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# Higher order chromaticity correction in the FCC-ee arcs – first approach

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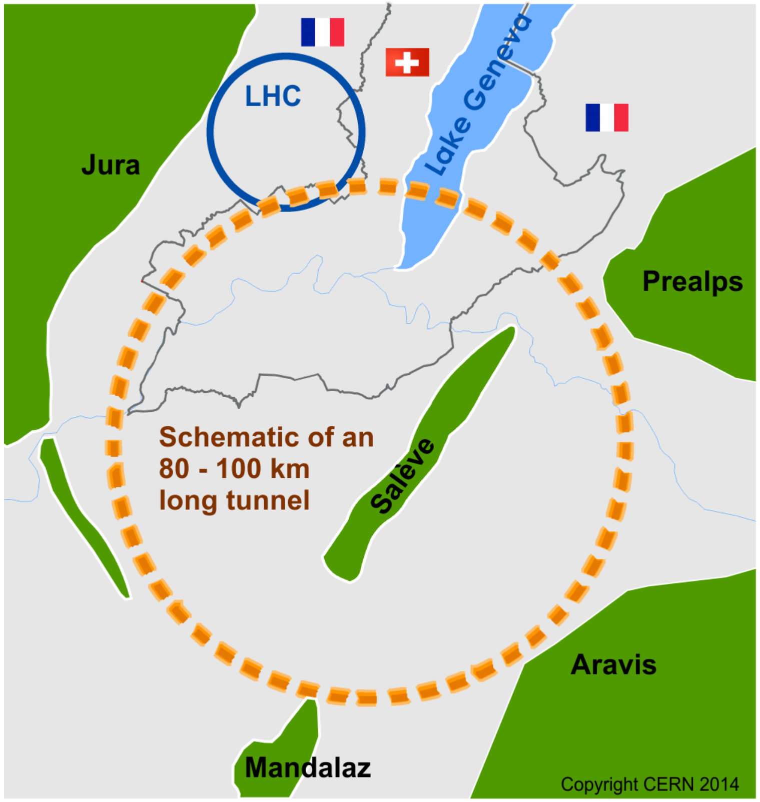
# FCC-ee

One part of the
**Future Circular Collider Study**

- 100 km e+/e-ring collider
- Precision studies of Z, W, H, t
  → Beam energies up to 175 GeV




- Beamstrahlung: mom. acceptance required: $\delta = \pm 2\%$
- Design luminosity: $L = O(10^{35}\ cm^{-2}s^{-1})$
  → Strong focusing in final doublet quadrupoles ($\beta^*_y = 1$ mm)
    → Very high chromaticity!

# Chromaticity

- Change of the tune with energy deviation

- Textbook*: $\Delta Q = \xi \cdot \Delta p / p$

- In our case not precise enough: $(\delta = \Delta p / p)$

$$Q(\delta) = Q_0 + \frac{\partial Q}{\partial \delta}\delta + \frac{1}{2}\frac{\partial^2 Q}{\partial \delta^2}\delta^2 + \frac{1}{6}\frac{\partial^3 Q}{\partial \delta^3}\delta^3 + \ldots$$

*K. Wille: The Physics of Particle Accelerators

# FCC-ee: Natural Chromaticities

| | 4 IRs | ΔQ (δ=1.5 %) |
|---|---|---|
| $Q_x$ | 502.16 | |
| $Q_x'$ | -603.80 | -9.06 |
| $Q_x''$ | -8258.29 | -0.93 |
| $Q_x'''$ | -1.4e+08 | -79.31 |
| $Q_x''''$ | -2.1e+12 | -4.43e+03 |
| $Q_y$ | 334.28 | |
| $Q_y'$ | -2044.43 | -30.67 |
| $Q_y''$ | -8.4e+06 | -944.12 |
| $Q_y'''$ | -2.0e+11 | -1.10e+05 |
| $Q_y''''$ | -6.5e+15 | -1.37e+07 |

- 1st order correction
  → Straight forward…
- Higher orders
  → First approach:
  Montague formalism

# Montague functions

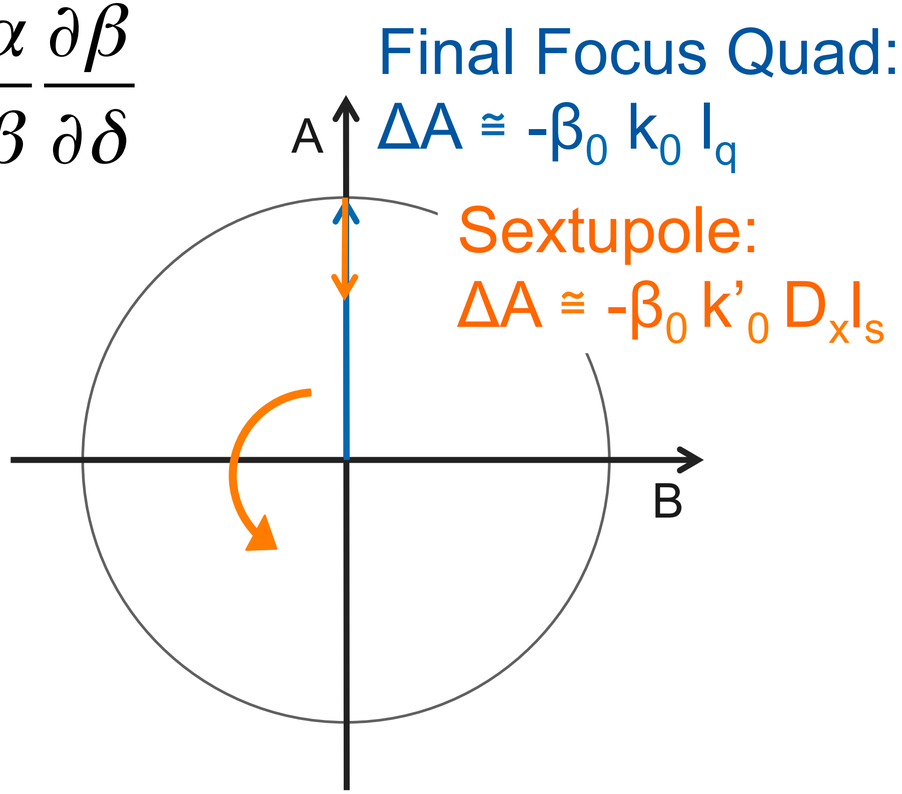
- Chromatic variables

$$B = \frac{1}{\beta}\frac{\partial \beta}{\partial \delta} \qquad A = \frac{\partial \alpha}{\partial \delta} - \frac{\alpha}{\beta}\frac{\partial \beta}{\partial \delta}$$

- W-vector

$$\vec{W} = \frac{1}{2}\left(B + iA\right)$$

$$= \frac{1}{2}\sqrt{A^2 + B^2}\, e^{i2\psi}$$




Oscillates with twice the phase advance!

