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Higher order chromaticity correction for the future low emittance collider FCC-ee

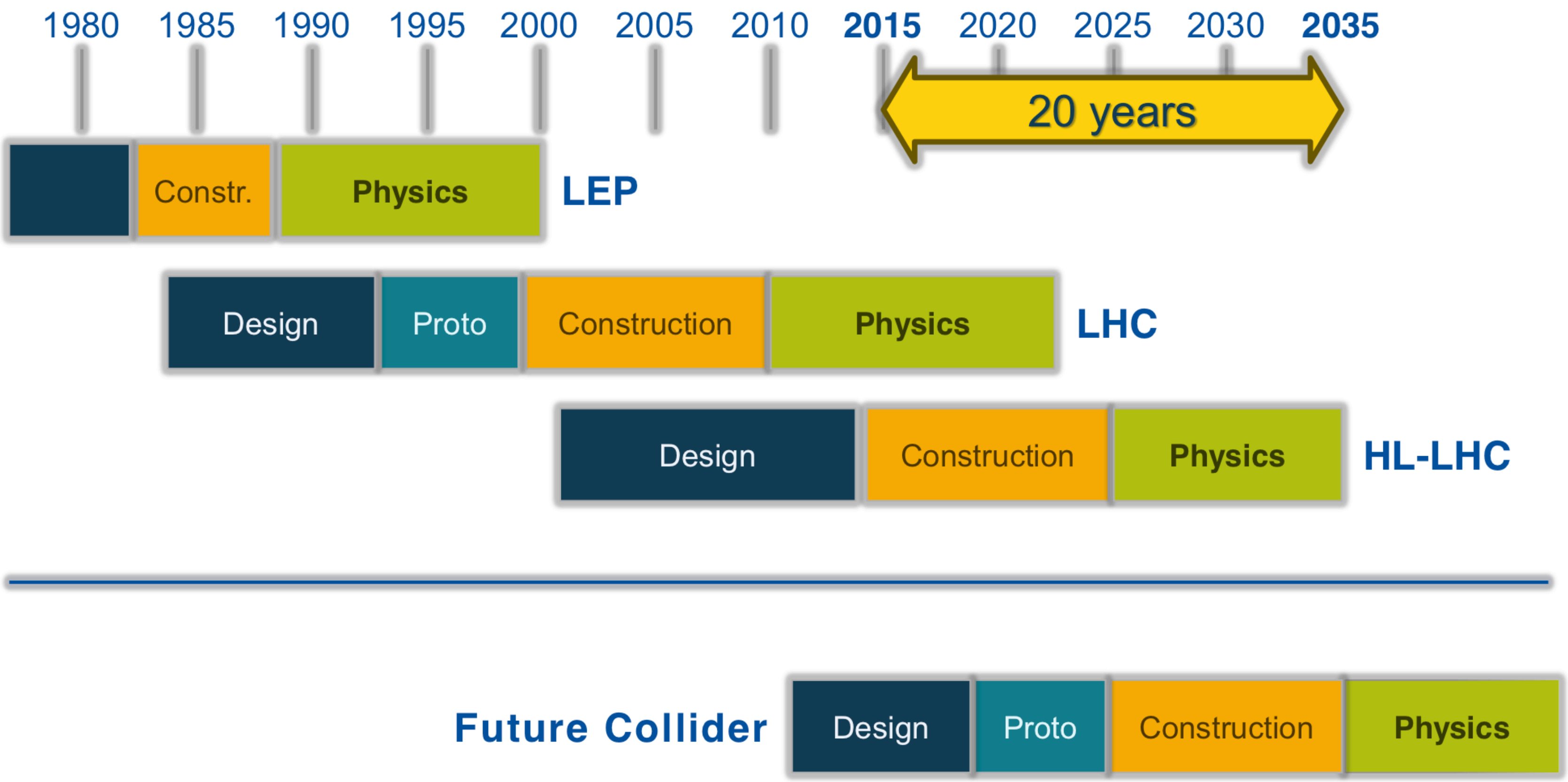
Bastian Haerer (CERN, Geneva; KIT, Karlsruhe) for the FCC-ee lattice design team

Future Circular Collider Study

- **European Strategy Group for Particle Physics 2013:**

  "…to propose an ambitious post-LHC accelerator project….., CERN should undertake design studies for accelerator projects in a global context,…with emphasis on proton-proton and electron-positron high-energy frontier machines.…."

CERN Circular Colliders

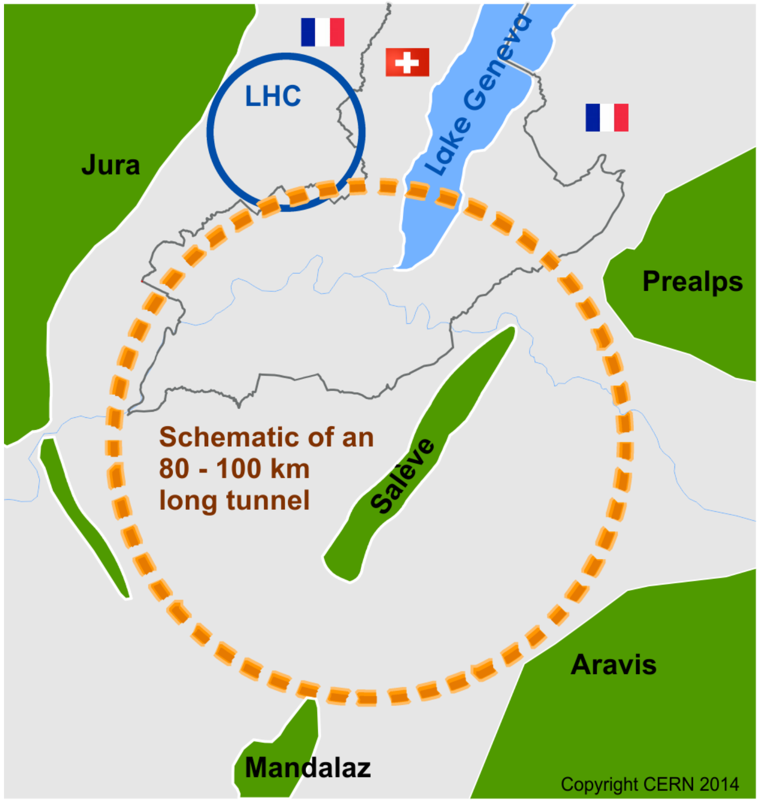


Conceptual Design Report will be delivered at the next meeting of the European Strategy Group for High Energy Physics in 2018

