm{Tr}(T^a T^b) = T_R \delta^{ab}$ for each line and clearly do not vanish when the interference with the leading-order amplitude is computed. We show some of the nonvanishing contributions in Fig. 2. Furthermore, it is easy to see that nonfactorizable contributions at NNLO can involve neither diagrams with gluon self-energy corrections nor non-Abelian QCD vertices. This latter feature renders all nonfactorizable corrections QED-like and leads, as we will discuss later in more detail, to a simple infrared structure of such contributions. We will now consider the various contributions to the NNLO QCD nonfactorizable corrections and review the construction of the subtraction terms.

A. Double-real emission contribution

We begin with the *nonfactorizable* contributions to the double-real emission process

$$q(p_1) + Q(p_2) \to q'(p_3) + Q'(p_4) + g(p_5) + g(p_6) + H(p_H). \tag{1}$$

All such contributions to the *amplitude squared* carry the same color factor given by

$$\sum_{a,b} \mathrm{Tr}(T^a T^b)^2 = T_R^2 (N_c^2 - 1), \tag{2}$$

where $T_R = 1/2$, $N_c = 3$, a and b are the color indices of gluons p_5 and p_6, respectively, and the summation over quark colors has been performed. Since the color factor is always the same, it is convenient to work with *color-stripped* amplitudes and restore the overall color factor at the end.

We write the relevant color-stripped amplitudes as[1]

$$A_0^{ij}(1_q, 2_Q, 3_{q'}, 4_{Q'} | 5_g, 6_g), \tag{3}$$

[1]Dependence of the amplitude on the Higgs boson momentum p_H is not shown because it is not relevant for the present discussion.

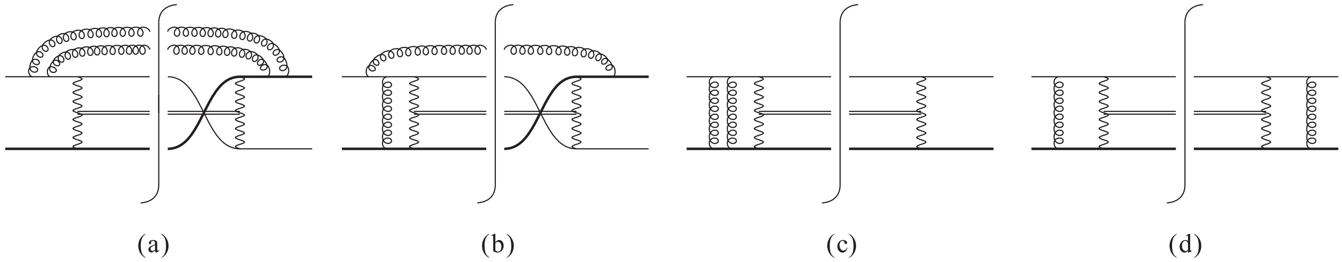

FIG. 2. Schematic examples of nonvanishing contributions to the nonfactorizable (a) double-real, (b) real-virtual, and (c)-(d) double-virtual amplitude squared. It is easy to see that the color factor for each contribution is $T_R^2(N_c^2-1)$, as stated in the main text. To distinguish the two *massless* quark lines, one is printed in bold.

where superscript $i(j) \in \{1,2\}$ refers to one of the two quark lines from which gluon 5(6) is emitted (see Fig. 1). We emphasize again that only Abelian diagrams contribute to A_0^{ij} and that, to obtain them, the color generators in quark-gluon vertices are to be removed. Similarly, we define *color-stripped* amplitudes A_0^i for a single gluon emission from line $i \in \{1,2\}$, and A_0 for the amplitude of the process without additional gluons.

Following Ref. [12] we define

$$\begin{aligned} F_{\rm LM}^{\rm nf}(1_q,2_Q,3_{q'},4_{Q'}|5_g,6_g) &\equiv \mathcal{N}\int {\rm dLips}_{34H}\hat{\mathcal{O}}(\{p_{i=1,\ldots,6},p_H\})(2\pi)^d\delta^{(d)} \\ &\times\left(p_1+p_2-p_H-\sum_{i=3}^{6}p_i\right) \\ &\times 2{\rm Re}\left[A_0^{11}A_0^{22\star}+A_0^{12}A_0^{21\star}\right](1,2,3,4|5,6), \end{aligned} \tag{4}$$

where ${\rm dLips}_{34H}$ is the Lorentz-invariant phase space of the two final-state fermions and the Higgs boson, $\mathcal{N} = 1/(4N_c^2)$ includes spin and color-averaging factors, $\hat{\mathcal{O}}(\{p_{i=1,\ldots,6},p_H\})$ is an arbitrary infrared-safe observable, and $d = 4-2\epsilon$ is the spacetime dimension.

To obtain the partonic differential cross section we restore color charges and write

$$d\sigma_{\rm rr}^{\rm nf} = \frac{T_R^2(N_c^2-1)}{2s}\left\langle F_{\rm LM}^{\rm nf}(1,2,3,4|5,6)\right\rangle, \tag{5}$$

where $s = 2p_1\cdot p_2$. We also define $\langle F_{\rm LM}^{\rm nf}(1,2,3,4|5,6)\rangle$ as an integral over the two-gluon phase space[2]

$$\left\langle F_{\rm LM}^{\rm nf}(1,2,3,4|5,6)\right\rangle \equiv \int [dp_5][dp_6]\theta(E_5-E_6) \times F_{\rm LM}^{\rm nf}(1,2,3,4|5,6). \tag{6}$$

[2]We choose to order gluon emissions in energy and, therefore, do not include the factor $1/2!$ to account for identical final states. This has to be kept in mind when comparing to Ref. [10] where the gluons were not ordered.

Note that we dropped the subscripts indicating the parton type for brevity; we will continue to use this shortened notation in what follows, unless parton type becomes relevant. The phase-space element $[dp_k]$ is defined as

$$[dp_k] \equiv \frac{d^{d-1}p_k}{(2\pi)^{d-1}2E_k}\theta(E_{\rm max}-E_k), \tag{7}$$

where $E_{\rm max}$ is a parameter that should be equal to or greater than the maximal energy that a final-state parton can have because of momentum conservation.