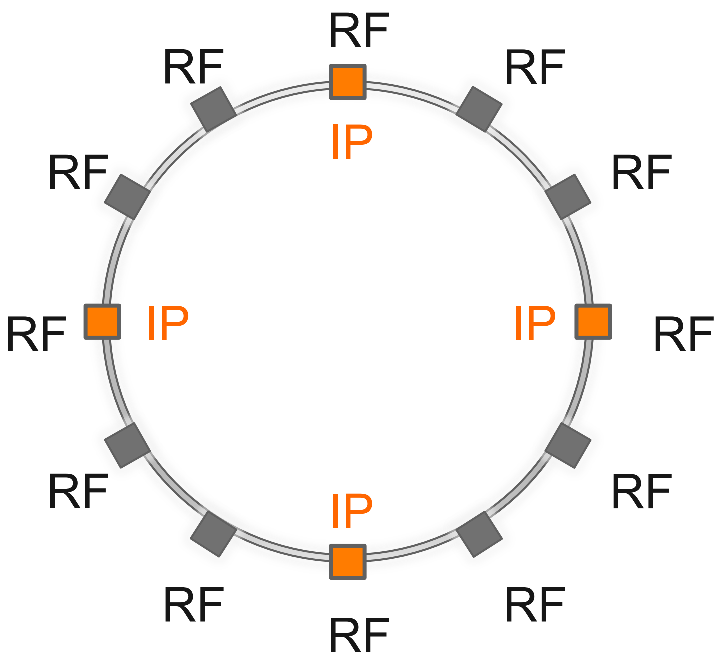
# FCC-ee Lattice

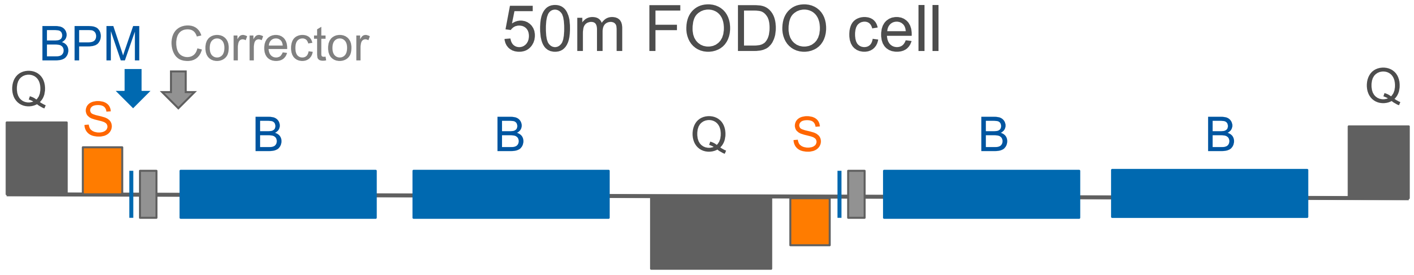
Circumference: 100 km

Arc length: 6.8 km

Straight section length: 1.5 km

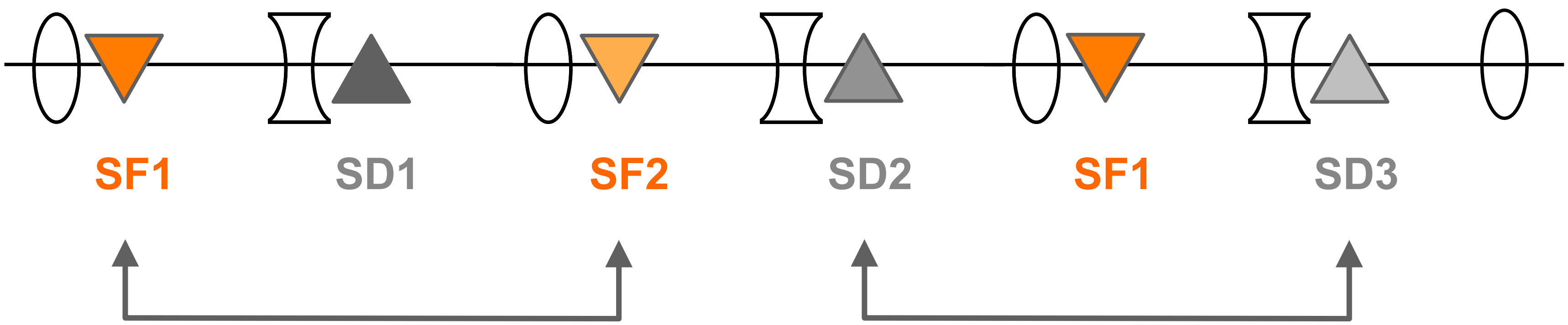
4 mini-beta insertions (IPs)




B = bending magnet, Q = quadrupole, S = sextupole

# FCC-ee sextupole scheme




$\mu_x = 90° = \pi/2$

$\mu_y = 60° = \pi/3$

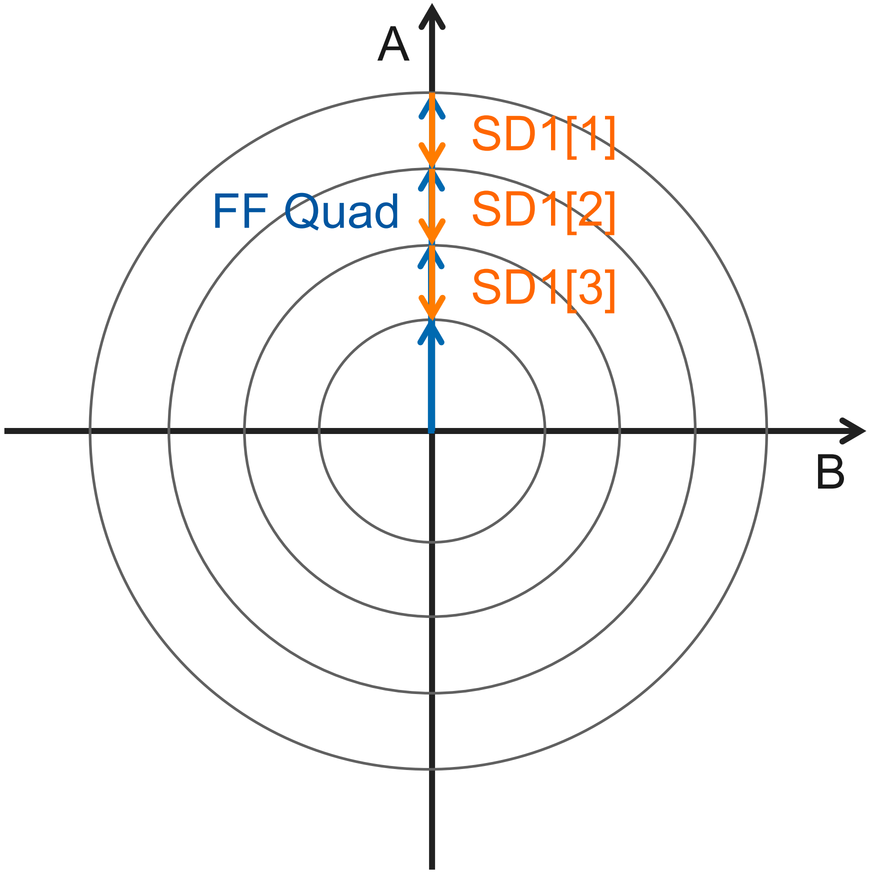
$\mu_x = 180° = \pi$ (→ -I transformation)

Even number of sextupoles per family!