Michael Benedikt

FCC-ee

One part of the
Future Circular Collider Study



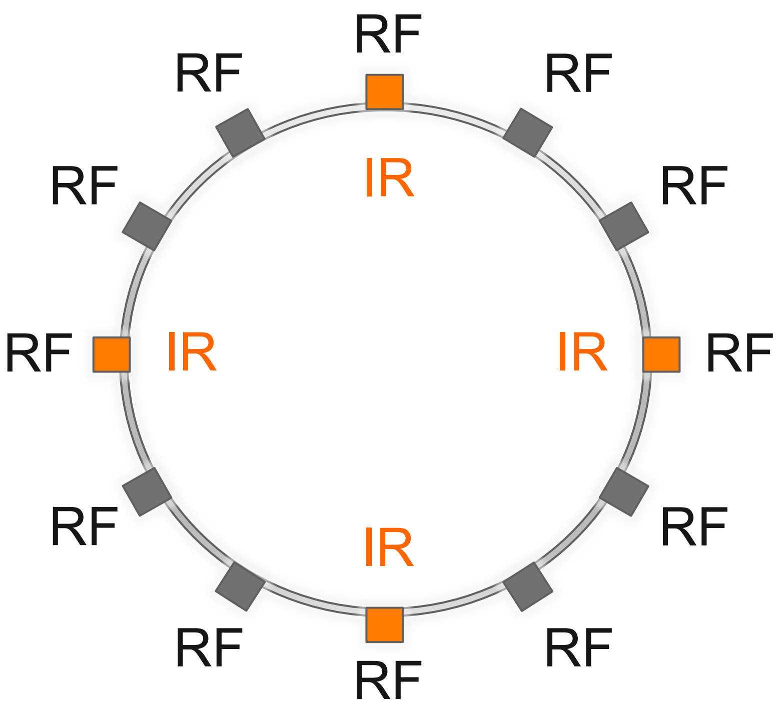
- 100 km e+/e- storage ring collider
- Precision studies of Z, W, H, t
  → Beam energies up to 175 GeV

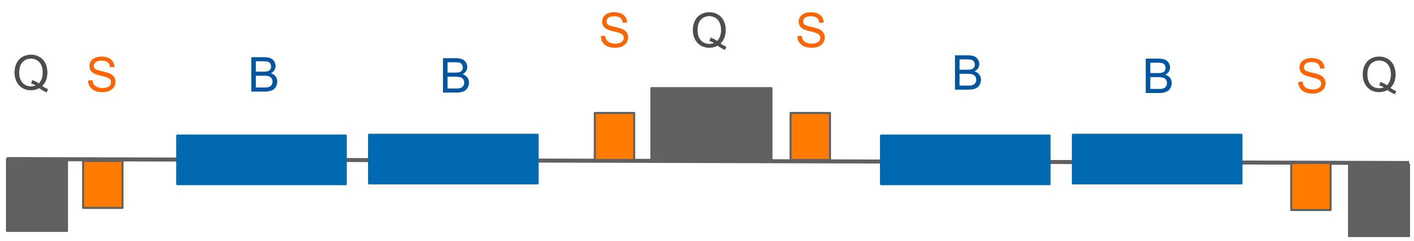
- Beamstrahlung: mom. acceptance required: $\delta = \pm 2\%$
- Design luminosity: $L = O(10^{35}\ cm^{-2}s^{-1})$
  → Strong focusing in final doublet quadrupoles ($\beta^*_y = 1$ mm)
    → Very high chromaticity! **($Q'_y < -2000$)**

12-fold layout

Circumference: 100 km
Arc length: 6.8 km
Straight section length: 1.5 km

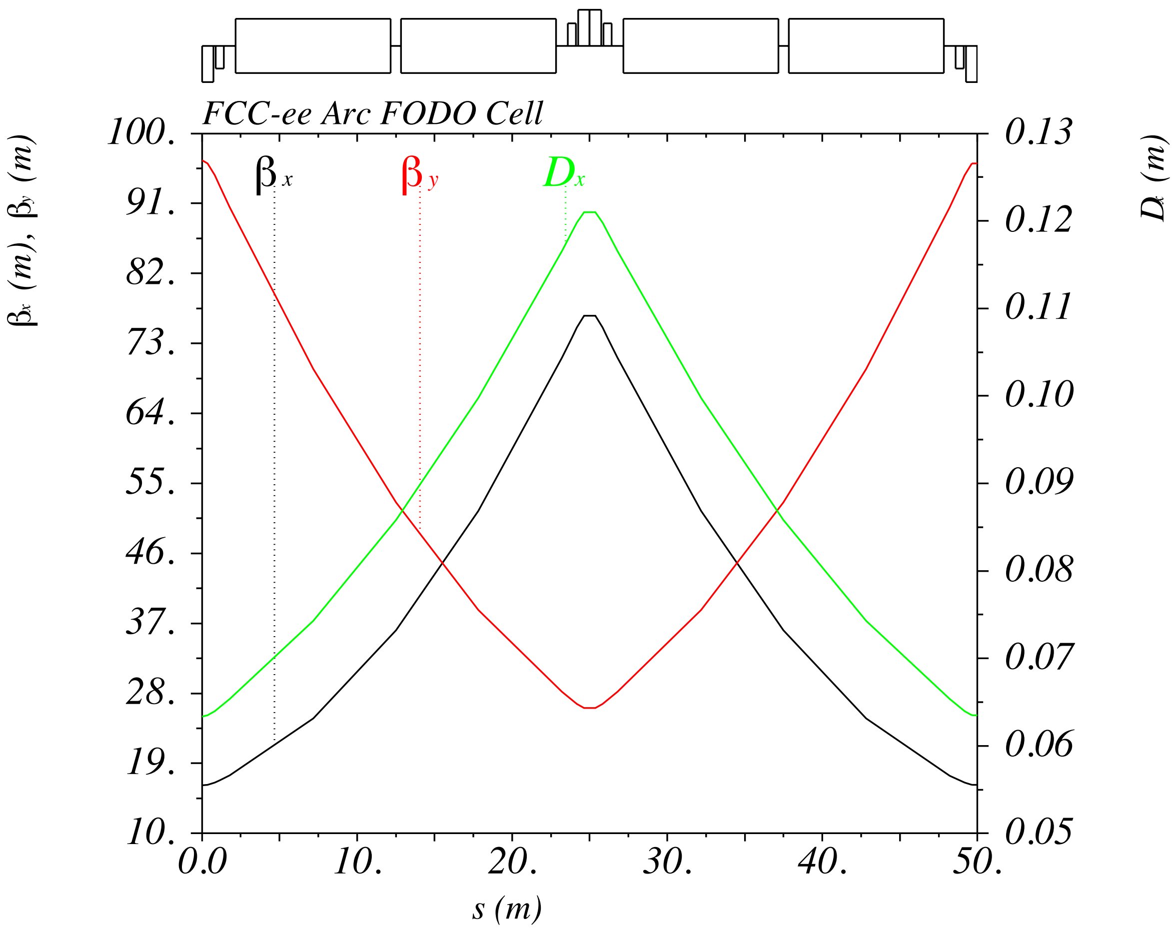
4 interaction regions (IR)
with mini-beta insertions





