

Thermal simulations on a spinning rotor gauge to improve systematic uncertainties for viscosity measurements

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ABSTRACT

Experimental values for the viscosity of the radioactive hydrogen isotope tritium (T_2) are currently unavailable in literature. The value of this material property over a wide temperature range is of interest for applications in the field of fusion, neutrino physics, as well as to test ab initio calculations. To measure the viscosity of tritium, a measurement setup has been built utilizing a spinning rotor gauge inside a Dewar. Temperature control is of high importance to produce accurate and precise values. To ensure high quality measurement results, prior to the tritium measurements, thermal simulations are done on measurements with protium and deuterium inside the sample gas volume of the spinning rotor gauge.

1. Introduction

The viscosity is one of the fundamental material properties of gases, needed for gas dynamic calculations. Therefore, the viscosity of many gases has been measured over a wide temperature range [1–6]. For many simple systems like noble gases [7] or molecular hydrogen [8,9], ab initio calculations have been performed. Simple molecules which have not been covered widely due to their radioactivity are the tritiated isotopologues of hydrogen. Ab initio calculations exist [10], but since they have been carried out using a classical approach and neglect quantum effects, they are only suitable for temperatures of 300 K and above. Calculated values for the viscosity of tritium are currently only carried out by the extrapolation from protium and deuterium by utilizing their mass ratio. These calculations have an estimated uncertainty of 10%.

The cryogenic region of viscosity covered by neither theoretical calculations nor experiment is of interest for several applications such as the closed tritium cycle design, development and operation for nuclear fusion and experimental neutrino physics. The Karlsruhe Tritium Neutrino Experiment (KATRIN) is one such experiment which aims to measure the electron antineutrino mass, using the electron spectrum of the tritium β -decay. The windowless gaseous tritium source (WGTS) is a 10 m long stainless steel tube, where the tritium is circulating in a closed cycle. For the simulation of the column density profile inside the WGTS, the viscosity of tritium is needed, to reduce the systematic effect on the KATRIN results. The KATRIN collaboration has published a new experimental upper limit on the neutrino mass of 0.8 eV (90%CL) [11]. In fusion fuel cycle the viscosity of tritium is needed for example to

simulate the tokamak exhaust process, isotope separation systems based on gas chromatography and others.

In order to measure the viscosity of tritium down to temperatures relevant to the KATRIN experiment, which is currently operated at 80 K, we have built a viscosity measurement apparatus (ViMA) based on a spinning rotor gauge (SRG) [12]. By using the SRG as a viscometer, the temperature of the sample gas builds up a temperature gradient, caused by the heating of the rotating sphere. To reduce the impact on the viscosity measurement uncertainty, this gradient has to be known. For safety reasons, these studies have to be done prior to the tritium measurements with protium and deuterium. Only after proof of principle (see [13]) and validation of the results, the setup is ready to measure with tritium. The highest impact on the uncertainty of the viscosity measurements with this kind of setup is given by the temperature. The viscosity is in first approximation proportional to \sqrt{T} , leading to a deviation of nearly 1% for a difference from expected to measured temperature of 1 K. In this paper we report on simulation results, where the temperature difference inside the SRG has been calculated to reduce the current measurement uncertainty from 2% down to the targeted uncertainty of 1%.

2. Measurement procedure of the spinning rotor gauge (SRG)

The viscosity of gases can generally be measured by the so called spinning body viscometry. A rotor, shaped as a round plate or a sphere, fixed on a thin wire or held in suspension inside a sample gas volume by