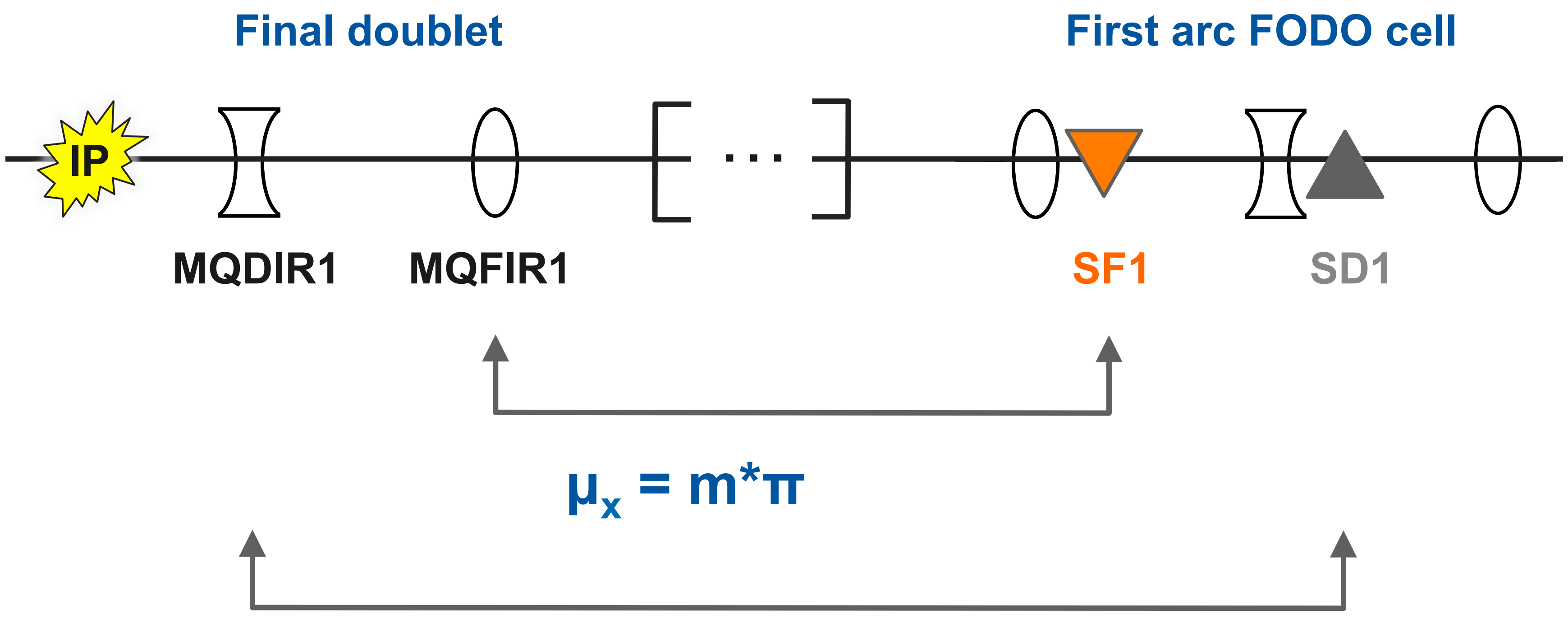
# -I transformation

- Sextupoles of each family are in phase

→ W-vector rotates by 2π



# Phase advance FD – 1st Sext.

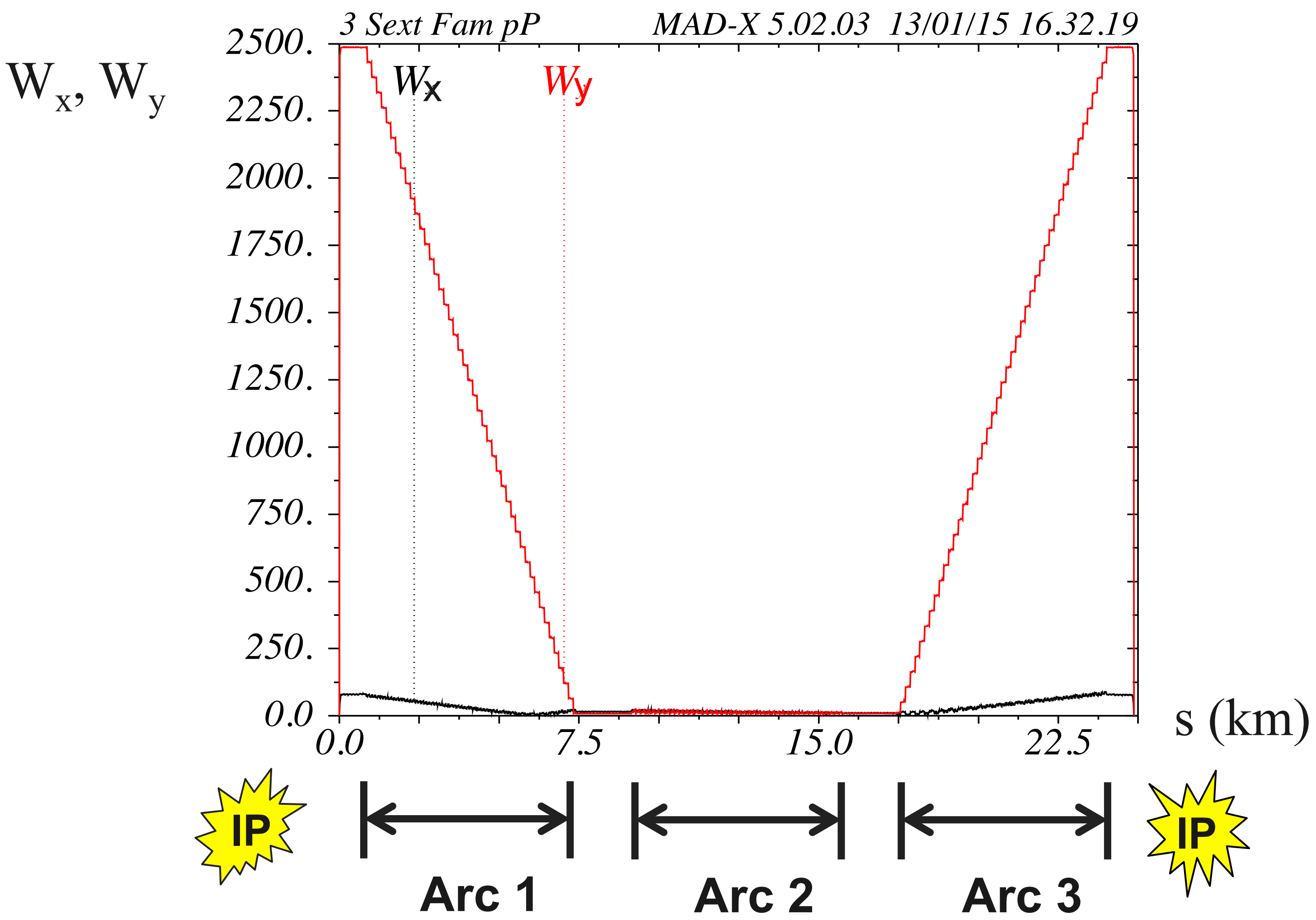



$\mu_x = m*\pi$

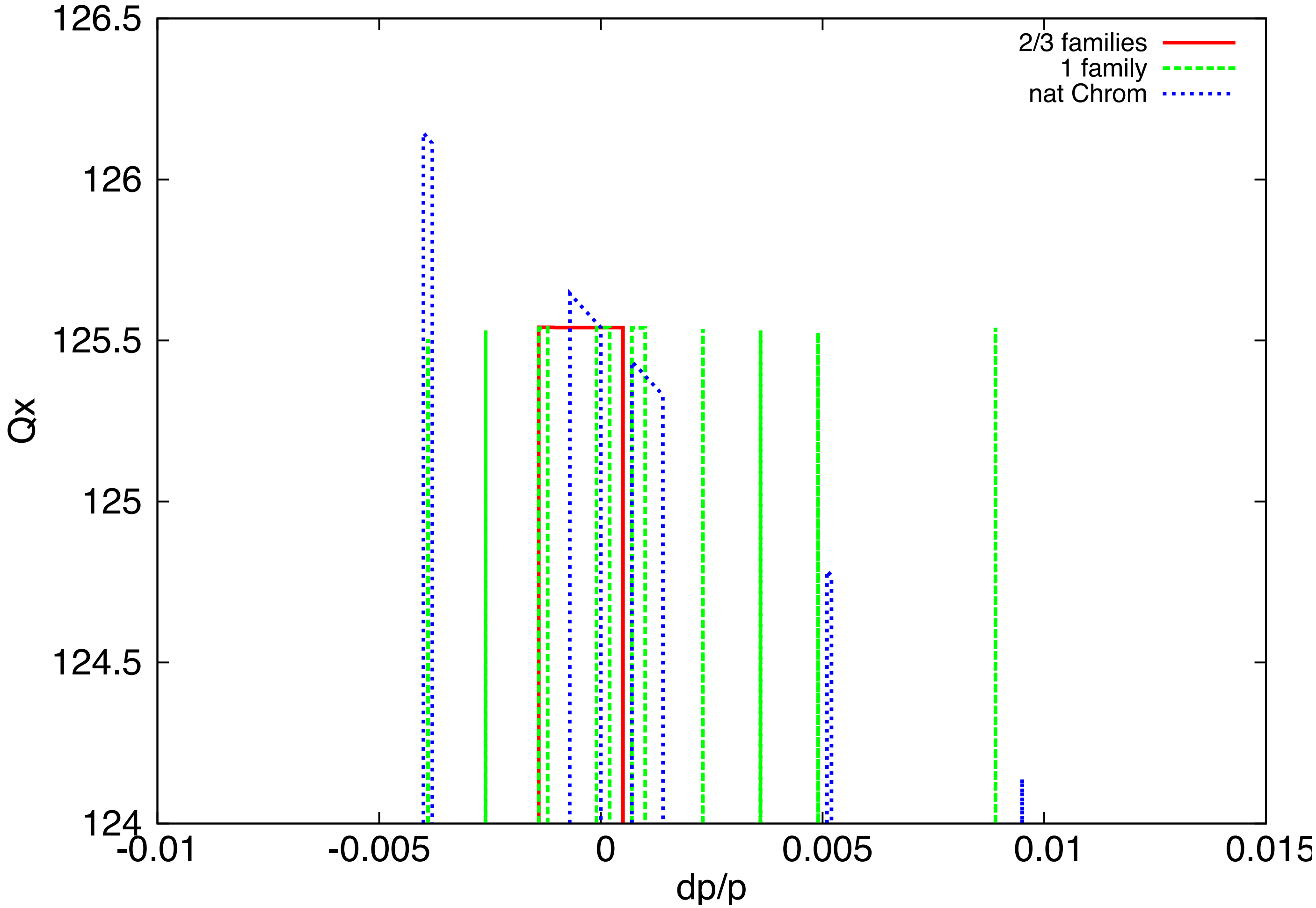
$\mu_y = n*\pi$

**(m,n integer)**

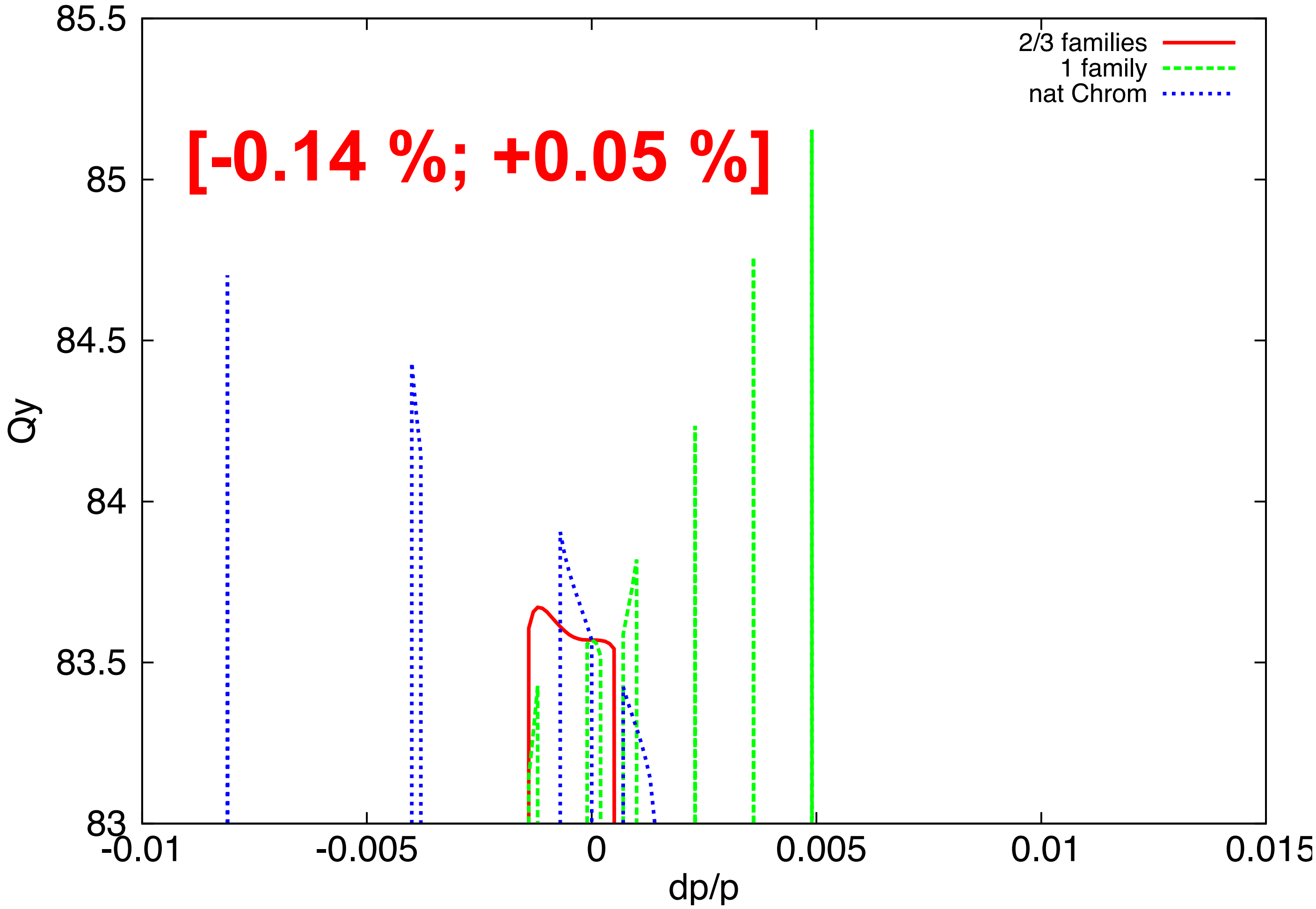
# W functions for 1 quarter



# Momentum acceptance in x



# Momentum acceptance in y

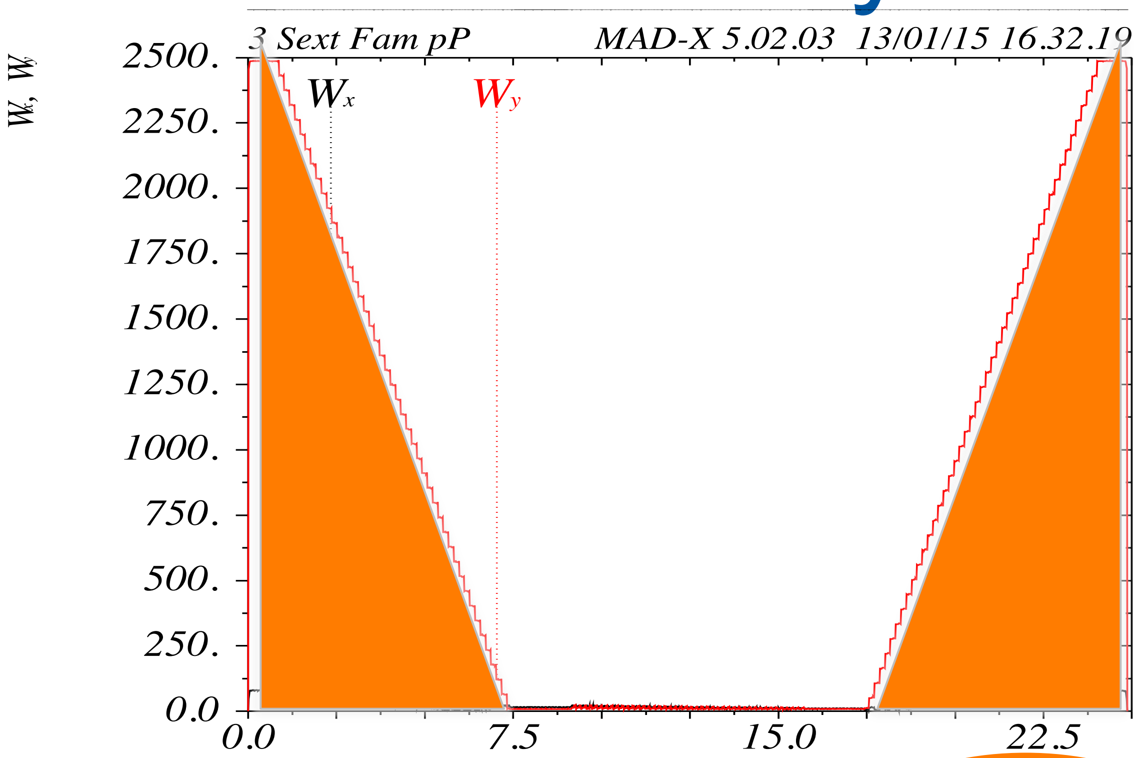


# "Corrected" Chromaticity

| | Nat. Chrom. | Corr. Chrom. | ΔQ (0.05 %) |
|---|---|---|---|
| $Q_x$ | 502.16 | 502.16 | |
| $Q_x'$ | -603.80 | 5.7e-05 | 2.83e-08 |
| $Q_x''$ | -8.3e+03 | 3.5e+03 | 4.41e-04 |
| $Q_x'''$ | -1.4e+08 | -5.5e+05 | -1.14e-05 |
| $Q_x''''$ | -2.1e+12 | -8.5e+09 | -2.20e-05 |
| $Q_y$ | 334.28 | 334.28 | |
| $Q_y'$ | -2044.43 | 2.8e-01 | 1.39e-04 |
| $Q_y''$ | -8.4e+06 | -1.2e+04 | -1.53e-03 |
| $Q_y'''$ | -2.0e+11 | -3.4e+09 | -7.00e-02 |
| $Q_y''''$ | -6.5e+15 | 3.6e+10 | 9.25e-05 |

(…using the whole ring.)