B = bending magnet, Q = quadrupole, S = sextupole

FODO & low emittance



$$\varepsilon_x = \frac{C_g}{J_x}\gamma^2\theta^3 F$$ (*)

L = 50 m, $\mu_{x/y}$ = 90°/60°

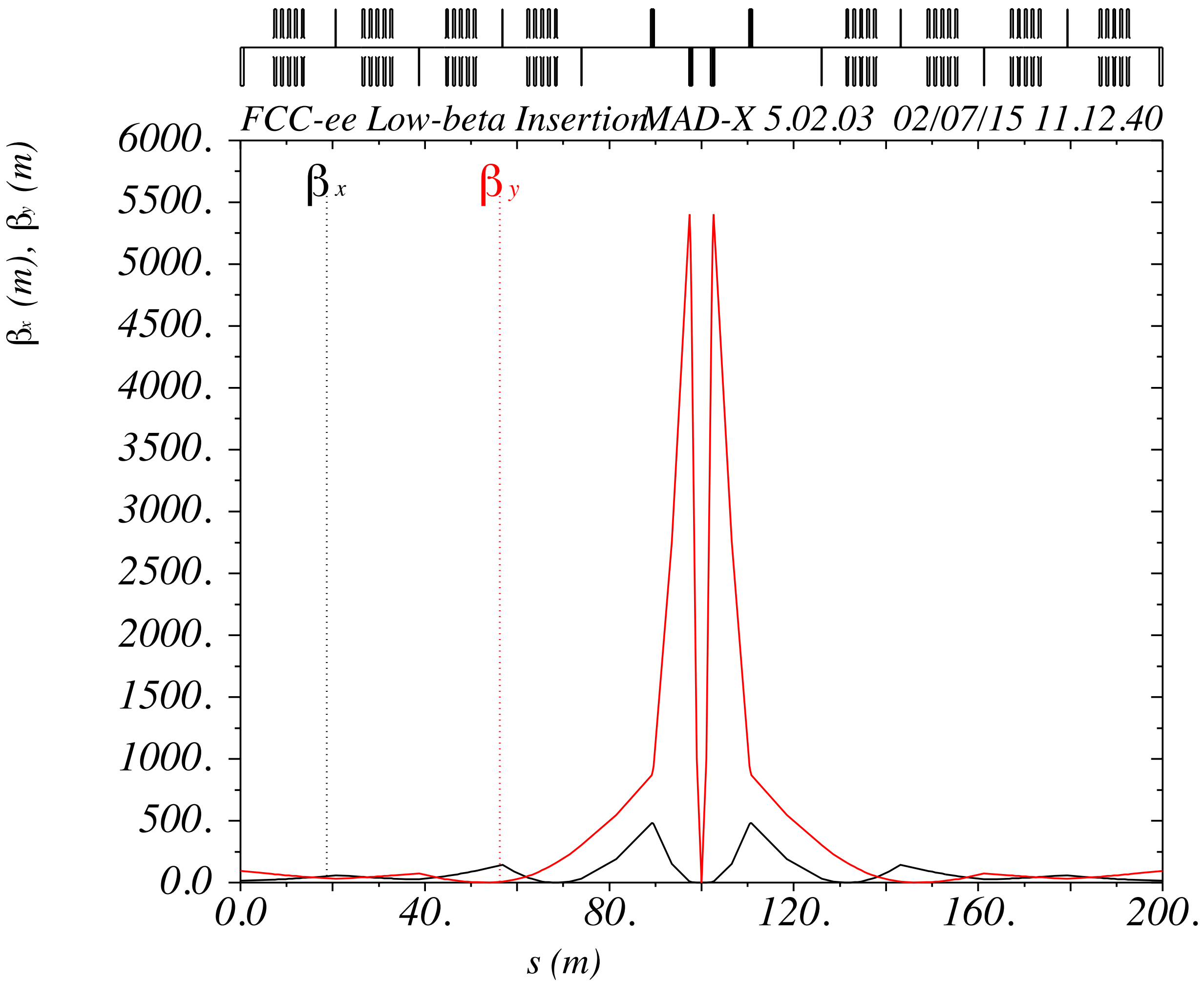
- γ = 342466 (175 GeV)
- θ = 1.96 mrad
- F = 3.125

MADX Emit:

- **ε_x = 1.00 nm rad**
- **U_0 = 8.05 GeV/turn**

(*) L.C.Teng: Minimizing the Emittance in Designing the Lattice of an Electron Storage Ring

Mini-beta insertion



No local chromaticity correction scheme!