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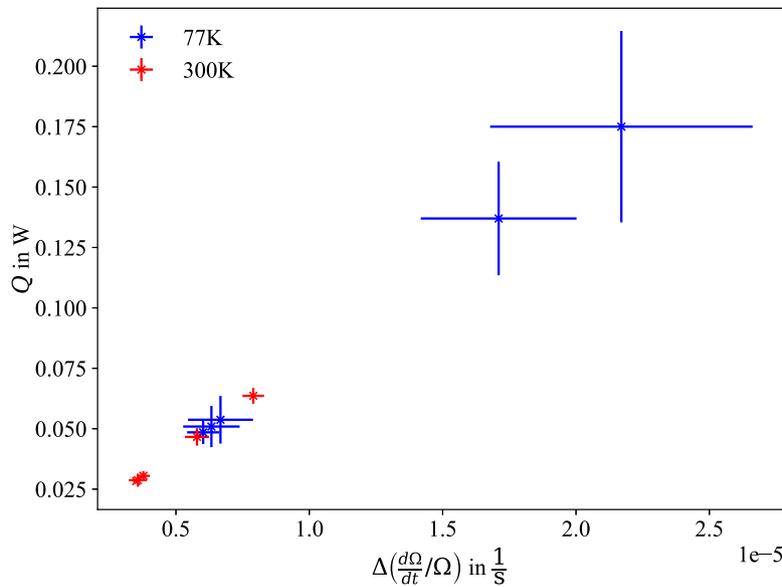


Fig. 1. Heat produced in dependence on the normalized deceleration rate. Here every cross corresponds to one measurement at one pressure setpoint. As defined, this looks fairly linear.

a magnet system and is circularly accelerated. When the acceleration by the magnet system is stopped, the deceleration, caused mainly by the surrounding gas is measured. In this case, a spinning rotor gauge (MKS SRG-3) is used, where the spinning body is a stainless steel sphere inside an evacuated thimble. By measuring the normalized deceleration rate in dependence of the pressure inside the evacuated finger, the viscosity can be calculated by a linear fit, as long as the system is in the slip regime of gas flows. The formula used is derived in [14]:

$$\frac{1}{D/\Omega} = \frac{1}{8\pi a_1^3 C_0 \mu} + \frac{1}{p} \cdot \sqrt{\frac{2k_B T}{m}} \left(\frac{c_m}{8\pi a_1^3 C_0} \left(\frac{3}{a_1} + \frac{1}{a_2} \right) \right). \quad (1)$$

where D is the torque on the sphere, Ω the angular speed of the sphere, C_0 is a calibration constant, a_1 and a_2 are the radii of the rotating sphere and the thimble surrounding it respectively. The viscosity μ can then be extracted from the y-axis-intersection of the fit function. For the temperature dependent measurement of the viscosity, the system is thermally cycled. First gas is filled inside the evacuated finger at a defined pressure, the spinning rotor gauge is started and after reaching the equilibrium, since the rotor is heated up during acceleration, the system is cooled down. For the analysis lastly the warm-up curve is used. This is then repeated with different pressure setpoints, to fit Eq. (1) for every measured temperature.

3. Thermal effects inside the SRG evacuated finger

Usually SRGs are used as pressure sensors for ultra high vacuum. In this case it takes hours for the sphere to be decelerated from 440 Hz to 420 Hz, which are the standard operation parameters for our device. Since the viscosity is measured in the slip regime, meaning approx. 20 Pa to 1500 Pa, it takes only a few seconds until the sphere has to be re-accelerated. In the slip regime, the interactions between the molecules and the cylinder walls are not negligible anymore, compared to the continuous flow regime, where the behavior of the fluid is dominated by the interactions of the molecules with each other. This is caused by the fact, that in the slip flow regime, the Knudsen-number becomes comparable to the mean free path of the molecules. By repeatedly accelerating the sphere, more energy is brought into the system, heating up the sphere by eddy currents. The question was, if this effect is measurable. Since a temperature measurement close to the rotating sphere might have caused issues with the deceleration measurement itself, a simulation has been done, to show a worst case scenario. To

get a grip on the energy accumulated inside the sphere, the pirouette effect is used. The pirouette effect couples the rotation frequency with the expansion of the sphere. The smaller the radius of the sphere, the faster it rotates at the same energy. Since a heating of the sphere will increase its radius slightly, the temperature of the sphere can be extracted from the pirouette effect. The thermal expansion of the sphere is directly coupled to the temperature, described by the thermal expansion coefficient α . The general equation for heat is given by

