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Heuristic reoptimization of time-extended multi-robot task allocation problems

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Abstract

Providing high quality solutions is crucial when solving NP-hard time-extended multi-robot task allocation (MRTA) problems. Reoptimization, that is, the concept of making use of a known solution to an optimization problem instance when the solution to a similar problem instance is sought, is a promising and rather new research field in this application domain. However, so far no approximative time-extended MRTA solution approaches exist for which guarantees on the resulting solution's quality can be given. We investigate the reoptimization problems of inserting as well as deleting a task to/from a time-extended MRTA problem instance. For both problems, we can give performance guarantees in the form of an upper bound of 2 on the resulting approximation ratio for all heuristics fulfilling a mild assumption. We furthermore introduce specific solution heuristics and prove that smaller and tight upper bounds on the approximation ratio can be given for these heuristics if only temporal unconstrained tasks and homogeneous groups of robots are considered. A conclusory evaluation of the reoptimization heuristic demonstrates a near-to-optimal performance in application.

KEYWORDS

approximation ratio, performance guarantees, reoptimization, task deletion, task insertion, time-extended multi-robot task allocation

1 | INTRODUCTION

Multi-robot systems have gained increasing attention in recent years within various application domains including agriculture [30], cleaning work [14] and planetary exploration [28, 29]. For the success of these systems, a purposeful coordination of the robotic team is essential. To address the coordination problem, multi-robot task allocation (MRTA) solves the combinatorial optimization problem of defining which task to allocate to which robot, given a set of tasks and a set of robots to perform these tasks [16, 20]. If the scheduling of the tasks is also sought, so called time-extended MRTA problems arise. Time-extended MRTA problems are NP-hard [16], which makes the determination of exact solutions untraceable for larger problem instances. Therefore, literature reports a plethora of heuristic and meta-heuristic solution approaches for solving these kinds of problems in a centralized manner including for example, genetic algorithms [21], memetic algorithms [24] or local search [6].

Besides solving time-extended MRTA optimization problems from scratch, the concept of reoptimization, that is, reusing a solution to an optimization problem instance to solve a slightly modified problem instance, has recently gained attention for application to MRTA problems [7, 17]. The usage of reoptimization techniques seems especially promising in application domains where changes in the problem instance may occur. This may for example be the case in automated warehouses, when an order is canceled or additional orders arrive. Also in application domains in which the tasks to be performed by the robotic team are defined by a human operator, as this is for example the case in planetary exploration [28, 29], it can occur that the operator adds or deletes tasks after a solution to the initial coordination problem had already been computed. In such cases, the initially determined solution might still contain information relevant for the modified problem instance such that using reoptimization instead of optimization techniques can be beneficial w.r.t. calculation time or recognizability of the initial solution.

No matter whether optimization or reoptimization techniques are used, they are only relevant for practical application, if they consider all constraints of the MRTA problem instance and meet application specific quality requirements. Solutions not considering all constraints might not be executable in practice and highly sub-optimal solutions can be associated with significantly increased mission duration, energy consumption and/or other costs. Therefore, efficient solution approaches for which guarantees on the resulting solution quality can be given, are desirable. To the best of our knowledge, no approximative time-extended MRTA solution approach with proven guarantees on the solution quality exists.

In this article, we investigate reoptimization both for inserting a new task into an initial solution as well as deleting a task out of an initial solution to time-extended MRTA problems. For both modifications, we derive guarantees on the resulting solution quality in the form of upper bounds on the resulting approximation ratios, that is, the ratio between the costs of the solution generated by the heuristic reoptimization and the costs of a globally optimal solution of the same problem instance. In order to account for a variety of application areas, we conduct this analysis both for homogeneous groups of robots as well as for heterogeneous groups w.r.t. driving velocity and task capabilities. Furthermore, we also consider MRTA problems with precedence and synchronization constraints between tasks.

1.1 | Related work

Reoptimization in the context of MRTA is a rather new field of research and to the best of our knowledge so far only few solution approaches exist. For unmanned underwater vehicles Giger [17] introduced a replanning framework based on a genetic algorithm for reassigning tasks. However, only homogeneous groups of agents were considered. In [7] Bischoff et al. proposed a reoptimization framework for time-extended MRTA problems considering heterogeneous groups of robots and timely ordering constraints. The framework is based on a genetic algorithm and studies various problem instance modifications like adding and deleting a task, robot, or timely constraint. For both approaches however, no guarantees on the resulting solution quality are given.

More research in reoptimization has been conducted in the area of the vehicle routing problem (VRP), a coordination problem related to time-extended MRTA problems. It is defined on weighted graphs in which for each vehicle a circuit (route) through the graph must be found such that all customer nodes are visited exactly once. The reoptimization problem in this area, the so-called dynamic vehicle routing problem (DVRP), strives to incorporate dynamically arising problem modifications online into the existing routing plan. DVRPs with various characteristics have been studied, including the consideration of differing transportation modes, varying logistical contexts like pickup and/or delivery problems, the consideration of application specific constraints, different modifications like the insertion of new customer requests or changes in travel time, and others [27]. A great variety of solution methods for these NP-hard problems [27] has been proposed including tabu search [15, 22], genetic algorithms [1], local search approaches [23], particle swarm optimization [19] and insertion heuristics [13]. Nevertheless, for all of the above approaches guarantees on the resulting solution quality are either impossible to give due to the nature of the applied metaheuristics or have not been investigated by the research.

In contrast to this, reoptimization heuristics with guarantees on the resulting solution quality have already been studied for the traveling salesman problem (TSP), that is, the problem of optimally routing one mobile entity to visit all nodes within a graph exactly once at minimal costs. The TSP correlates to the special case of a time-extended MRTA with only one robot which has to perform all given tasks. The guarantees on the solution quality are given in the form of upper bounds on the resulting approximation ratios. Archetti et al. [2] proved the TSP reoptimization problems of adding a task to remain NP-hard. They studied the so-called *cheapest insertion procedure* to insert an additional task to the prior solution and proved the resulting approximation ratio to be bounded above by $3/2$ in the case of metric distances. The same reoptimization problem was studied by Aussiello et al. [3, 4]. In [4], they proposed a new heuristic which chooses the best solution between the one generated by the Christofides algorithm and the cheapest insertion procedure and proved it to lead to a guaranteed approximation ratio of $4/3$. Another composed reoptimization heuristic with lower complexity leading to the same guaranteed approximation ratio was later presented by Monnot [26]. The deletion of a node has also already been studied for the TSP. Archetti et al. [2] prove the corresponding reoptimization problem to be NP-hard. They introduce a deletion heuristic and guarantee the resulting approximation ratio to be bounded above by 3/2 for metric and symmetric edge costs. In the case of asymmetric edge costs, Ausiello et al. [3] prove the approximation ratio to be bounded above by 2. Besides the reoptimization problem of inserting or deleting a task of a TSP problem instance, also reoptimization heuristics with guarantees on the solution quality for changing single or multiple edge weights have been investigated [5, 8–10]. Furthermore, reoptimization approaches for the TSP with deadlines have also been studied [8, 10–12].

Despite these promising theoretical insights into heuristic reoptimization allowing for guaranteed worst-case approximation ratios when applied to the TSP, so far no performance guarantees can be given when reoptimization is applied to routing and scheduling problems considering more than one mobile entity, including a group of mobile entities with heterogeneous capabilities and heterogeneous tasks.

1.2 | Our contribution

In this article, we investigate heuristic reoptimization solutions to the time-extended MRTA reoptimization problems of inserting and deleting a task and focus on theoretical insights on worst-case performance ratios. We consider an optimal solution to a time-extended MRTA problem instance to be known. A problem instance and its solution are modeled graph-based where tasks and initial robot positions are represented by nodes. For modeling the reoptimization problems, we add a node representing the new task to the respective initial problem instance or delete a node out of the initial problem instance respectively and search for a solution to the associated modified problem instances.

For the task insertion problem, we derive a condition under which we can guarantee an upper bound of 2 on the approximation ratio. This guarantee holds independently of the applied reoptimization heuristic, as long as the respective condition is fulfilled. We then introduce and investigate the *cheapest maximum cost insertion heuristic*, a modification of the cheapest insertion heuristic to become expressive and applicable for the MRTA problems under consideration, and prove it to always fulfill the condition an thus to guarantee the respective approximation ratio. Three MRTA problem configurations are introduced, differing in whether heterogeneous groups of robots and temporal task constraints are considered. For temporally unconstrained problem instances smaller upper bounds on the approximation ratios are derived, the smallest one being 3/2 for homogeneous groups of robots. We furthermore prove the given bounds to be tight, meaning that no smaller upper bounds for the respective heuristics exist.

For the task deletion reoptimization problem, we prove the approximation ratio to be bounded above by 2 under weak assumptions. To solve the reoptimization problem, we apply a simple task deletion heuristic, for which we prove the bound to be tight. For the special case of homogeneous robots and a common depot, we guarantee the approximation ratio to be bounded above by $3/2$. The proposed reoptimization heuristics are examined in simulation which reveals on the one hand that the worst case guarantees are satisfied and on the other hand that the approximation ratios in the evaluated scenarios are even much lower than the worst case guarantees.

The article is structured as follows: In Section 2, we give a formal definition of the underlying MRTA optimization problem and of the considered reoptimization problems. In Sections 3 and 4, we present our main contributions: In Section 3, we derive a generally valid upper bound of 2 on the approximation ratio for the MRTA task insertion problem under a weak assumption. Thereafter, we introduce and analyze a specific reoptimization heuristic for which we subsequently derive even smaller upper bounds on the approximation ratio if temporally unconstrained MRTA reoptimization problems are considered. In Section 4 the respective analysis is conducted for the MRTA task deletion problem. This includes generally valid results on the approximation ratio as well as an analysis of a proposed heuristic. The results of the practical investigation are given in Section 5.

2 | PROBLEM DEFINITION

In this section, the general heterogeneous MRTA optimization problem and the notation used throughout the article is presented, followed by the definition of the reoptimization problem and its different configurations under consideration.

2.1 | Heterogeneous MRTA optimization problem

In an instance $\mathcal{I} = \{T, \mathcal{R}, S, V, \mathcal{T}, A, P, G\}$ of the the heterogeneous MRTA problem, a set of tasks $T = \{t_1, \dots, t_N\}$, $N \in \mathbb{N}$, is to be conducted by a heterogeneous set of mobile robots $\mathcal{R} = \{r^1, \dots, r^K\}$, $K \in \mathbb{N}$. Each robot $r^k \in \mathcal{R}$ starts and ends its route at its individual depot $s^k \in S = \{s^1, \dots, s^K\}$ and moves with a constant individual velocity $v^k \in \mathbb{R}^{>0}$, that is, $V = \{v^1, \dots, v^K\}$ defines the set of robot's velocities. The problem is defined on a complete graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where the set of nodes $\mathcal{V} = T \cup S$ contains all task nodes and robot depots. The edges $\mathcal{E}$ are associated with metric edge costs described by distances $d : \mathcal{V} \times \mathcal{V} \to \mathbb{R}^{\geq 0}$. The set of task durations $\mathcal{T} = \{\tau_1, \dots, \tau_N\}$, $\tau_i \in \mathbb{R}^{\geq 0}\ \forall \tau_i \in \mathcal{T}$, defines for each task a base task duration. Taking into account a capability $a_i^k \in [0, 1] \subset \mathbb{R}$ for every robot $r^k \in \mathcal{R}$, that is, $A = \left\{ a_i^k \middle| t_i \in T, r^k \in \mathcal{R} \right\}$, a robot-dependent task duration d_i^k follows. It is given by $d_i^k = \tau_i / a_i^k$ if $a_i^k > 0$ and is set to $d_i^k = \infty$ elsewise. Analogously, a robot dependent edge cost $d_{i,j}^k = d(i,j)/v^k$ resembling the driving time needed by robot $r^k \in \mathcal{R}$ to cover the distance $d(i,j)$ can be defined. The heterogeneity of the robotic team is thereby described by the set of velocities V and the set of robot-dependent task capabilities A.