L*	= 2 m
β^*_x	= 1 m
β^*_y	= 0.001 m

Chromaticity

- Change of the tune with energy deviation
- Textbook: $\Delta Q = \xi \cdot \Delta p / p$
- In our case not precise enough: $\left(\delta = \Delta p / p\right)$

$$Q(\delta) = Q_0 + \frac{\partial Q}{\partial \delta}\delta + \frac{1}{2}\frac{\partial^2 Q}{\partial \delta^2}\delta^2 + \frac{1}{6}\frac{\partial^3 Q}{\partial \delta^3}\delta^3 + \ldots$$

FCC-ee: Natural Chromaticities

	4 IRs	ΔQ (δ=1.5 %)
Q_x	506.16	
Q_x'	-6.22×10^2	-9.33
Q_x''	-1.37×10^4	-1.53
$Q_x^{(3)}$	-2.25×10^8	-126.30
$Q_x^{(4)}$	-1.86×10^{13}	-39328.54
Q_y	334.28	
Q_y'	-2.06×10^3	-30.90
Q_y''	-8.50×10^6	-956.75
$Q_y^{(3)}$	-1.94×10^{11}	-109157.99
$Q_y^{(4)}$	-6.40×10^{15}	-13501679.77

- 1st order correction
  → Straight forward…
- Higher orders
  → First approach:
  Montague formalism

Montague functions

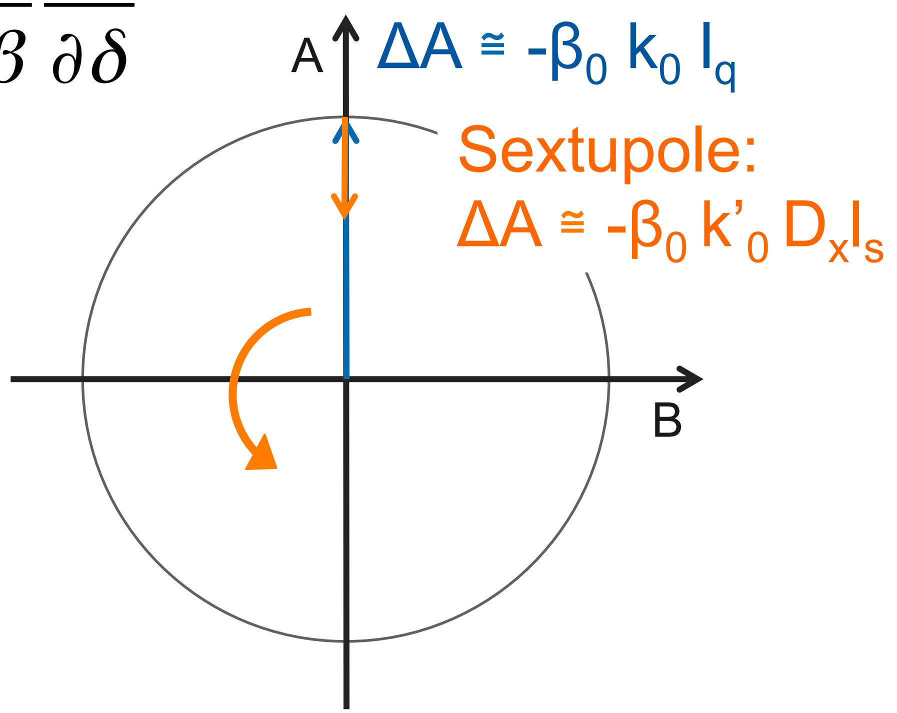
- Chromatic variables

$$B = \frac{1}{\beta}\frac{\partial \beta}{\partial \delta} \qquad A = \frac{\partial \alpha}{\partial \delta} - \frac{\alpha}{\beta}\frac{\partial \beta}{\partial \delta}$$

- W-vector

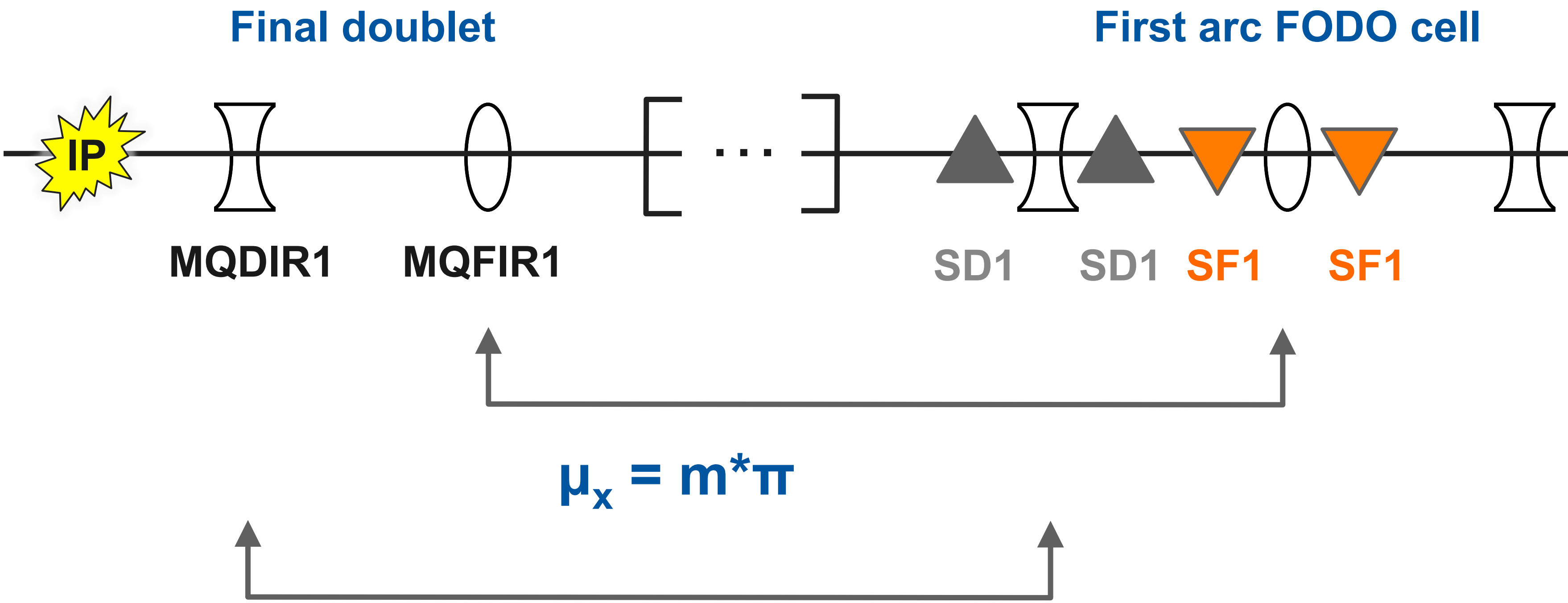
$$\vec{W} = \frac{1}{2}(B + iA)$$

$$= \frac{1}{2}\sqrt{A^2 + B^2}\, e^{i2\mu}$$



Rotates with twice the phase advance!