# 3rd order chromaticity




~W²

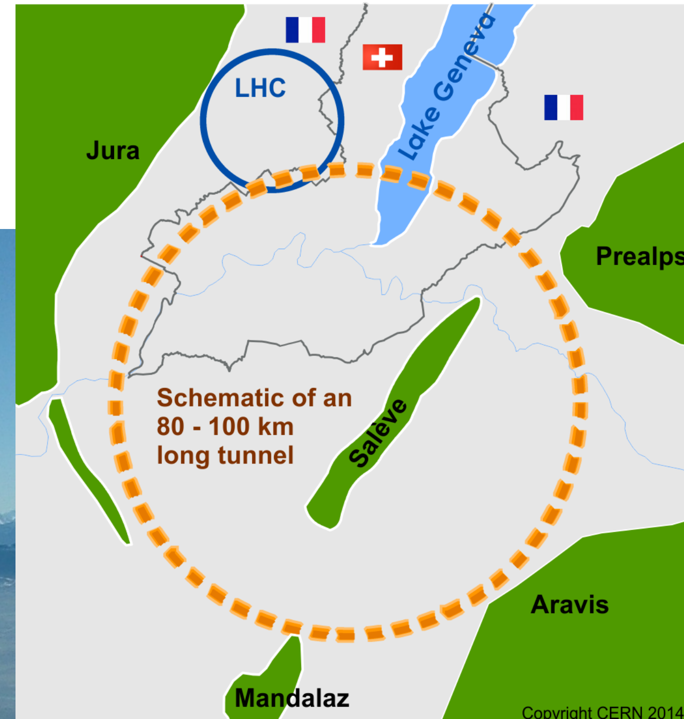
$$\frac{\partial^3 \varphi_y}{\partial \delta^3} = 6\frac{\partial \varphi_y}{\partial \delta} - \int_0^{\Pi} \beta_y (K_1 - K_2\eta_0)(a_{y,1}^2 + b_{y,1}^2) ds + $$
$$+ \; 3\int_0^{\Pi} \beta_y (K_2\eta_1 - K_2\eta_2) ds + \frac{3}{2}\int_0^{\Pi} \beta_y b_{y,2}(K_1 - K_2\eta_0) ds$$

Anton Bogomyagkov

# Take home message

- The higher orders of the chromaticity have a big influence in our machine.
- They need to be well corrected in order to obtain ± 2% momentum acceptance.
- We are applying and investigating methods to do that.

# Thank you for your attention!

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# Chromaticity

$$\varphi(\delta) = \varphi_0 + \frac{\partial \varphi}{\partial \delta}\delta + \frac{\partial^2 \varphi}{\partial \varphi^2}\delta^2 + \ldots$$

The first three orders, as derived by A. Bogomyagkov:

$$\frac{\partial \varphi_y}{\partial \delta} = \frac{1}{2}\int_0^{\Pi} \beta_y (K_1 - K_2\eta_0)\, ds\,,$$

$$\frac{\partial^2 \varphi_y}{\partial \delta^2} = -2\frac{\partial \varphi_y}{\partial \delta} - \int_0^{\Pi} \beta_y K_2 \eta_1\, ds + \frac{1}{2}\int_0^{\Pi} \beta_y b_{y,1}(K_1 - K_2\eta_0)\, ds\,,$$

$$\frac{\partial^3 \varphi_y}{\partial \delta^3} = 6\frac{\partial \varphi_y}{\partial \delta} - \int_0^{\Pi} \beta_y (K_1 - K_2\eta_0)(a_{y,1}^2 + b_{y,1}^2)\, ds + $$

$$+ \; 3\int_0^{\Pi} \beta_y (K_2\eta_1 - K_2\eta_2)\, ds + \frac{3}{2}\int_0^{\Pi} \beta_y b_{y,2}(K_1 - K_2\eta_0)\, ds\,.$$

(A. Bogomyagkov: “Crab waist interaction region for FCC-ee and the arc second attempt”, presentation in the FCC-ee meeting no. 13, 09 Febuary 2015)

# Chromaticity

- Change of the tune with energy deviation

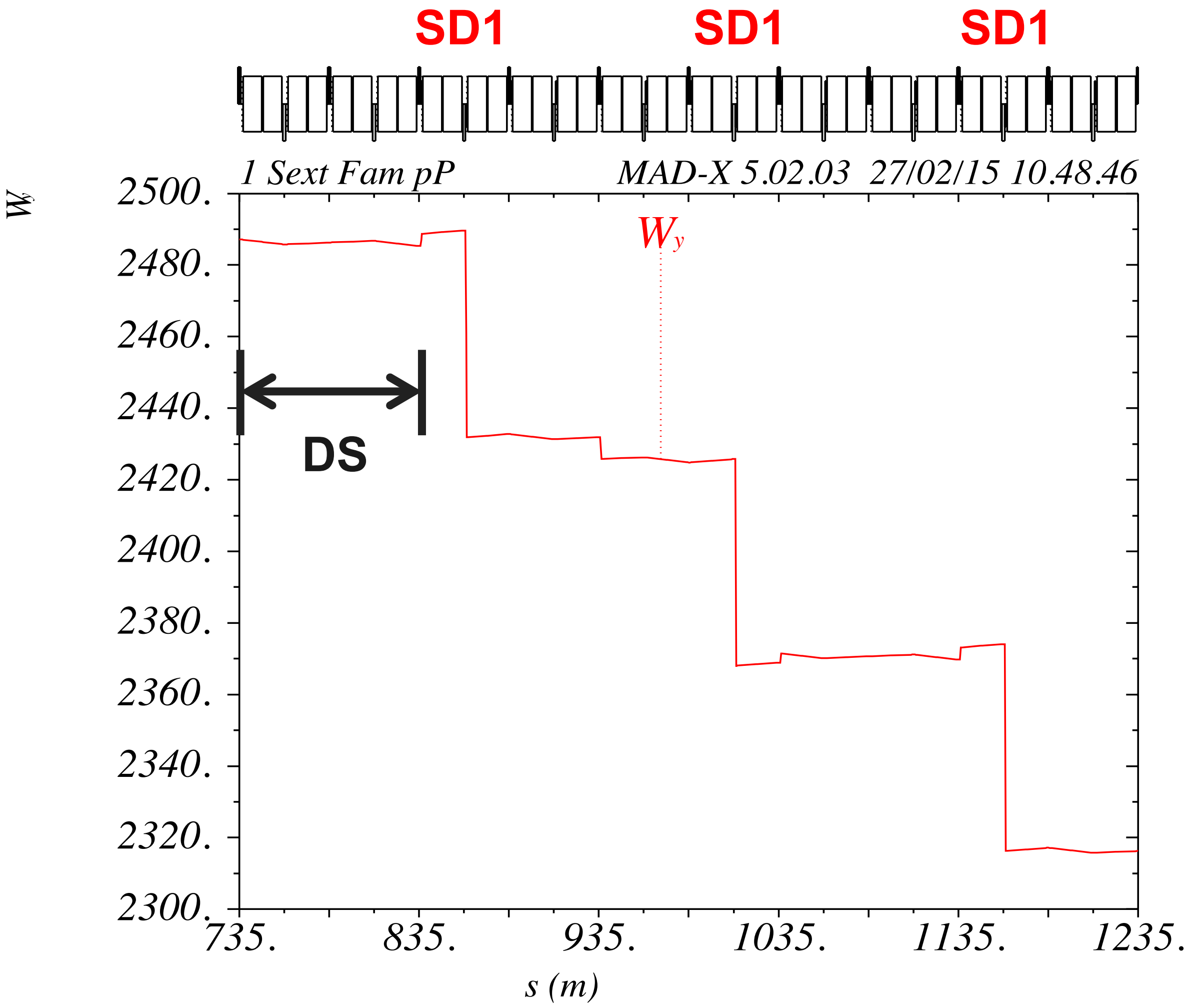
- Textbook*: $\xi = \frac{\Delta Q}{\Delta p / p} = \frac{1}{4\pi} \oint \left( K_1 + K_2 D_x \right) \beta(s) \, ds$