To construct the subtraction terms, we need to understand the singularities of the matrix element in Eq. (6). Although, in general, such singularities can arise when the emitted gluons are either soft or collinear to other partons, the case of nonfactorizable corrections is special *because only soft singularities are possible*. However, since we order gluons in energy and since the matrix element fully factorizes in the double-soft $E_5 \sim E_6 \to 0$ limit because of the Abelian nature of nonfactorizable corrections, it is sufficient to write

$$\begin{aligned} \left\langle F_{\rm LM}^{\rm nf}(1,2,3,4|5,6)\right\rangle &= \left\langle [I-S_6]F_{\rm LM}^{\rm nf}(1,2,3,4|5,6)\right\rangle \\ &+ \left\langle S_6 F_{\rm LM}^{\rm nf}(1,2,3,4|5,6)\right\rangle \end{aligned} \tag{8}$$

to obtain a fully regulated double-real emission contribution. We remind the reader that an operator S_i extracts the leading behavior of the function $F_{\rm LM}^{\rm nf}$ in the limit where the energy of parton i vanishes; see Ref. [12] for additional details.

We now turn our attention to the subtraction term containing the single soft singularity, i.e. the second term on the right-hand side of Eq. (8). It is given by

$$\begin{aligned} S_6 F_{\rm LM}^{\rm nf}(1_q,2_Q,3_{q'},4_{Q'}|5_g,6_g) &= -2g_{s,b}^2\kappa_{qQ}\int [dp_6]\theta(E_5-E_6){\rm Eik}_{\rm nf}(1_q,2_Q,3_{q'},4_{Q'}|6_g) \\ &\times F_{\rm LM}^{\rm nf}(1_q,2_Q,3_{q'},4_{Q'}|5_g), \end{aligned} \tag{9}$$

where $\kappa_{qQ} = +1$ if both q and Q are either quarks or antiquarks, and $\kappa_{qQ} = -1$ otherwise. The eikonal function in Eq. (9) reads

$$\mathrm{Eik_{nf}}\left(1_q, 2_Q, 3_{q'}, 4_{Q'}|6_g\right) = \sum_{\substack{i\in\{1,3\}\\ j\in\{2,4\}}} \frac{\lambda_{ij}(p_i\cdot p_j)}{(p_i\cdot p_6)(p_j\cdot p_6)}, \quad (10)$$

with $\lambda_{ij} = +1$ if both i and j are either incoming or outgoing, and $\lambda_{ij} = -1$ otherwise. We also note that in Eq. (9) we have introduced a nonfactorizable, single-gluon emission contribution

$$F_{\mathrm{LM}}^{\mathrm{nf}}(1,2,3,4|5) \equiv \mathcal{N}\int \mathrm{dLips}_{34H}\hat{\mathcal{O}}(\{p_{i=1,\dots,5},p_H\}) \times (2\pi)^d\delta^{(d)}\left(p_1+p_2-p_H-\sum_{i=3}^{5}p_i\right) \times 2\mathrm{Re}\left[A_0^1A_0^{2\star}\right](1,2,3,4|5). \quad (11)$$

The normalization factor $\mathcal{N}$ and integration measure were introduced in Eq. (4).

Integration of the eikonal factor over the gluon momentum p_6 in Eq. (9) has already been discussed in the literature; see e.g. Ref. [13]. We obtain

$$\left\langle S_6F_{\mathrm{LM}}^{\mathrm{nf}}(1,2,3,4|5,6)\right\rangle = -2[\alpha_{s,b}]\kappa_{qQ}\left\langle (2E_5)^{-2\epsilon}K_{\mathrm{nf}}(1,2,3,4)F_{\mathrm{LM}}^{\mathrm{nf}}(1,2,3,4|5)\right\rangle. \quad (12)$$

The function $K_{\mathrm{nf}}(1,2,3,4)$ can be found in Appendix A and $[\alpha_{s,b}]$ is defined as follows:

$$[\alpha_{s,b}] \equiv \frac{g_{s,b}^2}{8\pi^2}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}. \quad (13)$$

There is still a soft singularity, $E_5\to 0$, in the function $F_{\mathrm{LM}}^{\mathrm{nf}}(1,2,3,4|5)$ in Eq. (12) that needs to be extracted. Analogous to Eq. (8), we do this by subtracting and adding the soft limit of gluon g_5. We find

$$\left\langle S_6F_{\mathrm{LM}}^{\mathrm{nf}}(1,2,3,4|5,6)\right\rangle = -2[\alpha_{s,b}]\kappa_{qQ}\left\langle [I-S_5](2E_5)^{-2\epsilon}K_{\mathrm{nf}}(1,\dots,4) \times F_{\mathrm{LM}}^{\mathrm{nf}}(1,2,3,4|5)\right\rangle - 2[\alpha_{s,b}]\kappa_{qQ}\left\langle S_5(2E_5)^{-2\epsilon} \times K_{\mathrm{nf}}(1,\dots,4)F_{\mathrm{LM}}^{\mathrm{nf}}(1,2,3,4|5)\right\rangle. \quad (14)$$

The limit of the color-stripped single-real emission amplitude is similar to Eq. (9) and reads

$$S_5(2E_5)^{-2\epsilon}F_{\mathrm{LM}}^{\mathrm{nf}}(1,2,3,4|5) = -2g_{s,b}^2\kappa_{qQ}(2E_5)^{-2\epsilon} \times \mathrm{Eik_{nf}}(1,2,3,4|5) \times F_{\mathrm{LM}}^{\mathrm{nf}}(1,2,3,4), \quad (15)$$

where we introduced

$$F_{\mathrm{LM}}^{\mathrm{nf}}(1,2,3,4) \equiv \mathcal{N}\int \mathrm{dLips}_{34H}\hat{\mathcal{O}}(\{p_{i=1,\dots,4},p_H\}) \times (2\pi)^d\delta^{(d)}(p_1+p_2-p_H-p_3-p_4) \times |A_0|^2(1,2,3,4) \quad (16)$$

to describe the leading-order process. Upon integration over the unresolved phase space of gluon g_5 we find

$$\left\langle S_5(2E_5)^{-2\epsilon}F_{\mathrm{LM}}^{\mathrm{nf}}(1,2,3,4|5)\right\rangle = -[\alpha_{s,b}](2E_{\mathrm{max}})^{-4\epsilon}\left\langle K_{\mathrm{nf}}F_{\mathrm{LM}}^{\mathrm{nf}}(1,2,3,4)\right\rangle, \quad (17)$$

where we suppressed the dependence of the function K_{nf} on the Born momenta.