Phase advance FD – 1st Sext.

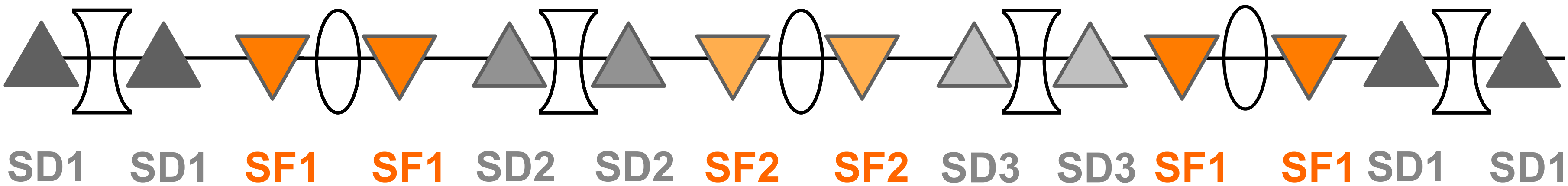


$\mu_x = m*\pi$

$\mu_y = n*\pi$

(m,n integer)

FCC-ee sextupole scheme



$\mu_x = 90° = \pi/2$

$\mu_y = 60° = \pi/3$

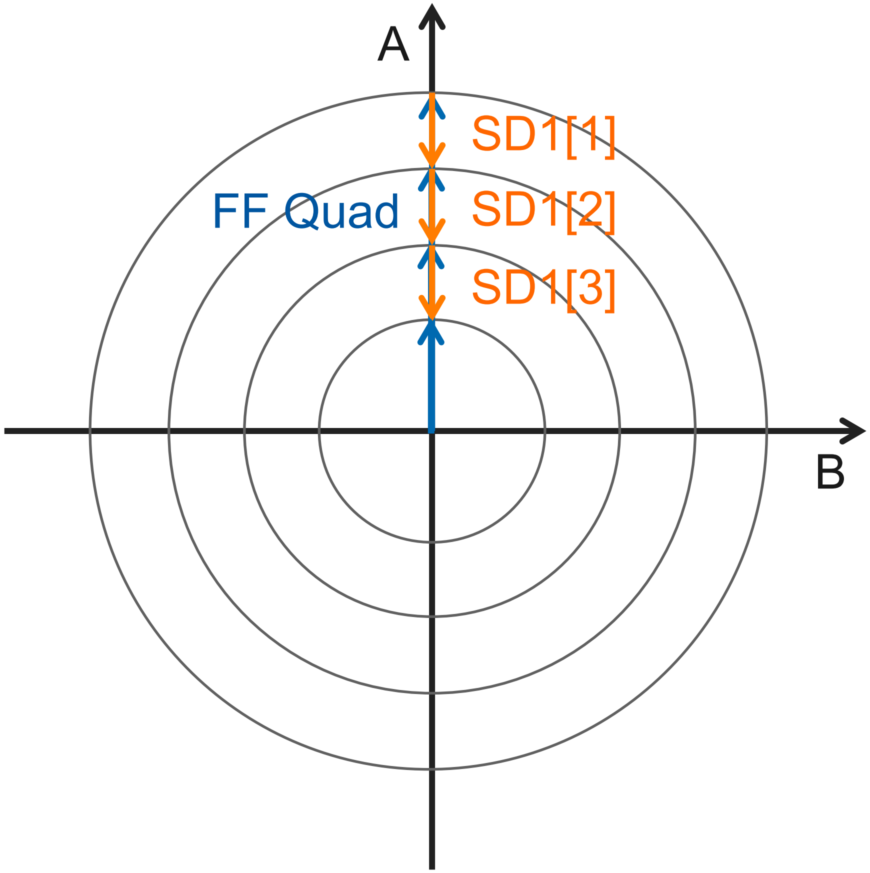
$\mu_x = 180° = \pi$ (→ -I transformation)

Even number of sextupoles per family!