Furthermore, the order of task execution can be timely constrained. The set of precedence constraints P contains all constraints $p_{i,j} \in \{0,1\}$, $i,j \in T$, $i \neq j$, where $p_{i,j} = 1$ determines that task t_j must not start before task t_i has been finished. The elements $g_{i,j} \in \{0,1\}$, $i,j \in T$, $i \neq j$, of the synchronization set G constrain the execution of tasks t_i and t_j, for which $g_{i,j} = 1$ holds, to start simultaneously. The optimization objective is to determine a robot-task allocation and task sequence respecting all constraints and minimizing the weighted sum of task execution, driving and waiting times. A solution to an MRTA problem is defined by the routing and timing information contained in the set $X = \left\{ x_{i,j}^k \middle| i,j \in \mathcal{V}, k \in \{1, \ldots, K\} \right\} \cup \{ z_i | i \in \mathcal{V}\}$, $x_{i,j}^k \in \{0,1\}$, $z_i \in \mathbb{R}^{\geq 0}$. The binary decision variables $x_{i,j}^k$ are equal to one if the edge (i,j) lies on route of robot r^k and are zero elsewise. For the task nodes $\mathcal{T}$, the real valued decision variables z_i denote the starting time of task $i \in T$, and for the depots, z_{s^k}, denotes the time the robot r^k leaves its depot $s^k \in S$.

Using these definitions, the sum of driving times $c_{\mathcal{I}}^d(X)$ is given by $c_{\mathcal{I}}^d(X) = \sum_{r^k \in \mathcal{R}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} d_{i,j}^k x_{i,j}^k$, the sum of task execution times $c_{\mathcal{I}}^e(X)$ is determined by $c_{\mathcal{I}}^e(X) = \sum_{r^k \in \mathcal{R}} \sum_{i \in T} \left(d_i^k \sum_{j \in \mathcal{V}} x_{i,j}^k \right)$ and the sum of waiting times $c_{\mathcal{I}}^w(X)$ can be calculated by $c_{\mathcal{I}}^w(X) = \sum_{r^k \in \mathcal{R}} \sum_{i \in \mathcal{V}} \sum_{j \in T} x_{i,j}^k \left(z_j - z_i - d_i^k - d_{i,j}^k \right)$. However, this determination of the waiting times $c_{\mathcal{I}}^w(X)$ is nonlinear. In order to obtain a linear model, additional decision variables $w_{i,j} \in \mathbb{R}^{\geq 0}$, $\forall\, i,j \in \mathcal{V}$, representing the waiting time between two nodes i and j, are introduced. Thus, $c_{\mathcal{I}}^w(X)$ can be linearly expressed as $c_{\mathcal{I}}^w(X) = \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} w_{i,j}$.

With these definitions in place, a linear mixed-integer optimization problem results, representing the heterogeneous MRTA problem with synchronization and precedence constraints. A 3-index flow formulation is used to formulate the respective optimization problem.

Problem 1 (heterogeneous MRTA optimization problem)**.** For a given problem instance $\mathcal{I}$, the optimization problem to be solved is given by

$$\min_X J_{\mathcal{I}} = \min_X \Bigg\{ \underbrace{\sum_{r^k \in \mathcal{R}} \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} d_{i,j}^k x_{i,j}^k}_{:= c_{\mathcal{I}}^d(X)} + \gamma \underbrace{\sum_{r^k \in \mathcal{R}} \sum_{i \in T} \left(d_i^k \sum_{j \in \mathcal{V}} x_{i,j}^k \right)}_{:= c_{\mathcal{I}}^e(X)} + \epsilon \underbrace{\sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} w_{i,j}}_{:= c_{\mathcal{I}}^w(X)} \Bigg\}, \tag{1}$$

subject to

$$\sum_{k \in \{1,\ldots,K\}} \sum_{i \in \mathcal{V}} x_{i,j}^k = 1, \qquad \forall\, j \in \mathcal{V}, \tag{2a}$$

$$\sum_{i \in \mathcal{V}} x_{i,s^k}^k = 1, \qquad \forall\, k \in \{1, \ldots, K\}, \tag{2b}$$

$$\sum_{k \in \{1,\ldots,K\}} x_{i,i}^k = 0, \qquad \forall\, i \in T, \tag{2c}$$

$$\sum_{i \in \mathcal{V}} x_{i,j}^k = \sum_{i \in \mathcal{V}} x_{j,i}^k, \qquad \forall\, j \in \mathcal{V}, k \in \{1, \ldots, K\}, \tag{2d}$$

$$w_{ij} \geq z_j - z_i - \sum_{k \in \mathcal{R}} x_{ij}^k \left(d_i^k + d_{ij}^k \right) - \left(1 - \sum_{k \in \mathcal{R}} x_{ij}^k \right) T_{\max}, \qquad \forall i \in \mathcal{V}, j \in \mathcal{T}, \tag{2e}$$

$$z_j - z_i - \sum_{k \in \mathcal{R}} x_{i,j}^k \left(d_i^k + d_{i,j}^k \right) \geq \left(\sum_{k \in \mathcal{R}} x_{i,j}^k - 1 \right) T_{\max}, \qquad \forall\, i \in \mathcal{V}, j \in \mathcal{T}, \tag{2f}$$

$$p_{i,j} \left(z_i + \sum_{k \in \{1,\ldots,K\}} \sum_{l \in \mathcal{V}} \left(x_{i,l}^k d_i^k \right) - z_j \right) \leq 0, \qquad \forall\, i,j \in T, \tag{2g}$$

$$g_{i,j} \left(z_i - z_j \right) = 0, \qquad \forall\, i,j \in T, \tag{2h}$$

$$x_{i,j}^k \in \{0,1\} \qquad \forall\, i,j \in \mathcal{V}, k \in \{1, \ldots, K\}, \tag{2i}$$

$$z_i \in \mathbb{R}^{\geq 0} \qquad \forall\, i \in \mathcal{V}, \tag{2j}$$

$$w_{i,j} \in \mathbb{R}^{\geq 0}, \qquad \forall\, i,j \in \mathcal{V}, \tag{2k}$$

The objective function $J_{\mathcal{I}}$ to be minimized (1) comprises three parts, which are the sum of all driving times $c^d_{\mathcal{I}}(X)$, the sum of all robot-dependent task execution times $c^e_{\mathcal{I}}(X)$ and the sum of all waiting times $c^w_{\mathcal{I}}(X)$ associated with a solution X. The parameters $\gamma \in \mathbb{R}^{\geq 0}$ and $\epsilon \in \mathbb{R}^{\geq 0}$ can be used to weight the execution and waiting times on the objective function value relatively to the driving times. This can for example be desirable in applications where waiting times are associated with less energy consumption. Constraint (2a) ensures that the robot's routes visit all nodes exactly once, constraint (2b) forces the routes of all robots to drive into their depot, constraint (2c) inhibits loops in task nodes and constraint (2d) ensures that robots' routes leave all nodes they visit.

If an edge (i,j) lies on the route of any robot, that is, $\sum_{k\in\mathcal{R}} x^k_{i,j} = 1$, constraint (2e) ensures that the waiting time between the two nodes cannot become smaller than the starting time of task t_j minus the sum of the starting time of node i, it's robot-dependent duration of execution and the driving time to task t_j. In case edge (i,j) is not taken by any robot, that is, $\sum_{k\in\mathcal{R}} x^k_{i,j} = 0$, a sufficiently large choice of the constant $T_{\max}$ ensures $w_{i,j}$ to be bounded below by a negative value which becomes negligible due to constraint (2k). In the same way, equations (2f) and (2j) ensure the consistency of starting times. Constraints (2g) and (2h) ensure the fulfillment of the precedence and synchronization constrains, respectively. Constraint (2i) ensures the routing variables to be binary and non-negative starting and waiting times are assured by constraints (2j) and (2k), respectively.

We denote an optimal solution of a problem instance $\mathcal{I}$ by $X^*_{\mathcal{I}} := \arg\min J_{\mathcal{I}}(X)$ and the associated optimal value of the objective function by $J^*_{\mathcal{I}} := J_{\mathcal{I}}\left(X^*_{\mathcal{I}}\right)$.

2.2 | MRTA task insertion reoptimization problem

The MRTA task insertion problem we are investigating in this article is, given an optimal solution $X^*_{\mathcal{I}}$ to an initial problem instance $\mathcal{I}$, how to use this solution to solve a related problem instance $\mathcal{I}^+$ in which an additional task t_{N+1} has to be considered.

Problem 2 (MRTA task insertion reoptimization problem)**.** We are given an initial MRTA problem instance $\mathcal{I}$ according to Problem 1 and an optimal solution $X^*_{\mathcal{I}}$. The modified problem instance $\mathcal{I}^+$ is given by $\mathcal{I}^+ = \left\{T^+, \mathcal{R}, S, \mathcal{T}^+, V, A^+, P^+, G^+\right\}$, where $T^+ = T \cup \{t_{N+1}\}$, $\mathcal{T}^+ = \mathcal{T} \cup \{\tau_{N+1}\}$, $A^+ = A \cup \left\{a^1_{N+1}, \dots, a^K_{N+1}\right\}$, $P^+ = P \cup \left\{p_{i,N+1} = p_{N+1,i} = 0 \middle| \forall\, t_i \in T\right\}$ and $G^+ = G \cup \left\{g_{i,N+1} = g_{N+1,i} = 0 \middle| \forall\, t_i \in T\right\}$, that is, the modified problem instance $\mathcal{I}^+$ contains an additional temporarily unconstrained task t_{N+1} compared to the initial problem instance $\mathcal{I}$. The task insertion reoptimization problem aims at finding a solution $X^R_{\mathcal{I}^+}$ to the modified problem instance $\mathcal{I}^+$ by inserting the task t_{N+1} to the optimal solution of the initial problem instance $X^*_{\mathcal{I}}$ such that the resulting modified solution $X^R_{\mathcal{I}^+}$ fulfills all constraints according to equations (2) of the modified problem instance $\mathcal{I}^+$ and optimizes objective function (1).

2.3 | MRTA task deletion reoptimization problem

Besides the consideration of an additional task, we furthermore consider the MRTA reoptimization problem of deleting an task out of the initial problem instance, that is, given an optimal solution $X^*_{\mathcal{I}}$ to an initial problem instance $\mathcal{I}$, how to use this solution to solve a related problem instance $\mathcal{I}^-$ which differs from the initial instance by containing exactly one task less.

Problem 3 (MRTA task deletion reoptimization problem)**.** We are given an initial MRTA problem instance $\mathcal{I}$ according to Problem 1 and an optimal solution $X^*_{\mathcal{I}}$. The modified problem instance $\mathcal{I}^-$ is given by $\mathcal{I}^- = \{T^-, \mathcal{R}, S, \mathcal{T}^-, V, A^-, P^-, G^-\}$, where the modified problem instance $\mathcal{I}^-$ differs from the initial problem instance $\mathcal{I}$ by one missing task t_q, that is, $T^- = T \setminus \{t_q\}$ with $t_q \in T$, $\mathcal{T}^- = \mathcal{T} \setminus \{\tau_q\}$, $A^- = A \setminus \left\{a^1_q, \dots, a^K_q\right\}$, $P^- = P \setminus \left\{p_{i,q}, p_{q,i} \middle| \forall\, i \in T\right\}$ and $G^- = G \setminus \left\{g_{i,q}, g_{q,i} \middle| \forall\, i \in T\right\}$. The aim of the task deletion reoptimization problem is finding a solution $X^R_{\mathcal{I}^-}$ to the modified problem instance $\mathcal{I}^-$ by modifying the optimal solution of the initial problem instance such that the resulting modified solution $X^R_{\mathcal{I}^-}$ does not contain the deleted task t_q and fulfills all constraints according to equations (2) of the modified problem instance $\mathcal{I}^-$ and optimizes objective function (1).