Finally, we combine Eqs. (8), (14), (17) and replace

$$[\alpha_{s,b}] \to \frac{\tilde{\alpha}_s}{2\pi}\mu^{2\epsilon}, \quad (18)$$

where $\tilde{\alpha}_s = \alpha_s(\mu)e^{\epsilon\gamma_E}/\Gamma(1-\epsilon)$, to express the result through the strong coupling defined in the $\overline{\mathrm{MS}}$ scheme. The result is the fully regulated representation of the double-real contribution to nonfactorizable corrections

$$\left\langle F_{\mathrm{LM}}^{\mathrm{nf}}(1,2,3,4|5,6)\right\rangle = \left\langle [I-S_6]F_{\mathrm{LM}}^{\mathrm{nf}}(1,2,3,4|5,6)\right\rangle - 2\left(\frac{\tilde{\alpha}_s}{2\pi}\right)\kappa_{qQ} \times \left\langle [I-S_5]\left(\frac{2E_5}{\mu}\right)^{-2\epsilon}K_{\mathrm{nf}}F_{\mathrm{LM}}^{\mathrm{nf}}(1,2,3,4|5)\right\rangle + 2\left(\frac{\tilde{\alpha}_s}{2\pi}\right)^2\left(\frac{2E_{\mathrm{max}}}{\mu}\right)^{-4\epsilon}\left\langle K_{\mathrm{nf}}^2F_{\mathrm{LM}}^{\mathrm{nf}}(1,2,3,4)\right\rangle. \quad (19)$$

B. Real-virtual contribution

Next, we consider the real-virtual contribution to the NNLO QCD nonfactorizable corrections. It arises from the one-loop corrections to the process with an additional gluon in the final state

$$q(p_1)+Q(p_2)\to q'(p_3)+Q'(p_4)+g(p_5)+H(p_H). \quad (20)$$

The real-virtual contribution to the nonfactorizable correction is also proportional to the color factor shown in Eq. (2). Hence, following the discussion of the double-real contribution, we define a *color-stripped* amplitude A_1^i as a sum of Abelian diagrams where a virtual gluon is exchanged between the two quark lines and a real gluon is emitted from line i. Using this amplitude, we write the real-virtual contribution as

$$\begin{aligned} F_{\rm LV}^{\rm nf}&(1,2,3,4|5) \\ &\equiv \mathcal{N}\int {\rm dLips}_{34H}\hat{\mathcal{O}}(\{p_{i=1,\ldots,5},p_H\}) \\ &\times (2\pi)^d\delta^{(d)}\left(p_1+p_2-p_H-\sum_{i=3}^{5}p_i\right) \\ &\times 2{\rm Re}\left[A_0^1A_1^{2\star}+A_0^2A_1^{1\star}\right](1,2,3,4|5). \end{aligned} \tag{21}$$

The only singularity present in $F_{\rm LV}^{\rm nf}(1,2,3,4|5)$ arises in the soft, $E_5\to 0$ limit. To regulate it, we write

$$\begin{aligned} \left\langle F_{\rm LV}^{\rm nf}(1,2,3,4|5)\right\rangle &= \left\langle [I-S_5]F_{\rm LV}^{\rm nf}(1,2,3,4|5)\right\rangle \\ &\quad+\left\langle S_5F_{\rm LV}^{\rm nf}(1,2,3,4|5)\right\rangle. \end{aligned} \tag{22}$$

Although the first term in the above equation is fully regular inasmuch as the real emission is concerned, it contains an explicit infrared $1/\epsilon$ pole which arises as a result of the integration over the loop momentum. We extract it by writing [14]

$$\begin{aligned} F_{\rm LV}^{\rm nf}(1,2,3,4|5) &= \frac{\tilde{\alpha}_s}{2\pi}2\kappa_{qQ}I_1(\epsilon)F_{\rm LM}^{\rm nf}(1,2,3,4|5) \\ &\quad+F_{\rm LV,fin}^{\rm nf}(1,2,3,4|5), \end{aligned} \tag{23}$$

where

$$I_1(\epsilon)\equiv\frac{1}{\epsilon}\ln\left(\frac{p_1\cdot p_4p_2\cdot p_3}{p_1\cdot p_2p_3\cdot p_4}\right), \tag{24}$$

$F_{\rm LM}^{\rm nf}(1,2,3,4|5)$ is the color-stripped single-real emission contribution defined in Eq. (11) and $F_{\rm LV,fin}^{\rm nf}(1,2,3,4|5)$ is the $\mathcal{O}(\epsilon^0)$ coefficient in the ϵ-expansion of Eq. (21).