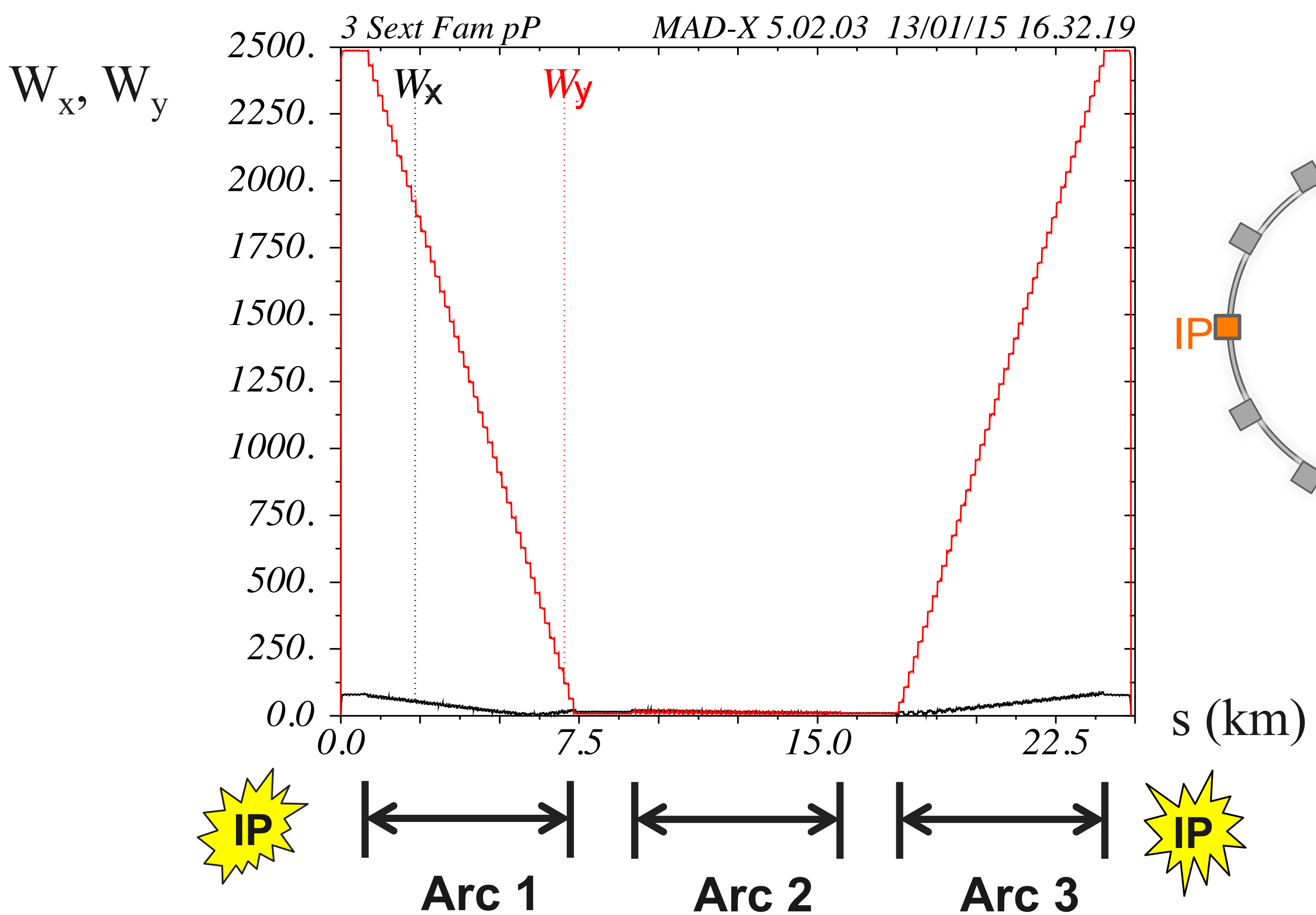
-I transformation

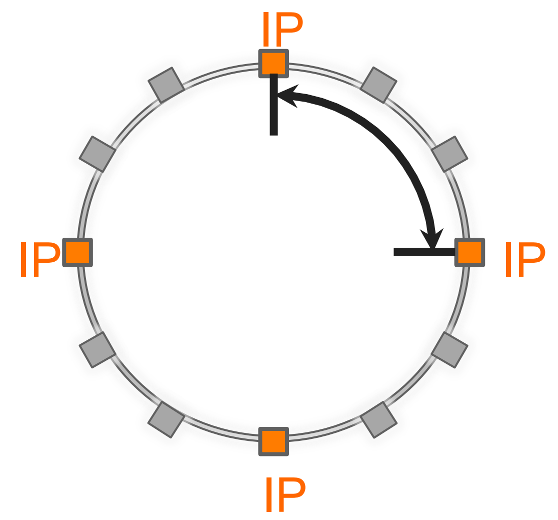
- Sextupoles of each family are in phase

→ W-vector rotates by 2π



W functions in the 1st quarter



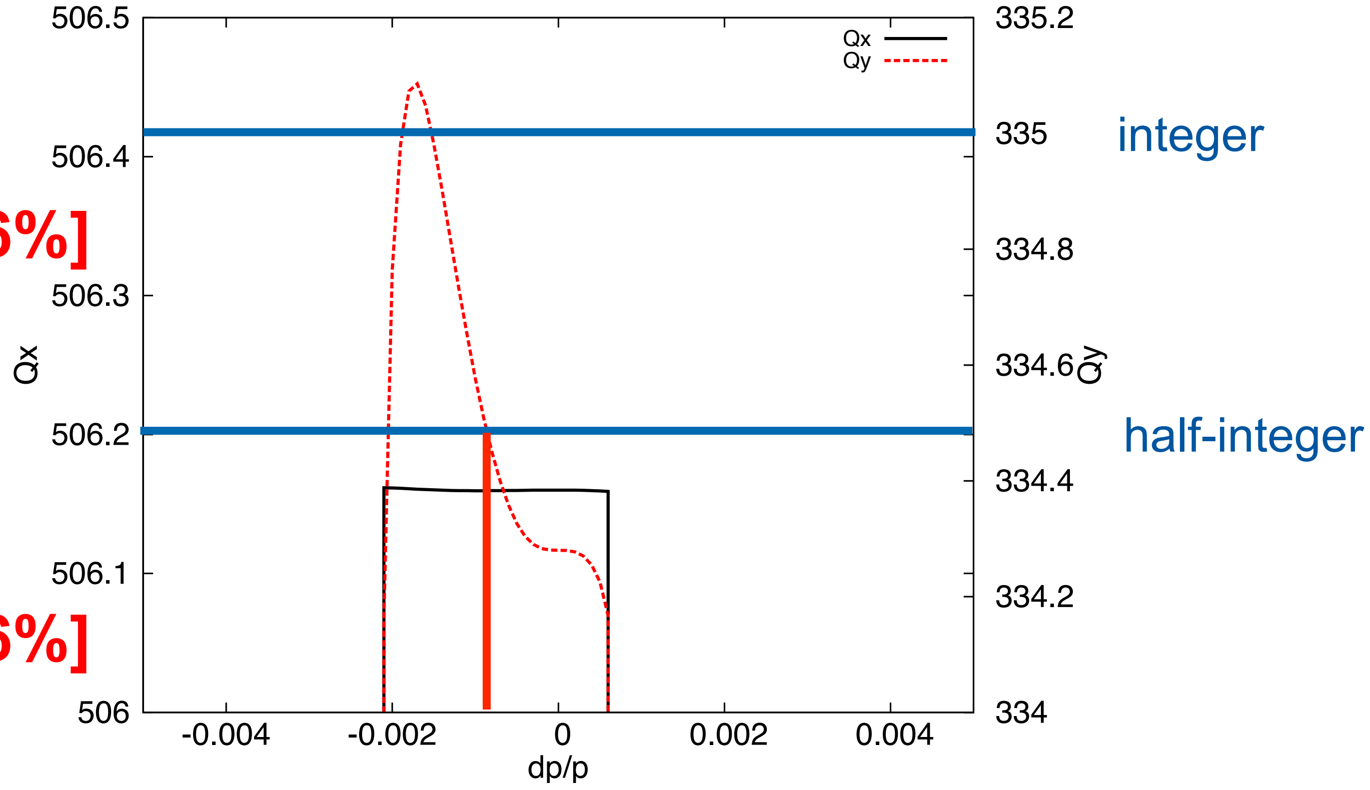


Momentum acceptance

[-0.22%, 0.06%]