$$Q = c \cdot m \cdot \Delta T, \quad (2)$$

where c is the specific heat of the rotor, m is its mass and ΔT is the temperature difference. By calculating the mass of the rotor from the given dimensions of the sphere and its density, the only left quantity is the temperature difference. This can be extracted by equation 16 from chapter 24.3 in [15], which leads then to

$$Q = c \cdot \rho \cdot V \cdot \frac{\Delta(\frac{d\Omega}{dt}/\Omega)}{2\alpha}, \quad (3)$$

were, ρ is the density of the rotor, V is its Volume and $\Delta(\frac{d\Omega}{dt}/\Omega)$ is the difference in the normalized deceleration rate, extracted from the measurements. So $\Delta(\frac{d\Omega}{dt}/\Omega)$ is the difference between the highest and the lowest normalized deceleration rate from one acceleration cycle. The values for Q logically differ between the measurements, depending on the pressure, the temperature at which the measurement is conducted and the sample gas. Fig. 1 shows the values for Q calculated with Eq. (3). By definition, Q is linear with $\Delta(\frac{d\Omega}{dt}/\Omega)$ Knowing this and extracting the values for $\Delta(\frac{d\Omega}{dt}/\Omega)$ from the measurements, a simple thermal simulation with ANSYS® 2020 R2 can be done, to get an upper limit on the temperature difference inside the sample gas around the rotor. From Monte Carlo Simulations of the error budget we know, that an increased temperature around the rotor of 2 K would cause an error of 2%, which dominates all other systematic uncertainties by a factor of 10.

4. Thermal simulation of the SRG

ANSYS® provides many different Simulation tools, mainly finite elements methods (FEM). For the first simulations ANSYS® 2022 R2 Mechanic is used with a transient thermal simulation. To keep the simulation simple and time efficient, only the thimble, the sphere and the gas inside are simulated. The gas-flow inside the thimble of the

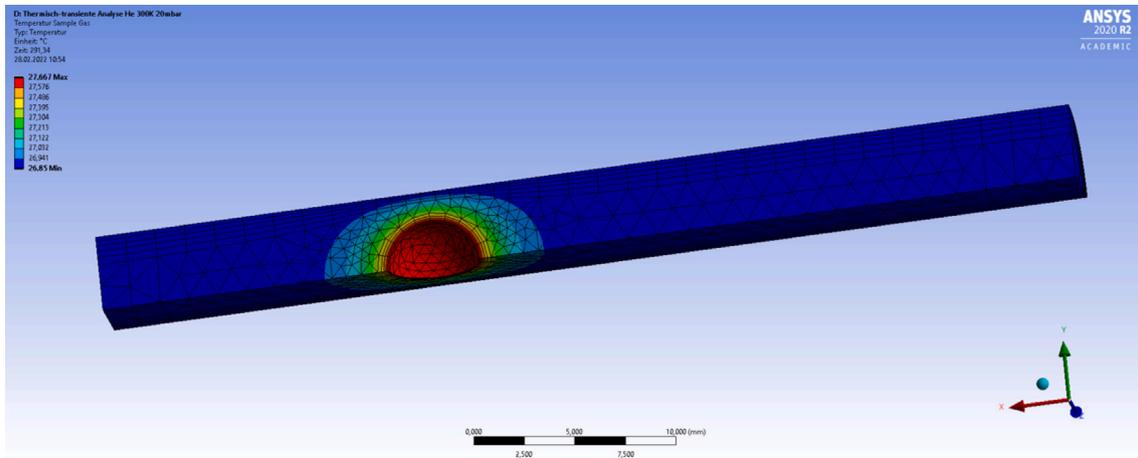


Fig. 2. Thermal simulation of the SRG. The temperature difference, generated with this simulation of the heat inside the SRG at 300 K and 2000 Pa is ≈ 1 K over a distance of 3 mm. Example for the temperature difference inside the sample gas volume. The Cylinder and the rotor are included in the simulation, but excluded on the graph for optical reasons. This picture shows the simulation result for helium at 300 K and 2000 Pa.

