Unified Description, Cycle Time Models and Optimization of a Double-Deep Shuttle-Based Storage and Retrieval System with Two Differently Sized Unit Loads

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Kurzfassung

In der Lagerlogistik wird immer mehr Flexibilität und Skalierbarkeit gefordert. Shuttle Systeme mit Behälterlift gewinnen daher zunehmend an Bedeutung. Aufgrund der Entkopplung der horizontalen und vertikalen Bewegungen können bei diesem System gleichzeitig Lager- und Förderprozesse durchgeführt werden. Dadurch lässt sich ein sehr hoher Durchsatz erzielen. Aufgrund der unterschiedlichen Kundenanforderungen, aber auch dem Ziel der Leistungssteigerung, werden bestehende Shuttle Systeme von den Herstellern kontinuierlich angepasst und weiterentwickelt. Dies führt jedoch dazu, dass die technische Entwicklung in der Praxis der wissenschaftlichen Betrachtung dieser Systeme oftmals voraus ist. Infolgedessen fehlen oft methodische und analytische Modelle für bereits bestehende Systeme.

Eine immer häufiger in der Praxis anzutreffende Konfiguration sind **Shuttle Systeme** in Kombination mit einem **doppeltiefen Lager**, einem **Doppel-Lastaufnahmemittel** und der Lagerung von **unterschiedlich großen Ladeeinheiten**. Eine wissenschaftliche Betrachtung und Bewertung dieses Systems liegt jedoch noch nicht vor. Ziel dieser Arbeit ist es, diese Forschungslücke zu schließen.

Aufgrund der Komplexität von Shuttle Systemen ist eine detaillierte und allgemeingültige Beschreibung zwingend erforderlich. Um alle relevanten Daten eines Shuttle Systems zu erfassen, wird ein methodischer und allgemeingültiger Ansatz, bestehend aus acht Schritten, vorgestellt. Darüber hinaus wird ein Ansatz für die Beschreibung des Layouts, Materialflusses und der Steuerungsstrategien mit Hilfe von Blockdiagrammen, angepassten „Unified Modeling Language"-Diagrammen und angepassten "Activity Swim Lane"-Diagrammen präsentiert.

Anschließend werden für einige Standardkonfigurationen und für ein Shuttle System mit doppeltiefer Lagerung, einem Doppel-Lastaufnahmemittel und der Lagerung von zwei unterschiedlich großen Ladeeinheiten analytische Modelle zur Berechnung der Fahrtzeit, Spielzeit und dem Durchsatz entwickelt. Die entwickelten analytischen Modelle werden so angepasst, dass diese auch für unterschiedliche Lagerbetriebsstrategien gültig sind. Die entwickelten Modelle werden mittels einer Simulation validiert. Mithilfe der analytischen Modelle werden anschließend unterschiedliche Shuttle System Konfigurationen und verschiedene Lagerbetriebsstrategien miteinander verglichen und bewertet. Es wird gezeigt, dass ein steigender Lagerfüllgrad und eine steigende Anzahl kleiner Ladeeinheiten zu einer Erhöhung der Spielzeit führt. Darüber hinaus wird empfohlen, immer zwei kleine Ladeeinheiten zusammen auf derselben Ebene ein- und auszulagern, um einen höheren Durchsatz zu erzielen.

Im nächsten Teil der Arbeit wird untersucht, inwieweit die Spielzeit durch eine optimale Sequenzierung der Ein- und Auslageraufträge reduziert werden kann. Dazu wird ein Optimierungsmodell (0/1 ganzzahlige Optimierung) mithilfe der Software IBM ILOG CPLEX Optimization Studio aufgestellt mit dem Ziel, die optimale Reihenfolge eines Blocks an Ein- und Auslagerungsaufträgen mit minimaler Fahrtzeit zu erhalten. Unter Verwendung des entwickelten Optimierungsmodells wird anschließend die Leistungssteigerung gegenüber der Abarbeitung nach „First Come First Served“ bewertet. Eine optimierte Reihenfolge führt zu einer durchschnittlichen Verbesserung von 4.72 % (Fehlergrenze des Mittelwerts, 95 % Konfidenzintervall 0,49 %) gegenüber „First Come First Served“. Den größten Nutzen hat das Optimierungsmodell für Shuttle Systeme mit einem hohen Lagerfüllgrad und einer großen Anzahl an kleinen Ladeeinheiten.

Abschließend wird mithilfe der Materialflusssimulationssoftware AnyLogic ein ereignisdiskretes und agentenbasiertes Simulationsmodell aufgebaut, um die Auswirkungen verschiedener Parameter und Lagerbetriebsstrategien auf die Systemleistung zu bewerten. Es wird gezeigt, dass der Durchsatz eines Shuttlesystems von der Anzahl der Pufferplätze abhängt. Der Durchsatz kann mit zunehmender Anzahl von Pufferplätzen erhöht werden, da sich die Wartezeiten vom Lift und von Shuttlefahrzeugen vor dem Eingangs- und Ausgangspuffer verringern. Ein Puffer mit sehr hoher Kapazität erhöht den Durchsatz, ist aber auch mit höheren Investitionskosten und Platzbedarf verbunden. Um das für den gewünschten Anwendungsfall am besten geeignete Shuttle System zu finden, muss am Ende das bestmögliche Verhältnis von Investitionskosten, Durchsatz und anderen relevanten Messgrößen gewählt werden. Dies kann mit Hilfe der entwickelten Simulationsmodelle und analytischen Modelle erfolgen.

Abstract

Shuttle-based storage and retrieval systems with tote lifts are becoming more popular for warehouse logistics due to their increased flexibility and scalability. By decoupling the horizontal and vertical movements, lift and shuttle vehicle movements can be performed simultaneously. This allows a very high throughput to be achieved. Due to the different customer requirements, but also the goal of increasing performance, existing shuttle systems are continuously adapted and further developed by the manufacturers. However, this leads to the fact that the technical development in practice is often ahead of the scientific study of these systems. As a result, methodical and analytical models for already existing systems are often missing.

An increasingly common configuration in practice is a **shuttle system** with **double-deep storage**, a **dual-load handling device** and the storage of **differently sized unit loads**. However, a scientific study and evaluation of this system is not yet available. The aim of this work is to close this research gap.

The large number of independently operating systems, like lifts and shuttle vehicles, different rack shapes and the use of different control strategies make shuttle systems very complex. Therefore, a detailed description of the characteristics of a shuttle system is mandatory. In order to capture all relevant data of a shuttle system, a methodological and generally valid approach consisting of eight steps is presented. In addition, an approach is presented for a universal representation of the design, material flow, and control strategies using block diagrams, adapted “unified modeling language”-diagrams and adapted “activity swim lane” diagrams.

Subsequently, for a shuttle system with double-deep storage, a dual-load handling device and the capability to store two differently sized unit loads, analytical models for travel time, cycle time, and throughput are developed. In addition, the analytical models are adapted to be valid for different control strategies. Using the analytical models, different shuttle system configurations and different control strategies are then compared and evaluated. It is shown, that an increasing filling degree and an increasing number of small unit loads lead to an increase of the cycle time. For the lift system, it is shown that two small unit loads should always be stored and retrieved together on the same tier, to achieve a higher throughput.

In the next part of the thesis the optimization potential is investigated by optimizing the sequence of storage and retrieval requests. For this purpose, a zero-one integer programming model is developed, using the software IBM ILOG CPLEX Optimization Studio, with the objective of obtaining the optimal sequence of a block of storage and retrieval requests with minimum cycle time. Using the developed optimization model, the performance improvement compared to processing the requests after "first come first served" is then evaluated. It is shown, that optimized sequencing leads to an average improvement of the total cycle time over "first come first served" of 4.72 % (Error Bound for the Mean - 95 % confidence interval 0.49 %). The most impact of optimization occurs in environments that have a high filling degree and a large number of small unit loads.

Finally, a discrete event agent-based simulation model is constructed, using the software AnyLogic, to evaluate the impact of different parameters and control strategies on system performance. It is shown that the throughput of a shuttle system depends on the number of buffer locations. The throughput can be increased with an increasing number of buffer locations, since waiting times of the lift and shuttle vehicles in front of the inbound and outbound buffer are decreased. A buffer with very high capacity increases throughput, but is also associated with higher investment costs and space requirements. For the most suitable shuttle system for the desired use case, the best possible ratio of investment costs, throughput, and other relevant key performance indicators must be selected in the end. This can be done with the help of the developed simulation and analytical models.

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1 Introduction

In the era of e-commerce and online retail, the requirements for warehouse logistics have undergone significant changes. There is a growing demand for increased flexibility because the whole process is becoming more dynamic. Predicting and planning the height and frequency of demand peaks have become challenging. Furthermore, the range of products is increasing while product life cycles are decreasing. As a result, there is a need for flexible and scalable automated storage and retrieval systems. This is why shuttle-based storage and retrieval systems (SBS/RSs) are installed more frequently in warehouses.

The main characteristics of a SBS/RS are the separation of the horizontal transportation from the vertical transportation of unit loads (ULs). The vertical transportation is performed using the lift and the horizontal transportation is performed using the shuttle vehicles.

The requirements for SBS/RSs include maximizing storage efficiency and optimizing throughput for the specific application. As a result, existing SBS/RSs are adapted and new configurations are developed.

To increase the efficiency and performance of SBS/RSs several adaptions can be made:

(i) Using a double-deep rack system to achieve a higher space utilization.
(ii) Using multi-load handling devices for lift and shuttle vehicles to transport several ULs in one command cycle.
(iii) Using different sizes of ULs to store a large number of different products.

In practice this type of SBS/RS is already installed and in use. However, there has been no scientific study and evaluation of such a system.

The goal of this thesis is to build new analytical tools and gain more knowledge about a double-deep SBS/RS storing two different sizes of ULs while lifts and shuttle vehicles are equipped with a dual-load handling device (for a detailed description of the design of the considered shuttle system see Figure 2.6). For this purpose, generally valid **analytical equations** are developed **to calculate travel times, cycle times, and throughput** for different SBS/RS configurations and control strategies. Additionally, with the goal to improve throughput, an **optimization model** with the objective to get the optimized sequence of a block of storage and retrieval requests with minimum cycle time is developed and used to quantify the impact of such sequencing policies in a double-deep SBS/RS system. Finally, different SBS/RS configurations and control strategies are evaluated by using **simulation**.

1.1 Problem Description

In this thesis, several research questions are answered:

> **First research question: Which data is required to describe a SBS/RS in a comprehensible way?**

Cycle time models, simulation models, and optimization models are developed for a specific SBS/RS configuration and its control strategies. In order to apply the models described in many publications and the derived recommendations in practice, a detailed description of the SBS/RS is crucial because they are only valid for the specific use case. For this reason, a concise and generally valid description is mandatory. Therefore, a methodical approach is provided, on the basis of which a SBS/RS can be described in a comprehensible way.

Second research question: How to describe the design, the material flow and the applied control strategies of a SBS/RS generally?

The description of the design, material flow and applied control strategies can be very complex and challenging. At the moment, there is no uniform description method available and everyone uses their own methods. Therefore, it is crucial to have a clear and universally understandable mapping method that can be used to describe the layout, material flow, and control strategies of a SBS/RS.

Third research question: How can the expected travel time, cycle time, and throughput of a double-deep SBS/RS, equipped with a dual-load handling device and the storage of two different sizes of ULs, be determined?

Moreover, there are many SBS/RS configurations for which analytical models do not yet exist. In this work, analytical models are developed specifically for the storage and retrieval process of two ULs of different sizes in a multi-load handling SBS/RS in double-deep storage. These models can then be used to calculate the travel time, cycle time, and throughput.

Fourth research question: How to sequence storage and retrieval requests, to increase performance?

In many cases, the storage and retrieval requests are processed in the sequence “first come first served” (FCFS). By combining several storage and retrieval requests into blocks, the so-called block sequencing problem can be solved. The aim is to bring the storage and retrieval requests into an optimal sequence, that minimizes the cycle time. Still though, existing models do not consider the use case of the storage and retrieval of two ULs of different sizes in a multi-load handling SBS/RS in double-deep storage. This is more challenging because relocation is also needed to create empty storage columns to store large ULs. Additionally, three different command cycles need to be considered (dual, triple and quadruple command cycles) which also lead to different load handling and travel times. The focus therefore is to develop an optimization model that solves the joint optimization problem of storage location assignment and storage and retrieval scheduling. This allows to process all storage and retrieval requests of each block in the shortest time.

1.2 Organization of the Thesis

The overall structure of this thesis is depicted in Figure 1.1.

In **chapter 1** the motivation for this thesis, and the research questions being answered are described.

Chapter 2 provides a description of the characteristics of a SBS/RS. Beside the description of different SBS/RS configurations, the technical design, flow of material, and applied control strategies that characterize a SBS/RS are described. For this, a methodical approach is provided. In the second part of this chapter generally applicable models for describing the layout, material flow, and control strategies of a SBS/RS are presented.

In the following **chapter 3**, a review on existing analytical models mainly for SBS/RSs but also for other storage systems that consider multi-deep storage and multi-load handling devices is given. The literature review shows the need of analytical models for the storage and retrieval process of two different sizes of ULs in a multi-load handling SBS/RS in double-deep storage. For different lift and shuttle vehicle configurations, finally travel time, cycle time, and throughput models are developed and different control strategies are applied.

An optimization model is developed in **chapter 4** to solve the block sequencing problem of storage and retrieval requests. First, a literature review on scheduling and sequencing models for automated storage and retrieval systems is done. It underlines the need of an optimization model for the storage and retrieval process of two different sizes of ULs in a multi-load handling SBS/RS in double-deep storage. In the next step, an optimization model to solve the joint optimization problem of storage location assignment and storage and retrieval scheduling is developed. Afterwards, the performance of the optimization model is evaluated. Finally, some experiments using the software CPELX are performed and recommendations for action are derived.

In **chapter 5** a small simulation experiment to gain knowledge on different control strategies and different SBS/RS configurations is carried out. While waiting times are neglected in the analytical models, they are taken into account in the simulation model. For this a parameterizable simulation model using the software AnyLogic is built. Performance measures are carried out for various input parameters and recommendations for action are derived for different SBS/RS configurations and control strategies.

Finally, the main results are summarized in **chapter 6** and a short outlook on future research is given.

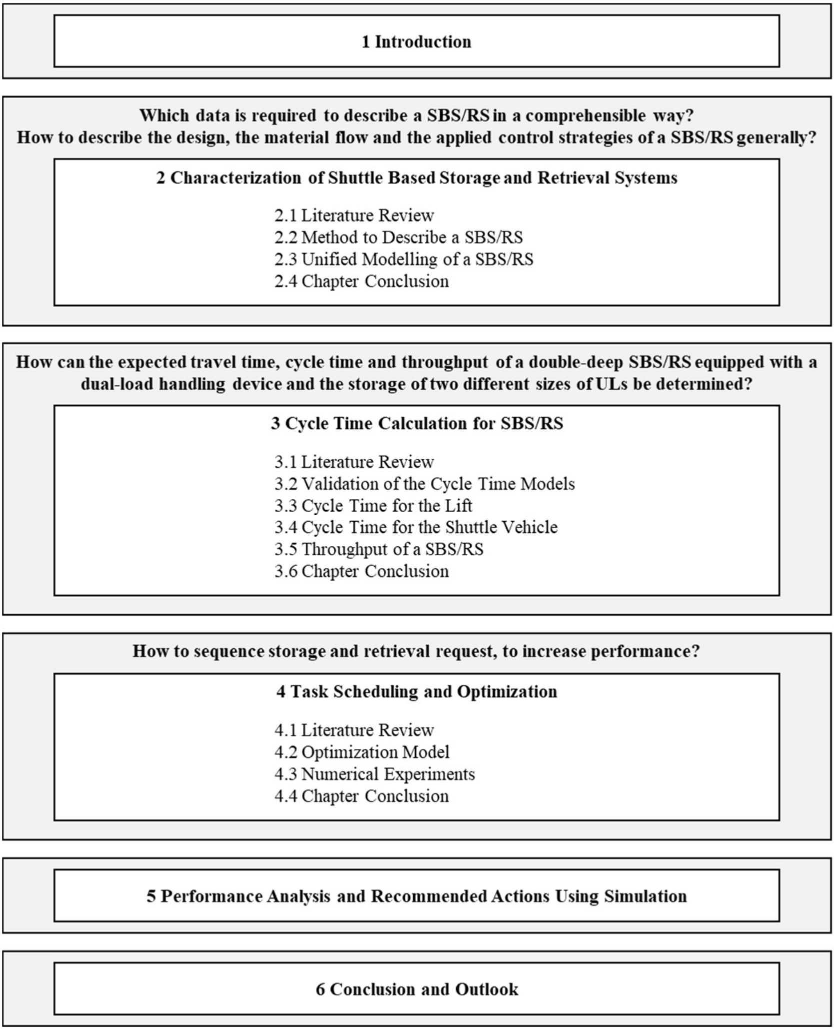


Figure 1.1: Structure of this thesis

2 Characterization of Shuttle-Based Storage and Retrieval Systems

As shown in Figure 2.1, different automated storage and retrieval systems (AS/RS) use different types of material handling equipment and thus achieve different levels of throughput. The earliest forms are AS/RSs with **stacker cranes**, which have a single crane in an aisle that conducts both vertical and horizontal movement (Figure 2.1 left) to store and retrieve ULs within the high-bay rack system. Next generation systems have adopted shuttle systems, which decouple horizontal and vertical movements. In so called shuttle-based storage and retrieval systems (SBS/RSs) mainly **one-level shuttle vehicles** (Figure 2.1 right) are used to store and retrieve ULs within the high-bay rack system. The shuttle vehicles travel in horizontal direction in the rack. Every shuttle vehicle is equipped with at least one load handling device to pick up and drop off ULs. The vertical movement of the ULs or shuttle vehicles between the tiers and the input/output (I/O) point of the shuttle system is done by using a lift. Due to the decoupling of the vertical movement (lift) and horizontal movement (shuttle vehicle), storage and retrieval tasks and movement operations can be performed simultaneously and independently. This makes the SBS/RS very flexible and scalable. In addition, the number of shuttle vehicles and lifts can be adapted as desired which leads to a higher throughput, compared to a storage system with stacker crane. All tiers and aisles can have a different layout and allow the best possible use of the available space (VDI 2692 2015). A third considered material handling equipment is a **multi-level shuttle vehicle** (Figure 2.1 middle). Multi-level shuttle vehicles are, strictly speaking, stacker cranes in compressed form. Several multi-level shuttle vehicles are placed above each other. Each multi-level shuttle vehicle can reach several storage levels without changing tiers. The access to the pre-storage zone is achieved via a lift system.

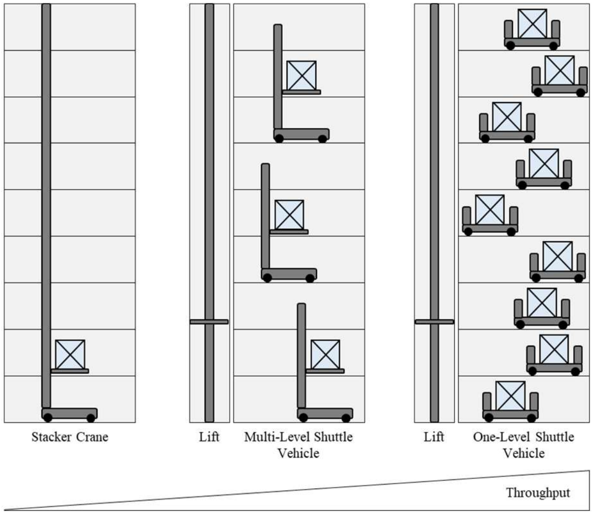


Figure 2.1: Comparison of different automated storage and retrieval systems and the achievable throughput (extended from Klinkhammer Intralogistics 2022)

For this work, the following terms and definitions apply (some are based on VDI 2692 2015):

- **Automated storage and retrieval systems (AS/RS):** The entire system consists of at least one vehicle or machine to store and retrieve goods and a high bay rack system to store and buffer goods.
- **Buffer:** Buffers for ULs are located at every tier to decouple the movements of the shuttle vehicle (horizontal direction) and the lift (vertical direction).
- **Cycle time:** This is the total duration for one command cycle which is the sum of the travel times, load handling times and waiting times.
- **Lift system:** The movement of ULs and/or shuttle vehicle in vertical direction is realized by using a lift.

- **Load handling device**: Each vehicle is equipped with at least one load handling device, that is needed to pick up and drop off the unit loads (UL).
- **Miniload Warehouse**: A miniload warehouse is a fully automated storage system for small and light parts, stored in totes or on trays.
- **Multi-level shuttle vehicle:** A vehicle that moves horizontally in an aisle, has a low lifting height and can reach several storage levels without changing tiers.
- **One-level shuttle vehicle:** A vehicle that moves horizontally in an aisle and can reach only one storage level.
- **Rack system:** The rack is used to store and buffer the ULs. A classic high-bay racking system with narrow aisles between is usually used. The shuttle vehicles can travel on the rails mounted on the rack.
- **Shuttle-based storage and retrieval system (SBS/RS)** or **shuttle system:** The entire system consists of at least one shuttle vehicle, the high-bay rack system and at least one vertical conveyor (in the following also named as lift or elevator).
- **Shuttle vehicle:** Vehicles that move horizontally along the aisle and cross-aisle and are used for transportation and for storage and retrieval of unit loads.
- **Stacker crane:** A vehicle that moves horizontally and vertically in an aisle and can reach every storage level.
- **System axes:** Movement axes of the shuttle vehicle and lift (based on VDI 2692 2015)
    - x – axis (horizontal direction - longitudinal aisle)
    - y – axis (vertical direction)
    - z – axis (horizontal direction - cross-aisle direction and storage channel direction)
- **Travel time:** Movement time of lift and shuttle vehicle.
- **Unit load (UL):** Physical transport unit consisting of load carrier e.g., pallet, small load carrier, tote, tray, carton, bin and goods

Figure 2.2 depicts a schematic view of design issues and their independencies for all kind of storage systems where shuttle vehicles are used. The characteristics of SBS/RSs will be described more in detail in the following pages.

On the very top the interaction between the SBS/RS and **connected systems** is depicted. The performance of storage systems is dependent and influenced by all the other systems in the warehouse, especially the systems in the pre-storage zone e.g., conveyor technology or order picking stations. The number of I/O points between the pre-storage zone and the rack, the required number of storage locations, the required throughput, the space available (footprint) and the used control strategy defines the design of the storage system.

The middle box describes the order and article structure and the used load carrier which define the system configuration and the individual system components. Depending on the **article structure**, the type of goods that need to be stored, the required **load carrier** is defined. The type, dimension and total mass of the ULs are decisive for the choice of the suitable rack system and the possible storage and retrieval machine (e.g., shuttle vehicle, pallet shuttle vehicle, stacker crane, etc.). The **order structure**, the arrival rate of the orders, number of orders and items per order, determine the throughput to be achieved and the required number of storage locations.

The different **system configurations** can be classified according to the technical equipment used for transport, storage and retrieval tasks. Classical miniload SBS/RS consist of lift systems and shuttle vehicles. In storage systems for pallets there are several different configurations possible, mainly dependent on the achievable throughput. Systems with pallet shuttle vehicles and lift systems, but also systems with stacker cranes or external forklifts, that place the pallet shuttle vehicle in the storage channels, are possible. In the last years, new SBS/RS configurations for miniload storage systems have been developed. The newest development are shuttle vehicles which can travel in vertical and horizontal direction by its own, using a special chassis and rack system (Azadeh et al. 2019).

Each storage configuration consists of a number of **dynamic components** (e.g., shuttle vehicle, pallet shuttle vehicle, lift, stacker crane, load handling device, forklift) and a **static component** (e.g., rack). Each of the dynamic and static system components can be uniquely described by all its properties. For the user, in addition to the geometric dimensions and capacity, kinematic properties of the components are of importance, since acceleration and maximum velocity defines the required travel duration. The rack system is more complex in SBS/RSs than in classical high-bay rack systems with stacker cranes. The rack system needs to include rails systems on each tier, on which the shuttle vehicles can move.

Last but not least, it is pivotal to evaluate **performance** of each storage system for the best possible outcome. The performance of such a storage system is also dependent on the **control strategy** used. The large number of shuttle vehicles and lifts systems which operate simultaneously and independently require complex control strategies. Both, the physical design and the used control strategies of the connected systems and the storage system itself influence the overall system performance.

Depending on all the above mentioned points and the objective function – e.g., maximum throughput, minimum energy consumption, minimum foot print – the decision of the design of the storage system and the used control strategies can be made.

In the following, only the classic shuttle system with a high-bay rack, shuttle vehicle, telescopic arm (as a load handling device) and lift system is considered (see red box in Figure 2.2).

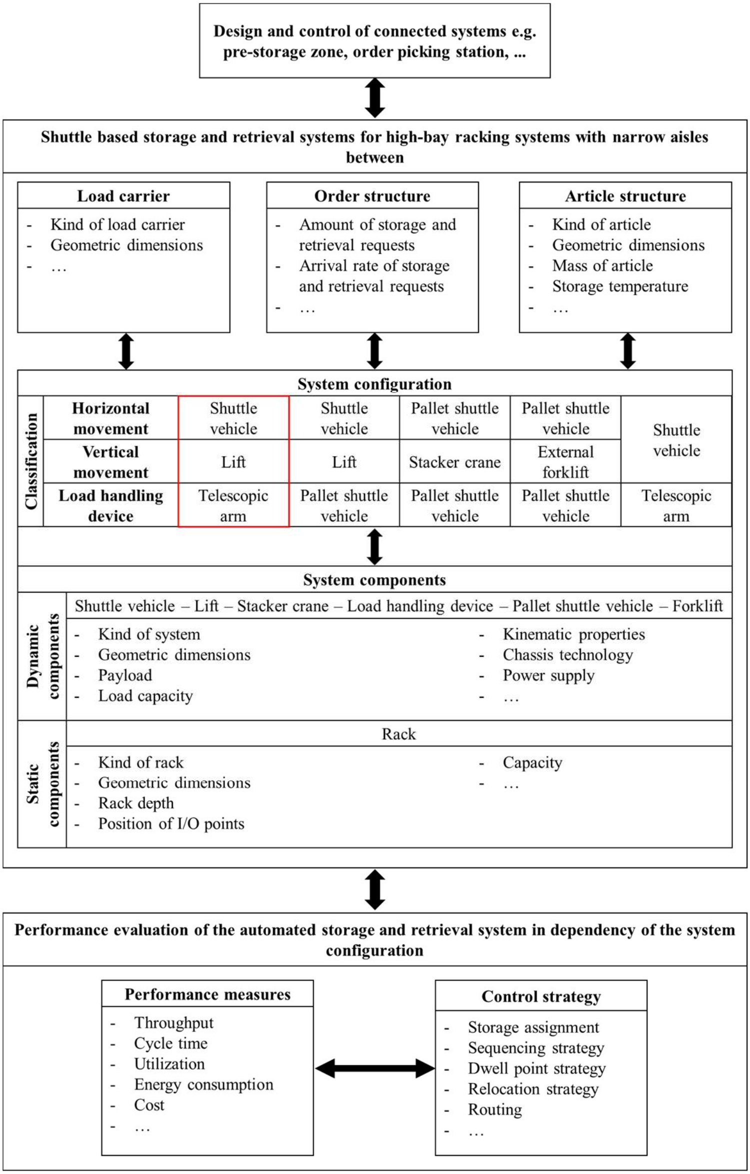


Figure 2.2: Characterization of a SBS/RS (extended from Roodbergen and Vis (2009), Epp (2018) and Azka et al. (2021))

2.1 Fundamentals and Literature Review

A storage system is especially necessary when incoming and outgoing processes can not be synchronized either technically or organizationally. This means that the material flow must be interrupted and a storage system must be integrated into the overall system e.g., warehouse. The storage system thus serves to store and buffer goods. (Arnold and Furmans 2019, p. 175)

Depending on the application, industry, and which type of goods will be handled, different storage systems are required. On the German manufacturers' webpages (well-known manufacturers here include DAMBACH Lagersysteme GmbH & Co. KG, GEBHARDT Intralogistics Group, psb intralogistics GmbH, SSI Schäfer) a detailed description of different automated storage systems and storage technologies can be found. On their webpages, there is often a detailed description available of the storage systems (e.g., rack design and load handling technology) and the technical specifications (e.g., geometrical dimensions, velocity and acceleration of shuttle vehicles).

There are also numerous publications in the literature that deal with the subject of storage systems. These can be divided into the following three categories:

(i) publications that describe the processes and technology of the entire warehouse,
(ii) publications that describe the various racking systems, their functions and areas of application, and
(iii) publications that examine and analyze individual storage systems (e.g., automated storage and retrieval system (AS/RS) and shuttle-based storage and retrieval system (SBS/RS)).

A general overview of warehouse processes and a description of various warehouse and storage systems and their operation (including warehouse management, warehouse operation strategies, utilization, requirements, etc.) can be found in Gudehus (2010) and Arnold and Furmans (2019). Ten Hompel et al. (2018 pp. 57) present an overview of the different techniques (floor storage, static and dynamic rack storage and storage on conveyors) and the storage technology used. Baker and Canessa (2009) provide an overview of the current literature on the overall methodology of warehouse design. The output of their work is a general framework of steps, with specific tools, that can be used for the development of a comprehensive methodology for warehouse design.

There are also some publications that classify storage systems and racks according to their function, goods to be stored and the used handling technology. Ten Hompel et al. (2018 pp. 57) present an overview and categorization of the different storage systems techniques (floor storage, static and dynamic rack storage and storage on conveyors) and the storage technology used.

Martin (2016) presents a systematic classification of storage systems depending on the access type (direct access and indirect access to the goods and ULs) and gives an overview of different storage systems techniques and control strategies. Roodbergen and Vis (2009) present a classification of various AS/RS options using the three classes: crane, handling and rack.

In addition, there are many publications considering the design of different racking systems. The main focus of these publications is the static and dynamic dimensioning, the design of the racking system and steel construction or the seismic behavior of racking systems (see for example Ungermann and Schulze Bertelsbeck (2016), Castiglioni (2016)). There are also numerous standards that deal with the rack design (some examples are DIN EN 15512 (2022), DIN EN 15878 (2010), DIN EN 15629 (2010), DIN EN 15620 (2021)).

There is a large number of different system options that exist for automated storage and retrieval systems. The most known basic version is a high-bay racking system with an automated stacker crane. However, during the last years or decades, many new and more complex storage systems have been developed, such as SBS/RS, AutoStore or puzzle-based storage systems.

Since the focus of this work lies on SBS/RSs, in what follows, only literature on SBS/RSs will be discussed.

Shuttle systems are becoming increasingly important and have now become an alternative to the classic high-bay storage systems with a stacker crane due to their advantages, higher and scalable performance and better utilization of space. Shuttle systems have modular structure and can be easily adapted to changing requirements. This allows companies to react flexibly to new logistical challenges (Diehn 2015). However, the flexibility is bought with a higher complexity, than with the classic high-bay warehouse with stacker crane. Depending on the application, different storage system geometries can be realized, as well as the number of shuttle vehicles or lifts can be varied. This diversity is not yet covered by the standards and guidelines VDI 2692 (2015) and FEM 9.860 (2017). Both only consider standard configurations and simplified analytical models for the calculation of cycle time.

Azadeh et al. (2019) review several automated and robotic handling systems, such as SBS/RSs, shuttle-based compact storage systems, and robotic mobile fulfillment systems. They present a classification of automated picking systems and analyze for each system the system itself, design optimization and operations planning and control.

In addition, a systematic literature search has revealed that there are over 200 publications on SBS/RSs. Authors or research institutions involved in many publications on SBR/RSs include T. Lerher (University of Maribor, Slovenia), B.Y. Ekren (Yasar University, Turkey), C. Malmborg (Rensselaer Polytechnic Institute, USA) M. Eder & G. Kartnig (Vienna University of Technology, Austria), F. Schloz & K.-H. Wehking (University of Stuttgart), and T. Kriehn & M. Fittinghoff (Heilbronn university of applied science).

Most of these publications deal with the performance determination of SBS/RSs (e.g., cycle time, throughput, utilization or energy demand calculation) using analytical-mathematical or simulation models. Mostly, different storage configurations and different control strategies are compared and subsequently evaluated.

The literature review shows that there already exists a multitude of publications on automated storage and retrieval systems.

However, during the detailed analysis of the publications, it was noticed that in many publications the examined storage systems are not illustrated and described in a comprehensible way, relevant information for a more detailed understanding is missing and each author is using a different methodology to describe storage systems. But the existing literature is only helpful for the reader, if the analyzed system is described clearly and understandably.

The challenge is therefore, as systems become more and more complex, to depict and describe them for the reader and user in a generally valid and understandable way, with all the characteristics, relevant parameters and control strategies. There are already numerous publications for the classification and categorization of different racking configurations. All previous publications lack a method and approach for the detailed description of SBS/RSs or AS/RSs and an overview on which data should be specified.

For this reason, the necessity is recognized to develop a methodical procedure that describes each automatic storage system clearly and universally. In chapter 2.2 and chapter 2.3, therefore, an approach and the method are presented and applied for the example of SBS/RSs. However, since this thesis is not intended to be a textbook in the end, not all possible configuration, parameters, strategies, etc. have been listed and described in detail. This work can be seen as the basis for a generally valid description method, which can be adapted and extended for other warehouse systems (e.g., AS/RS, AutoStore).

2.1.1 Approaches and Methods to Describe Systems and Processes

In the literature there can already be found different approaches and methods on how systems and processes can be described and mapped in an understandable way.

- Unified modeling language (UML) is used to provide system architects, software engineers, and software developers with tools for analysis, design, and implementation of software-based systems as well as for modeling similar processes. With the help of different diagram types, the system structures, object interactions and system behavior can be mapped. (OMG 2017)

- Systems modeling language (SysML) is an extension of the UML standard which provides additional extensions needed to address requirements in the UML for systems engineering. (OMG 2019)
- Ontologies in data management, are a formally ordered representation of the structure of a system i.e., the relevant entities and the relationship that exists between them. (Guarino et al. 2009)
- Morphological analysis is a method for investigating the totality of relationships contained in multi-dimensional problem complexes. Thus all possible combinations and solutions for a system can be mapped. (Ritchey 1998)
- Engineering drawing is a drawing giving all information about a part for its production or a drawing of an assembly with all the components and the correct orientation to each other. (DIN 199-1 2021)
- Flowcharts use a normed set of symbols to describe workflows, processes or algorithms. (DIN 66001 1983)

2.1.2 Applied Approaches and Methods for Logistics and Production Processes

For the description and mapping of logistics and production processes, some of these above mentioned methods are already applied to describe logistics and production processes.

Ontologies are already being used to describe logistics processes. For example ontologies are used to model the information flow in a warehouse (Bakkali et al. 2015), offer a unified representation of the logistics domain and facilitate the formal, semantic description of logistics services (Hoxha et al. 2010) or offer a unified representation of the different packaging in logistics (Kowalski and Quink 2013).

SysML is mostly used for modeling complex physical systems. SysML is also used in logistics to describe systems and processes. Sprock et al. (2017) develop a cost model for a warehouse to support the design process. Rehm et al. (2010) model a discrete production line in the context of automatic simulation model generation, and use SysML to analyze and structure its significant properties. Limère et al. (2010) create a tool to model the in-plant logistics processes of an automotive assembly line for analyzing the costs of the material supply. McGinnis et al. (2014) develop engineering methods and tools to support warehouse design and use SysML for the formal documentation of all phases of the warehouse design.

The other methods are also used to describe logistic processes. In many publications, flowcharts are used to describe processes and strategies (e.g., Ekren and Heragu (2012)), engineering drawings to describe the layout and design of complex systems (e.g., Lerher et al. (2014); Lerher (2017)), but also morphological analysis to show all possible combinations and solutions (e.g., Kutzner et al. (2020)).

2.1.3 Conclusion on Literature for Approaches and Methods to Describe Systems and Processes

The literature review shows that it is possible to describe complex systems and processes in a generally understandable way. Despite the wide variety of methods available, there is little research about unified description models for logistic systems. No existing approach for describing and modeling automated storage and retrieval system generally in literature can be found.

In this thesis, the knowledge obtained through the literature review is used to develop and define a method that can be used for the unified description of automated storage and retrieval systems. The goal is to develop a unified description method that is easy to understand and applicable to all automated storage systems. Since it is not possible to describe storage systems generally only by using one method, the approach created in this thesis consists of a combination and linking of different methods and approaches. Depending on the application e.g., simulation study, mathematical calculation models or optimization models, different software tools can be used for the subsequent implementation.

2.2 Method to Describe a SBS/RS

To design complex systems, multiple stakeholders and experts are involved and a large number of input data is required. To be able to model these complex systems, it is helpful to divide the overall system into subsystems and to consider the layout and the control strategy properties separately. This requires a method that supports this procedure and meets the requirement of universal applicability. This implies that this method is applicable for describing any shuttle based storage and retrieval system.

To describe storage systems with all their characteristics, a method is presented in Figure 2.3. Once the mentioned steps have been answered, each storage system is then precisely defined.

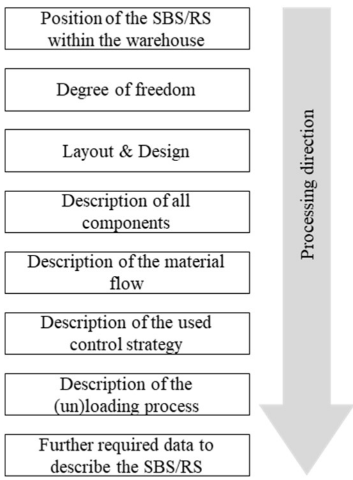


Figure 2.3: Method to describe a SBS/RS

In the boxes at the end of the subchapters 2.2 and 2.3, the SBS/RS considered in this work is described and the relevant data needed for this work is summarized. It should be mentioned that only examples are given, and not all possible data are listed in the boxes. If someone would like, e.g., to calculate the energy consumption of the SBS/RS or run a finite element analysis of the rack structure, different input data would be required and should then be described in more detail.

2.2.1 Position of the SBS/RS within the Warehouse

For the overall understanding of the system under consideration, it is of particular importance how the SBS/RS is integrated in the warehouses. Incoming goods are usually prepared for storage in the upstream process and conveyor systems usually transport the ULs to the input point of the storage system. The ULs are stored in the storage system until a customer requests certain goods. From the output point of the SBS/RS, the retrieved ULs are usually transported using conveyor systems to the picking stations or downstream processes and get prepared for shipping. Figure 2.4 depicts the position of the SBS/RS within the warehouse.

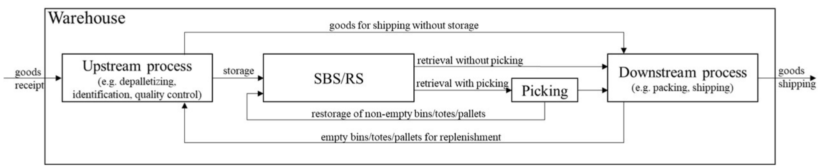


Figure 2.4: Position of the SBS/RS within the warehouse (adapted from Epp (2018))

In the first step, it is particularly important to define the system boundaries of the SBS/RS under consideration. If only the SBS/RS is considered, it is sufficient to describe only the SBS/RS with all its properties. In a next step, the interfaces to the pre-storage zone (the number of I/O points), arrival rates of storage and retrieval requests, and control strategies used, must also be specified.

Besides the SBS/RS, there are also relevant upstream and downstream processes (including picking). These processes must also be described in detail and all system-relevant data and properties must be included.

- In this work a miniload SBS/RS is considered.
- The SBS/RS is considered in isolation from upstream and downstream processes.

2.2.2 Degree of Freedom of the SBS/RS

In the next step, if the boundaries of the system are known, it is necessary to define the considered SBS/RS based on the degree of freedom.

Shuttle systems are available on market in many different configurations. They can be classified based on the type of transport used in the vertical and horizontal direction:

- If the transport of ULs in vertical direction is performed by means of an UL lift (= tote lift), the shuttle vehicles can not change tiers (known as a **tier-captive** system). In this case, there is at least one shuttle vehicle on each tier. In tier-captive systems, the horizontal transport of the ULs is performed by the shuttle vehicles which move along the x- and z-axis. At each tier, there are transfer buffers to decouple the lift system and the shuttle vehicles. The lift moves along the y-axis and transports the ULs in vertical direction between the I/O point of the storage system and the various tiers.
- If the lift moves the shuttle vehicles vertically up and down between different tiers, then it is called vertical roaming using a shuttle lift (also known as a **tier-to-tier** system). In this case, every shuttle vehicle can reach every storage location in this aisle. Tier-to-tier systems are usually operated with fewer shuttle vehicles than tier-captive systems. In extreme cases a shuttle system can even be operated with only one shuttle vehicle in each aisle (tier-to-tier), each

tier (aisle-to-aisle) or in the total shuttle system (aisle-to-aisle and tier-to-tier).

- Depending on the configuration of the rack and shuttle vehicle, the shuttle vehicles can also change aisles (known as **aisle-to-aisle** system). For this purpose, shuttle vehicles must have a chassis with which they can travel in both, x- and z-directions. In addition, the rack must be designed in such a way that, besides the movement of the shuttle vehicle in x-direction, the movement between the aisles in z-direction – using a cross-aisle – is also possible.

This results in a total of four different categories for shuttle systems, that emerge from the different movement axes of the shuttle vehicle. Figure 2.5 provides an overview of the four different categories. For each of the four categories, different configurations are depicted.

		1	2	3	4
Degree of freedom	Change of aisle	No	No	Yes	Yes
	Change of tier	No	Yes	No	Yes
	Configuration	aisle-captive tier-captive	aisle-captive tier-to-tier	aisle-to-aisle tier-captive	aisle-to-aisle tier-to-tier
	Movement axis of the shuttle vehicle	x	x – y	x – z	x – y – z
	Movement axis of the lift	y	y	y	y

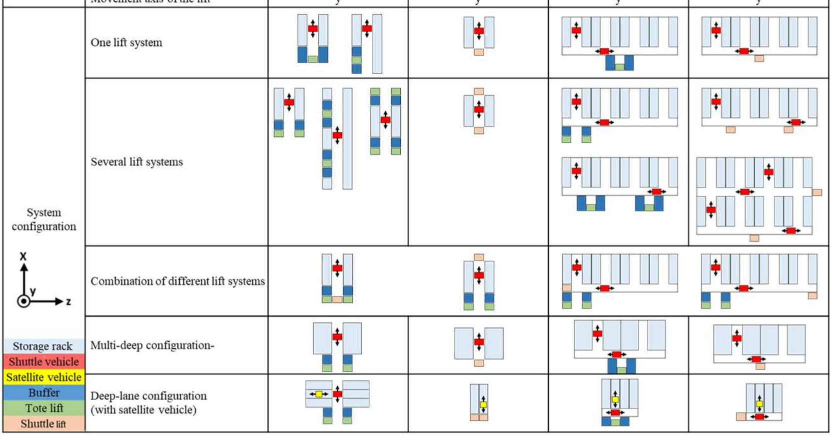


Figure 2.5: Top view of different SBS/RS configurations

Aisle-captive and tier-captive systems (1) are the most common configurations for miniload SBS/RSs and achieve the greatest throughput, since horizontal and vertical movement can be performed in parallel.

All the three other categories (2, 3, 4) in most cases achieve less throughput. Although each shuttle vehicle can reach any position in the system, it requires long travel distances and thus long travel times. In all systems, however, additional vehicles and lifts can be added. This increases the overall throughput but requires more complex control strategies.

A combination of different lift systems enables completely new SBS/RS configurations. These configurations can no longer be clearly assigned to one of the four categories but are rather a combination of different categories. An example could be, that the shuttle vehicles are tier-captive and there is one shuttle vehicle on each tier. If the utilization of this tier is very high at a certain time, then an additional shuttle vehicle can travel to this tier to support, using the shuttle lift.

If several ULs can be stored in one storage channel (number of rows $n_r > 1$) then it is called a multi-deep rack or deep-lane rack. Depending on the depth of the storage channels, different equipment for load handling is needed to (un)load ULs or pallets. For small storage depths (in most cases $n_r \leq 2$), shuttle vehicles can be equipped with a classic telescopic arm that can reach the first and second storage row. For storage depths with $n_r > 2$ often a satellite vehicle is used. Satellite vehicles (also called daughter vehicles) travel in the individual lanes to (un)load the ULs (mostly pallets). For the transport of the satellite vehicle in the horizontal - longitudinal or horizontal - transverse aisle, another shuttle vehicle (also called mother vehicle or satellite vehicle carrier) is required. The satellite vehicle carrier picks up the satellite vehicle and transports it to the next destination.

- The considered SBS/RS is aisle-captive and tier-captive with two lift systems.
- Each tier is equipped with one one-level shuttle vehicle.
- At the end of each aisle, two independent lifts (one inbound lift and one outbound lift) are mounted.

2.2.3 Layout and Design of the Considered SBS/RS

The next step is to specify the structure of the SBS/RS with all geometric dimensions.

A visualization of the SBS/RS with the help of technical drawings is the simplest way to illustrate the system. A technical drawing is a documentation which contains all necessary information about the SBS/RS and informs about functionalities, in written form as well as by illustrations. Thus, the structure, functionality and geometric dimensions can be illustrated very well with it.

For the visualization of the warehouse, a 3D image is always very helpful but unsuitable for engineers and system planners because important information, such as geometric dimensions and interactions can not be mapped or displayed. The dimensions of the entire SBS/RS, the dimensions of each individual storage location and the position of the lift and buffer must be clearly described, as well as the position of the individual objects in relation to each other.

At least two views are relevant for a clear description of a simple SBS/RS: the top view and the side view. For more complex SBS/RSs, however, it may be relevant to include several side views in addition to the top view. This is especially necessary if there are I/O points on each side of the SBS/RS between the lift and the pre-storage zone. For the calculation, e.g., of the travel times, the exact position of the I/O points is important.

If the individual tiers have different layouts, a top view drawing with all dimensions must be created for each tier. This is especially the case if the individual storage locations on each tier have different dimensions.

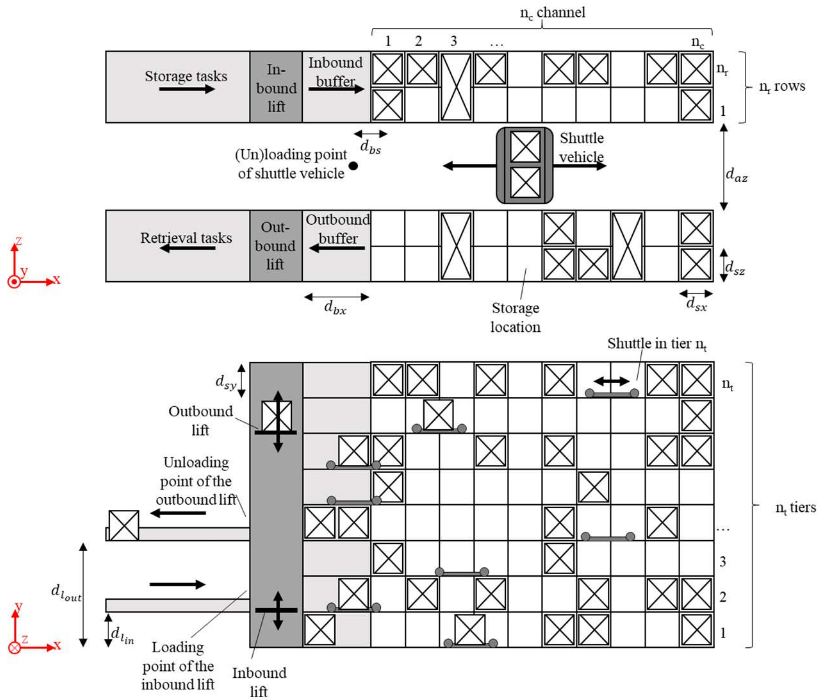


Figure 2.6: Top view and side view of the considered SBS/RS

In Figure 2.6 the tier-captive and aisle-captive SBS/RS is depicted by a top view and side view drawing. All geometric dimensions for the rack and the movement directions of the lift, shuttle vehicle and the inbound and outbound buffer are specified.

The following points should be included in the technical drawing:

- Top view and side view of the SBS/RS
- All geometric dimensions of the rack
- Exact position and number of the I/O points between the SBS/RS and the pre-storage zone and the (un)loading points for shuttle vehicles

- Movement direction of the individual dynamic components, e.g., lift, shuttle vehicle, pallet shuttle vehicle
- Movement direction of, e.g., conveyers of static components, like buffers

Rack

- The rack consists of one aisle with n_t tiers.
- Each tier on each side of the aisle has n_c storage channels.
- Every storage channel consist of n_r rows. If $n_r > 1$, it is a multi-deep storage rack, otherwise a single-deep storage rack.
- In this work a double-deep storage rack is considered ($n_r = 2$).
- All tiers have the same height and the storage channels of any tier are equally sized.
- Each storage channel can hold two small ULs or one large UL.
- The size of one storage location is given by length d_{sx}, width d_{sz} and height d_{sy}.
- The width of the aisle is given by d_{az}.

Lift

- The rack has two independent tote lifts at the end of the aisle, where one lift is for the incoming ULs and one lift is for the outgoing ULs.
- The inbound lift provides the vertical transport of the incoming ULs between the input point of the aisle and the inbound buffer of the target tier.
- The outbound lift provides the vertical transport of the outgoing ULs between the outbound buffer of the tier and the output point of the aisle.
- In vertical direction, the distance between the loading point of the pre-storage zone and the inbound lift is given by $d_{l_{in}}$.
- In vertical direction, the distance between the unloading point of the pre-storage zone and the outbound lift is given by $d_{l_{out}}$.

Buffer

- There is one buffer on each side of the rack, one for incoming ULs and one for outgoing ULs.
- The length of the buffer is given by d_{bx}.
- Each buffer can hold a predefined number of small and large ULs.

Shuttle vehicle

- The shuttle vehicle provides the horizontal transport between the buffer and the storage channels.
- The (un)loading point of the shuttle vehicle is in the middle of each storage channel (x-axis).
- The (un)loading point of the shuttles vehicles is between the two buffers.
- The distance between the (un)loading points of shuttle vehicle and buffer and the (un)loading point at the first storage channel is d_{bs}.

2.2.4 Description of the Components of a SBS/RS

In the next step, the individual system components must be described in their technical characteristics.

Depending on which data and information are relevant for the SBS/RS, a very detailed description of the individual technical components and their properties may be necessary. Depending on the required level of detail, not only the entire assembly (e.g., shuttle vehicle) but also all sub-assemblies (e.g., chassis, control unit, energy supply unit) up to the listing of all necessary individual parts (e.g., flat-head screw DIN-EN-ISO 10642 8.8 M3 plated) can be described.

For a clear and uniform approach that describes the individual system components, the four categories listed below can be used (see Figure 2.7). The three categories geometric dimensions, load capacity and kinematic properties are sufficient for the description in most cases. If individual points can not be assigned here, they can be listed under additional properties.

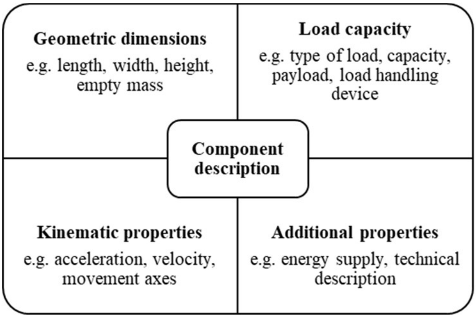


Figure 2.7: Categories for detailed description of all components

In the following the components (i) storage rack, (ii) buffer, (iii) lift, (iv) shuttle vehicle and (v) unit load are described, using the four categories for component description.

(i) Storage Rack

The rack is primary used for the storage of the ULs. The rack system of the SBS/RS can be uniquely described by the number, position and geometrical dimensions of the storage locations, number and position of tiers and number and position of aisles (also cross-aisles).

The description of the design of the SBS/RS has already been depicted in Figure 2.6. Additional attachments to the racking, such as rails on which the shuttle vehicles can travel, maintenance tiers, energy supply or sprinkler system are not considered in detail, since they are not relevant for the determination for travel times or throughput.

In the following, only the relevant parameters are given, which are necessary for the later calculation of the travel time of the shuttle vehicle and lift.

Geometric dimensions

- Detailed description see Figure 2.6
- Total length (x-axis) of the rack: $n_c \cdot d_{sx}$
- Total height (y-axis) of the rack: $n_t \cdot d_{sy}$
- Total width of the rack (z-axis): $2 \cdot n_r \cdot d_{sz} + d_a$

Load capacity

- The total number of storage location of one aisle with the rack on both sides of the aisle is: $2 \cdot n_r \cdot n_c \cdot n_t$
- The capacity of each storage location is one. This means that each storage location can hold one UL with the capacity of one.
- The capacity of each storage channel is two. This means that each storage channel exists of two storage locations in a row. Thus, each storage channel can hold one UL with the capacity of two or two ULs with the capacity of one each.

Kinematic properties

- Not applicable

Additional properties

- No more data is needed for travel time determination. If a finite element method analysis of the rack is to be carried out, all profiles, the exact rack structure, and also all attachments must be specified and described, as these are necessary to calculate the static and dynamic forces.

(ii) Buffer

In tier-captive systems there are transfer buffers to decouple lift and shuttle vehicles. The buffers are integrated in the rack system mostly between the storage locations and the lift. The buffers are mostly equipped with a rolling conveying system.

For storage tasks, the inbound lift unloads the ULs at the inbound buffer. Depending of the size of the buffer, there will be a travel inside the buffer to the loading point of the shuttle vehicle. For retrieval tasks, the shuttle vehicle unloads the ULs at the unloading point in front of the outbound buffer. From this point, the ULs travel to the loading point of the outbound lift.

Geometric dimensions
- Total length of the buffer: d_{bx}
- Distance between (un)loading point shuttle vehicle – buffer and the middle of the first storage channel: d_{bs}

Load capacity
- Capacity of the inbound buffer: $c_{b_{in}}$
- Capacity of the outbound buffer: $c_{b_{out}}$
- Example 1: If the capacity of the buffer is one ($c_{b_{in}} = 1, c_{b_{out}} = 1$), the buffer can hold one UL with the capacity of one.
- Example 2: If the capacity of the buffer is two ($c_{b_{in}} = 2, c_{b_{out}} = 2$), the buffer can hold one UL with the capacity of two or two ULs with the capacity of one each.

Kinematic properties
- If the buffer is very large, acceleration and velocity of the roller conveyor system is needed to calculate the travel time of the ULs inside the buffer.

Additional properties
- Not needed

(iii) Lift

A SBS/RS requires at least one lift for operation. The lift is composed of a lift rack or a lift shaft, which can be standalone (and is connected to the rack system) or integrated into the rack. The lift moves up and down within the shaft. Depending on the specific rack configuration and throughput needs, multiple lifts can be installed at various positions within the rack system. The placement of the lifts typically depends on the location of the interfaces to the pre-storage zone (I/O point of the SBS/RS). Lifts are usually located at the end of the aisle.

Several different lift systems are available, depending on the kind of SBS/RS configuration and the required capacity (see Figure 2.8). The type of lift system depends on the degree of freedom, whether it is tier-to-tier or tier-captive. In tier-captive SBS/RSs a tote lift (a lifting platform equipped with a roller conveyor) is used to transport the ULs in vertical direction. For storage tasks, the inbound lift loads the ULs at the I/O point with the pre-storage zone and travels to the selected tier, where it unloads the UL to the inbound buffer. For retrieval tasks, the outbound lift loads the UL at the tier from the outbound buffer, travels to the I/O point, and unloads the UL to the pre-storage zone. In most SBS/RSs, separated lifts for storage and retrieval tasks are used, since a higher throughput can be achieved. But it is also possible to only use one lift for the SBS/RS, for processing all storage and retrieval tasks.

In tier-to-tier SBS/RSs, a shuttle lift (lifting table with rails mounted on it) is installed which transports the shuttle vehicle in vertical direction. For storage tasks, the inbound lift loads the shuttle vehicle at the I/O point with the pre-storage zone and travels to the selected tier. At the tier the lift unloads the shuttle vehicle and then the shuttle vehicle continues its travel to the selected storage location. The retrieval process operates in exactly the opposite way.

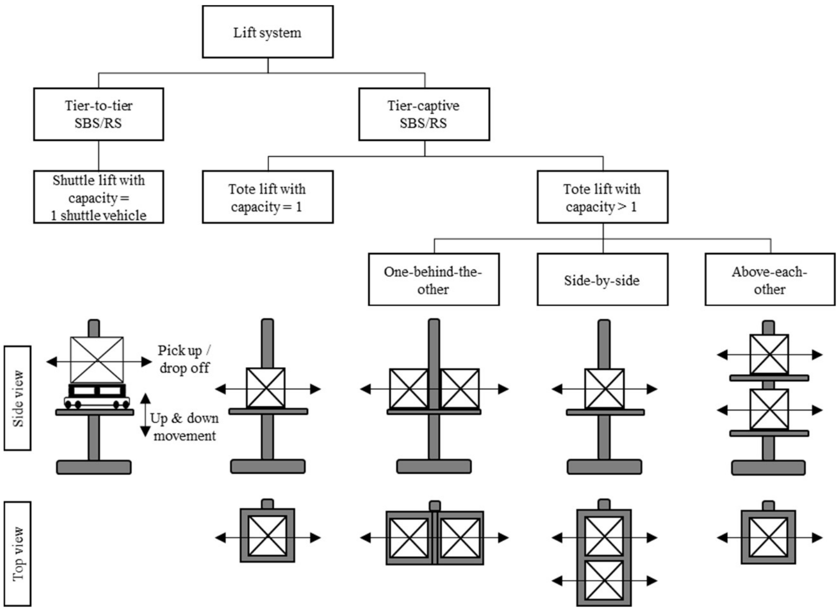


Figure 2.8: Different lift systems (side view and top view)

Depending on the required capacity, different designs of the lift are possible. Thus, only one UL or several ULs can be transported at the same time. For the case that several ULs can be transported at the same time, the arrangement of the ULs on the lifting table defines the design of the lift system. ULs can be transported:

- One-behind-the-other
- Side-by-side
- Above-each-other

Geometric dimensions

- If it is required e.g., to get the footprint of the SBS/RS, length, width and height of the lift system have to be specified.
- In vertical direction, the distance between the loading point between the pre-storage zone and the inbound lift is given by $d_{l_{in}}$.
- In vertical direction, the distance between the unloading point between the pre-storage zone and the outbound lift is given by $d_{l_{out}}$.