We now discuss the second term on the right-hand side of Eq. (22). The soft-gluon limit of any one-loop QCD amplitude is known [15]. It contains two terms—the product of the tree-level eikonal current and a one-loop amplitude without the soft gluon, as well as the product of a one-loop correction to the eikonal current and the relevant tree-level amplitude. Since the one-loop correction to the eikonal current is purely non-Abelian, it plays no role in the computation of nonfactorizable corrections. We discard it and write

$$\begin{aligned} S_5F_{\rm LV}^{\rm nf}(1,2,3,4|5) &= -2g_{s,b}^2\kappa_{qQ}\int[{\rm d}p_5]{\rm Eik}_{\rm nf}(1,2,3,4|5) \\ &\quad\times F_{\rm LV}^{\rm nf}(1,2,3,4), \end{aligned} \tag{25}$$

where we introduced a color-stripped one-loop virtual contribution

$$\begin{aligned} F_{\rm LV}^{\rm nf}(1,2,3,4)\equiv\mathcal{N}&\int{\rm dLips}_{34H}\times\hat{\mathcal{O}}(\{p_{i=1,\ldots,4},p_H\}) \\ &\times(2\pi)^d\delta^{(d)}(p_1+p_2-p_H-p_3-p_4) \\ &\times 2{\rm Re}\left[A_0A_1^\star\right](1,2,3,4). \end{aligned} \tag{26}$$

The integral over unresolved momentum p_5 in Eq. (25) evaluates to

$$\begin{aligned} \left\langle S_5F_{\rm LV}^{\rm nf}(1,2,3,4|5)\right\rangle &= -2\kappa_{qQ}\frac{\tilde{\alpha}_s}{2\pi}\left(\frac{2E_{\rm max}}{\mu}\right)^{-2\epsilon} \\ &\quad\times\left\langle K_{\rm nf}(1,2,3,4)F_{\rm LV}^{\rm nf}(1,2,3,4)\right\rangle. \end{aligned} \tag{27}$$

To proceed further, we note that $F_{\rm LV}^{\rm nf}(1,2,3,4)$ contains infrared poles from the loop integration. We make them explicit by writing

$$\begin{aligned} F_{\rm LV}^{\rm nf}(1,2,3,4) &= \frac{\tilde{\alpha}_s}{2\pi}2\kappa_{qQ}I_1(\epsilon)F_{\rm LM}^{\rm nf}(1,2,3,4) \\ &\quad+F_{\rm LV,fin}^{\rm nf}(1,2,3,4). \end{aligned} \tag{28}$$

The function $I_1(\epsilon)$ has already appeared in Eq. (24).

Combining Eqs. (22), (23), (27), and (28), we obtain the final result for the real-virtual contribution to the nonfactorizable corrections

$$\begin{aligned} &\left\langle F_{\rm LV}^{\rm nf}(1,2,3,4|5)\right\rangle \\ &\quad=\frac{\tilde{\alpha}_s}{2\pi}\kappa_{qQ}\left\langle 2I_1(\epsilon)[I-S_5]F_{\rm LM}^{\rm nf}(1,2,3,4|5)\right\rangle \\ &\quad\quad+\left\langle[I-S_5]F_{\rm LV,fin}^{\rm nf}(1,2,3,4|5)\right\rangle \\ &\quad\quad-4\left(\frac{\tilde{\alpha}_s}{2\pi}\right)^2\left(\frac{2E_{\rm max}}{\mu}\right)^{-2\epsilon}\left\langle I_1(\epsilon)K_{\rm nf}F_{\rm LM}^{\rm nf}(1,2,3,4)\right\rangle \\ &\quad\quad-2\frac{\tilde{\alpha}_s}{2\pi}\kappa_{qQ}\left(\frac{2E_{\rm max}}{\mu}\right)^{-2\epsilon}\left\langle K_{\rm nf}F_{\rm LV,fin}^{\rm nf}(1,2,3,4)\right\rangle. \end{aligned} \tag{29}$$

C. Double-virtual contribution

The last contribution that we need to consider is the double-virtual nonfactorizable correction to the process

$$q(p_1)+Q(p_2)\to q'(p_3)+Q'(p_4)+H(p_H). \tag{30}$$

We write the double-virtual amplitude of this process separating the $1/\epsilon$ infrared poles from the finite remainder using the results in Ref. [16]. Since the nonfactorizable corrections are Abelian, the divergent structure of the loop amplitude is fully determined by the square of $I_1(\epsilon)$; cf. Eq. (24). We write

$$\left\langle F^{\rm nf}_{\rm LVV}(1,2,3,4)\right\rangle = \left(\frac{\tilde{\alpha}_s}{2\pi}\right)^2\left\langle 2I_1(\epsilon)^2 F^{\rm nf}_{\rm LM}(1,2,3,4)\right\rangle + \frac{\tilde{\alpha}_s}{2\pi}\kappa_{qQ}\left\langle 2I_1(\epsilon)F^{\rm nf}_{\rm LV,fin}(1,2,3,4)\right\rangle + \left\langle F^{\rm nf}_{\rm LVV,fin}(1,2,3,4)\right\rangle, \quad (31)$$

where $F^{\rm nf}_{\rm LVV,fin}$ contains the finite result for the interference of the two-loop and tree-level amplitudes as well as the one-loop amplitude squared.

D. Explicit pole cancellation and IR finite result

The final result for the cross section is obtained by combining the double-real, real-virtual, and double-virtual contributions given in Eqs. (19), (29), and (31), respectively. We write the partonic cross section as

$$\begin{aligned} d\sigma^{\rm nf}_{\rm nnlo} &= \frac{T_R^2(N_c^2-1)}{2s}\left[\left\langle F^{\rm nf}_{\rm LM}(1,2,3,4|5,6)\right\rangle + \left\langle F^{\rm nf}_{\rm LV}(1,2,3,4|5)\right\rangle + \left\langle F^{\rm nf}_{\rm LVV}(1,2,3,4)\right\rangle\right] \\ &= \frac{T_R^2(N_c^2-1)}{2s}\Bigg[\left\langle [I-S_6]F^{\rm nf}_{\rm LM}(1,2,3,4|5,6)\right\rangle - 2\frac{\tilde{\alpha}_s}{2\pi}\left\langle [I-S_5]\mathcal{W}(E_5;1,\dots,4)F^{\rm nf}_{\rm LM}(1,2,3,4|5)\right\rangle \\ &\quad + 2\left(\frac{\tilde{\alpha}_s}{2\pi}\right)^2\left\langle \mathcal{W}(E_{\rm max};1,\dots,4)^2 F^{\rm nf}_{\rm LM}(1,2,3,4)\right\rangle + \left\langle [I-S_5]F^{\rm nf}_{\rm LV,fin}(1,2,3,4|5)\right\rangle \\ &\quad - 2\frac{\tilde{\alpha}_s}{2\pi}\left\langle \mathcal{W}(E_{\rm max};1,\dots,4)F^{\rm nf}_{\rm LV,fin}(1,2,3,4)\right\rangle + \left\langle F^{\rm nf}_{\rm LVV,fin}(1,2,3,4)\right\rangle\Bigg]. \end{aligned} \quad (32)$$