Table 1
 $\Delta(\frac{d\bar{v}}{d\bar{\omega}})$ in s^{-1} from the measurements with hydrogen, deuterium and helium for 77 K and 300 K.

p in Pa	$\Delta(\frac{d\bar{v}}{d\bar{\omega}})$ in s^{-1} in dependence of gastype and temperature					
	H2		He		D2	
	300 K	77 K	300 K	77 K	300 K	77 K
100	5.79E-06	6.02E-06	2.40E-06	3.40E-06	3.10E-06	1.40E-06
300	7.90E-06	6.67E-06	4.30E-06	4.00E-06	2.60E-06	1.70E-06
500	3.51E-06	6.33E-06	3.20E-06	3.40E-06	2.60E-06	2.30E-06
1000	3.55E-06	8.30E-06	2.70E-06	4.20E-06	2.40E-06	nan
2000	3.78E-06	1.71E-05	2.80E-06	1.17E-05	2.60E-06	nan

SRG is neglected. To mimic the cooling system, the outer surface of the thimble is set to 77 K or 300 K for the respective measurements. The thermal energy is generated as a heat flux through the surface of the sphere, to simulate a perfect heat transfer from the sphere to the sample gas. The initial temperature of the system is set to 77 K or 300 K respectively. The heat load, calculated with Eq. (3), is activated for 1 s to 2 s, afterwards the system has 6 s to 12 s to cool down again. The heating and cooling time is dependent on the deceleration rate. These time values, as well as the difference in the deceleration rate $\Delta(\frac{d\bar{v}}{d\bar{\omega}})$, see Table 1, are extracted from the raw data of the measurements, so the simulation can only be done after the measurement. This is repeated for 5 min until the system reaches equilibrium. Gas parameters like density, thermal conductivity, heat capacity and viscosity are adjusted to the current measurement, meaning 77 K to 300 K and 20 Pa to 2000 Pa. The same is done for helium, hydrogen and deuterium. The gas molecules are set static, so the results gained in this way are only upper limits, since the cooling effect of moving gas is neglected. As the sphere is rotating, there will be some movement of the gas surrounding it. To get an impression of the deviation in the results caused by moving gas, a fluent simulation is done for a few single setpoints, see Section 5. The temperature difference derived from the simple thermal simulation ranges from 0.8 K to 2.5 K. An example of such a gradient can be seen in Fig. 2. From these values, we can conclude that the likely reason for the systematic discrepancy between our measurement results for the viscosity and literature values is the heating of the SRG rotor, which, without correction, causes us to attribute measured viscosity values to systematically lower temperatures [12].

5. Fluid simulation of the SRG

The gas inside the thimble is not static but moving around the sphere, colliding with the inner wall of the thimble. This suggests the assumption, that the heat transfer inside the gas might be faster than

for the static case. To make this influence visible a simulation has to be done, which implements the gas motion caused by the rotating sphere. For this, ANSYS® 2022 R2 Fluent is used. The input parameters are the same as for the transient thermal simulation. In addition to the heat load on the sphere, its rotation is given as a boundary condition. These simulations work with smaller timesteps (1×10^{-3} s) to resolve the particle movement. After every timestep the temperature is evaluated before the next fluent timestep is started. If only the temperature had to be simulated, like in the transient thermal simulation with Mechanic, as described in the previous section, the timesteps could be in the order of seconds. As the particle movement influences the thermal stability and equilibrium of the system, they cannot be neglected. But since the movement tends to be fast – the sphere rotates with 440 Hz, accelerating the molecules – the timesteps have to be chosen much smaller, causing the computing time to increase from only a few minutes to a day for one simulation of 5 min real time. This shows, that the fluent simulations are much more time consuming. For this reason, only 500 Pa are simulated for helium, hydrogen and deuterium.