3 | REOPTIMIZATION OF THE MRTA TASK INSERTION PROBLEM

In this section, we study the MRTA task insertion reoptimization problem and investigate bounds on the worst-case performance of reoptimization heuristics. These bounds are given by a guaranteed maximum approximation ratio, that is, the ratio between the objective function values of the approximate reoptimized solution and the optimal solution to the modified problem instance $\mathcal{I}^+$. We prove that any task insertion solution approach that fulfills a certain assumption, leads to an approximation ratio bounded above by 2. We introduce the *cheapest maximum cost insertion heuristic* (CMI) which we prove to fulfill the respective assumption. Subsequently, we analyze the application of the CMI to temporally unconstrained MRTA reoptimization problems and prove reduced worst case approximation ratios.

3.1 | Analysis of an solution heuristic independent performance guarantee for the MRTA task insertion reoptimization problem

Given an optimal solution $X^*_{\mathcal{I}}$ to the initial problem instance $\mathcal{I}$, any MRTA task insertion reoptimization approach that uses the initial solution $X^*_{\mathcal{I}}$ to solve the modified problem instance $\mathcal{I}^+$, will yield a reoptimized solution $X^R_{\mathcal{I}^+}$ that adds a heuristic- and problem instance dependent, non-negative cost increase $\Delta^+ \geq 0$ to the optimal costs $J^*_{\mathcal{I}}$ of the initial problem instance, that is,

$$J^R_{\mathcal{I}^+} = J^*_{\mathcal{I}} + \Delta^+. \tag{3}$$

Depending on the specific reoptimization method used to solve the task insertion reoptimization problem, the cost increment Δ^+ resulting from the application of the respective method to a reoptimization problem instance may differ. For the following analysis we assume the cost increment Δ^+ to have a value at most as high as the costs $J^*_{\mathcal{I}^+}$ of the optimal solution to the modified problem instance.

Assumption 1. *Let the cost inclement Δ^+ resulting from solving the MRTA task insertion reoptimization problem according to Problem 2 be bounded above by the costs $J^*_{\mathcal{I}^+}$ of an optimal solution to the modified problem instance, that is,*

$$\Delta^+ \leq J^*_{\mathcal{I}^+}. \tag{4}$$

We now prove that for any reoptimization solution approach solving an instance of the MRTA task insertion problem such that Assumption 1 holds, performance guarantees can be given according to the following Theorem.

Theorem 1. *For any MRTA task insertion reoptimization heuristic solving an instance of Problem 2 such that Assumption 1 holds, the resulting approximation ratio $\alpha = J^R_{\mathcal{I}^+}/J^*_{\mathcal{I}^+}$ cannot become greater than 2, that is,*

$$\alpha = \frac{J^R_{\mathcal{I}^+}}{J^*_{\mathcal{I}^+}} \leq 2. \tag{5}$$

Proof. It holds that, since the modified problem instance $\mathcal{I}^+$ only differs from the initial problem instance $\mathcal{I}$ by having one additional task t_{N+1}, the unknown costs $J^*_{\mathcal{I}^+}$ of an optimal solution to the modified problem instance cannot be smaller than the costs $J^*_{\mathcal{I}}$ of the optimal solution of the initial problem instance,

$$J^*_{\mathcal{I}^+} \geq J^*_{\mathcal{I}}. \tag{6}$$

Thus, it holds for the approximation ratio

$$\alpha = \frac{J^R_{\mathcal{I}^+}}{J^*_{\mathcal{I}^+}} = \frac{J^*_{\mathcal{I}} + \Delta^+}{J^*_{\mathcal{I}^+}} \leq 2.$$

■

Remark. We note that the performance guarantee given by Theorem 1 is independent of the specific reoptimization approach used to solve an MRTA task insertion reoptimization problem instance. Any reoptimization method fulfilling Assumption 1 for an specific reoptimization problem instance cannot yield an reoptimization solution $X^R_{\mathcal{I}^+}$ having costs $J^R_{\mathcal{I}^+}$ of more than twice the costs $J^*_{\mathcal{I}^+}$ of an optimal solution to the modified problem instance $\mathcal{I}^+$.

In the next sections we will introduce and analyze a specific heuristic to solve the MRTA task insertion reoptimization problem.

3.2 | Cheapest maximum cost insertion heuristic

We introduce a heuristic solution called the *cheapest maximum cost insertion heuristic* (CMI) to solve the MRTA task insertion reoptimization problem (Problem 2). It is inspired by the cheapest insertion heuristic which was proposed by Archetti et al. [2] for the TSP and chooses to insert the new task to the initial solution such that the resulting cost increment, that is, objective function value increment, is minimized. While the cheapest insertion heuristic is based on the determination of the exact cost increment, the CMI uses an overapproximation of the cost increase associated with inserting the new task t_{N+1} on an edge of an robot's route of the initial solution.

This is due to the fact that precedence and synchronization constraints can be present within a MRTA problem instance, which may cause waiting times $c_{\mathcal{I}}^{w} \neq 0$ that influence the objective function (1). These waiting times must therefore be incorporated when determining the cost increment. Even though the task added t_{N+1} is not associated with any ordering constraint, the tasks following within the same route might have to consider precedence or synchronization constraints and their temporal shifts can therefore cause an increase in waiting times in the routes of other robots. In the same manner, the temporal task shifts caused by these additional waiting times might cause further additional waiting times and associated task starting time shifts within the routes of even more robots. This effect makes the exact determination of exact insertion costs much more challenging which is why we propose a less complex overestimation of the insertion costs called the *maximum insertion costs* $\Delta_{\max}^{i,j,k}$. Using these, the CMI is defined as follows:

Definition 1 (cheapest maximum cost insertion heuristic (CMI)). *The cheapest maximum cost insertion heuristic applied to MRTA reoptimization problems of adding a task t_{N+1} to the optimal solution $X_{\mathcal{I}}^{*}$ of an initial problem instance is given by*:

1. *For each edge (i,j) that is part of the route of any robot k in the optimal solution of the initial problem instance $X_{\mathcal{I}}^{*}$, that is, $\forall \left\{ (i,j,k) \middle| x_{i,j}^{k} \in X_{\mathcal{I}}^{*} \wedge x_{i,j}^{k} = 1 \right\}$ calculate the maximum insertion costs $\Delta_{\max}^{i,j,k}$, that is, the maximum cost increment resulting from the insertion of the new task t_{N+1} on the respective edge of the respective robot's route.*
2. *Insert the task t_{N+1} on the route of robot $\hat{k}$ between the nodes $\hat{\imath}$ and $\hat{\jmath}$ which correspond to the minimum of the above determined maximum insertion costs $\Delta_{\max}^{i,j,k}$. The resulting optimal maximum insertion costs $\Delta_{\max}^{*}$ are given by*

$$\Delta_{\max}^{*} = \Delta_{\max}^{\hat{\imath},\hat{\jmath},\hat{k}} := \min_{\left\{ (i,j,k) \middle| x_{i,j}^{k} \in X_{\mathcal{I}}^{*} \wedge x_{i,j}^{k} = 1 \right\}} \Delta_{\max}^{i,j,k}. \tag{7}$$

The maximum insertion costs $\Delta_{\max}^{i,j,k}$ of inserting the new task t_{N+1} on an edge of a robot's route in the initial solution $X_{\mathcal{I}}^{*}$ take into account the resulting additional driving and task execution times as well as the maximum additional waiting times that might result. To determine the overestimation of the waiting times, we consider the initial solution $X_{\mathcal{I}}^{*}$ and let $\beta_j \in \{1, \dots, K\}$ be the number of routes that might be affected by a temporal shift of task j, $j \in T$. This implies that the temporal shift of task j might in the worst case cause additional waiting times of the same amount as the initial temporal shift of task j in the routes of $\beta_j - 1$ robots other than the one task j is assigned to. Consequently, the maximal insertion cost resulting from inserting task t_{N+1} on edge (i,j) in the route of robot k is given by the additional driving and task execution times for robot k which are associated with this insertion plus the same amount of waiting time in at most $\beta_j - 1$ routes, that is,

$$\begin{aligned}
\Delta_{\max}^{i,j,k} &= \begin{cases} d_{i,N+1}^{k} + d_{N+1,j}^{k} - d_{i,j}^{k} + \gamma d_{N+1}^{k} + \epsilon\left(\beta_j - 1\right)\left(d_{i,N+1}^{k} + d_{N+1,j}^{k} - d_{i,j}^{k} + d_{N+1}^{k}\right) & \text{if } a_i^k \neq 0 \\ \infty & \text{if } a_i^k = 0 \end{cases} \\
&= \begin{cases} \frac{d(i,t_{N+1})}{v^k} + \frac{d(t_{N+1},j)}{v^k} - \frac{d(i,j)}{v^k} + \gamma \frac{\tau_{N+1}}{a_{N+1}^{k}} + \epsilon\left(\beta_j - 1\right)\left(\frac{d(i,t_{N+1})}{v^k} + \frac{d(t_{N+1},j)}{v^k} - \frac{d(i,j)}{v^k} + \frac{\tau_{N+1}}{a_{N+1}^{k}}\right) & \text{if } a_{N+1}^{k} \neq 0 \\ \infty & \text{if } a_{N+1}^{k} = 0 \end{cases} \\
&\forall \left\{ (i,j,k) \middle| x_{i,j}^{k} \in X_{\mathcal{I}}^{*} \wedge x_{i,j}^{k} = 1 \right\}.
\end{aligned} \tag{8}$$

Remark. Since the cheapest maximum insertion cost heuristic chooses the edge on which to insert the new task based on a potential overestimation of the actual cost increment, that is, $\Delta_{\max}^{*} \geq \Delta^{+}$, the cost of the solution resulting from the application of the maximum insertion cost heuristic $J_{\mathcal{I}^{+}}^{\text{CMI}}$ is bounded above by

$$J_{\mathcal{I}^{+}}^{\text{CMI}} \leq J_{\mathcal{I}}^{*} + \Delta_{\max}^{*}. \tag{9}$$

Remark–Determination of β: Consider the directed graph $\tilde{\mathcal{G}}_{X_{\mathcal{I}}^{*}} = \left(\mathcal{V}, \tilde{E}_{X_{\mathcal{I}}^{*}}\right)$ which can be obtained easily from the initial solution $X_{\mathcal{I}}^{*}$ and the initial problem instance $\mathcal{I}$. The node set $\mathcal{V} = T \cup S$ contains the set of tasks T and the set of depots S. The arcs $\tilde{E}_{X_{\mathcal{I}}^{*}}$ are determined by the initial solution $X_{\mathcal{I}}^{*}$ and the synchronization and precedence constraints within the problem instance $\mathcal{I}$, that is,

$$\tilde{E}_{X_{\mathcal{I}}^{*}} = \left\{ (i,j) \middle| \left\{ \exists k \in K : x_{i,j}^{k} \in X_{\mathcal{I}}^{*} \wedge x_{i,j}^{k} = 1 \right\} \vee \left\{ p_{i,j} \in P \wedge p_{i,j} = 1 \right\} \vee \left\{ g_{i,j} \in G \wedge g_{i,j} = 1 \right\} \right\}.$$

Then β_i, $\forall i \in T$, is is given by the number of robots $k \in \mathcal{R}$ for which there exists a path from task i to the robot's depot s_k within the digraph $\tilde{\mathcal{G}}_{X_{\mathcal{I}}^{*}}$, that is,

$$\beta_i = \left| \left\{ k \in \mathcal{R} : \exists \text{ path}(i, \dots, s_k) \text{in } \tilde{\mathcal{G}}_{X_{\mathcal{I}}^{*}} \right\} \right|,$$

and can for example be determined using a backwards breadth first search within the graph $\tilde{\mathcal{G}}_{X_{\mathcal{I}}^{*}}$.

In the next section, we start the analysis of the worst-case approximation ratios resulting from the application of the CMI to MRTA task insertion reoptimization problems with an assumption and a relevant assertion.