Load capacity

- Tote lift with one-behind-the-other configuration.
- Capacity of the inbound lift: $c_{l_{in}}$
- Capacity of the outbound lift: $c_{l_{out}}$
- Example: If the capacity of the lift is two ($c_{l_{in}} = 2$, $c_{l_{out}} = 2$), the lift can hold one UL with the capacity of two or two ULs with the capacity of one each.

Kinematic properties

- Velocity of the inbound lift: $v_{l_{in}}$
- Acceleration and deceleration of the inbound lift: $a_{l_{in}}$
- Velocity of the outbound lift: $v_{l_{out}}$
- Acceleration and deceleration of the outbound lift: $a_{l_{out}}$
- Both lifts travel along the y-axis.

Additional properties

- The considered lift for this SBS/RS are tote lifts.
- No more data is needed for travel time determination, but if e.g., the energy consumption needs to be calculated, further data is required.

(iv) Shuttle Vehicle

Shuttle vehicles travel in horizontal direction on the rails mounted on the rack and transport the ULs. Depending on the shuttle system configuration, shuttle vehicles are assigned to one tier (tier-captive) and can not change tiers or shuttle vehicles can change tiers (tier-to-tier) using the shuttle lift.

The classical shuttle vehicle is an one-level shuttle vehicle which means, that it can just reach the storage location which is on the same tier as the shuttle vehicle that it travels on (see Figure 2.1). If the shuttle vehicles have an integrated lifting device, the shuttle vehicle can reach several storage levels without changing tiers. These types of shuttle vehicles are called multi-level shuttle vehicles.

On each shuttle vehicle a load handling device is mounted on top of the shuttle vehicle which is used to (un)load the ULs between the shuttle vehicle and buffer and storage locations. Depending on the kind of UL and the SBS/RS configuration, there are different shuttle vehicle designs with different transfer techniques (compare Figure 2.9):

Shuttle vehicles for pallets:

- Lifting mechanism for pallets: the vehicle (pallet shuttle or satellite) drives under the pallet and lifts it up so that it can be transported

Shuttle vehicles for small load carrier, tote, tray, carton, etc.:

- Telescopic arm or telescopic gripper
- Drawing technology or gripping technology
- Classical conveyor technology e.g., roller conveyor for the transfer of ULs between pre-storage zone – lift – buffer but also shuttle vehicles with roller conveyors are possible

The design of the shuttle vehicle also differs in the number of load handling devices and in the arrangement of the load handling device(s) on the shuttle vehicle. In addition to the transport of one UL, several ULs can also be transported at the same time. This requires either a multi-load handling device or several load handling devices on one shuttle vehicle.

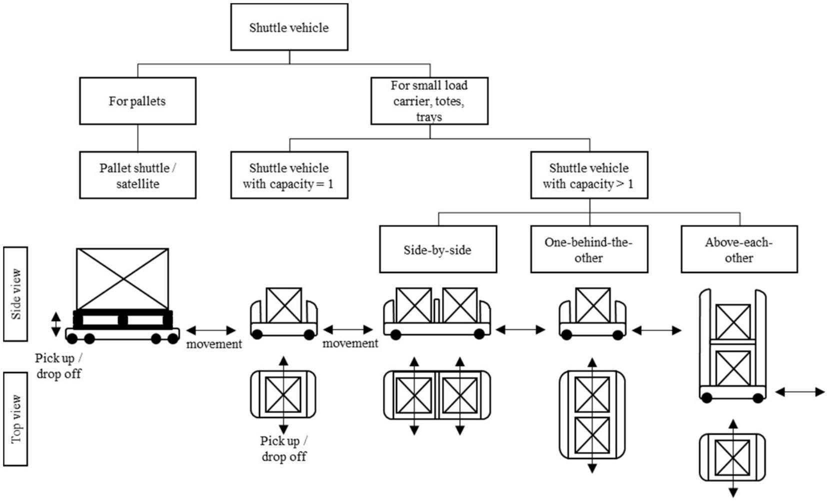


Figure 2.9: Different kind of shuttle vehicles with different load handling devices arrangements (side view and top view)

Shuttle vehicles with capacity larger than one can be classified again into three different types:

- The load handling devices are arranged **side-by-side**. Several ULs can be transported at the same time. In this case, the vehicle is longer, but the aisle width remains the same. The control strategy is very simple because this is comparable to two shuttle vehicles with single-load handling devices.
- Several ULs can be transported **one-behind-the-other** on the shuttle vehicle. Only one load handling device is required. With this configuration it is possible to pick up several ULs from one storage channel at the same time. This load handling device is also very suitable, if the ULs have different length. However, this means a wider aisle between the racks. Additionally, sequencing the requests is getting more difficult compared to all the other load handling devices (see Figure 2.10). If both ULs are to be stored on the same side of the rack, UL number 1 must be unloaded first and then UL

number 2. It is not possible to unload UL number 2 first. Therefore, the planning of the sequence to avoid additional travel times is more complex.

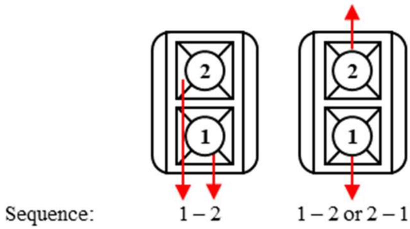


Figure 2.10: Possible sequence for one-behind-the-other load handling device

- Several ULs can be transported **above-each-other**. This load handling device configuration is only applicable for SBS/RS configurations with multi-level shuttle vehicles.

For all shuttle vehicles with a capacity larger than one the classic sequence of processing the storage and retrieval requests e.g., quadruple cycle S1-S2-R1-R2, no longer has to be adhered to (S stands for storage operation and R for retrieval operation). From the time when applies

free capacity on the shuttle vehicle ≥ required capacity for the next retrieval request

the sequence could be changed to e.g., quadruple cycle S1-R1-S2-R2, which could lead to an overall shorter travel time, to complete this quadruple cycle.

Geometric dimensions
- If it is required, length, width and height of the shuttle vehicle can be specified.
- If it is required, the weight of the shuttle vehicle can be specified.

Load capacity
- Shuttle vehicle with one-behind-the-other configuration
- Capacity of the shuttle vehicle: c_s
- Example: If the capacity of the shuttle vehicle is two ($c_s = 2$), the shuttle vehicle can hold one UL with the capacity of two or two ULs with the capacity of one each.

Kinematic properties
- Velocity of the shuttle vehicle: v_s
- Acceleration/deceleration of the shuttle vehicle: a_s
- Shuttle vehicle travels along the x-axis

Additional properties
- Shuttle vehicle for small load carrier
- One-level shuttle vehicle

(v) Unit Load

The kind of UL and the size and weight of the UL defines the SBS/RS configuration and the geometrical dimensions of the required rack system, lift, buffer and shuttle vehicle.

This data is not relevant for the travel time calculation, but at least the type of UL should be named (e.g., small load carrier, pallet).

Geometric dimensions
- If it is required, length, width and height of the UL can be specified.
- If it is required, the weight of the UL can be specified.

Load capacity
- Not applicable

Kinematic properties
- Not applicable

Additional properties
- Type of UL: small load carrier

2.2.5 Description of the Material Flow

After a complete design description of the SBS/RS and all system components, the next step is to look at the material flow and the control logic of the SBS/RS. While there is still a direct correlation between technical parameters and system performance in the classic AS/RS with stacker crane (higher velocity leads to higher throughput), this correlation is no longer necessarily true for SBS/RSs. Maximum system performance can only be achieved if the subsystems (lift and shuttle vehicle) are precisely matched to each other. Due to the complexity of SBS/RSs, it is therefore of particular importance to describe the material flow and the control logic used for the whole storage system, but also for the individual components in detail.

The goal is to understand the interactions and interdependencies of the individual components. The easiest way to illustrate this is with the help of graphics and flowcharts.

Queuing networks are particularly well suited for illustrating and modeling material flow systems and for answering questions related to performance, because they can capture the interactions between different servers and consider stochastic effects and queues (Furmans 2000). These figures do not only contribute to a better understanding, but also serve as preparatory work for the later modeling of SBS/RSs as a queuing network.

Figure 2.11 depicts the material and data flow of a tier-captive SBS/RS. The UL arrives in front of the inbound lift and gets served by the inbound lift. If the inbound lift is busy, ULs wait in front of the inbound lift until the lift is idle and then the ULs can be processed. The shuttle vehicles on each level handle both, the storage and retrieval tasks of the respective level. Storage and retrieval requests that are to be processed in the moment of a busy shuttle vehicle, wait in the queue until the shuttle vehicle is idle again. All retrieval tasks are processed by the same outbound lift. If the outbound lift is busy, the ULs wait until the lift is available and can then be processed.

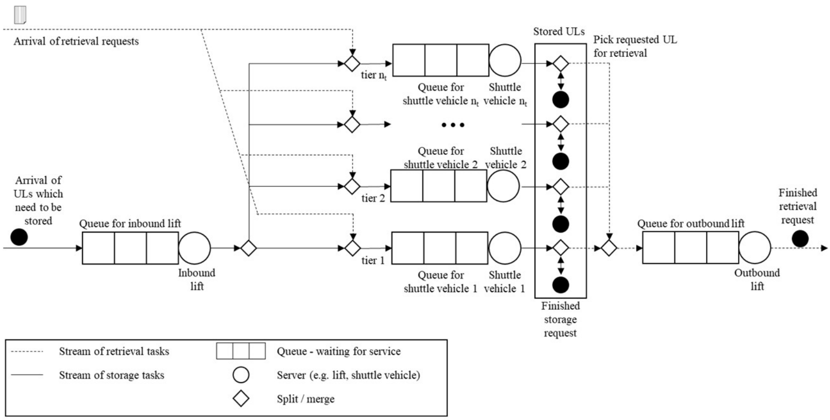


Figure 2.11: Queuing network for the tier-captive SBS/RS

In addition to the overall process, the lift and the shuttle vehicle can also be modelled as independent material flow elements in a SBS/RS. This enables a much more detailed representation of the individual processes. Important for the investigation of the system behavior and the performance evaluation is also the knowledge about the arrival rates and processing times (=service times).

Example for the queuing model for the inbound lift (see Figure 2.12 (a))

The UL arrivals occur according to a random process, which can be described by the inter-arrival time distribution (statistical distribution of the time between two arriving ULs). If the inter-arrival time is known, the arrival rate (average number of UL arrivals per time unit) can also be specified. If the inbound lift is empty, the arriving UL can be picked up directly by the lift and transported to the pre-selected tier. The overall travel time is also randomly distributed. The service rate specifies the maximum number of ULs that can be processed per time unit. In addition, the inter-departure time distribution can be output as a result. The inter-departure time distribution on one specific tier is then also the inter-arrival time of the ULs for the shuttle vehicle on this tier. If the lift is occupied, the UL must wait in front of the lift. Depending on the selected strategy and the system configuration, the next UL is then selected from the queue. In SBS/RSs, the orders are usually processed according to the FCFS principle.

The other three processes, the storage process using the shuttle vehicle (see Figure 2.12 (b)), the retrieval process using the shuttle vehicle (see Figure 2.12 (c)) and the retrieval process using the lift (see Figure 2.12 (d)), can also be described as queueing models.

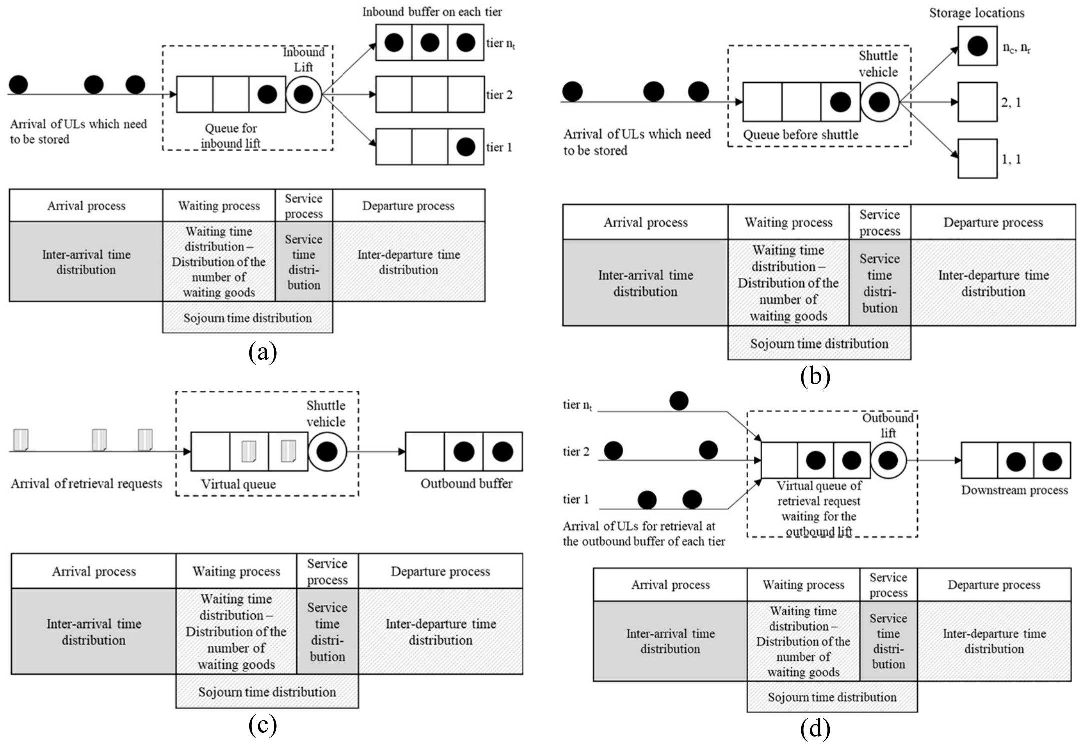


Figure 2.12: Queuing models for (a) inbound lift (storage operation), (b) shuttle vehicle (storage operation), (c) shuttle vehicle (retrieval operation), (d) outbound lift (retrieval operation)

After the material flow has been represented in simplified form as a queueing model, the next step is to describe the material flow in more detail. The simplest way to depict this, is with the help of flowcharts. In the following, the UML "activity" diagram serves as the basis for the visualization (OMG 2017). The "activity" diagram can be used to model a process or the object flow as a sequence of different decision and process steps (see Figure 2.13). Every process starts and ends with the **Start / End block**. The **Decision block** allows to divide the flow into different branches, depending on the condition. In the **Process block** some processes are carried out. In storage systems, only three processes can occur: wait (blue), travel (orange) and load handling (green). The color visualization makes it clear which type of process is carried out.

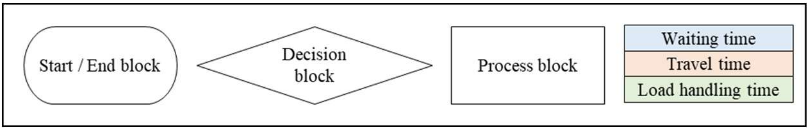


Figure 2.13: Used elements for the material flow description

The "activity" diagram is combined with a so called "swim lane" diagram. This kind of diagram provides clarity by placing the process steps within the vertical "swim lanes". It delineates "who does what" and thus enables a clear assignment of the individual process steps. The following "swim lanes" can be used for a SBS/RS: pre-storage zone, lift, buffer, shuttle and rack - storage location. If there are transitions between the individual "swim lanes", e.g., (un)loading of ULs, process blocks can also be placed directly on the transition lines. Figure 2.14 depicts the material flow of one UL for the storage process. The "activity-swim lane" diagram describes the storage process for the UL in great detail.

An advantage of this type of mapping is that the equation for the required time for the whole storage process of one UL can also be derived directly from the "activity-swim lane" diagram. The required time for the storage process would be in this case the sum of all process blocks. If a process block, e.g., waiting time block has not been run through, then the value for this block is set equal to zero.

Figure 2.15 depicts the material flow of one UL for the retrieval process. If it is helpful for clarity, individual blocks can also be described in more detail in an additional “activity” or “activity-swim lane” diagram. For example, the relocation process is described in more detail in Figure 2.16. Further, Figure 2.17 depicts a detailed description of the travel time of each UL in the inbound lift. If the inbound lift processes two small ULs and the total travel time of the lift for the second UL is desired, this "activity" diagram can be used to determine the total travel time.

Further “activity” diagrams for the detailed description of the travel times of the ULs can be found in appendix A.

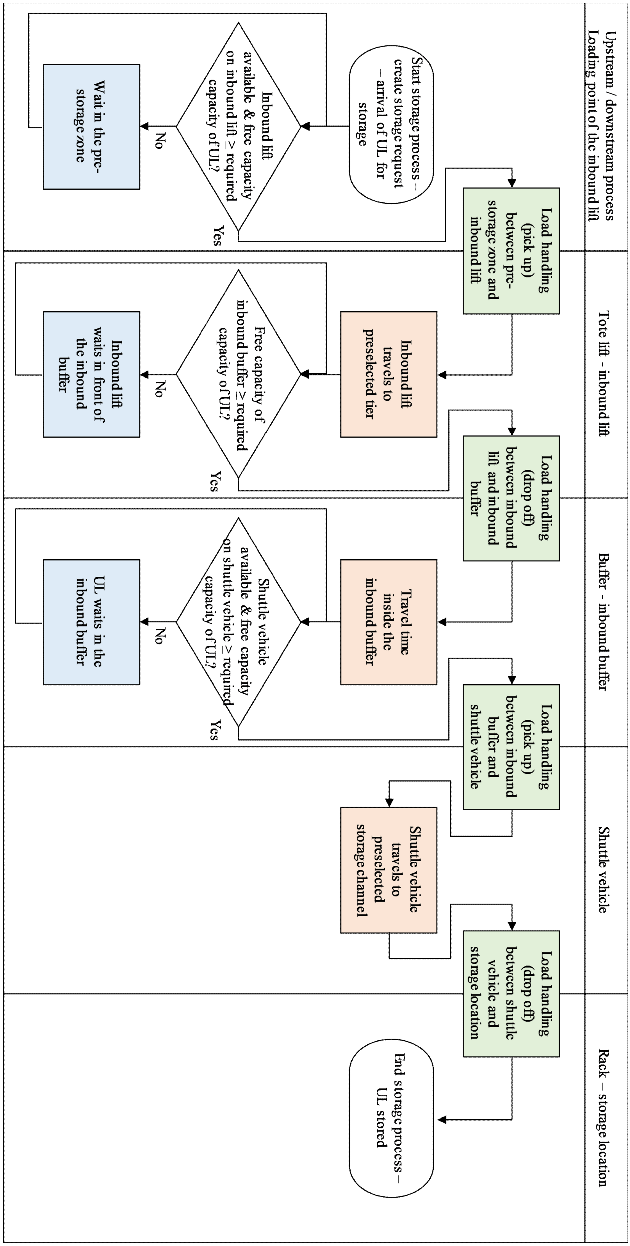


Figure 2.14: Material flow of the storage process for one UL

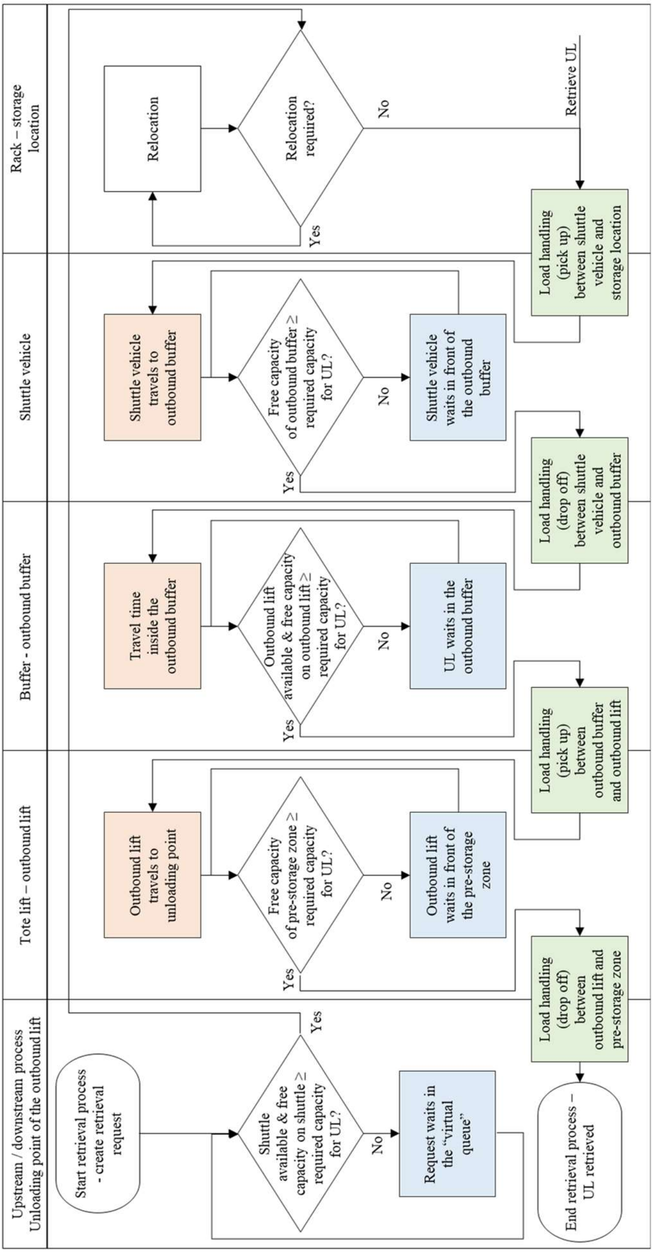


Figure 2.15: Material flow of the retrieval process for one UL

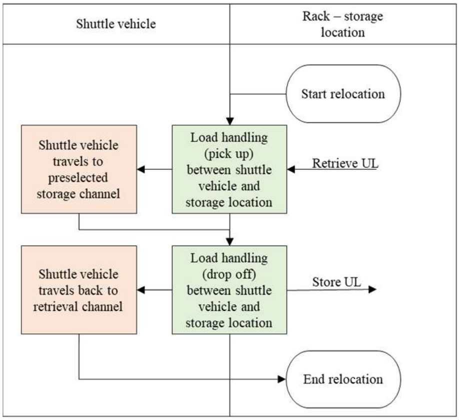


Figure 2.16: Material flow of the relocation process

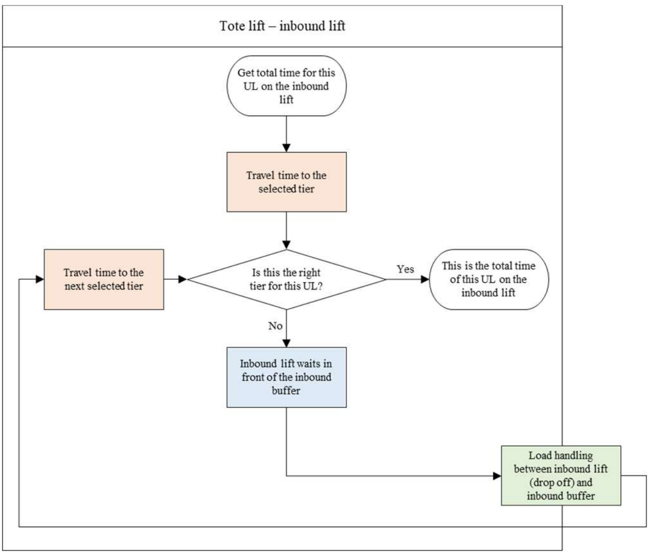


Figure 2.17: Detailed process description for the travel time of one UL on the inbound lift (storage process)

Material flow

- To describe the material flow, the SBS/RS can be described as a queueing model.
- The material flow of each UL is known and with the help of the "activity swim lane" diagrams the material flow is well described. In addition, the required time of an UL for the storage process and the retrieval process can be derived from the diagrams.

2.2.6 Description of the Used Control Strategy

So far, only the material flow for the ULs has been specified. In addition, however, it is also necessary to describe the control logic of the individual components in detail. The control logic of lift and shuttle vehicle can also be specified using "activity" diagrams. From the "activity" diagrams the cycle time of lift and shuttle vehicle can be derived. Depending on the SBS/RS configuration and the control strategy used, a different path through the "activity" diagram can be chosen. The sum of the blocks passed through is then the cycle time for this cycle. The "activity" diagrams have been created in such a general way that they can be adapted and therefore can be applied to any SBS/RS configuration. As an example Figure 2.18 depicts the control logic of the inbound lift. The control logic of the shuttle vehicle and the outbound lift can be found in appendix A.

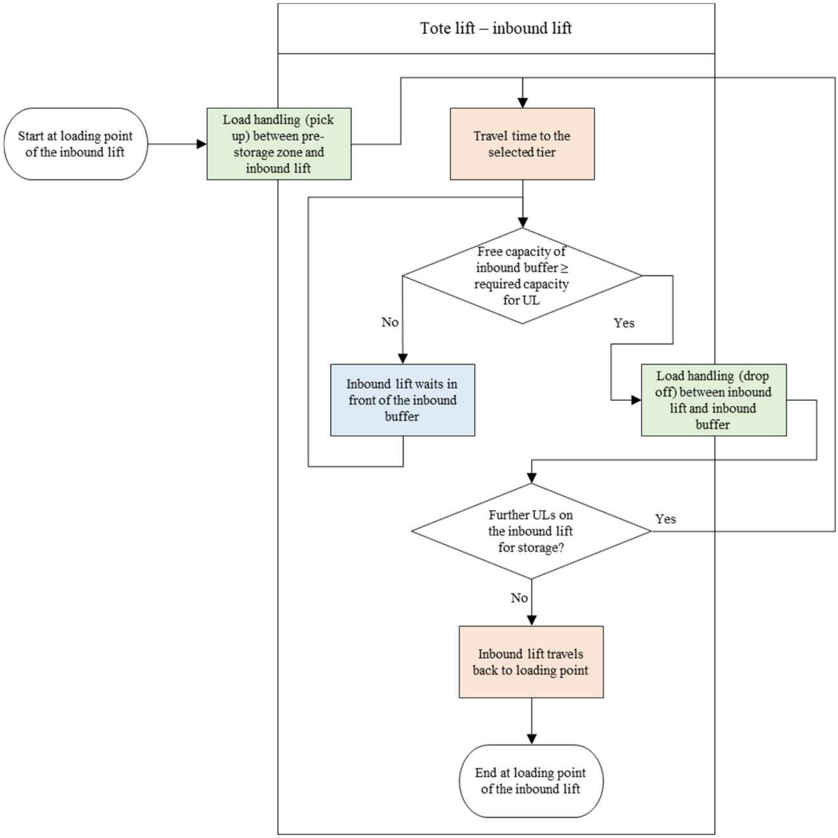


Figure 2.18: Control logic of the inbound lift

After the material flow and the control logic are known, the next step is to specify the control strategies used for the SBS/RS and for each lift and shuttle vehicle. In addition to the design of the storage system, the control strategies, also have a decisive impact on the performance. Depending on the control strategy, processes are carried out differently. There is no general best strategy, but it must be selected individually based on the use case and the requirements.

The control policies for SBS/RSs are very similar to the control policies for AS/RS. The main difference is that the strategies must apply to the lift as well as to the shuttle vehicle (and satellite vehicle). Also, the interaction of the individual system components must be controlled. Roodbergen and Vis (2009) have already described possible control strategies for AS/RSs in great detail. Epp (2018) has summarized control strategies for SBS/RSs and additionally considered routing strategies. In both publications, however, the consideration of multi-deep racks, in which relocation operations are necessary, is missing. Thus, in this work additional relocation strategies are considered.

The control strategies for SBS/RSs can be classified into the following six subjects:

(i) Storage assignment
(ii) Sequencing strategy
(iii) Dwell point strategy
(iv) Relocation and repositioning strategy
(v) Resource assignment
(vi) Routing

The various control strategies are explained in more detail below.

(i) Storage assignment

The storage assignment determines the storage locations in which items (or ULs) are stored. The following storage assignment strategies are frequently used:

- Dedicated storage assignment strategy: Every item has its fixed number of dedicated storage locations. This means that the storage location of this item is always known. However, the disadvantage is that the storage locations can remain empty over a long time if an item is out of stock.

- Random storage assignment strategy: Every item can be stored in any empty storage location, which results in a lower number of storage locations and therefore in a smaller storage system.
- Class based storage assignment strategy: With zoning, fixed storage areas are reserved for each item group, within random storage assignment strategy is applied. The areas are usually defined according to turnover rates, so that items with a high turnover value can be accessed faster. Therefore, every item is assigned to one of these areas, based on its turnover rate.
- Turnover based storage assignment strategy: Items with the highest turnover values are stored closest to the I/O point. This reduces the travel distances of the material handling equipment and allows faster access to these items.
- Closest open location storage assignment strategy: Items are stored in the first empty storage location closest to the I/O point. However, this can lead to, items with a low turnover value being stored close to the I/O point and items with a high turnover value being stored far away from the I/O point. This can then cause long travel distances.

Storage assignment

- Random storage assignment strategy is applied.

(ii) Sequencing strategy

Sequence strategies determine the sequence in which storage and retrieval operations are to be carried out. The aim is to achieve the highest possible throughput through an optimized sequence.

Several basic decisions have to be made before the sequence strategy can be defined:

- Are all requests in the queue or just a fixed number of requests considered (=block sequencing)?
- What is the updating interval of the sequence? Is the sequence updated every time a new request is added to the queue, or is a fixed number of requests sequenced and completed before considering the newly added ones?
- Which command cycles are considered? The more complex the system under consideration, the more complex the command cycles can get. In a SBS/RS mostly single- and dual-command cycles are carried out. Multi-command cycles can be carried out with a multi-load handling device. In a single-command cycle only one UL is stored or retrieved. E.g., in a single-command cycle of a storage operation, the shuttle vehicle loads the UL at the I/O point, travels to the storage locations, unloads the UL and travels back to the I/O point. In a dual-command cycle, either one storage and one retrieval operation, two storage or two retrieval operations are carried out. Figure 2.19 shows all possible command cycles, for the case that the shuttle vehicle has a dual-load handling device and two different sizes of ULs are considered. S stands for storage operation and R for retrieval operation.
- If a strict command cycle strategy is applied, this means that, e.g., in the case of a dual-command cycle, with one storage and one retrieval operation, the shuttle vehicle must wait for the retrieval operation even if a storage request is already available. Hence, exceptions are often made for such cases, e.g., that instead of a dual-command cycle a single-command cycle is performed.

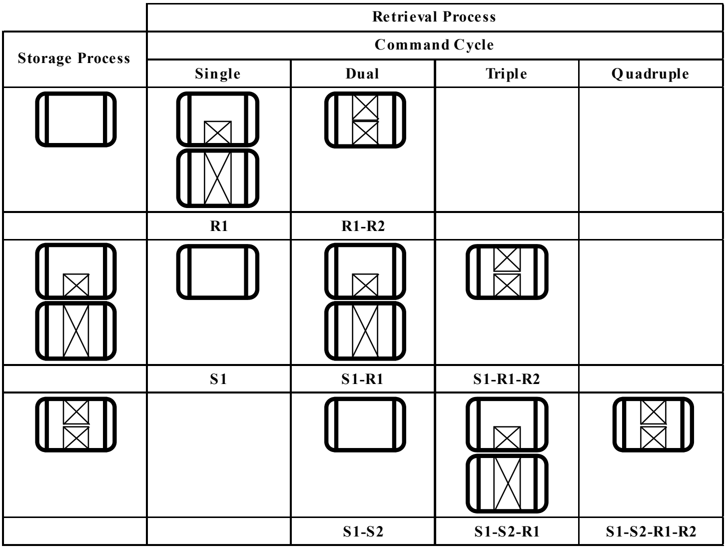


Figure 2.19: Different command cycles for a shuttle vehicle with an one-behind-the-other load handling device with the capacity of two (capacity for two small ULs or one large UL)

- Which sequence strategy is considered? The next step is to define how the orders are to be processed. Considering the overall storage system, storage requests that arrive physically in front of the lift are usually processed according to the FCFS principle because they are not able to pass each other in the queue. The sequence is therefore fixed. Of course, there is also the possibility that the storage requests in the pre-zone are brought into a desired sequence with the help of conveyor technology. This, though is time-consuming and expensive. In contrast, the retrieval requests are in a virtual queue and can be processed in any sequence as long as no restrictions are given (e.g., priority rules or due dates). Since there are a number of queues in shuttle systems,

(i) **physical queues**: storage requests in front of the lift, storage request in the inbound buffer, retrieval requests in the outbound buffer and

(ii) **virtual queues**: retrieval request waiting for the shuttle vehicle, retrieval requests waiting for the lift,

sequencing is getting a very complex task. Thus, requests that physically arrive in a queue are processed according to FCFS. Requests arriving in a virtual queue can be processed in any sequence (e.g., FCFS, shortest total travel distance, shortest total completion time), taking restrictions into account.

Sequencing is getting more complex, if, e.g., “due dates” for retrieval operations or “best before dates” for, e.g., food products, priority rules or complex shuttle system configurations with several I/O points are considered. To solve this, either complex optimization models or heuristics need to be developed.

Sequencing strategy

- Different command cycles are considered.
- Storage and retrieval requests are processed according to FCFS, but also a presorting strategy is considered.

(iii) Dwell Point strategy

The dwell point is the location of the load handling equipment (e.g., shuttle vehicle, lift) after completion of a storage or retrieval operation for the case that it is idle and no storage or retrieval requests are available. The aim is, to minimize the travel time for the next operation after idling. The following dwell point strategies are frequently used:

- Point of service completion (POSC): The load handling equipment remains at the location, where the last storage or retrieval operation has been completed (Bozer and White 1984).
- Return to I/O Point (RIO): The load handling equipment returns to the I/O point after completion of the last storage or retrieval operation (Bozer and White 1984).

- Static dwell point: It is also possible that every possible location in the storage system is chosen, e.g., exactly the mid-point of each aisle (Bozer and White 1984).
- Dynamic (optimal) dwell point: Depending on the performed storage and retrieval operations, any position in the storage system can be chosen. This position can change after each storage and retrieval operation and must therefore always be redetermined, depending on factors such as number of storage and retrieval operations performed, the probability of occurrence of the next storage or retrieval request and required travel time. (Egbelu 1991 and Peters et al. 1996) Still, there do not exist many publications on this topic and therefore further research has to be done on dwell point strategies, focusing on SBS/RSs.

Dwell Point strategy

- Dwell point strategy for the inbound lift: RIO (I/O point with pre-storage zone)
- Dwell point strategy for the outbound lift: POSC (I/O point with pre-storage zone)
- Dwell point strategy for the shuttle vehicle: POSC

(iv) Relocation and Repositioning strategy

A relocation process is the combination of a retrieval process plus a storage process. During a relocation process an UL is retrieved and directly stored at a different storage location. Since only the shuttle vehicles perform retrieval operations from storage locations, the relocation strategies are only applicable for the shuttle vehicle. There are several reasons why a relocation is necessary:

- An UL needs to be retrieved but is blocked by another UL. Therefore, this UL (blocker) needs to be relocated first, to get access to the UL which can then be retrieved. This is only the case for multi-deep storage channels.

- If all storage channels are either half-full or full and a large UL needs to be stored, a relocation process is needed. Therefore, a small UL of a half-full storage channel needs to be relocated to another half-full storage channel to get an empty and a full storage channel.
- Storage locations that were previously still very suitable for an UL, may no longer be optimal. This is the case, for example, for ULs with a high turnover value, since the demand frequency changes over time. These ULs are stored close to the I/O point and when storage locations close to the I/O point become empty, repositioning of these ULs may become necessary. The same applies to products with a seasonal influence.
- It is also possible to increase the performance of the storage system through relocation tasks. The idle time of the load handling equipment can be used to relocate ULs in such a way that storage and retrieval times for subsequent orders can be minimized.

All above listed storage assignment strategies also can be used for the case of relocation. Additionally, the nearest neighbor strategy can be applied. This strategy is often applied to relocate the blocker to the closest empty storage location to minimize travel times.

Relocation and Repositioning strategy
- Relocation, if small ULs are blocked for retrieval.
- Relocation, if large ULs need to be stored, but all storage channels are half-full or full.
- Relocations strategy: Random storage assignment strategy and nearest neighbor is applied.

(v) Resource assignment

In simple SBS/RS configurations, where shuttle vehicles can not change tiers (tier-captive) or aisles (aisle-captive) and there is only one shuttle vehicle per tier, all requests are assigned to one shuttle vehicle. In more complex SBS/RS configurations where multiple shuttle vehicles could process a storage or retrieval request, a strategy must be defined, which vehicle processes which request. Possible resource assignment strategies are:

- Random resource assignment strategy: A random shuttle is assigned to process the next request. Depending on the state of the shuttle vehicle (idle or busy), longer waiting times may occur until the request can be processed. In addition, long travel distances may occur, if the randomly selected shuttle vehicle is at the other end of the storage system.
- FCFS: The shuttle vehicle that is idle first, processes the next request.
- Shortest travel distance: The shuttle vehicle with the shortest travel distance, processes the next request.
- Optimal solution: All shuttle vehicles can be assigned to the individual requests with the objective function, e.g., shortest processing time over all requests. However, an optimization model is required to solve this problem.

The same applies for the lifts. If just one lift or two separated lifts (one inbound lift and one outbound lift) are available, all the requests are assigned to these lifts. When it comes to multiple lifts, it is important to develop strategies for determining which requests should be processed by which lift.

Resource assignment

- Since there is only one inbound lift, one outbound lift and only one shuttle vehicle per tier in the SBS/RS under consideration, the ULs are always clearly assigned to a resource and no special assignment strategy needs to be applied.

(vi) Routing

In simple SBS/RS configurations, where shuttle vehicles can not change tiers (tier-captive) or aisles (aisle-captive) and there is only one shuttle vehicle per tier, the shortest travel route strategy to the next request is usually selected. However, if vehicles can change tiers (tier-to-tier) or aisles (aisle-to-aisle) or several vehicles can travel simultaneously at one tier or use the same cross aisle at the same time, then routing or dead lock handling is of particular importance to run these systems efficiently and robustly.

The problem that must be solved is, where and when which vehicle may take which path to reach the specified target position without the vehicles blocking each other or, in the worst case, leading to a deadlock. Therefore, special routing strategies or "deadlock avoidance" strategies must be applied. The literature is very short on routing strategies for SBS/RSs, which could be a future field of research.

One strategy is routing with time windows. There must be a free time window, that is long enough to allow the shuttle vehicle to perform the storage or retrieval request. Therefore, the warehouse is divided into many small areas. All areas on the travel path to the next storage or retrieval request are reserved. This makes it possible to see directly which vehicle is in which area at which time and which time windows are free for other shuttle vehicles. At a certain time, only one vehicle is allowed to be in one specific area. (Lienert and Fottner 2017)

Routing

- Since there is only one inbound lift, one outbound lift and only one shuttle vehicle per tier in the SBS/RS under consideration, no routing strategy needs to be applied.

In summary, it can be stated that the individual control strategies for SBS/RSs are interdependent. Most strategies are transferred directly from AS/RSs. Due to the complexity of SBS/RSs, however, there is still a great need for further research, also with regard to performance increase through optimized control strategies for SBS/RSs. Thus, the development of SBS/RS specific control strategies represents a future field of research.

2.2.7 Description of the (Un)Loading Process and Load Handling Times

To calculate the throughput of the SBS/RS, it is necessary to specify the load handling times for (un)loading, which includes the drop-off and pick-up of ULs. Depending on the configuration of the SBS/RS, either only ULs or the entire shuttle vehicle (with or without loading) may be transferred. The following figure shows all possible load handling times, indicated by arrows that represent the transfer direction. A single arrow denotes that ULs can only be transferred in the direction of the arrow, while a double arrow denotes that ULs can **be transferred in both directions.**

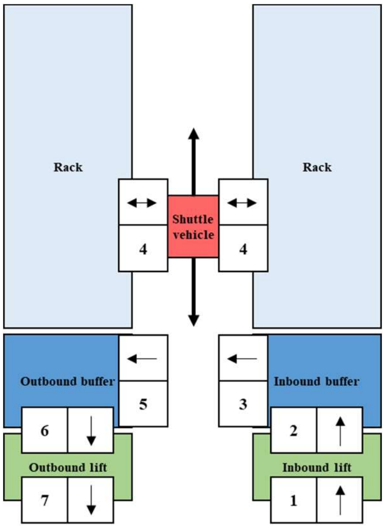


Figure 2.20: All load handling processes with direction of load handling

For this SBS/RS configuration (tier-captive and aisle-captive), seven load handling times must be taken into account:

- 1 and 7 indicate the load handling times between the pre-storage zone and the lift.
- 2 and 6 indicate the load handling times between the lift and the buffer.
- 3 and 5 indicate the load handling times between the buffer and the shuttle vehicle.
- 4 indicates the load handling time between the shuttle vehicle and the storage location.

Load handling times 1, 2, 6 & 7 and 3 & 5 are often the same in many SBS/RSs. Therefore, in most shuttle systems, only the following three load handling times need to be specified:

- Load handling time to/from lift
- Load handling time between buffer and shuttle vehicle
- Load handling time between shuttle vehicle and storage location

Specifying load handling times for simple SBS/RS configurations is straightforward. However, for more complex systems and when considering different load handling cases, a detailed description of each case becomes necessary.

The following three different cases can occur:

(i) Dropping off **or** picking up one UL: In this case, the total load handling time is clearly defined.

$$t_{total} = t_n \tag{2.1}$$

(ii) Dropping off **or** picking up several ULs simultaneously: In this case, all individual-load handling times are known, and the total load handling time is the maximum of all individual values.

$$t_{total} = max(t_1, t_2, \dots, t_n) \tag{2.2}$$

Simultaneously picking up or dropping off multiple ULs reduces the total load handling time compared to transferring individual ULs one after the other.

(iii) Dropping off **and** picking up one or several ULs at the same time: When ULs are dropped off and picked up simultaneously, the total duration of the load handling time is the maximum of all individual values.

$$t_{total} = max\ (t_1, t_2, \ldots, t_n) \quad (2.3)$$

The following provides an overview of several possible cases:

The **first use case** illustrates different pick up options and the duration required for transferring from the pre-storage zone onto the inbound lift. The individual illustrations are grouped into the two categories mentioned above: (i) picking up one UL, and (ii) picking up several ULs simultaneously. Since the transfer of the ULs is "row-wise", the same load handling times are required for all cases.

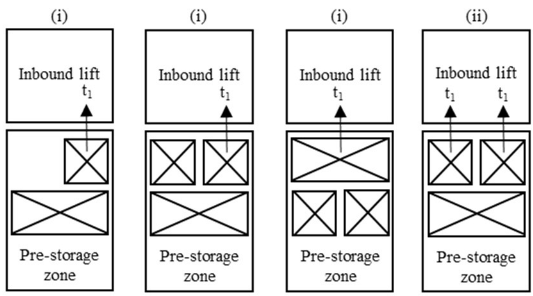


Figure 2.21: Load handling between pre-storage zone and inbound lift

The **second use case** demonstrates different drop off options and the duration required for dropping off ULs to storage locations. In this use case a shuttle vehicle with a dual-load handling device side-by-side and a double-deep storage system is considered. The load handling times vary depending on whether the storage is in the front or back row, with $t_2 < t_3$.

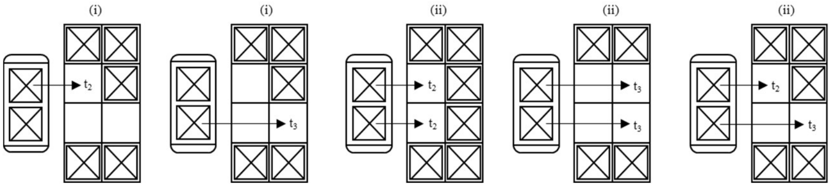


Figure 2.22: Unloading (drop off) between shuttle vehicle and storage location – for a side-by-side load handling device

The **third use case** demonstrates different drop off options and the duration required for dropping off ULs to storage locations. In this use case a shuttle vehicle with a dual-load handling device one-behind-the-other and a double-deep storage system is considered. The load handling times vary depending on whether the storage is in the front or back row, with $t_2 < t_3$.

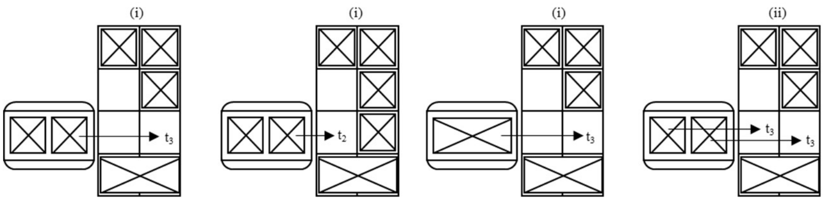


Figure 2.23: Unloading (drop off) between the shuttle vehicle and storage location – for a one-behind-the-other load handling device

The **fourth use case** illustrates a simultaneous drop off and pick up process, which is only possible if the shuttle vehicle's technical capabilities allow for such operations. This occurs only when the pick up and drop off storage locations are precisely opposite each other or at the loading and unloading point between the shuttle vehicle and the inbound and outbound buffer. The load handling times vary depending on whether the storage is in the front or back row, with $t_2 < t_3$.

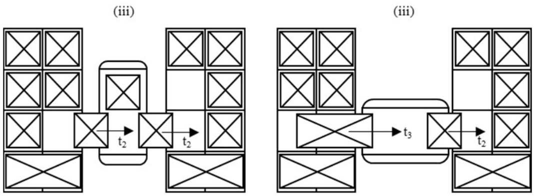


Figure 2.24: Simultaneous pick up and drop off between shuttle vehicle and storage location

The literature often only considers the load handling of one UL after the other, with more complex cases being mostly overlooked. First approaches to address multi-load handling devices and simultaneous pick up and drop off of several ULs can be found in Seemüller (2006). However, developing methods for more complex use cases represent a future field of research.

- For more complex SBS/RSs, such as those using multi-load handling devices or multi-deep storage racks, it is best to describe the different load handling times using figures.
- Load handling time between pre-storage zone – lift – buffer is t_1, regardless of whether a large UL, a small UL or two small ULs are transferred. The load handling time is always t_1.
- If one small UL is stored in or retrieved from the front row, the load handling time is t_2.
- If one small UL is stored in or retrieved from the back row, the load handling time is t_3.
- If two small UL are stored in or retrieved from the same storage channel at the same time, the load handling time is t_3.
- If one large UL is stored in or retrieved from one storage channel, the load handling time is t_3.

2.2.8 Further Required Input Data

Depending on the specific SBS/RS being analyzed and the considered use case, additional data has to be specified in order to provide a comprehensive description of the SBS/RS. The following parameters are essential for providing such a description:

- **Filling degree:** The filling degree is a key metric that provides insight into the current occupation of storage locations. It represents the proportion of occupied storage locations to the total available (empty and occupied) storage locations. In practice the filling degree changes over time. Based on the considered storage system and used control strategies, the cycle time will also change over time. To evaluate the performance of a storage system, best would be to provide a time series data of the filling degree.

$$Filling\ degree = \frac{occupied\ storage\ locations}{all\ storage\ locations} \tag{2.4}$$

- **Article structure:** The article structure (composition of the assortment) and the order structure (the composition and sequence of orders) are crucial factors in determining the efficiency of a SBS/RS. Depending on the industry, there may be fluctuations due to seasonal effects, trends, promotions, or changes over the product life cycle.

In order to simulate a SBS/RS, a detailed description of the article and order structure is necessary. This can be achieved by either using real data or, in a simplified manner, by describing the item and order structure using a probability distribution or ratio.

- The filling degree is given by z.
- Two different sizes of ULs (small ULs and large ULs) are considered.
- A small UL has a capacity of one and a large UL has a capacity of two.
- A large UL has the same size as two small ULs.
- The percentage of small ULs is given by P_{small}.
- Storage and retrieval requests are randomly generated.

2.2.9 Key Performance Indicators

Various performance measures can be employed to evaluate different SBS/RS configurations and control strategies, depending on the specific objective. Key performance indicators (KPIs) are useful in comparing different SBS/RS configurations, optimizing the design of the system and its components, and determining the best control strategy. The following are some of the most relevant performance measures for SBS/RSs:

- **Throughput:** Throughput defines the number of completed storage and retrieval tasks within a specified time period.
- **Utilization:** Utilization defines the proportion of time that resources such as shuttle vehicles and lifts are actively used compared to the total available time.
- **Mean travel time**: Indicates the average travel time of shuttle vehicles and lifts. In addition, the travel time distribution can also be specified.
- **Mean cycle time:** Indicates the average cycle time of the shuttle vehicles and lifts. In addition, the cycle time distribution (service time distribution) can also be specified.
- **Duration for the retrieval process:** Indicates how long it takes for an UL to be retrieved - from the start of the retrieval operation of the UL until the arrival of the UL in the pre-storage zone.
- **Number of waiting storage request:** Indicates the number of ULs waiting to be stored in front of the inbound lift within the pre-storage zone.

- **Number of waiting retrieval requests:** Indicates the number of retrieval requests waiting in the virtual queue to be processed and retrieved.
- **Waiting times and waiting time distributions:** In the SBS/RS, where waiting times occur (see chapter 2.2.5 and chapter 2.2.6), the number of waiting requests or ULs, the distribution of the number of waiting requests or ULs, the waiting time and the waiting time distribution can be specified.
- **Inter-arrival time distribution:** Indicates the time between the arrival of two successive requests or ULs. In addition, the inter-arrival time distribution (service time distribution) can be specified.
- **Inter-departure time distribution**: Indicates the time between the departure of two ULs after they have been processed by a lift or shuttle vehicle. In addition, the inter-departure time distribution can be specified.
- **Costs:** Costs are important for economical evaluation. On the one hand there are initial costs for a SBS/RS and on the other hand there are ongoing costs in operation. Both costs need to be considered, when designing a new SBS/RS.
- **Energy consumption:** Indicates how high the energy consumption is. It can be measured in a variety of ways, such as the energy consumption per storage location or the average energy consumption per storage or retrieval operation.
- **Footprint of the SBS/RS:** Indicates the required space of the used SBS/RS configuration.

- The above KPIs can be used to evaluate the performance of a SBS/RS. The choice of relevant KPIs depends on the objectives of evaluation.

2.3 Unified Modeling of the SBS/RS

Having presented the method for describing SBS/RSs in the preceding section, chapter 2.3 introduces a unified modeling approach. The major challenge is to create a universally applicable mapping of SBS/RSs. The main focus is on a unified description of the layout and material flow of SBS/RSs.

For a unified storage system description, the method described in chapter 2.2 must still be run through. However, the description of the layout and individual components can be done via the unified model description method.

2.3.1 Entity Relationship Diagram

The initial step involves mapping the relationships and number of all individual components in a SBS/RS. This is accomplished by creating an entity relationship diagram (ER diagram) which illustrates the relationships between various "entities" such as shuttle vehicles, lifts, buffers, and the rack system. The ER diagram serves as the basis for unifying the necessary data for the SBS/RS. (Chen 1976)

Figure 2.25 presents the entity relationship diagram for the SBS/RS illustrated in Figure 2.6. The diagram depicts the relationships between the various entities and the number of each component required. The actual values for n can be substituted in the diagram if they are known. The diagram can also be expanded by including additional entities or describing the entities in more detail (e.g., instead of lift, this could be extended to inbound lift and outbound lift).

Here are two examples of how to interpret the diagram:

- A SBS/RS can be equipped with $\boldsymbol{n_{shuttles}}$ shuttle vehicles, but **1** shuttle vehicle is assigned exactly to one SBS/RS.
- A SBS/RS has **1** storage rack and this storage rack is assigned exactly to **1** SBS/RS.

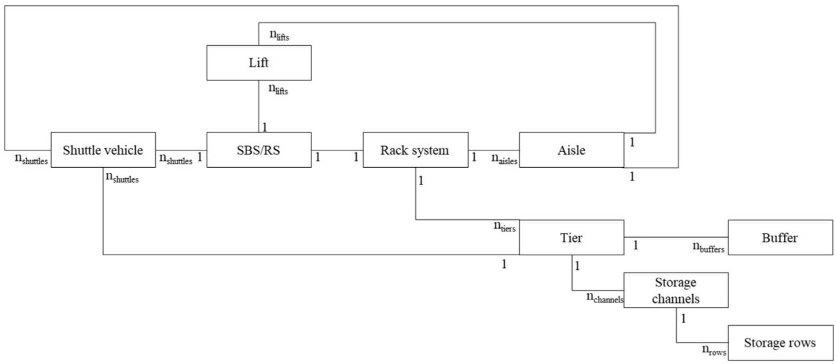


Figure 2.25: Simplified entity relationship diagram of a SBS/RS

- The entity relationship diagram is an effective way to represent the relationships between individual components within a SBS/RS, while also specifying the required number of each entity.

2.3.2 Block Layout Diagram

The ER diagram shows the connections and relationships of the individual entities, but no layout can be derived from it. In the next step, the exact layout must be described. This chapter therefore presents one way of describing the layout and design of SBS/RSs. With the help of this method, it is possible to represent all SBS/RSs described in Figure 2.5.

All components and entities (e.g., lift, shuttle vehicle, buffer, rack) in a storage system can be represented as rectangular objects. By linking individual rectangular blocks, each component can be described uniquely. A rectangle always represents the capacity of one. This means that the smallest UL that has to be stored, has the capacity of one. In most storage systems one UL with capacity one is stored in one storage location with the capacity of one. E.g., if the UL has the capacity of two, two rectangle blocks, each with the capacity of one, are required.

Figure 2.26 shows the top view and side view of a double-deep storage rack with four tiers. All 80 storage locations are clearly described. Since such a representation is only possible for small rack sizes, the illustration of all rack configurations has been reduced to 4 blocks (see Figure 2.27). Therefore, only the rectangle blocks are indicated in each case. This is completely sufficient, since they indicate the position and orientation of the first and last block. The number of blocks in between can be derived from the indices number of channels (x), number of tiers (y), and number of rows (z). Depending on the complexity of the rack configuration, further side views or top views may be necessary.

Each block is clearly described in space by the coordinate system and the side and top view. In addition, the geometrical dimension must be specified for each block. Since each block has a unique ID, length, width and height can be specified for each block. If all blocks have the same dimensions, it is sufficient to specify just one block. If the blocks have different dimensions, e.g., the tiers have different heights or the storage channels have different widths, the individual dimension must be specified for each block. With all this data, the storage rack can be clearly described in its entity.

Like the rack system, lift, shuttle vehicle, buffer, etc. can be described using the same method.

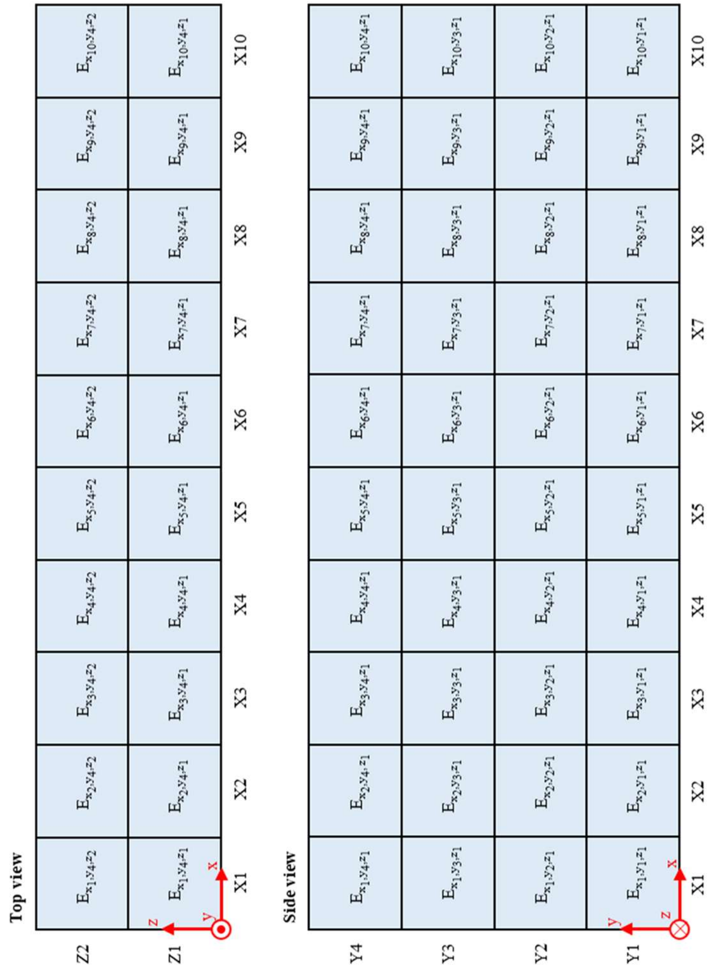


Figure 2.26: Description of the storage rack

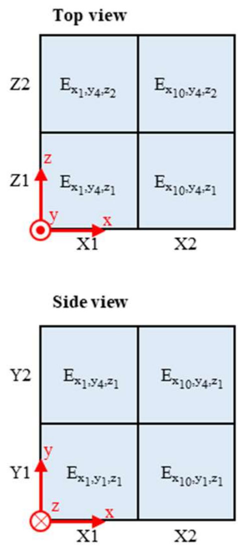


Figure 2.27: Simplified description of the storage rack

After the individual components of the storage system have been described, the next step is to combine the individual blocks into one overall system. For this purpose, the blocks can be placed in the coordinate system in such a way that the used layout of the storage system is mapped.

Figure 2.28 depicts the description of the layout of the SBS/RS from Figure 2.6. For a clear definition of the storage system, the top view and both side views are needed. The green blocks represent the lift, the dark blue blocks the buffer and the light blue blocks the rack system. The rails for the shuttle vehicles are shown in red. The white blocks are, for example, pre-storage zones or aisles. With the help of this block layout, the SBS/RS can be clearly described and the interfaces of the individual blocks are known.