6. Simulation results

In both simulations the temperature difference inside the sample gas volume is approximately 3 mm wide and flattens at the thimble. This flattening is simply caused by the fact, that for the simulation the outer wall of the thimble is set to a fix temperature, either 77 K or 300 K. About 3 mm of distance to the sphere is too short, to be able to measure an effect with a simple temperature sensor, without disturbing the electromagnetic field of the SRG. What is even more, at this distance a temperature sensor would disturb the gas flow around the rotor, thereby influencing the result of the measurement. The simulation results show, that the heat generated during the measurement increases linearly with $\Delta(\frac{d\bar{v}}{d\bar{\omega}})$, and is independent of the gas temperature, as had been expected from the formula used to derive the heat load on

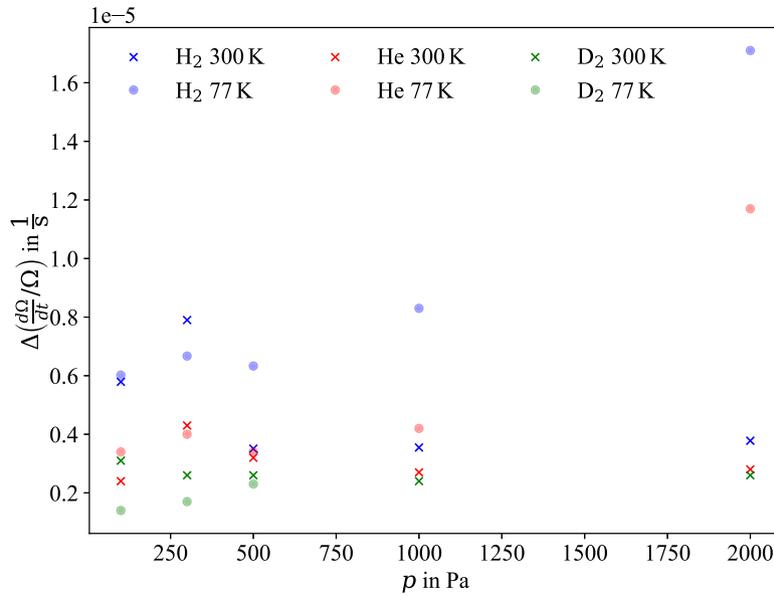


Fig. 3. Difference in the normalized deceleration rate in dependence of the pressure. It can be seen, that no matter the relation of the deceleration rate and the pressure is, the difference in the normalized deceleration rate, so the delta between the highest and lowest deceleration rate, is not constantly growing with the pressure.

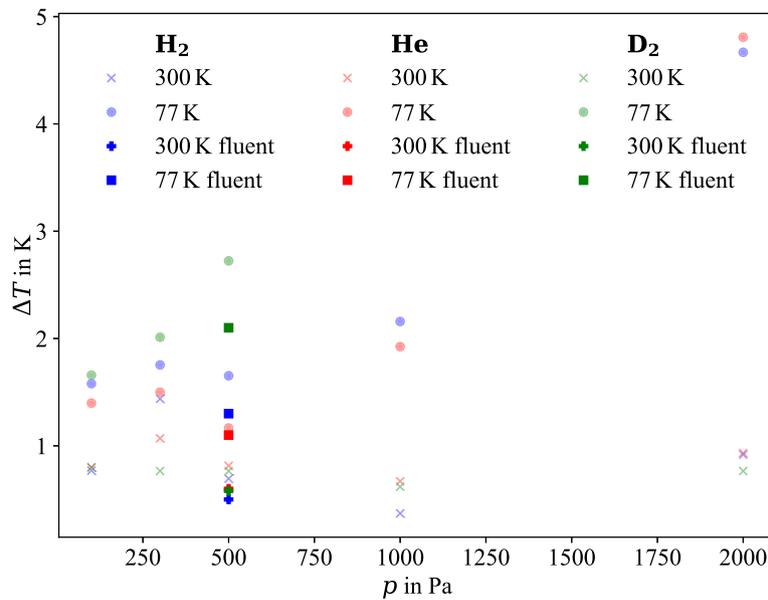


Fig. 4. Temperature difference in dependence of the pressure of the sample gas. The deceleration rate is not completely linear dependent on the pressure, but shows some structure. But from the fluent simulations at 500 Pa it can be seen, that the thermal simulation overestimates the temperature difference inside the SRG.