In Eq. (32) we introduced a finite function $\mathcal{W}(E;1,2,3,4)$ defined as[3]

$$\begin{aligned} \mathcal{W}(E;1,2,3,4) &\equiv \kappa_{qQ}\left[\left(\frac{2E}{\mu}\right)^{-2\epsilon}K_{\rm nf}(\epsilon) - {\rm I}_1(\epsilon)\right] \\ &= \kappa_{qQ}\Bigg[-2\ln\left(\frac{2E}{\mu}\right)\ln\left(\frac{p_1\cdot p_4 p_3\cdot p_2}{p_1\cdot p_2 p_3\cdot p_4}\right) \\ &\quad + \sum_{\substack{i\in\{1,3\}\\ j\in\{2,4\}}}\lambda_{ij}\left(\frac{1}{2}\ln^2(\eta_{ij}) + {\rm Li}_2(1-\eta_{ij})\right)\Bigg] \\ &\quad + \mathcal{O}(\epsilon), \end{aligned} \quad (33)$$

where $\eta_{ij} = 1-\cos\theta_{ij}$ with angles defined in the partonic center-of-mass frame. The representation of the partonic cross section given in Eq. (32) makes the cancellation of all $1/\epsilon$ poles manifest and allows us to take the $\epsilon\to 0$ limit right away. Note that upon doing so, the coupling constant $\tilde{\alpha}_s$ becomes $\alpha_s(\mu)$, the standard $\overline{\rm MS}$ coupling constant.

III. NUMERICAL IMPLEMENTATION

The numerical implementation of the nonfactorizable contribution Eq. (32) requires double-real amplitudes as well as finite parts of real-virtual amplitudes and double-virtual amplitudes. To obtain the required double-real amplitudes, we extend the calculation of the factorizable NNLO QCD corrections reported in Ref. [4].

To compute the real-virtual contributions, we require nonfactorizable one-loop amplitudes for the processes $q+Q\to q'+Q'+H$ and $q+Q\to q'+Q'+H+g$. These amplitudes were computed in Ref. [11], and we employ them in our numerical implementation. Extracting the nonfactorizable contribution from the existing code requires only minor changes.[4] However, it turns out to be nontrivial to achieve stable and reliable numerical results close to singular limits.

The existing implementation uses numerical Passarino-Veltman reduction point by point and the ONELOOP library [17] for the evaluation of scalar integrals. To reach sufficient numerical accuracy we limit catastrophic cancellation by working with scaleless $\mathcal{O}(1)$ quantities. This is achieved by scaling out the energy of the incoming partons in all momenta and masses in each phase space point and reintroducing it at the very end of the calculation.

Furthermore, we find it necessary to work with quadruple precision. With these two measures we achieve agreement with the infrared pole prediction in Eq. (24) to more than ten digits for most phase space points. In addition to checking the amplitude's pole structure, we also find a satisfactory agreement between the exact six-point amplitude and its expected limit when the energy of the final-state gluon becomes small; see Eq. (25). Obviously, this last feature is a necessary requirement for being able to use Eq. (32) for phenomenological studies.

For the finite remainder of the two-loop amplitude, $F^{\rm nf}_{\rm LVV,fin}$, we use the results of Ref. [8]. These results are obtained in the eikonal approximation which provides the leading term in the expansion of this amplitude in $p_\perp/\sqrt{s}$ where $p_\perp$ is a typical transverse momentum of the

[3]The ϵ-expansion of function $K_{\rm nf}$ can be found in Appendix A; see Eq. (A2).

[4]We are grateful to T. Figy for making the code used for the computations reported in Ref. [11] available to us.

final-state tagging jets. This approximation is motivated by typical WBF signatures and the fiducial selection cuts derived from them.[5]

As a final comment we note that the finite remainder of the double-virtual amplitude [8] that we use in this computation is an approximation to the exact result which, so far, remains unknown. In particular, the amplitude computed in the eikonal approximation [8] is infrared finite which means that there is no connection between the first two terms on the right-hand side of Eq. (31), required to cancel divergences in the double-real and real-virtual contributions, and $\langle F^{\rm nf}_{\rm LVV,fin}(1,2,3,4)\rangle$. However, as we will show in Sec. IV, it is quite unlikely that the missing parts of the finite remainder of the double-virtual amplitude that are linked to the cancellation of infrared divergences can impact the phenomenology of weak boson fusion in a significant way. We provide reference evaluations of the finite remainder in an ancillary file [19].

IV. RESULTS

The goal of this section is to compute the nonfactorizable NNLO QCD corrections to Higgs boson production in weak boson fusion and to compare them to the factorizable ones. To do that, we adopt standard parameters and kinematic selection criteria from Refs. [5,13]; we reproduce them here for completeness.

We consider 13 TeV proton-proton collisions. The Higgs boson is chosen to be stable with a mass of $m_H = 125$ GeV. Vector boson masses are taken to be $M_W = 80.398$ GeV and $M_Z = 91.1876$ GeV with widths $\Gamma_W = 2.105$ GeV and $\Gamma_Z = 2.4952$ GeV, respectively. Weak couplings are derived from the Fermi constant $G_F = 1.16639\times 10^{-5}\ {\rm GeV}^{-2}$, and the Cabibbo–Kobayashi–Maskawa matrix (CKM) is set to the identity matrix.