- In our case not precise enough: $\left( \delta = \Delta p / p \right)$

$$Q(\delta) = Q_0 + \frac{\partial Q}{\partial \delta} \delta + \frac{1}{2} \frac{\partial^2 Q}{\partial \delta^2} \delta^2 + \frac{1}{6} \frac{\partial^3 Q}{\partial \delta^3} \delta^3 + \ldots$$

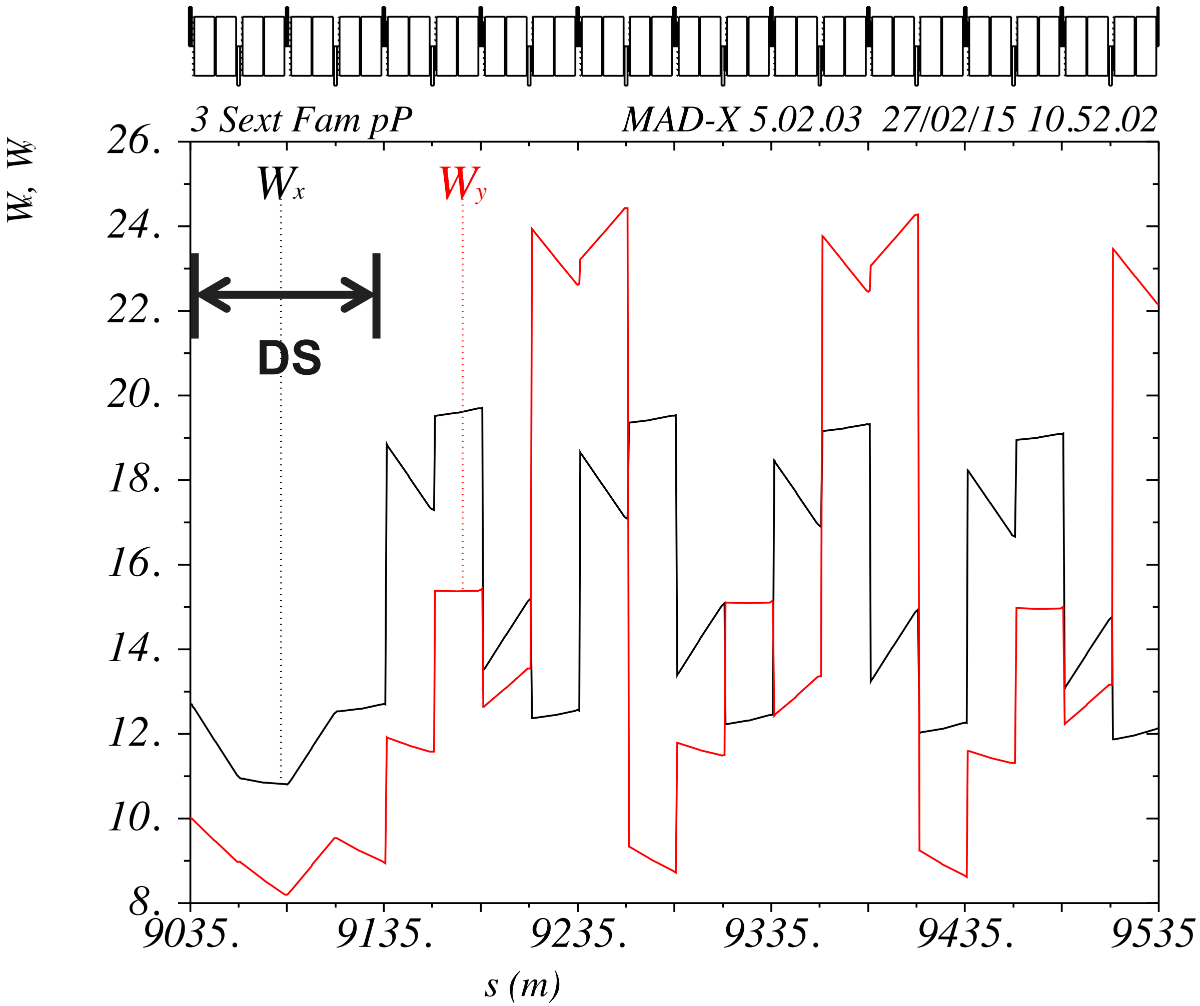
*K. Wille: The Physics of Particle Accelerators

# Vertical W-function in Arc 1




k2sd1 = -18.44 1/m$^3$
k2sd2 = -0.11 1/m$^3$
k2sd3 = -0.19 1/m$^3$

# W-functions in Arc 2




k2sd1 = -6.22 1/m$^3$
k2sd2 = -6.21 1/m$^3$
k2sd3 = -6.29 1/m$^3$

# “Corrected” Chromaticity

| | Nat. Chrom. | Corr. Chrom. |
|---|---|---|
| $Q_x$ | 502.16 | 502.16 |
| $Q_x'$ | -603.80 | 5.7e-05 |
| $Q_x''$ | -8.3e+03 | 3.5e+03 |
| $Q_x'''$ | -1.4e+08 | -5.5e+05 |
| $Q_x''''$ | -2.1e+12 | -8.5e+09 |
| $Q_y$ | 334.28 | 334.28 |
| $Q_y'$ | -2044.43 | 2.8e-01 |
| $Q_y''$ | -8.4e+06 | -1.2e+04 |
| $Q_y'''$ | -2.0e+11 | -3.4e+09 |
| $Q_y''''$ | -6.5e+15 | 3.6e+10 |

- $1^{st}$ order
  → corrected ✔
- Higher orders
  → Better, but not good enough!

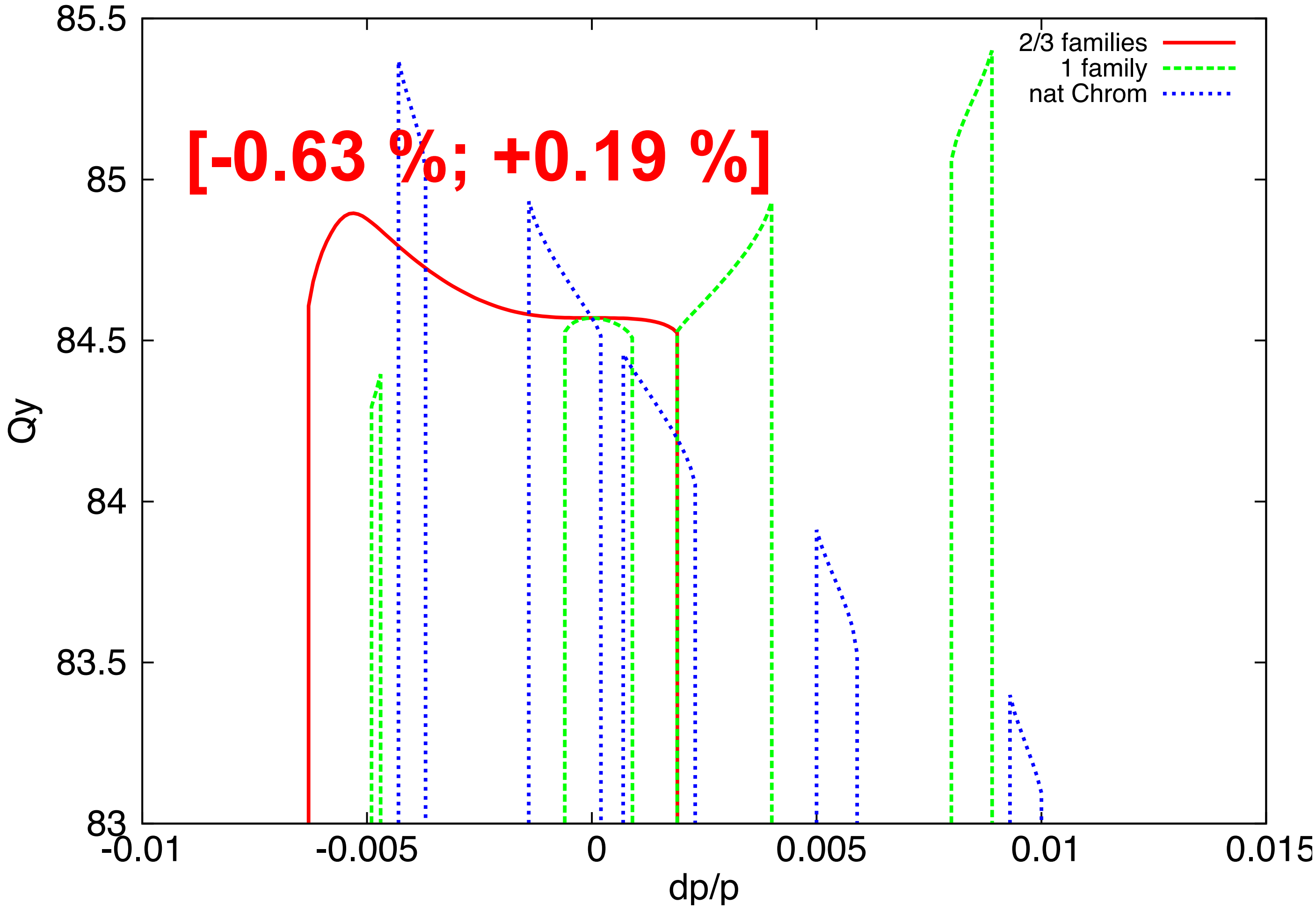
# Next steps I

… according to the FCC-ee work plan:

- Keep the layout as ideal as possible
  → 12-fold ring with 4 equally distributed mini-betas

- Variation of β* with LEP-like mini-beta module
  → How far can we get without local CCS?
  → LEP parameters: Can we achieve the same momentum acceptance?