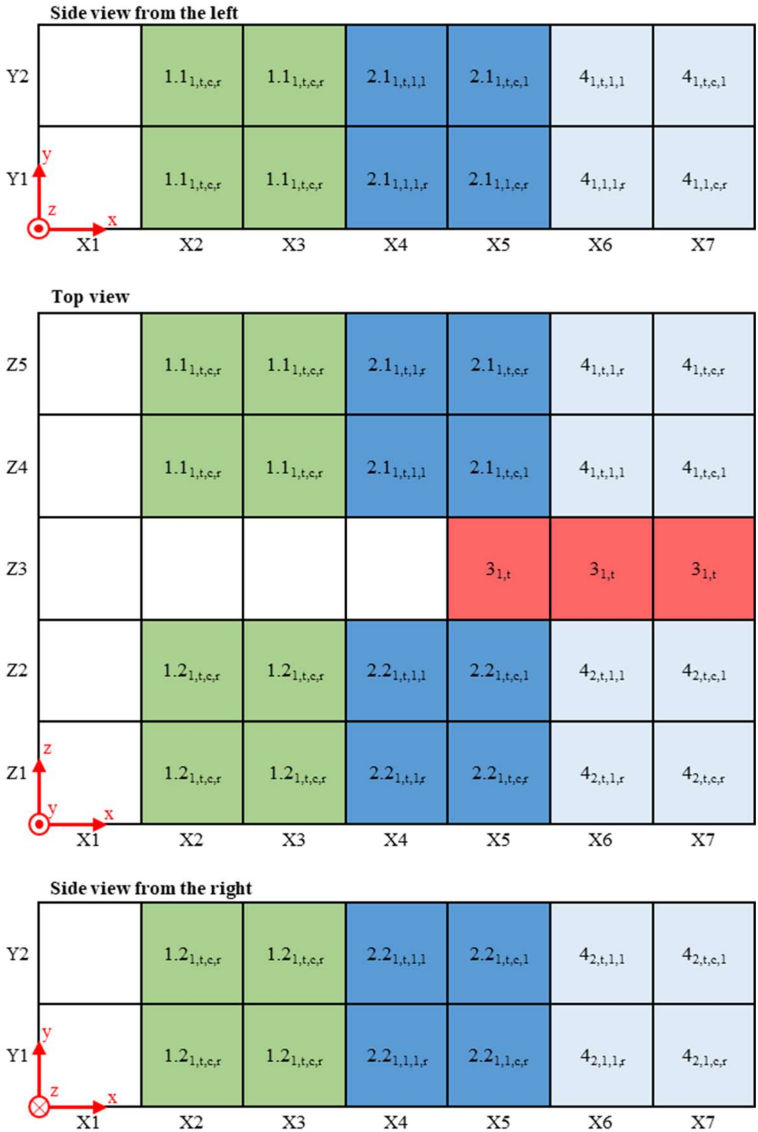


Figure 2.28: Description of the layout of the considered SBS/RS

The following notation (see Table 2.1) is used to describe the SBS/RS. If other systems are considered, additional parameters can be added. In more complex systems the number of indices may not be sufficient. In this case additional indices should be added. In order to describe each block uniquely, an ID is added. For example, when several inbound lifts are installed in a SBS/RS, then each lift has a unique ID and position. These must be clearly distinguished from each other because each lift can have a different dimension and control strategy.

Table 2.1: Notation used for the block layout diagram

Notation	Description
$1.1_{ID,tier,channel,row}$	Tote lift - inbound lift
$1.2_{ID,tier,channel,row}$	Tote lift - outbound lift
$2.1_{ID,tier,channel,row}$	Inbound buffer
$2.2_{ID,tier,channel,row}$	Outbound buffer
$3_{ID,tier}$	Rail for shuttle vehicle
$4_{ID,tier,channel,row}$	Rack system - storage location

The next step is to assign the geometrical dimensions to the individual blocks (see Table 2.2). If the blocks of one entity (e.g., lift, rack, buffer) have the same dimensions, it is sufficient to specify just one block. If the blocks have different dimensions, e.g., the tiers have different heights or the storage channels have different widths, the individual dimension must be specified for each individual block.

Since in this example all blocks of an entity have the same dimensions, the dimensions are always given for only one block of each entity. The following table shows the dimensions of the individual blocks.

Table 2.2: Geometrical dimension of the individual blocks

Block ID	Description	Dimension in x-direction [m]	Dimension in y-direction [m]	Dimension in z-direction [m]
$1.1_{1,tier,channel,row}$ $\forall\, tier \in n_t$, $\forall\, channel \in n_c$, $\forall\, row \in n_r$	Tote lift – inbound lift	d_{lx}	d_{sy}*	d_{lz}
$1.2_{1,tier,channel,row}$ $\forall\, tier \in n_t$, $\forall\, channel \in n_c$, $\forall\, row \in n_r$	Tote lift - outbound lift	d_{lx}	d_{sy}*	d_{lz}
$2.1_{1,tier,channel,row}$ $\forall\, tier \in n_t$, $\forall\, channel \in n_c$, $\forall\, row \in n_r$	Inbound buffer	d_{bx}	d_{sy}	d_{bz}
$2.2_{1,tier,channel,row}$ $\forall\, tier \in n_t$, $\forall\, channel \in n_c$, $\forall\, row \in n_r$	Outbound buffer	d_{bx}	d_{sy}	d_{bz}
$3_{1,tier,}$ $\forall\, tier \in n_t$	Rail for shuttle vehicle	d_{ax}	d_{ay}**	d_{az}
$4_{1,tier,channel,row}$ $\forall\, tier \in n_t$, $\forall\, channel \in n_c$, $\forall\, row \in n_r$	Storage rack system - storage location	d_{sx}	d_{sy}	d_{sz}

*The total height of the lift d_{ly} is the sum of the individual heights of each tier
**If the shuttle vehicle can reach several storage levels, the height is a multiple of the height of a storage location

For the (un)loading point between the storage system and the pre-storage zone, the exact positions must be specified (see Table 2.3). If a lift has several (un)loading points, these must all be specified individually by adding additional rows.

Table 2.3: Position of the (un)loading points of the lifts

Lift ID	Loading point with the pre-storage zone – incoming ULs for storage (Yes / No)	Unloading point with pre-storage zone – outgoing ULs for retrieval (Yes / No)	Position in y-direction [m]
1.1_1	Yes	No	$d_{l_{in}}$
1.2_1	No	Yes	$d_{l_{out}}$

- With the help of the block layout diagram and the tables with the geometrical dimensions, the layout and geometric dimensions of each storage system can be mapped generally.
- The arrangement of the individual blocks in a coordinate system clearly defines their orientation and position in relation to one another.
- For very complex layouts, the position of each group of four rectangle blocks can be precisely described in relation to the origin. For this purpose, only the coordinate of the lower left corner must be specified.
- The travel distance of the lift and shuttle vehicle can be determined by the sum of the individual length, width and height of the individual blocks.

2.3.3 Material Flow Description

After the design of the storage system has been described in great detail, the next step is to describe the interaction of the blocks. For this purpose, the block layout diagram depicted in Figure 2.28 is used as a basis and extended to include the material flow. Both, the material flow within the blocks of an entity and the material flow between the blocks of different entities must be described. By linking the individual blocks, the interaction of the individual blocks and thus the material flow can be specified (see Figure 2.29).

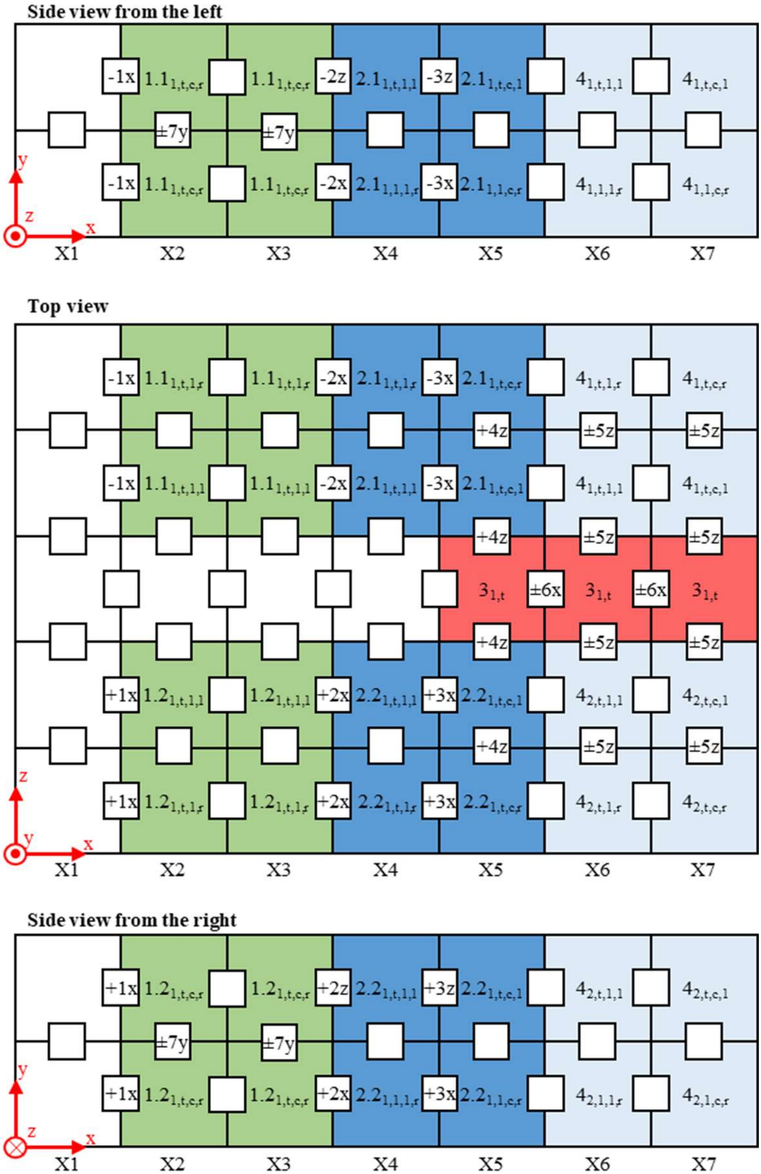


Figure 2.29: Description of the interaction between the individual blocks by using linking blocks

Each linking block is composed of three digits. The first part is +, - or ± and indicates the direction of movement. + is in the same direction as the arrows of the coordinate system, - is in the opposite direction and ± means, that movement is possible in both directions. The second part is a number. This is the unique ID of each linking block. The last digit is x, y, or z and indicates the axis of movement.

Additionally, the linking blocks are differentiated according to the type of movement. The following three types of movement are possible:

(i) Movement of lift and shuttle vehicle: The movement axis is specified on which the lift and shuttle vehicle move.
(ii) Load handling and (un)loading operations: The movement axis is specified on which the UL is transferred between different entities.
(iii) Movement of UL: The movement axis is specified on which the ULs move within an entity.

The following table lists the description of the used linking blocks:

Table 2.4: Description of the linking blocks

Notation	Description	Category
empty interaction blocks	No material flow between these blocks	Load handling axis of UL
1x	(Un)loading between upstream/downstream process and lift	Load handling axis of UL
2x	(Un)loading between lift and buffer	Load handling axis of UL
3x	Movement of ULs from one buffer location to the next buffer location (e.g., roller conveyor)	Movement axis of UL
4z	(Un)loading between buffer and shuttle vehicle	Load handling axis of UL
5z	(Un)loading between shuttle vehicle and storage location	Load handling axis of UL
6x	Movement of the shuttle vehicle	Movement axis of shuttle vehicle
7y	Movement of the lift	Movement axis of lift

With the help of the IDs of the individual blocks and the notation of the linking blocks, the entire material flow and the control strategies can be mapped. Figure 2.30 uses the example of the relocation process to describe how this can be mapped using the new concept with blocks and entities. Both entities, shuttle vehicle and storage locations, are uniquely described and known via the ID. The transfer of the ULs can be specified via the linking blocks (±5z) and the travel between the individual storage locations via the linking block (±6x). Since the shuttle vehicle is assigned to one tier and aisle, the travel route is known. In this example, it is the "rail for shuttle vehicle" 3_1. The same procedure can be applied to all material flow diagrams and control strategies described in chapter 2.2.

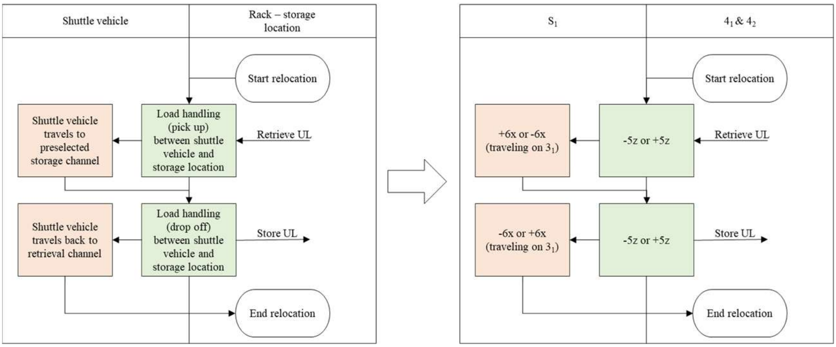


Figure 2.30: Material flow of the relocation process using IDs of the entities

- The interaction of the individual blocks and the material flow can be clearly mapped by linking the individual blocks. Therefore, the large blocks are linked together using small linking blocks. By assigning unique IDs to each entity, they can be easily linked and distinguished.
- For diagonal movements, small blocks can also be placed at the respective corners.

2.3.4 Component Description

In addition to the layout of the storage system, the individual components such as the lift system and shuttle vehicle can also be described in detail using the block method. As an example, Figure 2.31 depicts three different shuttle vehicle configurations. For the representation of the layout and the material flow the same procedure as above is chosen. The following properties apply to the description of the lift system and shuttle vehicle:

- In addition to the top view, a side view is also required.
- Each block has the capacity of one.
- Small lift and shuttle vehicle configurations can be mapped using one or two blocks. If larger configurations are considered, the rectangle blocks must be specified.
- For the description of the blocks the following notation is used here: $S_{ID,x,y,z}$, where S stands for shuttle vehicle (or L for lift), ID for a unique identification, x, y and z for the dimensions in each axis direction.
- If relevant, further properties like length, width and height of the blocks can be specified.
- Direction of movement of lift and shuttle vehicle can be taken from Figure 2.29.
- For the interactions between the block, additional notations need to be added (e.g., ±8z, which is the movement of ULs from one side of the shuttle vehicle to the other side of the shuttle vehicle).

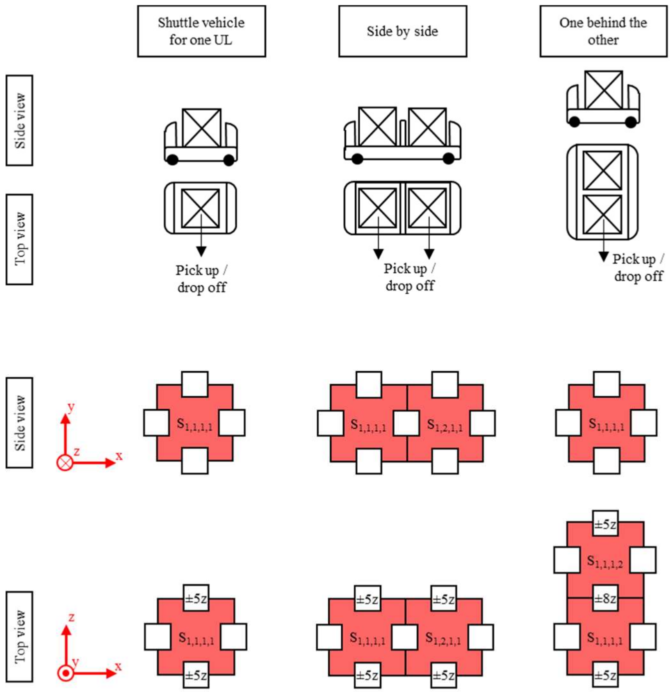


Figure 2.31: Description of different shuttle vehicle configurations

- With the help of the block layout diagram and tables (with the geometrical dimensions), the design, geometric dimensions and the material flow of each lift and shuttle vehicle can be described generally.
- The arrangement of the individual blocks in a coordinate system clearly defines their orientation and position in relation to one another.

2.3.5 Attribute Description

To provide a more comprehensive description, individual entities can be assigned properties, also known as attributes. These attributes can be specified using an extended UML class diagram as shown in Figure 2.32. To improve clarity, the attributes are organized into different categories. The block representing an entity is a rectangle with five rows. It includes all the attributes that apply to the entity. The top row contains the name of the entity (class). The second row contains the attributes (properties) of the entity. If the entity is a dynamic object (e.g., lift, shuttle vehicle, conveyor system), then the third row contains the movement axes. The fourth row contains the control strategy of this entity. For example, for a dynamic object, the aisles and tiers on which the vehicle travels must be assigned (fifth row).

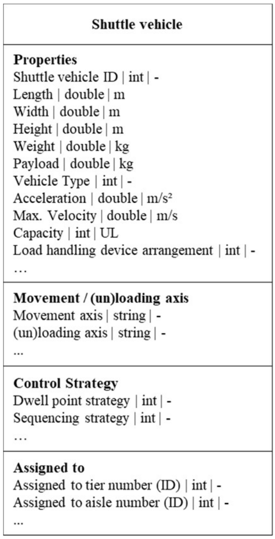


Figure 2.32: Attributes of the entity shuttle vehicle

When several shuttle vehicles are used in a SBS/RS and each vehicle has different properties, an individual attribute list can be created for each vehicle. If all vehicles have the same attributes, one list is sufficient. Depending on the shuttle vehicle more attributes can be added.

- With the help of the attribute list, each entity can be described uniquely and all properties can be specified.

As described in chapter 2.2, further data must be specified and assigned to the individual entities. For example, load handling times need to be specified. The data for the load handling times can be linked directly to the linking blocks and are thus clearly assigned. Other data that must be specified includes the article structure, filling degree, and KPIs.

2.4 Chapter Conclusion

In conclusion, a method with eight steps was developed to describe every SBS/RS configuration in more detail. When following all eight steps, this results in a clear description of the SBS/RS with all its complexity. The developed method can be applied to any other storage system as well. Additionally, the method can be extended as needed, for instance, to include other components or control strategies.

Based on this, a generally applicable method for describing SBS/RSs was presented. Blocks can be used to represent the SBS/RS's layout as well as the shuttle vehicle and lift design. Linking blocks can be used to map the connections between individual blocks and describe their interactions. Through the use of notations and IDs, each entity and process can be uniquely described and mapped. Each block can be defined by its unique ID, allowing for the assignment of individual attributes to each block. With this method, the functionality and geometric dimensions of individual entities can be described. This data is essential for calculating travel and cycle times, developing simulation models, or providing a comprehensive and universally valid mapping of a SBS/RS.

3 Cycle Time for a SBS/RS

SBS/RSs have become an alternative to AS/RSs due to their advantages of higher and scalable performance and better space utilization. However, they have a higher complexity due to the separation of the horizontal and vertical transportation process. This is due to the two independently operating systems, the lift and the shuttle vehicle.

There is a direct correlation between technical parameters and system performance. Higher velocity leads to higher throughput. However, this is only valid if all the individual subsystems (lifts and shuttle vehicles) are precisely matched to each other. Whether the lift, that often represents the bottleneck, achieves the required performance, depends among other things on the performance of the shuttle vehicles in the individual tiers. Therefore, for the performance evaluation of a SBS/RS, the knowledge of the travel times and cycle times is of relevance. These values can then be used to determine the expected total throughput.

The performance of an automated storage system can be determined either by measurements on the real system, by simulation or through analytical models:

- **Simulation**: In order to determine key performance indicators, parametric simulation models are developed that represent the SBS/RS as realistic as possible. With a parameter variation, different system configurations can be simulated and compared. In addition to the cycle time, throughput, energy consumption, waiting times, etc. can also be output here. However, the creation of simulation models is very time-intensive.
- **Analytical or mathematical models**: Mathematical models are used to determine the performance of various SBS/RS configurations. These are mostly approximate equations for calculating the expected values, with a number of made assumptions and limitations. Nevertheless, it is possible to obtain only minimally deviating

results compared to the simulation. The calculation of waiting times and buffer sizes is also possible with queuing models. Once the models are developed, they can be used for fast performance calculation of SBS/RSs. However, for each new configuration or different used control strategy, a new model must be developed.

Especially in the design phase of new storage systems, it is important to obtain reliable results quickly and easily. Although simulation models provide a very detailed description of the real system and can provide more realistic results, it is very time consuming to create these simulation models. Analytical models have the advantage that expected values can be calculated very quickly and different use cases can be compared.

3.1 Fundamentals and Literature Review

There already exist many publications on SBS/RSs. Most of these publications deal with the performance determination of SBS/RSs (e.g., travel time-, cycle time-, throughput-, utilization-, or energy demand calculation) using analytical-mathematical or simulation models. Mostly, different storage configurations and different control strategies are compared and subsequently evaluated.

There are several review papers available, which give an overview and discuss different storage systems configurations, but do not describe any cycle time models in detail (e.g., Azadeh et al. (2019), Roodbergen and Vis (2009), van den Berg (1999), Vasili et al. (2012), Kalyanaraman and Keerthika (2016), Kosanić et al. (2018)).

3.1.1 Standards and Guidelines for SBS/RS

The standard VDI 2692 (2015) explains the SBS/RS technology and simplified cycle time models for the standard configuration. Based on VDI 2692, the guideline FEM 9.860 (2017) was developed, which additionally considers cycle time equations for multi-load handling devices and double-deep storage systems. The approach in the guideline has different shortcomings: Multi-load handling devices are just considered for the lift. For double-deep storage, the relocation distance and the relocation probability is simplified. The rearrangement distance is set to one storage channel for filling degrees between 50 and 80 %, to two for 80 - 90 %, and to three for 90 - 95 %.

3.1.2 Literature on Different Models for SBS/RSs

Malmborg (2002) is the first to develop analytical models for SBS/RSs. He presents a system utilization model for a tier-to-tier SBS/RS. Ekren and Heragu (2012) and Ekren et al. (2013) use a semi open queuing network (SOQN) to model a tier-to-tier SBS/RS. The advantage of queuing models is, that waiting times and queue length between lift and shuttle vehicles can be considered. A good literature review on queuing network models for SBS/RSs can be found in Epp (2018). Epp (2018) uses an open queueing network and a decomposition approach to get probability distributions for different performance values for a SBS/RS with multi-level shuttle vehicles. Roy et al. (2015) define the optimal dwell point location and the optimal location of the cross aisle, using a SOQN. Lerher, Ekren, Dukic, et al. (2015) and Lerher (2016a) present analytical models for different performance values for tier-captive SBS/RS.

Simulations models are a good approach to compare different SBS/RS configurations and control strategies and get answers on cycle time, throughput, waiting times, and other KPIs (e.g., Marchet et al. (2013), Lerher, Ekren and Sari (2015), Lerher et al. (2017)).

There are also several publications which deal with the optimization of SBS/RSs, to increase the throughput or reduce blocking effects. Roy et al. (2016) use simulation models to evaluate blocking effects of several shuttle vehicles within one tier. Schloz et al. (2019) develop a deep reinforcement learning approach to sequence two shuttle vehicles for retrieval tasks within a tier of a SBS/RS with a cross aisle, that does not allow blocking effects. For more literature about optimization of SBS/RSs see the literature review in chapter 4.1.

Table 3.1: Selection on existing literature on SBS/RSs

Publication	System*	Model**
Ekren and Heragu (2012)	SBS/RS – 1 – 1 – 1, CA, TT	SOQN, simulation
Ekren et al. (2013)	SBS/RS – 1 – 1 – 1, CA, TT	SOQN, simulation
Epp (2018)	SBS/RS – 1 – 1 – 1, TT, TC, MLS	OQN, DA
Lerher, Ekren, Dukic, et al. (2015)	SBS/RS – 1 – 1 – 1, TC	Analytical cycle time model
Lerher, Ekren and Sari (2015)	SBS/RS – 1 – 1 – 1, TC	Simulation
Lerher (2016a)	SBS/RS – 1 – 1 – 1, TC	Analytical cycle time model
Lerher et al. (2017)	SBS/RS – 1 – 1 – 1, TC	Simulation
Malmborg (2002)	SBS/RS – 1 – 1 – 1, CA, TT	Analytical cycle time model
Marchet et al. (2013)	SBS/RS – 1 – 1 – 1, TC	Simulation
Roy et al. (2015)	SBS/RS – 1 – 1 – 1, CA, TT	SOQN
Roy et al. (2016)	SBS/RS – 1 – 1 – 1, CA, TC	Simulation
Schloz et al. (2019)	SBS/RS – 1 – 1 – 1, CA, TC	Optimization, DRL
This work	**SBS/RS – 2 – 2 – 2, TC**	**Analytical cycle time model, optimization, simulation**

*storage system – storage cell depth – number of load handling devices – size of ULs, cross aisle (CA), tier-to-tier (TT), tier-captive (TC), multi-level shuttle (MLS)

**semi open queueing network (SOQN), open queueing network (OQN), decomposition approach (DA), deep reinforcement learning (DRL)

This brief overview demonstrates the breadth of research on SBS/RSs. In the following subchapters, the state of research on multi-deep storage system, multi-load handling devices and various UL sizes will be discussed.

3.1.3 Fundamentals on Multi-Load Handling Devices

Meanwhile, there are many manufacturers in the market that offer multi-load handling devices for stacker cranes (AS/RSs) as well as shuttle systems (SBS/RSs).

With multi-load handling devices or multiple transport positions per vehicle (capacity of the vehicle > 1), the simultaneous transport of several ULs is possible. It allows to perform larger command cycles and to process batches. This results into reduced cycle time and higher throughput.

Many publications consider the sequencing of storage and retrieval requests. While the sequence of processing the storage and retrieval requests is clearly defined for single-load and dual-load handling devices, more complex sequencing strategies are required for larger load handling devices.

The following two questions are usually answered:

- Storage assignment: Which UL should be stored in which empty storage location?
- Storage and retrieval scheduling: In which sequence should the storage and retrieval requests be processed?

For this purpose, optimization models are often developed with the aim of minimizing the maximum travel distance or travel time. An overview of publications dealing with scheduling and sequencing of multi-load handling devices are listed in Table 3.2. Chapter 4.1 describes the individual publications in more detail.

Most publications on travel time and cycle time models for storage systems consider single-load handling devices with a capacity of one. In previous research, the development of travel time and cycle time models for multi-load handling devices hardly find any attention.

3.1.4 Literature on Multi-Load Handling Devices

Sarker et al. (1991) are the first ones who calculate the travel time for a side-by-side dual-load handling device. Keserla and Peters (1994) present in their paper an above-each-other load handling device. Both load handling devices operate independently, with individual ULs being stored and retrieved. Taking Bozer and White (1984) as a basis, they develop a simplified model for calculating the cycle time for a dual-load handling device performing a quadruple cycle. Azzi et al. (2011) extend Bozer and White (1984) and develop new travel time models for a dual-load handling device that follows the FEM 9.851 standard (FEM 9.851 2003). Meller and Mungwattana (1997) integrate two analytical models, a FCFS model and a nearest neighbor model, to get the travel time for a side-by-side load handling device. Schenone et al. (2019) adapt this equation by replacing the constant factor and multiplying the travel time by a coefficient, which is dependent on the rack configuration and the input and output points. Lerher et al. (2011) show a throughput improvement for a triple-load handling system compared to a dual-load handling system. Therefore, they develop an analytical model using the nearest neighbor strategy and determine the performance of multi-load handling systems using simulation. Potrč et al. (2004) develop a cycle time model for random storage assignment and present a heuristic that minimizes the travel distance and travel time.

In his work Seemüller (2006) develops several cycle time models for double-deep storage system and multi-load handling devices. The author considers the simultaneous storage of several ULs next to each other or behind each other. However, Seemüller uses a simplified approximation method for the relocation travel distance and time. The same applies for the travel distance between two potential storage locations when applying the nearest neighbor strategy. Dörr (2018), Dörr and Furmans (2016a), and Dörr and Furmans (2016b) develop cycle time models for a double-deep storage system with a side-by-side dual-load handling device. They use the equation from Lippolt (2003) for double-deep storage systems and extend these equations for the dual-load handling device and different control strategies. Finally, recommendations are derived using the results of the equations and simulations. Xu et al. (2015) present cycle time models for a quadruple-command cycle for a FCFS and a nearest neighbor control strategy. For each cycle they define nine different cases in which a cycle is performed and define the occurrence probability of each case to calculate the average cycle time.

Table 3.2: Existing literature on multi-load handling devices

Publication	System*	Model	Type of load handling device**
Cunkas and Ozer (2019)	AS/RS – 1 – 2 – 1	Scheduling, sequencing	Side-by-side
Dooly and Lee (2008)	AS/RS – 1 – 2 – 1	Scheduling, sequencing	Side-by-side
Kazemi et al. (2019)	AS/RS – 1 – n – 1	Scheduling, sequencing	Side-by-side
Kazemi et al. (2021)	AS/RS – 1 – n – 1	Scheduling, sequencing	Side-by-side
Peng and Yang (2015)	AS/RS – 1 – n – 1	Scheduling, sequencing	Side-by-side
Popović et al. (2014)	AS/RS – 1 – 3 – 1	Scheduling, sequencing	Side-by-side
Sarker et al. (1994)	AS/RS – 1 – 2 – 1	Scheduling, sequencing	Side-by-side
Shunji Tanaka (2007)	AS/RS – 1 – n – 1	Scheduling, sequencing	Side-by-side
Wauters et al. (2016)	AS/RS – 1 – 2 – 1	Scheduling, sequencing	Side-by-side
Yang, Miao, Xue and Qin (2015)	AS/RS – 1 – n – 1	Scheduling, sequencing	Side-by-side
Yang, Miao, Xue and Ye (2015)	AS/RS – 1 – n – 1	Scheduling, sequencing	Side-by-side
Yang et al. (2017)	AS/RS – 1 – n – 1	Scheduling, sequencing	Side-by-side
Azzi et al. (2011)	AS/RS – 1 – 2 – 1	Travel time model	Side-by-side
Dörr (2018)	AS/RS – 2 – 2 – 1	Cycle time model	Side-by-side
Dörr and Furmans (2016a)	AS/RS – 2 – 2 – 1	Cycle time model	Side-by-side
Dörr and Furmans (2016b)	AS/RS – 2 – 2 – 1	Cycle time model	Side-by-side
Keserla and Peters (1994)	AS/RS – 1 – 2 – 1	Cycle time model, scheduling, sequencing	Above-each-other
Lerher et al. (2011)	AS/RS – 1 – n – 1	Travel time model, simulation	Side-by-side
Meller and Mungwattana (1997)	AS/RS – 1 – n – 1	Travel time model	Side-by-side
Potrč et al. (2004)	AS/RS – 1 – n – 1	Cycle time, simulation	Above-each-other, one-behind-the-other
Sarker et al. (1991)	AS/RS – 1 – 2 – 1	Travel time, scheduling, sequencing	Side-by-side
Schenone et al. (2019)	AS/RS – 1 – 2 – 1	Travel time model	Side-by-side
Seemüller (2006)	AS/RS – 2 – n – 1	Cycle time model	Side-by-side, one-behind-the-other
Xu et al. (2015)	AS/RS – 2 – 2 – 1	Cycle time model	Side-by-side
This work	**SBS/RS – 2 – 2 – 2**	**Cycle time, scheduling, sequencing, simulation**	**One-behind-the-other**

*storage system – storage cell depth – number of load handling devices – size of ULs

**for the different shuttle vehicle configurations see Figure 2.9

3.1.5 Fundamentals on Multi-Deep Storage Systems

The advantage of **multi-deep storage systems** is, that the storage volume can be used more efficiently and thus a higher space utilization is achieved. In addition, multi-deep storage systems reduce the number of vehicles required, as there are fewer aisles for the same number of storage locations, compared to single-deep storage systems. In the case of multi-deep storage systems, however, it is not possible to have immediate access to every storage location. If the rearmost UL needs to be retrieved and is blocked by one or more ULs that are stored in front, these ULs must be relocated first to access the rearmost UL. There are already a number of publications in the literature that deal with multi-deep storage systems.

Flow racks are also multi-deep storage racks. ULs are loaded on the "storage side" and travel within the storage channel (mostly on rollers) to the other end of the rack to the "retrieval side". The storage and retrieval process is automated using a stacker crane.

Deep-lane configurations exist both in AS/RSs and SBS/RSs. The difference between both systems is, that for the transportation in longitudinal or transverse aisle direction in an AS/RS the stacker crane is used and in a SBS/RS the shuttle vehicle is used.

3.1.6 Literature on Multi-Deep Storage Systems

Ghomri and Sari (2015) develop a mathematical model to calculate the average travel time for the retrieval process under random storage. Sari et al. (2005) develop closed form travel time equations. Those can be used to evaluate the throughput of flow racks for different design configurations.

The guideline VDI 4480, Part 4 (2002) presents a method of how to calculate the throughput of multi-deep AS/RS and deep-lane AS/RS. The shortcoming of this guideline is, that it does not present equations for the cycle time as well as does not consider relocation processes.

Lippolt (2003) is the first one who develops cycle time models for double-deep AS/RSs with an exact determination of the relocation probabilities and relocation channel distance. Dörr and Furmans (2016a), Dörr and Furmans (2016b), and Dörr (2018) use the equation from Lippolt (2003) for double-deep AS/RSs and extend these equations for a dual-load handling device and different control strategies. Lehmann and Hußmann (2021) and Lehmann and Hußmann (2022) extend Lippolt (2003) for multi-deep AS/RS. They develop cycle time models for multi-deep AS/RS, which can be used to determine the average travel time for single-command and dual-command cycles, the relocation probability, and number of expected relocations for different storage allocation strategies. Lehmann and Knötgen (2020) present an analytical model for the determination of the optimal filling degree, depending on the cycle time and space utilization for a double-deep AS/RS. In his work, Seemüller (2006) develops several cycle time models for double-deep storage systems and multi-load handling devices. He considers the simultaneous storage of several ULs next to each other or behind each other, using a simplified approximation method for the relocation travel distance and relocation time. In the developed cycle time model of Lerher et al. (2010) the storage location assignment is simplified. First all storage locations in the back row are filled and then the storage rows in the front. Therefore, there is no relocation process required for filling degrees between 0 % and 50 %. Lerher (2016b) applies the same principle also to a SBS/RS.

Xu et al. (2015) present cycle time models for a quadruple-command cycle for a FCFS and a nearest neighbor control strategy. They also make the assumption that all storage locations in the back row are occupied before storage locations in the front row get filled. In a second model they assume, if an empty storage channel is available, both ULs can be stored in the same storage channel. As an input for their model they need the exact number of half-filled storage channels. The paper, though, lacks on equations of how to get this value. Also, the required duration for a relocation process and required relocation channel distance is not specified in this work. Xu et al. (2016) consider three different relocation strategies (relocation to the nearest neighbor, a random location, a deterministic point) and assume that relocation is only required for filling degrees larger than 50 %. Xu et al. (2019)

develop a cycle time model for a class based storage policy. They only consider single-command cycles and ignore load handling times. For relocation, they assume that all blocking ULs are relocated to a buffer and after finishing the retrieval process, the ULs are stored back in the origin storage channel.

Guerrazzi et al. (2019) calculate the travel time and energy consumption for the storage and retrieval process in a deep-lane SBS/RS configuration but do not consider possible relocation processes. Eder and Kartnig (2016) present a mathematical model to calculate the cycle time of a multi-deep SBS/RS and use simulation for the performance evaluation. In several publications Eder (2022), Eder (2020a), and Eder (2020b) applies an open queuing model (Markov queue with limited capacity) to discuss the interaction between shuttle vehicles and lifts. D'Antonio et al. (2018), and D'Antonio and Chiabert (2019) consider a SBS/RS deep-lane system with four different classes of ULs. Each storage channel can only be filled with the same class of ULs. As a consequence, no relocation processes are required. Manzini et al. (2016) adapt the same principle, that each storage channel contains ULs of the same class. For the same system Bruno and D'Antonio (2018) consider the "last in first out" (LIFO) strategy, that does not require relocation operations. Fan et al. (2015) use simulations to evaluate deep-lane SBS/RSs. Marolt et al. (2022) focus on analyzing the efficiency of the satellite vehicle carrier for nine different storage and relocation strategies by using simulation. Since the focus was on the satellite vehicle carrier, they do not consider the lift operations in their simulation model.

Also there is some research on optimization of double-deep storage systems. The focus lies on the development of optimization models and not of cycle time models. The detailed description of these papers (Wang et al. (2019) and Zhan et al. (2020)) is given in chapter 4.1.

Table 3.3: Existing literature on multi-deep storage systems

Publication	System*	Model
Ghomri and Sari (2015)	Flow rack, AS/RS	Cycle time model
Sari et al. (2005)	Flow rack, AS/RS	Cycle time model
Dörr (2018)	AS/RS – 2 – 2 – 1	Cycle time model
Dörr and Furmans (2016a)	AS/RS – 2 – 2 – 1	Cycle time model
Dörr and Furmans (2016b)	AS/RS – 2 – 2 – 1	Cycle time model
Lehmann and Hußmann (2021)	AS/RS – n – 1 – 1, SV	Cycle time model
Lehmann and Hußmann (2022)	AS/RS – n – 1 – 1, SV	Cycle time model
Lehmann and Knötgen (2020)	AS/RS – 2 – 1 – 1	Cycle time model
Lippolt (2003)	AS/RS – 2 – 1 – 1	Cycle time model
Lerher et al. (2010)	AS/RS – 2 – 1 – 1	Cycle time model
Seemüller (2006)	AS/RS – 2 – n – 1	Cycle time model
Xu et al. (2015)	AS/RS – 2 – 2 – 1	Cycle time model
Xu et al. (2016)	AS/RS – 2 – 1 – 1, CA, SV	Cycle time model
Xu et al. (2019)	AS/RS – n – 1 – 1, CA, SV	Cycle time model
Bruno and D'Antonio (2018)	SBS/RS – n – 1 – 1, CA, TT, SV	Cycle time model, simulation
D'Antonio and Chiabert (2019)	SBS/RS – n – 1 – 1, CA, TT, SV	Cycle time model
D'Antonio et al. (2018)	SBS/RS – n – 1 – 1, CA, TT, SV	Cycle time model
Fan et al. (2015)	SBS/RS – n – 1 – 1, CA, TC, SV	Simulation
Guerrazzi et al. (2019)	SBS/RS – n – 1 – 1, CA, TC, SV	Travel time model, energy model
Eder (2022)	SBS/RS – n – 1 – 1, TC	Open queueing system
Eder (2020a)	SBS/RS – n – 1 – 1, TC	Open queueing system
Eder (2020b)	SBS/RS – n – 1 – 1, TC, MLS	Open queueing system
Eder and Kartnig (2016)	SBS/RS – n – 1 – 1, TC	Cycle time model, simulation
Lerher (2016b)	SBS/RS – 2 – 1 – 1, TC	Cycle time model
Manzini et al. (2016)	SBS/RS – n – 1 – 1, CA, TC, SV	Cycle time model
Marolt et al. (2022)	SBS/RS – n – 1 – 1, TC, SV	Cycle time model
This work	**SBS/RS – 2 – 2 – 2, TC**	**Cycle time, scheduling, sequencing, simulation**

*storage system – storage cell depth – number of load handling devices – size of ULs, cross aisle (CA), tier-to-tier (TT), tier-captive (TC), satellite vehicle (SV), multi-level shuttle (MLS)

3.1.7 Literature on Different Sizes of ULs

Warehouses can store ULs of different sizes. Load handling devices are designed in such a way that ULs with different footprint and height can be picked up and dropped off with one and the same load handling device. In the guideline VDI 4480, Part 4 (2002) ULs with the same footprint but different heights are considered. This makes it possible to have different heights for the individual storage tiers and thus achieve a higher space utilization. Cardona and Gue (2019) show that the potential space saving of using multiple tier heights in the same storage system is between 29 % and 45 %. Further publications that deal with the same topic are Cardona and Gue (2020) and Lee et al. (2005). In all these publications only the optimized rack design is defined and no cycle time models are developed.

A different field of research is the shelf space allocation that considers a large number of different products with different dimensions. Especially in supermarkets, this is the process of determining the exact storage location and the required space for each product in the rack, based on the total number of each product to be stored.

Table 3.4: Existing literature on storage systems with different sizes of ULs

Publication	System*	Model
Cardona and Gue (2019)	Storage Rack – 1 – 1 – n	Optimization model
Cardona and Gue (2020)	Storage Rack – 1 – 1 – n	Design method, simulation
Lee et al. (2005)	Storage Rack – 1 – 1 – n	Optimization model
This work	**SBS/RS – 2 – 2 – 2, TC**	**Cycle time, scheduling, sequencing, simulation**

*storage system – storage cell depth – number of load handling devices - size of ULs, tier-captive (TC)

3.1.8 Summary on the Literature Review

The literature review shows that there already exists a large number of publications on SBS/RSs. However, no one has yet considered the combination of a double-deep SBS/RS, lift and shuttle vehicle with a dual-load handling device and two differently sized ULs. With more systems coming on the market that store ULs of different sizes, there is a need for analytical models to evaluate the performance of such systems.

The literature study also reveals that for the calculation of the travel times currently mainly the following two methods are used:

(i) Summing up the individual travel times using the case distinction from appendix B (whether the maximum velocity v_{max} is reached or not) and deriving the expected value from it by dividing through the number of all possible cycles. (e.g., VDI 2692 (2015), FEM 9.860 (2017), Eder (2020a))

(ii) No consideration of the case distinction and use of the two approximation equations (3.1) and (3.2). (e.g., Eder and Kartnig (2016), Arnold and Furmans (2019))

In this work, analytical models are developed under the assumption that the maximum velocity v_{max} can always be reached. In addition, it is taken into account that in some cases no travel operation is required because the ULs are directly transferred (e.g., from the pre-storage zone to the first tier that are on the same level, using the lift). The travel time models also consider the option, that two ULs are transferred together which only requires a single-command cycle instead of a dual-command cycle. Using probabilities, the different cases are weighted and therefore an approximation equation for the travel and cycle time can be derived. This approach is superior to the above described methods (i) and (ii). It has the advantage of faster and simplified calculation while improving accuracy. This approach has the advantage over case (i), that the calculation is faster and simpler, but not as accurate. Compared to case (ii) this approach achieves more accurate results.

The aim of this work is to provide a set of different cycle time equations for the shuttle vehicle and the lift, covering a very large number of different configurations, which can be used for the design of new SBS/RSs. In the next chapters the required travel and cycle time equations are derived for a double-deep SBS/RS, with a dual-load handling device and two different sizes of ULs. For some cases also multi-load handling devices are considered. The respective travel times, cycle times and throughput models are derived separately for lift and shuttle vehicle. For the sake of completeness, the equations are first derived for simple standard storage configurations and then these equations are used as a basis for more complex configurations. Additionally, for the lift, four different cases with different positions of the transfer position to the pre-storage zone are considered. For both the lift and the shuttle vehicle, different control strategies are considered and then compared with each other. At the end of this chapter, throughput models for the entire SBS/RS are derived. Although waiting times are not taken into account in this chapter, the throughput models for the lift and shuttle vehicle can still be used to determine the overall throughput of the SBS/RS.

3.2 Validation of the Developed Analytical Equations

To ensure the correctness of the developed analytical equations the results are compared with the results of the simulation model. Based on the simulation runs performed and the results obtained, it can be assumed with a high degree of confidence that the equations are error-free and correct.

For the validation, the simulation model in AnyLogic described in chapter 5 is used. Further details about the chosen warm-up phase and number of replications can be found in appendix F.

3.3 Cycle Time Calculation for the Lift

In general, each aisle in a SBS/RS is equipped with two independent lift systems: one for storage operations and one for retrieval operations. For a detailed description of the inbound lift process, refer to Figure 2.1, and for the outbound lift process, refer to appendix A, Figure A.5.

Various cycles can be performed based on the lift capacity, as shown in Figure 2.19. The following equations are derived to calculate the travel and cycle time of the lift, depending on the lift capacity, the ratio of large to small ULs, and the I/O point's positions from the lift to the pre-storage zone.

For the analytical models it is assumed that the maximum velocity is always reached. This leads to a minor deviation to the actual value for short travel distances. For more details, see appendix B.

The average travel time, including the time component for acceleration and deceleration, from the starting position to any storage position is:

$$E(t) = \frac{1}{2}\frac{L}{v} + \frac{v}{a} \tag{3.1}$$

with L (length of on tier or height of the storage rack), v (maximum velocity) and a (acceleration / deceleration).

For the average travel time between any two random storage locations i and j the following equation can be used:

$$E(t) = \frac{1}{3}\frac{L}{v} + \frac{v}{a} \tag{3.2}$$

For more details on the derivation and description of the two travel time models, refer to Arnold and Furmans (2019).

3.3.1 Single-Command Cycle

The average travel time for the lift is determined using equation (3.1) with H_{lift} (height of the storage rack), v_{lift} (maximum velocity of the lift) and a_{lift} (maxium acceleration / deceleration of the lift). To calculate the travel time for a single-command cycle, both, the travel time to the tier and the return travel time to the I/O point, must be considered. Therefore, the entire equation (3.1) must be multiplied by two.

$$E(t_{SC\_Lift}) = 2 \cdot \left(\frac{1}{2}\frac{H_{lift}}{v_{lift}} + \frac{v_{lift}}{a_{lift}}\right) = \frac{H_{lift}}{v_{lift}} + 2\frac{v_{lift}}{a_{lift}} \tag{3.3}$$

This new equation provides the average travel time for a single-command cycle for the lift. To determine the time required for one complete cycle, additional factors such as load handling times (i.e., the time required for loading (t_{load}) and unloading (t_{unload}) off the ULs) and dead time (i.e., reaction time (t_0)) must be incorporated.

$$E(SC_{Lift}) = E(t_{SC\_Lift}) + t_{load} + t_{unload} + t_0 \tag{3.4}$$

Various lift configurations are possible for SBS/RSs, resulting in different lift travel times. The position of the I/O point determines which equation should be used. If the I/O point is at the same level as a tier, the UL can be transferred directly from the pre-storage zone to that tier's buffer, without the need for lift travel.

Equation (3.3) provides a good approximation for calculating the travel time. However, for the precise calculation of different configurations for a single-command cycle, the subsequent equations should be used. Four different use cases are considered.

Case 1: The I/O point (which is the pre-storage zone connected to the lift), is located at the same level as the first tier $n_{I/O} = 1$. This means that no lift travel is necessary for the first tier.

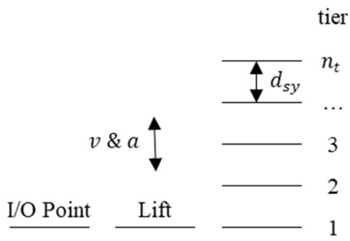


Figure 3.1: Case 1

The following equation can be used to calculate the average travel time and is valid for:

- The lift performs storage or retrieval operations
- Lift capacity: $c_{lift} = 1$
- Capacity requirement of an UL is one
- Lift configuration: Case 1
- Type of command cycle: Single-command cycle $k = 1$

$$E\left(t_{SC\_Lift}\right) = \frac{\frac{2}{v_{lift}} \cdot \left(\sum_{x=0}^{n_t-1}\left(\left|x d_{s_y}\right|\right)\right) + 2(n_t-1)\frac{v_{lift}}{a_{lift}}}{n_t} \tag{3.5}$$

$$E\left(t_{SC\_Lift}\right) = \frac{(n_t - 1)d_{sy}}{v_{lift}} + \left(2 - \frac{2}{n_t}\right)\frac{v_{lift}}{a_{lift}} \tag{3.6}$$

If equation (3.6) (exact approximation equation under the assumption that the maximum velocity is always reached) is compared with equation (3.3) (approximation equation), the calculated expected value deviates by $\frac{2}{n_t} \cdot \frac{v_{lift}}{a_{lift}}$, whereby equation (3.3) is overestimating the actual travel time. The more tiers are considered, the smaller is the percentage deviation.

The cycle time is shown in Figure 3.2. The numerical values can be found in appendix D. As the number of tiers increases, the cycle time increases linearly. In addition, the calculated values are compared and validated with the results of the simulation. The relative error is calculated by

$$relative\ error = 1 - \frac{analytical\ value}{simulation\ value} \tag{3.7}$$

The results indicate (see Figure 3.3) that both equations overestimate the results of the simulation but that equation (3.6) achieves better results, particularly when considering a larger number of tiers. For a small number of tiers, the relative error is very high.

The reason for the large error with a low number of tiers is that it is assumed that the maximum velocity is always reached, also for very short travel distances. This results in an overestimation of the actual travel times (for more details see appendix B). However, with a larger number of tiers or a larger distance between the tiers, the relative error becomes smaller. If both cases would be considered, that the lift 1) reaches and 2) does not reach the maximum velocity, the deviation between the analytical model and simulation would be zero, also for a small number of tiers. In this case, though, the travel time to each tier needs to be calculated using the equations from appendix B, summing up all travel times and dividing by the number of tiers.

Table 3.5: Input values

Parameter	Value
d_{sy}	0.5 m
n_t	1 - 50 tiers
v_{lift}	4 m/s
a_{lift}	3 m/s²
t_{load}	4 s
t_{unload}	4 s
t_0	0 s (for simplification)

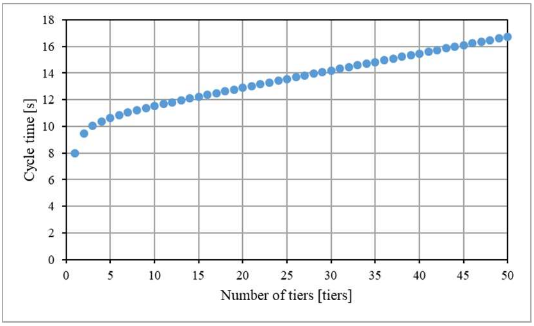


Figure 3.2: Cycle time (travel time exact equation (3.6))

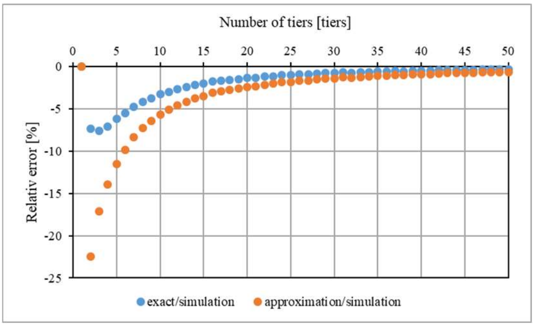


Figure 3.3: Relative error (travel time exact equation (3.6) and travel time approximation equation (3.3))

Case 2: The I/O point (pre-storage zone – lift) is below tier one, which means, that an additional travel distance d_{I/O_y} needs to be traveled.

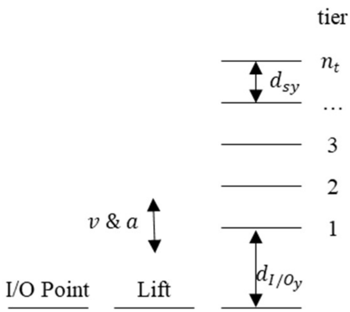


Figure 3.4: Case 2

The following equation can be used to calculate the average travel time and is valid for:

- The lift performs storage or retrieval operations
- Lift capacity: $c_{lift} = 1$
- Capacity requirement of an UL is one
- Lift configuration: Case 2
- Type of command cycle: Single-command cycle $k = 1$

$$E(t_{SC\_Lift}) = \frac{\frac{2}{v_{lift}} \cdot \left(\sum_{x=0}^{n_t - 1} \left(\left| d_{I/O_y} + x d_{sy} \right| \right) \right) + \frac{2 n_t v_{lift}}{a_{lift}}}{n_t} \tag{3.8}$$

$$E(t_{SC\_Lift}) = \frac{(n_t - 1) d_{sy}}{v_{lift}} + 2 \frac{v_{lift}}{a_{lift}} + 2 \frac{d_{I/O_y}}{v_{lift}} \tag{3.9}$$

If equation (3.9) (exact approximation equation) is compared with equation (3.3) (approximation equation), the calculated expected value deviates by $\frac{d_{I/O_y}}{v_{lift}}$, whereby equation (3.3) is underestimating the actual travel time. The larger the value d_{I/O_y}, the larger the deviation. The more tiers are considered, the smaller the percentage deviation.

The cycle time is shown in Figure 3.5. As the number of tiers increases, the cycle time increases linearly and the larger the value d_{I/O_y}, the larger the cycle time. Additionally, the cycle times, using equation (3.9) for travel time, are compared and validated with the results of the simulation (see Figure 3.6). The results indicate that the equation overestimates the results of the simulation, especially for a small value of d_{I/O_y}. As the number of tiers or the value of d_{I/O_y} increases, the relative error approaches zero. In this example, the critical length is 5.33 meters. If the distance traveled is large than 5.33 meters, the maximum velocity is always reached. For such cases the relative error is zero.

Table 3.6: Input values

Parameter	Value
d_{sy}	0.5 m
$d_{I/Oy}$	1, 2, 6 m
n_t	1 - 50 tiers
v_{lift}	4 m/s
a_{lift}	3 m/s²
t_{load}	4 s
t_{unload}	4 s
t_0	0 s (for simplification)

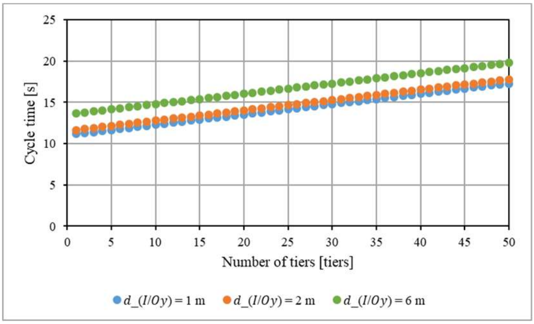


Figure 3.5: Cycle time (cycle time equation (3.9))

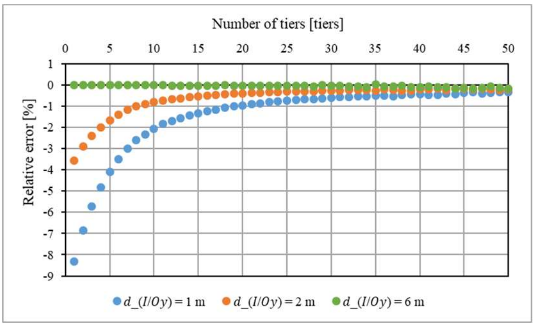


Figure 3.6: Relative error (cycle time equation (3.9))

Case 3: The I/O point (pre-storage zone – lift) is at the same level as tier $n_{I/O}$, which means, that for tier $n_{I/O}$ no lift travel is needed.

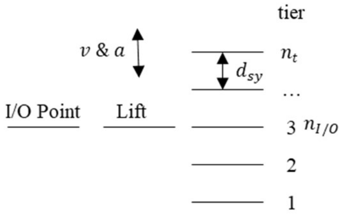


Figure 3.7: Case 3

The following equation can be used to calculate the average travel time and is valid for:

- The lift performs storage or retrieval operations
- Lift capacity: $c_{lift} = 1$
- Capacity requirement of an UL is one
- Lift configuration: Case 3
- Type of command cycle: Single-command cycle $k = 1$

$$E\left(t_{SC\_Lift}\right) = \frac{\frac{2}{v_{lift}} \cdot \left(\sum_{x=0}^{n_t-1}\left(\left|-(n_{I/O}-1)d_{sy}+xd_{sy}\right|\right)\right)}{n_t} + \frac{2(n_t-1)\frac{v_{lift}}{a_{lift}}}{n_t} \tag{3.10}$$

$$E\left(t_{SC\_Lift}\right) = \frac{(n_{I/O}-1)\left(\frac{n_{I/O} \cdot d_{sy}}{v_{lift}}+2\frac{v_{lift}}{a_{lift}}\right)}{n_t} + \frac{(n_t-n_{I/O}+1)\left(\frac{(n_t-n_{I/O})d_{sy}}{v_{lift}}+\left(2-\frac{2}{n_t-n_{I/O}+1}\right)\frac{v_{lift}}{a_{lift}}\right)}{n_t} \tag{3.11}$$

Figure 3.8 shows the correlation between cycle time and the tier of the I/O point. The shortest cycle time can be achieved when $n_{I/O}$ is exactly equal to $\frac{n_t}{2}$. The further away the I/O point is from $\frac{n_t}{2}$, the higher is the average cycle time. Therefore, the highest throughput can be achieved when the I/O point is at $\frac{n_t}{2}$. As the $n_{I/O}$ position moves further towards the middle, the relative error between the analytical model and the simulation increases due to the higher number of short distances traveled (see Figure 3.9).

In practice, it is essential to conduct an efficiency analysis in addition to calculating the average cycle time. While having the I/O point at exactly $\frac{n_t}{2}$, this results in the shortest average cycle time and the highest throughput. Installing the conveyor technology at a height of $\frac{n_t}{2}$ can result in significantly higher costs. Moreover, it may not be technically feasible to install the conveyor technology at that height. Therefore, a trade-off analysis between cycle time, throughput, and installation costs must be conducted to determine the optimal I/O point position.

Table 3.7: Input values

Parameter	Value
d_{sy}	0.5 m
$n_{I/O}$	tier 1 – tier 50
n_t	50 tiers
v_{lift}	4 m/s
a_{lift}	3 m/s²
t_{load}	4 s
t_{unload}	4 s
t_0	0 s (for simplification)

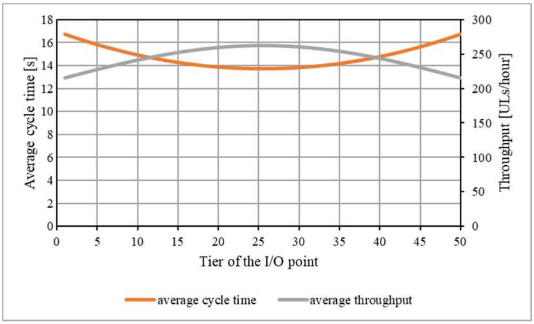


Figure 3.8: Impact of the position of the I/O point tier on the average cycle time (cycle time equation (3.11)) and the maximum throughput (appendix E, equation (E.1))

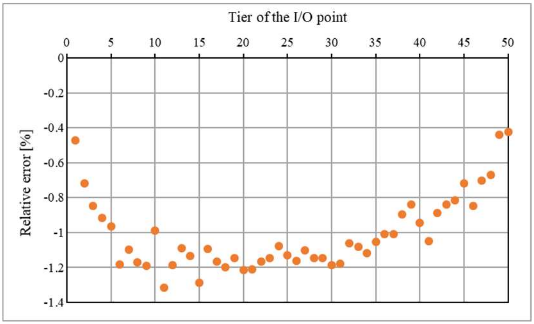


Figure 3.9: Relative error of the cycle time (cycle time equation (3.11))

Case 4: The I/O point (pre-storage zone – lift) is not at the same level as one tier, which means, that always a lift travel is needed. I/O point is between tier 1 and tier n_t. n_1 is the number of tiers below the I/O point and d_1 is the distance between the I/O point level and the tier below.