↓

[-0.08%, 0.06%]

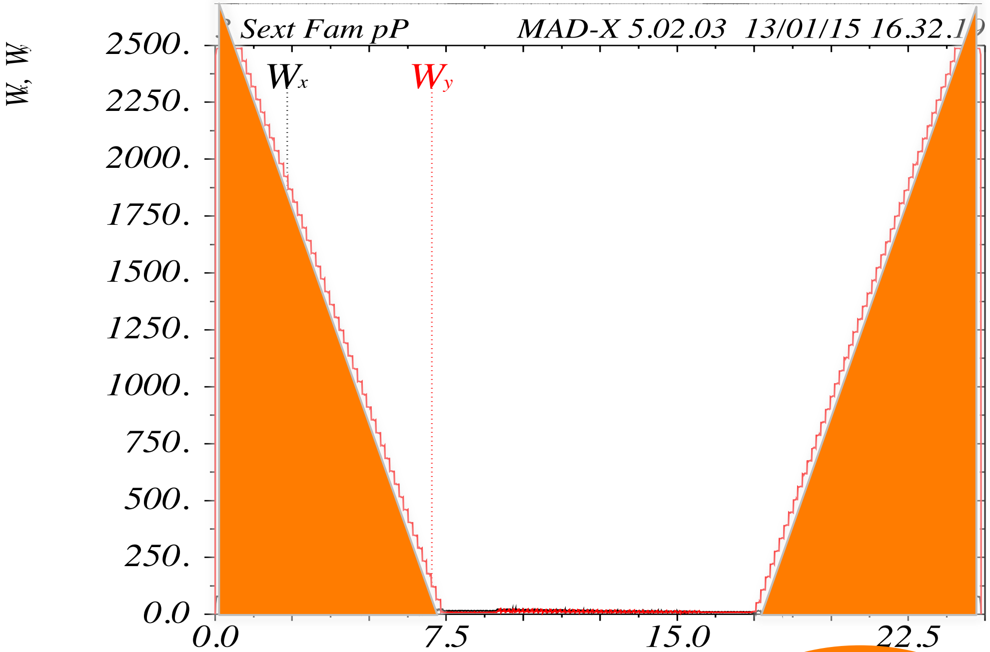


Corrected Chromaticity

	Natural Chromaticities	Corrected Chromaticities	ΔQ (δ=0.05 %)
Q_x	506.16	506.16	
Q_x'	-6.22×10^2	-3.47×10^{-6}	0.000
Q_x''	-1.37×10^4	-2.70×10^3	0.000
$Q_x^{(3)}$	-2.25×10^8	-8.24×10^6	0.000
$Q_x^{(4)}$	-1.86×10^{13}	4.55×10^{12}	0.012
Q_y	334.28	334.28	
Q_y'	-2.06×10^3	-1.22×10^{-5}	0.000
Q_y''	-8.50×10^6	9.07×10^3	0.001
$Q_y^{(3)}$	-1.94×10^{11}	-2.17×10^9	-0.045
$Q_y^{(4)}$	-6.40×10^{15}	4.87×10^{13}	0.126

3rd order chromaticity

$$b_2 = \frac{1}{\beta}\frac{\partial^2 \beta}{\partial \delta^2}$$



$$\frac{d^3\phi}{d\delta^3} = 6\frac{d\phi}{d\delta} + \int_0^\mu \beta^2(K_1 - K_2\eta_0)(a_1^2 + b_1^2)d\phi$$
$$- 3\int_0^\mu \beta^2\left(K_2\eta_1 + K_3\frac{\eta_0^2}{2} - K_2\eta_2 + K_3\eta_0\eta_1\right)d\phi$$
$$- \frac{3}{2}\int_0^\mu \beta^2 b_2(K_1 - K_2\eta_0)d\phi$$