We use `NNPDF31-nnlo-as-118` parton distribution functions [20] and $\alpha_s(M_Z) = 0.118$ for all calculations reported below. The evolution of both parton distribution functions and the strong coupling constant is obtained directly from LHAPDF [21]. The dynamical renormalization and factorization scales are set equal, $\mu_R = \mu_F = \mu$, with the central value [3]

$$\mu_0 = \sqrt{\frac{m_H}{2}\sqrt{\frac{m_H^2}{4} + p_{\perp,H}^2}}. \tag{34}$$

To define the WBF fiducial volume we employ the inclusive anti-$k_\perp$ jet algorithm [22] with $R = 0.4$. Events are required to contain at least two jets with transverse momenta $p_{\perp,j} > 25$ GeV and rapidities $|y_j| < 4.5$. The two leading-$p_\perp$ jets must have well-separated rapidities, $|y_{j_1} - y_{j_2}| > 4.5$, and their invariant mass should be larger than 600 GeV. In addition, the two leading jets must be in separate hemispheres in the laboratory frame; this is enforced by requiring that the product of their rapidities in the laboratory frame is negative, $y_{j_1} y_{j_2} < 0$.

The analysis of the double-virtual contribution to the nonfactorizable correction to Higgs boson production in weak boson fusion has already been performed in Refs. [8,9]. The new elements that we add to this analysis are the double-real and real-virtual contributions. Although typically one expects that all types of contributions are comparable in magnitude, we find that for Higgs production in WBF this is not the case.

For example, computing the nonfactorizable NNLO QCD corrections to the fiducial WBF cross section for central values of the renormalization and factorization scales and for values of parameters as described above, we find

$$\sigma_{\rm nf} = -3.1^{+0.7}_{-0.9}\ {\rm fb}. \tag{35}$$

The subscript and superscript correspond to results by varying the dynamic scale in Eq. (34) by a factor of 0.5 and 2, respectively. We do not show the Monte Carlo integration error; note, however, that it is negligible and at the level of a few per mille.[6]

We note that the significant scale uncertainty stems from the fact that nonfactorizable corrections appear at NNLO *for the very first time* and there is no mechanism to e.g. compensate the change in the strong coupling constant when the renormalization scale is modified. For this reason it is not surprising that we find $\mathcal{O}(40\%)$ uncertainty in $\sigma_{\rm nf}$ upon varying μ_R and μ_F within an interval $[\mu_0/2, 2\mu_0]$. We also note that $\sigma_{\rm nf}$ provides $\mathcal{O}(0.5)\%$ correction to the fiducial cross section computed through NNLO QCD in the factorization approximation [4] and is about a factor of 10 smaller than the factorizable NNLO QCD corrections.

As we already mentioned, one would normally expect that double-virtual, real-virtual, and real-real corrections provide comparable contributions to $\sigma_{\rm nf}$. However, it turns out that this is not the case and that *only* 0.01% of $\sigma_{\rm nf}$ comes from the real-virtual and the double-real contributions, whereas the dominant 99.99% comes from the double-virtual one.

This relation between the double-virtual and all the other contributions holds for all kinematic distributions that we considered. To give some examples, in Fig. 3 we show the different contributions to the transverse momentum distributions of the hardest jet and the distribution of the invariant mass of the pair of leading jets.

[5]We note that fully analytic result for the leading eikonal approximation is available in Ref. [18].

[6]The discrepancy with the result reported in Ref. [8] is due to a different choice of dynamical scale and the use of an approximate tree-level amplitude in that reference.

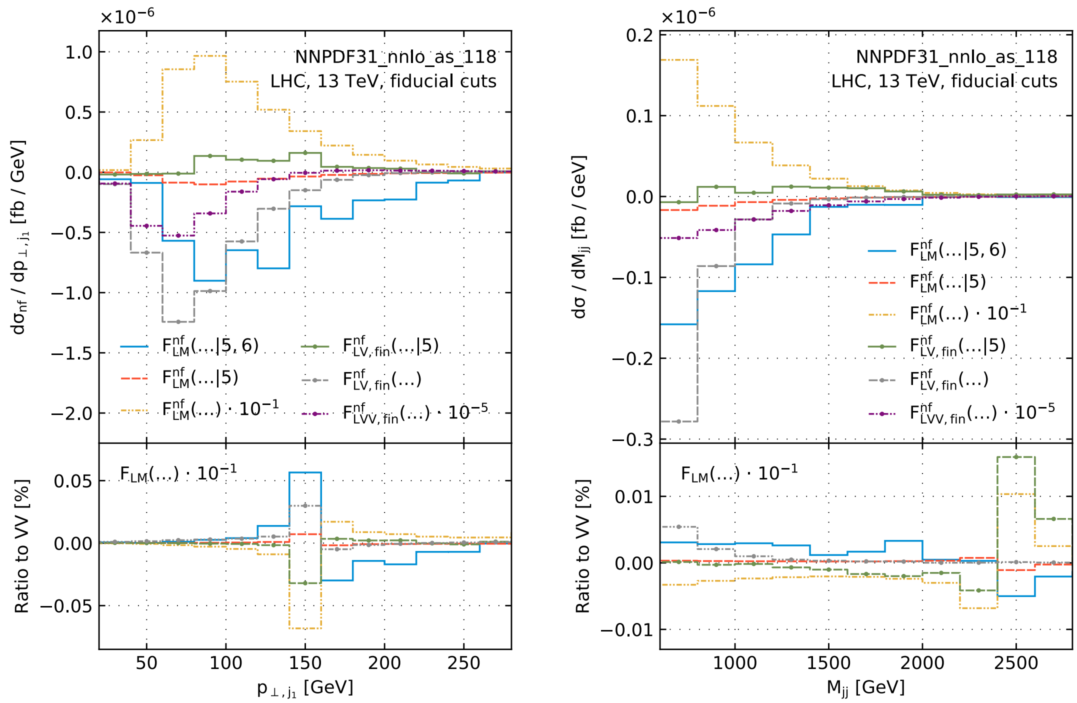

FIG. 3. Nonfactorizable contribution to the transverse momentum distributions of the leading jet (left) and to the distribution of the invariant mass of the tag-jet system (right). Contributions are shown individually for different terms on the right-hand side of Eq. (32), and we label them with the present matrix element; e.g. the plot label $F^{\rm nf}_{\rm LM}(1,2,3,4|5)$ refers to the contribution of the *full* second term. Note that in the plots we use ellipses for the sequence of Born momenta, 1,2,3,4, for representational purposes. For each plot (and differently in upper and lower panes) contributions are scaled to be of similar orders. The lower pane shows the ratio with respect to double-virtual contributions. See text for further details.