In order to analyze the performance of the above defined CMI reoptimization heuristic, we make the following assumption:

Assumption 2. *A feasible solution to the MRTA reoptimization problem given according to Problem 2 can be found by the application of the cheapest maximum insertion cost heuristic.*

Assumption 2 ensures that the application of the CMI heuristic leads to a solution $X_{\mathcal{I}^+}^{\text{CMI}}$ that fulfills all constraints of the heterogeneous MRTA optimization problem instance $\mathcal{I}^+$ as given by equations (2). Assumption 2 is fulfilled if at least one of the robots $k \in \mathcal{R}$ is capable to perform task t_{N+1}, that is, $\exists\ k \in \mathcal{R}$: $a_{N+1}^k > 0$.

3.3 | Performance guarantee for the Cheapest Maximum Insertion Cost Heuristic

For the analysis of the approximation ratios resulting from the application of the previously introduced CMI heuristic, the following Lemma will be relevant.

Lemma 1. *When a node t_{N+1} is inserted on edge $(i,j) \in \mathcal{E}$, it holds for the distances that*

$$d(i, t_{N+1}) + d(t_{N+1}, j) - d(i,j) \leq 2d(i, t_{N+1}). \tag{10}$$

Proof. Since metric distances are considered, the triangle inequality $d(t_{N+1}, j) \leq d(t_{N+1}, i) + d(i,j)$ holds. Together with the symmetry of the distances, that is, $d(t_{N+1}, i) = d(i, t_{N+1})$, the assertion follows, that is:

$$\begin{aligned} d(t_{N+1}, j) \leq d(t_{N+1}, i) + d(i,j) &\Leftrightarrow d(t_{N+1}, j) + d(t_{N+1}, i) - d(i,j) \leq 2d(t_{N+1}, i) \\ &\Leftrightarrow d(t_{N+1}, j) + d(t_{N+1}, i) - d(i,j) \leq 2d(i, t_{N+1}) \end{aligned}$$ ■

We now prove the CMI to always fulfill Assumption 1 and thus to guarantee the resulting approximation ratio to be bounded above by 2.

Theorem 2. *Let Assumption 2 hold. Solving the MRTA task adding reoptimization problem, Problem 2, by the application of the cheapest maximum insertion cost heuristic with the maximum insertion costs as defined in (8) leads to an approximation ratio*

$$\alpha = \frac{J_{\mathcal{I}^+}^{\text{CMI}}}{J_{\mathcal{I}^+}^*} \leq 2. \tag{11}$$

Proof. Consider the optimal solution $X_{\mathcal{I}^+}^*$ of $\mathcal{I}^+$. Within $X_{\mathcal{I}^+}^*$, the task t_{N+1} is assumed to be allocated to robot $k^* \in \mathcal{R}$. Then the objective function value $J_{\mathcal{I}^+}^*$ of the unknown optimal solution $X_{\mathcal{I}^+}^*$ can be written as

$$J_{\mathcal{I}^+}^* = \phi_{\mathcal{R}\setminus r^{k^*}} + \phi_{r^{k^*}} \tag{12}$$

where $\phi_{r^{k^*}}$ denotes the costs of the route of robot k^* and $\phi_{\mathcal{R}\setminus r^{k^*}}$ denotes the sum of the costs of the routes of all other robots. Since robot k^* is at least assigned to task t_{N+1}, its route starts and ends at its depot s^{k^*} and the triangle inequality holds for the distances d, we know that

$$\phi_{r^{k^*}} \geq 2\frac{d(s_{k^*}, t_{N+1})}{v^{k^*}} + \gamma \frac{\tau_{N+1}}{a_{N+1}^{k^*}}. \tag{13}$$

Using the definition of the maximum insertion costs (8) and (7) as well as (10) from Lemma 1, it holds for the optimum maximum insertion costs

$$\Delta_{\max}^* \overset{(10)}{\leq} 2\frac{d(t_{N+1}, j)}{v^k} + \gamma \frac{\tau_{N+1}}{a_{N+1}^k} + \epsilon\left(\beta_j - 1\right)\left(2\frac{d(t_{N+1}, j)}{v^k} + \frac{\tau_{N+1}}{a_{N+1}^k}\right)\forall \left\{(j,k)\middle| \exists x_{i,j}^k \in X_{\mathcal{I}}^* : x_{i,j}^k = 1\right\}. \tag{14}$$

Equation (14) holds for inserting task t_{N+1} on any edge (i,j) of the route of an robot $k \in \mathcal{R}$ that was chosen in the initial solution $X_{\mathcal{I}}^*$. Thus, it also holds for inserting task t_{N+1} as the last task before the depot into the route of robot k^*, that is, for $j = s^{k^*}$ and $k = k^*$. We know that inserting task t_{N+1} at this position cannot introduce additional waiting times to the schedule of any other robot other than k^* since there is no subsequent task in robot k^*'s route with potential ordering constraints and therefore $\beta_{s^{k^*}} = 1$. Inserting these values in (14) yields

$$\Delta_{\max}^* \leq 2\frac{d(t_{N+1}, s^{k^*})}{v^{k^*}} + \gamma \frac{\tau_{N+1}}{a_{N+1}^{k^*}}. \tag{15}$$

Using (13) and (15), $\Delta^*_{\max} \leq \phi_{r^{k*}}$ follows. Since $\Delta^*_{\max}$ potentially overestimates the actual insertion costs, that is, $\Delta^+ \leq \Delta^*_{\max}$ and according to (12) it furthermore holds that $J^*_{\mathcal{I}^+} \leq \phi_{r^{k*}}$, we know that Assumption 1 ($\Delta^+ \leq J^*_{\mathcal{I}^+}$) is always fulfilled when an MRTA task insertion reoptimization problem instance is solved using the CMI. Consequently, applying Theorem 1, the assertion follows. ∎

Proposition 1. *For the the application of the CMI heuristic with the maximum insertion costs as defined in (8) to the MRTA task adding reoptimization problem, Problem 2, the approximation ratio of $\alpha \leq 2$ is a tight bound, that is, no lower upper bound on the resulting approximation ratio exists.*

Proof. To prove that 2 is the smallest possible upper bound on the approximation ratio resulting from the application of the CMI heuristic to MRTA task insertion reoptimization problems, we give a MRTA reoptimization problem instance for which the approximation ratio resulting from the CMI heuristic, $\alpha = J^{\mathrm{CMI}}_{\mathcal{I}^+}/J^*_{\mathcal{I}^+}$, is exactly 2. The respective instance is depicted in Figure 1. It depicts an initial instance $\mathcal{I}$ with two robots with their depots s^1 and s^2 and task 1 to execute. Robot 1 has a velocity of $v_1 = 1$ while robot 2 has a velocity of $v_2 = 1/2$ and has half the distance to cover to reach task 1 compared to robot 1. Both robots are fully capable to execute task 1, that is, $a^1_1 = a^2_1 = 1$. Since both possible solutions to this instance, that is, either allocating task 1 to robot 1 or to robot 2, have the same objective function value, one of them will be chosen by chance and it is assumed that robot 2 is chosen to execute task 1 in the initial solution $X^*_{\mathcal{I}}$. In the modified instance $\mathcal{I}^+$, task 2 is added at the same position as task 1. Since only robot 1 is capable of executing task 2, that is, $a^1_2 = 1$ and $a^2_2 = 0$, task 2 will be assigned to robot 1 in the reoptimized solution. The optimal solution however would be to assign both tasks to robot 1 which would lead to an optimal objective function value half as big as the one of the reoptimized solution, that is, the resulting approximation ratio equals $\alpha = 2$. ∎

3.4 | Consideration of temporal unconstrained MRTA task adding reoptimization problems

We now consider MRTA reoptimization problems without any precedence or synchronization constraints such that no waiting times can arise in the solutions.

3.4.1 | Considered problem configurations

To reveal the difference in performance guarantees for the reoptimization problem of adding a task to an MRTA problem instance (Problem 2) depending on the properties of the MRTA instance, we introduce three problem configurations. They differ with respect to the features of the corresponding MRTA problems as given in Table 1. The full problem configuration considered so far is denoted by $\mathrm{P}_{\mathrm{t,het}}$ and allows for the consideration of fully heterogeneous teams of robots as well as both types of task ordering constraints, that is, precedence and synchronization constraints, and therefore imposes no restrictions on the MRTA reoptimization problem (Problem 2). In contrast to this, the other configurations $\mathrm{P}_{\mathrm{hom}}$ and $\mathrm{P}_{\mathrm{het}}$ assume tasks to be unconstrained w.r.t. their order of task execution, that is, no precedence and synchronization constraints are considered. The reoptimization problem with a heterogeneous group of robots $\mathrm{P}_{\mathrm{het}}$ allows for robots that differ in both driving velocities as well as task capabilities. The homogeneous MRTA reoptimization problem $\mathrm{P}_{\mathrm{hom}}$ however, considers a homogeneous group of robots

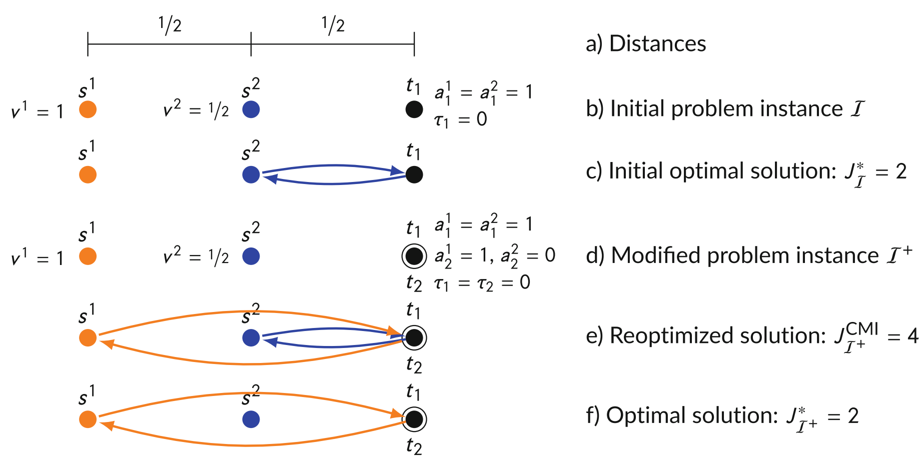

FIGURE 1 Example of an MRTA task insertion reoptimization instance with an approximation ratio of 2 for the application of the CMI heuristic.

TABLE 1 Problem configurations considered for reoptimization.

Problem configuration	Assumptions on Velocities V	Task capabilities A	Precedence constraints P	Synchronization constraints G
$\mathrm{P}_{\mathrm{t,het}}$	—	—	—	—
$\mathrm{P}_{\mathrm{het}}$	—	—	$p_{i,j} = 0, \forall p_{i,j} \in P, p_{i,j} \in P^+$	$g_{i,j} = 0, \forall g_{i,j} \in G, g_{i,j} \in G^+$
$\mathrm{P}_{\mathrm{hom}}$	$v^k = 1, \forall v^k \in V$	$a_i^k = 1, \forall a_i^k \in A, a_i^k \in A^+$	$p_{i,j} = 0, \forall p_{i,j} \in P, p_{i,j} \in P^+$	$g_{i,j} = 0, \forall g_{i,j} \in G, g_{i,j} \in G^+$

w.r.t. driving velocities and task capabilities. We note that setting the homogeneous velocities and capabilities each to the value one can be done without loss of generality, since any reasonable problem instance with homogeneous velocities and capabilities can be transferred to this form by normalization.