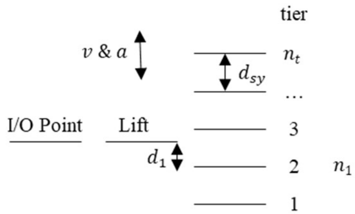


Figure 3.10: Case 4

The following equation can be used to calculate the average travel time and is valid for:

- The lift performs storage or retrieval operations
- Lift capacity: $c_{lift} = 1$
- Capacity requirement of an UL is one
- Lift configuration: Case 4
- Type of command cycle: Single-command cycle $k = 1$

$$E\left(t_{SC\_Lift}\right) = \frac{\frac{2}{v_{lift}} \cdot \left(\sum_{x=0}^{n_t-1}\left(\left|-\left((n_1-1)d_{sy}+d_1\right)+xd_{sy}\right|\right)\right)+2n_t\frac{v_{lift}}{a_{lift}}}{n_t} \tag{3.12}$$

$$E\left(t_{SC\_Lift}\right) = \frac{n_1\left(\frac{(n_1-1)d_{sy}}{v_{lift}}+2\frac{v_{lift}}{a_{lift}}+2\frac{d_1}{v_{lift}}\right)}{n_t} + \frac{(n_t-n_1)\left(\frac{(n_t-n_1-1)d_{sy}}{v_{lift}}+2\frac{v_{lift}}{a_{lift}}+2\frac{d_{sy}-d_1}{v_{lift}}\right)}{n_t} \tag{3.13}$$

3.3.2 Dual-Command Cycle

In addition to a single-command cycle, it is also possible to perform a dual-command cycle with two different scenarios:

(i) In the first scenario, each lift performs either storage or retrieval tasks. The lift has the capacity to handle two ULs and can perform two storage or retrieval operations.
(ii) In the second scenario, each lift performs both storage and retrieval tasks. The lift has the capacity to handle one UL, but it performs first one storage and then one retrieval operation.

In both scenarios, the travel time for the lift can be calculated using equations (3.1) and (3.2). The dual-command cycle consists of the following three portions: travel time from the I/O point to the first tier, travel time between the first tier and second tier, and travel time back to the I/O point.

$$E\left(t_{DC\_Lift}\right) = 2 \cdot \left(\frac{1}{2}\frac{H_{lift}}{v_{lift}} + \frac{v_{lift}}{a_{lift}}\right) + \frac{1}{3}\frac{H_{lift}}{v_{lift}} + \frac{v_{lift}}{a_{lift}} = \frac{4}{3}\frac{H_{lift}}{v_{lift}} + 3\frac{v_{lift}}{a_{lift}} \tag{3.14}$$

This equation is commonly used in literature to approximate the travel time of the lift for a dual-command cycle. However, for higher accuracy, more exact approximation equations are derived for the average travel and cycle times for the four cases described above.

To calculate the total time required for one dual-command cycle, it is important to take into account the load handling times, which include the time required for loading (picking up) and unloading (dropping off) the ULs, as well as any dead time due to reaction time or other factors. There are two different cases to consider when accounting for load handling times:

(i) When both ULs on the lift are transferred to the same tier simultaneously, which occurs with a probability of $\left(\frac{1}{n_t}\right)$.

(ii) When the ULs have different target tiers and therefore must be transferred individually on different tiers, which occurs with a probability of $\left(1-\frac{1}{n_t}\right)$.

Therefore, the expected cycle time for a dual-command cycle for the storage operation of the inbound lift is:

$$E(DC_{Lift}) = E(t_{DC\_Lift}) + t_{load} + 2\left(1-\frac{1}{n_t}\right)t_{unload} + \left(\frac{1}{n_t}\right)t_{unload} + t_0 \tag{3.15}$$

The probability of load handling times can vary, depending on the lift system and load handling concept used. The following equations apply specifically to the side-by-side lift system and assumes a drop off time of t_{unload}.

The same equation can be also used to calculate the retrieval operation of the outbound lift, but then t_{load} and t_{unload} have to be switched.

Case 1: To calculate the average travel time, three different cases must be considered, each with their respective probabilities, which are included in the total travel time.

(i) Both ULs on the lift are transferred to the same tier. In this case, equation (3.6) can be used to calculate the travel time. This occurs with a probability of $\left(\frac{n_t}{n_t^2}\right)$.

(ii) When one UL's target tier is the first tier and the second UL's target tier is any other tier, only two acceleration processes are required. In this case, equation (3.3) or (3.9) can be used to calculate the travel time. This occurs with a probability of $\left(\frac{2n_t-2}{n_t^2}\right)$.

(iii) When both ULs' target tiers are randomly chosen, but not tier one or the same tier for both ULs, equation (3.14) can be used to calculate the travel time. This occurs with a probability of $\left(1-\frac{n_t}{n_t^2}-\frac{2n_t-2}{n_t^2}\right)$.

For this case the average travel time between two randomly chosen tiers can be calculated using the following equations $\frac{d_{sy}}{v} \cdot \frac{\left(\sum_{x=2}^{n_t} \sum_{y=2}^{n_t} |x-y|\right)}{n_t{}^2-3n_t+2} + \frac{v_{lift}}{a_{lift}}$ or $\frac{1}{3}\frac{n_t d_{sy}}{v_{lift}} + \frac{v_{lift}}{a_{lift}}$.

The following equation can be used to calculate the average travel time and is valid for:

- The lift performs storage or retrieval operations
- Lift capacity: $c_{lift} = 2$
- Capacity requirement of an UL is one
- Lift configuration: Case 1
- Type of command cycle: Dual-command cycle $k = 2$

$$E(t_{DC\_Lift}) = \frac{n_t}{n_t{}^2}\left(\frac{(n_t - 1)d_{sy}}{v_{lift}} + \left(2 - \frac{2}{n_t}\right) \cdot \frac{v_{lift}}{a_{lift}}\right) + \left(\frac{2n_t - 2}{n_t{}^2}\right) \cdot \left(\frac{n_t d_{sy}}{v} + 2\frac{v_{lift}}{a_{lift}}\right) + \left(1 - \frac{n_t}{n_t{}^2} - \frac{2n_t - 2}{n_t{}^2}\right) \cdot \left(\frac{4}{3} \cdot \frac{n_t d_{sy}}{v_{lift}} + 3\frac{v_{lift}}{a_{lift}}\right) \tag{3.16}$$

If equation (3.16) (exact approximation equation) is compared with equation (3.14) (approximation equation), equation (3.14) is overestimating the actual travel time. The more tiers are considered, the smaller the percentage deviation.

The subsequent figure shows the relationship between cycle time and number of tiers. Larger distances between the tiers result in longer cycle times. Additionally, as the number of tiers increases, the cycle time increases linearly. Furthermore, Figure 3.12 presents the relative error. As the number of tiers increases, the error decreases. In addition, the error becomes smaller as the distance between the tiers (i.e., the distance traveled) increases.

Table 3.8: Input values

Parameter	Value
d_{sy}	0.5 m
n_t	50 tiers
v_{lift}	4 m/s
a_{lift}	3 m/s²
t_{load}	4 s
t_{unload}	4 s
t_0	0 s (for simplification)

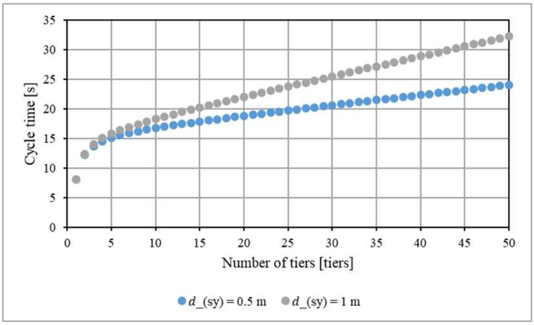


Figure 3.11: Cycle time (travel time exact equation (3.16))

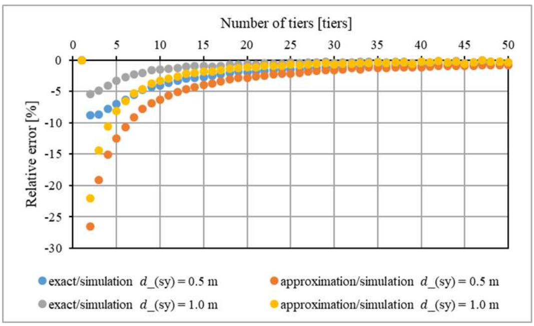


Figure 3.12: Relative error (travel time exact equation (3.16) and travel time approximation equation (3.14))

Case 2: To calculate the average travel time, two different cases must be considered, each with their respective probabilities, which are included in the total travel time.

(i) Both ULs on the lift are transferred to the same tier. In this case, equation (3.9) can be used to calculate the travel time. This occurs with a probability of $\left(\frac{1}{n_t}\right)$.

(ii) Both ULs' target tiers are different tiers. This occurs with a probability of $\left(1-\frac{1}{n_t}\right)$. For this case, the average travel time between two randomly chosen tiers can be calculated, using the following equations $\frac{d_{sy}}{v}\cdot\frac{\left(\sum_{x=1}^{n_t}\sum_{y=1}^{n_t}|x-y|\right)}{{n_t}^2-n_t}+\frac{v_{lift}}{a_{lift}}$ or $\frac{1}{3}\frac{(n_t+1)d_{sy}}{v_{lift}}+\frac{v_{lift}}{a_{lift}}$.

The following equation can be used to calculate the average travel time and is valid for:

- The lift performs storage or retrieval operations
- Lift capacity: $c_{lift}=2$
- Capacity requirement of an UL is one
- Lift configuration: Case 2
- Type of command cycle: Dual-command cycle $k=2$

$$E\left(t_{DC\_Lift}\right)=\frac{1}{n_t}\left(\frac{(n_t-1)d_{sy}}{v_{lift}}+2\frac{v_{lift}}{a_{lift}}+2\frac{d_{I/Oy}}{v_{lift}}\right)+\left(1-\frac{1}{n_t}\right)\left(\frac{(n_t-1)d_{sy}}{v_{lift}}+2\frac{v_{lift}}{a_{lift}}+2\frac{d_{I/Oy}}{v_{lift}}+\frac{1}{3}\frac{(n_t+1)d_{sy}}{v_{lift}}+\frac{v_{lift}}{a_{lift}}\right) \tag{3.17}$$

Case 3: To calculate the average travel time, three different cases must be considered, each with their respective probabilities, which are included in the total travel time.

(i) Both ULs on the lift are transferred to the same tier. In this case, equation (3.11) can be used to calculate the travel time. This occurs with a probability of $\left(\frac{1}{n_t}\right)$.

(ii) When one UL's target tier is on the same level as the I/O point and the second UL's target tier is any other tier, only two acceleration processes are required. This occurs with a probability of $\left(\frac{2n_t-2}{{n_t}^2}\right)$.

(iii) Both ULs' target tiers are different tiers. This occurs with a probability of $\left(1-\frac{1}{n_t}-\frac{2n_t-2}{{n_t}^2}\right)$. For this case the average distance between two randomly chosen tiers can be calculated, using the following equations $\frac{d_{sy}}{v_{lift}} \cdot \frac{\left(\sum_{x=1}^{n_t}\sum_{y=1}^{n_t}|x-y|\right)-2\cdot\sum_{y=1}^{n_t}|n_{I/O}-y|}{{n_t}^2-3n_t+2}+\frac{v_{lift}}{a_{lift}}$.

The following equation can be used to calculate the average travel time and is valid for:

- The lift performs storage or retrieval operations
- Lift capacity: $c_{lift}=2$
- Capacity requirement of an UL is one
- Lift configuration: Case 3
- Type of command cycle: Dual-command cycle $k=2$

$$E\left(t_{DC_{Lift}}\right)=\frac{1}{n_t}\left(2\frac{d_{sy}}{v_{lift}}\frac{\left(\sum_{x=1}^{n_t-n_{I/O}}x+\sum_{x=1}^{n_{I/O}-1}x\right)}{n_t}+\left(2-\frac{2}{n_t}\right)\frac{v_{lift}}{a_{lift}}\right)+\frac{2n_t-2}{{n_t}^2}\cdot \left(2\cdot 2\frac{d_{sy}}{v_{lift}}\frac{\left(\sum_{x=1}^{n_t-n_{I/O}}x+\sum_{x=1}^{n_{I/O}-1}x\right)}{n_t}+2\frac{v_{lift}}{a_{lift}}\right)+\left(1-\frac{1}{n_t}-\frac{2n_t-2}{{n_t}^2}\right)\cdot \left(2\cdot 2\frac{d_{sy}}{v_{lift}}\frac{\left(\sum_{x=1}^{n_t-n_{I/O}}x+\sum_{x=1}^{n_{I/O}-1}x\right)}{n_t}+2\frac{v_{lift}}{a_{lift}}+\frac{d_{sy}}{v_{lift}}\cdot \frac{\left(\sum_{x=1}^{n_t}\sum_{y=1}^{n_t}|x-y|\right)-2\cdot\sum_{y=1}^{n_t}|n_{I/O}-y|}{{n_t}^2-3n_t+2}+\frac{v_{lift}}{a_{lift}}\right) \quad (3.18)$$

Case 4: To calculate the average travel time, two different cases must be considered, each with their respective probabilities, which are included in the total travel time.

(i) Both ULs on the lift are transferred to the same tier. In this case, equation (3.12) can be used to calculate the travel time. This occurs with a probability of $\left(\frac{1}{n_t}\right)$.

(ii) Both ULs' target tiers are different tiers. This occurs with a probability of $\left(1-\frac{1}{n_t}\right)$. For this case the average distance between two randomly chosen tiers can be calculated, using the following equations $\frac{d_{sy}}{v_{lift}} \cdot \frac{\left(\sum_{x=1}^{n_t}\sum_{y=1}^{n_t}|x-y|\right)}{n_t{}^2-n_t} + \frac{v_{lift}}{a_{lift}}$.

The following equation can be used to calculate the average travel time and is valid for:

- The lift performs storage or retrieval operations
- Lift capacity: $c_{lift} = 2$
- Capacity requirement of an UL is one
- Lift configuration: Case 4
- Type of command cycle: Dual-command cycle $k = 2$

$$E\left(t_{DC_{Lift}}\right) = \frac{1}{n_t}\left(\frac{2}{v_{lift}} \cdot \frac{\left(\sum_{x=0}^{n_t-1}\left(\left|-\left((n_1-1)d_{s_y}+d_1\right)+xd_{s_y}\right|\right)\right)}{n_t} + 2\frac{v_{lift}}{a_{lift}}\right) + \left(1-\frac{1}{n_t}\right) \cdot \left(\frac{2}{v_{lift}} \cdot \frac{\left(\sum_{x=0}^{n_t-1}\left(\left|-\left((n_1-1)d_{s_y}+d_1\right)+xd_{s_y}\right|\right)\right)}{n_t} + 2\frac{v_{lift}}{a_{lift}} + \frac{d_{sy}}{v_{lift}} \cdot \frac{\left(\sum_{x=1}^{n_t}\sum_{y=1}^{n_t}|x-y|\right)}{n_t{}^2-n_t} + \frac{v_{lift}}{a_{lift}}\right) \tag{3.19}$$

3.3.3 Using one Lift for Storage and Retrieval Tasks

If only one lift is installed at one aisle, the lift has to perform storage and retrieval tasks. Therefore, the same equations from chapter 3.3.1 and 3.3.2 can be used.

The technical implementation of the lift system may make it impossible to transfer the UL for storage and the UL for retrieval simultaneously. Therefore, even if they are transferred on the same tier, two load handling times must always be considered for the dual-command cycle. Furthermore, the load handling time between the pre-storage zone and the lift system must be taken into account twice, once for the storage operation and once for the retrieval operation.

The following equation can be used to calculate the average cycle time for a single-command cycle and is valid for:

- The lift performs storage or retrieval operations
- Lift capacity: $c_{lift} = 1$
- Capacity requirement of an UL is one
- Lift configuration: Case 1 - 4
- Type of command cycle: Single-command cycle $k = 1$

$$E(SC_{Lift}) = E(t_{SC\_Lift}) + t_{load} + t_{unload} + t_0 \tag{3.20}$$

The following equation can be used to calculate the average cycle time for a dual-command cycle and is valid for:

- The lift performs a combined storage and retrieval operation
- Lift capacity: $c_{lift} = 1$
- Capacity requirement of an UL is one
- Lift configuration: Case 1 - 4
- Type of command cycle: Dual-command cycle $k = 2$

$$E(DC_{Lift}) = E(t_{DC\_Lift}) + 2 \cdot t_{load} + 2 \cdot t_{unload} + t_0 \tag{3.21}$$

3.3.4 Multi-Command Cycle

The next step is to calculate the travel time required for a multi-command cycle, which occurs when:

(i) The lift has a capacity equal to or greater than one ($c_{lift} \geq 1$) and is capable of performing combined storage and retrieval operations.
(ii) The lift has a capacity equal to or greater than two ($c_{lift} \geq 2$) and is capable of performing only storage or retrieval operations.

In both cases, the lift travels to multiple tiers in each command cycle. The number of travels required between any two random storage locations (i and j) increases with the lift's capacity. Moreover, as more tiers need to be reached, the travel time also increases.

The following equation provides a good approximation for calculating the cycle time of the lift for a multi-command cycle, where c_{lift} represents the maximum capacity of the lift. This equation is valid for:

- The lift performs storage or retrieval operations
- Lift capacity: $c_{lift} \in \{1, 2, 3, 4, \dots\}$
- Capacity requirement of an UL is one
- Lift configuration: Case 1
- Type of command cycle: Multi-command cycle $k \in \{1, 2, 3, 4, \dots\}$
- Sequencing strategy: FCFS sequencing

$$E\left(MC_{Lift\_FCFS\_sequencing}\right) = 2 \cdot \left(\frac{1}{2}\frac{H_{lift}}{v_{lift}} + \frac{v_{lift}}{a_{lift}}\right) + \left(c_{lift} - 1\right) \cdot \left(\frac{1}{3}\frac{H_{lift}}{v_{lift}} + \frac{v_{lift}}{a_{lift}}\right) + E\left(n_{load\_k}\right)t_{load} + c_{lift}t_{unload} + t_0 \tag{3.22}$$

If the individual requests are not pre-sorted and are processed according to the FCFS method, there may be a significant number of up and down movements of the lift (see Figure 3.13 FCFS sequencing). On the other hand, if the orders are pre-sorted and processed in an optimized sequence, the lift only needs to travel from the bottom (from the I/O point) to the top and back down to the I/O point once (see Figure 3.13 optimized sequencing). In both cases, there are five acceleration and deceleration processes. However, if the orders are processed after FCFS, the lift must travel a longer distance. If the orders are processed in an optimized sequence, the maximum travel distance is up to the tier that is furthest away from the I/O point in this cycle and back to the I/O point.

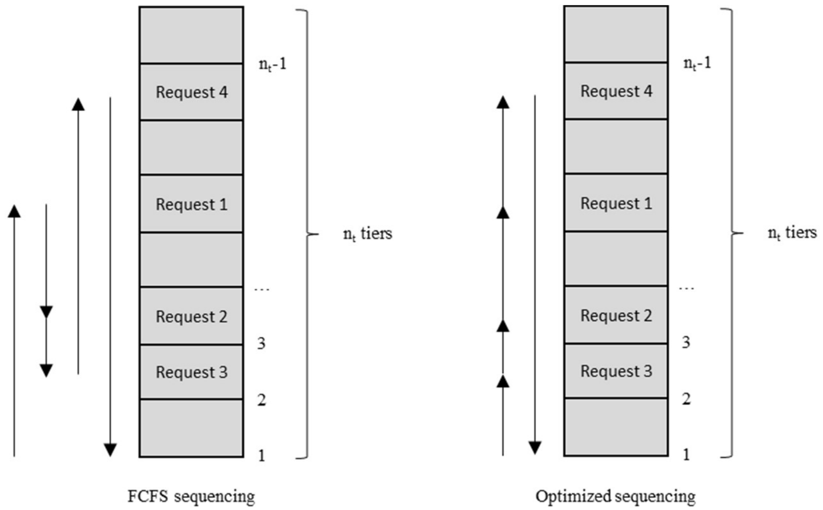


Figure 3.13: Multi-command cycles for the lift with different sequencing strategies

If an optimized sequence is considered, it is necessary to calculate the average maximum travel distance. Since the lift is only capable of performing storage or retrieval operations, c_{lift} equals the total number of randomly selected tiers.

The equation for the expected value of the maximum of n uniform random variables can be found in McCammon (2017). With this equation, the average maximum travel distance is computed using the following equation:

$$E(H_{liftMax}) = \frac{c_{lift}}{c_{lift}+1} \cdot H_{lift} \tag{3.23}$$

The equation can be specified either with the lift capacity or with the number of command cycles performed. The following applies:

- c_{lift}: if the lift performs storage or retrieval operations
- $2c_{lift}$: if the lift performs combined storage and retrieval operations

- $\frac{c_{lift}}{2}$: if two small ULs are transported to the same tier (only applies for $c_{lift} \in \{2, 4, 6, 8, 10, \dots\}$
- k: applies to all cases

The total cycle time for a multi-command cycle can be described as follows and is valid for:

- The lift performs storage or retrieval operations
- Lift capacity: $c_{lift} \in \{1, 2, 3, 4, \dots\}$
- Capacity requirement of an UL is one
- Lift configuration: Case 1
- Type of command cycle: Multi-command cycle $k \in \{1, 2, 3, 4, \dots\}$
- Sequencing strategy: Optimized sequencing

$$E\left(MC_{Lift\_optimized\_sequencing\_approximation}\right) = 2 \cdot \frac{c_{lift}}{c_{lift}+1} \cdot \frac{H_{lift}}{v_{lift}} + (c_{lift} + 1)\frac{v_{lift}}{a_{lift}} + E\left(n_{load\_k}\right)t_{load} + c_{lift}t_{unload} + t_0 \quad (3.24)$$

The equation provided is a good approximation for estimating the cycle time of a multi-command cycle with the I/O point at the lowest tier. It is based on the assumptions that each UL is transported to a different tier and that multiple ULs are not transferred to a tier simultaneously.

Consequently, the assumed number of acceleration and deceleration operations in a real system is actually smaller than $c_{lift} + 1$. While the total time for picking up ULs at the I/O point (between pre-storage zone and lift) can be calculated exactly, the load handling time between the tiers and the lift depends on the considered command cycle. Depending on the considered command cycle, t_{load} is multiplied by the parameter $E\left(n_{load\_k}\right)$, where $E\left(n_{load\_k}\right)$ depends on the lift configuration and the performed command cycle (e.g., side-by-side lift system with two ULs side by side: single-command cycle $E\left(n_{load\_k}\right) = 1$, dual-command cycle $E\left(n_{load\_k}\right) = 1$, triple-command cycle $E(n_{load\_k}) = 2$, etc.). Furthermore, the unloading times in this equation are also an approximation, as ULs can be transferred individually or as a batch depending on the lift configuration. If several ULs are transferred simultaneously as a batch or if each UL is transferred individually, different unloading times can occur. However, the equation assumes, that the same unloading time applies to each UL.

For the exact calculation of the travel and cycle time for a multi-command cycles, the following approach can be used:

Algorithm to get the maximum of n uniform random variables

1: Total number of tiers n_t

2: Type of command cycle k

3: Calculate the expected value

3.1: Create all possible combinations

$$\text{for } (x_1 = 1\ ;\ x_1 \le n_t\ ;\ x_1 + +)$$
$$\text{for } (x_2 = 1\ ;\ x_2 \le n_t\ ;\ x_2 + +)$$
$$\dots$$
$$\text{for } (x_k = 1\ ;\ x_k \le n_t\ ;\ x_k + +)$$

3.2: Get the maximum of each combination and add this value to a collection list (size of the collection is ${n_t}^k$)

3.3: Get the average value of all entries of the collection. This is the average value of the maximum of each combination $E(n_{tMax\_kt})$.

4: Depending on the lift configuration, the average travel distance can now be calculated, for example by multiplying the value determined in step 3.3 by the distance between the two tiers. The average travel distance for the case, that the I/O point with the pre-storage zone is at the lowest tier, would be: $E(H_{liftMax}) = (E(n_{tMax\_kt}) - 1) \cdot d_{sy}$

Note: (i) single-command cycle ($k = 1$), dual-command cycle ($k = 2$), …

(ii) The number of “for loops” in 3.1 is the size of k

The next step involves obtaining the exact distribution of all combinations. The total number of all combinations, which are “permutations with repetition and order matters”, is ${n_t}^k$. The total number of all combinations, which are “combinations with repetition and order does not matter”, is $\frac{(k+n_t-1)!}{k!(n_t-1)!}$. Once all combinations are known, a frequency distribution can be generated. Additionally, for each combination, it is necessary to define the number of acceleration and deceleration operations required and the load handling time that will occur. Using these values, the average number of acceleration/deceleration operations, $E(n_{acceleration\_kt})$, and the average number of unloading operations, $E(n_{unload\_kt})$, can be defined. As an example, for Case 1 with a side-by-side lift system and processing only storage requests, the total cycle time for a multi-command cycle can be described as follows and is valid for:

- The lift performs storage or retrieval operations
- Lift capacity: $c_{lift} \in \{1, 2, 3, 4, \dots\}$
- Capacity requirement of an UL is one
- Lift configuration: Case 1
- Type of command cycle: Multi-command cycle $k \in \{1, 2, 3, 4, \dots\}$
- Sequencing strategy: Optimized sequencing

$$E(MC_{Lift\_optimized\_sequencing\_exact}) = 2(E(n_{tMax\_kt}) - 1) \cdot \frac{d_{sy}}{v} + E(n_{acceleration\_kt}) \frac{v_{lift}}{a_{lift}} + E(n_{load\_k})t_{load} + E(n_{unload\_kt})t_{unload} + t_0 \tag{3.25}$$

If only one lift is used for storage and retrieval operations, the same equation can be used. However, in practice, storage operations must be executed prior to retrieval operations. In this case, storage operations are performed on the travel to the tier furthest away from the I/O point, and retrieval operations are performed on the travel back to the I/O point.

Since the calculation of the exact solution requires a high computing time, the individual expected values can be calculated in advance. Depending on the type of command cycle k and the number of tiers n_t, the corresponding values can then be selected from the table and inserted in equation (3.25). The exact average travel time of the multi-command cycle can then be calculated very easily for different configurations. Only the values for velocity, acceleration and load handling times have to be added.

For instance, in the table below, the exact values have been computed for various configurations. It can be expanded as necessary.

Table 3.9: Input values for equation (3.25) in dependency of the type of command cycle k and the number of tiers n_t

		Type of command cycle k						
Number of tiers n_t		**single k=1**	**dual k=2**	**triple k=3**	**quad-ruple k=4**	**quin-tuple k=5**	**sextu-ple k=6**	**septu-ple k=7**
n_t =1	**Average maximum tier** $E(n_{tMax\_kt})$	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	Average number of acceleration operations $E(n_{acceleration\_kt})$	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Average number of unloading operations $E(n_{unload\_kt})$	1.00	1.00	2.00	2.00	3.00	3.00	4.00
n_t =5	**Average maximum tier** $E(n_{tMax\_kt})$	3.00	3.80	4.20	4.43	4.58	4.69	4.76
	Average number of acceleration operations $E(n_{acceleration\_kt})$	1.60	2.40	2.94	3.36	3.69	3.95	4.16
	Average number of unloading operations $E(n_{unload\_kt})$	1.00	1.80	2.48	3.09	3.65	4.19	4.72
n_t =10	**Average maximum tier** $E(n_{tMax\_kt})$	5.50	7.15	7.98	8.47	8.79	9.02	9.19
	Average number of acceleration operations $E(n_{acceleration\_kt})$	1.80	2.70	3.44	4.10	4.69	5.22	5.70
	Average number of unloading operations $E(n_{unload\_kt})$	1.00	1.90	2.72	3.48	4.18	4.84	5.48
n_t =20	**Average maximum tier** $E(n_{tMax\_kt})$	10.50	13.83	15.49	16.48	17.15	17.62	17.97
	Average number of acceleration operations $E(n_{acceleration\_kt})$	1.90	2.85	3.71	4.52	5.30	6.03	6.73
	Average number of unloading operations $E(n_{unload\_kt})$	1.00	1.95	2.86	3.72	4.55	5.34	6.11
n_t =50	**Average maximum tier** $E(n_{tMax\_kt})$	25.50	33.83	38.00	40.49	42.16	43.35	44.24
	Average number of acceleration operations $E(n_{acceleration\_kt})$	1.96	2.94	3.88	4.80	5.71	6.59	7.46
	Average number of unloading operations $E(n_{unload\_kt})$	1.00	1.98	2.94	3.88	4.81	5.72	6.61
Average number of loading operations $E(n_{load\_k})$		1.00	1.00	2.00	2.00	3.00	3,00	4.00

If equation (3.25) (exact equation) is compared with equation (3.24) (approximation equation), equation (3.24) is overestimating the actual cycle time. This is mainly due to the fact that the number of acceleration operations and unloading operations is overestimated.

Now three different control strategies for the same input values are compared:

(i) FCFS sequencing,
(ii) optimized sequencing,
(iii) optimized sequencing and two ULs are unloaded together at the same tier (in the following named "Strategy 1"). Two ULs can always be transferred simultaneously by using a side-by-side lift configuration. Optimal efficiency can be achieved by always transferring two ULs to the same tier, thereby minimizing the number of trips and (un)loading operations.

For Strategy 1, equation (3.25) has been adapted and is valid for:

- The lift performs storage or retrieval operations
- Lift capacity: $c_{lift} \in \{1, 2, 3, 4, \dots\}$
- Capacity requirement of an UL is one
- Lift configuration: Case 1
- Type of command cycle: Multi-command cycle $k \in \{1, 2, 3, 4, \dots\}$
- Sequencing strategy: Optimized sequencing
- Two small ULs are unloaded together at the same tier

$$\begin{aligned} E\left(MC_{Lift\_Strategy1}\right) = 2 \cdot \frac{E\left(n_{load\_k}\right)}{E\left(n_{load\_k}\right)+1} \cdot \frac{H_{lift}}{v_{lift}} + \\ \left(E\left(n_{load\_k}\right) + 1\right) \frac{v_{lift}}{a_{lift}} + \\ E\left(n_{load\_k}\right) t_{load} + \\ E\left(n_{load\_k}\right) t_{unload} + t_0 \end{aligned} \tag{3.26}$$

It is assumed that the I/O point is at the lowest tier and that two ULs can be transferred simultaneously during the loading phase (picking up the ULs from the pre-storage zone). In Figure 3.14 cycle times for various lift capacities and different control strategies are compared.

Table 3.10: Input values

Parameter	Value
d_{sy}	0.5 m
n_t	50 tiers
v_{lift}	4 m/s
a_{lift}	3 m/s²
t_{load}	4 s
t_{unload}	4 s
t_0	0 s (for simplification)

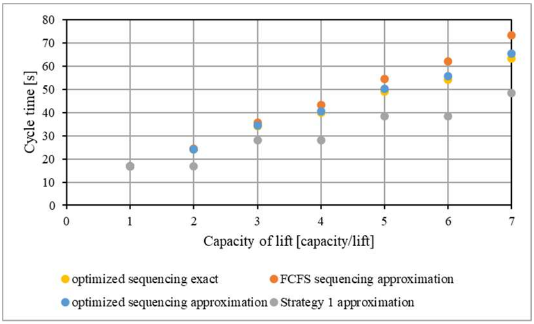


Figure 3.14: Cycle time for three different control strategies (cycle time FCFS sequencing equation (3.22), cycle time optimized sequencing approximation equation (3.24), cycle time optimized sequencing exact equation (3.25), cycle time Strategy 1 equation (3.26))

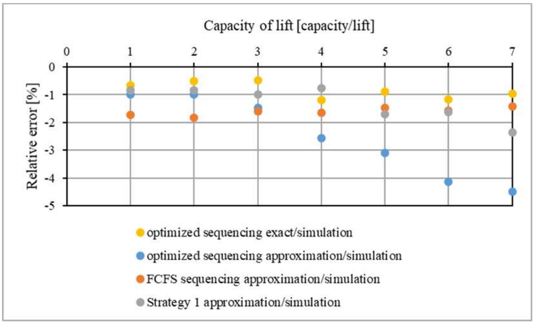


Figure 3.15: Relative error (cycle time FCFS sequencing equation (3.22), cycle time optimized sequencing approximation equation (3.24), cycle time optimized sequencing exact equation (3.25), cycle time Strategy 1 equation (3.26))

Figure 3.15 shows that there is minimal deviation between the simulation results and the results calculated with the exact equation (with considering optimized sequencing). For small command cycles the approximation equation (with considering optimized sequencing) gives very good results and overestimates only slightly the exact result. The larger the number of command cycles, the larger the deviation between the results. As described above, the reason is the overestimation of the number of acceleration and deceleration operations and the number of unloading operations, but also the short travel distances.

Processing storage and retrieval operations after FCFS results in the longest cycle time, whereas optimized sequencing can reduce the cycle time. The cycle time can be decreased by always transferring two ULs to the same tier. The cycle time remains the same whether one or two small ULs are transported, since in both cases only one tier needs to be approached. The same applies to three and four small ULs (two tiers must be approached), five and six (three tiers must be approached), etc. The impact of the different strategies on cycle time increases with larger lift capacities.

Figure 3.16 compares the throughput for various lift capacities for the three different control strategies (throughput equations see appendix E, equation (E.5)). Strategy 1 achieves the highest throughput. In many cases a higher capacity results in lower throughput. While capacity may increase, the data reveals that it often results in a decrease in throughput (only applicable for Strategy 1). This only applies to lifts with odd capacities since an extra travel and transfer operation are required for the additional UL. For example, with a lift capacity of two, only one travel to one tier is needed, while a lift capacity of three requires two tiers to be traveled to, increasing travel and load handling times. However, only one more UL is transferred. Thus, it is recommended to perform sequencing with increasing lift capacity, fully utilizing the lift and choosing lift configurations with even capacity.

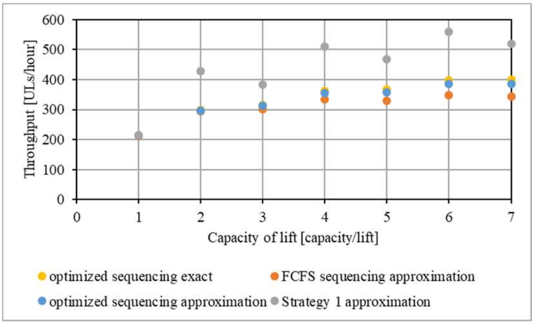


Figure 3.16: Throughput for the three different control strategies (cycle time FCFS sequencing equation (3.22), cycle time optimized sequencing approximation equation (3.24), cycle time optimized sequencing exact equation (3.25), cycle time Strategy 1 equation (3.26))

3.3.5 Two Different Sizes of ULs and a Lift Capacity of Two

In the next step, the cycle time for the lift, transporting small and large ULs, is defined.

Depending on the ratio of large to small ULs, different command cycles need to be considered:

(i) If only large ULs are stored, only single-command cycles are performed.
(ii) If only small ULs are stored, dual-command cycles are performed.
(iii) If both small and large ULs are stored, then depending on the ratio of large to small ULs, single and dual-command cycles are performed.

The percentage of small ULs is P_{small} and the percentage of large ULs is $1 - P_{small}$. Two small ULs are always grouped together. In the next step the percentage of operations performed for large and small ULs also needs to be determined. The probability, depending on P_{small}, that an operation is performed with small ULs is $\frac{\frac{P_{small}}{2}}{\frac{P_{small}}{2}+1-P_{small}}$ and the probability that an operation is performed with large ULs is $\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}}$. With the help of these probabilities, the individual time blocks can be weighted.

Thus, the following equation can be derived for the cycle time and is valid for:

- The lift performs storage or retrieval operations
- Lift capacity: $c_{lift} = 2$
- Capacity requirement of small ULs is one
- Capacity requirement of large ULs is two
- Two small ULs are unloaded together at the same tier
- Lift configuration: Case 1
- Type of command cycle: Single/Dual-command cycle $k \in \{1, 2\}$
- Sequencing strategy: Optimized sequencing

$$E\left(MC_{TwoSizesOfUL\_Lift\_Random}\right) = \left(\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}}\right) E\left(SC_{Lift}\right) + \left(\frac{\frac{P_{small}}{2}}{\frac{P_{small}}{2}+1-P_{small}}\right) E\left(DC_{Lift}\right) + t_{load} + \left(\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}}\right) t_{unload} + \left(\frac{\frac{P_{small}}{2}}{\frac{P_{small}}{2}+1-P_{small}}\right) \cdot \left(\left(1-\frac{1}{n_t}\right) 2t_{unload} + \left(\frac{1}{n_t}\right) t_{unload}\right) + t_0 \tag{3.27}$$

The average travel time can be calculated using different equations, depending on the lift configuration and the position of the I/O point. The corresponding equations have already been specified in chapter 3.3.1 and chapter 3.3.2.

In the following, cycle time and throughput are calculated as a function of P_{small} for Case 1. If $P_{small} = 0$, only large ULs are stored and only single-command cycles are performed. The throughput is lowest here, since just one large UL is transported per cycle. As P_{small} increases, more small ULs are stored and more dual-command cycles are performed. As a result, the average cycle time increases. Despite the longer cycle time, there has been an increase in the total throughput because one large or two small ULs are transported per cycle.

Table 3.11: Input values

Parameter	Value
d_{sy}	0.5 m
n_t	50 tiers
v_{lift}	4 m/s
a_{lift}	3 m/s²
t_{load}	4 s
t_{unload}	4 s
t_0	0 s (for simplification)

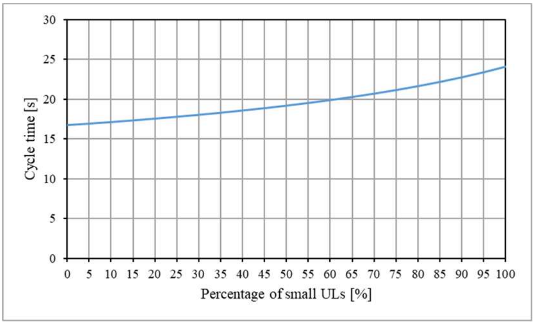


Figure 3.17: Cycle time (cycle time equation (3.27))

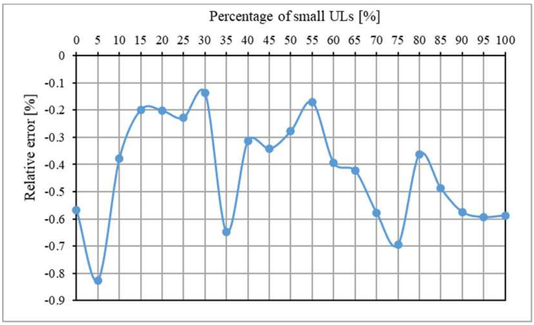


Figure 3.18: Relative error (cycle time equation (3.27))

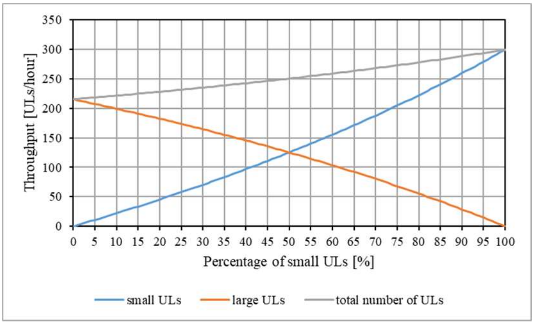


Figure 3.19: Throughput in dependency of the percentage of small ULs (appendix E, equation (E.6) with cycle time random equation (3.27))

If Strategy 1 is used, single-command cycles are always performed (lift configuration: side-by-side with $c_{lift} = 2$), regardless of the percentage of small ULs. Only the throughput depends on the percentage of small ULs. The equation is valid for:

- The lift performs storage or retrieval operations
- Lift capacity: $c_{lift} = 2$
- Capacity requirement of small ULs is one
- Capacity requirement of large ULs is two
- Two small ULs are unloaded together at the same tier
- Lift configuration: Case 1
- Type of command cycle: Single-command cycle $k = 1$
- Sequencing strategy: Strategy 1

$$E\left(MC_{TwoSizesOfUL\_Lift\_Strategy1}\right) = E\left(SC_{Lift}\right) + t_{load} + t_{unload} + t_0 \quad (3.28)$$

Using the equation of Case 1 for $E\left(SC_{Lift}\right)$, the average cycle time is 16.74 s with a maximum relative error of -0.62 %. In this case, the average cycle time is independent of the percentage of small ULs.

Comparing the achieved throughput for the control strategy random (equation (3.27)) with Strategy 1 (equation (3.28)), a higher throughput is achievable with Strategy 1 because the number of trips and load handling times is reduced (see Figure 3.20).

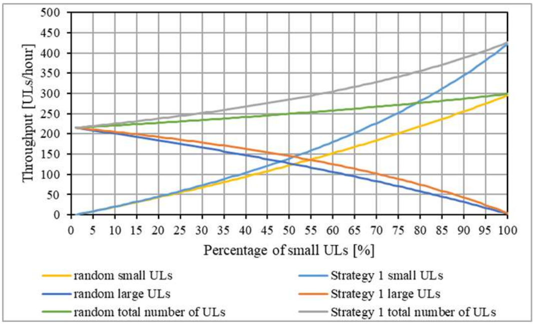


Figure 3.20: Throughput in dependency of the percentage of small ULs (appendix E, equation (E.6) with cycle time random equation (3.27) and cycle time Strategy 1 equation (3.28))

3.3.6 Two Different Sizes of ULs and a Lift Capacity Larger than Two

If the lift has a capacity of four, there are four possible ways in which the lift can be occupied, with two different sizes of ULs. The figure below illustrates these four scenarios. Depending on the percentage of small ULs, the probability of occurrence of each state can be determined and thus, the travel time, cycle time, and throughput can be calculated.

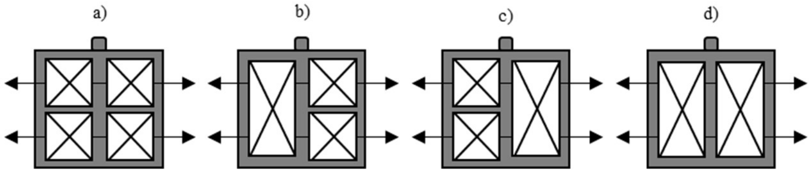


Figure 3.21: Side-by-side lift configuration with a capacity of four

The equation is valid for:

- The lift performs storage or retrieval operations
- Lift capacity: $c_{lift} = 4$
- Capacity requirement of small ULs is one
- Capacity requirement of large ULs is two
- Lift configuration: side-by-side
- Lift configuration: Case 1
- Type of command cycle: Multi-command cycle $k \in \{2, 3, 4, \ldots\}$
- Control strategy: Optimized sequencing

In case a) it is a quadruple-command cycle. The equation (3.29) can be used for the calculation of the cycle time:

$$Part1 = \left(\frac{\frac{P_{small}}{2}}{\frac{P_{small}}{2}+1-P_{small}}\right) \cdot \left(\frac{\frac{P_{small}}{2}}{\frac{P_{small}}{2}+1-P_{small}}\right) \cdot \left(2 \cdot \frac{4}{5}\frac{H_{lift}}{v_{lift}} + 5\frac{v_{lift}}{a_{lift}} + 2t_{load} + 4t_{unload} + t_0\right) \quad (3.29)$$

In case b) and c) it is a triple-command cycle. The equation (3.30) can be used for the calculation of the cycle time:

$$Part2 = \left(\frac{\frac{P_{small}}{2}}{\frac{P_{small}}{2}+1-P_{small}}\right) \cdot \left(\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}}\right) \cdot \left(2 \cdot \frac{3}{4}\frac{H_{lift}}{v_{lift}} + 4\frac{v_{lift}}{a_{lift}} + 2t_{load} + 3t_{unload} + t_0\right) \quad (3.30)$$

In case d) it is a dual-command cycle. The equation (3.31) can be used for the calculation of the cycle time:

$$Part3 = \left(\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}}\right) \cdot \left(\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}}\right) \cdot \left(2 \cdot \frac{2}{3}\frac{H_{lift}}{v_{lift}} + 3\frac{v_{lift}}{a_{lift}} + 2t_{load} + 2t_{unload} + t_0\right) \quad (3.31)$$

To calculate the total cycle time, the durations of the four time shares are added up.

$$E\left(MC_{TwoSizesOfUL\_Lift\_OptimizedSequencing}\right) = Part1 + 2 \cdot Part2 + Part3 \quad (3.32)$$

In the following, the cycle time is calculated as a function of P_{small} (see Figure 3.22). The relative error increases with the number of small ULs (see Figure 3.23). As mentioned in chapter 3.3.4, the reason is the overestimation of the number of acceleration and deceleration operations and the number of unloading operations, as well as the short travel distances. However, it is also possible to use the exact equation for a multi-command cycle, as described in chapter 3.3.4. The relative error is then reduced.

Table 3.12: Input values

Parameter	Value
d_{sy}	0.5 m
n_t	50 tiers
v_{lift}	4 m/s
a_{lift}	3 m/s²
t_{load}	4 s
t_{unload}	4 s
t_0	0 s (for simplification)

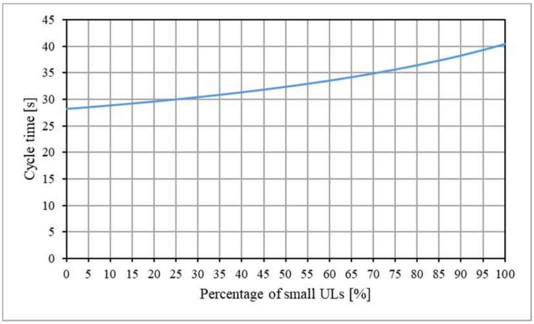


Figure 3.22: Cycle time (cycle time equation (3.32))

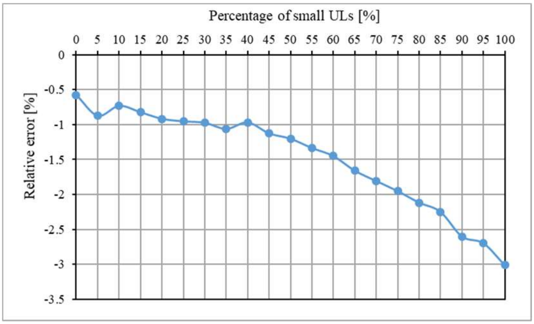


Figure 3.23: Relative error (cycle time equation (3.32))

If multi-command cycles are performed, it is particularly important that the ULs are sequenced in such a way that additional upward and downward travels are avoided. If one lift is used for storage or retrieval operations, the ULs must be sorted according to size. For example, all small ULs are processed on the travel up and the large ULs are processed on the travel down.

Figure 3.16 demonstrates that Strategy 1 yields a higher throughput than assigning each UL to a different tier. It is therefore recommended to group two small ULs together and assign them to a single tier. The following section provides a generally applicable equation for this case.

For this lift configuration, the general equation for a multi-command cycle with two different sizes of ULs can be derived.

The equation is valid for:

- The lift only performs storage or retrieval requests
- Lift capacity: $c_{lift} \in \{2, 4, 6, 8, 10, \dots\}$
- Capacity requirement of small ULs is one
- Capacity requirement of large ULs is two
- Two small ULs are unloaded together at the same tier
- Lift configuration: side-by-side
- Lift configuration: Case 1
- Type of command cycle: Multi-command cycle $k \in \{1, 2, 3, 4, \dots\}$
- Control strategy: Strategy 1
- Two small ULs are unloaded together at the same tier

$$E\left(MC_{TwoSizesOfUL\_Lift\_Strategy1\_New}\right) = 2 \cdot \frac{\frac{c_{lift}}{2}}{\frac{c_{lift}}{2}+1} \cdot \frac{H_{lift}}{v_{lift}} + \left(\frac{c_{lift}}{2} + 1\right)\frac{v_{lift}}{a_{lift}} + \frac{c_{lift}}{2} \cdot t_{load} + \frac{c_{lift}}{2} \cdot t_{unload} + t_0 \quad (3.33)$$

In practice, it is important to consider the feasibility of different lift sizes. While a larger lift can increase throughput, it also comes with higher investment costs and space requirements. It is equally important to assess the overall system, the interaction between lift and shuttle vehicles, in conjunction with the chosen lift.

3.4 Cycle Time Calculation for the Shuttle Vehicle

In this subchapter the cycle time models for a tier-captive and aisle-captive SBS/RS are defined. Each tier of a SBS/RS is equipped with one shuttle vehicle. The shuttle vehicle can perform storage and retrieval operations. For a detailed process description of the control logic see appendix A - Figure A.4.

Depending on the capacity of the shuttle vehicle, the number of different UL sizes, and the capacity of each storage channel, different command cycles can be performed (see Figure 2.19). In this chapter, equations are derived for calculating the travel and cycle time for the shuttle vehicle, depending on the shuttle vehicle's capacity, the percentage of small ULs, and the capacity of each storage channel.

Typically, a tier-captive and aisle-captive SBS/RS consists of two tote lifts (one for storage operations and one for retrieval operations) and two buffers to decouple horizontal and vertical movements. This system can be classified into two different configurations:

(i) The lift is located at the end of the aisle. In this case, the equations of chapter 3.3 (Case 2) can be used as a basis.
(ii) The lift is located in the middle of the aisle. In this case, the equations of chapter 3.3 (Case 4) can be used as a basis.

In the following only Case 2 will be applied for the shuttle vehicle, where the distance between the buffer and the first storage channel is known ($= d_{bs}$). The transfer point between the shuttle vehicle and the buffer is the starting and ending point of each command cycle.

Only equations that have not yet been derived for the lift system in chapter 3.3 will be derived in detail in this chapter.

Still though, due to the large number of possible shelf configurations and shuttle vehicle configurations, it is not feasible to derive an equation for every configuration in this work. Instead, the focus lies on the most common and important SBS/RS configurations.

3.4.1 Single-Command Cycle and Single-Deep Storage

The equation is valid for:

- The shuttle vehicle performs storage or retrieval operations
- Shuttle vehicle capacity: $c_s = 1$
- Capacity requirement of an UL is one
- Type of command cycle: Single-command cycle $k = 1$
- Storage depth (total number of rows in one storage channel): $n_r = 1$

Exact approximation equation for the average travel time:

$$E\left(t_{SC\_Shuttle}\right) = \frac{(n_c - 1)d_{s_x}}{v_s} + 2\frac{v_s}{a_s} + 2\frac{d_{bs}}{v_s} \tag{3.34}$$

Approximation equation for the average travel time:

In addition to the rack length, the distance between the buffer and the first storage channel can be added. In most cases, the distance is equal to the distance between two storage channels. For simplification, it can be assumed that the travel distance d_{bs} between the buffer and the first storage channel is very small, compared to the total rack length L_{Rack}, and can therefore be neglected. Therefore, only the total length of the rack L_{Rack} is considered in the equation. If the distance d_{bs} is very large, $L_{Rack} + d_{bs}$ should be considered as the travel distance in the equation.

$$E\left(t_{SC\_Shuttle}\right) = 2 \cdot \left(\frac{1}{2}\frac{L_{Rack}+d_{bs}}{v_s} + \frac{v_s}{a_s}\right) = \frac{L_{Rack}+d_{bs}}{v_s} + 2\frac{v_s}{a_s} \tag{3.35}$$

Equation for the average cycle time:

This is the sum of the travel time, the (un)loading time between the shuttle vehicle and the buffer ($t_{(un)loadSB}$), and the (un)loading time between the shuttle vehicle and the storage location ($t_{(un)load}$).

$$E(SC_{Shuttle}) = E\left(t_{SC\_Shuttle}\right) + t_{(un)loadSB} + t_{(un)load} + t_0 \tag{3.36}$$

3.4.2 Dual-Command Cycle and Single-Deep Storage

The shuttle vehicle performs a combined storage and retrieval operation, starting with the storage operation followed by the retrieval operation. The equation is valid for:

- The shuttle vehicle performs storage and retrieval operations
- Shuttle vehicle capacity: $c_s = 1$
- Capacity requirement of an UL is one
- Type of command cycle: Dual-command cycle $k = 2$
- Storage depth (total number of rows in one storage channel): $n_r = 1$

Exact approximation equation for the average travel time:

$$E(t_{DC\_Shuttle}) = \frac{1}{n_r}\left(\frac{(n_c-1)d_{s_x}}{v_s} + 2\frac{v_s}{a_s} + 2\frac{d_{bs}}{v_s}\right) + \left(1 - \frac{1}{n_c}\right) \cdot \left(\frac{(n_c-1)d_{s_x}}{v_s} + 2\frac{v_s}{a_s} + 2\frac{d_{bs}}{v_s} + \frac{1}{3}\frac{(n_c+1)d_{s_x}}{v_s} + \frac{v_s}{a_s}\right) \quad (3.37)$$

Approximation equation for the average travel time:

$$E(t_{DC\_Shuttle}) = 2 \cdot \left(\frac{1}{2}\frac{L_{Rack}}{v_s} + \frac{v_s}{a_s}\right) + \frac{1}{3}\frac{L_{Rack}}{v_s} + \frac{v_s}{a_s} = \frac{4}{3}\frac{L_{Rack}}{v_s} + 3\frac{v_s}{a_s} \quad (3.38)$$

Equation for the average cycle time:

Since the shuttle vehicle has the capacity of one and is equipped with one load handling device, it is only possible to drop off and pick up ULs one after the other (e.g., drop off one UL at the right side and then pick up one UL from the opposite side). Therefore, for each UL, independent load handling times (one loading and one unloading time) have to be considered.

$$E(DC_{Shuttle}) = E(t_{DC\_Shuttle}) + 2t_{(un)loadSB} + 2t_{(un)load} + t_0 \quad (3.39)$$

In the following the cycle times (equation (3.39)), using equation (3.37) and equation (3.38) for the travel time are compared (see Figure 3.24). For the travel distance, $L_{Rack} + d_{bs}$ is inserted in equation (3.38). As the number of storage channels increases, the cycle time increases linearly. The larger the distance d_{bs}, the higher the cycle time. The reason for the large relative error when having a low number of storage channels lies in the assumption that the maximum velocity of the shuttle vehicle is always reached, including for small distances (see Figure 3.25).

Table 3.13: Input values

Parameter	Value
d_{sx}	0.5 m
d_{bs}	0.5, 1.0, 2.0 m
n_c	100 storage channels
v_S	2.5 m/s
a_S	1.5 m/s²
$t_{(un)load}$	4 s
$t_{(un)loadSB}$	4 s
t_0	0 s (for simplification)

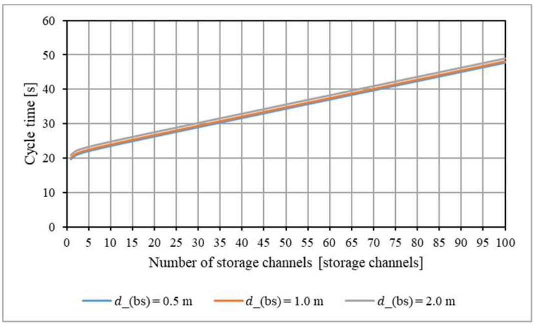


Figure 3.24: Cycle time (cycle time exact equation (3.37))

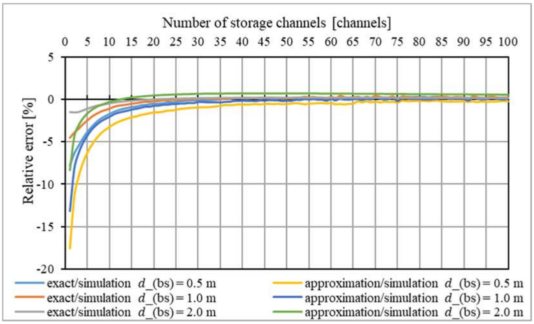


Figure 3.25: Relative error (cycle time exact equation (3.37) and cycle time approximation equation (3.38))

3.4.3 Multi-Command Cycle and Single-Deep Storage

The shuttle vehicle performs several storage and retrieval operations. The number of storage operations is directly proportional to the number of ULs that the shuttle vehicle can carry. Consequently, an equal number of retrieval operations must be performed. All storage operations are carried out first, and then all retrieval operations.

The shuttle vehicle can be a side-by-side, one-behind-the-other or a combination of both configurations. Depending on the shuttle vehicle configuration, it may be possible to hand over multiple ULs simultaneously. However, to determine the exact solution, individual probabilities need to be calculated, based on the shuttle vehicle configurations (such as the probability that two ULs are handed over simultaneously, three ULs are handed over simultaneously, etc.). Depending on the filling degree, the probability can be calculated that one storage channel is empty, that two storage channels next to each other are empty, that three storage channels next to each other are empty, etc. The number of ULs that can be transferred simultaneously determine the average number of acceleration and deceleration operations and the average number of (un)loading operations. For simplicity, it is assumed that only one UL can be transferred between the shuttle vehicle and the storage locations at a time.

The equation is valid for:

- The shuttle vehicle performs storage and retrieval operations
- Shuttle vehicle capacity: $c_s \in \{1, 2, 3, 4, \ldots\}$
- Capacity requirement of an UL is one
- Type of command cycle: Multi-command cycle $k \in \{1, 2, 3, 4, \ldots\}$
- Storage depth (total number of rows in one storage channel): $n_r = 1$
- Sequencing strategy: FCFS sequencing and optimized sequencing
- All ULs are (un)loaded simultaneously between the buffer and the shuttle vehicle

It is assumed that all ULs are transferred simultaneously between the shuttle vehicle and the buffer, regardless of the number of ULs. Therefore, the load handling time is fixed at $t_{(un)loadSB}$.

Approximation equation for the average cycle time – FCFS sequencing:

The following equation is a good approximation for calculating the cycle time of the shuttle vehicle for a multi-command cycle, whereby c_s is the maximum capacity of the shuttle vehicle.