To understand the reason for this unusual suppression of the double-real and the real-virtual contributions, consider the quantity

$$L(1,2,3,4) = \ln\left(\frac{p_1\cdot p_4 p_3\cdot p_2}{p_1\cdot p_2 p_3\cdot p_4}\right), \quad (36)$$

which arises upon integration of the eikonal current describing single gluon emission. We note that this quantity appears in the integrated subtraction term described by the function $\mathcal{W}(E;1,2,3,4)$ defined in Eq. (33).

For instance, to estimate the contribution of two soft gluons to the nonfactorizable corrections in the presence of fiducial WBF cuts, we consider the following integral:

$$\sigma_{RR} \sim \left(\frac{\tilde{\alpha}_s}{2\pi}\right)^2 N_c^2 \langle L^2(1,2,3,4) F^{\rm nf}_{\rm LM}(1_q,2_q,3_q,4_q)\rangle. \quad (37)$$

To proceed we use the fact that in the relevant phase-space region p_3 and p_4 are nearly collinear to p_1 and p_2, respectively, and compute the function L in this limit. To this end, we write

$$p_3 = \alpha_3 p_1 + \beta_3 p_2 + p_{3,\perp},$$
$$p_4 = \alpha_4 p_1 + \beta_4 p_2 + p_{4,\perp}, \quad (38)$$

where $\alpha_3, \beta_4 \sim 1$ and

$$p_{i,\perp}\cdot p_1 = p_{i,\perp}\cdot p_2 = 0, \quad (39)$$

for $i\in\{3,4\}$. From the mass-shell condition for outgoing quarks, we obtain

$$\beta_3 \sim \frac{p^2_{3,\perp}}{s} \ll 1, \qquad \alpha_4 \sim \frac{p^2_{4,\perp}}{s} \ll 1. \quad (40)$$

We thus find

$$L(1,2,3,4) = -\ln\left(1+\frac{\beta_3\alpha_4}{\alpha_3\beta_4} - \frac{2\vec{p}_{3,\perp}\cdot\vec{p}_{4,\perp}}{s\alpha_3\beta_4}\right) \approx \frac{2\vec{p}_{3,\perp}\cdot\vec{p}_{4,\perp}}{s}. \quad (41)$$

A typical transverse momentum in Higgs production in weak boson fusion is ~60 GeV, and a typical partonic center-of-mass energy is approximately $\sqrt{s}\approx 600$ GeV. Therefore, $L\sim 10^{-2}$ in the relevant region of the partonic phase space, and we find

$$\sigma_{RR} \sim \left(\frac{\tilde{\alpha}_s}{2\pi}\right)^2 N_c^2 \left\langle L^2(1,2,3,4) F_{\rm LM}^{\rm nf}(1_q,2_q,3_q,4_q)\right\rangle \sim \left(\frac{\tilde{\alpha}_s}{2\pi}\right)^2 10^{-4}\sigma_{\rm LO}, \tag{42}$$

where we used $N_c^2\langle F_{\rm LM}^{\rm nf}(1_q,2_q,3_q,4_q)\rangle = \sigma_{\rm LO}$.

In comparison, virtual corrections do not vanish in the forward region. In fact, as shown in Ref. [8], they are characterized by a phase-space dependent function $\chi_{\rm nf}$ which is $\mathcal{O}(\pi^2)$ in the forward region. We then estimate

$$\sigma_{VV} \sim \left(\frac{\tilde{\alpha}_s}{2\pi}\right)^2 N_c^2 \left\langle \chi_{\rm nf}(1,2,3,4) F_{\rm LM}^{\rm nf}(1,2,3,4)\right\rangle \approx \left(\frac{\tilde{\alpha}_s}{2\pi}\right)^2 10\sigma_{\rm LO}, \tag{43}$$

where we used $\pi^2 \approx 10$. Taking the ratio, we obtain

$$\frac{\sigma_{RR}}{\sigma_{VV}} \sim 10^{-5}, \tag{44}$$

which is consistent with the results of the explicit computation presented earlier in this section.

We have checked that the extraordinarily strong suppression of the double-real and real-virtual corrections is a consequence of the fiducial cuts which are used to identify events when the Higgs boson is produced in weak boson fusion. If the cuts are relaxed so that one does not require strong rapidity separation of the two tagging jets and a strong constraint on their invariant mass, the double-real and real-virtual contributions increase by several orders of magnitude. In fact, they become comparable to the virtual corrections which only grow by an $\mathcal{O}(1)$ factor.

V. CONCLUSIONS

In this paper we extended the calculation of nonfactorizable contributions to Higgs boson production in weak boson fusion at $\mathcal{O}(\alpha_s^2)$ by combining the results for the double-virtual contributions in the eikonal approximation [8] with nonfactorizable real-virtual and double-real QCD corrections. We observed that, thanks to the fiducial cuts used to identify WBF events, and a peculiar enhancement of the double-virtual contributions, the nonfactorizable NNLO QCD corrections are entirely dominated by virtual effects. We have checked that the striking dominance of the two-loop virtual corrections extends to all major kinematic distributions relevant for Higgs production in WBF.

Outside the fiducial region the relative importance of the various contributions levels out. However, the eikonal approximation will also start to break down. It would, therefore, be interesting to understand how to go beyond the eikonal approximation for the double-virtual amplitude and estimate the impact of nonvanishing transverse momenta of the final-state jets on the two-loop correction. This question may be of some relevance for studies that select harder Higgs bosons which happens, for example, when one considers Higgs decays into a b-quark pair. Furthermore, identical-flavor contributions are expected to increase when relaxing the fiducial cuts. We leave these questions for future investigations.