The CMI used for the general task adding reoptimization problem $\mathrm{P}_{\mathrm{t,het}}$ uses a potential overestimation of the waiting times resulting from a task insertion for the determination of the smallest maximum insertion costs. However, when neither precedence nor synchronization constraints have to be considered, as this is the case for problem configurations $\mathrm{P}_{\mathrm{hom}}$ and $\mathrm{P}_{\mathrm{het}}$, no waiting times arise and for these configurations the maximum insertion costs $\Delta_{\max}^{i,j,k}$ therefore resemble the actual insertion costs $\Delta^{i,j,k}$ of inserting the new task t_{N+1} on an edge (i,j) of the route of a robot $k \in \mathcal{R}$. In other words, the maximum number of routes that can be affected by a temporal shift of any task $i \in T$ is equal to one, that is, $\beta_i = 1\ \forall i \in T$. Using this equivalence in (8) yields the actual insertion costs $\Delta^{i,j,k}$, which are given by

$$\mathrm{P}_{\mathrm{hom}}, \mathrm{P}_{\mathrm{het}} : \Delta_{\max}^{i,j,k} = \Delta^{i,j,k} = \begin{cases} d_{i,N+1}^k + d_{N+1,j}^k - d_{i,j}^k + \gamma d_{N+1}^k = \frac{d(i,t_{N+1})}{v^k} + \frac{d(t_{N+1},j)}{v^k} - \frac{d(i,j)}{v^k} + \gamma \frac{\tau_{N+1}}{a_{N+1}^k} & \text{if } a_i^k \neq 0 \\ \infty & \text{if } a_i^k = 0 \end{cases} \tag{16}$$
$$\forall \left\{ (i,j,k) \middle| x_{i,j}^k \in X_{\mathcal{I}}^* \wedge x_{i,j}^k = 1 \right\}.$$

Remark. Since no overestimation of the insertion costs takes place if the CMI is applied to task insertion reoptimization problems of configuration $\mathrm{P}_{\mathrm{hom}}$ or $\mathrm{P}_{\mathrm{het}}$, the optimal maximum insertion costs $\Delta_{\max}^*$ are equivalent to the actual cost increase for problems of these configurations, that is, $\mathrm{P}_{\mathrm{hom}}, \mathrm{P}_{\mathrm{het}} : \Delta_{\max}^* = \min\{\Delta^{i,j,k} \big| x_{i,j}^k \in X_{\mathcal{I}}^* \wedge x_{i,j}^k = 1\} = \Delta^+$.

3.5 | Performance guarantees for temporal unconstrained MRTA task adding reoptimization problems

We start the investigation of the performance of the CMI for MRTA task insertion reoptimization problems without synchronization and precedence constraints by considering the configuration $\mathrm{P}_{\mathrm{het}}$ with heterogeneous robotic teams. For that purpose, the maximum and minimum velocity of the robots in the problem instance $\mathcal{I}$ are of interest. We denote them by $v^{\max} := \max\left\{v^k \in V\right\}$ and $v^{\min} := \min\left\{v^k \in V\right\}$, respectively. Furthermore, the maximum and minimum of the real valued capabilities of the robots to perform the new task t_{N+1}, denoted by $a_{N+1}^{\max} := \max\left\{a_{N+1}^k \in A\right\}$ and $a_{N+1}^{\min} := \min\left\{a_{N+1}^k \in A\right\}$ will be relevant. With these definitions in place, the following performance guarantee for the application of the CMI heuristic to MRTA task insertion reoptimization problems of configuration $\mathrm{P}_{\mathrm{het}}$ can be given.

Theorem 3. *Let Assumption 2 hold. For the MRTA reoptimization problem with heterogeneous teams of robots* $\mathrm{P}_{\mathrm{het}}$, *the application of the CMI with the insertion costs as defined in (16), leads to an approximation ratio*

$$\mathrm{P}_{\mathrm{het}} : \quad \alpha = \frac{J_{\mathcal{I}^+}^{\mathrm{CMI}}}{J_{\mathcal{I}^+}^*} \leq \min\left\{ \frac{3}{2} \frac{v^{\max} a_{N+1}^{\max}}{v^{\min} a_{N+1}^{\min}}, 2 \right\}. \tag{17}$$

Proof. All problems of configuration $\mathrm{P}_{\mathrm{het}}$ are a subset of problems of configuration $\mathrm{P}_{\mathrm{t,het}}$. Theorem 2 therefore also holds for problems of configuration $\mathrm{P}_{\mathrm{het}}$, that is, the approximation ratio for configuration $\mathrm{P}_{\mathrm{het}}$ is bounded above by 2. We furthermore show that $\alpha \leq \left(3v^{\max} a_{N+1}^{\max}\right) / \left(2v^{\min} a_{N+1}^{\min}\right)$ holds.

Without loss of generality, we normalize the robots' velocities and their capabilities to perform the new task t_{N+1}, as well as the distances and base task durations, that is,

$$\bar{v}^k := \frac{v^k}{v^{\max}} \quad \Rightarrow \quad \bar{v}^{\max} = 1, \quad \bar{v}^{\min} = \frac{v^{\min}}{v^{\max}} \quad \text{and} \quad \bar{d}(i,j) := \frac{d(i,j)}{v^{\max}} \tag{18}$$

$$\overline{a}^k_{N+1} := \frac{a^k_{N+1}}{a^{\max}_{N+1}} \quad \Rightarrow \quad \overline{a}^{\max}_{N+1} = 1, \quad \overline{a}^{\min}_{N+1} = \frac{a^{\min}_{N+1}}{a^{\max}_{N+1}} \quad \text{and} \quad \overline{\tau}_{N+1} := \frac{\tau_{N+1}}{a^{\max}_{N+1}}. \tag{19}$$

Using the definition of the insertion costs (16) and Lemma 1, it follows

$$\begin{aligned}\Delta^* &\le \frac{1}{\overline{v}^k}\left(\overline{d}(i,t_{N+1}) + \overline{d}(t_{N+1},j) - \overline{d}(i,j)\right) + \gamma \frac{\overline{\tau}_{N+1}}{\overline{a}^k_{N+1}} && \forall \left\{(i,j,k)\middle| x^k_{i,j} \in X^*_{\mathcal{I}} \wedge x^k_{i,j} = 1\right\} \\ &\overset{(10)}{\le} \frac{2\overline{d}(i,t_{N+1})}{\overline{v}^k} + \gamma \frac{\overline{\tau}_{N+1}}{\overline{a}^k_{N+1}} && \forall \left\{(i,k)\middle| \exists j \in \mathcal{V} : x^k_{i,j} \in X^*_{\mathcal{I}} \wedge x^k_{i,j} = 1\right\}\end{aligned} \tag{20}$$

$$\le \frac{2\overline{d}(i,t_{N+1})}{\overline{v}^{\min}} + \gamma \frac{\overline{\tau}_{N+1}}{\overline{a}^{\min}_{N+1}} \qquad \forall\, i \in \mathcal{V}. \tag{21}$$

Let i^* and j^* be the predecessor and follower nodes of task t_{N+1} within the unknown optimal solution $X^*_{\mathcal{I}^+}$ of $\mathcal{I}^+$, where task t_{N+1} is, as previously, assumed to be allocated to robot k^*. Evaluating (21) for the nodes i^* and j^* yields

$$\Delta^* \le \frac{\overline{d}(i^*,t_{N+1})}{\overline{v}^{\min}} + \frac{\overline{d}(t_{N+1},j^*)}{\overline{v}^{\min}} + \gamma \frac{\overline{\tau}_{N+1}}{\overline{a}^{\min}_{N+1}}. \tag{22}$$

Within the optimal solution $J^*_{\mathcal{I}^+}$ of $\mathcal{I}^+$, let $\varphi_{r^{k^*}}(i^*,j^*)$ denote the costs of the route of robot k^* without the part between the nodes i^* and j^*. Then $J^*_{\mathcal{I}^+}$ can be written as

$$J^*_{\mathcal{I}^+} = \phi_{\mathcal{R}\setminus r^{k^*}} + \varphi_{r^{k^*}}(i^*,j^*) + \frac{\overline{d}(i^*,t_{N+1})}{\overline{v}^{k^*}} + \frac{\overline{d}(t_{N+1},j^*)}{\overline{v}^{k^*}} + \gamma \frac{\overline{\tau}_{N+1}}{\overline{a}^{k^*}_{N+1}}. \tag{23}$$

We know that robot k^* is assigned to task t_{N+1} in the optimal solution $X^*_{\mathcal{I}^+}$ with task t_{N+1} being located between the nodes i^* and j^*. Since robot k^* starts and ends its route at its depot and since the triangle inequality holds, we know that $d^{k^*}_{i^*,j^*} \le \varphi_{r^{k^*}}(i^*,j^*)$ holds. Using this relation it follows that

$$J^*_{\mathcal{I}} \le \phi_{\mathcal{R}\setminus r^{k^*}} + \varphi_{r^{k^*}}(i^*,j^*) + d^{k^*}_{i^*,j^*} \le 2\left(\phi_{\mathcal{R}\setminus r^{k^*}} + \varphi_{r^{k^*}}(i^*,j^*)\right). \tag{24}$$

Equation (23) together with (24) yields

$$J^*_{\mathcal{I}^+} \ge \frac{J^*_{\mathcal{I}}}{2} + \frac{\overline{d}(i^*,t_{N+1})}{\overline{v}^{k^*}} + \frac{\overline{d}(t_{N+1},j^*)}{\overline{v}^{k^*}} + \gamma \frac{\overline{\tau}_{N+1}}{\overline{a}^{k^*}_{N+1}}. \tag{25}$$

Let us now consider two cases to finish the proof:

CASE 1: $\overline{d}(i^*,t_{N+1}) + \overline{d}(t_{N+1},j^*) + \gamma\overline{\tau}_{N+1} \ge \frac{J^*_{\mathcal{I}}}{2}$

$$\alpha = \frac{J^*_{\mathcal{I}} + \Delta^*}{J^*_{\mathcal{I}^+}} \overset{(22),(25)}{\le} \frac{J^*_{\mathcal{I}} + \frac{1}{\overline{v}^{\min}}\left(\overline{d}(i^*,t_{N+1}) + \overline{d}(t_{N+1},j^*)\right) + \gamma \frac{1}{\overline{a}^{\min}_{N+1}}\overline{\tau}_{N+1}}{\frac{J^*_{\mathcal{I}}}{2} + \frac{1}{\overline{v}^{k^*}}\left(\overline{d}(i^*,t_{N+1}) + \overline{d}(t_{N+1},j^*)\right) + \gamma \frac{1}{\overline{a}^{k^*}_{N+1}}\overline{\tau}_{N+1}} \tag{26}$$

$$\overset{\overline{v}^{\max}=\overline{a}^{\max}_{N+1}=1}{\le} \frac{J^*_{\mathcal{I}} + \frac{1}{\overline{v}^{\min}}\left(\overline{d}(i^*,t_{N+1}) + \overline{d}(t_{N+1},j^*)\right) + \gamma \frac{1}{\overline{a}^{\min}_{N+1}}\overline{\tau}_{N+1}}{\frac{J^*_{\mathcal{I}}}{2} + \left(\overline{d}(i^*,t_{N+1}) + \overline{d}(t_{N+1},j^*)\right) + \gamma\overline{\tau}_{N+1}} \tag{27}$$

$$\le \frac{1}{\overline{v}^{\min}\,\overline{a}^{\min}_{N+1}} \frac{J^*_{\mathcal{I}} + \left(\overline{d}(i^*,t_{N+1}) + \overline{d}(t_{N+1},j^*)\right) + \gamma\overline{\tau}_{N+1}}{\frac{J^*_{\mathcal{I}}}{2} + \left(\overline{d}(i^*,t_{N+1}) + \overline{d}(t_{N+1},j^*)\right) + \gamma\overline{\tau}_{N+1}} \tag{28}$$

$$\overset{(\text{Case1})}{\le} \frac{3}{2}\frac{1}{\overline{v}^{\min}\,\overline{a}^{\min}_{N+1}} = \frac{3}{2}\frac{v^{\max}a^{\max}_{N+1}}{v^{\min}a^{\min}_{N+1}}. \tag{29}$$

CASE 2: $\overline{d}(i^*,t_{N+1}) + \overline{d}(t_{N+1},j^*) + \gamma\overline{\tau}_{N+1} < \frac{J^*_{\mathcal{I}}}{2}$

$$\begin{aligned}\alpha = \frac{J^*_{\mathcal{I}} + \Delta^*}{J^*_{\mathcal{I}^+}} &\overset{(6),(22)}{\le} \frac{J^*_{\mathcal{I}} + \frac{1}{\overline{v}^{\min}}\left(\overline{d}(i^*,t_{N+1}) + \overline{d}(t_{N+1},j^*)\right) + \gamma \frac{1}{\overline{a}^{\min}_{N+1}}\overline{\tau}_{N+1}}{J^*_{\mathcal{I}}} \\ &\le \frac{1}{\overline{v}^{\min}\,\overline{a}^{\min}_{N+1}} \frac{J^*_{\mathcal{I}} + \overline{d}(i^*,t_{N+1}) + \overline{d}(t_{N+1},j^*) + \gamma\overline{\tau}_{N+1}}{J^*_{\mathcal{I}}} \\ &\overset{(\text{Case2})}{<} \frac{3}{2}\frac{1}{\overline{v}^{\min}\,\overline{a}^{\min}_{N+1}} = \frac{3}{2}\frac{v^{\max}a^{\max}_{N+1}}{v^{\min}a^{\min}_{N+1}}.\end{aligned} \tag{30}$$

Since both, $\alpha \le \frac{3}{2}\frac{v^{\max}a^{\max}_{N+1}}{v^{\min}a^{\min}_{N+1}}$ as well as $\alpha \le 2$ (Theorem 2) must hold, (17) follows. ■

Remark. The problem instance shown in Figure 1, which has an approximation ratio of $\alpha = 2$ is of configuration level $\mathrm{P}_{\mathrm{het}}$, with $\mathrm{P}_{\mathrm{het}}$ being a subset of the general problem instances $\mathrm{P}_{\mathrm{t,het}}$. Therefore, $\alpha \leq 2$ is also a tight bound on the approximation ratio for adding a task to temporal unconstrained MRTA problems of configuration $\mathrm{P}_{\mathrm{het}}$.