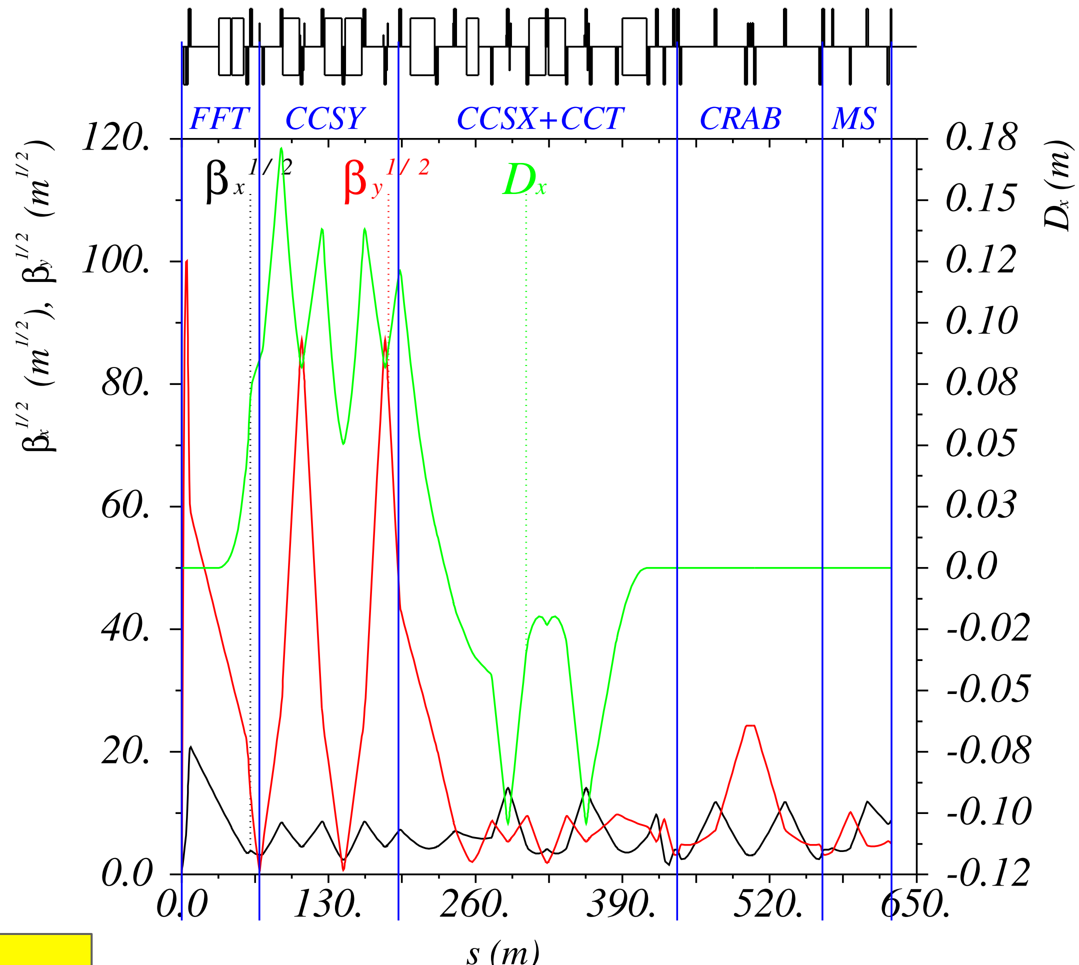
~W²

Put Sextupole where b_2 is large

Octupoles

Anton Bogomyagkov

IR with local chromaticity correction



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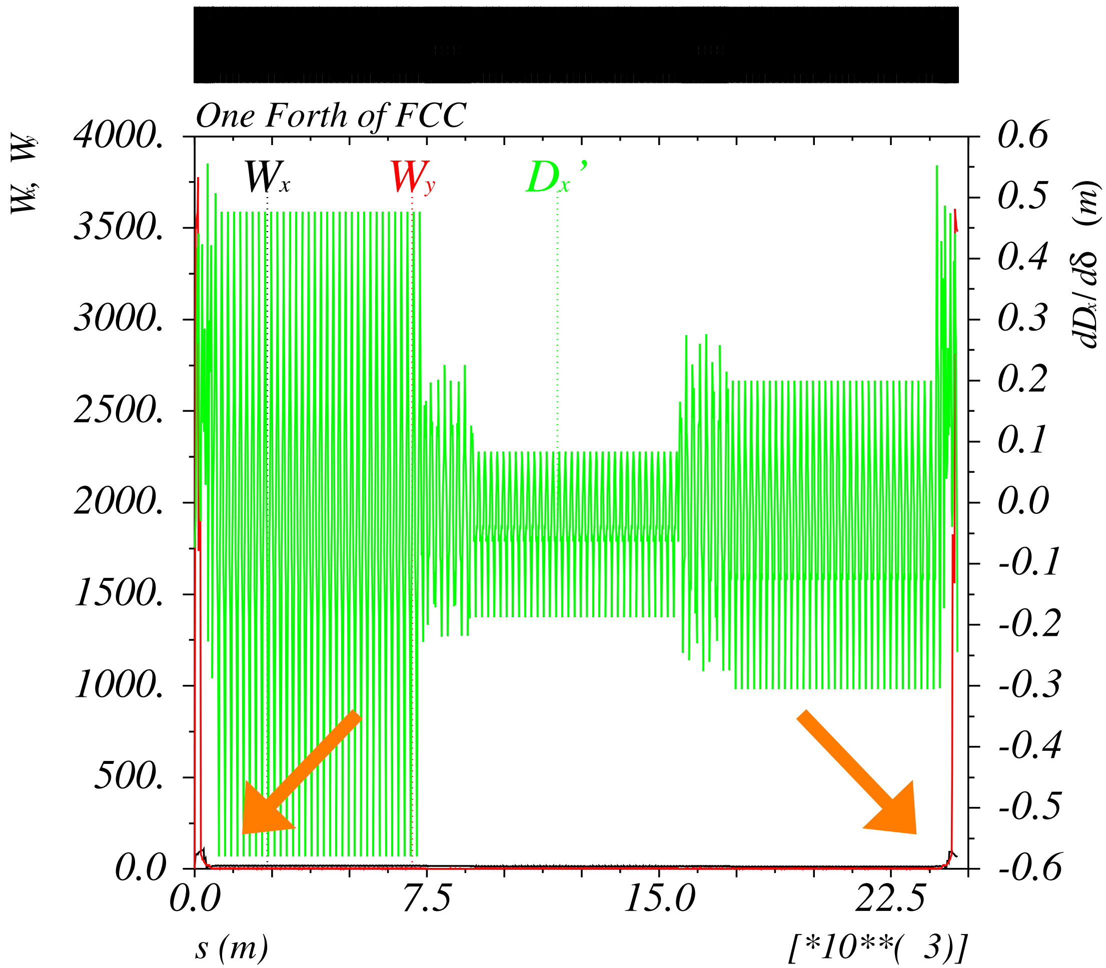
Advantage of local CCS



	Value	$\Delta Q(2\%)$
Q_x	124.54	
Q'_x	0	0
Q''_x	170	0.034
Q'''_x	$-4.5 \cdot 10^4$	-0.059
Q''''_x	$-5.3 \cdot 10^6$	-0.035
Q_y	84.57	
Q'_y	0	0
Q''_y	387	0.077
Q'''_y	$-5.3 \cdot 10^5$	-0.7
Q''''_y	$-4.3 \cdot 10^6$	-0.029

3 orders of magn. smaller!!!

Anton Bogomyagkov

Next steps

- Get 3rd order chromaticity under control
  → Analyse higher order derivatives of β functions

- Optimise bandwidth and dynamic aperture using more sextupole families

- Try non-interleaved sextupole scheme

- Combine optimised arc and local chromaticity correction scheme for best performance

Discussions

- Which experience do you have with higher order chromaticity control?
- Which method and software do you use to increase energy acceptance and DA?

Thank you for your attention!