# Mom. accept. for LEP parameters




$\beta_x^* = 1.75$ m
$\beta_y^* = 0.07$ m

# Next steps II

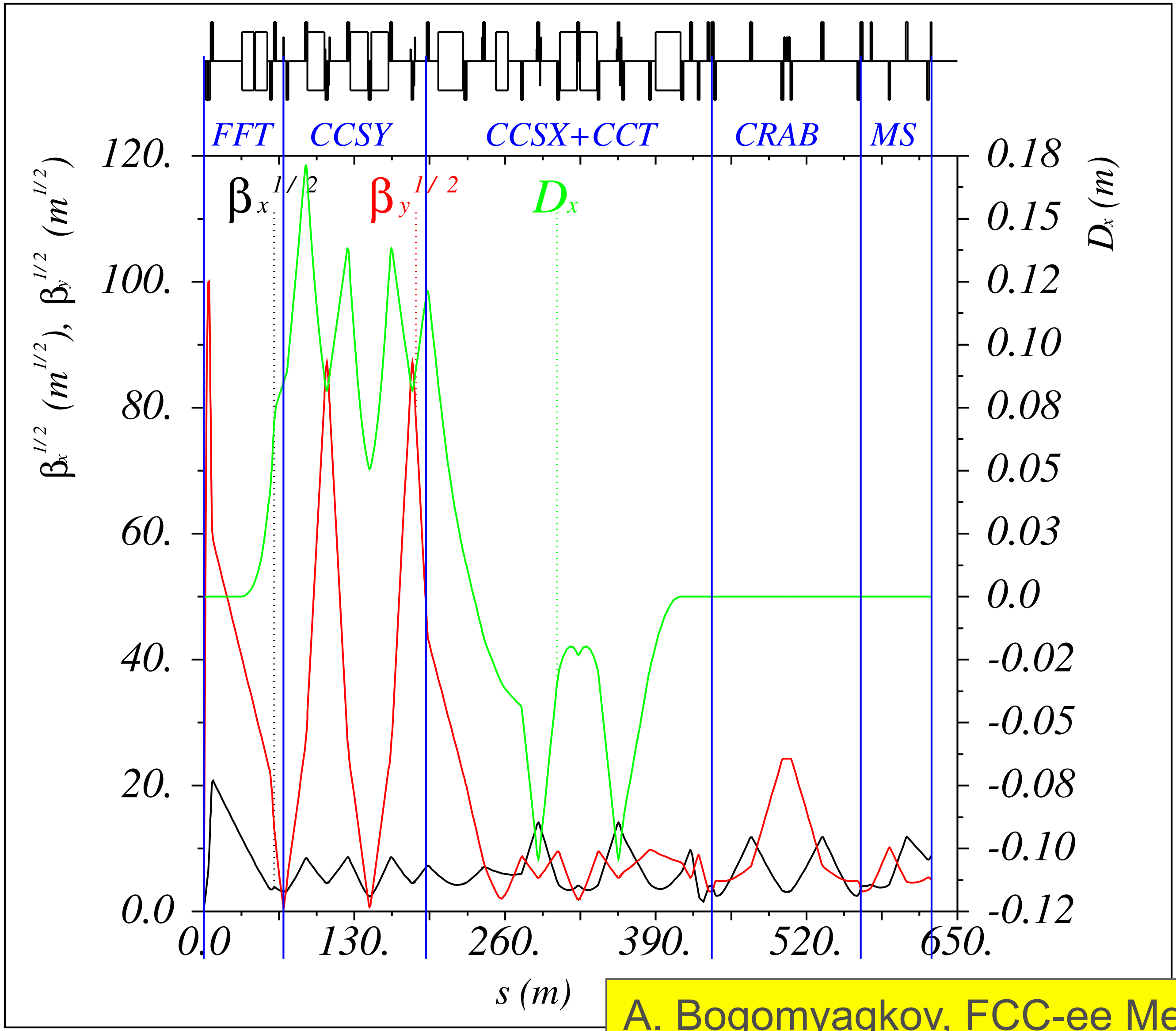
… according to the FCC-ee work plan:

- Correction of the actual chromaticities
  → … not just the W functions

- Best possible correction of the 3rd order chromaticity

- (Anton proposed to switch to 135° phase advance
  → Scheme with 4 sextupole families might be able to correct 4th order chromaticity)

# Interaction Region optical functions: New

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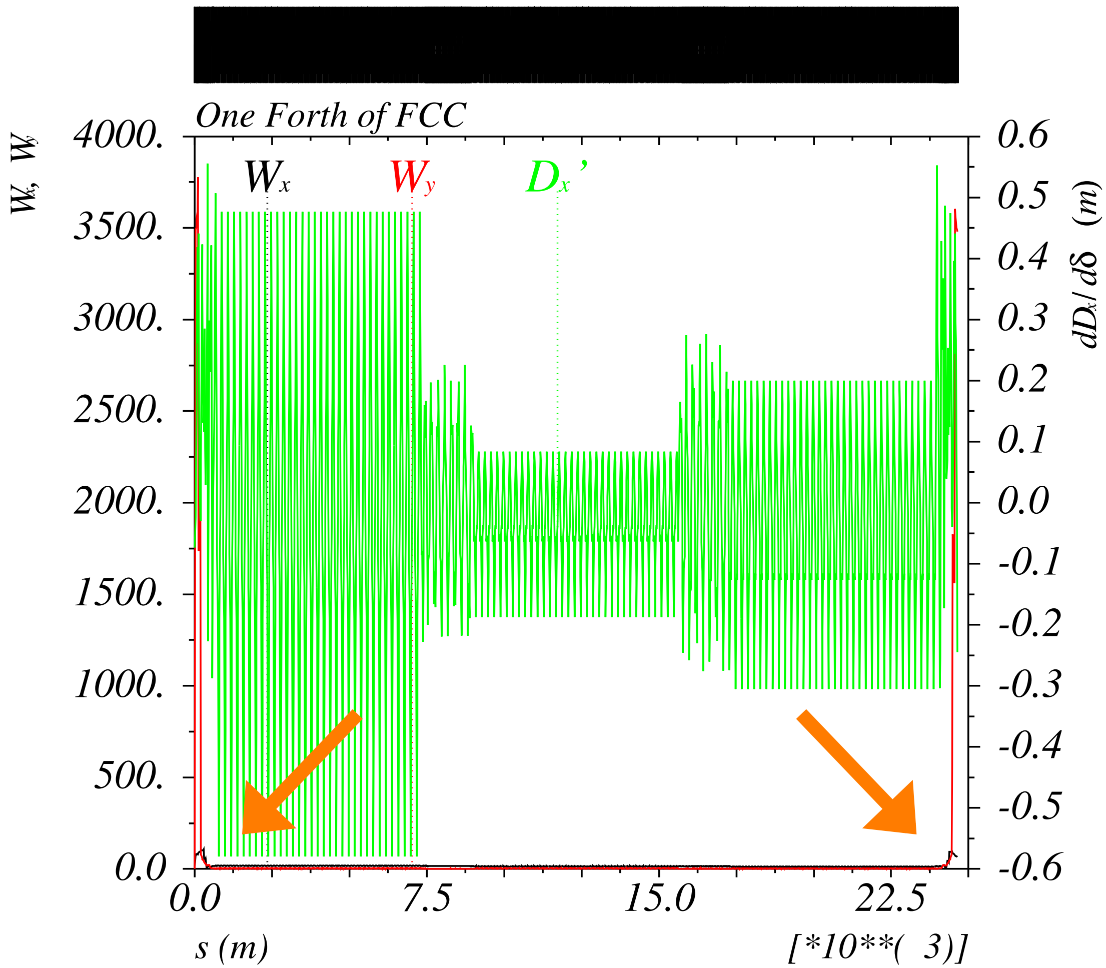
# Advantage of local CCS