Corollary 1. *Let Assumption 2 hold. For the homogeneous MRTA reoptimization Problem* $\mathrm{P}_{\mathrm{hom}}$, *the application of the CMI with the insertion costs as defined in (16), leads to an approximation ratio*

$$\mathrm{P}_{\mathrm{hom}}: \quad \alpha = \frac{J_{\mathcal{I}^+}^{\mathrm{CMI}}}{J_{\mathcal{I}^+}^{*}} \leq \frac{3}{2}. \tag{31}$$

Proof. Since all problems of configuration $\mathrm{P}_{\mathrm{hom}}$ also fulfill the requirements of configuration $\mathrm{P}_{\mathrm{het}}$, Theorem (3) holds for these problems. By definition of configuration $\mathrm{P}_{\mathrm{hom}}$, the driving velocities and task execution capabilities of all robots are equal, that is, $v^{\max} = v^{\min}$ and $a_{N+1}^{\max} = a_{N+1}^{\min}$. By inserting this into (17) of Theorem (3) the assertion follows. ■

Proposition 2. *For the the application of the CMI with the insertion costs as defined in (16) to the homogeneous MRTA reoptimization Problem* $\mathrm{P}_{\mathrm{hom}}$, *the approximation ratio of* $\alpha \leq 3/2$ *is a tight bound.*

Proof. A problem instance of configuration $\mathrm{P}_{\mathrm{hom}}$ for which the approximation ratio $\alpha = J_{\mathcal{I}^+}^{\mathrm{CMI}}/J_{\mathcal{I}^+}^{*}$ converges to $3/2$ is depicted in Figure 2. It is assumed that the task durations are equal to zero for both tasks, that is, $\tau_1 = \tau_2 = 0$. In the initial instance $\mathcal{I}$, two robots with their respective depots s^1 and s^2 are available to execute task 1. In the modified instance $\mathcal{I}^+$, task 2 is added. For $\delta \to 0$, the approximation ratio approaches $\alpha \to 3/2$. ■

Table 2 summarizes the results on the approximation ratios for the application of the CMI for all problem configurations considered in this article. For the general problem configuration $\mathrm{P}_{\mathrm{t,het}}$ a tight upper bound of $\alpha \leq 2$ was proven in Theorem 2 and Proposition 1. The upper bound on the approximation ratio for problem instances of configuration $\mathrm{P}_{\mathrm{het}}$ without temporal constraints and heterogeneous robotic teams depends on the features of the robotic team (Theorem 3) and is bounded above by $\alpha \leq 2$, which is furthermore a tight bound (see preceding Remark). A constant and tight upper bound of $3/2$ can be guaranteed for problem instances of configuration $\mathrm{P}_{\mathrm{hom}}$ without temporal constraints and homogeneous robots, as given by Corollary 1 and Proposition 2.

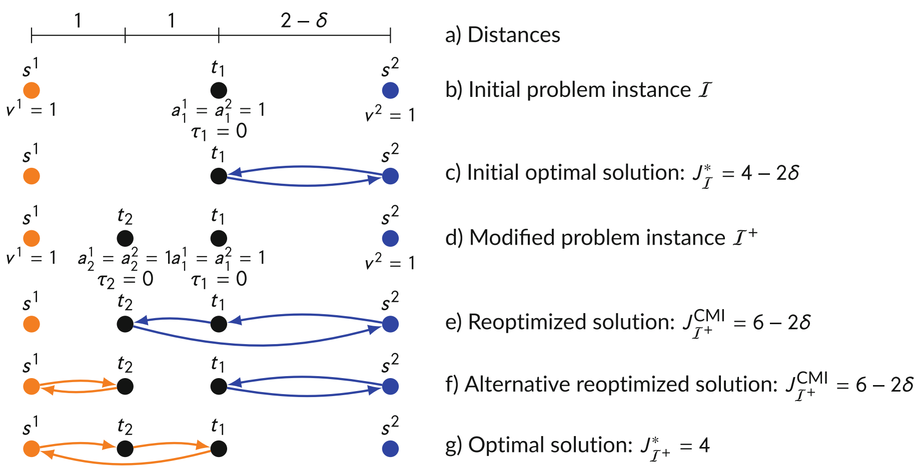

FIGURE 2 Example of an MRTA task insertion reoptimization instance of the configuration $\mathrm{P}_{\mathrm{hom}}$ with an approximation ratio of $\alpha \to \frac{3}{2}$ for $\delta \to 0$ for the application of the CMI heuristic.

TABLE 2 Guaranteed upper bounds on the approximation ratios for the application of the CMI to MRTA task insertion reoptimization problems.

Problem configuration	Approximation ratio α	Tight bound
$\mathrm{P}_{\mathrm{hom}}$	$\leq \frac{3}{2}$	Yes
$\mathrm{P}_{\mathrm{het}}$	$\leq \min\left\{\frac{3v^{\max}a_{N+1}^{\max}}{2v^{\min}a_{N+1}^{\min}}, 2\right\}$	Tight for $\alpha \leq 2$
$\mathrm{P}_{\mathrm{t,het}}$	≤ 2	Yes

4 | REOPTIMIZATION OF THE MRTA TASK DELETION PROBLEM

In this section, the MRTA task deletion reoptimization problem, that is, Problem 3, is investigated. We prove an upper bound on the approximation ratio for the MRTA task deletion reoptimization problem under an assumption that is fulfilled for the majority of problem instances and propose a heuristic to solve these reoptimization problems. Subsequently, we derive a reduced bound for the approximation ratio for the proposed heuristic when a team of homogeneous robots originating from the same start and end position is considered.

4.1 | Analysis of a solution heuristic independent performance guarantee for the MRTA task deletion reoptimization problem

To analyze the approximation ratios resulting for the task deletion reoptimization problem, that is, Problem 3, the following considerations are relevant: When solving a task deletion reoptimization problem, an optimal solution $X^*_{\mathcal{I}}$ of the initial problem instance $\mathcal{I}$ is used to derive a heuristic solution to the modified problem instance $\mathcal{I}^-$ which differs from the initial instance by not containing task t_q as defined in Problem 3. The cost of the generated reoptimization solution of the modified problem instance $\mathcal{I}^-$ and the costs of the initial solution differ by a cost difference Δ^-, that is,

$$J^R_{\mathcal{I}^-} = J^*_{\mathcal{I}} - \Delta^-. \tag{32}$$

Depending on the specific reoptimization method used to solve the task deletion reoptimization problem, the cost difference Δ^- resulting from the application of the respective method to a reoptimization problem instance may differ. Since any task deletion reoptimization approach removes a task from the initial solution, the following assumption will hold for most reasonable task deletion reoptimization approaches:

Assumption 3. *Let the task deletion reoptimization approach, the initial problem instance $\mathcal{I}$ and the modified problem instance $\mathcal{I}^-$ be defined such that the cost reduction Δ^- of the reoptimized solution compared to the initial solution is non-negative, that is,*

$$\Delta^- \geq 0. \tag{33}$$

Consider the task insertion problem, Problem 2, for the exact same problem instances, that is, an optimal solution $X^*_{\mathcal{I}^-}$ of the problem instance $\mathcal{I}^-$ is known and any feasible reoptimization approach is applied to insert task t_q to derive a reoptimized solution $X^R_{\mathcal{I}}$. Then the costs of the optimal solution of the problem instance $J^*_{\mathcal{I}^-}$ and of the reoptimized solution to the problem instance containing task t_q, that is, $J^R_{\mathcal{I}}$, differ by a non-negative cost increment Δ^+, that is,

$$J^R_{\mathcal{I}} = J^*_{\mathcal{I}^-} + \Delta^+ \quad \text{with} \quad \Delta^+ \geq 0. \tag{34}$$

To guarantee the performance quality of a task deletion reoptimization approach, we make the following assumption:

Assumption 4. *Let the initial problem instance $\mathcal{I}$, the modified problem instance $\mathcal{I}^-$ and the task deletion approach be defined such that there exists a task insertion approach in such a way that the difference between the cost increment Δ^+ of the task insertion approach and the cost reduction Δ^- of the task deletion approach does not exceed the optimal costs of the modified problem instance $\mathcal{I}^-$, that is,*

$$\Delta^+ - \Delta^- \leq J^*_{\mathcal{I}^-}. \tag{35}$$

Remark. Numerous examples and problem instances investigated have shown that Assumption 4 does not impose strong limitations on the problem instances. In most problem instances with several tasks the cost increment Δ^+ itself will be smaller than the optimal costs $J^*_{\mathcal{I}^-}$ of the problem instance without t_q, that is, $\Delta^+ < J^*_{\mathcal{I}^-}$ will hold. But even in cases for which $\Delta^+ \geq J^*_{\mathcal{I}^-}$ holds, we can show that Assumption 4 must still be fulfilled if the problem instances are such that the worst task insertion approximation ratio of $\alpha = 2$ is reached. In these cases

$$\alpha = \frac{J^*_{\mathcal{I}^-} + \Delta^+}{J^*_{\mathcal{I}}} \leq \frac{J^*_{\mathcal{I}^-} + \Delta^+}{\Delta^+} \overset{\Delta^+ \geq J^*_{\mathcal{I}^-}}{\leq} 2$$

holds according to Assumption 1.[1] Consequently, the approximation ratio of 2 can only be reached if $\Delta^+ = J^*_{\mathcal{I}^-}$ holds and thus also in these cases Assumption 4 must be fulfilled.

[1]Please note that for the consideration of inserting task t_q to problem instance $\mathcal{I}^-$, instance $\mathcal{I}$ corresponds to $\mathcal{I}^+$ in the notation used for the task insertion reoptimization problem. Thus, using the notation of the task deletion reoptimization problem, Assumption 1 is given by $\Delta^+ \leq J^*_{\mathcal{I}}$.

For any task deletion approach fulfilling Assumptions 3 and 4, the resulting approximation ratio is bounded above as given by the following theorem.