The storage and retrieval requests are processed on a FCFS basis. Storage requests are processed in the sequence in which they are picked up by the shuttle vehicle, while retrieval requests are processed in the sequence in which they enter the virtual waiting queue. The processing sequence begins with all storage requests, followed by all retrieval requests.

$$E(MC_{Shuttle}) = 2 \cdot \left(\frac{1}{2}\frac{L_{Rack}}{v_S} + \frac{v_S}{a_S}\right) + (2c_s - 1)\left(\frac{1}{3}\frac{L_{Rack}}{v_S} + \frac{v_S}{a_S}\right) + 2t_{(un)loadSB} + 2c_s \cdot t_{(un)load} + t_0 \tag{3.40}$$

Approximation equation for the average cycle time – optimized sequencing:

The following equation is a good approximation for calculating the cycle time of the shuttle vehicle for a multi-command cycle. The storage and retrieval requests are processed in an optimized sequence. That means, the shuttle vehicle must travel only once from the inbound buffer to the storage channel furthest away and back to the outbound buffer. First all storage requests are processed and then all retrieval requests.

$$E(MC_{Shuttle}) = 2 \cdot \frac{2c_s}{2c_s+1} \cdot \frac{L_{Rack}}{v_s} + (2c_s + 1)\frac{v_s}{a_s} + 2t_{(un)loadSB} + 2c_s \cdot t_{(un)load} + t_0 \quad (3.41)$$

In Figure 3.26, the cycle time is calculated for both cases (FCFS sequencing and optimized sequencing). The results show, that an optimized sequence achieves the smaller cycle time. The larger the capacity of the shuttle vehicle, and the more storage and retrieval operations are performed, the larger the deviation between the two cases. The reason for the increased relative error (see Figure 3.27) for the optimized sequence strategy is the large number of short travel distances because it is assumed that the maximum velocity is always reached (which is not always the case for short distances).

It is recommended to sequence the requests and process the storage orders from the front to the back and the retrieval orders on the way back. Depending on the shuttle vehicle configuration, the orders must be presorted, or if this is not possible, the reserved places of the ULs must be swapped with each other to avoid dead lock situations.

Table 3.14: Input values

Parameter	Value
d_{sx}	0.5 m
d_{bs}	0.5 m
n_c	100 storage channels
v_S	2.5 m/s
a_S	1.5 m/s²
$t_{(un)load}$	4 s
$t_{(un)loadSB}$	4 s
t_0	0 s (for simplification)

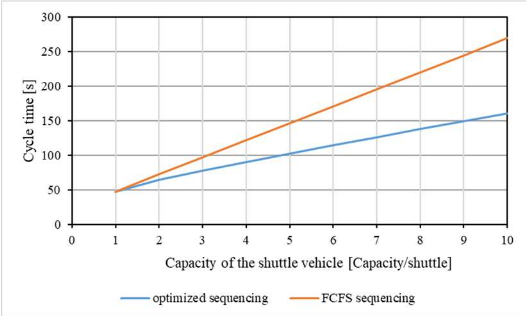


Figure 3.26: Cycle time (cycle time FCFS sequencing equation (3.40) and cycle time optimized sequencing equation (3.41))

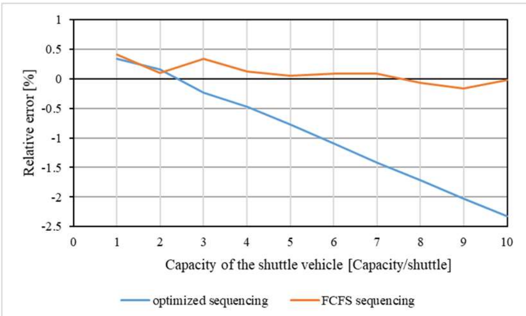


Figure 3.27: Relative error (cycle time FCFS sequencing equation (3.40) and cycle time optimized sequencing equation (3.41))

3.4.4 Single-Command Cycle and Double-Deep Storage

This subchapter presents cycle time models for double-deep storage systems. In such systems, ULs can be stored in either empty or half-full storage channels. Retrieval is possible from both, half-full and full storage channels. However, when retrieving the UL from the back row of full storage channels, the blocking UL, which is stored in the front row, must be relocated first.

For double-deep storage systems, the time required for the relocation operation must be considered. Therefore, it is important to know the states of individual storage channels, empty, half-full, and full (see Figure 4.5), since they affect the load handling times and the occurrence probability of the relocation process. All of these states and probabilities should be specified as a function of the filling degree.

Lippolt (2003) determined the individual probabilities of occurrence for all possible storage and retrieval operations and used a Markov chain to determine the transition probability between the three states of storage channels (empty, half-full and full). The storage channels must always be filled from the back to the front. Therefore, it is not possible that only the front row is occupied. In addition, the probability of a required relocation operation can be determined.

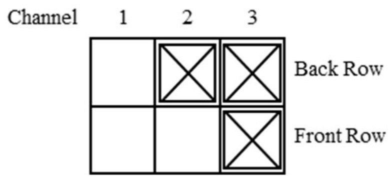


Figure 3.28: State of storage channels: (1) empty, (2) half-full (back row is occupied and front row is empty), (3) full (back and front row are occupied)

In the following, the probabilities for the states of individual channels and the probability for relocation are listed as a function of the filling degree z (see also Lippolt (2003)).

$$P_{empty}(z) = \frac{1-z}{1+z} \tag{3.42}$$

$$P_{halffull}(z) = \frac{2z(1-z)}{1+z} \tag{3.43}$$

$$P_{full}(z) = \frac{2z^2}{1+z} \tag{3.44}$$

$$P_{relocation}(z) = \frac{z}{1+z} \tag{3.45}$$

In the following, the probabilities for all possible storage and retrieval operations are listed (see also Lippolt (2003)):

Storage of an UL in an empty storage channel:

$$P_{S1}(z) = \frac{P_{empty}(z)}{P_{empty}(z)+P_{halffull}(z)} = \frac{1}{2z+1} \tag{3.46}$$

Storage of an UL in a half-full storage channel:

$$P_{S2}(z) = \frac{P_{halffull}(z)}{P_{empty}(z)+P_{halffull}(z)} = \frac{2z}{2z+1} \tag{3.47}$$

Retrieval of an UL from a half-full storage channel:

$$P_{R1}(z) = \frac{P_{halffull}(z)}{2P_{full}(z)+P_{halffull}(z)} = \frac{1-z}{1+z} \tag{3.48}$$

Retrieval of an UL from the front row of a full storage channel:

$$P_{R2}(z) = \frac{P_{full}(z)}{2P_{full}(z)+P_{halffull}(z)} = \frac{z}{1+z} \tag{3.49}$$

Retrieval of an UL from the back row of a full storage channel and relocation of the blocking UL into the front row of a half-full storage channel:

$$P_{R3}(z) = \frac{P_{full}(z)}{2P_{full}(z)+P_{halffull}(z)} \cdot \frac{P_{halffull}(z)}{P_{empty}(z)+P_{halffull}(z)} = \frac{2z^2}{2z^2+3z+1} \tag{3.50}$$

Retrieval of an UL from the back row of a full storage channel and relocation of the blocking UL into the back row of an empty storage channel:

$$P_{R4}(z) = \frac{P_{full}(z)}{2P_{full}(z)+P_{halffull}(z)} \cdot \frac{P_{empty}(z)}{P_{empty}(z)+P_{halffull}(z)} = \frac{z}{2z^2+3z+1} \tag{3.51}$$

If the UL has to be retrieved from the back row of a full storage channel, the UL in the front row has to be relocated. From a random retrieval location, the distance to a potential relocation channel must be determined. A potential relocation channel is an empty or a half-full storage channel. The average distance between two randomly selected storage channels is $\frac{1}{3}L_{Rack}$. This means, the mean relocation distance is between 0 (opposite side) and $\frac{1}{3}L_{Rack}$. The next step is to determine the average relocation distance. Three scenarios are to be distinguished:

(i) Random relocation channel

A relocation channel is randomly selected from all empty storage locations in this tier. The mean relocation distance is the mean distance between two randomly selected storage channels. Therefore, the expected value for the distance between the storage channels is $\frac{1}{3}L_{Rack}$. This expected value is always the same, regardless of the filling degree.

$$E(L_{relocation}) = \frac{1}{3}L_{Rack} \tag{3.52}$$

The expected time needed for a relocation operation can be calculated by multiplying the sum of the required travel time and the load handling times with the probability that a relocation operation will occur.

$$\begin{aligned} E(t_{relocation}) = P_{relocation}(z) \cdot \Big(2 \cdot \Big(\frac{E(L_{relocation})}{v_S} + \frac{v_S}{a_S}\Big) + \\ t_{(un)loadfront} + P_{S1}(z)t_{(un)loadback} + \\ P_{S2}(z)t_{(un)loadfront}\Big) \end{aligned} \tag{3.53}$$

(ii) Nearest neighbor – nearest relocation channel on the same rack side

A relocation channel is searched closest to the retrieval channel on the same side of the rack. The first relocation channel is the storage channel closest to the retrieval channel that is empty or half-full. If several potential relocation channels have the same distance, one of them is selected randomly.

There are exactly two channels at a distance of one storage channel each, one to the right and the other to the left of the retrieval channel. The probability that there is a relocation channel with a distance of one storage channel, is the counter probability that the storage channel is fully occupied.

$$P(1) = 1 - P_{full}(z)^2 \tag{3.54}$$

There are exactly two channels at a distance of two channels. The probability of finding a relocation channel at a distance of two channels is thus:

$$P(2) = \left(1 - \left(1 - P_{full}(z)^2\right)\right)\left(1 - P_{full}(z)^2\right) \tag{3.55}$$

From this, the general equation can be derived:

$$P(i) = P_{full}(z)^{2i-2} - P_{full}(z)^{2i}, \qquad \forall i \geq 1 \tag{3.56}$$

If the weighted values of the probabilities of the relocation channel distance are summed up with the relocation channel distance, the result is the mean relocation channel distance:

$$E(L_{relocation}) = \left(\sum_{i=1}^{\infty}\left(P_{full}(z)^{2i-2} - P_{full}(z)^{2i}\right) \cdot i\right) \cdot d_{sx} \tag{3.57}$$

Using the expected value of the mean relocation channel distance the expected relocation time can be calculated:

$$E(t_{relocation}) = P_{relocation}(z) \cdot \left(2 \cdot \left(\frac{E(L_{relocation})}{v_S} + \frac{v_S}{a_S}\right) + t_{(un)loadfront} + P_{S1}(z)t_{(un)loadback} + P_{S2}(z)t_{(un)loadfront}\right) \tag{3.58}$$

If only one side is considered for relocation, this can lead to a situation where there is no empty storage location on the same side as the actual blocking UL. In this case, an “intelligent” control strategy must be developed in order to avoid possible deadlocks.

(iii) Nearest neighbor – nearest relocation channel considering both rack sides

A relocation channel is searched close to the retrieval location on both sides of the tier. This doubles the number of potential relocation channels. In addition, it is also possible to transfer the UL exactly to the opposite storage channel.

For the opposite storage channel, it applies:

$$P(0) = 1 - P_{full}(z) \tag{3.59}$$

For all other channels, it applies:

$$P(i) = P_{full}(z)^{4i-3} - P_{full}(z)^{4i+1}, \qquad \forall i \geq 1 \tag{3.60}$$

Mean relocation channel distance:

$$E(L_{relocation}) = \left(\sum_{i=1}^{\infty}\left(P_{full}(z)^{4i-3} - P_{full}(z)^{4i+1}\right) \cdot i\right) \cdot d_{sx} \tag{3.61}$$

If there is the relocation to the opposite storage channel, there will be no travel time. This scenario is already included in the expected relocation channel distance. The acceleration component $\frac{v_s}{a_s}$ only applies when there is no relocation to the opposite side of the rack. This occurs with a probability of $1 - \left(1 - P_{full}(z)\right) = P_{full}(z)$. All in all, the expected relocation time is calculated as follows:

$$E(t_{relocation}) = P_{relocation}(z) \cdot \left(2 \cdot \left(\frac{E(L_{relocation})}{v_s} + P_{full}(z)\frac{v_s}{a_s}\right) + t_{(un)loadfront} + P_{S1}(z)t_{(un)loadback} + P_{S2}(z)t_{(un)loadfront}\right) \tag{3.62}$$

For a filling degree of 90 %, the probabilities for both cases (relocation on one side and relocation on both sides) are listed in the following figure. When considering both rack sides, the probability of finding a potential relocation channel directly in the adjacent storage channels is significantly higher than compared to considering only one side of the rack. As the distance from the actual retrieval channel increases, the probability of occurrence decreases.

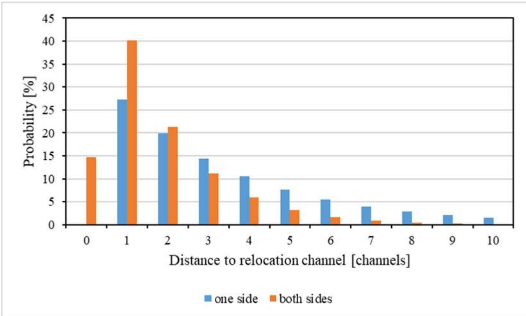


Figure 3.29: Probability for the distance of the potential relocation channel when searching in one rack side (equation (3.56)) and when searching in both rack sides ((equation (3.59) and (3.60)), at a filling degree of 90%

The following figure shows the expected mean travel distance for the relocation process as a function of the filling degree. With a low filling degree, it is possible to relocate in the adjacent or opposite storage channel directly. The higher the filling degree, the larger the distance that needs to be traveled for a relocation operation. The graph also shows, that a potential relocation channel should always be searched on both sides of the rack.

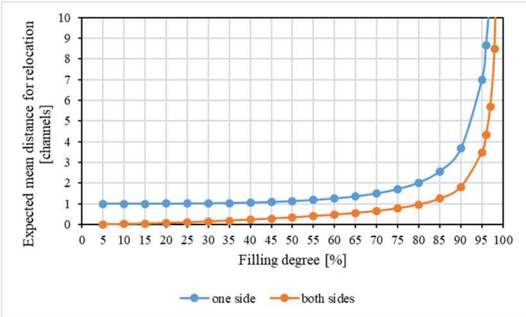


Figure 3.30: Expected mean distance (one side equation (3.57), both sides equation (3.61))

In the following, cycle time models are presented for this SBS/RS configuration:

- The shuttle vehicle performs storage and/or retrieval operations
- Shuttle vehicle capacity: $c_s = 1$
- Capacity requirement of an UL is one
- Type of command cycle: Single-command cycle $k = 1$
- Storage depth (total number of rows in one storage channel): $n_r = 2$

Cycle time model for storage operation

Depending on whether ULs need to be stored in the front row or the back row, different load handling times occur. $t_2 = t_{(un)loadfront}$ for the front row and $t_3 = t_{(un)loadback}$ for the back row (see also Figure 2.22 and Figure 2.23). Using the occurrence probabilities for the storage operations, the load handling times can be weighted.

$$E\left(SC_{Shuttle\_Storage}\right) = 2 \cdot \left(\frac{1}{2}\frac{L_{Rack}}{v_S} + \frac{v_S}{a_S}\right) + t_{(un)loadSB} + P_{S1}(z)t_{(un)loadback} + P_{S2}(z)t_{(un)loadfront} + t_0 \quad (3.63)$$

Cycle time model for retrieval operation

In the case of retrieval, the required time for the relocation process must also be taken into account.

$$E\left(SC_{Shuttle\_Retrieval}\right) = 2 \cdot \left(\frac{1}{2}\frac{L_{Rack}}{v_S} + \frac{v_S}{a_S}\right) + t_{(un)loadSB} + \left(1 - P_{R2}(z)\right)t_{(un)loadback} + P_{R2}(z)t_{(un)loadfront} + E\left(t_{relocation}\right) + t_0 \quad (3.64)$$

3.4.5 Dual-Command Cycle and Double-Deep Storage

The shuttle vehicle performs a combined storage and retrieval operation, starting with the storage operation followed by the retrieval operation.

- The shuttle vehicle performs storage and retrieval operations
- Shuttle vehicle capacity: $c_s = 1$
- Capacity requirement of an UL is one
- Type of command cycle: Dual-command cycle $k = 2$
- Storage depth (total number of rows in one storage channel): $n_r = 2$

$$E\left(DC_{Shuttle}\right) = 2 \cdot \left(\frac{1}{2}\frac{L_{Rack}}{v_S} + \frac{v_S}{a_S}\right) + \frac{1}{3}\frac{L_{Rack}}{v_S} + \frac{v_S}{a_S} + 2t_{(un)loadSB} + P_{S1}(z)t_{(un)loadback} + P_{S2}(z)t_{(un)loadfront} + \left(1 - P_{R2}(z)\right)t_{(un)loadback} + P_{R2}(z)t_{(un)loadfront} + E\left(t_{relocation}\right) + t_0 \quad (3.65)$$

The total cycle time is calculated, by considering the three different relocation strategies: random, one side, and both sides (see Figure 3.31). Overall, it can be seen that the choice of relocation strategy has only a minor impact on the total cycle time. The shortest cycle time is achieved when relocation is done on both sides, followed by one side and then random. With a filling degree close to 100 % there is almost no difference in the cycle times, no matter which relocation strategy is chosen. This is due to the fact that at a very high filling degree, the number of possible relocation channels get close to zero. Thus, the same relocation channels are eligible for relocation regardless of the chosen relocation strategy. The relative error is shown in Figure 3.32. For the strategies relocation on one side and relocation on both sides the graph can be explained as follows: With an increasing filling degree, the number of relocations and relocation distance increases. With an increasing number of relocations, more short distances have to be traveled (increasing relative error). At a certain point, the relative error decreases, because the relocation distances increase.

Table 3.15: Input values

Parameter	Value
d_{sx}	0.5 m
d_{bs}	0.5 m
n_c	100 storage channels
v_S	2.5 m/s
a_S	1.5 m/s²
$t_{(un)load\_front}$	4 s
$t_{(un)load\_back}$	5 s
$t_{(un)loadSB}$	5 s
t_0	0 s (for simplification)

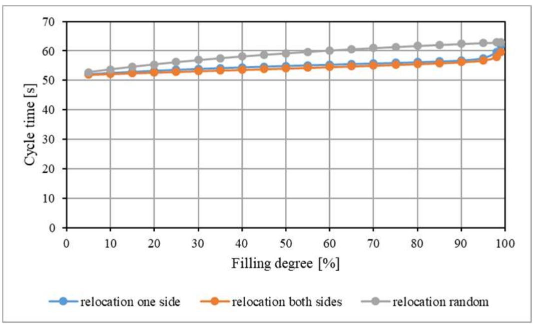


Figure 3.31: Cycle time calculation using equation (3.65) with the following equations for the mean relocation channel distance (relocation one side equation (3.57), relocation both sides equation (3.61), relocation random equation (3.52))

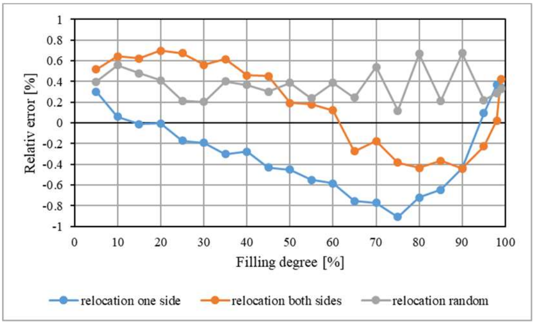


Figure 3.32: Relative error (Cycle time g equation (3.65) with the following equations for the mean relocation channel distance (relocation one side equation (3.57), relocation both sides equation (3.61), relocation random equation (3.52))

3.4.6 Multi-Command Cycle and Double-Deep Storage

The following section describes the cycle time model for multi-command cycles using the sequencing strategy. Initially, the shuttle vehicle moves from the inbound buffer to the back of the tier to store all the ULs. Subsequently, the shuttle vehicle moves from the back to the outbound buffer to process the retrieval requests.

- The shuttle vehicle performs storage and retrieval operations
- Shuttle vehicle capacity: $c_s \in \{1, 2, 3, 4, \ldots\}$
- Shuttle vehicle configuration: side-by-side
- Capacity requirement of an UL is one
- Type of command cycle: Multi-command cycle $k \in \{2, 3, 4, \ldots\}$
- Storage depth (total number of rows in one storage channel): $n_r = 2$
- Sequencing strategy: Optimized sequencing
- All ULs are (un)loaded simultaneously between the buffer and the shuttle vehicle
- Relocation strategy: Nearest neighbor – nearest relocation channel considering both rack sides

$$E(MC_{Shuttle}) = 2 \cdot \frac{2c_s}{2c_s+1} \cdot \frac{L_{Rack}}{v_s} + (2c_s + 1)\frac{v_s}{a_s} + 2t_{(un)loadSB} + c_s P_{S1}(z) t_{(un)loadback} + c_s P_{S2}(z) t_{(un)loadfront} + c_s\big(1 - P_{R2}(z)\big) t_{(un)loadback} + c_s P_{R2}(z) t_{(un)loadfront} + c_s E(t_{relocation}) + t_0 \quad (3.66)$$

In the following, the cycle time for various shuttle vehicle capacities (see Figure 3.33) is defined. On the one hand, as the shuttle vehicle capacity increases, the cycle time also increases. However, despite longer cycle times, throughput also increases with higher shuttle vehicle capacities (see Figure 3.35). On the other hand, as the filling degree increases, cycle time increases while throughput decreases. This is because the distance to a potential relocation channel increases and the frequency of relocation operations increases.

At a high filling degree, the cycle time increases very sharply. This is due to the sharp increase in relocation distances (see Figure 3.30). When using a shuttle vehicle with a large capacity there must be a large number of empty storage locations, otherwise the shuttle vehicle can not get fully loaded. With a shuttle vehicle capacity of $c_s = 4$, there must be at least five empty storage locations (four empty storage locations for storage and one for a possible relocation operation). For example, for one tier with 200 storage channels and a filling degree of 99 %, there are four empty storage locations. This means, that only three ULs can be stored and retrieved and thus the shuttle vehicle with a capacity of four does not bring any advantages. Another possibility is, that the filling degree should not exceed 98.75 %. With a filling degree of 98.75 %, there are five empty storage locations. The shuttle vehicle with a capacity of four can than always fully loaded. This means, that four storage operations followed by four retrieval operations can be processed. The relative error (see Figure 3.34) can be explained in the same way as in chapter 3.4.5.

Table 3.16: Input values

Parameter	Value
d_{sy}	0.5 m
d_{bs}	0.5 m
n_t	100 storage channels
v_S	2.5 m/s
a_S	1.5 m/s²
$t_{(un)load\_front}$	4 s
$t_{(un)load\_back}$	5 s
$t_{(un)loadSB}$	5 s
t_0	0 s (for simplification)

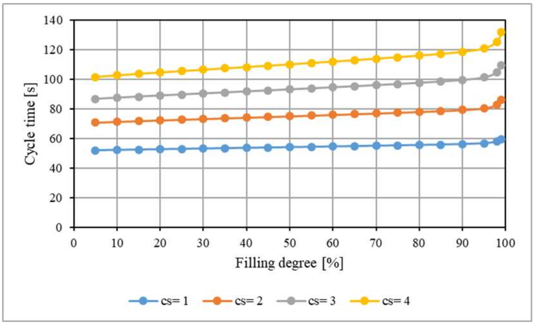


Figure 3.33: Cycle time calculation (equation (3.66))

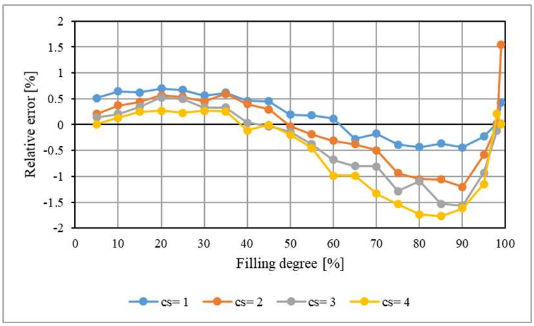


Figure 3.34: Relative error (equation (3.66))

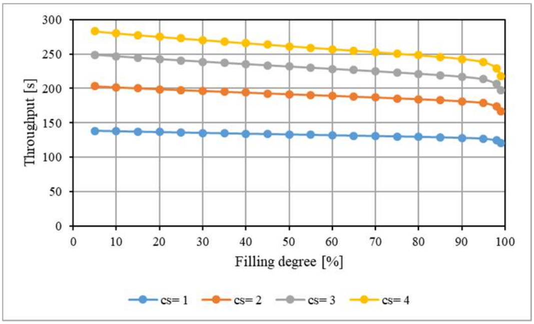


Figure 3.35: Throughput (appendix E, equation (E.14))

If the shuttle vehicle configuration is side-by-side, then it is possible to pick up and drop off ULs on both sides, independent of the shuttle vehicle occupancy. However, if the shuttle vehicle configuration is one-behind-the-other, then two cases need to be considered for the relocations process, under the assumption that only two small ULs are placed behind each other:

- If the shuttle vehicle is empty and the first relocation process is performed, then the possible relocation channel can be on both sides.
- If the shuttle vehicle is occupied, then the next relocation process can only be on the same side.

In the case of a high filling degree, it may not always be possible to relocate on the same side. Therefore, an intelligent deadlock avoidance strategy should be implemented.

For the use case of a one behind the other shuttle vehicle configurations, the following equation can be used to calculate the cycle time:

$$\begin{aligned}E(MC_{Shuttle}) = 2 \cdot \frac{2c_S}{2c_S+1} \cdot \frac{L_{Rack}}{v_S} + (2c_s + 1)\frac{v_S}{a_S} + 2t_{(un)loadSB} +\\ c_S P_{S1}(z) t_{(un)loadback} + c_S P_{S2}(z) t_{(un)loadfront} +\\ c_s\big(1 - P_{R2}(z)\big) t_{(un)loadback} +\\ c_S P_{R2}(z) t_{(un)loadfront} +\\ x_{oneside} E(t_{relocationOneSide}) +\\ x_{bothsides} E(t_{relocationBothSides}) + t_0\end{aligned} \tag{3.67}$$

Depending on the shuttle vehicle capacity c_s, the relocation process may occur on both sides or only on one side. In both situations different values for the number of relocation processes are obtained.

Table 3.17: Values for the number of relocation processes on one side and both sides

	$x_{oneside}$	$x_{bothsides}$
$c_s = 1$	0	1
$c_s \in \{2, 4, 6, \dots\}$	$\frac{c_S}{2}$	$\frac{c_S}{2}$
$c_s \in \{3, 5, 7, \dots\}$	$\frac{c_S}{2} - 1$	$\frac{c_S}{2} + 1$

In the following figure, the two shuttle vehicle configurations side-by-side and one-behind-the-other are compared, using the two equations (3.66) and (3.67). To evaluate both configurations the deviation is calculated by $deviation = 1 - \frac{equation\ (3.66)}{equation\ (3.67)}$. For the case, that the relocation process is always possible on both sides (side-by-side shuttle vehicle configuration), a minimally shorter cycle time can be achieved.

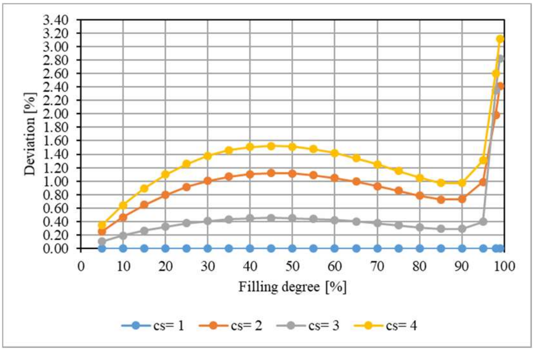


Figure 3.36: Deviation of equation (3.66) and equation (3.67)

3.4.7 Two Different Sizes of ULs, Dual-Load Handling Device and Double-Deep Storage

If the shuttle vehicle has a capacity of two and two different sizes of ULs have to be stored, there are four possible command cycles. It is assumed that always a combined storage and retrieval operation is performed and that the shuttle vehicle is always fully loaded. The figure below illustrates these four scenarios. Depending on the percentage of small ULs, the probability of occurrence of each state can be determined as well as travel time, cycle time, and throughput.

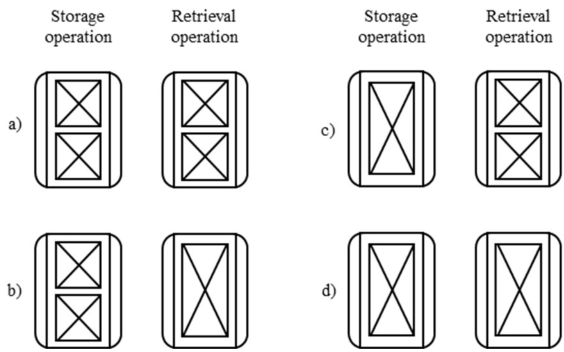


Figure 3.37: One-behind-the-other shuttle vehicle configuration with a capacity of two for combined storage and retrieval operations

The equation is valid for:

- The shuttle vehicle performs storage and retrieval operations
- Shuttle vehicle capacity: $c_s = 2$
- Capacity requirement of small ULs is one
- Capacity requirement of large ULs is two
- Type of command cycle: Multi-command cycle $k \in \{2, 3, 4\}$
- Storage depth (total number of rows in one storage channel): $n_r = 2$
- Sequencing strategy: Optimized sequencing
- All ULs are (un)loaded simultaneously between the buffer and the shuttle vehicle

The cycle time is derived based on the filling degree z and the percentage of small ULs P_{small}. Therefore, the following steps need to be performed:

Step 1) Determine the filling degree for small ULs

Based on the percentage of small and large ULs and the filling degree, the storage channels are filled with small and large ULs. All storage channels that are not occupied by large ULs are used for the storage of small ULs. The total number of possible storage channels for small ULs must first be calculated. The new filling degree for small ULs is the expected number of small ULs divided by the expected number of possible storage channels.

$$z_{forSmallULs} = \frac{n_C \cdot z \cdot \left(\frac{\frac{P_{small}}{2}}{\frac{P_{small}}{2} + 1 - P_{small}} \right)}{n_C - n_C \cdot z \cdot \left(\frac{1 - P_{small}}{\frac{P_{small}}{2} + 1 - P_{small}} \right)} \tag{3.68}$$

Step 2) Determine the states of the individual channels

The probabilities for the states of individual channels and the relocation probability are calculated as a function of the filling degree $z_{forSmallULs}$ using the equations from chapter 3.4.4. Only the states of the storage channels in which small ULs can be stored are considered.

$$P_{empty}(z_{forSmallULs}) = \frac{1 - z_{forSmallULs}}{1 + z_{forSmallULs}} \tag{3.69}$$

$$P_{halffull}(z_{forSmallULs}) = \frac{2 z_{forSmallULs}(1 - z_{forSmallULs})}{1 + z_{forSmallULs}} \tag{3.70}$$

$$P_{full}(z_{forSmallULs}) = \frac{2 {z_{forSmallULs}}^2}{1 + z_{forSmallULs}} \tag{3.71}$$

$$P_{relocation}(z_{forSmallULs}) = \frac{z_{forSmallULs}}{1 + z_{forSmallULs}} \tag{3.72}$$

Step 3) Determine the probabilities for storage and retrieval operations

Since only small ULs have different pick up and drop off times based on their storage location (front or back), it is necessary to recalculate the individual probabilities in dependency of the filling degree for small ULs $z_{forSmallULs}$.

In the following, the probabilities for the relevant storage and retrieval operations are listed:

Storage of an UL in an empty storage channel:

$$P_{S1}(z_{forSmallULs}) = \frac{1}{2z_{forSmallULs}+1} \tag{3.73}$$

Storage of an UL in a half-full storage channel:

$$P_{S2}(z_{forSmallULs}) = \frac{2z_{forSmallULs}}{2z_{forSmallULs}+1} \tag{3.74}$$

Retrieval of an UL from the front row of a full storage channel:

$$P_{R2}(z_{forSmallULs}) = \frac{z_{forSmallULs}}{1+z_{forSmallULs}} \tag{3.75}$$

Step 4) Determine the mean relocation distance and the expected relocation time

The next step is to determine the relocation channel distance for small ULs. Since both small and large ULs are stored, the relocation channel distance for small ULs increases with a growing number of large ULs being stored. Therefore, it is necessary to determine the probability of a full storage channel $P_{fullNew}$, which includes all fully occupied storage channels with small and large ULs.

The probability of a full storage channel is the total number of full storage channels occupied with small ULs plus the total number of storage channels occupied with large ULs divided by the number of all storage channels.

$$P_{fullNew}(z) = \frac{1}{n_c}\left(P_{full}(z_{forSmallULs}) \cdot \left(n_c - n_c \cdot z \cdot \left(\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}}\right)\right) + n_c \cdot z \cdot \left(\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}}\right)\right) \tag{3.76}$$

When calculating the probability of a full storage channel $P_{fullNew}(z)$, the mean relocation channel distance can be determined using the equation from chapter 3.4.4.

Mean relocation channel distance for the relocation on one side:

$$E(L_{relocationOneSideNew}) = \left(\sum_{i=1}^{\infty}\left(P_{fullnew}(z)^{2i-2} - P_{fullnew}(z)^{2i}\right)\cdot i\right)\cdot d_{sx} \tag{3.77}$$

Expected relocation time for the relocation on one side:

$$E(t_{relocationOneSideNew}) = P_{relocation}(z_{forSmallULs})\cdot \left(2\left(\frac{E(L_{relocationOneSideNew})}{v_S} + \frac{v_S}{a_S}\right) + t_{(un)loadfront} + P_{S1}(z_{forSmallULs})t_{(un)loadback} + P_{S2}(z_{forSmallULs})t_{(un)loadfront}\right) \tag{3.78}$$

Mean relocation channel distance for the relocation on both sides:

$$E(L_{relocationBothSidesNew}) = \left(\sum_{i=1}^{\infty}\left(P_{fullnew}(z)^{4i-3} - P_{fullnew}(z)^{4i+1}\right)\cdot i\right)\cdot d_{sx} \tag{3.79}$$

Expected relocation time for the relocation on both sides:

$$E(t_{relocationBothSidesNew}) = P_{relocation}(z_{forSmallULs})\cdot \left(2\left(\frac{E(L_{relocationBothSidesNew})}{v_S} + P_{fullnew}(z)\frac{v_S}{a_S}\right) + t_{(un)loadfront} + P_{S1}(z_{forSmallULs})t_{(un)loadback} + P_{S2}(z_{forSmallULs})t_{(un)loadfront}\right) \tag{3.80}$$

Step 5) Determine the cycle time

Finally, the equation for the expected cycle time can be derived. Depending on the percentage of small ULs, the probability of occurrence of each state (see Figure 3.37) can be determined as well as the cycle time, and throughput.

In scenario a) it is a quadruple-command cycle (storage of two small ULs and retrieval of two small ULs). For this, the equation (3.81) is used for the calculation of the cycle time:

$$Part1 = \left(\frac{\frac{P_{small}}{2}}{\frac{P_{small}}{2}+1-P_{small}}\right)\cdot\left(\frac{\frac{P_{small}}{2}}{\frac{P_{small}}{2}+1-P_{small}}\right)\cdot \left(2\cdot\frac{4}{5}\frac{L_{Rack}}{v_S}+5\frac{v_S}{a_S}+2t_{(un)loadSB}+ 2P_{S1}\left(z_{forSmallULs}\right)t_{(un)loadback}+ 2P_{S2}\left(z_{forSmallULs}\right)t_{(un)loadfront}+2\cdot \left(1-P_{R2}\left(z_{forSmallULs}\right)\right)t_{(un)loadback}+ 2P_{R2}\left(z_{forSmallULs}\right)t_{(un)loadfront}+ E\left(t_{relocationOneSideNew}\right)+ E\left(t_{relocationBothSidesNew}\right)+t_0\right) \tag{3.81}$$

In scenario b) it is a triple-command cycle (storage of two small ULs and retrieval of one large UL). For this, the equation (3.82) is used for the calculation of the cycle time:

$$Part2 = \left(\frac{\frac{P_{small}}{2}}{\frac{P_{small}}{2}+1-P_{small}}\right)\cdot\left(\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}}\right)\cdot\left(2\cdot\frac{3}{4}\frac{L_{Rack}}{v_S}+ 4\frac{v_S}{a_S}+2t_{(un)loadSB}+2P_{S1}\left(z_{forSmallULs}\right)t_{(un)loadback}+ 2P_{S2}\left(z_{forSmallULs}\right)t_{(un)loadfront}+t_{(un)loadback}+t_0\right) \tag{3.82}$$

In scenario c) it is a triple-command cycle (storage of one large UL and retrieval of two small ULs). For this, the equation (3.83) is used for the calculation of the cycle time:

$$Part3 = \left(\frac{\frac{P_{small}}{2}}{\frac{P_{small}}{2}+1-P_{small}}\right) \cdot \left(\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}}\right) \cdot \left(2 \cdot \frac{3}{4}\frac{L_{Rack}}{v_s} + 4\frac{v_s}{a_s} + 2t_{(un)loadSB} + t_{(un)loadback} + 2\left(1 - P_{R2}\left(z_{forSmallULs}\right)\right) t_{(un)loadback} + 2P_{R2}\left(z_{forSmallULs}\right)t_{(un)loadfront} + E\left(t_{relocationOneSideNew}\right) + E\left(t_{relocationBothSidesNew}\right) + t_0\right) \tag{3.83}$$

In scenario d) it is a dual-command cycle (storage of one large UL and retrieval of one large UL). For this, the equation (3.84) is used for the calculation of the cycle time:

$$Part4 = \left(\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}}\right) \cdot \left(\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}}\right) \cdot \left(2 \cdot \frac{2}{3}\frac{L_{Rack}}{v_s} + 3\frac{v_s}{a_s} + 2t_{(un)loadSB} + 2t_{(un)loadback} + t_0\right) \tag{3.84}$$

To calculate the total cycle time, the durations of the four time shares are added up.

$$E\left(MC_{TwoSizesOfUL\_Shuttle\_optimizedSequencing}\right) = Part1 + Part2 + Part3 + Part4 \tag{3.85}$$

For the case of $P_{small} = 0\ \%$ only large ULs are stored and retrieved. In this situation, the equation above can not be used because a deviation by 0 is not possible. This case is the classical dual-command cycle for which equation (3.40) can be used.

$$E\left(MC_{TwoSizesOfUL\_Shuttle\_optimizedSequencing}\right) = \begin{cases} \text{equation (3.38) \& equation (3.39)} & for\ P_{small} = 0\ \% \\ \text{equation (3.85)} & for\ P_{small} > 0\ \% \end{cases} \tag{3.86}$$

The following points are not taken into account in the cycle time equation developed in this chapter:

- Two small ULs are stored simultaneously in the same storage channel.
- Two small ULs are retrieved simultaneously from the same storage channel.
- One small UL is stored and one small UL is retrieved simultaneously from the opposite storage channel.
- Relocation before storage: For the case that a large UL needs to be stored, but there is no empty storage channel available, but several half-filled channels with small ULs, one small UL of a half-filled storage channel must be relocated to another half-filled storage channel to get one full and one empty storage channel.

In the following it is shown that the probability for the relocation before storage is very small and therefore can be neglected.

It is only necessary to relocate small ULs, if there is no empty storage channel for a large UL. The probability for an empty storage channel is:

$$P_{emptyNew}(z) = \frac{1}{n_c}\left(P_{empty}\left(z_{forSmallULs}\right) \cdot \left(n_c - n_c \cdot z \cdot \left(\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}}\right)\right)\right) \tag{3.87}$$

In the following figure, the probability of an empty storage channel is plotted against the filling degree, along with the percentage of small ULs. The higher the filling degree and the higher the percentage of small ULs, the lower the probability of finding an empty storage channel.

With a SBS/RS size of 200 storage channels and a filling degree of 99 %, there are on average 1.075 empty storage channels ($P_{emptyNew}(99\ \%) = 0.5375\ \%$), regardless of the percentage of small ULs. This means that on average one empty storage channel can be expected. The larger the storage system, the more empty storage locations are available on average and no relocation operations before the storage of large ULs is required. At least three empty storage locations (≙ 1.5 storage channels) should always be available per tier. Two empty storage locations are required for one large UL or two small ULs for storage, and one empty storage location is required for a potential relocation operation. Thus, the filling degree of a SBS/RS should be chosen in such a way that at least three empty storage locations are available on each tier to achieve the highest performance.

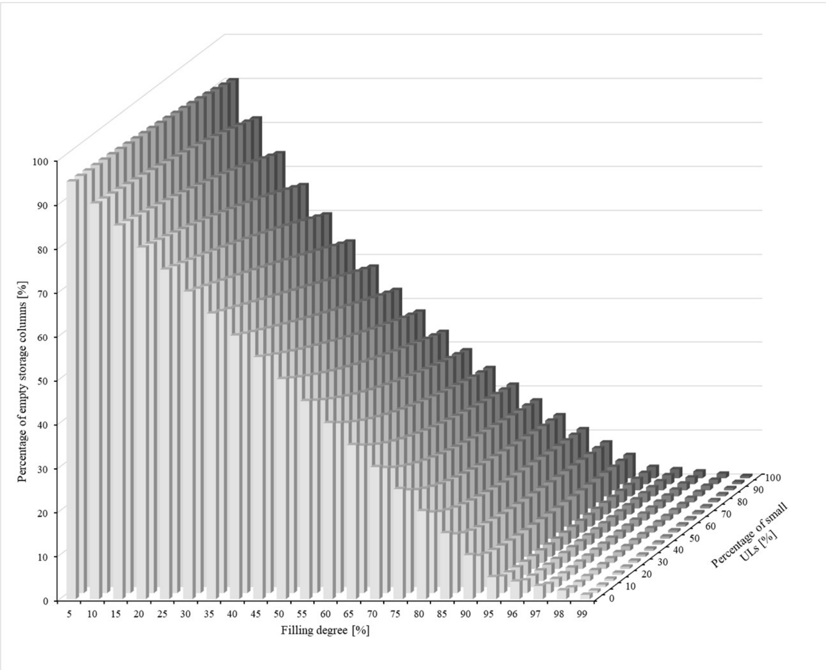


Figure 3.38: The probability for an empty storage channel (equation (3.87))

In the following, the cycle time, the relative error, and the throughput are determined. As the percentage of small ULs and the filling degree increase, the cycle time also increases. While at a low percentage of small ULs mainly dual-command cycles are performed, at a high percentage of small ULs mainly quadruple cycles are performed. The latter leads to a higher cycle time and an increase of relocation operations. At the same time, more short travel distances are traveled, which leads to a higher relative error.

All in all, the highest throughput is achieved when the percentage of small ULs is 100 % and the filling degree is close to 0 %. The higher the filling degree and the higher the percentage of small ULs, the lower the total throughput.

Table 3.18: Input values

Parameter	Value
d_{sx}	0.5 m
d_{bs}	0.5 m
n_c	100 storage channels
v_S	2.5 m/s
a_S	1.5 m/s²
$t_{(un)load\_front}$	4 s
$t_{(un)load\_back}$	5 s
$t_{(un)loadSB}$	5 s
t_0	0 s (for simplification)

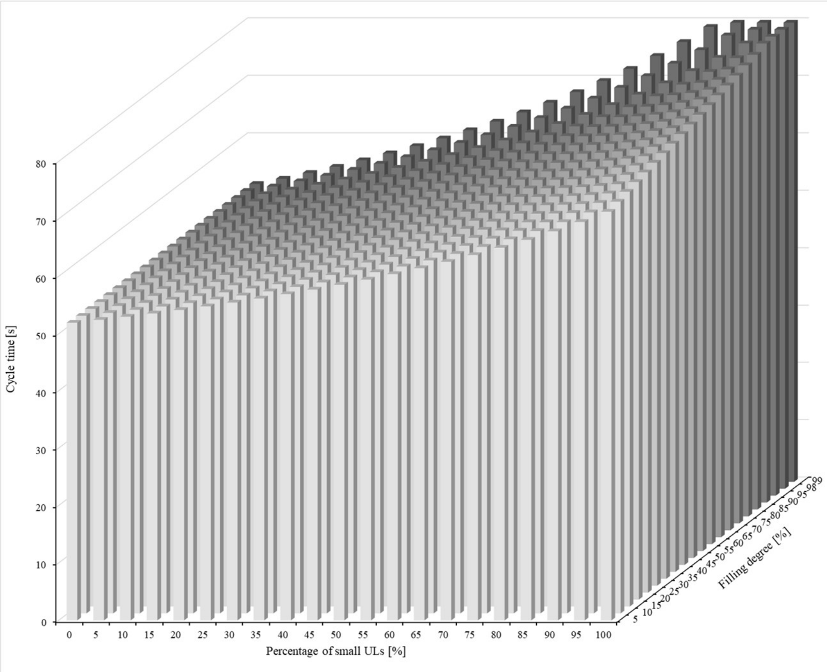


Figure 3.39: Cycle time calculation (equation (3.66))

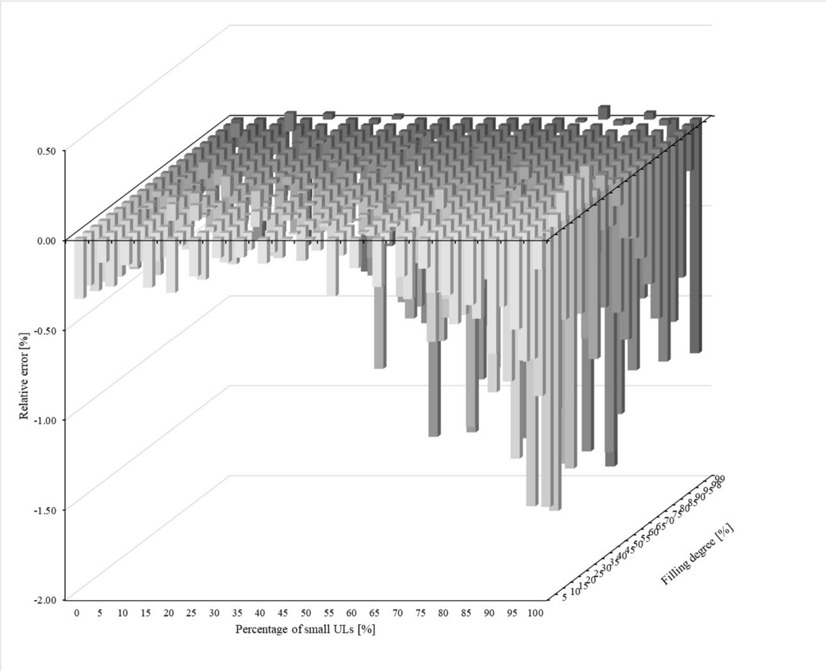


Figure 3.40: Relative error (equation (3.66))

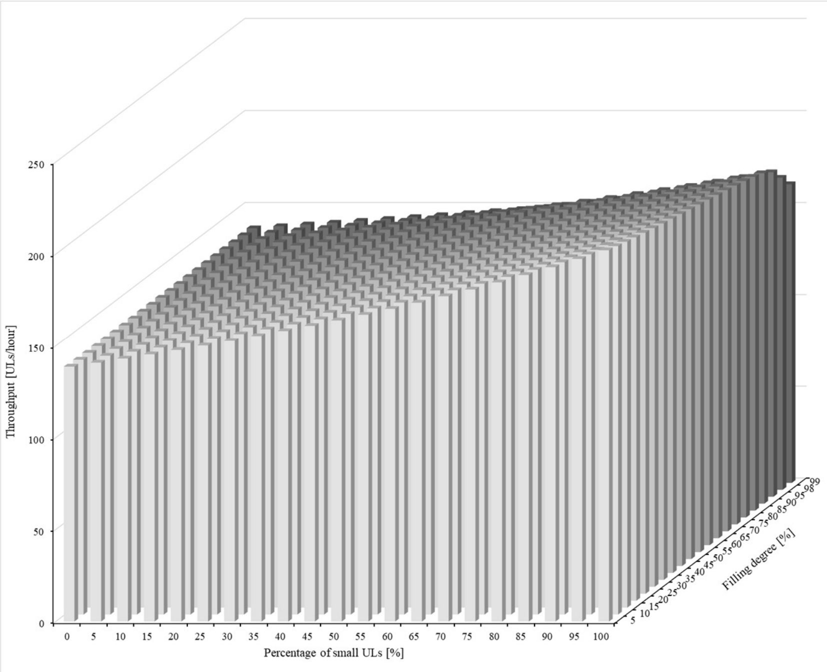


Figure 3.41: Throughput (appendix E, equation (E.15))

3.5 Throughput of the SBS/RS

In the two previous chapters, the equations for the travel time and cycle time of lift and shuttle vehicle were derived. In order to assess the performance of a SBS/RS, though, it is crucial to determine the maximum achievable throughput.

In addition to cycle time, there are also various waiting times that occur in a SBS/RS. Those depend on the system configuration and the time distribution of the inter-arrival times.

- $E(t_{InboundLiftWaitsForStorageRequest})$: The inbound lift waits at the transfer point between the lift and the pre-storage zone for incoming ULs for storage.
- $E(t_{InboundLiftWaitsInFrontOfInboundBuffer})$: The inbound lift waits at the transfer point between the lift and the inbound buffer for an empty buffer location.
- $E(t_{ShuttleVehicleWaitsForStorageRequest})$: The shuttle vehicle waits at the transfer point between the shuttle vehicle and the inbound buffer for incoming ULs for storage.
- $E(t_{ShuttleVehicleWaitsForRetrievalRequest})$: The shuttle vehicle waits at a predefined location in the tier for a relocation request.
- $E(t_{ShuttleVehicleWaitsInFrontOfOutboundBuffer})$: The shuttle vehicle waits at the transfer point between the shuttle vehicle and the outbound buffer for an empty buffer location.
- $E(t_{OutboundLiftWaitsForRetrievalRequest})$: The outbound lift waits at a predefined location for incoming ULs for retrieval.
- $E(t_{OutboundLiftWaitsInFrontOfPreStorageZone})$: The outbound lift waits at the transfer point between the lift and the pre-storage zone for an empty location to drop off the ULs. However, this waiting time almost never occurs because arriving ULs will be transported to the next downstream station directly.

3.5.1 Throughput Calculation Considering Waiting Times

To determine the expected throughput per hour, the number of cycles per hour for the lift and the shuttle vehicle must be determined. Therefore, the average time is determined, in which the lifts and the shuttle vehicles are idle and busy.

For a SBS/RS with separate lifts for storage and retrieval and one shuttle vehicle per tier, the following equations can be used for the throughput calculation. The throughput equations for all other cases from chapter 3.3 and chapter 3.4 are described in detail in appendix E.

Idle and busy for the inbound lift:

$$E(t_{InboundLiftBusy}) = E(t_{InboundLiftCycleTime}) + E(t_{InboundLiftWaitsInFrontOfInboundBuffer}) \tag{3.88}$$

$$E(t_{InboundLiftIdle}) = E(t_{InboundLiftWaitsForStorageRequest}) \tag{3.89}$$

Idle and busy for the outbound lift:

$$E(t_{OutboundLiftBusy}) = E(t_{OutboundLiftCycleTime}) + E(t_{OutboundLiftWaitsInFrontOfPreStorageZone}) \tag{3.90}$$

$$E(t_{OutboundLiftIdle}) = E(t_{OutboundLiftWaitsForRetrievalRequest}) \tag{3.91}$$

Idle and busy for the shuttle vehicle:

$$E(t_{ShuttleVehicleBusy}) = E(t_{ShuttleVehicleCycleTime}) + E(t_{ShuttleVehicleWaitsInfrontOfOutboundBuff}) + E(t_{ShuttleVehicleWaitsForRetrievalRequest}) \tag{3.92}$$

$$E(t_{ShuttleVehicleIdle}) = E(t_{ShuttleVehicleWaitsForStorageRequest}) \tag{3.93}$$

The throughput can then be determined. Depending on the lift and shuttle vehicle configuration and the percentage of small ULs, the number of command cycles in one hour is multiplied by the expected number of ULs that are stored and retrieved per cycle.

Throughput of the inbound lift:

$$\lambda_{InboundLift} = \frac{3600}{E(t_{InboundLiftBusy})+E(t_{InboundLiftIdle})} \cdot E(\#InboundLiftULs) \tag{3.94}$$

Throughput of the outbound lift:

$$\lambda_{OutboundLift} = \frac{3600}{\frac{E(t_{OutboundLiftBusy})+E(t_{OutboundLiftIdle})}{E(\#OutboundLiftULs)}} \tag{3.95}$$

Throughput of one shuttle vehicle:

This is the expected total throughput (the sum of all stored and retrieved ULs per hour).

$$\lambda_{ShuttleVehicle} = \frac{3600}{\frac{E(t_{ShuttleVehicleBusy})+E(t_{ShuttleVehicleIdle})}{E(\#ShuttleVehicleULs)}} \tag{3.96}$$

In a balanced system, the number of stored ULs is equal to the number of retrieved ULs over time.

$$\lambda_{ShuttleVehicleStorage} = \frac{1}{2}\lambda_{ShuttleVehicle} \tag{3.97}$$

$$\lambda_{ShuttleVehicleRetrieval} = \frac{1}{2}\lambda_{ShuttleVehicle} \tag{3.98}$$

The throughput of all shuttle vehicles is the throughput of one shuttle vehicle multiplied by the number of tiers.

$$\lambda_{AllShuttleVehicles} = \lambda_{ShuttleVehicle} \cdot n_t \tag{3.99}$$

Since there is an interaction between the inbound lift, shuttle vehicle, and the outbound lift, the expected throughput of the SBS/RS is the “bottleneck” of the inbound lift, shuttle vehicle, and outbound lift.

$$\lambda_{SBS/RS} = MIN\left[\lambda_{InboundLift};\ \frac{1}{2}\lambda_{AllShuttleVehicles};\ \lambda_{OutboundLift}\right] \tag{3.100}$$

3.5.2 Simplified Calculation of the Throughput

If the impact of waiting times, the buffer capacity, and inter-arrival times are taken into account, then the SBS/RS must be modelled as a queueing model (see chapter 2.2.5).

However, since waiting times and their distributions are often not available and modeling SBS/RS as queueing models is very complex, a simplified solution is presented. For a simplified determination of the maximum throughput of the SBS/RS, the following assumptions can be made:

- The system is in a state of equilibrium, i.e., the number of storage operations is equal to the number of retrieval operations over time.
- There are always enough storage and retrieval requests to be processed, which reduces the following waiting times to zero:

$$E(t_{InboundLiftWaitsForStorageRequest}) = 0$$

$$E(t_{ShuttleVehicleWaitsForRetrievalRequest}) = 0$$

- The buffers between lift and shuttle vehicle have such a large capacity that arriving ULs for storage and retrieval can always be handed over directly and there are no waiting times.

$$E(t_{InboundLiftWaitsInFrontOfInboundBuffer}) = 0$$

$$E(t_{ShuttleVehicleWaitsInFrontOfOutboundBuffer}) = 0$$

- The conveyor system connected to the SBS/RS is dimensioned in such a way that ULs to be retrieved are transferred directly without waiting.

$$E(t_{OutboundLiftWaitsInFrontOfPreStorageZone}) = 0$$

- Lift and shuttle vehicles are considered to be independent from each other and there are always enough storage and retrieval requests waiting in the buffers at each tier. This reduces the following waiting times to zero.

$$E\left(t_{ShuttleVehicleWaitsForStorageReques}\right) = 0$$

$$E\left(t_{OutboundLiftWaitsForRetrievalRequest}\right) = 0$$

Applying these assumptions and simplifications, the throughput can be calculated without considering waiting times and only by using the cycle time of the lift and shuttle vehicles.

Finally, an example is used to determine the throughput for a SBS/RS. Therefore, the throughput of the inbound and outbound lift (equation (3.28) and appendix E, equation (E.6)) and shuttle vehicle (equation (3.85) and appendix E, equation (E.15)) is calculated. The total throughput (equation (3.99) and equation (3.100)) of the SBS/RS is then calculated.

The following use case has been selected to calculate the throughput:

- Lift configuration: Case 1
- The inbound lift performs storage operations and the outbound lift retrieval operations
- Lift capacity: $c_{lift} = 2$
- Capacity requirement of small ULs is one
- Capacity requirement of large ULs is two
- Two small ULs are unloaded together at the same tier
- The shuttle vehicle performs storage and retrieval operations
- Shuttle vehicle capacity: $c_s = 2$
- Storage depth (total number of rows in one storage channel): $n_r = 2$
- Lift sequencing strategy: Strategy 1
- Shuttle vehicle sequencing strategy: Optimized sequencing
- All ULs are (un)loaded simultaneously between the buffer and the shuttle vehicle
- Relocation strategy: Nearest neighbor – nearest relocation channel considering both rack sides
- Capacity of inbound and outbound buffer each is 100

Table 3.19: Input values for the SBS/RS

Parameter	Value
d_{sx}	0.5 m
d_{bs}	0.5 m
v_S	2.5 m/s
a_S	1.5 m/s²
$t_{(un)load\_front}$	4 s
$t_{(un)load\_back}$	5 s
$t_{(un)loadSB}$	5 s
d_{sy}	0.5 m
v_{lift}	4 m/s
a_{lift}	3 m/s²
t_{load}	4 s
t_{unload}	4 s
t_0	0 s (for simplification)
z	95 %
P_{small}	66.67 %

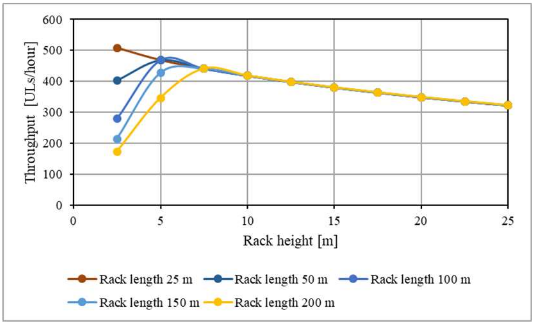


Figure 3.42: Throughput of the SBS/RS (equation (3.100))

The results show (see Figure 3.42 and appendix D.16) that for low but long racks, the shuttle vehicle determines the maximum throughput of the SBS/RS (shuttle vehicle is the bottleneck). However, as the shelf height increases, the inbound/outbound lift determines the maximum throughput (lift is the bottleneck).