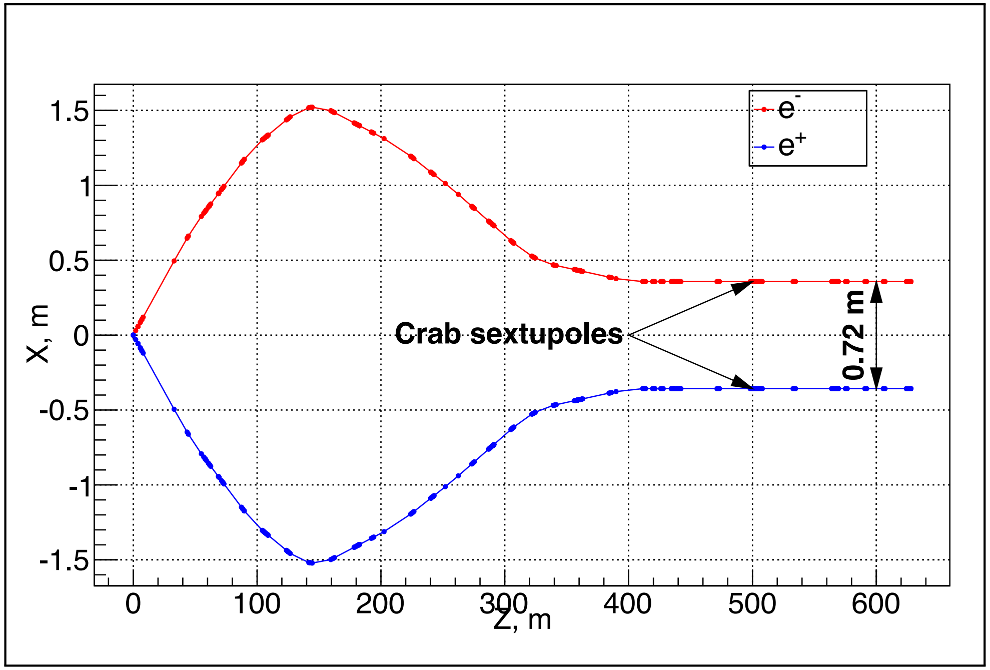



| | Value | $\Delta Q(2\%)$ |
|---|---|---|
| $Q_x$ | 124.54 | |
| $Q'_x$ | 0 | 0 |
| $Q''_x$ | 170 | 0.034 |
| $Q'''_x$ | $-4.5 \cdot 10^4$ | $-0.059$ |
| $Q''''_x$ | $-5.3 \cdot 10^6$ | $-0.035$ |
| $Q_y$ | 84.57 | |
| $Q'_y$ | 0 | 0 |
| $Q''_y$ | 387 | 0.077 |
| $Q'''_y$ | $-5.3 \cdot 10^5$ | $-0.7$ |
| $Q''''_y$ | $-4.3 \cdot 10^6$ | $-0.029$ |

**3 orders of magn. smaller!!!**

Anton Bogomyagkov

# Interaction Region layout: New




Energy loss $\Delta U = 0.1$ GeV

| | L [m] | B [T] | $\phi$ [mrad] |
|---|---|---|---|
| B0 | 10.5 | 0.06 | 1 |
| B1 | 10.5 | 0.17 | 3 |
| B2 | 14.5 | 0.17 | 4.2 |
| B3 | 15 | 0.22 | 5.6 |
| B4 | 15 | 0.22 | 5.6 |
| B5 | 21.5 | 0.06 | 2.2 |
| B6 | 10.5 | 0.04 | 0.7 |
| B7 | 14.5 | -0.11 | -2.7 |
| B8 | 14.5 | -0.11 | -2.7 |
| B9 | 21.5 | -0.05 | -1.8 |

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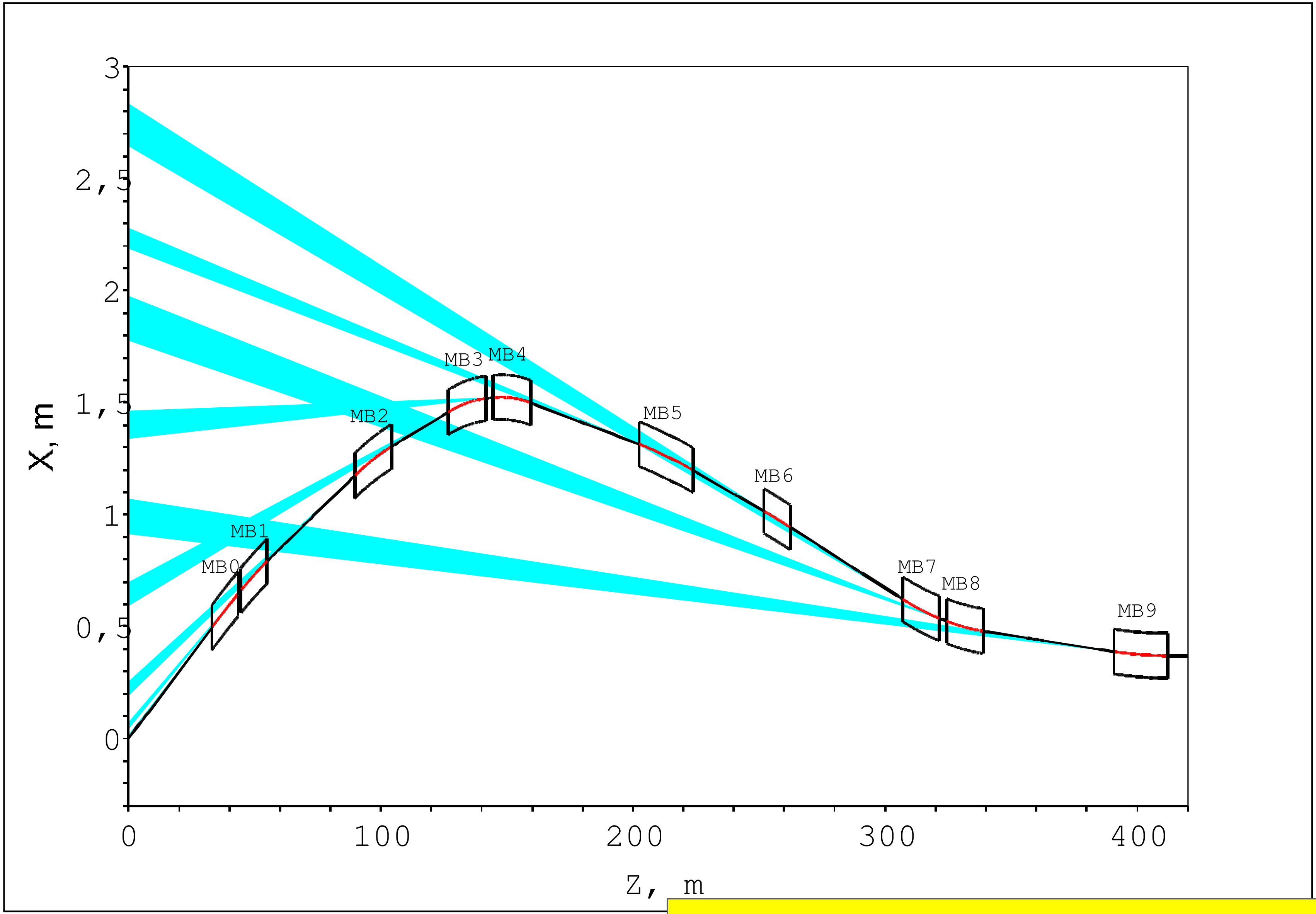
# New synchrotron radiation fans from S. Glukhov

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