Theorem 4. *For any MRTA task deletion reoptimization approach solving an instance of Problem 3 such that Assumption 3 and Assumption 4 hold, the resulting approximation ratio* $\alpha = J^R_{\mathcal{I}^-}/J^*_{\mathcal{I}^-}$ *cannot exceed 2, that is,*

$$\alpha = \frac{J^R_{\mathcal{I}^-}}{J^*_{\mathcal{I}^-}} \leq 2. \tag{36}$$

Proof. We know that $J^*_{\mathcal{I}^-} \leq J^*_{\mathcal{I}}$ as well as (32), (33) and (34) hold. It thereby follows that

$$J^*_{\mathcal{I}^-} \leq J^R_{\mathcal{I}^-} \leq J^*_{\mathcal{I}} \leq J^R_{\mathcal{I}}. \tag{37}$$

Using (34), (37) and (35) from Assumption 4 it follows for the approximation ratio:

$$\begin{aligned}\alpha &= \frac{J^R_{\mathcal{I}^-}}{J^*_{\mathcal{I}^-}} = \frac{J^*_{\mathcal{I}} - \Delta^-}{J^*_{\mathcal{I}^-}} \\ &\overset{(37)}{\leq} \frac{J^R_{\mathcal{I}} - \Delta^-}{J^*_{\mathcal{I}^-}} \overset{(34)}{=} \frac{J^*_{\mathcal{I}^-} + \Delta^+ - \Delta^-}{J^*_{\mathcal{I}^-}} = 1 + \frac{\Delta^+ - \Delta^-}{J^*_{\mathcal{I}^-}} \\ &\overset{(36)}{\leq} 2.\end{aligned} \tag{38}$$

■

Consequently, any task deletion reoptimization approach that, when applied to an instance of Problem 3, does not augment the initial costs (Assumption 3) and furthermore fulfills Assumption 4, is guaranteed to generate a solution of costs at most twice as high as the costs of an optimal solution to the modified problem instance $\mathcal{I}^-$.

4.2 | Task deletion heuristic

To solve the MRTA task deletion reoptimization problem 3, we propose the *task deletion heuristic* (TD). The heuristic is derived from the deletion procedure of Archetti et al. [2] for the TSP. The heuristic erases a task from the tour of the executing robot by deleting the edges connecting the deleted task to the route and inserting a new edge pairing the predecessor and successor task. Hence, the heuristic for task deletion is defined as follows:

Definition 2 (task deletion heuristic (TD))**.** *The task deletion heuristic to remove a task* t_q *from an optimal solution* $X^*_{\mathcal{I}}$ *of an initial problem instance is given by*:

1. *Within the tour of robot* k^**, that is, the robot task* t_q *is assigned to by the initial solution* $X^*_{\mathcal{I}}$*, remove the edges* (n_p, t_q) *and* (t_q, n_s) *connecting the task* t_q *to its predecessor node* n_p *and successor node* n_s *within* $X^*_{\mathcal{I}}$.
2. *Insert the edge* (n_p, n_s) *to the route of robot* k^*.

Remark. The TD heuristic is applicable to MRTA problems of full complexity, that is, of configuration $\mathrm{P}_{\mathrm{t,het}}$. If the deleted task t_q is associated to any precedence or synchronization constraints, the respective constraints cannot be fulfilled by any solution to the modified problem instance $\mathcal{I}^-$. Hence, the respective temporal constraints should be deleted as well. If this is done, the application of the TD heuristic ensures the resulting solution to be feasible, that is, all constraints as given by equations (2) will still be fulfilled.

The cost deviation $\Delta^{\mathrm{TD}-}$ caused by the application of the TD heuristic includes the reduction of driving times Δ_{c^d} and task execution times Δ_{c^e} for robot k^*, the robot the deleted task t_q was assigned to in the initial solution, as well as a waiting time deviation Δ_{c^w}, that is,

$$J^{\mathrm{TD}}_{\mathcal{I}^-} = J^*_{\mathcal{I}} - \Delta^{\mathrm{TD}-} \quad \text{with} \tag{39}$$

$$\Delta^{\mathrm{TD}-} = \underbrace{d^{k^*}_{n_p,t_q} + d^{k^*}_{t_q,n_s} - d^{k^*}_{n_p,n_s}}_{\Delta_{c^d}} + \gamma \underbrace{d^{k^*}_{t_q}}_{\Delta_{c^e}} + \epsilon\Delta_{c^w}. \tag{40}$$

4.3 | Analysis of performance guarantees for the task deletion heuristic

According to Theorem 4, the approximation ratio resulting from the application of the TD heuristic is bounded above by 2 for every task deletion reoptimization instance in which the TD heuristic fulfills Assumptions 3 and 4. As analyzed previously, Assumption 4 is fulfilled for a vast majority of problem instances and can be guaranteed to be fulfilled even for problem instances in which a reoptimized solution to the respective task insertion problem would yield the worst possible approximation ratio of $\alpha = 2$.

To analyze the fulfillment of Assumption 3 for the TD heuristic, we investigate the summands of the total cost deviation $\Delta^{\text{TD}-}$. The driving time deviation Δ_{c^d} cannot become negative due to the metric distances between nodes and the positive driving velocities. Also the task execution time deviation Δ_{c^e} must by definition of the base task durations and the capabilities always be greater or equal to zero. The only summand to the total cost deviation that might become negative is the waiting time deviation Δ_{c^w}. Therefore, the following cases can be differentiated concerning the fulfillment of Assumption 3 for the TD heuristic:

1. When the reoptimization problem is of configuration P_{hom} or P_{het}, that is, no precedence or synchronization constraints have to be considered, no waiting times arise in these instances. Therefore, also the waiting time deviation Δ_{c^w} must be equal to zero, that is, $\Delta_{c^w} = 0$. The overall cost deviation $\Delta^{\text{TD}-}$ therefore only contains non-negative summands and consequently always fulfills Assumption 3.
2. When the reoptimization problem is of configuration $\text{P}_{\text{t,het}}$ and precedence and/or synchronization constraints are present in the initial problem instance, waiting time deviations $\Delta_{c^w} \neq 0$ may arise. These waiting time deviations can both be positive or negative.

   a. A positive waiting time deviation $\Delta_{c^w} > 0$ occurs, if the total amount of waiting time within the reoptimized solution $X_{\mathcal{I}^-}^{\text{TD}}$ to the modified problem instance $\mathcal{I}^-$ is smaller than the sum of waiting times within the initial solution $J_{\mathcal{I}}^*$. In these cases, the reduction of waiting time is caused by robots having to wait shorter due to robot k^* reaching node n_s (and potentially following nodes) earlier than in the initial solution. In these cases, however, the waiting time deviation is another positive summand to the overall cost deviation $\Delta^{\text{TD}-}$ and thus Assumption 3 is always fulfilled.
   b. A negative waiting time deviation $\Delta_{c^w} < 0$ may occur, if robot k^* spends part of the time that was reserved within the initial solution for driving to and executing the deleted task t_q, with waiting within the reoptimized solution. Thus, if the total task execution time deviation Δ_{c^e} as well as the total reduction of driving times Δ_{c^d} is time spend waiting by robot k^* within the modified solution $X_{\mathcal{I}^-}^{\text{TD}}$, the total cost deviation becomes the smallest, that is,

   $$\Delta^{\text{TD}-} \geq \Delta_{c^d} + \gamma\Delta_{c^e} - \epsilon(\Delta_{c^d} + \Delta_{c^e}).$$

   For some choices of the weighting parameters, especially if $\epsilon \leq 1$ and $\epsilon \leq \gamma$, the fulfillment of Assumption 3 can thus also be guaranteed for negative waiting time deviations for the TD heuristic. Only if the waiting time weighting parameter ϵ is chosen to be comparatively high, instances in which negative waiting time deviations occur by the application of the TD heuristic, might not fulfill Assumption 3.

In summary it can be said that an approximation ratio of $\alpha = J_{\mathcal{I}^-}^{\text{TD}}/J_{\mathcal{I}^-}^* \leq 2$ can be guaranteed for the vast majority of task deletion reoptimization instances since Assumptions 3 and 4 are fulfilled by the TD heuristic in the majority of cases. With the following proposition we furthermore prove $\alpha \leq 2$ to be a tight bound for these instances.

Proposition 3. *For the the application of the TD heuristic to an instance of the MRTA task deletion reoptimization problem in which Assumption 3 and Assumption 4 are fulfilled, the approximation ratio of $\alpha \leq 2$ is a tight bound.*

Proof. In Figure 3 a MRTA task deletion problem instance with an approximation ratio converging toward $\alpha = 2$ for $\delta \to 0$ with $\delta > 0$, is depicted. In the initial problem instance two robots starting at different positions are available to perform two tasks. Both robots are fully capable for both task executions and differ in their driving velocity, that is, $v_1 = 1/2$ and $v_2 = 1$. The optimal solution of this instance assigns both tasks to robot r_2. In the modified problem instance, task t_2 is deleted. The resulting TD solution has an objective function value of $J_{\mathcal{I}^-}^{\text{TD}} = 4 - 2\delta$. The total cost deviation is therefore given by $\Delta^{\text{TD}-} = J_{\mathcal{I}^-}^{\text{TD}} - J_{\mathcal{I}}^* = 0$ and thus Assumption 3 is fulfilled. The optimal solution to the modified instance would be to assign task t_1 to the other robot which would yield an objective function value of $J_{\mathcal{I}^-}^* = 2$. Consequently, for $\delta \to 0$, the approximation ratio converges toward $\alpha \to 2$. Using the CMI with $J_{\mathcal{I}^-}^*$ as initial solution would result in assigning task t_2 to robot r_2 and thus $\Delta^+ - \Delta^{\text{TD}-} = 2 - 2\delta \leq 2 = J_{\mathcal{I}^-}^*$, that is, also Assumption 4 is fulfilled. ■

4.4 | Performance guarantee for the TD heuristic in MRTA task deletion problems with homogeneous groups of robots and a single depot

For the special case MRTA problems with a homogeneous team of robots which moreover all start and end their tour at the same position, that is, problems of configuration P_{hom} for which additionally $s^1 = s^2 = \cdots = s^K$ holds, a smaller upper bound on the approximation ratio can be proven for the TD heuristic.

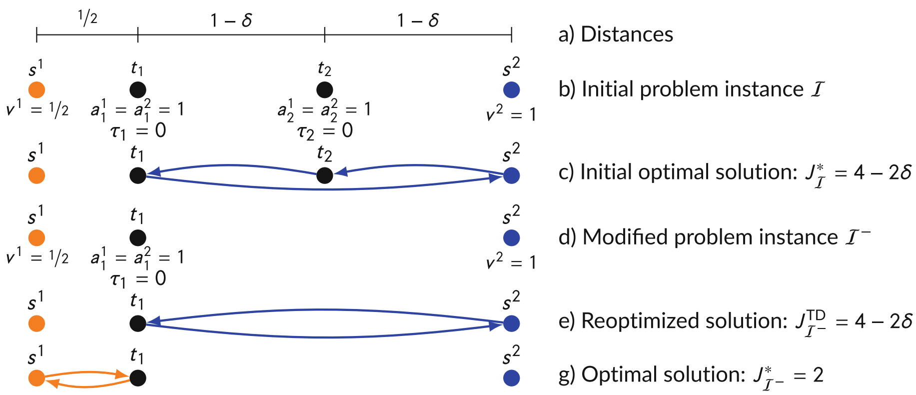

FIGURE 3 Example of a MRTA task deletion reoptimization instance with an approximation ratio $\alpha \rightarrow 2$ for $\delta \rightarrow 0$.