Furthermore, it was found during the comparison with the simulation that:

- The maximum relative error between the analytical model and simulation is 3.62 %. In this case, the analytical model underestimates the actual throughput.
- In a balanced system with the same control strategy and configuration for the inbound and outbound lift, the inbound lift and outbound lift achieve exactly the same throughput. However, the outbound lift has lower utilization than the inbound lift because the outbound lift's waiting time, $E\left(t_{OutboundLiftWaitsInFrontOfPreStorageZone}\right)$, is assumed to be zero.

3.6 Chapter Conclusion

In this chapter, travel time, cycle time and throughput models for different SBS/RS configurations and control strategies were presented. The main goal was to develop equations for a double-deep SBS/RS with a shuttle vehicle equipped with a dual-load handling device, a lift equipped with a multi-load handling device and the storage of two different sizes of ULs. In the first step, the travel time, and cycle time equations were derived for different lift configurations and different control strategies. In the second step, these equations were transferred to the shuttle vehicle. Finally, the equations for the maximum throughput were derived under the assumption that no waiting of the lift and shuttle vehicles, but also no waiting times of orders for processing occur.

By comparing the analytical models with simulation (without considering waiting times), it was shown that the derived equations provide very good results and deviate only slightly from the results of the simulation. In most cases, the analytical equations overestimated the simulation results. This is due to the assumption (made for the analytical models) that the maximum velocity is always reached, even for short distances.

To avoid this deviation, an additional case distinction can be made based on this work (see appendix B for the case distinction that the maximum velocity is not reached and maximum velocity is reached). However, this case requires additional equations and higher computational effort. Due to the small deviation between the analytical model and simulation, the case distinction can be neglected.

The performance analysis determined the maximum achievable throughput. It was based on a simplified assumption that excluded waiting times of lift and shuttle vehicles. In reality, waiting times can negatively impact the system's throughput, and vary depending on factors, such as system configuration, and the inter-arrival times of storage and retrieval requests. To accurately calculate the performance of the SBS/RS with analytical models that consider waiting times, the presented models must be expanded by using queuing models. The simulation study in chapter 5.2 considers waiting times and in chapter 5.3 the results of the simulation study are compared with the results of the analytical models.

Based on the achieved results from the analytical models, the following recommendations can be made:

- For the lift it is recommended to pre-sort orders and process them in an optimized sequence. Thus the lift only needs to travel from the bottom (from the I/O point) to the top and back down to the I/O point once (see Figure 3.13 optimized sequencing). This reduces travel distance and travel time and increases the throughput.
- If the lift has a capacity larger than one and small and large ULs are processed, it is recommended to presort the requests so that always two small ULs can get processed together (see Strategy 1). In addition, two small ULs should always be (un)loaded together at the same tier. This reduces additional travels and if two small ULs can (un)loaded simultaneously, this will reduce load handling times.
- For the shuttle vehicles it is recommended to sequence the storage and retrieval requests and process the storage orders from the front

(from the I/O point) to the back and the retrieval orders on the way back to the I/O point.

- If in double deep shuttle systems, a relocation operation (relocation of blocking ULs) is required, the closest empty or half full storage channel should be chosen on both sides, as a potential relocation channel. This minimizes the relocation time.
- In addition, it is shown that an increasing filling degree and an increasing number of small ULs, lead to an increase of the cycle time.

4 Task Scheduling and Optimization

Customers want storage systems with high storage density, high turnover rates, and fast access times. SBS/RSs have been developed precisely for this application. However, in many SBS/RSs either the lifts or the shuttle vehicles are the bottleneck and limit the maximum possible performance (e.g., throughput). There are various factors that determine which component, e.g., the lift or shuttle vehicle, is the bottleneck in the overall system. The following figure shows the relation between the throughput and the number of tiers respectively length of aisle.

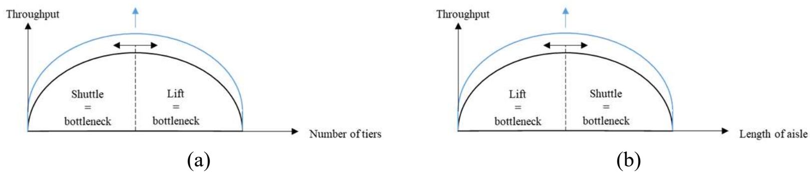


Figure 4.1: (a) relationship between throughput and number of tiers, (b) relationship between throughput and length of aisle

The more tiers a SBS/RS has, the more likely it is that the lift will be the bottleneck because it has to serve more tiers. In this case the lift is utilized 100 %. The utilization of each shuttle vehicle is lower than 100 % because waiting times for the shuttle vehicles occur. The same also applies, if the aisles are very long. As the aisle length increases, the likelihood of the shuttle vehicle being the bottleneck also increases, as it has to travel longer distances, resulting in waiting times for the lift. An optimal ratio is achieved when the shuttle vehicle and lift operate at 100 % utilization, without any waiting times for either component.

The point at which either the lift or shuttle vehicle becomes the bottleneck can shift in either direction. However, the main goal is to minimize the impact of the bottleneck within the current system and optimize the overall shuttle system. This can be achieved through several adjustments, such as:

- the acceleration and maximum velocity of lift and shuttle vehicle can be increased,
- the capacity of the lift and shuttle vehicle can be increased, or
- additional lifts and shuttle vehicles can be added.

The challenge is to reduce the impact of the bottleneck in an existing SBS/RS without making any design changes or structural measures while also enhancing the overall performance of the system (see blue throughput curve in Figure 4.1). This can only be achieved by implementing an improved control strategy.

An optimization model is typically used to reduce the impact of the bottleneck, particularly when the shuttle vehicle is the bottleneck. Furthermore, it can be assumed that there will be a performance increase through the use of an optimized control strategy.

The aim of this chapter is to study the joint optimization problem of storage location assignment and storage and retrieval scheduling in a double deep SBS/RS with dual-load handling device and two differently sized ULs, with the objective to complete all storage and retrieval request in the shortest time.

4.1 Fundamentals and Literature Review

Many research projects have been carried out in recent decades to analyze the optimization of storage systems, including the changing of parameters and the selection of the best set of parameters.

The following three topics about optimization of automated storage and retrieval systems are studied in literature, with the aim of finding the best configuration with minimum costs, minimum energy consumption or with maximum throughput:

- Cost based: minimize warehouse costs e.g., by saving in stock levels, energy consumption etc. (e.g., Rajkovic et al. (2017), Accorsi et al. (2017)).
- Energy based: minimize the total amount of energy consumption (while maximizing throughput) (e.g., Lerher (2016a), Ekren and Lerher (2016), Eder and Kartnig (2018)).
- Time based: minimize the mean travel time or mean cycle time and maximize the throughput.

To increase the throughput of a SBS/RS it is also possible to place storage and retrieval requests in an optimal sequence in which they will then be processed. The problem of optimally sequencing a given list of retrieval orders is similar to the well-known traveling salesman problem (Han et al. 1987) which is proved to be NP-complete by Papadimitriou (1977). Han et al. (1987), Mahajan et al. (1998) and Lee and Schaefer (1997) deal with the sequencing of retrieval orders in AS/RSs with a single-load handling device, to improve the throughput. They show that the sequencing methods can reduce the total travel time and thereby, increase throughput. Chung and Lee (2008) solve the joint optimization of the storage location assignment and storage and retrieval scheduling strategy. They consider the scenario that each product is stored in multiple storage locations. They use a meta-search method based on genetic algorithm to solve the optimization task. Tanaka and Araki (2009) study an AS/RS with separated in- and out-points. Their objective is to find the optimal travel route for processing of storage and retrieval requests, in order to minimize total travel time. Hachemi et al. (2012) study a sequencing problem where the objective is, to minimize the travel time of a dual-command cycle. They consider the scenario that for a retrieval request, each product can be stored in multiple storage locations.

There are many publications studying AS/RSs with more than one load handling device. Compared to a single-load handling device, more complex command cycles can be carried out. It is no longer necessary that all storage requests must be finished before retrieval requests can be performed. However, at the end of each cycle, all selected storage and retrieval requests must be processed. Sarker et al. (1991) solve the sequencing problem of a block of retrieval requests with the objective of minimizing travel distance between the storage and retrieval requests. Shunji Tanaka (2007) apply a hybrid algorithm to minimize the total traveling time of an AS/RS with a multi-load handling device. Dooly and Lee (2008) study an AS/RS with a dual-load handling device and consider different types of command cycles (different number of storage and retrieval tasks in one cycle). The objective is to minimize the total traveling time. To achieve this, they apply a polynomial optimal method and compare it with two heuristic methods (FCFS and nearest neighbor). Popović et al. (2014) develop a genetic algorithm for sequencing orders in a triple-shuttle AS/SR that operates in a class-based storage with a modified sextuple-command cycle and a planning horizon of multiple successive cycles. Yang, Miao, Xue and Qin (2015) study the joint optimization problem of storage location assignment and storage and retrieval scheduling in an AS/RS with a multi-load handling device. Furthermore, they consider block sequencing (all requests considered in the planning horizon are available) and that storage and retrieval request are processed FCFS. They use a two-phase tabu search algorithm and a genetic algorithm plus a modified nearest neighbor heuristic to solve this problem. Yang, Miao, Xue and Ye (2015) consider the same system but use a shared storage strategy. Empty storage locations caused by retrieval of one UL can be directly used to store the next UL. They apply an integer programming model for small sized problems and a variable neighborhood search heuristic for large problems. Yang et al. (2017) apply for the same use cases two tabu search algorithms, in combination with a FCFS and nearest neighbor algorithm, to generate initial solutions. Small sized problems are solved optimally using an integer programming model. Peng and Yang (2015) study the sequencing of retrieval orders where storage orders are served under FCFS policy and random storage policy using an integer programming model and tabu search algorithm. Wauters et al. (2016) study an AS/RS with a dual-load handling device in which a set of storage and retrieval requests must be scheduled such that the prioritized waiting time is minimized. They propose a decomposition approach (decomposing it into a location assignment and sequencing problem). They apply different heuristic strategies for making the assignments. They use a general mathematical model and efficient branch and bound procedure for optimizing the sequence. For more information about scheduling in AS/RS, see the survey paper from Boysen and Stephan (2016). This paper gives an overview on the scheduling of storage and retrieval requests in an AS/RS.

In addition to the classic AS/RS, there are also some publications on optimization of SBS/RSs in the literature. SBS/RSs are more complex than AS/RSs, since storage and retrieval tasks can be processed in parallel. Therefore, Carlo and Vis (2012) study the scheduling of two non-passing lifts of a SBS/RS with the objective of assigning a set of pre-defined requests to the lifts and to schedule the lifts such that the total time required to serve all requests is minimized. Kriehn et al. (2017) and Kriehn et al. (2018) apply different control strategies and a sequencing strategy for retrieval requests for a SBS/RS and demonstrate the potential of throughput improvement by using simulation. Wang et al. (2015) solve the multi-objective optimization function in the task scheduling problem between shuttle vehicles and lifts with the objective of minimum shuttle vehicle waiting time, minimum lift idle time and minimum outbound operation time by using a genetic algorithm (non-dominated sorting genetic algorithm (NSGA-II)). Another publication on scheduling of retrieval tasks in SBS/RSs is presented by Wang et al. (2019). They use a modified simulated annealing algorithm to solve scheduling task for a double-deep SBS/RS and minimize the total time of retrieval tasks. Zhan et al. (2020) also study the task scheduling problem (solved with a genetic algorithm) and developed a multi objective optimization model that minimizes the total working time and carbon emission. Habl et al. (2020) present a SBS/RS with several shuttle vehicles moving along the same rail. To prevent collision and blockings between the shuttle vehicles, a scheduling algorithm is applied to assign the extracted storage and retrievals tasks to the vehicles and select the schedule with the lowest completion time.

Table 4.1: Existing literature on scheduling and optimization on AS/RSs and SBS/RSs

Publication	System*	Objective	Considered Method**
Han et al. (1987)	AS/RS – 1 – 1 – 1	Maximize throughput	H (NN)
Lee and Schaefer (1997)	AS/RS – 1 – 1 – 1	Minimize total travelling time	LAP, H
Mahajan et al. (1998)	AS/RS – 1 – 1 – 1	Maximize throughput	H (NN)
Tanaka and Araki (2009)	AS/RS – 1 – 1 – 1	Maximize throughput	MIP
Chung and Lee (2008)	AS/RS – 1 – 1 – 1	Minimize total travelling time	GA
Hachemi et al. (2012)	AS/RS – 1 – 1 – 1	Minimize total travelling time	IP (CPLEX)
Sarker et al. (1991)	AS/RS – 1 – 2 – 1	Minimum travel distance	H
Shunji Tanaka (2007)	AS/RS – 1 – n – 1	Minimize total travelling time	TS, MILP (CPLEX)
Dooly and Lee (2008)	AS/RS – 1 – 2 – 1	Minimize total travelling time	H
Popović et al. (2014)	AS/RS – 1 – 3 – 1	Minimize total time to serve all requests	GA, GH
Peng and Yang (2015)	AS/RS – 1 – n – 1	Minimize total travelling time	TS, IP (CPLEX)
Yang, Miao, Xue and Ye (2015)	AS/RS – 1 – n – 1	Minimize total travelling time	VNS, IP (CPLEX)
Yang, Miao, Xue and Qin (2015)	AS/RS – 1 – n – 1	Minimize total travelling time	TS, GA, H, IP (CPLEX)
Yang et al. (2017)	AS/RS – 1 – n – 1	Minimize total travelling time	TS, IP (CPLEX)
Wauters et al. (2016)	AS/RS – 1 – 2 – 1	Minimize prioritized waiting time	H, MIP (CPLEX)
Carlo and Vis (2012)	SBS/RS – Lift	Minimize total time to serve all requests	H (integrated look ahead heuristic)
Kriehn et al. (2018)	SBS/RS – 1 – 1 – 1	Throughput optimization	Simulation
Kriehn et al. (2017)	SBS/RS – 1 – 1 – 1	Throughput optimization	Simulation
Habl et al. (2020)	SBS/RS – n – 1 – 1	Minimize completion time	Algorithm Simulation
Zhan et al. (2020)	SBS/RS – 2 – 1 – 1	Minimizing the total working time	GA
Wang et al. (2015)	SBS/RS – 1 – 1 – 1	Minimize shuttle vehicle waiting time, minimize lift idle time and minimize outbound operation time	GA
Wang et al. (2019)	SBS/RS – 2– 1 – 1	Minimize the total time of retrieval tasks	SA
This work	**SBS/RS – 2 – 2 – 2**	**Minimize total time to serve all requests**	**IP (CPLEX)**

*storage system – storage sell depth – number of load handling devices – size of ULs
**genetic algorithm (GA), greedy heuristics (GH), tabu search (TS), heuristic e.g., nearest neighbor heuristic, look ahead heuristic, etc. (H), variable neighbor search (VNS), mixed integer programming (MIP), integer programming (IP), linear assignment problem (LAP), simulated annealing (SA)

In this chapter, an overview of optimization and sequencing in different research and application areas is given. In total three different optimization areas (cost, energy, and time based) are identified for automated storage and retrieval systems. In Table 4.1 the considered literature is summarized and classified in respect to the considered system, objective function and used methods. The literature presented is only a selection with a focus on the time based optimization that sequences storage and retrieval requests for better performance.

The existing literature has several limitations. There is no optimization model for optimized sequencing of storage and retrieval requests for a SBS/RS, with a double-deep storage system, a dual-load handling device, and storing two different sizes of ULs. In this work, the identified research gap is closed by creating an optimization model for this configuration.

4.2 System Description

The double-deep storage rack and the shuttle vehicle equipped with a one-behind-the-other dual-load handling device, is analyzed in this chapter (see Figure 4.2). The shuttle vehicle with a dual-load handling device can carry multiple ULs. The capacity of the shuttle vehicle is two, which means that two small ULs or one large UL can be carried at a time. Therefore, the performance of multiple storage and retrieval operations at one command cycle is feasible. The increase of storage and retrieval operations performed with a dual-load handling device can lead to an increase in the total throughput of the SBS/RS, compared to a system with a single-load handling device. The main reason for this is the reduction of travel distances, empty runs, and load handling times.

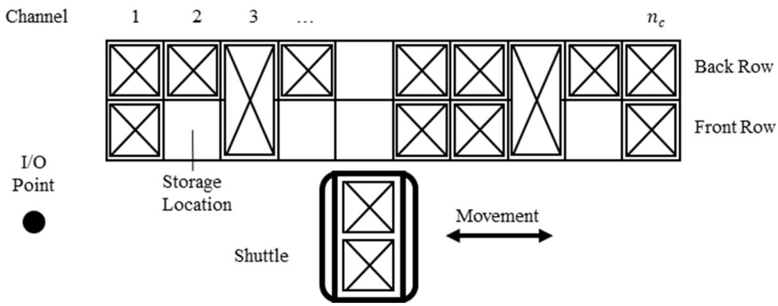


Figure 4.2: The simplified model for the optimization model

In many cases, storage and retrieval requests are processed according to the FCFS principle. This means, that the storage and retrieval requests must be processed exactly in the predetermined sequence, although there may be a better sequence in terms of total processing time and throughput. The aim is to plan the storage location assignment and the scheduling of storage and retrievals requests in advance. For this purpose, block sequencing can be applied.

The list of storage and retrieval requests changes over time as old requests are completed and new requests appear. Depending on whether the block size is fixed or variable, there are two different approaches for sequencing storage and retrieval requests:

(i) select each block of storage and retrieval requests individually, sequence it and when the blocks are completed select the next block (Han et al. 1987).
(ii) resequencing the list after every command cycle, if a new request has been added. For this case, an additional control priority strategy needs to be added, so that requests at the end of the aisle are also processed. (Han et al. 1987)

In this research the block size is fixed. This means that there is a known set of ULs available to be stored and ULs to be retrieved. All storage and retrieval requests are partitioned into several blocks, whereby each block contains the same number of command cycles. Each block needs to have the exact number of ULs to perform the predefined number of command cycles. This means that every set of storage and retrieval requests of one block can have different sizes because two storage or retrieval requests of small ULs are equal to one storage or retrieval request of a large UL. The blocks are selected FCFS for sequence. When one block is completed, the next block can be processed (see Figure 4.3).

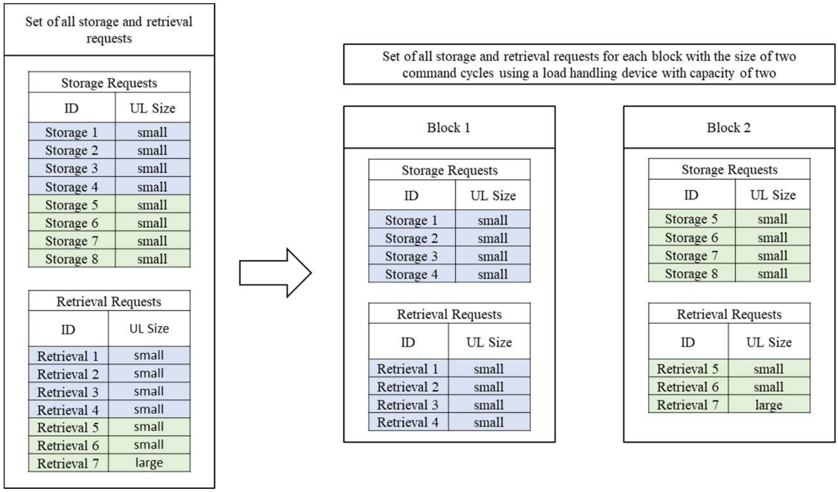


Figure 4.3: Dividing the storage and retrieval request into the individual blocks

The aim of block sequencing is to determine in where to store storage requests and in which order to process storage and retrieval requests in the planning horizon. Storage requests are assigned to empty storage locations that comply with specific restrictions, such as process-related sequences or categories. The challenge, however, is to additionally bring the storage requests into an optimal sequence with the retrieval requests. The advantage over FCFS is that the storage and retrieval requests can be combined in all possible sequences in each block. The assignment of the storage requests to an empty storage location and the sequence in which the storage and retrieval requests are processed should be optimal with respect to the objective of the shortest possible processing time.

The process of block sequencing can be described the following way: Block sequencing starts with the first block consisting of a pre-defined number of command cycles. Each command cycle corresponds to a travel of the shuttle vehicle from the starting point (I/O point) fully loaded with two small or one large UL to the selected empty storage location. After all the ULs are stored and the shuttle vehicle is emptied, the processing of the retrieval requests can start. When all retrieval requests are completed, the fully loaded shuttle vehicle travels back to the I/O point. This process is repeated until all requests of this block have been processed. Then the next block can be started (see Figure 4.4).

According to the example above (see Figure 4.3), each block consists of two command cycles. Thus, for each block, this process must be run through twice. Then the next block can be started.

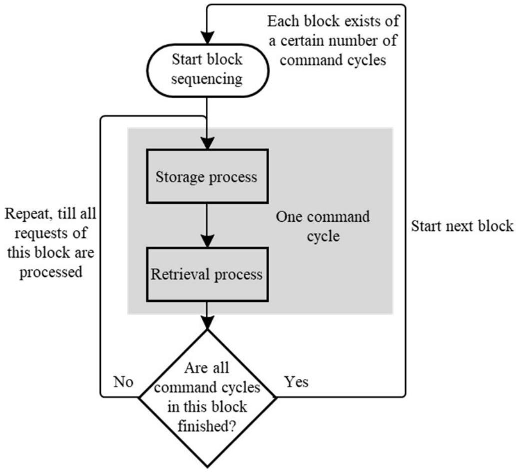


Figure 4.4: The considered process flow of block sequencing

Based on the size of the ULs and the state of the storage channel (see Figure 4.5), different storage and retrieval processes are possible. A storage location is empty, if no UL is stored, and it is occupied, if an UL is stored.

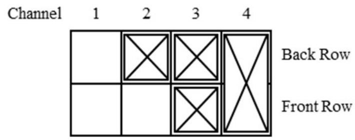


Figure 4.5: State of storage channels: (1) empty, (2) half-full (back row is occupied and front row is empty), (3) and (4) full (back and front row are occupied)

In the storage process (see Figure 4.6), the ULs are stored in an empty storage location. It is assumed that the shuttle vehicle starts the storage process fully loaded with two small ULs or one large UL at the I/O point. Different cases can be distinguished here. Depending on the size of the ULs and the storage location, different load handling times must be taken into account. Storage of small ULs is always possible in the back row of an empty storage channel. Storage of small ULs in the front row is only possible, if the back row is occupied in this storage channel (storage channel is half-full). Large ULs can only be stored in an empty storage channel. If all storage channels are either half-full or full and a large UL needs to be stored, a relocation process is needed. Therefore, a small UL of a half-full storage channel needs to be relocated to another half-full storage channel to get an empty and a full storage channel. Then a storage channel is empty and the large UL can be stored.

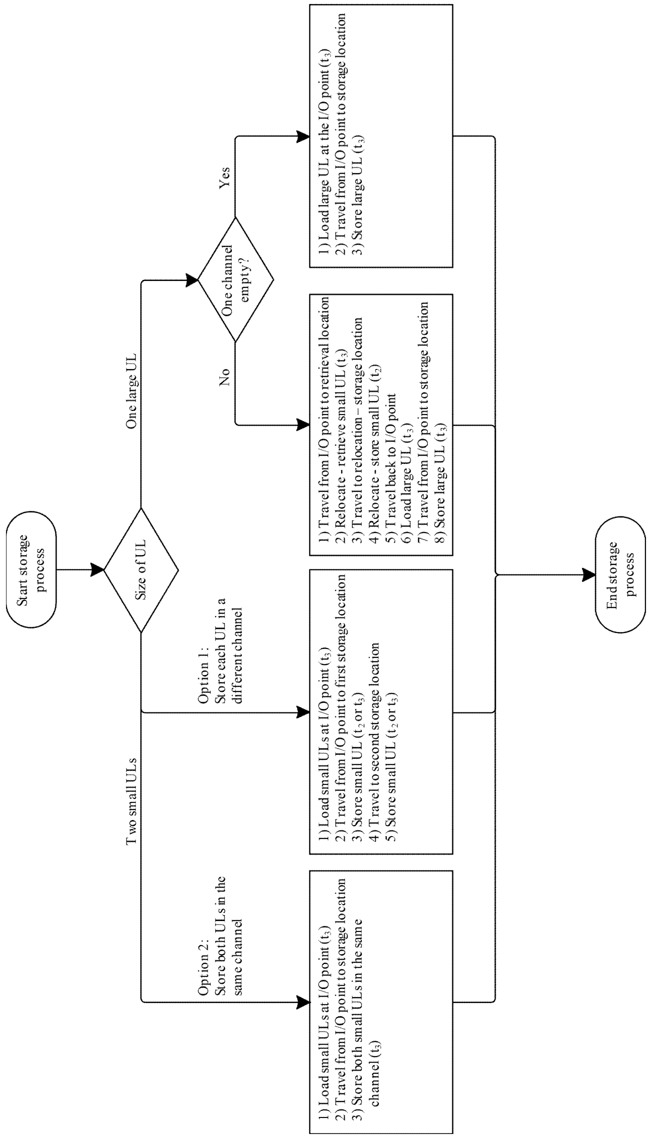


Figure 4.6: Detailed description of the storage process with load handling times (t_2 and t_3)

For the retrieval process (see Figure 4.7), the ULs are retrieved from storage. Different cases can be distinguished here. Depending on the size of the ULs and the storage location, different load handling times must be taken into account. It is assumed that the shuttle vehicle finishes the retrieval process fully loaded with two small ULs or one large UL at the I/O point. Retrieval of a small UL of the front row is directly possible. This is the same process as for single-deep storage. Retrieval of small ULs of the back row is directly possible, if the front row of this storage channel is empty. In the case that the UL in the back row of a storage channel needs to be retrieved and the front row of this channel is occupied by a different UL, the UL in the front row needs to be relocated to another empty storage location first. Therefore, the UL in the front row needs to be retrieved and stored at any empty storage location. Then it is possible to retrieve the UL from the back row.

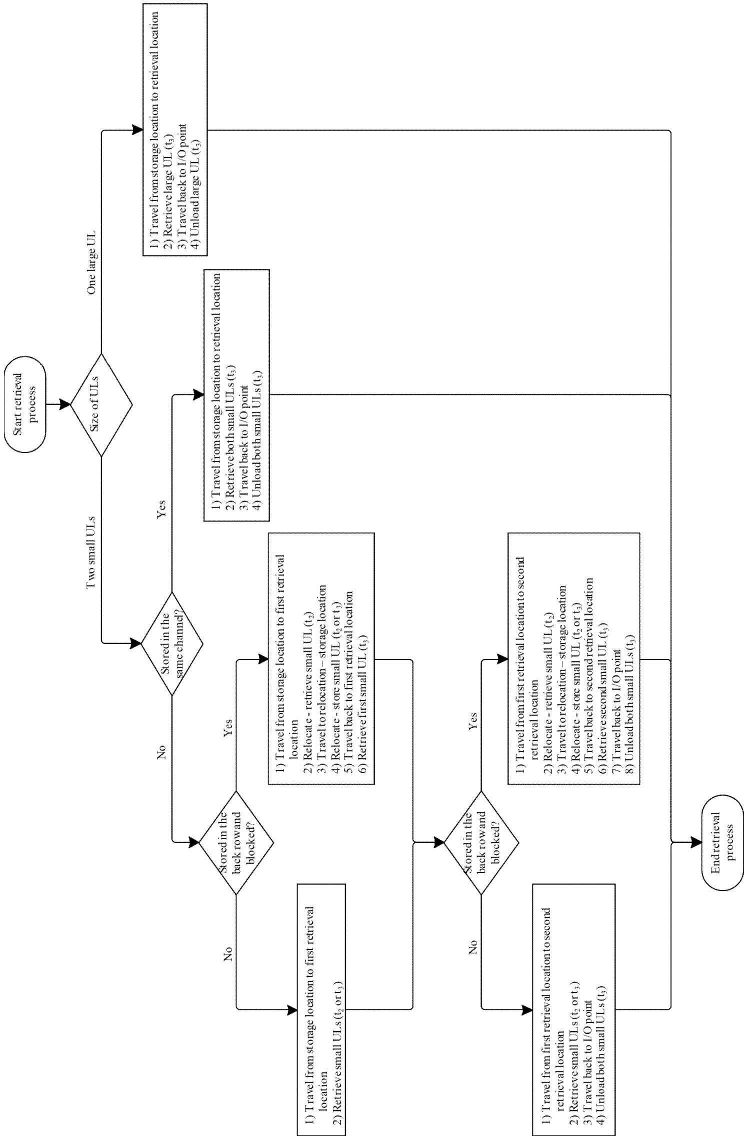


Figure 4.7: Detailed description of the retrieval process with load handling times (t_2 and t_3)

After each storage, retrieval, and relocation process, the state of each storage location is updated from empty to occupied or from occupied to empty. In the following the different states are listed:

- After the storage process of an UL, the storage location is set to an occupied location. As long as this UL is stored at this storage location, this storage location is occupied.
- After the retrieval process of an UL, the location is not occupied anymore and is set to an empty location. For future storage requests, this location can be used as a possible storage location.

A relocation process is the combination of a retrieval process plus a storage process. During a relocation process of a small UL, the UL is retrieved and directly stored again at a different storage location. In the following the different states are listed:

- After the relocation – retrieval process of one UL: The location is not occupied anymore and is set to an empty storage location. For future storage requests this location can be used as a possible storage location.
- After the relocation – storage process of one UL: The new storage location is occupied and set to an occupied storage location.

Depending on the size of the ULs and the storage location, different load handling times must be taken into account. The load handling time to store or retrieve an UL in/from the front row is shorter than the load handling time to store or retrieve an UL in/from the back row. Therefore, it applies $t_2 < t_3$.

In the following the two different load handling times are listed (see Figure 4.8):

- If one small UL is stored or retrieved in/from the front row, the load handling time is t_2.
- If one small UL is stored or retrieved in/from the back row, the load handling time is t_3.

- If two small UL are stored or retrieved in/from the same storage channel at the same time, the load handling time is t_3.
- If one large UL is stored or retrieved in/from one storage channel, the load handling time is t_3.

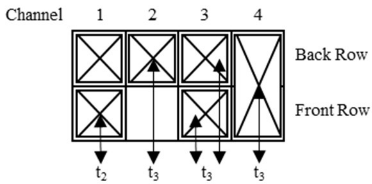


Figure 4.8: Load handling time depending on the size of the ULs and the storage process

Depending on the size of the ULs, different command cycles can be performed. The capacity of the shuttle vehicle is two, which means that two small ULs or one large UL can be carried at a time. To achieve the highest possible throughput, it is assumed that the shuttle vehicle starts the storage process fully loaded (loaded with two small ULs or with one large UL) and finishes the retrieval process fully loaded (loaded with two small ULs or with one large UL). Thus, four different command cycles are possible for double-deep storage and a shuttle vehicle equipped with a dual-load handling device and two different sizes of ULs. The size of a large UL is twice the size of a small UL. Figure 4.9 shows the considered command cycles with the feasible operational sequences, where r stands for relocation, S for storage and R for retrieval. The relocation process of small ULs is displayed in this table. This process, though, is only relevant, if the storage process of a large UL or the retrieval of a small UL is not possible and a relocation process is needed.

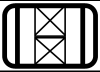
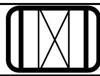
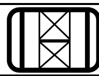
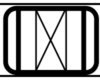
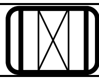
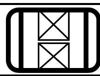
	Retrieval Process			
	Command Cycle			
Storage Process	Single	Dual	Triple	Quadruple
				
			S1-S2-R1	S1-S2-r1-R1-r2-R2
				
		r0-S1-R1	r0-S1-r1-R1-r2-R2	

Figure 4.9: Considered command cycles

The optimization model considered is a classical offline optimization. All input data (set of storage requests, set of retrieval requests, and the initial state of each storage location) are known at the beginning of the planning horizon. The sequencing can be calculated in advance for each individual block. After the optimal sequencing for this block has been determined and the new state of every storage location is known, the sequence for the next block can be determined. The storage and retrieval requests that are continuously added to the system over time are not considered until the previous storage and retrieval requests have been processed. Depending on the number of existing storage and retrieval requests in the queue, different block sizes can be considered for optimization. It is also possible that each block has a different size. The minimum size of a block is one command cycle. This means that the requests are processed according to the FCFS principle and no optimization is achieved. The maximum size of the block can only amount the number of ULs being currently stored. This means that all current ULs are retrieved from storage at once.

4.2.1 Made Assumptions

For the convenience of formulating the optimization problem, the following assumptions are made:

Assumptions in general:

- All storage requests which are considered for block sequencing are known and available.
- All the retrieval request which are considered for block sequencing are known and available.
- The initial state of each storage location, empty or occupied, is known.
- The filling degree must be less than 100 %. At least three storage locations must always be empty to run the system deadlock free. Two empty storage locations are required for the storage process and one empty storage location is required for a possible relocation process.

ULs:

- Two different sizes of ULs are considered, whereas the size of a large UL is twice the size of a small UL.

Storage System - Rack:

- For simplification just one side of the rack is considered (see Figure 4.2).
- The rack is double-deep, which means that every storage channel has two storage locations.
- Either two small ULs or one large UL can be stored in each storage channel.
- All the storage locations are of the same size.
- The distance between the I/O point and the storage channels are known.
- The distance between the individual storage channels are known.

- The distance from the I/O point to the first storage channel is the same as the distance between the individual storage channels.
- The distance between the storage channels is the width of one storage channel.
- The I/O point is located at the end of the aisle.
- The capacity of the inbound and outbound buffer is large enough to hold all ULs which are stored and retrieved, thus, there is no necessity to consider waiting times.
- The initial state of each storage location, either empty or occupied, is known.

Shuttle System & Load Handling Device:

- The shuttle vehicle just moves horizontally.
- The specifications of the shuttle vehicle such as acceleration, deceleration and velocity are known.
- Acceleration and deceleration have the same value.
- The shuttle vehicle can carry a maximum of two small ULs or one large UL at a time.

The calculation of the total processing time is the sum of all travel times between the storage channels and the travel times between the storage channels and the I/O point. In addition, each load handling time (t_2 and t_3) for the storage and retrieval operations but also for the relocation operation is added. Since the load handling time at the I/O point is always the same, t_3 in each case, it is not considered because it has no influence on the optimal solution.

For the calculation of the travel time, the following equation is used, with d (travel distance), v_s (maximum velocity of the shuttle vehicle) and a_s (maximum acceleration and deceleration of the shuttle vehicle):

$$t(d) = \frac{|d|}{v_s} + \frac{v_s}{a_s} \tag{4.1}$$

Control Strategy – General Assumptions:

- Only the four command cycles described in Figure 4.9 are considered.
- There will always be another block ready for processing and each block starts with a storage operation or a relocation operation before a storage operation.
- All storage and retrieval requests of one block are known and ready for processing.
- Each storage location and each UL has an assigned category. An UL may only be stored in the same category. In this optimization model, ULs of an assigned category can only be stored in a storage location of the same category. If all storage locations of this category are occupied, then neither storage nor relocation of this category is possible. In practice, however, this is handled differently. If all storage locations of the required category are occupied, the UL can be stored in the storage location of another category. However, clear rules must be defined in advance.
- An UL can be stored in any empty storage location of the same category.
- After each storage, retrieval or relocation process is performed, the state of the storage locations is updated to empty or occupied.
- Relocation before storage and before retrieval is considered.
- ULs which need to be retrieved in this block, can not be relocated.

4.2.2 Required Input Date for the Optimization Model

To carry out the optimization, some input data is required in advance. All the required input data for the developed optimization model is listed below.

- Information of the storage requests: Size and category of ULs
- Information of the retrieval requests: Size, category, and the storage location of this UL (channel and row)
- The initial state of each storage location: Empty or occupied

- The category of every storage location
- Dimensions of each storage location (to calculate the total travel distance of the shuttle vehicle)
- Load handling times: t_2 and t_3
- Acceleration and deceleration and maximum velocity of the shuttle vehicle (to calculate the travel time)
- Number of command cycles (to get the number of storage and retrieval request of each block)

4.2.3 Results of the Optimization Model

This joint optimization problem refers to the following two decisions:

(i) which empty storage locations are selected to store ULs and
(ii) how to sequence the storage and the retrieval requests of one block?

The objective of the joint optimization problem is to find an optimal solution that:

- All ULs, which need to be stored, are stored in empty storage locations.
- All ULs, which need to be retrieved, are retrieved from their assigned storage location.
- The total cycle time to process all storage and retrieval requests of one block is minimized.

The main objective is to minimize the total cycle time of all storage and retrieval requests of one block. It is also possible to optimize for the shortest total travel distance to process all storage and retrieval requests of one block. However, minimization of total travel time does not guarantee minimization of total cycle time. The reasons for this are different load handling times for the front row, back row, small ULs and large ULs. For example, to store two small ULs simultaneously in the same storage channel leads to a shorter total load handling time than to store these two ULs in different storage channels. For this case it could be better to travel a longer distance to an empty storage channel to achieve the shortest possible cycle time. Another output is the detailed step by step process description of the sequence in which the storage and retrieval requests are processed in one block. In addition, the state of each storage location (empty or occupied) is updated after each storage and retrieval step. The updated list with all the states of every storage location can then be used as an input for the planning process of the next block.

4.2.4 Optimization Model

In this chapter the developed 0-1 integer programming model is described. This chapter is structured as follows: At the beginning, the required data sets, parameters, and decision variables are described. Afterwards, the objective function and the model's constraints are described.

Sets

Table 4.2: Overview of all sets employed to the optimization model

Notation	Description
S	Set of storage requests, indexed on $s \in S$ (the number of storage requests in the current block is given by $\|S\|$)
R	Set of retrieval requests, indexed on $r \in R$ (the number of retrieval requests in the current block is given by $\|R\|$)
$R \subset A$	R is a proper subset of A
A	Set of all storage locations of the considered storage system, indexed on $a \in A$ (the number of storage locations is given by $\|A\|$)
N	Set of UL categories (only required if zoning is considered. Without zoning, all ULs have category 1), indexed on $n \in N$ (the number of categories is given by $\|N\|$)
L	Set of processed positions in one command cycle $L = \{0, 1, 2, 3, 4, 5, 6\}$ Where the individual positions stand for the following processes: 0) Relocation before storage 1) First storage process 2) Second storage process 3) Relocation before first retrieval 4) First retrieval process 5) Relocation before second retrieval 6) Second retrieval process
$l_x \subset L$	Storage requests are processed on position $\{1, 2\}$ of each com-

	mand cycle, l_x is a proper subset of L
$l_y \subset L$	Retrieval requests are processed on position {4, 6} of each command cycle, l_y is a proper subset of L
$l_r \subset L$	Relocation operation are processed on position {0, 3, 5} of each command cycle, l_r is a proper subset of L

Parameters

Table 4.3: Overview of all parameters employed to the optimization model

Category	Notation	Description
Storage Request	ID_s	The ID of storage request $s \in S$
	$Size\_s$	The size of the UL of storage request $s \in S$
	$Category\_s$	The category of storage request $s \in S$
	$SCategory\_n$	Number of ULs of category $n \in N$ in the current set of storage requests
Retrieval Request	ID_r	The ID of retrieval request $r \in R$
	$Channel\_r$	The channel of retrieval request $r \in R$
	Row_r	The row of retrieval request $r \in R$
	$Size\_r$	The size of the UL of retrieval request $r \in R$
	$Category\_r$	The category of retrieval request $r \in R$
Storage Location	ID_a	The ID of the storage location $a \in A$
	$Channel\_a$	The channel of the storage location $a \in A$
	Row_a	The row of the storage location $a \in A$
	$Size\_a$	The size of the storage location $a \in A$
	$Category\_a$	The category of the storage location $a \in A$
	$State\_a$	The state of the storage location (1 for occupied and 0 for empty)
	d_{sx}	Width of one storage location (channel)
Shuttle system	a_s	Acceleration of shuttle vehicle (acceleration and deceleration are equal)
	v_s	Velocity of the shuttle vehicle
	t_2	Load handling time for front row
	t_3	Load handling time for back row
Command cycle	m	Number of operation cycles (number of command cycles) for each block
Travel distance	d_{0a}	Travel distance between the I/O point and storage location a
	d_{ab}	Travel distance between storage location a and storage location b
	d_{b0}	Travel distance between storage location b and the I/O point

Decision variables

Binary decision variable to determine storage sequence order

$$x_{a(p,w),l,k} = \begin{cases} 1, & \text{if a UL is stored in the storage location } a \text{ with channel } p \text{ and row } w \text{ at the } l^{th} \text{ point in the } k^{th} \text{operation cycle} \\ 0, & \text{otherwise} \end{cases} \quad (4.2)$$

with $a \in A$; $l \in l_x$; $k = 1, \dots, m$

Binary decision variable to determine storage sequence order

$$e_{s,k} = \begin{cases} 1, & \text{if storage request } s \text{ is executed in the } k^{th} \text{ operation cycle} \\ 0, & \text{otherwise} \end{cases} \quad (4.3)$$

with $s \in S$; $k = 1, \dots, m$

Binary decision variable to determine retrieval sequence order:

$$y_{a(p,w)l,k} = \begin{cases} 1, & \text{if the UL of storage location } a \text{ with channel } p \text{ and row } w \text{ is retrieved at the } l^{th} \text{ point in the } k^{th} \text{ operation cycle} \\ 0, & \text{otherwise} \end{cases} \quad (4.4)$$

with $a \in A$; $l \in l_y$; $k = 1, \dots, m$

Binary decision variable to get the state of every storage location (occupied or empty):

$$z_{a(p,w),l,k} = \begin{cases} 1, & \text{if the location } a \text{ with channel } p \text{ and row } w \\ & \text{is occupied after performing an operation} \\ & \text{at the } l^{th} \text{ point in the } k^{th} \text{ operation}^{1} \text{ cycle} \\ 0, & \text{if the location } a \text{ with channel } p \text{ and row } w \\ & \text{is empty after performing an operation} \\ & \text{at the } l^{th} \text{ point in the } k^{th} \text{ operation cycle} \end{cases} \tag{4.5}$$

with $a \in A$; $l \in L$; $k = 1, \dots, m$

Binary decision variable to determine, if a relocation is executed and to get the relocation sequence order – relocation retrieval operation:

$$r_{a(p,w),l,k,1} = \begin{cases} 1, & \text{if an UL is relocated from storage} \\ & \text{location } a \text{ with channel } p \text{ and row } w \\ & \text{at the } l^{th} \text{ point in the } k^{th} \text{ operation cycle} \\ 0, & \text{otherwise} \end{cases} \tag{4.6}$$

with $a \in A$; $l \in l_r$; $k = 1, \dots, m$

Binary decision variable to determine if a relocation is executed and to get the relocation sequence order – relocation storage operation:

$$r_{a(p,w),l,k,2} = \begin{cases} 1, & \text{if an UL is relocated to storage} \\ & \text{location } a \text{ with channel } p \text{ and row } w \\ & \text{at the } l^{th} \text{ point in the } k^{th} \text{ operation cycle} \\ 0, & \text{otherwise} \end{cases} \tag{4.7}$$

with $a \in A$; $l \in l_r$; $k = 1, \dots, m$

[1] an operation is a storage, retrieval or a relocation process

Objective function

The objective is to minimize the total cycle time of all storage and retrieval requests of one block. The objective function has been divided into three parts which are summed up. The first part is necessary in case that an UL has to be relocated before storage. The second part is for the standard storage, retrieval, and relocation process and the third part considers the load handling times of the individual ULs.

$$\min \sum_{k=1}^{m} \begin{pmatrix} Part\ 1 \\ + \\ Part\ 2 \\ + \\ Part\ 3 \end{pmatrix} \tag{4.8}$$

Part 1

For the case, that a large UL can not be stored because all storage channels are half-filled or fully filled, one small UL of a half-filled storage channel must be relocated into another half-filled storage channel to generate an empty storage channel. This part is the required travel time for the relocation process.

from the I/O point to the relocation location (for retrieval)

$$\sum_{a\in A} r_{a(p,w),0,k,1} \cdot \left(\frac{d_{0a}}{v_s} + \frac{v_s}{a_s}\right)$$

from the relocation location to the new storage location

$$+\sum_{a\in A}\sum_{b\in A} r_{a(p,w),0,k,1} \cdot r_{b(p,w),0,k,2} \cdot \left(\frac{d_{ab}}{v_s} + \frac{v_s}{a_s}\right)$$

from the new storage location back to the I/O point

$$+\sum_{b\in A} r_{b(p,w),0,k,2} \cdot \left(\frac{d_{b0}}{v_s} + \frac{v_s}{a_s}\right)$$

Part 2

This part is the required travel time for the storage, retrieval, and the relocation process. The relocation process is only performed in the case, that a small UL of the back row must be retrieved but is occupied by an UL in the front row of the same storage channel.

from the I/O point to the first storage location

$$+\sum_{a \in A} x_{a(p,w),1,k} \cdot \left(\frac{d_{0a}}{v_s} + \frac{v_s}{a_s}\right)$$

from the first storage location to the second storage location

$$+\sum_{a \in A} \sum_{b \in A} x_{a(p,w),1,k} \cdot x_{b(p,w),2,k} \cdot \left(\frac{d_{ab}}{v_s} + \frac{v_s}{a_s}\right)$$

from the second storage location to the first retrieval location

$$+\sum_{b \in A} \sum_{r \in R} x_{b(p,w),2,k} \cdot y_{r(p,w),4,k} \cdot \left(\frac{d_{br}}{v_s} + \frac{v_s}{a_s}\right)$$

from the first retrieval location to the relocation location and back

$$+\sum_{a \in A} \sum_{b \in A} r_{a(p,w),3,k,1} \cdot r_{b(p,w),3,k,2} \cdot 2 \cdot \left(\frac{d_{ab}}{v_s} + \frac{v_s}{a_s}\right)$$

from the first retrieval location to the second retrieval location

$$+\sum_{r \in R} \sum_{t \in R} y_{r(p,w),4,k} \cdot y_{t(p,w),6,k} \cdot \left(\frac{d_{rt}}{v_s} + \frac{v_s}{a_s}\right)$$

from the second retrieval location to the relocation location and back

$$+\sum_{a \in A} \sum_{b \in A} r_{a(p,w),5,k,1} \cdot r_{b(p,w),5,k,2} \cdot 2 \cdot \left(\frac{d_{ab}}{v_s} + \frac{v_s}{a_s}\right)$$

from the second retrieval location back to the I/O point

$$+\sum_{t \in R} y_{t(p,w),6,k} \cdot \left(\frac{d_{to}}{v_s} + \frac{v_s}{a_s}\right)$$

Part 3

This part computes the required load handling time. The calculation of the load handling time is dependent on the storage row, front or back, and the size of the UL.

load handling time for the storage process in the front row (R1)

$$+\sum_{a\in A}\sum_{l\in l_x} x_{a(p,1),l,k}\cdot t_2$$

load handling time for the storage process in the back row (R2)

$$+\sum_{a\in A}\sum_{l\in l_x} x_{a(p,2),l,k}\cdot t_3$$

minus load handling time for the case two small ULs or one large UL are stored in the same storage channel

$$-\sum_{a\in A} x_{a(p,2),1,k}\cdot x_{a(p,1),2,k}\cdot t_2$$

load handling time for the retrieval process in the front row (R1)

$$+\sum_{a\in A}\sum_{l\in l_y} y_{a(p,1),l,k}\cdot t_2$$

load handling time for the retrieval process in the back row (R2)

$$+\sum_{a\in A}\sum_{l\in l_y} y_{a(p,2),l,k}\cdot t_3$$

minus load handling time for the case two small ULs or one large UL are retrieved in the same storage channel

$$-\sum_{a\in A} y_{a(p,1),4,k}\cdot y_{a(p,2),6,k}\cdot t_2$$

relocation - retrieval from the front row (R1) and storage in the front row (R1)

$$+\sum_{a\in A}\sum_{l\in l_r}\sum_{t=1}^{2} r_{a(p,1),l,k,t}\cdot t_2$$

relocation - storage in the back row (R2)

$$+\sum_{a\in A}\sum_{l\in l_r}\sum_{t=1}^{2} r_{a(p,2),l,k,t}\cdot t_3$$

To be able to solve the optimization problem, a large number of constraints are required. The constraints can be classified into four different groups:

- Constraints for the storage and retrieval process
- Constraints for the relocation process
- Constraints to get the state of each storage location after an operation
- Constraints for different control strategies and categories

Constraints for the storage and retrieval process

Each storage location a with channel p and row w is visited at most once while processing all storage requests.

$$\sum_{k=1}^{m} \sum_{l \in l_x} x_{a(p,w),l,k} \leq 1 , \qquad \forall a \in A \tag{4.9}$$

Each storage location of each retrieval request r with channel p and row w is visited once in the block.

$$\sum_{k=1}^{m} \sum_{l \in l_y} y_{r(p,w),l,k} = 1 , \qquad \forall r \in R \tag{4.10}$$

Just one empty storage location a with channel p and row w is visited for each position l in each cycle k.

$$\sum_{a \in \mathrm{A}} x_{a(p,w),l,k} = 1 , \qquad \forall l \in l_x \;\; ; \;\; \forall k = 1, \dots, m \tag{4.11}$$

Just one storage location a with channel p and row w is visited for each position l in each cycle k.

$$\sum_{a \in A} y_{a(p,w),l,k} = 1 , \qquad \forall l \in l_y \ ; \ \forall k = 1, \dots, m \tag{4.12}$$

ULs can only get stored in empty storage location a with channel p and row w. This is for the first storage position in each command cycle.

$$x_{a(p,w),1,k} \leq 1 - z_{a(p,w),0,k} , \qquad \forall a \in A \ ; \ \forall k = 1, \dots, m \tag{4.13}$$

ULs can only get stored in empty storage location a with channel p and row w. This is for the second storage position in each command cycle.

$$x_{a(p,w),2,k} \leq 1 - z_{a(p,w),1,k} , \qquad \forall a \in A \ ; \ \forall k = 1, \dots, m \tag{4.14}$$

Each storage request is just processed once.

$$\sum_{k=1}^{m} e_{s,k} = 1 , \qquad \forall s \in S \tag{4.15}$$

At each command cycle, two small ULs or one large UL need to be stored.

$$\sum_{s \in S} Size\_s \cdot e_{s,k} = 2 , \qquad \forall k = 1, \dots, m \tag{4.16}$$

A large UL must be stored in two storage locations in the same channel.

$$\left(\sum_{s\in S} e_{s,k}\right) - 1 \geq x_{a(p,1),2,k} - x_{a(p,2),1,k} + x_{a(p,2),2,k} + x_{a(p,1),1,k}\,, \qquad \forall a \in A\ ;\ \forall k = 1,\dots,m \tag{4.17}$$

At each command cycle, two small ULs or one large UL need to be retrieved.

$$\sum_{r\in R}\sum_{l\in l_y} y_{r(p,w),l,k} \cdot Size\_r = 2\,, \qquad \forall k = 1,\dots,m \tag{4.18}$$

A large UL must be retrieved from two storage locations in the same channel.

$$\left(\sum_{r\in R}\sum_{l\in ly} y_{r(p,w),l,k}\right) - 1 = y_{a(p,1),4,k} - y_{a(p,2),6,k} + y_{a(p,1),6,k} + y_{a(p,2),4,k}\,, \tag{4.19}$$

$\forall a \in A\ ;\ \forall k = 1, \ldots, m$

A retrieval process of the UL of the back row is only possible in cycle k, if the front row of the same channel is not occupied in cycle k. This is the constraint for the first retrieval process in cycle k.

$$y_{a(p,2),4,k} + z_{a(p,1),3,k} \leq 1\,, \qquad \forall a \in A\ ;\ \forall k = 1,\dots,m \tag{4.20}$$

A retrieval process of the UL of the back row is only possible in cycle k, if the front row of the same channel is not occupied in cycle k. This is the constraint for the second retrieval process in cycle k.

$$y_{a(p,2),6,k} + z_{a(p,1),5,k} \leq 1\,, \qquad \forall a \in A\ ;\ \forall k = 1,\dots,m \tag{4.21}$$

If in one channel both storage locations are empty, the UL needs to be stored first in the back row. This applies for every storage and relocation storage process.

$$z_{a(p,1),l,k} \leq z_{a(p,2),l,k}\,, \quad \forall a \in A\ ;\ \forall l \in (l_x\ \cup\ l_r)\ ;\ \forall k = 1,\ldots,m \tag{4.22}$$

Constraints for the relocation process

A relocation cycle consists of "from" one storage location "to" another empty storage location. For every relocation retrieval process there has to be a relocation storage process in cycle k.

$$\sum_{a \in A} r_{a(p,w),l,k,1} = \sum_{a \in A} r_{a(p,w),l,k,2}\,, \qquad \forall l \in l_r\ ;\ \forall k = 1,\ldots,m \tag{4.23}$$

A relocation operation of a small UL stored in the back row (R2) is not possible for the case that a relocation process is needed right before a retrieval operation of a small UL. A relocation operation of a small UL stored in the back row (R2) is only possible for the case that a relocation process is needed right before the storage operation of a large UL.

$$r_{a(p,2),l,k,1} = 0\,, \qquad \forall l \in\ \{3,5\};\ \forall k = 1,\ldots,m \tag{4.24}$$

An UL can only be relocated to an empty storage location in cycle k. This is the first relocation process in cycle k before the first retrieval process.

$$r_{a(p,w),3,k,2} \leq 1 - z_{a(p,1),2,k}\,, \qquad \forall a \in A\ ;\ \forall k = 1,\ldots,m \tag{4.25}$$

An UL can only be relocated to an empty storage location in cycle k. This is the second relocation process in cycle k before the second retrieval process.

$$r_{a(p,w),5,k,2} \leq 1 - z_{a(p,1),4,k}\,, \qquad \forall a \in A\ ;\ \forall k = 1,\ldots,m \tag{4.26}$$

An UL can only be relocated to an empty storage location in cycle k. This is the relocation process in cycle k before a storage process.

$$r_{a(p,w),0,k,2} \leq 1 - z_{a(p,6),4,k}\,, \qquad \forall a \in A \; ; \; \forall k = 1,\ldots,m \tag{4.27}$$

There can be at most one relocation process at each position l in each cycle k.

$$\sum_{a \in \mathrm{A}} r_{a(p,1),l,k,1} \leq 1\,, \qquad \forall l \in l_r \; ; \; \forall k = 1,..,m \tag{4.28}$$

The storage location of the relocation retrieval process has to be a different storage location than the storage location of the relocation storage process.

$$r_{a(p,1),l,k,1} + r_{a(p,1),l,k,2} \leq 1\,, \quad \forall a \in A \; ; \; \forall l \in l_r \; ; \; \forall k = 1,\ldots,m \tag{4.29}$$

A relocation process is only possible, if the front row of this channel is occupied and the UL of the back row of this channel needs to be retrieved in cycle k.

$$y_{a(p,2),l,k} \geq r_{a(p,1),l-1,k,1}\,, \quad \forall a \in A \; ; \; \forall l \in l_y \; ; \; \forall k = 1,\ldots,m \tag{4.30}$$

Only relocate, if there is no retrieval request for this UL. Relocation of an UL is just allowed, if this UL is not retrieved in this block.

$$r_{r(p,w),l,k,1} \leq \sum_{u \in l_y} \sum_{c=1}^{k} y_{r(p,w),u,c}, \quad \forall r \in R \; ; \; \forall l \in l_r \; ; \; \forall k = 1,\ldots,m \tag{4.31}$$

Constraints to get the state of each storage location after an operation

This equation sets the state of every storage location at cycle 0 (before the first command cycle starts) to empty or occupied (= initial state of z).

$$z_{a(p,w),l,0} = State\_a \ , \quad \forall a \in A \ ; \ \forall l \in L \tag{4.32}$$

State of every storage location after the relocation process (before the storage process) in cycle k.

$$z_{a(p,w),0,k} = z_{a(p,w),6,k-1} - r_{a(p,w),0,k,1} + r_{a(p,w),0,k,2}, \quad \forall a \in A \ ; \ \forall k = 1,\dots,m \tag{4.33}$$

State of every storage location after the first storage process in cycle k.

$$z_{a(p,w),1,k} = z_{a(p,w),0,k} + x_{a(p,w),1,k} \ , \quad \forall a \in A \ ; \ \forall k = 1,\dots,m \tag{4.34}$$

State of every storage location after the second storage process in cycle k.

$$z_{a(p,w),2,k} = z_{a(p,w),1,k} + x_{a(p,w),2,k} \ , \quad \forall a \in A \ ; \ \forall k = 1,\dots,m \tag{4.35}$$

State of every storage location after the first relocation process in cycle k.

$$z_{a(p,w),3,k} = z_{a(p,w),2,k} - r_{a(p,w),3,k,1} + r_{a(p,w),3,k,2} \ , \quad \forall a \in A \ ; \ \forall k = 1,\dots,m \tag{4.36}$$

State of every storage location after the first retrieval process in cycle k.

$$z_{a(p,w),4,k} = z_{a(p,w),3,k} - y_{a(p,w),4,k}, \quad \forall a \in A \ ; \ \forall k = 1,\dots,m \tag{4.37}$$

State of every storage location after the second relocation process in cycle k.

$$z_{a(p,w),5,k} = z_{a(p,w),4,k} - r_{a(p,w),5,k,1} + r_{a(p,w),5,k,2}\ , \quad \forall a \in A\ ;\ \forall k = 1,\dots,m \tag{4.38}$$

State of every storage location after the second retrieval process in cycle k.

$$z_{a(p,w),6,k} = z_{a(p,w),5,k} - y_{a(p,w),6,k}\ , \quad \forall a \in A\ ;\ \forall k = 1,\dots,m \tag{4.39}$$

Constraints for different control strategies and categories

A total of eight different control strategies are considered using this optimization model. In the first step, a decision must be made between consideration with or without categories. In the second step, a decision must be made between FCFS or optimal sequence order for the processing of the storage and retrieval requests. The constraints for the relocation process must always be considered.

The difference between FCFS and optimal sequence is:

- FCFS: All the requests in the considered block are processed in the sequence in which these requests were created. This means, that storage and retrieval requests are processed in a predetermined sequence.
- Optimal sequence: All the requests in the considered block are processed in an optimal sequence. All the requests can be presorted and placed in an optimal sequence to minimize the total cycle time of all storage and retrieval requests of one block.