ACKNOWLEDGMENTS

We thank S. Plätzer for useful conversations. We are grateful to T. Figy for providing a Fortran code to compute the one-loop amplitudes for the $qQ \to q'Q' + H + g$ process and for explaining to us how to use it. This research is partially supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Grant No. 396021762—TRR 257. The research of K. A. is supported by the United States Department of Energy under Grant Contract No. DE-SC0012704. The work of C. B. H. presented here is supported by the Carlsberg Foundation, Grant No. CF21-0486.

APPENDIX: INTEGRATED SOFT EIKONAL

In this appendix we present results for the integrated soft eikonal function that we have written in terms of the function $K_{\rm nf}$; cf. Eq. (12). The exact form of $K_{\rm nf}$ reads

$$K_{\rm nf}(1_q,2_Q,3_{q'},4_{Q'};\epsilon) = \frac{1}{\epsilon^2}\left[\frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\right] \sum_{\substack{i\in\{1,3\}\\ j\in\{2,4\}}} \lambda_{ij}\eta_{ij\,2}{\rm F}_1 \times (1,1;1-\epsilon;1-\eta_{ij}), \tag{A1}$$

where we use $\eta_{ij} \equiv 1-\cos\theta_{ij} \equiv (p_i\cdot p_j)/(2E_iE_j)$.

It may appear from Eq. (A1) that the function $K_{\rm nf}$ contains second-order poles in ϵ. This, however, cannot be the case since collinear singularities cannot appear in nonfactorizable contributions. An explicit computation yields the result that confirms this expectation. Expanding $K_{\rm nf}$ in ϵ, we obtain

$$K_{\rm nf}(1_q,2_Q,3_{q'},4_{Q'};\epsilon) = \frac{1}{\epsilon}\ln\left(\frac{p_1\cdot p_4 p_3\cdot p_2}{p_1\cdot p_2 p_3\cdot p_4}\right) + \sum_{\substack{i\in\{1,3\}\\ j\in\{2,4\}}} \lambda_{ij}\left(\frac{1}{2}\ln^2(\eta_{ij}) + {\rm Li}_2(1-\eta_{ij})\right) + \mathcal{O}(\epsilon). \tag{A2}$$

[1] T. Figy, C. Oleari, and D. Zeppenfeld, Phys. Rev. D **68**, 073005 (2003); E. L. Berger and J. M. Campbell, Phys. Rev. D **70**, 073011 (2004); T. Figy and D. Zeppenfeld, Phys. Lett. B **591**, 297 (2004).

[2] M. Ciccolini, A. Denner, and S. Dittmaier, Phys. Rev. Lett. **99**, 161803 (2007); Phys. Rev. D **77**, 013002 (2008); T. Figy, S. Palmer, and G. Weiglein, J. High Energy Phys. 02 (2012) 105.

[3] P. Bolzoni, F. Maltoni, S.-O. Moch, and M. Zaro, Phys. Rev. Lett. **105**, 011801 (2010); Phys. Rev. D **85**, 035002 (2012); M. Cacciari, F. A. Dreyer, A. Karlberg, G. P. Salam, and G. Zanderighi, Phys. Rev. Lett. **115**, 082002 (2015); **120**, 139901(E) (2018); J. Cruz-Martinez, T. Gehrmann, E. W. N. Glover, and A. Huss, Phys. Lett. B **781**, 672 (2018).

[4] K. Asteriadis, F. Caola, K. Melnikov, and R. Röntsch, J. High Energy Phys. 02 (2022) 046.

[5] K. Asteriadis, F. Caola, K. Melnikov, and R. Röntsch, Phys. Rev. D **107**, 034034 (2023).

[6] F. A. Dreyer and A. Karlberg, Phys. Rev. Lett. **117**, 072001 (2016).

[7] T. Chen, T. M. Figy, and S. Plätzer, Eur. Phys. J. C **82**, 704 (2022).

[8] T. Liu, K. Melnikov, and A. A. Penin, Phys. Rev. Lett. **123**, 122002 (2019).

[9] F. A. Dreyer, A. Karlberg, and L. Tancredi, J. High Energy Phys. 10 (2020) 131.

[10] C. Brønnum-Hansen, K. Melnikov, J. Quarroz, C. Signorile-Signorile, and C.-Y. Wang, J. High Energy Phys. 06 (2022) 061.

[11] F. Campanario, T. M. Figy, S. Plätzer, and M. Sjödahl, Phys. Rev. Lett. **111**, 211802 (2013).

[12] F. Caola, K. Melnikov, and R. Röntsch, Eur. Phys. J. C **77**, 248 (2017).

[13] K. Asteriadis, F. Caola, K. Melnikov, and R. Röntsch, Eur. Phys. J. C **80**, 8 (2020).

[14] W. T. Giele and E. W. N. Glover, Phys. Rev. D **46**, 1980 (1992); Z. Kunszt, A. Signer, and Z. Trocsanyi, Nucl. Phys. **B420**, 550 (1994); S. Catani and M. H. Seymour, Phys. Lett. B **378**, 287 (1996); Nucl. Phys. **B485**, 291 (1997); **B510**, 503(E) (1998).

[15] Z. Bern, V. Del Duca, W. B. Kilgore, and C. R. Schmidt, Phys. Rev. D **60**, 116001 (1999); D. A. Kosower and P. Uwer, Nucl. Phys. **B563**, 477 (1999); S. Catani and M. Grazzini, Nucl. Phys. **B591**, 435 (2000).

[16] S. Catani, Phys. Lett. B **427**, 161 (1998).

[17] A. van Hameren, Comput. Phys. Commun. **182**, 2427 (2011).

[18] L. Gates, arXiv:2305.04407.

[19] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevD.109.014031 for reference evaluations of the finite remainder of the double-virtual amplitude.

[20] R. D. Ball *et al.* (NNPDF Collaboration), Eur. Phys. J. C **77**, 663 (2017).

[21] A. Buckley, J. Ferrando, S. Lloyd, K. Nordström, B. Page, M. Rüfenacht, M. Schönherr, and G. Watt, Eur. Phys. J. C **75**, 132 (2015).

[22] M. Cacciari, G. P. Salam, and G. Soyez, J. High Energy Phys. 04 (2008) 063.