Theorem 5. *The application of the TD heuristic to MRTA task deletion reoptimization problems of configuration* P_{hom}, *for which additionally* $s^1 = s^2 = \cdots = s^K$ *holds, leads to an approximation ratio*

$$\alpha = \frac{J^{\text{TD}}_{\mathcal{I}^-}}{J^*_{\mathcal{I}^-}} \leq \frac{3}{2}. \tag{41}$$

Proof. The proof for this MRTA configuration is inspired by the proof made by Archetti et al. [2] for the TSP. Due to the problem configuration all velocities and capabilities are equal to one and no temporal constraints are considered. The costs of the reoptimized solution are therefore given by

$$J^{\text{TD}}_{\mathcal{I}^-} = J^*_{\mathcal{I}} - d(n_p, t_q) - d(t_q, n_s) + d(n_p, n_s) - \gamma\tau_q. \tag{42}$$

We furthermore know that the optimal solution to the initial problem instance $\mathcal{I}$ has costs at least as high as the costs of the optimal solution to the modified problem instance to which the task t_q was added to the tour of any robot by connecting it via two edges to any node, that is, $J^*_{\mathcal{I}} \leq J^*_{\mathcal{I}^-} + 2d(n, t_q) + \gamma\tau_q,\ \forall\ n \in \mathcal{V}$. Evaluating for nodes n_p and n_s and adding it up yields $J^*_{\mathcal{I}} \leq J^*_{\mathcal{I}^-} + d(n_p, t_q) + d(t_q, n_s) + \gamma\tau_q$. Inserting this inequality into (42) we get

$$J^{\text{TD}}_{\mathcal{I}^-} \leq J^*_{\mathcal{I}^-} + d(n_p, n_s). \tag{43}$$

Since at least nodes n_p and n_s are part of the modified problem instance, the homogeneous robots start and end in the same position and the triangle inequality holds, it holds for the optimal solution to the modified problem instance

$$J^*_{\mathcal{I}^-} \geq 2d(n_p, n_s). \tag{44}$$

Using (43) and (44), it follows for the approximation ratio

$$\alpha = \frac{J^{\text{TD}}_{\mathcal{I}^-}}{J^*_{\mathcal{I}^-}} \overset{(43)}{\leq} \frac{J^*_{\mathcal{I}^-} + d(n_p, n_s)}{J^*_{\mathcal{I}^-}} = 1 + \frac{d(n_p, n_s)}{J^*_{\mathcal{I}^-}} \overset{(44)}{\leq} 1 + \frac{d(n_p, n_s)}{2d(n_p, n_s)} = \frac{3}{2}. \tag{45}$$

■

Proposition 4. *For the application of the TD heuristic to the MRTA task deletion reoptimization problem of configuration* P_{hom} *with a common start and end position* $s^1 = s^2 = \cdots = s^K$, *the approximation ratio of* $\alpha \leq \frac{3}{2}$ *is a tight bound.*

Proof. The deletion heuristic applied by Archetti et al. [2] to the TSP task deletion reoptimization problem corresponds to the TD if only one robot is considered. Archetti et al. prove the corresponding upper bound on the approximation ratio to be bounded above by $\frac{3}{2}$ and prove the bound to be tight (Archetti et al. [2, Remark 2]). Since the TSP node deletion problem corresponds to the special case of the MRTA task deletion reoptimization problem of configuration P_{hom} with a common start and end position, $\alpha \leq 3/2$ is a tight bound for the respective MRTA reoptimization problem. ■

5 | SIMULATIVE EVALUATION

In the previous sections, we proved the approximation ratios resulting from the application of the CMI heuristic to the MRTA task insertion reoptimization problem as well as from the application of the TD heuristic to the MRTA task deletion reoptimization problem to be bounded above. In order to get an impression of the performance of the proposed MRTA reoptimization heuristics in the application to MRTA reoptimization problems other than the special examples in which the upper bounds of the approximation ratios are reached (cf. Figures 1–3), simulative evaluations were conducted. The experimental setup and the results of these investigations are presented in the following.

5.1 | Experimental setup

The evaluation of the proposed reoptimization heuristics was conducted both for the task insertion as well as for the task deletion reoptimization problem on problem instances of the previously introduced problem configurations $\mathrm{P}_{\mathrm{hom}}$, $\mathrm{P}_{\mathrm{het}}$ and $\mathrm{P}_{\mathrm{t,het}}$. For each configuration, a dataset with 100 MRTA reoptimization problem instances was generated, each considering three robots. Each instance of the task insertion problem contained eight tasks in the initial problem instance, that is, the ninth task was to be added by reoptimization. Each instance of the task deletion reoptimization problem contained nine tasks in the initial problem instance of which one was chosen randomly to be deleted.

The metric distances between tasks and robot's depots are calculated with the Euklidean metric from locations in the plane with the base area for the nodes to be located being a square with 100 units of length. The depots of the three robots are fixed to points with a distance of 15 units to the center of the base area and equal distances to each other, that is, $s^1 = [50, 65], s^2 = [37.01, 42.5], s^3 = [62.99, 42.5]$. For each reoptimization problem instance, the locations of all tasks are drawn uniformly distributed within the base area. For the configurations $\mathrm{P}_{\mathrm{hom}}$ and $\mathrm{P}_{\mathrm{het}}$, the parameter values given in Table 1 apply as before. The remaining parameters are chosen as follows: We consider three task types A-C which are mapped to the task locations by drawing from a discrete uniform distribution. Each task type corresponds to a base task duration τ_i and is associated with task- and robot-dependent capabilities. Their values and the three robots' velocities are stated in Table 3. For problem instances of the configuration $\mathrm{P}_{\mathrm{t,het}}$ precedence and synchronization constraints are described by uniformly drawn pairs of task indices which, viewed as undirected graph edges, do not contain self loops or repeated edges. Their number is limited to at most two constraints of each type and at least one constraint in total and is uniformly drawn as well. To also allow for the modeling of direct cooperation of several robots on the same task location, we generated an additional dataset, denoted as $\mathrm{P}^*_{\mathrm{t,het}}$. In this dataset tasks which are linked by a synchronization constraint share the same location and are of the additional task type D (also listed in Table 3). The common location is randomly chosen out of the initially assigned task locations. Since all robots are assigned the same capability to perform tasks of type D, the synchronous completion of the tasks is ensured. This allows for the modeling of the direct cooperation of any group of robots. For the evaluation of the application of the TD heuristic to MRTA problems with homogeneous robots and a single depot (see Section 4.4), we investigated an additional dataset for the MRTA task deletion reoptimization problem, which is denoted as $\mathrm{P}^*_{\mathrm{hom}}$. It differs from $\mathrm{P}_{\mathrm{hom}}$ only in the location of the depots, which is located in the center of the base area for all robots.

The reoptimization problem instances were solved using the reoptimization heuristics according to the respective configuration as described in Sections 3 and 4. Total enumeration was used both to obtain the initial optimal solutions, as well as to obtain the optimal solutions to the modified problem instances necessary to determine the respective approximation ratios. The methods were implemented in Python 3 and the calculations were performed on an Intel® Core i7-9800K processor.

5.2 | Results

The results of the simulative evaluation for the task insertion as well as for the task deletion reoptimization problem are given in Table 4 and Table 5, respectively. For each problem configuration, the average and the worst, that is, maximum, approximation ratio of the reoptimization instances of the respective dataset are given as well as the number of reoptimization instances per dataset for which the application of the proposed reoptimization heuristic yielded an optimal solution, that is, an approximation ratio of $\alpha = 1$.

TABLE 3 Parametrization of the base task durations and of the robots' velocities and capabilities for the configurations $\mathrm{P}_{\mathrm{het}}$, $\mathrm{P}_{\mathrm{t,het}}$ and $\mathrm{P}^*_{\mathrm{t,het}}$. For the configuration $\mathrm{P}_{\mathrm{hom}}$, the velocities and capabilities given in Table 1 apply as defined.

		Task type A	Task type B (used in $\mathrm{P}_{\mathrm{het}}$, $\mathrm{P}_{\mathrm{t,het}}$ and $\mathrm{P}^*_{\mathrm{t,het}}$)	Task type C	Task type D (used in $\mathrm{P}^*_{\mathrm{t,het}}$)
	Base task duration $\tau_i \in \mathcal{T}$	1	10	40	25
	Velocity $v^k \in V$		Capabilities $a_i^k \in A$		
Robot r^1	0.7 (1 in $\mathrm{P}_{\mathrm{hom}}$)	1	1	1	1
Robot r^2	1	1	0.5	0	1
Robot r^3	1	1	0	0.5	1

TABLE 4 Results of the simulative evaluation of the proposed reoptimization insertion heuristics.

		P_{hom}	P_{het}	$P_{t,het}$	$P^*_{t,het}$
Approximation ratio	∅	1.009	1.012	1.016	1.015
	max	1.070	1.189	1.120	1.205
	Guaranteed upper bound	1.5	$\in [1.5, 2.0]$	2.0	2.0
Number of optimally solved instances ($J^{CMI}_{I^+} = J^*_{I^+}$)		69	77	51	52

TABLE 5 Results of the simulative evaluation of the proposed reoptimization deletion heuristics.

		P_{hom}	P^*_{hom}	P_{het}	$P_{t,het}$	$P^*_{t,het}$
Approximation ratio	∅	1.011	1.009	1.007	1.023	1.045
	max	1.103	1.218	1.079	1.303	1.333
	Upper bound	2.0	1.5	2.0	2.0	2.0
Number of optimally solved instances ($J^{TD}_{I^-} = J^*_{I^-}$)		67	71	74	58	53

All reoptimization problem instances generated observe the guarantees on the approximation ratios derived previously. For the datasets of both reoptimization problems of all problem configurations, the average approximation ratio is just slightly above one and more than half of the problem instances were solved optimally by the application of the respective reoptimization heuristic. Also the worst approximation ratios, which lie between 1.07 for configuration P_{hom} and 1.205 for configuration $P^*_{t,het}$ with cooperative tasks for the task insertion problem and between 1.103 and 1.333 for the task deletion problem, are lower than the previously derived tight upper bounds on the approximation ratios for the respective configurations. Consequently, the simulative results validate that the proposed reoptimization heuristics observe the previously derived tight upper bounds on the approximation ratio, and also indicate that in many problem instances they yield even optimal or close-to-optimal solutions.

Remark. Producing solutions to the modified problem instance such that the modified solution resembles the initial one is a favorable trait in application domains where humans are involved in evaluating the solutions generated [25]. The stability of reoptimization approaches which describes the property of producing a solution to the modified problem instance as similar as possible to the initial solution [18] can be measured by means of the sum over the Levenshtein distances of all robot's schedules [7]. For comparison we evaluated the Levenshtein distances of the optimal solutions to the initial one for the task deletion problem, which lay in a range of 1 to 17. By definition of the proposed reoptimization heuristics, the reoptimized solutions all have the minimal possible Levenshtein distance of 1 to the initial solution and thus resemble the initial solution as much as possible.

Besides the positive results w.r.t. solution quality, the application of the proposed reoptimization heuristics furthermore are promising w.r.t. calculation time, with the average calculation times being 2.3 ms for the CMI and 1.6 ms for the TD heuristic.

6 | CONCLUSION

In this article, we have studied the reoptimization problems of both inserting and deleting a task from an optimal solution to a time-extended MRTA problem. We are the first to derive guarantees on the resulting solution for the application of reoptimization in coordination problems with more than one mobile entity. For the MRTA task insertion reoptimization problem we derive an upper bound of 2 for the approximation ratio for all reoptimization approaches that fulfill a weak assumption. Moreover, we introduce the cheapest maximum cost insertion heuristic (CMI) for which we prove the assumption to always be fulfilled and furthermore verify the resulting upper bound on the approximation ratio of $\alpha \leq 2$ to be tight. Further investigations of temporal unconstrained MRTA task insertion problems reveal the CMI to guarantee lower approximation ratios for these kinds of problems with the smallest upper bound on the approximation ration being $3/2$ for homogeneous groups of robots. Also for the task deletion reoptimization problem we prove an upper bound of 2 for the approximation ratio for all reoptimization approaches that fulfill weak assumptions. For the task deletion heuristic (TD) applied to solve the task deletion reoptimization problem, we prove the upper bound on the approximation ratio of $\alpha \leq 2$ to be tight. In the special case of temporal unconstrained tasks and an homogeneous group of robots all having the same start and end position, we prove a tight upper bound on the approximation ratio of $\alpha \leq 3/2$ for the TD. The theoretically derived upper bounds on the approximation ratios are validated by a practical evaluation which reveals the proposed heuristics to yield close-to-optimal approximation ratios for the majority of problem instances evaluated.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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