Depending on the considered control strategy, different constraints apply.

Table 4.4: Required constraints for control strategies without considering categories

<table>
<tr><th colspan="5">Without Categories
-
ULs can be stored in every empty storage location</th></tr>
<tr><th colspan="2">Storage</th><th colspan="2">Retrieval</th><th rowspan="2">Relocation</th></tr>
<tr><th>FCFS (F)</th><th>Optimal (O)</th><th>FCFS (F)</th><th>Optimal (O)</th></tr>
<tr><td>ULs can be stored in every empty storage location.

A small UL requires one empty storage location.

A large UL requires one empty storage channel.

Storage requests are processed after FCFS.</td><td>ULs can be stored in every empty storage location.

A small UL requires one empty storage location.

A large UL requires one empty storage channel.

Storage requests are processed in an optimal sequence.</td><td>Retrieval requests are processed after FCFS.</td><td>Retrieval requests are processed in an optimal sequence.</td><td>Relocation is possible to any empty storage location.

Relocation of a small UL before a retrieval process is possible to any empty storage location.

Relocation of a small UL before a storage process is possible to any empty storage location.</td></tr>
<tr><td colspan="5">All constraints from (4.9) until (4.39) are required</td></tr>
<tr><td>Constraint (4.40) is also required.</td><td>No additional constraints required.</td><td>Constraints (4.45), (4.46) and (4.47) are also required.</td><td>No additional constraints required.</td><td>No additional constraints required.</td></tr>
</table>

Table 4.5: Required constraints for control strategies with considering categories

With Categories – ULs can be stored in every empty storage location of the same category				
Storage		**Retrieval**		**Relocation**
FCFS (F)	**Optimal (O)**	**FCFS (F)**	**Optimal (O)**	
ULs can be stored in every empty storage location of the same category. A small UL requires one empty storage location. A large UL requires one empty storage channel. Storage requests are processed after FCFS.	ULs can be stored in every empty storage location of the same category. A small UL requires one empty storage location. A large UL requires one empty storage channel. Storage requests are not processed in an optimal sequence.	Retrieval requests are processed after FCFS.	Retrieval requests are not processed in an optimal sequence.	Relocation is possible to any empty storage location of the same category. Relocation of a small UL before a retrieval process is possible to any empty storage location. Relocation of a small UL before a storage process is possible to any empty storage location.
All constraints from (4.9) until (4.39) are required				
Constraints (4.40) and (4.44) are also required.	Constraints (4.41) and (4.42) are also required.	Constraints (4.45), (4.46) and (4.47) are also required.	No additional constraints required.	Constraint (4.43) is also required.

Storage requests are processed FCFS regarding their size.

$$\sum_{c=1}^{k} e_{p,c} \leq \sum_{c=1}^{k} e_{p-1,c}\ , \qquad \forall p = 2, \dots, |S|\ ;\ \forall k = 1, \dots, m \tag{4.40}$$

Number of ULs of category $n \in N$ must be the same as the number of ULs finally stored in category n.

$$\sum_{a \in A\,:\,Category\_a = n}\ \sum_{l \in l_x} \sum_{k=1}^{m} x_{a(p,w),l,k} = \mathrm{SCategory\_n}\ , \qquad \forall n = 1, \dots, |\mathrm{N}| \tag{4.41}$$

The category of the stored UL must be the same category as the storage location in cycle k.

$$\sum_{a \in A} \sum_{l \in l_x} x_{a(p,w),l,k} \cdot Category\_a = \sum_{s \in S} e_{s,k} \cdot Category\_s \cdot Size\_s, \qquad \forall k = 1, \dots, m \tag{4.42}$$

An UL must be relocated to the same category.

$$\sum_{a \in A} r_{a(p,w),l,k,2} \cdot Category\_a = \sum_{a \in A} r_{a(p,w),l,k,1} \cdot Category\_a\ , \qquad \forall l \in l_r\ ;\ \forall k = 1, \dots, m \tag{4.43}$$

Storage requests are processed FCFS within their command cycle. For this constraint the big M method is used with $M = 1000$.

$$1000 * \left(2 - e_{s,k} - e_{s-1,k}\right) \geq \left| Category\_s - \sum_{a \in A} x_{a(p,w),1,k} \cdot Category\_a \right|, \qquad \forall s = 1, \dots, |S| - 1\ ;\ \forall k = 1, \dots, m \tag{4.44}$$

Retrieval requests are processed FCFS – Constraint 1.

$$\sum_{c=1}^{k}\sum_{l\in l_y} y_{r(p,w),l,c} \leq \sum_{c=1}^{k}\sum_{l\in l_y} y_{r-1(p,w),l,c}, \qquad (4.45)$$
$$\forall r \in R : r > 1 \; ; \; \forall k = 1, \dots, m$$

Retrieval requests are processed FCFS – Constraint 2.

$$y_{r(p,w),4,k} \leq y_{r-1(p,w),6,k-1} + y_{r-1(p,2),6,k-1}, \qquad (4.46)$$
$$\forall r \in R : r > 1 \; ; \; \forall k = 2, \dots, m$$

Retrieval requests are processed FCFS – Constraint 3. Constraint 3 is only needed for the case, if one command cycle is considered.

$$y_{r(p,w),6,k} + y_{r-1(p,w),4,k} \leq y_{r(p,w),4,k} + y_{r-1(p,2),6,k}, \qquad (4.47)$$
$$\forall r \in R : r = \{2, 4, 6..\infty\} \; ; \; \forall k = 2, \dots, m$$

4.3 Numerical Experiments

In the following section, a large number of experiments are carried out to analyze and evaluate the efficiency of the optimization model and the different control strategies. By varying several dependent input parameters, the system's behavior is analyzed and general statements are made on how the performance behave in relation to the input parameters and the chosen control strategy. By comparing the results of the experiments, recommendations are given.

The optimization model is implemented in IBM ILOG CPLEX Optimization Studio (OPL). The numerical experiments are carried out using IBM ILOG CPLEX Optimization Studio 12.9.00 with preset settings and solved with the CPLEX mixed integer optimizer.

The experiments were conducted on a machine with CPU (AMD Epyc 7002 P with 64 cores and 128 Threads), RAM (128 GB DDR4 3200) and GPU (Nvidia Geforce 2080 RTX with 8 GB GDDR6).

For the experiments, one rack side of one tier of a SBS/RS is considered. The rack is double-deep and can hold one large UL or two small ULs per storage channel. Each storage channel has a width of $h = 0.5\,m$. It is assumed that the shuttle vehicle accelerates and decelerates with $a = 1.5\,m/s^2$ and reaches a maximum velocity of $v = 2.5\,m/s$. The load handling time to store and retrieve ULs from the front row is $t_2 = 4\,s$ and for the back row is $t_3 = 5\,s$. The distance between the storage channel centers is used to represent the distance between two storage channels.

Table 4.6: Input for the optimization model experiments

Category	Description	Notation	Value
Shuttle vehicle	Acceleration of shuttle vehicle (acceleration and deceleration are equal)	a_s	1.5 m/s^2
	Velocity of the shuttle vehicle	v_s	2.5 m/s
	Load handling time for front row	t_2	4 s
	Load handling time for back row	t_3	5 s
Storage system	Distance between storage channels (from center to center) = dimension of one storage location in x-direction	d_{sx}	0.5 m
	Total number of storage locations	$\|A\|$	200 storage locations
	Number of categories	$\|N\|$	1 category
Further input data	Filling degree	-	25 % - 95 %
	Percentage of small ULs	-	0 % - 100 %
	Size of ULs – stacking boxes (footprint: length x width)	-	600 mm x 400 mm 300 mm x 400 mm

In the planning horizon, the storage and retrieval requests are served according to the block sequencing policy and processed either FCFS (F) or in optimal sequence (O). In the following, the strategies are named F - F (Storage request are processed FCFS and retrieval request are processed FCFS), O - O (Storage request are processed in optimal sequence and retrieval request are processed in optimal sequence), F - O (Storage request are processed FCFS and retrieval request are processed in optimal sequence) and O - F (Storage request are processed in optimal sequence and retrieval request are FCFS). All orders are known in advance, as well as the current occupancy of all storage locations.

At the beginning, the process sequence for both control strategies F - F and O - O are briefly explained using a simple example. A large UL and two small ULs need to be stored and a large UL and two small ULs need to be retrieved. Arrival sequence of the storage requests for FCFS is: large UL, small UL, small UL. Arrival sequence of the retrieval requests for FCFS is: small UL (storage location 8), small UL (storage location 6), large UL (storage location 11/12). The following figure shows the two process flows.

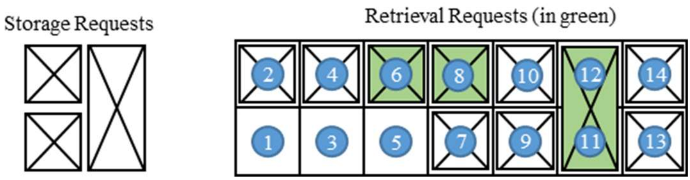


Figure 4.10: Considered storage system for the example

By comparing the results of the two different control strategies, the advantages of using the optimization model become apparent. The total distance and the total cycle time to finish all requests of this block is decreasing. If the requests are processed according to FCFS, the following assignment would result (see Figure 4.11 left side). Since there is no free storage channel, a relocation has to take place first. Only then it is possible to store the large UL. Two additional relocations are needed to retrieve the two small ULs. With the optimized sequence (see Figure 4.11 right side), the requests are placed in such a sequence that the number of travels between the different storage channels and the number of relocations is reduced.

Strategy	F - F		
Total travel distance to complete all requests [m]	14		
Total time to complete all requests [s]	84.93		
Cycle	Process step	Storage location	Size of UL
1	relocation from	2	small
	relocation to	3	small
	storage 1	1/2	large
	relocation from	7	small
	relocation to	5	small
	retrieval 1	8	small
	relocation from	5	small
	relocation to	8	small
	retrieval 2	6	small
2	storage 2	6	small
	storage 3	5	small
	retrieval 3	11/12	large

Strategy	O - O		
Total travel distance to complete all requests [m]	11		
Total time to complete all requests [s]	61.40		
Cycle	Process step	Storage location	Size of UL
1	storage 1	1	small
	storage 2	3	small
	retrieval 1	6	small
	relocation from	7	small
	relocation to	6	small
	retrieval 2	8	small
2	storage 3	7/8	large
	retrieval 3	11/12	large

Figure 4.11: Results of the example

4.3.1 Computing Time

In this section, the required computing time to solve the optimization model is analyzed. For optimization models, the question of computing time is always asked, namely whether the computing time for solving the optimization problem is acceptable or not.

In the first series of experiments, the impact of the storage filling degree and the percentage of small ULs on the computing time is analyzed. Therefore, different block sizes (one command cycle, two command cycles, three command cycles, and four command cycles) are considered. A total number of 20 runs (randomly generated filled storage system and randomly generated storage and retrieval requests) for each configuration is performed.

In the first experiment (see Table 4.7), the stopping criterion is analyzed. It is studied how the value of the objective function changes, if the process of finding a solution is stopped after 1, 2, 3, 4, 5, 10, 30, 60 or 600 minutes, respectively. This means that the solving process of the optimization model is stopped and the best result of the objective function determined up to this point is used as a result. The last two columns of the table list the number of runs which could get solved optimally or not solved optimally until the stop criterion is reached. For each configuration, 20 runs are performed for a filling degree of 75 % and a percentage of small ULs of 100 %, respectively.

The average value over all runs for the computing time and the result of the objective function is output, as well as the error bound for the mean (EBM) of the 95 % confidence interval. The 95 % confidence interval is calculated based on the Student t-distribution. O - O is chosen as the strategy for the first experiment.

With a block size of one command cycle, an approved optimal solution is found in an average of 2.46 seconds.

With a block size of two command cycles, the first stop criterion is after one minute. For six runs the optimal solution is obtained. Since not all runs were solved optimally, the stop criterion is increased until all runs are solved optimally. In order to solve all runs optimally, a mean computing time of 162.62 seconds (EBM - 95 % confidence interval: 47.38 seconds) is required. If the mean total cycle time of the optimal solution is compared with the results of the stop criterion of one minute, an improvement of 0.10 % (EBM - 95 % confidence interval: 0.19 %) is achieved. If this deviation is within an acceptable range for the user, there is no need to get the approved optimal solution. In this case the solving process can be stopped after one minute. When stopping all runs after two minutes, there is no difference in the value for the objective function. If it is acceptable although the optimality has not yet been confirmed, the stopping criterion can be chosen at one or two minutes.

With a block size of three command cycles, even after 60 minutes, 17 runs can not be approved to be solved optimally. To solve all 20 runs optimal, the mean computing time is 28440.08 seconds, which are 7.9 hours (EBM - 95 % confidence interval: 12063.95 seconds, which are 3.35 hours). If the mean total cycle time after one minute is compared with the optimal approved solution, an improvement of 2.17 % (EBM - 95 % confidence interval: 0.63 %) is achieved. If the results after two minutes are also compared with the optimal approved solution, an improvement of 1.26 % (EBM - 95 % confidence interval: 0.55 %) is achieved. In both cases, this shows that the solution achieved is very close to the optimal solution but that the optimality can not yet be confirmed for all runs.

With a block size of four command cycles, even after 600 min, no run can be approved to be solved optimally. The improvement of the achieved mean total cycle time between 1 min and 600 min is 5.68 % (EBM - 95 % confidence interval: 1.04 %). It can be shown that the improvement achieved increases with increasing computing time but also levels off. When analyzing the 20 runs, it can also be seen that between a stop criterion of 60 min and 600 min, only 4 runs achieve an improvement. Considering this, it can be concluded that the optimum solution has already been found but the final proof is still outstanding.

In summary, it can be stated that the larger the block size (more command cycles are considered) the higher the computing time to solve the optimization problem because more storage and retrieval requests have to be processed and more options need to be considered. Even if the optimum has not been approved, the already achieved result does not deviate very much from the optimum result. If this deviation is within an acceptable range for the user and no confirmation is needed, then this is the optimal solution and the solving process can be stopped prematurely. In this way, an approximately optimal solution can be found even for a very short computing time. In order to decrease the computing time a stop criterion can also be determined for each configuration. This will typically get a result close to the actual optimum.

For the following experiments, a block size of three command cycles and a stop criterion of 5 minutes is chosen.

Table 4.7: Evaluation of the computing time – different block sizes

# of command cycles	Mean total cycle time [s]	EBM - 95 % confi-dence interval [s]	Mean compu-ting time [s]	EBM - 95 % confi-dence interval [s]	Stop criterion [min]	# of runs optimal solved - approved	# of runs stopped after time
1	55.43	5.05	2.46	0.44	1	20	0
2	105.89	7.29	51.52	6.99	1	6	14
	105.80	7.32	91.69	19.52	2	8	12
	105.80	7.32	118.59	29.96	3	13	7
	105.80	7.32	133.96	38.16	4	16	4
	105.80	7.32	160.29	45.70	5	17	3
	105.80	7.32	162.62	47.38	10	20	0
3	164.21	11.67	61.97	0.28	1	0	20
	162.76	11.72	121.53	0.61	2	0	20
	162.48	11.63	181.71	0.41	3	0	20
	162.41	11.62	242.83	0.50	4	0	20
	162.02	11.53	303.83	1.31	5	0	20
	161.28	11.37	604.26	1.20	10	0	20
	160.70	11.22	1715.13	140.46	30	2	18
	160.54	11.11	3317.16	392.66	60	3	17
	160.54	11.11	28440.08	12063.95	none	20	0
4	224.43	14.67	62.53	0.21	1	0	20
	219.00	14.15	122.54	0.49	2	0	20
	217.33	13.84	182.08	0.72	3	0	20
	216.58	13.65	241.80	0.53	4	0	20
	216.52	13.61	303.43	0.63	5	0	20
	215.79	13.70	603.71	1.01	10	0	20
	212.39	13.07	1809.27	2.14	30	0	20
	211.39	12.81	3620.81	2.40	60	0	20
	210.91	12.76	36045.78	9.89	600	0	20

The next experiment (see Table 4.8) compares the computing time between the four different applicable control strategies. The results show that the shortest computing time is needed, if storage and retrieval requests are processed according to FCFS policy (F - F). For this control strategy, just an empty storage location has to be selected for every storage request and relocation task and the requests are then processed after FCFS. A higher computing time is needed for the policy O - F. Again a higher computing time is needed for the strategy F - O. In addition to selecting an empty storage location for every storage request and relocation task, the retrieval requests are processed in an optimized sequence. The highest computing time is required when both storage and retrieval requests are processed in an optimal sequence (O - O).

Table 4.8: Experiment to evaluate the computing time – different control strategies

# of command cycles	Filling degree [%]	Percentage of small ULs [%]	Control strategy	Mean total cycle time [s]	EBM - 95 % confidence interval [s]	Mean computing time [s]	EBM - 95 % confidence interval [s]	Stop criterion [min]	# of runs optimal solved	# of runs stopped after time
3	95	75	F - F	196.18	13.26	1.97	0.30	5	20	0
3	95	75	F - O	182.55	10.31	26.71	14.78	5	20	0
3	95	75	O - F	190.33	11.98	2.09	0.39	5	20	0
3	95	75	O - O	176.39	9.16	49.59	27.51	5	20	0

In the next experiment (see Table 4.9), the impact of the filling degree on the computing time is analyzed. The results show, that for a small filling degree (up to 80 %), no optimum solution can be reached in five minutes. Only at higher filling degrees, an optimal solution can be approved in less than five minutes. At a filling degree of 95 %, an optimal solution can be approved for all 20 runs, with an average computing time of 49.59 seconds (EBM - 95 % confidence interval: 27.51 seconds). In summary, the higher the filling degree, the shorter the computing time. This is due to the fact that there are fewer empty storage locations available for every storage request and relocation task, and thus fewer options have to be considered in the problem-solving process. This means that for high filling degrees, the optimization model is very well applicable in finding an optimal solution within a short computing time.

Table 4.9: Experiment to evaluate the computing time – different filling degrees

# of command cycles	Filling degree [%]	Percentage of small ULs [%]	Control strategy	Mean computing time [s]	EBM - 95 % confidence interval [s]	Stop criterion [min]	# of runs optimal solved	# of runs stopped after time
3	25	75	O - O	304.69	1.64	5	0	20
3	50	75	O - O	302.01	0.55	5	0	20
3	75	75	O - O	302.48	0.61	5	0	20
3	80	75	O - O	303.10	0.69	5	0	20
3	85	75	O - O	297.51	7.60	5	2	18
3	90	75	O - O	166.37	55.68	5	14	6
3	95	75	O - O	49.59	27.51	5	20	0

In the next experiment (see Table 4.10), the percentage of small ULs is increased from 0 % to 100 %. The higher the number of small ULs, the higher the computing time. At 0 % small ULs, only three large ULs are stored and retrieved and no relocation process can occur. At 100 % small ULs, six small ULs are stored and six small ULs are retrieved. In addition, six relocation processes can occur. This means that the more ULs are stored and retrieved, the more possible options must be considered and the higher the computing time.

Table 4.10: Experiment to evaluate the computing time – different percentage of small ULs

# of command cycles	Filling degree [%]	Percentage of small ULs [%]	Control strategy	Mean computing time [s]	EBM - 95 % confidence interval [s]	Stop criterion [min]	# of runs optimal solved	# of runs stopped after time
3	95	0	O - O	1.19	0.03	5	20	0
3	95	25	O - O	2.65	1.04	5	20	0
3	95	50	O - O	19.23	11.71	5	20	0
3	95	75	O - O	49.59	27.51	5	20	0
3	95	100	O - O	106.63	33.83	5	19	1

If this optimization model is applied in practice, the stop criterion should be defined in advance for the SBS/RS. Depending on the number of possible combinations (depending on number of storage locations, block size, filling degree, and percentage of small ULs), the stop criterion must be defined. Even if the problem-solving process is stopped and the optimum has not yet been approved, an increase in performance (higher throughput due to shorter cycle times) is still achieved.

4.3.2 Compare Different Control Strategies

In this chapter, the different control strategies are compared with each other. For this purpose, a total number of 20 runs (randomly generated filled storage system and randomly generated storage and retrieval requests) for each configuration is performed. The stop criterion is set to five minutes.

In this experiment (see Table 4.11), the two strategies F - F and O - O are compared for different block sizes (one command cycle, two command cycles, three command cycles and four command cycles). The filling degree is 95 % and the percentage of small ULs is 75 %.

Table 4.11: Compare different control strategies for different block sizes

			Control strategy F - F		Control strategy O - O		Improvement between F - F and O - O	
# of command cycles	Filling degree [%]	Percentage of small ULs [%]	Mean total cycle time [s]	EBM - 95 % confidence interval [s]	Mean total cycle time [s]	EBM - 95 % confidence interval [s]	Improvement mean total cycle time [%]	EBM - 95 % confidence interval [%]
1	95	75	63.62	8.75	63.36	8.74	0.43	0.48
2	95	75	134.84	10.60	122.35	9.69	8.83	3.73
3	95	75	196.18	13.26	176.39	9.16	9.42	3.46
4	95	75	257.39	14.78	225.57	12.04	12.14	2.25

The results show that in all cases, processing the storage and retrieval request in an optimized sequence (O - O), leads to a shorter total cycle time, compared to processing the requests according to the FCFS strategy (F - F).

Even with a block size of one, an improvement of 0.43 % (EBM - 95 % confidence interval of 0.48 %) can be achieved. In this case, a maximum of two small ULs need to be stored and retrieved. With FCFS, the requests must be processed exactly in a predetermined sequence. When using the O - O policy, the requests can be processed in an optimized sequence with a minimized total travel distance or minimized total cycle time. The larger the blocks considered, the more storage and retrieval requests have to be processed and more sequence options are possible. By optimizing the processing sequence, the required total cycle time can be reduced compared to processing the requests according to FCFS. The larger the considered block size, the higher the optimization achieved (in percentage terms) compared to FCFS.

In the next experiment (see Table 4.12), the impact of different filling degrees is analyzed and the two strategies F - F and O - O are compared. For the experiment, a block size of three command cycles is considered. The filling degree is varied (50 %, 75 %, 80 %, 85 %, 90 %, and 95 %) and the percentage of small ULs is 75 %.

Table 4.12: Compare different control strategies for different filling degrees

			Control strategy F - F		Control strategy O - O		Improvement between F - F and O - O	
# of command cycles	Filling degree [%]	Percentage of small ULs [%]	Mean total cycle time [s]	95 % confidence interval [s]	Mean total cycle time [s]	EBM - 95 % confidence interval [s]	Improvement mean total cycle time [%]	EBM - 95 % confidence interval [%]
3	50	75	146.25	8.40	138.79	8.19	4.94	2.60
3	75	75	157.06	11.92	149.34	11.36	4.99	2.48
3	80	75	164.46	12.09	154.95	9.90	5.39	2.34
3	85	75	173.95	9.31	159.85	7.79	7.87	2.32
3	90	75	181.18	11.05	164.60	9.99	8.98	2.48
3	95	75	196.18	13.26	176.39	9.16	9.42	3.46

The results show that there is a higher optimization potential with increasing filling degree. Compared to processing according to FCFS, an improvement of about 5 % for a filling degree of 50 %, 75 %, and 80 %, can be achieved by applying O - O. The highest improvement can be reached with a filling degree of 95 % with 9.42 % (EBM - 95 % confidence interval of 3.46 %). With an increase of the filling degree, less empty storage locations are available, to store ULs. Thus, it is not always possible that there are empty storage locations on the travel route to the first retrieval request. As a consequence, additional and longer travels have to be performed. If the requests are processed according to FCFS, it may occur that the shuttle vehicle has to travel back and forth several times to process all requests. Processing the requests in an optimized sequence, additional travel back and forth can be reduced. An attempt is also made to process the retrieval requests in such a sequence that the new generated empty storage locations can be used as storage locations in a subsequent cycle of this block. This achieves considerable improvements over processing after FCFS, especially with high filling degrees because additional or longer trips are mostly avoided.

In the next experiment (see Table 4.13), the impact of different percentages of small ULs is analyzed. For this, the two strategies F - F and O - O are compared for a block size of three command cycles. The filling degree is 85 % and the percentage of small ULs is varied (0 %, 25 %, 50 %, 75 %, and 100 %).

Table 4.13: Compare different control strategies for different percentages of small ULs

			Control strategy F - F		Control strategy O - O		Improvement between F - F and O - O	
# of command cycles	Filling degree [%]	Percentage of small ULs [%]	Mean total cycle time [s]	EBM - 95 % confidence interval [s]	Mean total cycle time [s]	EBM - 95 % confidence interval [s]	Improvement mean total cycle time [%]	EBM - confidence interval [%]
3	95	0	108.46	8.25	108.44	8.27	0.03	0.06
3	95	25	127.41	11.94	125.30	10.64	1.29	1.54
3	95	50	149.06	11.97	145.51	12.56	2.51	1.45
3	95	75	167.17	11.48	157.85	9.95	5.31	2.09
3	95	100	183.20	10.56	171.39	11.88	6.61	2.37

The results show, with an increasing percentage of small ULs, the optimization achieves a higher improvement than processing according to FCFS. Even if only large ULs are processed (percentage of small ULs = 0 %), the requests can be processed in such an optimized sequence that an improvement can be achieved. The more small ULs need to be processed in one block, the more options are possible, and therefore the optimization potential is increasing.

In the last experiment, for a block size of three command cycles, 540 runs were performed with different filling degrees (between 25 % and 95 %) and different percentage of small ULs (between 0 % and 100 %). All input data is randomly generated. For each configuration, all 4 control strategies are considered.

- Total number of runs: 540 runs
- 5 runs could not get solved for the control strategy F - F because of a deadlock. This occurs, when an UL stored in the back row needs to be retrieved before the UL stored in the same storage channel in the front row. This use case can not be modeled with this optimization model.

- Mean filling degree over all 540 runs: 81.48 % (EBM - 95 % confidence interval 1.23 %)
- Mean percentage of small ULs over all 540 runs: 51.85 % (EBM - 95 % confidence interval 2.93 %)

Table 4.14: Results of different control strategies

	Mean total travel distance		Mean total cycle time		Mean relocation before		Mean relocation between		Mean all relocations	
Control strategy	Mean total travel distance [m]	EBM - 95 % confidence interval [m]	Mean total cycle time [s]	EBM - 95 % confidence interval [s]	Mean relocation before [-]	EBM - 95 % confidence interval [-]	Mean relocation between [-]	EBM - 95 % confidence interval [-]	Mean all relocations EBM - [-]	EBM - 95 % confidence interval [-]
F - F	190.35	4.32	153.84	2.94	0.10	0.03	1.15	0.11	1.24	0.12
F - O	173.17	3.75	146.61	2.56	0.07	0.02	1.15	0.11	1.22	0.12
O - F	188.79	4.31	152.77	2.90	0.05	0.02	1.15	0.11	1.20	0.11
O - O	172.17	3.71	145.53	2.50	0.03	0.02	1.14	0.11	1.18	0.11

Table 4.15: Compare different control strategies

	Improvement mean total travel distance		Improvement mean total cycle time		Improvement mean relocation before		Improvement mean relocation between		Improvement mean all relocations	
Control strategy	Mean total travel distance [%]	EBM - 95 % confidence interval [%]	Mean total cycle time [%]	EBM - 95 % confidence interval [%]	Mean relocation before [%]	EBM - 95 % confidence interval [%]	Mean relocation between [%]	EBM - 95 % confidence interval [%]	Mean all relocations [%]	EBM - 95 % confidence interval [%]
F-F – F-O	8.08	0.83	4.12	0.43	2.15	1.22	0.00	0.00	1.46	0.91
F-F – O-F	0.81	0.30	0.63	0.21	4.49	1.76	0.00	0.00	2.28	1.02
F-F – O-O	**8.54**	**0.86**	**4.72**	**0.49**	**6.07**	**2.09**	**0.05**	**0.09**	**3.69**	**1.37**
O-F – O-O	7.79	0.81	4.13	0.42	1.59	1.17	0.05	0.09	1.42	0.94
F-O – O-O	0.52	0.22	0.64	0.20	3.93	1.73	0.05	0.09	2.23	1.05
O-F – F-O	7.28	0.82	3.48	0.44	-2.34	1.83	0.00	0.00	-1.53	1.53

When comparing the results of the different control strategies, it becomes evident that optimizing the sequence of the retrieval requests offers more potential for optimization than optimizing the sequence of the storage requests. Comparing F - F with O - O, an improvement of 4.72 % (EBM - 95 % confidence interval 0.49 %) is achieved for the mean total cycle time. If only the retrieval requests are processed in an optimized sequence (F - O), an improvement of 4.12 % (EBM - 95 % confidence interval 0.43) is reached. If only the storage requests are processed in an optimized sequence (O - F), an improvement of 0.63 % (EBM - 95 confidence interval 0.21) is observed.

In addition to the time improvement, the mean total travel distance can also be reduced. This is an important parameter, especially with regard to energy consumption.

By optimizing the sequence of the storage and retrieval requests, the total number of relocations is also reduced, which has a positive impact on the total cycle time and the total distance traveled. A shorter total distance traveled corresponds with a reduction in energy consumption.

Relocations before storage usually are only needed, if there is no empty storage channel and a large UL has to be stored next. By optimizing the sequence of the storage requests, a reduction of the required relocation tasks is achieved (O - F). A further reduction is obtained, if both storage and retrieval requests are processed in an optimal sequence (O - O).

The analysis of the results also shows that in some cases a relocation process before storage is performed, although there are empty storage channels for the storage of large ULs. However, the empty storage channels are at the end of the aisle. In this case it takes less time to carry out a relocation operation at the beginning of the aisle to get an empty storage channel, instead of traveling to the end of the aisle for the storage operation.

The number of relocation tasks before a relocation process is slightly reduced. This is due to the fact that a relocation of an UL is only required for blocked ULs which need to be retrieved. Also, if the sequence is optimized, the UL is still blocked and a relocation task is needed. The only exception is when two small ULs have to be retrieved from the same storage channel. With optimized sequencing, both ULs can be retrieved at the same time and a necessary relocation is avoided.

4.4 Chapter Conclusion

This chapter provides a summary of the findings from the conducted experiments and derives actionable recommendations.

Based on the results, the following scope of application can be defined:

- The developed optimization model can be used to define the optimized sequence of storage and retrieval requests in a SBS/RS.
- The higher the filling degree, the percentage of small ULs, and the block size, the more optimization potential can be achieved by using this optimization model with the control strategy F - O, O - F or O - O.
- Despite the high computing time for large-sized optimization problems, this model is practical and can be used to achieve an improvement in travel and cycle time.
- In a SBS/RS, the capacity of the inbound and outbound buffer at each tier is limited. A block size of three command cycles requires a buffer capacity for at least six small ULs on each tier. Thus, only those ULs would be considered on each tier that are present at the moment in the buffer. The storage requests have to be processed according to FCFS and can be combined with the retrieval requests in an optimized sequence (F - O). As the results showed, only a small improvement is achieved, if the storage requests are processed in an optimal sequence (O - O).

With the use of the optimization model, the following advantages can be achieved:

- Get an optimized sequence.
- Increase the throughput of the system, due to minimized cycle times.
- Reduce the number of relocation tasks.
- Reduce the energy consumptions, due to minimized travel distances.

If the optimization model is to be applied, the following procedure should be chosen:

1) Storage and retrieval requests and the state, empty or occupied, of every storage location, must be known in advance.
2) Define the size of the considered block. How many command cycles should be considered in one block?
3) Determine a control strategy: F - F, F - O, O - F or O - O
4) If an optimum solution can not be found in an acceptable time, define the stop criterion.
5) Based on the results, storage and retrieval requests can now be processed in the determined sequence.

The following recommended actions for the control of a SBS/RS can be derived from the achieved results:

With a high filling degree, fewer empty storage locations are available into which ULs can be stored. Thus, it is not always possible that there are empty storage locations for the ULs which need to be stored on the travel to the storage location of the relocation request. This means that additional and longer travels have to be made. If this is done according to FCFS, it can happen that the shuttle vehicle travels back and forth several times in order to process all requests in this cycle. If the sequence is optimized, then an attempt is made to process the requests in such a way that the shuttle vehicle only has to travel once from the I/O point to storage channel furthest away in this command cycle and then back to the I/O point. This avoids additional travel. In addition, an attempt is made to process the requests in such a way that the storage locations – which got empty through a relocation process - can be used as storage location for subsequent storage requests in the same block. This avoids additional or longer travels.

The following example (see Figure 4.12) shows the optimized processing sequence. A quadruple-command cycle is considered, with S1 (first storage request), S2 (second storage request), R1 (first retrieval request) and R2 (second retrieval request). If the requests are processed according to FCFS, the following travel would result (see Figure 4.12 (a)). To process all requests, several travels back and forth are necessary. If the processing sequence is now optimized (minimizing the total travel distance), the necessary travels are reduced and thus the total cycle time required (see Figure 4.12 (b)).

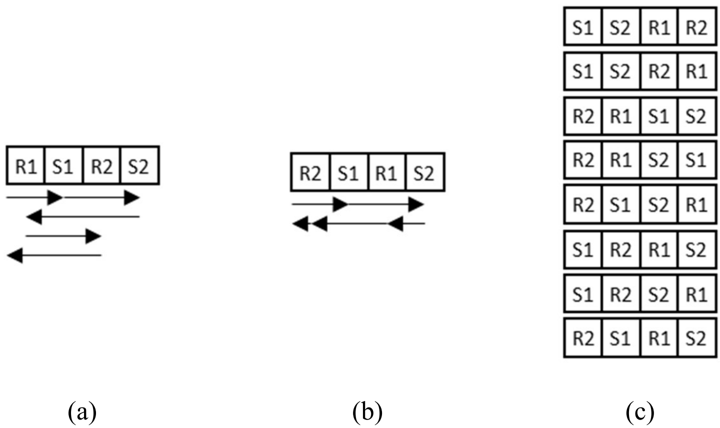


Figure 4.12: Sequence for minimized travel distance (a) sequence for F-F, (b) optimized sequence, (c) all possible optimized sequences

The results of the experiment show that it is best, if the shuttle vehicle travels from the I/O point to the back while processing all storage requests and on the travel back to the I/O point processing all retrieval requests. The furthest point away from the I/O point that must be reached can be an empty storage location for the last storage operation or the first retrieval request. Since retrieval request must always be processed, it is best that the furthest point away is the storage channel of the retrieval request. Figure 4.12 (c) summarizes all possible scenarios, demonstrating minimized travel distance by traveling from the front to the back and then back to the front.

Further actions can be recommended:

- With each retrieval process, new empty storage locations are created into which ULs can get stored at a later time. Requests should therefore be processed in such a sequence that the empty storage locations can be used for subsequent storage requests.
- An UL should not be stored in a storage channel from which an UL needs to be retrieved at a later point in this block.
- If two small ULs are to be retrieved from one storage channel, they should be retrieved together.
- If possible, two small ULs should be stored together in an empty storage channel.
- ULs should be stored close to the I/O point, best before the storage location of the next retrieval request.
- Relocation tasks before the storage operation of large ULs should only be carried out, if it takes less time to carry out the relocation operation than traveling to the closest empty storage channel.
- If possible, relocations should be made to the closest available empty storage location, which is the nearest neighbor control strategy.
- Do not relocate an UL into a storage channel from which an UL needs to be retrieved at a later point in the same block.

Based on the gained knowledge, next steps can be derived:

- Since the computing time is very high, the optimization problem can not be solved in a reasonable time for large-sized problems. To profit from the advantages of optimization in practice, either a more powerful computing machine can be used, a low stop criterion chosen or the computing time reduced by using a heuristic. The achieved results and recommended actions should be incorporated into the development of a heuristic to reduce the computing time and to solve large-sized problems.
- With the created optimization model, different layout configurations as well as storage strategies (e.g., zoning) can be investigated. This is helpful to make recommendations on layout decisions.
- The objective function can be adjusted so that this optimization model is also applicable to an AS/RS.

Maximum throughput of a SBS/RS may only be obtained, if this optimization model is extended to the entire SBS/RS. This model can currently only be used to optimize each tier of an aisle individually and to increase the throughput of each individual tier. However, the interaction of all shuttle vehicles with the inbound lift and outbound lift is not taken into account. In the future, this model should be extended to consider joint optimization of the storage location assignment, the scheduling of the storage and retrieval requests, and the interaction of the shuttle vehicles and lifts.

In conclusion, the following statement can be made: If investing in an optimization model or in a better heuristic for sequencing storage and retrieval requests, higher throughput can be achieved.

5 Performance Analysis and Recommended Actions Using Simulation

The aim of this chapter is to create a parameterizable simulation model for analyzing the performance of a SBS/RS. The model is designed in such a way that different SBS/RS configurations can be simulated, analyzed, and evaluated. Finally, recommendations for action are derived from the results obtained.

The parameterizable discrete event agent-based simulation model is created using the AnyLogic simulation software version 8.8.1 University. In the simulation model, the dynamic system behavior, i.e., the movement of the shuttle vehicle, lift, and ULs is modelled using standard control and storage strategies. The relevant KPIs, e.g., throughput, cycle time, and capacity utilization, are then determined for different scenarios.

Different SBS/RS configurations and control strategies have already been compared in chapter 3 using analytical models. Therefore, only a few simulations are performed in this chapter, for SBS/RS configurations which are not yet considered in chapter 3.

5.1 Model Description

The considered SBS/RS for the simulation study is already described in detail in the boxes at the end of each subchapter in chapter 2.2. The layout of the SBS/RS is depicted in Figure 2.6. The material flow of the ULs and the control strategy of lift and shuttle vehicle are described in the following flowcharts:

- Material flow of the ULs: see Figure 2.14 (storage process), Figure 2.15 (retrieval process) and Figure 2.16 (relocation process)

- Control logic of the shuttle vehicle, inbound and outbound lift: see Figure 2.18 (inbound lift), Figure A.4 (shuttle vehicle) and Figure A.5 (outbound lift)

In the following, the system logic and the different control strategies are described in detail:

Initial filling of the storage system at the start of the simulation: At the start of the simulation, the storage system is randomly filled, depending on the filling degree and the percentage of small ULs.

Generation of storage request: Using a random generator, the ULs are generated depending on the percentage of small ULs and stored in a queue according to FCFS. There are always ULs waiting for storage. The waiting ULs can be processed (transferred to the lift) in two different ways:

- **FCFS**: Orders are processed according to FCFS. However, this can lead to a situation where two small ULs can not be transferred together (e.g., if the first UL is a small UL and the second UL is a large UL and the lift only has a capacity of two, only the small UL can be transferred). This means that in some cases the available capacity of the lift can not be fully utilized.
- **Batch:** ULs are processed according to FCFS. If the first waiting UL in the queue is a large UL, it is transferred directly. If the next UL is a small UL, it is checked whether there is another small UL in the waiting queue. If this is the case, this small UL is picked and the two small ULs are transferred together. If there is no other small UL in the queue, the small UL is transferred alone. The goal is always to transfer two small ULs together and fully utilize the lift. In practice, either a pre-sorting can be carried out that two small ULs always arrive as a batch in front of the lift or there are two separate queues, where one queue is only for small ULs and the other queue is only for large ULs.

Storage location assignment: Each UL must be assigned to a unique storage location. Before transferring the UL onto the lift, the UL is assigned to a unique storage location. If all storage locations are occupied, the UL waits in front of the lift until an empty storage location is available. A free storage location is selected and reserved from all free storage locations in the warehouse. Another possibility is that ULs are transferred to a randomly selected tier that has enough empty storage locations available. Before loaded onto the shuttle vehicle, the storage location is assigned to the UL. The following two cases need to be considered:

- A storage location can only be selected for small ULs, if at least two storage locations are empty on one tier. One empty storage location is required for the UL to be stored and one empty storage location is required for possible relocation processes.
- A storage location can only be selected for large ULs, if at least three storage locations are empty on one tier. Two empty storage locations are required for the UL to be stored and one empty storage location is required for possible relocation processes. However, if there are only three half-full storage channels on one tier, a rearrangement must first be carried out. Two half-full storage channels become one full storage channel and one empty storage channel.

Load handling times: The different load handling times are already described in chapter 2.2.7. For the load handling process and the load handling time between pre-storage zone – lift – buffer see Figure 2.21. The load handling process and the load handling time between shuttle vehicle – storage location is explained in Figure 2.23 and Figure 2.24 (right side).

Inbound lift process sequencing strategy: This determines the sequence in which the inbound lift processes the storage requests. Two different strategies are considered:

- **FCFS:** The inbound lift processes the ULs according to FCFS. The ULs are processed in the sequence in which they are transferred to the inbound lift. However, this can lead to many up- and down travels of the inbound lift, if the inbound lift can process more than two ULs in one command cycle.

- **Optimized:** The ULs are pre-sorted and the inbound lift moves once from the bottom to the top (highest tier in this command cycle) and back to the I/O point and processes all requests. On average, block sequencing leads to a shorter overall travel distance.

Relocation process – storage assignment: The two following relocation processes are considered in the simulation model:

- Relocation before retrieval: Relocate only, if small ULs are blocked and need to be retrieved.
- Relocation before storage: Relocate only, if large ULs need to be stored but all storage channels are half-full or full and no empty storage channel is available.

The ULs that are relocated need to be stored in a different storage location. Therefore, the two different storage assignment strategies are applied:

- **Random storage location assignment**: One empty storage location is randomly chosen.
- **Nearest neighbor on one side or both sides**: The closest empty storage location is chosen. If several storage locations have the same distance, then one of them is randomly chosen.

Dwell point strategy: For inbound lift, outbound lift, and shuttle vehicle a dwell point strategy must be defined.

- The inbound lift only performs storage tasks and after completion, the lift returns to the I/O point with the pre-storage zone (**ROI**).
- The outbound lift only performs retrieval tasks and after completion, the lift dwells at the I/O point with the pre-storage zone (**POSC**).
- The shuttle vehicle performs storage and retrieval tasks. The shuttle vehicle dwells at the point where the last task has been finished (**POSC**).

Shuttle vehicle process sequencing strategy: This determines the sequence in which the shuttle vehicle processes the storage and retrieval requests (see Figure 2.19)

The aim is to store all ULs that are loaded on the shuttle vehicle first and then start with the retrieval operation. Before the retrieval operation can start, the storage operation must be finished and the shuttle vehicle empty. The combined processing of storage and retrieval operations is intended to reduce the number of empty runs and achieve the highest possible throughput. Depending on the capacity of the shuttle vehicle, and UL size, different numbers of storage and retrieval operations can be performed. However, if there are no ULs for retrieval after a storage operation, another storage operation will be processed. The same applies for retrieval. If there are no ULs for storage after a retrieval operation, another retrieval operation is started.

Two different strategies can be considered:

- **FCFS:** The shuttle vehicle processes the ULs according to FCFS. The orders are processed in the sequence in which they are transferred to the shuttle vehicle. However, this can lead to many forwards and backwards travels, if the shuttle vehicle can process more than two ULs in one command cycle.
- **Optimized:** The orders are pre-sorted and the shuttle vehicle travels once from the I/O point to the storage location, which is the furthest away from the I/O point in this command cycle and then back to the I/O point. On average, block sequencing leads to a shorter overall travel distance than processing strictly after FCFS.

Generation of retrieval request: Only the ULs that are stored can be retrieved. From all stored ULs, the ULs are randomly selected for retrieval, depending on the selected inter-arrival time. The arrival rate is chosen so that there are always waiting ULs in the virtual queue for retrieval.

The generated retrieval requests can be processed in two different ways:

- **FCFS**: Retrieval requests are processed according to FCFS. When a retrieval request is generated, it is immediately released on the tier and can be processed directly by the shuttle vehicle. However, this can lead to the situation that two small ULs can not always be retrieved together. E.g., if the first UL is a small UL and the second UL is a large UL, and the shuttle vehicle has a capacity of two, only the small UL can be retrieved in this command cycle. This means that in some cases the available capacity of the shuttle vehicle can not be fully utilized.
- **Batch**: Processing also takes place according to FCFS. If the first retrieval request in the virtual queue is a large UL, it is immediately released on the tier. If the next request is a small UL, it is checked whether there is another request of a small UL in the virtual waiting queue. If this is the case, the two requests of the small ULs are released on the tier. If there is no further request for a small UL, the small UL is released alone. The goal is always to release two small ULs together and fully utilize the shuttle vehicle.

Outbound lift process sequencing strategy: This determines the sequence in which the outbound lift processes the retrieval requests. When the ULs arrive in the buffers, a virtual waiting queue (waiting queue for ULs waiting for the outbound lift) is created based on the arrival time in the outbound buffer. Two different strategies can be considered:

- **FCFS:** The outbound lift processes the ULs according to FCFS. The ULs are processed in the sequence in which they are collected in the virtual queue. However, this can lead to many up and down travels of the outbound lift, if the outbound lift can process more than two ULs in one command cycle.
- **Optimized:** Depending on the capacity of the lift, ULs are selected from the virtual queue after FCFS until the capacity of the lift is reached. These ULs are combined into a block. Afterwards these ULs are pre-sorted and the outbound lift moves once from the bottom to the highest tier in this command cycle and back to the I/O

point. Block sequencing on average leads to a shorter overall travel distance than processing strictly after FCFS.

5.1.1 Assumptions

Also some assumptions and simplifications are made. Those are listed below:

- The arrangement of the ULs on the shuttle vehicle is not taken into account. It is assumed that the ULs are arranged on the shuttle vehicle in such a way that they can always be transferred (dropped off and picked up) and no deadlock can occur. An example of a deadlock case would be: A shuttle vehicle with a one-behind-the-other load handling device, loaded with two small ULs. The UL loaded on the left side needs to be stored on the right side of the aisle and the UL loaded on the right side needs to be stored on the left side of the aisle. This is not possible. To avoid deadlocks,
  (i) ULs need to be presorted so that the ULs arrive in the buffer of each tier in the correct sequence or
  (ii) ULs on the shuttle vehicle can swap the reserved storage locations among themselves.
- Switching time, computing time, and evaluation of sensor signals, positioning time, etc. is not considered (dead time = 0 seconds)
- For the velocity-time dependency, triangular und trapezoid profiles are considered (see appendix B)
- For simplification acceleration and deceleration are linearized and the time for positioning is ignored (see also appendix B)
- The impact of fluctuating customer demand and incoming orders is not considered. The assumption is that there will consistently be a large number of pending orders, in order to fully utilize the available capacity of the shuttle vehicles and the lifts.

5.1.2 KPIs

To evaluate different SBS/RS configurations and different control strategies several performance measures can be used. In this work the following KPIs are used:

Table 5.1: Considered KPIs

KPI	Unit	Description
Throughput	ULs/hour	The average throughput of small and large ULs per hour that are stored and retrieved
Utilization	%	The utilization of inbound lift, outbound lift and shuttle vehicle in the considered time period
Mean cycle time	s	The mean cycle time of inbound lift, outbound lift, and shuttle vehicle
Average lead time of the ULs to be stored	s	Indicates how long it takes on average for an UL to be stored - from the pick up of the UL by the inbound lift until the drop off of the UL in the right storage location
Average lead time of the ULs to be retrieved	s	Indicates how long it takes on average for an UL to be retrieved - from the pick up of the UL by the shuttle vehicle until the arrival of the UL in the pre-storage zone
Average waiting time	s/command cycle	All waiting times are considered (see chapter 3.5)
Average buffer size	ULs/buffer	The average number of occupied buffer locations. This is always recorded when a new UL is dropped off into the buffer.

In addition to the average value, the 95 % quantile is calculated.

5.1.3 Validation

To ensure the correctness of the simulation model, several procedures are carried out. First, the individual submodels (inbound lift, outbound lift, and shuttle vehicle) are each checked for correctness before the submodels are merged. Afterwards, the final model is also checked for correctness. The following four methods have been carried out:

- **Fixed value test:** In the model, constant values are chosen for the inter-arrival times, processing times, and a fixed storage location. This results in a deterministic system, and travel and cycle times can be determined exactly because the storage location is known. (Rabe et al. 2008, p. 96)
- **Animation and Monitoring:** Using the "graphical user interface", the material flow can be tracked and the correctness of the model can be confirmed over a short period of time. (Rabe et al. 2008, p.96 ff.)
- **Dimension test:** In order to avoid errors in the equations used, a dimension test has been performed. By recalculating the dimensions over the units, all used equations were examined and proved to be correct. (Rabe et al. 2008, p.98)
- **Comparison with analytical models:** Comparison of the obtained results with the results of the derived analytical models in chapter 3.

Based on the methods performed and the results obtained, it can be assumed with a high degree of certainty that the simulation model is error-free and the simulation results are correct. For details about the warm-up phase and number of replications, see appendix F.

5.2 Simulation experiment

In this chapter, simulations are performed for a few specific cases that have not been considered by using the analytical models. The purpose of these experiments is to show the interaction between lifts, and shuttle vehicles and the impact of different buffer sizes. For this purpose, several KPIs for different configurations and control strategies are determined and evaluated. A total of 10 different experiments are carried out (see Table 5.2).

Table 5.2: Overview of the performed simulation runs

		1.1	1.2	2.1	2.2	2.3	2.4	3.1	3.2	3.3	3.4
Rack configuration	Single-deep	x	x								
	Double-deep			x	x	x	x	x	x	x	x
Lift configuration	Side-by-side				x	x	x	x	x	x	x
	One-behind-the-other	x	x	x							
Shuttle configuration	Side-by-side	x	x	x							
	One-behind-the-other				x	x	x	x	x	x	x
Size of ULs	Small	x	x	x	x	x	x	x	x	x	x
	Large							x	x	x	x
Generation of storage requests	FCFS	x	x	x	x	x	x				x
	Batch							x	x	x	
Storage location assignment	Random storage location in front of the inbound lift								x		
	Random tier in front of the inbound lift - random storage location in front of the shuttle vehicle	x	x	x	x	x	x	x		x	x
	Batch two small ULs						x	x	x		
Inbound lift process sequencing strategy	FCFS										
	Optimized	x	x	x	x	x	x	x	x	x	x
Shuttle vehicle sequencing strategy	FCFS										
	Optimized	x	x	x	x	x	x	x	x	x	x
Outbound lift sequencing strategy	FCFS										
	Optimized	x	x	x	x	x	x	x	x	x	x
Relocation strategy	Nearest neighbor			x	x	x	x	x	x	x	x
Shuttle vehicle starts the storage process only fully loaded			x			x	x	x	x	(x)	(x)
Generation of retrieval requests	FCFS	x	x	x	x	x	x				x
	Batch							x	x	x	

x: selected strategy

(x): selected strategy: This strategy can not get applied every time. If a small UL is followed by a large UL, first the small UL needs to be stored. For this case the shuttle vehicle is not fully loaded.

5.2.1 Input Parameters

All the required input parameters for the simulation model are listed in the following tables. Some of the values and input data were provided by a German shuttle system manufacturer. In addition, further data was collected through a market research and a representative average value was calculated.

For the simulation experiment the SBS/RS described in Figure 2.6 is considered with the following specifications:

- Lift configuration: Case 1
- The inbound lift performs storage operations and the outbound lift retrieval operations
- Capacity requirement of small ULs is one
- Capacity requirement of large ULs is two
- The shuttle vehicle performs storage and retrieval operations
- All ULs are (un)loaded simultaneously between the buffer and the shuttle vehicle

Table 5.3: Input parameters

Description	Notation	Value
Shuttle vehicle max. velocity	$v_{shuttle}$	2.5 m/s
Shuttle vehicle acceleration	a_s	1.5 m/s²
Shuttle vehicle deceleration	a_s	1.5 m/s²
Shuttle vehicle capacity	c_s	variable
Inbound lift max. velocity	v_{lift}	4 m/s
Inbound lift acceleration	$a_{l_{in}}$	3 m/s²
Inbound lift deceleration	$a_{l_{in}}$	3 m/s²
Inbound lift capacity	$c_{l_{in}}$	variable
Outbound lift velocity	v_{lift}	4 m/s
Outbound lift acceleration	$a_{l_{out}}$	3 m/s²
Outbound lift deceleration	$a_{l_{out}}$	3 m/s²
Outbound lift capacity	$c_{l_{out}}$	variable
Inbound buffer capacity	$c_{b_{in}}$	variable
Outbound buffer capacity	$c_{b_{out}}$	variable
Inter-arrival rate of storage requests	-	1 request/s
Inter-arrival rate of retrieval requests	-	1 request/s
Percentage of small ULs	P_{small}	variable
Filling degree	z	0.95 %
Load handling time lift	t_1	4 s
Load handling time shuttle vehicle – front row	t_2	4 s
Load handling time shuttle vehicle – back row	t_3	5 s
Total number of storage locations	$\|A\|$	variable
Number of tiers	n_t	25
Total number of storage channels (one tier, one side)	n_c	100
Number of rows in one storage channel	n_r	variable
Height of one tier	d_{sy}	0.5 m
Dimension of one storage location in x-direction	d_{sx}	0.5 m
Position of the inbound lift	$d_{l_{in}}$	0 m
Position of the outbound lift	$d_{l_{out}}$	0 m
Distance between the I/O point of the buffer to the middle of the first storage channel in x-direction	d_{bs}	0.5 m

5.2.2 Experiment 1

In the first experiment, a single-deep SBS/RS is considered. The capacity of the inbound and outbound lift, shuttle vehicle, and inbound and outbound buffer is varied. The start configuration is:

- Lift configuration: one-behind-the-other
- Shuttle vehicle configuration: side-by-side
- Generation of storage request: FCFS
- Storage location assignment: random tier - random storage location
- Lift and shuttle vehicle sequencing strategy: optimized
- Generation of retrieval requests: FCFS
- Shuttle vehicle does not need to be fully loaded to start the storage operation

To analyze the interaction between lifts and shuttle vehicles, several KPIs need to be analyzed and evaluated (see Table 5.4). Due to the interdependence of individual parameters, it is necessary to specify the corresponding waiting times as well (see Table 5.5).

The analysis of the results is listed below:

- Increasing the buffer size leads to a higher throughput, higher utilization of the lifts and shuttle vehicles, and waiting times are decreasing.
- The lifts are the bottleneck for this SBS/RS configuration (utilization = 100 %).
- If the capacity of the bottleneck component, in this case the lift, is increased, the throughput increases. If the capacity of the shuttle vehicle is increased, the overall throughput remains constant because the lift still determines the total throughput.
- An increase of the buffer capacity leads to decreasing waiting times of the lift in front of the inbound buffer and thus to higher throughput. At the same time, the waiting time of the shuttle vehicle for storage requests is reduced. By increasing the throughput of the lift, the throughput and utilization of the shuttle vehicle also increase.

- If the lift has a capacity of two, it can simultaneously transport two ULs. The storage operations can be either single-command cycles, where two ULs are assigned to the same tier, or dual-command cycles, where each UL is assigned to a different tier.
  - With a shuttle vehicle capacity of one, the outbound lift only performs dual-command cycles because it processes the incoming retrieval orders according to FCFS. This means that the cycle time of the outbound lift is larger than the cycle time of the inbound-lift. For this reason, the outbound buffers run full and the waiting times of the shuttle vehicle in front of the outbound buffers increase. With increasing buffer capacity, the time required to retrieve one UL also increases.
  - The same effect occurs with a shuttle vehicle capacity of two. This is also due to the chosen control strategy. Based on the random storage location assignment, usually only one UL arrives at a time on the same tier. If the shuttle vehicle is idle, it processes the incoming UL immediately. Thus the shuttle is not fully loaded and just performs dual-command cycles.
  - Another strategy would be for the shuttle vehicle to start the storage process only, if it is fully loaded. Thus, two ULs are always loaded and the shuttle vehicle performs quadruple-command cycles. The two retrieved ULs arrive at the same time at the outbound buffer and are processed together by the outbound lift. As a result, the outbound lift only performs single-command cycles and achieves a lower utilization. Total throughput stays the same, since the inbound lift is the bottleneck. At the same time, the time required to retrieve an UL is also reduced. For this scenario at least two buffer locations are required, since all ULs are (un)loaded simultaneously between the buffer and the shuttle vehicle. The results for this use case can be found in Table 5.6 and the waiting times in appendix G, Table G.1.

All in all, for this example it is recommended to have a lift capacity and buffer capacity of two and a shuttle vehicle capacity of one. Lift and shuttle vehicle should start the storage and retrieval operations only fully loaded.