the sphere, see Fig. 1. Fig. 3 shows, that the delta in the deceleration rate does not increase constantly with the pressure. Only at higher pressures beginning at 1000 Pa, $\Delta\Omega$ seems to increase rapidly. Knowing that in this region at room temperature, the slip regime ends, this might already show the upper border of usability of the setup. In Fig. 4 and Table 2, the resulting maximum temperature difference is shown in dependence of the pressure. Here, again it can be seen, that the results of the fluent simulation are all smaller than the ones from the corresponding transient thermal simulations. Taking only the values into account which are well in between the ranges of the slip regime for this setup, it can be approximated, that for room temperature, the temperature difference has a maximum value of 0.5 K and at 77 K the temperature difference reaches 1.7 K.

7. Summary

In this paper we showed, that by using a SRG in the slip regime to measure the viscosity of gases, thermal issues appear. The rotating sphere has to be re-accelerated every few seconds, increasing the temperature of the surrounding gas. This causes a difference between the measured temperature to the true temperature. We used two different types of simulation to show this effect and fix it quantitatively. The first simulations are done without gas motion, to have fast results and an upper limit. These simulations show, that ΔT stays below 3 K in the relevant pressure region. The second simulations are then only done for the highest allowed pressure of 500 Pa, since they are very time consuming. These show, that for all gases ΔT is at maximum at 2.1 K

Table 2

Temperature difference in K in dependence of the gastype, pressure and temperature. The lowest row shows the results of the fluent simulation. The deviation between the thermal simulation and the fluent simulation goes up to nearly 40%, leading to much lower temperature differences to be expected around the sphere.

p in Pa	ΔT in K in dependence of and temperature					
	H2		He		D2	
	300 K	77 K	300 K	77 K	300 K	77 K
100	0.769	1.579	0.801	1.398	0.797	1.659
300	1.438	1.754	1.069	1.499	0.765	2.012
500	0.694	1.653	0.814	1.166	0.765	2.723
1000	0.370	2.158	0.670	1.924	0.618	nan
2000	0.918	4.667	0.933	4.808	0.765	nan
500 ^a	0.50	1.30	0.60	1.10	0.58	2.10

^a Values from fluent simulation.

(deuterium at 77 K). For the measurements at 300 K the gradient is between 0.5 K and 0.6 K. This helps us to reduce the uncertainty on the temperature, by implementing a gas and temperature dependent offset on the measured data during the analysis. In this way, the total uncertainty on the viscosity of gases can be reduced from 2% to 1% [12].

List of symbols

C_0	Parameter describing the flow of a sphere rotating inside a cylinder with rotation axis perpendicular to the cylinder axis
D	Torque on the SRG rotor
Q	heat load of the SRG sphere
V	Volume of the SRG rotor
$\Delta(\frac{d\Omega}{dt}/\Omega)$	deviation on the normalized deceleration rate of the SRG rotor
Ω	The angular speed of the SRG rotor
α	Thermal expansion coefficient of the SRG rotor
μ	Viscosity
ρ	Density of the SRG rotor
a_1	Radius of the SRG rotor
a_2	Radius of the cylinder surrounding the SRG rotor
c	specific heat of the SRG rotor
m	The mass of the SRG rotor

List of acronyms

KATRIN	Karlsruhe Tritium Neutrino Experiment
SRG	spinning rotor gauge
ViMA	viscosity measurement apparatus
WGTS	windowless gaseous tritium source
cd	column density

ORCID iD authorship contribution statement

Johanna Wydra: Writing – original draft, Visualization, Validation, Supervision, Software, Project administration, Methodology, Investigation, Formal analysis, Conceptualization. **Simon Gentner:** Writing – review & editing, Validation, Software, Investigation, Formal analysis. **Robin Größle:** Writing – review & editing, Validation, Supervision. **Michael Sturm:** Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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