Table 5.4: Results of experiment 1.1 – single-deep, one-behind-the-other lift, side-by-side shuttle vehicle, random tier - random storage location, storage and retrieval requests are processed after FCFS, shuttle vehicle starts the storage process not fully loaded

Input			Output									
Lift capacity [capacity/lift]	**Shuttle vehicle capacity [capacity/shuttle]**	**Buffer capacity [capacity/buffer]**	**Inbound lift utilization [%]**	**Outbound lift utilization [%]**	**Shuttle vehicle utilization [%]**	**Throughput [retrieved ULs/h]**	**Average lead time of the ULs to be stored [s]**	**Average lead time of the ULs to be retrieved [s]**	**Average inbound buffer size [capacity/buffer]**	**95 % quantile average inbound buffer size [capacity/buffer]**	**Average outbound buffer size [capacity/buffer]**	**95 % quantile average outbound buffer size [capacity/buffer]**
1	**1**	**1**	100.00	94.95	21.12	254.62	53.51	67.27	1.00	1.00	1.00	1.00
1	**1**	**2**	100.00	99.66	22.13	267.64	54.04	86.51	1.02	1.00	1.04	1.00
1	**1**	**3**	100.00	99.97	22.22	268.42	53.84	98.04	1.03	1.00	1.08	2.00
1	**1**	**4**	100.00	100.00	22.20	268.42	54.49	118.81	1.03	1.00	1.17	2.00
1	**1**	**5**	100.00	100.00	22.20	268.42	54.49	118.81	1.03	1.00	1.17	2.00
2	**1**	**1**	100.00	94.97	23.38	281.97	57.74	71.03	1.00	1.00	1.00	1.00
2	**1**	**2**	100.00	99.82	25.14	302.98	58.49	120.92	1.03	1.00	1.17	2.00
2	**1**	**3**	100.00	100.00	26.66	304.04	61.60	354.93	1.05	1.00	1.94	3.00
2	**1**	**4**	100.00	100.00	28.55	304.30	68.66	577.88	1.07	2.00	2.66	4.00
2	**1**	**5**	100.00	100.00	30.64	304.41	79.09	812.97	1.10	2.00	3.39	5.00
1	**2**	**1**	100.00	95.28	21.23	256.12	53.32	67.49	1.00	1.00	1.00	1.00
1	**2**	**2**	100.00	99.86	21.90	267.83	52.08	88.90	1.02	1.00	1.06	2.00
1	**2**	**3**	100.00	99.99	21.93	268.63	52.19	99.28	1.02	1.00	1.09	2.00
1	**2**	**4**	100.00	100.00	21.95	268.71	52.30	107.60	1.02	1.00	1.12	2.00
1	**2**	**5**	100.00	100.00	21.95	268.71	52.30	107.60	1.02	1.00	1.12	2.00
2	**2**	**1**	100.00	94.97	23.38	281.97	57.74	71.03	1.00	1.00	1.00	1.00
2	**2**	**2**	100.00	99.89	24.79	304.17	56.11	103.21	1.03	1.00	1.11	2.00
2	**2**	**3**	100.00	100.00	26.19	304.85	58.25	329.13	1.04	1.00	1.86	3.00
2	**2**	**4**	100.00	100.00	26.60	305.08	59.39	498.45	1.04	1.00	2.41	4.00
2	**2**	**5**	100.00	100.00	25.55	304.99	57.69	531.60	1.04	1.00	2.58	5.00

Table 5.5: Results of experiment 1.1, waiting times – single-deep, one-behind-the-other lift, side-by-side shuttle vehicle, random tier - random storage location, storage and retrieval requests are processed after FCFS, shuttle vehicle starts the storage process not fully loaded

Input			Waiting times						
Lift capacity [capacity/lift]	Shuttle vehicle capacity [capacity/shuttle]	Buffer capacity [capacity/buffer]	Inbound lift waits for storage request [s]	Inbound lift waits in front of inbound buffer [s]	Shuttle vehicle waits for storage request [s]	Shuttle vehicle waits for retrieval request [s]	Shuttle vehicle waits in front of outbound buffer [s]	Outbound lift waits for retrieval request [s]	Outbound lift waits in front of pre-storage zone [s]
1	1	1	0.0000	0.7139	278.85	0.0000	0.1078	0.7138	0.0000
1	1	2	0.0000	0.0461	261.84	0.0000	0.0004	0.0462	0.0000
1	1	3	0.0000	0.0047	260.77	0.0000	0.0000	0.0046	0.0000
1	1	4	0.0000	0.0000	260.83	0.0000	0.0000	0.0005	0.0000
1	1	5	0.0000	0.0000	260.83	0.0000	0.0000	0.0005	0.0000
2	1	1	0.0000	1.9335	244.58	0.0000	0.1162	1.0887	0.0000
2	1	2	0.0000	0.1693	222.35	0.0000	0.1117	0.0432	0.0000
2	1	3	0.0000	0.0745	217.08	0.0000	4.5647	0.0000	0.0000
2	1	4	0.0000	0.0744	211.34	0.0000	10.005	0.0000	0.0000
2	1	5	0.0000	0.0593	205.05	0.0000	16.198	0.0000	0.0000
1	2	1	0.0000	0.6614	276.81	0.0000	0.1107	0.6630	0.0000
1	2	2	0.0000	0.0193	267.24	0.0000	0.0080	0.0192	0.0000
1	2	3	0.0000	0.0009	266.63	0.0000	0.0000	0.0010	0.0000
1	2	4	0.0000	0.0000	266.70	0.0000	0.0000	0.0004	0.0000
1	2	5	0.0000	0.0000	266.70	0.0000	0.0000	0.0004	0.0000
2	2	1	0.0000	1.9335	244.58	0.0000	0.1162	1.0887	0.0000
2	2	2	0.0000	0.0850	228.36	0.0000	0.0893	0.0258	0.0000
2	2	3	0.0000	0.0218	224.59	0.0000	4.5292	0.0000	0.0000
2	2	4	0.0000	0.0057	223.76	0.0000	6.0275	0.0000	0.0000
2	2	5	0.0000	0.0095	226.34	0.0000	2.5480	0.0000	0.0000

Table 5.6: Results of experiment 1.2 – single-deep, one-behind-the-other lift, side-by-side shuttle vehicle, random tier - random storage location, storage and retrieval requests are processed after FCFS, shuttle vehicle starts the storage process only fully loaded

Input			Output										
Lift capacity [capacity/lift]	Shuttle vehicle capacity [capacity/shuttle]	Buffer capacity [capacity/buffer]	Inbound lift utilization [%]	Outbound lift utilization [%]	Shuttle vehicle utilization [%]	Throughput [retrieved ULs/h]	Average lead time of the ULs to be stored [s]	Average lead time of the ULs to be retrieved [s]	Average inbound buffer size [capacity/buffer]	95 % quantile average inbound buffer size [capacity/buffer]	Average outbound buffer size [capacity/buffer]	95 % quantile average outbound buffer size [capacity/buffer]
2	2	1	-	-	-	-	-	-	-	-	-	-
2	2	2	100.00	90.09	16.22	303.07	199.17	78.22	1.50	2.00	2.00	2.00
2	2	3	100.00	90.72	16.36	305.15	198.05	79.16	1.50	2.00	2.00	2.00
2	2	4	100.00	90.76	16.33	305.08	198.68	78.67	1.50	2.00	2.00	2.00
2	2	5	100.00	90.76	16.33	305.08	198.68	78.67	1.50	2.00	2.00	2.00

5.2.3 Experiment 2

In the second experiment, a double-deep SBS/RS is considered. The capacity of the inbound and outbound lift, shuttle vehicle, and inbound and outbound buffer are varied. Compared with a single-deep SBS/RS, the number of storage locations is doubled. The start configuration is:

- Lift configuration: one-behind-the-other
- Shuttle vehicle configuration: side-by-side
- Generation of storage request: FCFS
- Storage location assignment: random tier - random storage location
- Lift and shuttle vehicle sequencing strategy: optimized
- Generation of retrieval requests: FCFS
- Shuttle vehicle does not need to be fully loaded to start the storage operation

Table 5.7: Results of experiment 2.1 – double-deep, one-behind-the-other lift, side-by-side shuttle vehicle, random tier - random storage location, storage and retrieval requests are processed after FCFS, shuttle vehicle starts the storage process not fully loaded

Input			Output									
Lift capacity [capacity/lift]	**Shuttle vehicle capacity [capacity/shuttle]**	**Buffer capacity [capacity/buffer]**	**Inbound lift utilization [%]**	**Outbound lift utilization [%]**	**Shuttle vehicle utilization [%]**	**Throughput [retrieved ULs/h]**	**Average lead time of the ULs to be stored [s]**	**Average lead time of the ULs to be retrieved [s]**	**Average inbound buffer size [capacity/buffer]**	**95 % quantile average inbound buffer size [capacity/buffer]**	**Average outbound buffer size [capacity/buffer]**	**95 % quantile average outbound buffer size [capacity/buffer]**
1	**1**	**1**	100.00	93.90	23.18	251.97	55.99	68.54	1.00	1.00	1.00	1.00
1	**1**	**2**	100.00	99.52	24.40	267.33	56.58	87.63	1.03	1.00	1.04	1.00
1	**1**	**3**	100.00	99.98	24.49	268.45	56.53	106.44	1.03	1.00	1.09	2.00
1	**1**	**4**	100.00	100.00	24.46	268.64	56.49	114.26	1.03	1.00	1.12	2.00
1	**1**	**5**	100.00	100.00	24.46	268.64	56.49	114.26	1.03	1.00	1.12	2.00
2	**1**	**1**	100.00	93.77	25.34	277.85	60.49	71.77	1.00	1.00	1.00	1.00
2	**1**	**2**	100.00	99.51	27.52	301.54	61.32	108.59	1.04	1.00	1.10	2.00
2	**1**	**3**	100.00	100.00	29.52	303.69	65.98	380.82	1.06	1.00	2.00	3.00
2	**1**	**4**	100.00	100.00	30.11	303.98	69.19	557.57	1.07	2.00	2.60	4.00
2	**1**	**5**	100.00	100.00	31.62	303.93	76.42	800.46	1.09	2.00	3.38	5.00
1	**2**	**1**	100.00	93.84	23.02	251.95	55.92	68.05	1.00	1.00	1.00	1.00
1	**2**	**2**	100.00	99.82	24.29	268.16	54.28	95.01	1.02	1.00	1.02	1.00
1	**2**	**3**	100.00	99.98	24.15	268.31	54.51	107.54	1.03	1.00	1.11	2.00
1	**2**	**4**	100.00	100.00	24.12	268.21	54.33	119.49	1.02	1.00	1.15	2.00
1	**2**	**5**	100.00	100.00	24.12	268.21	54.33	119.49	1.02	1.00	1.15	2.00
2	**2**	**1**	100.00	93.91	25.39	278.40	60.44	71.72	1.00	1.00	1.00	1.00
2	**2**	**2**	100.00	99.73	27.48	303.34	58.45	101.17	1.03	1.00	1.03	1.00
2	**2**	**3**	100.00	100.00	29.02	304.69	60.67	252.15	1.04	1.00	1.54	2.00
2	**2**	**4**	100.00	100.00	28.95	305.08	60.62	387.63	1.04	1.00	2.03	3.00
2	**2**	**5**	100.00	100.00	28.49	304.93	60.74	474.46	1.04	1.00	2.36	4.00

Table 5.8: Results of experiment 2.1, waiting times – double-deep, one-behind-the-other lift, side-by-side shuttle vehicle, random tier - random storage location, storage and retrieval requests are processed after FCFS, shuttle vehicle starts the storage process not fully loaded

Input			Waiting times						
Lift capacity [capacity/lift]	**Shuttle vehicle capacity [capacity/shuttle]**	**Buffer capacity [capacity/buffer]**	**Inbound lift waits for storage request [s]**	**Inbound lift waits in front of inbound buffer [s]**	**Shuttle vehicle waits for storage request [s]**	**Shuttle vehicle waits for retrieval request [s]**	**Shuttle vehicle waits in front of outbound buffer [s]**	**Outbound lift waits for retrieval request [s]**	**Outbound lift waits in front of pre-storage zone [s]**
1	**1**	**1**	0.0000	0.8715	274.3919	0.0000	0.0889	0.8714	0.0000
1	**1**	**2**	0.0000	0.0643	254.4951	0.0000	0.0000	0.0643	0.0000
1	**1**	**3**	0.0000	0.0030	253.1427	0.0000	0.0000	0.0030	0.0000
1	**1**	**4**	0.0000	0.0000	253.0740	0.0000	0.0000	0.0004	0.0000
1	**1**	**5**	0.0000	0.0000	253.0740	0.0000	0.0000	0.0004	0.0000
2	**1**	**1**	0.0000	2.3132	241.7707	0.0000	0.0921	1.3579	0.0000
2	**1**	**2**	0.0000	0.2734	216.3217	0.0000	0.0233	0.1138	0.0000
2	**1**	**3**	0.0000	0.0965	208.8439	0.0000	5.3031	0.0000	0.0000
2	**1**	**4**	0.0000	0.0853	206.8925	0.0000	7.0511	0.0000	0.0000
2	**1**	**5**	0.0000	0.0874	202.4746	0.0000	11.6948	0.0000	0.0000
1	**2**	**1**	0.0000	0.8795	274.9311	0.0000	0.0846	0.8795	0.0000
1	**2**	**2**	0.0000	0.0240	259.5623	0.0000	0.7766	0.0239	0.0000
1	**2**	**3**	0.0000	0.0033	260.1287	0.0000	0.0178	0.0033	0.0000
1	**2**	**4**	0.0000	0.0000	260.1648	0.0000	0.0000	0.0002	0.0000
1	**2**	**5**	0.0000	0.0000	260.1648	0.0000	0.0000	0.0002	0.0000
2	**2**	**1**	0.0000	2.2693	241.2343	0.0000	0.0942	1.3261	0.0000
2	**2**	**2**	0.0000	0.1348	221.5027	0.0000	1.1111	0.0631	0.0000
2	**2**	**3**	0.0000	0.0185	217.0657	0.0000	5.6305	0.0000	0.0000
2	**2**	**4**	0.0000	0.0046	216.5116	0.0000	5.1528	0.0000	0.0000
2	**2**	**5**	0.0000	0.0105	218.2135	0.0000	3.8360	0.0000	0.0000

The results show a similar behavior as for single-deep SBS/RS (results see Table 5.7 and Table 5.8). Due to the required relocation operation of blocked ULs, the cycle time of the shuttle vehicle is longer than in a single-deep

SBS/RS. The increase of the cycle time means also an increase of the utilization of the shuttle vehicle. However, this does not have any influence on the overall performance of the SBS/RS, since the lift is the bottleneck.

Now a side-by-side lift and one-behind-the-other shuttle vehicle is used. Still the same control strategies for the double-deep SBS/RS are applied (results see Table 5.9 and appendix G, Table G.2). For this configuration, the throughput increases, compared with the throughput of the one-behind-the-other lift and side-by-side shuttle vehicle. This is due to the fact that the (un)loading times between the pre-storage zone and the lift are halved from eight seconds to four seconds. The same occurs when two ULs are simultaneously (un)loaded between the buffer and the lift.

Table 5.9: Results of experiment 2.2 – double-deep, side-by-side lift, one-behind-the-other shuttle vehicle, random tier - random storage location, storage and retrieval requests are processed after FCFS, shuttle vehicle starts the storage process not fully loaded

Input			**Output**									
Lift capacity [capacity/lift]	**Shuttle vehicle capacity [capacity/shuttle]**	**Buffer capacity [capacity/buffer]**	**Inbound lift utilization [%]**	**Outbound lift utilization [%]**	**Shuttle vehicle utilization [%]**	**Throughput [retrieved ULs/h]**	**Average lead time of the ULs to be stored [s]**	**Average lead time of the ULs to be retrieved [s]**	**Average inbound buffer size [capacity/buffer]**	**95 % quantile average inbound buffer size [capacity/buffer]**	**Average outbound buffer size [capacity/buffer]**	**95 % quantile average outbound buffer size [capacity/buffer]**
2	**2**	**1**	100.00	91.26	28.97	309.52	66.38	66.72	1.00	1.00	1.00	1.00
2	**2**	**2**	100.00	99.52	33.42	365.11	62.83	96.66	1.05	2.00	1.06	2.00
2	**2**	**3**	100.00	100.00	37.82	368.40	70.18	284.35	1.09	2.00	1.70	3.00
2	**2**	**4**	100.00	100.00	38.41	369.03	72.90	437.22	1.11	2.00	2.33	4.00
2	**2**	**5**	100.00	100.00	38.29	368.99	74.69	580.98	1.11	2.00	2.94	4.00

In the next step the control strategy that the shuttle vehicle needs always to be fully loaded to start the storage operation is applied (results see Table 5.10 and appendix G, Table G.3). Here, the same system behavior can be seen as in experiment 1.2. The main difference to experiment 2.2 is: utilization of the outbound lift is decreased, throughput stays the same, and average lead time to retrieve one UL is reduced.

Table 5.10: Results of experiment 2.3 – double-deep, side-by-side lift, one-behind-the-other shuttle vehicle, random tier - random storage location, storage and retrieval requests are processed after FCFS, shuttle vehicle starts the storage process only fully loaded

Input			**Output**										
Lift capacity [capacity/lift]	**Shuttle vehicle capacity [capacity/shuttle]**	**Buffer capacity [capacity/buffer]**	**Inbound lift utilization [%]**	**Outbound lift utilization [%]**	**Shuttle vehicle utilization [%]**	**Throughput [retrieved ULs/h]**	**Average lead time of the ULs to be stored [s]**	**Average lead time of the ULs to be retrieved [s]**	**Average inbound buffer size [capacity/buffer]**	**95 % quantile average inbound buffer size [capacity/buffer]**	**Average outbound buffer size [capacity/buffer]**	**95 % quantile average outbound buffer size [capacity/buffer]**
2	**2**	**1**	-	-	-	-	-	-	-	-	-	-
2	**2**	**2**	100.00	78.64	23.11	362.01	177.39	68.90	1.50	2.00	1.99	2.00
2	**2**	**3**	100.00	80.15	23.61	369.81	174.98	73.68	1.53	2.00	2.00	2.00
2	**2**	**4**	100.00	81.15	23.50	369.77	175.43	71.81	1.53	2.00	2.00	2.00
2	**2**	**5**	100.00	78.94	23.62	369.92	174.91	67.75	1.53	2.00	1.99	2.00

In the next step, the storage assignment strategy is adapted in such a way that two small ULs are batched and transported to the same tier. By doing so, the inbound lift and the outbound lift only perform single-command cycles. This decreases the cycle time of the lifts and thus the total throughput can be increased. The results for this use case can be found in Table 5.11 and the waiting times in appendix G, Table G.4.

Table 5.11: Results of experiment 2.4 – double-deep, side-by-side lift, one-behind-the-other shuttle vehicle, random tier - random storage location, batch two small ULs and store them together on the same tier, storage and retrieval requests are processed after FCFS, shuttle vehicle starts the storage process only fully loaded

Input			**Output**										
Lift capacity [capacity/lift]	**Shuttle vehicle capacity [capacity/shuttle]**	**Buffer capacity [capacity/buffer]**	**Inbound lift utilization [%]**	**Outbound lift utilization [%]**	**Shuttle vehicle utilization [%]**	**Throughput [retrieved ULs/h]**	**Average lead time of the ULs to be stored [s]**	**Average lead time of the ULs to be retrieved [s]**	**Average inbound buffer size [capacity/buffer]**	**95 % quantile average inbound buffer size [capacity/buffer]**	**Average outbound buffer size [capacity/buffer]**	**95 % quantile average outbound buffer size [capacity/buffer]**
2	**2**	**1**	-	-	-	-	-	-	-	-	-	-
2	**2**	**2**	100.00	85.02	29.02	455.62	70.75	71.07	2.00	2.00	2.00	2.00
2	**2**	**3**	100.00	85.33	29.10	457.81	70.96	70.93	2.05	2.00	2.00	2.00
2	**2**	**4**	100.00	97.69	33.33	523.90	74.26	90.65	2.13	3.00	2.01	2.00
2	**2**	**5**	100.00	98.00	33.51	526.46	73.87	92.74	2.12	3.00	2.01	2.00

To sum up using a double-deep SBS/RS means a higher space utilization compared to a single-deep SBS/RS. Depending on the selected lift and shuttle vehicle configuration and the control strategy used, different performance of the SBS/RS can be achieved. The best performance can be achieved for the following SBS/RS configuration and control strategies:

- The lift should be a side-by-side configuration with a capacity of two. This configuration is only possible for double-deep storage systems. The advantage is that two ULs can be (un)loaded simultaneously and thus the load handling time can be reduced.
- If the lift is a side-by-side configuration, the shuttle vehicle should be a one-behind-the-other configuration with a capacity of two.

- The capacity of the buffer should always be a multiple of two. Thus a buffer with a capacity of four should be chosen. In comparison with a buffer capacity of two, a higher throughput can be achieved.
- The shuttle vehicle should be fully loaded before starting the storage operation. This leads to a smaller shuttle vehicle utilization.
- Two small ULs should be batched and always be stored on the same tier. This reduces the cycle time of the inbound lift and increases the throughput.

5.2.4 Experiment 3

In the third experiment a double-deep SBS/RS and the storage of small and large ULs is considered. Different control strategies are applied and the performance of the SBS/RS is evaluated. The start configuration is:

- Lift configuration: side-by-side
- Shuttle vehicle configuration: one-behind-the-other
- Generation of storage request: batch two small ULs
- Storage location assignment: batch to small ULs - random tier - random storage location
- Lift and shuttle vehicle sequencing strategy: optimized
- Generation of retrieval requests: batch two small ULs
- Shuttle vehicle needs to be fully loaded to start the storage process
- $P_{small} = 66.67$ %

Table 5.12: Results of experiment 3.1 – double-deep, small and large ULs, side-by-side lift, one-behind-the-other shuttle vehicle, random tier - random storage location, batch two small ULs and store them together on the same tier, storage and retrieval requests are batched, shuttle vehicle starts the storage process only fully loaded

Input			Output										
Lift capacity [capacity/lift]	**Shuttle vehicle capacity [capacity/shuttle]**	**Buffer capacity [capacity/buffer]**	**Inbound lift utilization [%]**	**Outbound lift utilization [%]**	**Shuttle vehicle utilization [%]**	**Throughput [retrieved ULs/h]**	**Average lead time of the ULs to be stored [s]**	**Average lead time of the ULs to be retrieved [s]**	**Average inbound buffer size [capacity/buffer]**	**95 % quantile average inbound buffer size [capacity/buffer]**	**Average outbound buffer size [capacity/buffer]**	**95 % quantile average outbound buffer size [capacity/buffer]**
2	**2**	**2**	100.00	89.74	25.73	361.43	64.58	70.59	2.00	2.00	2.00	2.00
2	**2**	**4**	100.00	98.62	28.25	396.46	66.46	90.74	2.09	2.00	2.04	2.00
2	**2**	**6**	100.00	99.88	28.60	402.64	66.46	111.47	2.10	2.00	2.13	4.00
2	**2**	**8**	100.00	99.98	28.58	402.72	66.26	121.22	2.10	2.00	2.19	4.00
2	**2**	**10**	100.00	100.00	28.55	402.60	66.51	123.29	2.10	2.00	2.21	4.00

With this selected configuration and the control strategies used, the highest throughput is achieved because the lift and shuttle vehicles are always fully loaded and the lifts always perform single command-cycles. The results show (see Table 5.12 and appendix G, Table G.5) that a higher buffer capacity leads to a decrease of waiting times and thus the performance of the SBS/RS can be increased.

In the next step, the storage location assignment strategy is changed to a random selected storage location. The final storage location is selected when the UL is loaded onto the inbound lift. Still two small ULs are batched and assigned to the same tier.

Table 5.13: Results of experiment 3.2 – double-deep, small and large ULs, side-by-side lift, one-behind-the-other shuttle vehicle, random storage location, batch two small ULs and store them together on the same tier, storage and retrieval requests are batched, shuttle vehicle starts the storage process only fully loaded

Input			Output									
Lift capacity [capacity/lift]	**Shuttle vehicle capacity [capacity/shuttle]**	**Buffer capacity [capacity/buffer]**	**Inbound lift utilization [%]**	**Outbound lift utilization [%]**	**Shuttle vehicle utilization [%]**	**Throughput [retrieved ULs/h]**	**Average lead time of the ULs to be stored [s]**	**Average lead time of the ULs to be retrieved [s]**	**Average inbound buffer size [capacity/buffer]**	**95 % quantile average inbound buffer size [capacity/buffer]**	**Average outbound buffer size [capacity/buffer]**	**95 % quantile average outbound buffer size [capacity/buffer]**
2	**2**	**2**	100.00	89.38	25.69	361.74	64.68	70.53	2.00	2.00	2.00	2.00
2	**2**	**4**	100.00	98.88	28.07	395.59	65.98	90.98	2.09	2.00	2.04	2.00
2	**2**	**6**	100.00	99.74	28.28	398.54	66.34	110.58	2.10	2.00	2.13	4.00
2	**2**	**8**	100.00	99.93	28.45	401.10	66.40	120.06	2.10	2.00	2.19	4.00
2	**2**	**10**	100.00	100.00	28.57	401.44	66.50	121.33	2.10	2.00	2.19	4.00

The results show (see Table 5.13 and appendix G, Table G.6) that for this example, it makes no difference whether the final storage location is chosen when loading the inbound lift or shuttle vehicle. However, it should be avoided to reserve too many storage locations on one tier, as this would increase the cycle time of the shuttle vehicle. Reserved storage locations can not be used as a relocation channel, thus the required time for a relocation operation increases. To avoid this, an intelligent storage strategy can be used, e.g., relocation is possible into reserved storage locations, but then a new storage location has to be reserved.

In the next step, the storage location assignment strategy is changed. A random storage tier is selected in front of the lift. The final storage location is then selected before the UL is loaded onto the shuttle vehicle (results see Table 5.14 and appendix G, Table G.7).

Since each UL is randomly distributed to the individual tiers, the two small ULs on the inbound lift are usually stored on two different tiers. This increases the cycle time of the inbound lift and thus decreases the throughput. If one small UL arrives in the inbound buffer, the shuttle waits for a second small UL, thus the shuttle can get fully loaded. For the case that a small UL is followed by a large UL, the shuttle vehicle can not be fully loaded and must start the storage operation not fully loaded.

Table 5.14: Results of experiment 3.3 – double-deep, small and large ULs, side-by-side lift, one-behind-the-other shuttle vehicle, random tier - random storage location, storage requests are batched, retrieval requests are batched, if possible shuttle vehicle starts the storage process only fully loaded

Input			Output										
Lift capacity [capacity/lift]	**Shuttle vehicle capacity [capacity/shuttle]**	**Buffer capacity [capacity/buffer]**	**Inbound lift utilization [%]**	**Outbound lift utilization [%]**	**Shuttle vehicle utilization [%]**	**Throughput [retrieved ULs/h]**	**Average lead time of the ULs to be stored [s]**	**Average lead time of the ULs to be retrieved [s]**	**Average inbound buffer size [capacity/buffer]**	**95 % quantile average inbound buffer size [capacity/buffer]**	**Average outbound buffer size [capacity/buffer]**	**95 % quantile average outbound buffer size [capacity/buffer]**
2	**2**	**2**	100.00	73.74	19.77	264.32	202.27	60.49	1.61	2.00	1.82	2.00
2	**2**	**4**	100.00	90.32	24.16	323.77	179.37	72.16	2.00	4.00	1.85	2.00
2	**2**	**6**	100.00	91.59	24.56	328.28	178.77	73.09	2.02	4.00	1.85	2.00
2	**2**	**8**	100.00	91.79	24.58	329.09	178.99	73.66	2.02	4.00	1.85	2.00
2	**2**	**10**	100.00	91.94	24.60	328.59	178.95	74.17	2.02	4.00	1.85	2.00

In the next step, storage and retrieval requests are processed after FCFS and are not batched. As a result, the inbound lift is not always fully loaded and the maximum possible throughput decreases (results see Table 5.15 and appendix Table G.8).

Table 5.15: Results of experiment 3.4 – double-deep, small and large ULs, side-by-side lift, one-behind-the-other shuttle vehicle, random tier - random storage location, storage and retrieval requests are processed after FCFS, if possible shuttle vehicle starts the storage process only fully loaded

Input			**Output**									
Lift capacity [capacity/lift]	**Shuttle vehicle capacity [capacity/shuttle]**	**Buffer capacity [capacity/buffer]**	**Inbound lift utilization [%]**	**Outbound lift utilization [%]**	**Shuttle vehicle utilization [%]**	**Throughput [retrieved ULs/h]**	**Average lead time of the ULs to be stored [s]**	**Average lead time of the ULs to be retrieved [s]**	**Average inbound buffer size [capacity/buffer]**	**95 % quantile average inbound buffer size [capacity/buffer]**	**Average outbound buffer size [capacity/buffer]**	**95 % quantile average outbound buffer size [capacity/buffer]**
2	**2**	**2**	100.00	72.36	19.32	258.52	204.22	59.47	1.60	2.00	1.82	2.00
2	**2**	**4**	100.00	86.65	23.17	310.11	184.35	66.95	1.99	4.00	1.83	2.00
2	**2**	**6**	100.00	87.86	23.56	314.66	182.48	67.49	2.00	4.00	1.84	2.00
2	**2**	**8**	100.00	87.81	23.47	314.29	181.91	67.69	2.00	4.00	1.84	2.00
2	**2**	**10**	100.00	87.81	23.47	314.29	181.91	67.69	2.00	4.00	1.84	2.00

In conclusion, the storage of small and large ULs in the same SBS/RS is a very challenging task. The implementation of the correct control strategy leads to a higher throughput. The best performance can be achieved for the following SBS/RS configuration and control strategies:

- Buffer capacity should be four
- Two small ULs should always be batched. Thus, the lift and shuttle vehicles can be fully loaded.

- Two small ULs should always be stored on the same tier. The inbound lift only needs to perform single-command cycles. A smaller cycle time of the inbound lift leads to a higher throughput.
- An empty tier should be assigned first to each UL before loaded onto the inbound lift. The final storage location should be assigned before the UL is loaded onto the shuttle vehicle.

5.3 Comparison of the Simulation Model with the Analytical Model

In this chapter, the simulation results of chapter 5.2 will be compared with the results of the analytical models presented in chapter 3. In practice, waiting times can occur during the operation. However, the analytical models in chapter 3 do not consider waiting times. It will now be demonstrated, under which configurations the analytical models hold validity and whether simulation experiments remain necessary.

The tables below present the simulation results, the outcomes derived from the analytical models (along with the used equations), and the calculated relative errors.

The analytical models developed in this work do not consider the number of buffer locations. Thus, the results are the same at each experiment, regardless of the number of buffer locations. Nevertheless, it's important to note that the number of buffer locations does impact the waiting times. A larger buffer leads to shorter waiting times since incoming ULs can be transferred directly into the buffer.

In experiment 1.1 and 1.2, the analytical models demonstrate excellent performance. With a buffer size of one, the deviation ranges from -3.66% to -6.93%. For larger buffer sizes, the deviation is close to 1% (deviation between 0.52 % and 1.2 %). As recommended in chapter 5.2.2, it is advisable to choose a minimum of two buffer locations for these SBS/RS configurations. This leads to shorter waiting times and a higher throughput. For these configurations (at least two buffer locations) the analytical model provides almost the same answers, compared with the simulation experiment.

For experiment 2.1, 2.2 and 2.3, the same statement can be made as for experiment 1.1 and 1.2. Starting from a buffer size of two, the analytical models provide almost the same answers, compared with the simulation experiment. The deviation for these configurations is only between 0 % and 1.3%. In experiment 2.4, Strategy 1 is employed for the lift. This entails the simultaneous transfer of two small ULs to the same tier. To attain the highest achievable throughput, it is advised to have a minimum of four buffer locations for this strategy. In this scenario, the analytical models yield highly accurate results starting from a buffer size of four compared with the simulation experiment. The deviation for a buffer size of four is -1.35 %.

For experiment 3.1 and 3.2, the same statement can be made as for experiment 2.4. For just two buffer locations the deviation is around -10 %. For buffer sizes larger than two the deviation is between -0.67 % and 1.11 %. The developed analytical models are not suitable for experiment 3.3. This is due to the strategy employed, which cannot be accurately depicted by the analytical models developed in this work. In this chosen configuration, the lift and shuttle vehicle are often not fully loaded. Thus the SBS/RS is not fully utilized and a lower throughput is achieved. To accurately model this use case, it would be necessary to define the probabilities of utilization for the lift and the shuttle vehicle. These probabilities could then be factored into the throughput calculation.

In summary, it has been demonstrated that the analytical models yield highly accurate results for calculating the maximum achievable throughput of the SBS/RS. This holds true under the assumption that, for the specified configurations, a minimum of two buffer locations are available, and four buffer locations are required when applying Strategy 1. If different SBS/RS configurations or control strategies are considered, it is crucial to reevaluate the validity of the analytical models. It may be necessary to modify and adjust the models to ensure their accuracy for these new configurations.

Table 5.16: Results of experiment 1 compared with analytical results

Experiment	Lift capacity [capacity/lift]	Shuttle vehicle capacity [capacity/shuttle]	Buffer capacity [capacity/buffer]	Throughput simulation [retrieved ULs/h]	Throughput analytical [retrieved ULs/h]	Relative error [%]	Equation travel time lift	Equation cycle time lift	Equation throughput lift	Equation travel time shuttle vehicle	Equation cycle time shuttle vehicle	Equation throughput shuttle vehicle	Equation throughput SBS/RS
1.1	1	1	1	254.62	265.49	-4.27	3.6	3.4	E.1	3.37	3.39	E.9	3.100
	1	1	2	267.64	265.49	0.80	3.6	3.4	E.1	3.37	3.39	E.9	3.100
	1	1	3	268.42	265.49	1.09	3.6	3.4	E.1	3.37	3.39	E.9	3.100
	1	1	4	268.42	265.49	1.09	3.6	3.4	E.1	3.37	3.39	E.9	3.100
	1	1	5	268.42	265.49	1.09	3.6	3.4	E.1	3.37	3.39	E.9	3.100
1.1	2	1	1	281.97	301.51	-6.93	3.16	3.15	E.1	3.37	3.39	E.9	3.100
	2	1	2	302.98	301.51	0.49	3.16	3.15	E.1	3.37	3.39	E.9	3.100
	2	1	3	304.04	301.51	0.83	3.16	3.15	E.1	3.37	3.39	E.9	3.100
	2	1	4	304.3	301.51	0.92	3.16	3.15	E.1	3.37	3.39	E.9	3.100
	2	1	5	304.41	301.51	0.95	3.16	3.15	E.1	3.37	3.39	E.9	3.100
1.1	1	2	1	256.12	265.49	-3.66	3.6	3.4	E.1	3.41	3.41	E.10	3.100
	1	2	2	267.83	265.49	0.87	3.6	3.4	E.1	3.41	3.41	E.10	3.100
	1	2	3	268.63	265.49	1.17	3.6	3.4	E.1	3.41	3.41	E.10	3.100
	1	2	4	268.71	265.49	1.20	3.6	3.4	E.1	3.41	3.41	E.10	3.100
	1	2	5	268.71	265.49	1.20	3.6	3.4	E.1	3.41	3.41	E.10	3.100
1.1	2	2	1	281.97	301.51	-6.93	3.16	3.15	E.2	3.41	3.41	E.10	3.100
	2	2	2	304.17	301.51	0.88	3.16	3.15	E.2	3.41	3.41	E.10	3.100
	2	2	3	304.85	301.51	1.10	3.16	3.15	E.2	3.41	3.41	E.10	3.100
	2	2	4	305.08	301.51	1.17	3.16	3.15	E.2	3.41	3.41	E.10	3.100
	2	2	5	304.99	301.51	1.14	3.16	3.15	E.2	3.41	3.41	E.10	3.100
1.2	2	2	1	-	-	-	-	-	-	-	-	-	-
	2	2	2	303.07	301.51	0.52	3.16	3.15	E.2	3.41	3.41	E.10	3.100
	2	2	3	305.15	301.51	1.19	3.16	3.15	E.2	3.41	3.41	E.10	3.100
	2	2	4	305.08	301.51	1.17	3.16	3.15	E.2	3.41	3.41	E.10	3.100
	2	2	5	305.08	301.51	1.17	3.16	3.15	E.2	3.41	3.41	E.10	3.100

Table 5.17: Results of experiment 2 compared with analytical results

Experiment	Lift capacity [capacity/lift]	Shuttle vehicle capacity [capacity/shuttle]	Buffer capacity [capacity/buffer]	Throughput simulation [retrieved ULs/h]	Throughput analytical [retrieved ULs/h]	Relative error [%]	Equation travel time lift	Equation cycle time lift	Equation throughput lift	Equation travel time shuttle vehicle	Equation cycle time shuttle vehicle	Equation throughput shuttle vehicle	Equation throughput SBS/RS
2.1	1	1	1	251.97	265.49	-5.36	3.6	3.4	E.1	3.65	3.65	E.13	3.100
	1	1	2	267.33	265.49	0.69	3.6	3.4	E.1	3.65	3.65	E.13	3.100
	1	1	3	268.45	265.49	1.10	3.6	3.4	E.1	3.65	3.65	E.13	3.100
	1	1	4	268.64	265.49	1.17	3.6	3.4	E.1	3.65	3.65	E.13	3.100
	1	1	5	268.64	265.49	1.17	3.6	3.4	E.1	3.65	3.65	E.13	3.100
2.1	2	1	1	277.85	301.51	-8.51	3.16	3.15	E.2	3.65	3.65	E.13	3.100
	2	1	2	301.54	301.51	0.01	3.16	3.15	E.2	3.65	3.65	E.13	3.100
	2	1	3	303.69	301.51	0.72	3.16	3.15	E.2	3.65	3.65	E.13	3.100
	2	1	4	303.98	301.51	0.81	3.16	3.15	E.2	3.65	3.65	E.13	3.100
	2	1	5	303.93	301.51	0.80	3.16	3.15	E.2	3.65	3.65	E.13	3.100
2.1	1	2	1	251.95	265.49	-5.37	3.6	3.4	E.1	3.66	3.66	E.14	3.100
	1	2	2	268.16	265.49	1.00	3.6	3.4	E.1	3.66	3.66	E.14	3.100
	1	2	3	268.31	265.49	1.05	3.6	3.4	E.1	3.66	3.66	E.14	3.100
	1	2	4	268.21	265.49	1.02	3.6	3.4	E.1	3.66	3.66	E.14	3.100
	1	2	5	268.21	265.49	1.02	3.6	3.4	E.1	3.66	3.66	E.14	3.100
2.1	2	2	1	278.4	301.51	-8.30	3.16	3.15	E.2	3.66	3.66	E.14	3.100
	2	2	2	303.34	301.51	0.60	3.16	3.15	E.2	3.66	3.66	E.14	3.100
	2	2	3	304.69	301.51	1.04	3.16	3.15	E.2	3.66	3.66	E.14	3.100
	2	2	4	305.08	301.51	1.17	3.16	3.15	E.2	3.66	3.66	E.14	3.100
	2	2	5	304.93	301.51	1.12	3.16	3.15	E.2	3.66	3.66	E.14	3.100
2.2	2	2	1	309.52	365.11	-17.96	3.16	3.15	E.2	3.67	3.67	E.14	3.100
	2	2	2	365.11	365.11	0.00	3.16	3.15	E.2	3.67	3.67	E.14	3.100
	2	2	3	368.4	365.11	0.89	3.16	3.15	E.2	3.67	3.67	E.14	3.100
	2	2	4	369.03	365.11	1.06	3.16	3.15	E.2	3.67	3.67	E.14	3.100
	2	2	5	368.99	365.11	1.05	3.16	3.15	E.2	3.67	3.67	E.14	3.100
2.3	2	2	1	-	-	-	-	-	-	-	-	-	-
	2	2	2	362.01	365.11	-0.86	3.16	3.15	E.2	3.67	3.67	E.14	3.100
	2	2	3	369.81	365.11	1.27	3.16	3.15	E.2	3.67	3.67	E.14	3.100
	2	2	4	369.77	365.11	1.26	3.16	3.15	E.2	3.67	3.67	E.14	3.100
	2	2	5	369.92	365.11	1.30	3.16	3.15	E.2	3.67	3.67	E.14	3.100
2.4	2	2	1	-	-	-	-	-	-	-	-	-	-
	2	2	2	455.62	530.97	-16.54	3.26	3.26	E.5	3.67	3.67	E.14	3.100
	2	2	3	457.81	530.97	-15.98	3.26	3.26	E.5	3.67	3.67	E.14	3.100
	2	2	4	523.9	530.97	-1.35	3.26	3.26	E.5	3.67	3.67	E.14	3.100
	2	2	5	526.46	530.97	-0.86	3.26	3.26	E.5	3.67	3.67	E.14	3.100

Table 5.18: Results of experiment 3 compared with analytical results

Experiment	Lift capacity [capacity/lift]	Shuttle vehicle capacity [capacity/shuttle]	Buffer capacity [capacity/buffer]	Throughput simulation [retrieved ULs/h]	Throughput analytical [retrieved ULs/h]	Relative error [%]	Equation travel time lift	Equation cycle time lift	Equation throughput lift	Equation travel time shuttle vehicle	Equation cycle time shuttle vehicle	Equation throughput shuttle vehicle	Equation throughput SBS/RS
3.1	2	2	2	361.43	398.23	-10.18	3.6	3.28	E.6	3.85	3.85	E.15	3.100
	2	2	4	396.46	398.23	-0.45	3.6	3.28	E.6	3.85	3.85	E.15	3.100
	2	2	6	402.64	398.23	1.10	3.6	3.28	E.6	3.85	3.85	E.15	3.100
	2	2	8	402.72	398.23	1.11	3.6	3.28	E.6	3.85	3.85	E.15	3.100
	2	2	10	402.6	398.23	1.09	3.6	3.28	E.6	3.85	3.85	E.15	3.100
3.2	2	2	2	361.74	398.23	-10.09	3.6	3.28	E.6	3.85	3.85	E.15	3.100
	2	2	4	395.59	398.23	-0.67	3.6	3.28	E.6	3.85	3.85	E.15	3.100
	2	2	6	398.54	398.23	0.08	3.6	3.28	E.6	3.85	3.85	E.15	3.100
	2	2	8	401.10	398.23	0.72	3.6	3.28	E.6	3.85	3.85	E.15	3.100
	2	2	10	401.44	398.23	0.80	3.6	3.28	E.6	3.85	3.85	E.15	3.100

5.4 Chapter Conclusion

In this chapter, a performance evaluation was carried out through simulations for selected SBS/RS configurations and control strategies. The advantage of the simulation is, that it is possible to consider the interaction between lifts, and shuttle vehicles and the impact of different buffer sizes, which is not the case for the analytical models in this thesis. Due to the interaction, there may be waiting times that can influence the performance of the SBS/RS. Given the numerous parallel processes in a SBS/RS, achieving optimal performance relies on the ideal combination of lift configuration, number of buffer locations, shuttle vehicle configuration, and control strategies. For example, a buffer with low capacity leads to lower throughput and higher waiting times. A buffer with very high capacity increases throughput, but is also associated with higher investment costs and space requirements. For the most suitable SBS/RS for the desired use case, the best possible ratio of investment costs, throughput, and other relevant KPIs must be selected in the end. This can be done with the help of simulations and analytical models.

The results indicate that a minimum of two buffer locations should be chosen for the specified configurations, and at least four buffer locations for Strategy 1. With this number of buffer locations, the waiting times for an empty buffer location are very low, enabling a high throughput to be achieved. While additional buffer locations further decrease waiting times, they only yield a marginal increase in performance. Furthermore, it could be demonstrated that the analytical models, developed in chapter 3, provide highly accurate results for these specific use cases with two and four buffer locations. The maximum deviation is ranging from -1.35 % to 1.11 % comparing the results of the analytical models with the simulation results.

Only a few selected SBS/RS configurations were simulated, analyzed, and evaluated in this study. However, in future simulation studies, a full factorial simulation can be conducted to determine the optimal SBS/RS configuration.

6 Conclusion

Shuttle-based storage and retrieval systems with double-deep storage, a dual-load handling device for lift and shuttle vehicle and two different sizes of unit loads are getting more popular and are more frequently installed in warehouses. However, a scientific consideration and evaluation of this system was not yet available. This work is closing this research gap.

6.1 Summary of the Thesis

At the beginning of this work, a generally applicable method for describing the characteristics of a SBS/RS was presented. In order to capture all relevant data, a methodological approach consisting of eight steps was presented. When following all eight steps, this results in a clear description of the SBS/RS. The output includes, among other things, a clear description of the design of the SBS/RS and all its sub systems, the material flow, the employed control strategies, load handing times etc.. In addition, an approach was presented for a universal representation of design, material flow, and control strategies. The layout of a SBS/RS, the design of the shuttle vehicle and the lift, and the flow of materials can be described by applying the developed block diagram. The flow of materials between the individual components, e.g., lift, shuttle vehicle, and storage location, can be described by small linking blocks. All other relevant data can be presented in tables, adapted UML diagrams and adapted "activity swim lane" diagrams. Thus, it was demonstrated that uniform mapping of the SBS/RS is achievable, and the presented approach is easily comprehensible and applicable to various SBS/RS configurations.

In the second part of this work, travel time, cycle time, and throughput equations were derived for different lift and shuttle vehicle configurations. Starting with single-deep, single-load handling device and one size of ULs, these models where extended to double-deep, multi-load handling device and two different sizes of ULs. Moreover, various control strategies such as processing storage and retrieval requests after FCFS or in an optimized sequence have been taken into account, leading to the derivation of different equations.

In the first step, analytical models were derived for four different lift configurations with different I/O points. **In the second step**, these models for a multi-load handling lift were extended, which leads to multi-command cycles. Therefore, two different control strategies were implemented. (1) Processing the requests after FCFS or (2) presorting the request and processing the request in an optimized sequence which requires the lift to only travel up and down once. It was shown that presorting the requests and processing them in an optimized sequence leads to shorter cycle times than processing after FCFS. **In the third step**, the use case that the lift can handle ULs of two different sizes was considered, whereby two small ULs require the same capacity on the lift as one large UL. Depending on the percentage of small ULs an analytical model was developed. It was shown that the more small ULs are stored and retrieved, the higher the cycle time and throughput. In addition, it was shown, that higher throughput can be reached, if in one command cycle two small ULs are stored together on the same tier.

Based on the analytical model for the lift, the analytical models were derived for the shuttle vehicle. **In the fourth step**, single-deep SBS/RSs were considered. **In the fifth step**, the models were extended for double-deep SBS/RSs and three relocation strategies (random, one side, both sides) were considered. Depending on the filling degree, the relocation probability and relocation channel distance was calculated. It was shown that the shortest relocation cycle time is achieved by applying the “both sides” relocation strategy. **In the sixth step**, a multi-load handling shuttle vehicle that can transport several ULs at the same time was considered. Two different shuttle vehicle configurations, one-behind-the-other and side-by-side, were compared. The results demonstrated, that the side-by-side configuration achieves a shorter cycle time because ULs can be relocated always on both sides. **Finally**, the cycle time for a double-deep SBS/RS equipped with a dual-load handling device and the storage of two different sized ULs, in dependency of the filling degree and the percentage of small ULs, was defined.

Last but not least, for all the configurations presented, equations were derived to determine the expected maximum throughput under the assumption that no waiting times occur. Subsequently, the analytical models for cycle time, travel time and throughput were validated by simulation. For this case a perfect system was assumed with no waiting times occurring. The results showed, that there is minimal deviation (relative error) between the simulation results and the results using the derived equations. The relative error increases for small rack sizes (less tiers and storage channels), for an increasing size of the load handling devices and for an increasing number of small ULs, because the number of short distances traveled is increasing. The reason for the increasing relative error is the assumption for the analytical equations that the maximum velocity is always reached, also for short travel distances. The evaluation of the results from the analytical models also revealed, that incoming orders for the lift and shuttle vehicle should be pre-sorted, and two small ULs should always be batched to achieve a higher throughput. Additionally, it is recommended to process the orders in an optimized sequence. This will lead to shorter travel times and a higher throughput. If in double deep shuttle systems, a relocation operation (relocation of blocking ULs) is required, the closest empty or half full storage channel should be chosen on both sides, as a potential relocation channel. This minimizes the relocation time and increases the throughput.

In the third part, a zero-one integer programming optimization model for solving the block sequencing problem of storage and retrieval requests for one tier of a double-deep SBS/RS, a shuttle vehicle with a dual load handling device and with two differently sized ULs was developed. The optimization model helped to increase the throughput of the shuttle vehicle by reducing travel distance and total cycle time. The output of the joint optimization model is the storage location assignment and the optimized sequence of a block of several storage and retrieval requests, with the objective of the shortest travel or cycle time.

In the first step, the computing time was analyzed for different block sizes, filling degrees, and percentage of small ULs. In summary, it can be stated that the larger the block size (more command cycles are considered), the lower the filling degree, and the higher the percentage of small ULs, the longer the computing time. If the solving process of the optimization model is stopped, even if the optimum has not been approved, the achieved results lead to an improvement of the total cycle time. The achieved results do not deviate very much from the approved optimum. Therefore, it is recommended to determine a stop criterion for each configuration in order to decrease the computing time, but to still get a result close to the approved optimum solution.

In the second step, different control strategies were compared and evaluated. Both, the storage and retrieval requests of one block, were processed either FCFS or in an optimized sequence. It was shown that processing the requests in an optimized sequence leads to a shorter total cycle time of the whole block. As a general rule it can be derived, that the larger the block size, the higher the filling degree, and the higher the percentage of small ULs, the higher the achieved improvement (lower cycle time) when using an optimized sequence instead of FCFS. The highest improvement is achieved, if both, storage and retrieval requests, are processed in an optimized sequence. It is shown, that optimized sequencing leads to an average improvement of the total cycle time over “first come first served” of 4.72 % (EBM - 95 % confidence interval 0.49 %). However, performing the storage requests after FCFS and the retrieval requests in an optimized sequence achieves only slightly worse results (improvement 4.12 % (EBM - 95 % confidence interval 0.43 %)). It is therefore recommended, to process the storage requests after FCFS, and not to presort them. The retrieval requests should be processed in an optimized sequence. Since the retrieval requests are only “stored” in a virtual queue, the processing sequence can be changed very easily.

All in all, it was shown that by solving the block sequencing problem optimally, the travel time and cycle time can be reduced and the throughput increased, compared to processing the requests after FCFS.

In the fourth part, a discrete event agent-based simulation model was constructed to evaluate the impact of different parameters and control strategies on system performance. Through simulation it was possible to model the interaction effects between the individual system components, lift, shuttle vehicle, and buffer and the impact of waiting times. A performance evaluation was carried out for various system configurations and control strategies, and recommendations for action were derived. It was shown that the throughput of a shuttle system depends on the number of buffer locations. The throughput of a SBS/RS can be increased with an increasing number of buffer locations, since waiting times of the lift and shuttle vehicles in front of the inbound and outbound buffer is decreased. The results indicated that a minimum of two buffer locations should be chosen for the specified configurations, and at least four buffer locations for Strategy 1. Additional buffer locations only result in a marginal performance improvement. Furthermore, it could be demonstrated that the analytical models, developed in chapter 3, provide highly accurate results for these specific use cases with two and four buffer locations, with a maximum deviation ranging from -1.35 % to 1.11 %, when compared to the simulation results. This demonstrates that analytical models can be applied (for these SBS/RS configurations) to calculate performance, even if they do not consider waiting times and the interactions between the lift and shuttle vehicles.

6.2 Outlook

This work has filled several research gaps. However, there are still open research questions that have not been answered in this work. Therefore, this chapter lists possible research topics which can be a future field of research:

- The generally valid description method can be extended to other storage systems. The aim should be to use this work as a basis to create a standard or guideline, and fasten the usage of uniform mapping and description procedure in future. Thus, all relevant data and each storage system is specified very clearly.
- The generally applicable model can be used to describe the layout, material flow, and control strategies of a SBS/RS. Future research areas can be:
  - How can a simulation model be automatically generated from this?
  - How can travel time, cycle time, and throughput be automatically determined from this?

SBS/RSs are very complex systems and the more complex the SBS/RSs become, the more important it is to develop control strategies specifically adapted to SBS/RSs. Maximum throughput can be achieved only by implementing the optimal control strategy. Future research areas can be:

- Once a combination of different control strategies has been found to be optimal, it does not necessarily have to remain optimal when article and order structure change. Therefore, a dynamic control system is necessary that permanently adapts its strategies to the changing article and order structure to continue to achieve optimal performance. Such a control approach for SBS/RSs does not yet exist.
- Development of an artificial intelligence based warehouse control system. Due to the large number of parallel but interdependent processes in a SBS/RS, it can be assumed that a higher throughput can

be achieved via a smart control system. Artificial intelligence can assist in capturing and analyzing the vast number of processes and data volumes, thus optimizing the control of the SBS/RS.

- In complex SBS/RSs with several lifts and shuttle vehicles, it must be determined which lift and which shuttle vehicle processes which request and in which sequence. If the individual shuttle vehicles can also change tiers (tier-to-tier) and aisles (aisle-to-aisle), additional deadlock avoidance strategies must be taken into account. In the future intelligent control strategies, deadlock avoidance strategies or optimization models can be developed that take these problems into account.
- The literature often only considers the load handling of one UL after the other, with more complex cases being mostly overlooked. Multi-load handling devices and simultaneous pick up and drop off of several ULs are mostly neglected. However, developing methods and control strategies for more complex use cases represents a future field of research.

In this work, many different analytical models for the calculation of travel time, cycle time, and throughput were presented. There are still further extension possibilities:

- How can the analytical models be extended to multi-deep storage systems and several different sizes of ULs?
- For simplification, it was assumed that the shuttle vehicle can only (un)load one UL after each other. In the future, the models can be extended to consider the simultaneous (un)loading process of several ULs.
- In this work, it was assumed that in each command cycle first all ULs are stored and then all ULs are retrieved (S1-S2-R1-R2). In the future, analytical models can be created for further processing sequences, such as (S1-R1-S2-R2).
- Extension of the analytical models for more complex SBS/RS configurations with several lifts and shuttle vehicles.

- Applying queuing models to consider waiting times and different inter-arrival times of storage and retrieval requests.
- Apply the developed models to the classical AS/RS.

The application of the optimization model leads to an optimized sequence of the storage and retrieval requests with the objective of the shortest cycle time. Future research can extend the developed optimization model as follows:

- Improvement of the computing time to solve large sized problems in an acceptable time, e.g., apply different approaches like heuristics or genetic algorithm which would reduce computing time.
- Extension of the developed optimization model to consider the interaction of all shuttle vehicles with the inbound lift and outbound lift. The model can be extended to consider joint optimization of the storage location assignment, the scheduling of the storage and retrieval requests, and the interaction of the shuttle vehicles and lifts.
- Extension of the model for other storage systems, e.g., AS/RSs with stacker crane. The objective function has to be extended only by the diagonal drive of the stacker crane.

With the increasing popularity of SBS/RSs, there is a compelling need to optimize these systems. Ultimately, this can result in improved throughput performance, cost reduction, and energy efficiency. The research questions provided here are merely a limited selection from a range of potential research topics that can be further explored in the future.

A Material Flow Description

In this chapter, further “activity” diagrams are depicted. For further details see chapter 2.2.5.

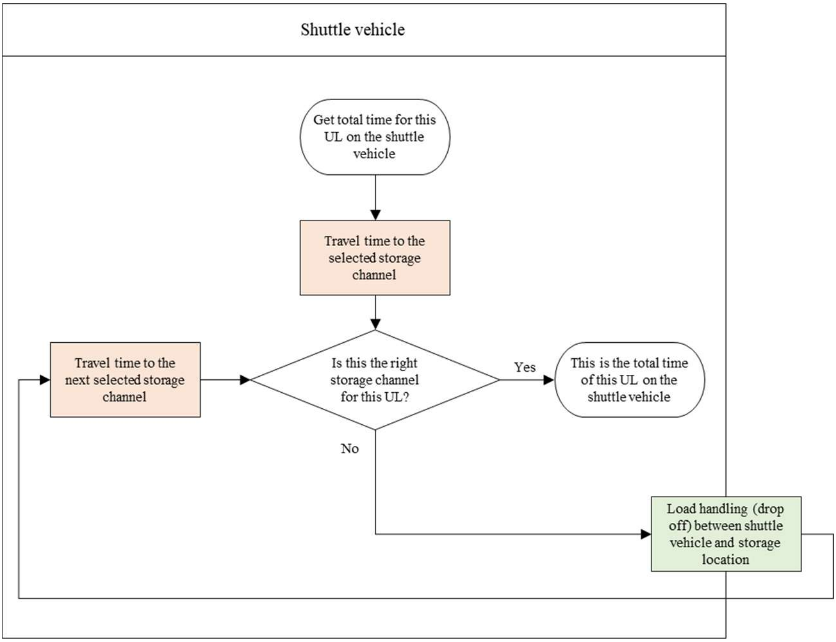


Figure A.1: Detailed process description for the travel time of one UL on the shuttle vehicle (storage process)

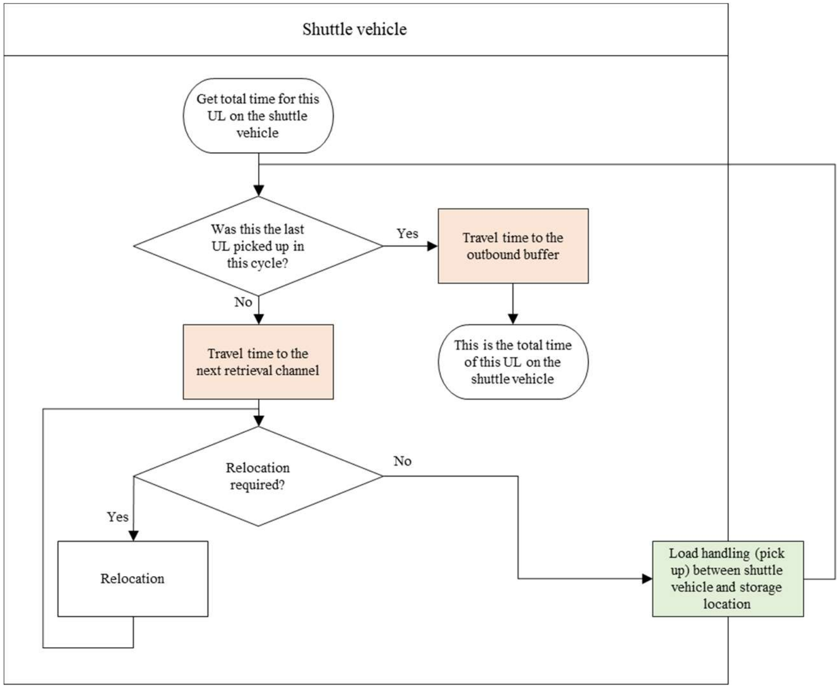


Figure A.2: Detailed process description for the travel time of one UL on the shuttle vehicle (retrieval process)

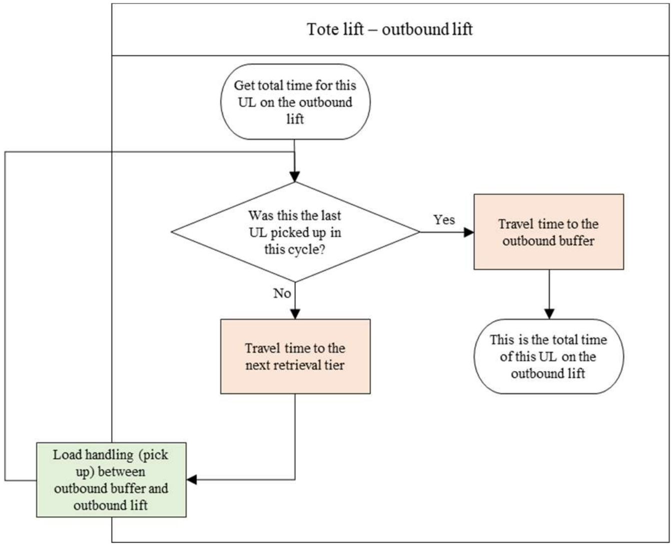


Figure A.3: Detailed process description for the travel time of one UL on the outbound lift (retrieval process)

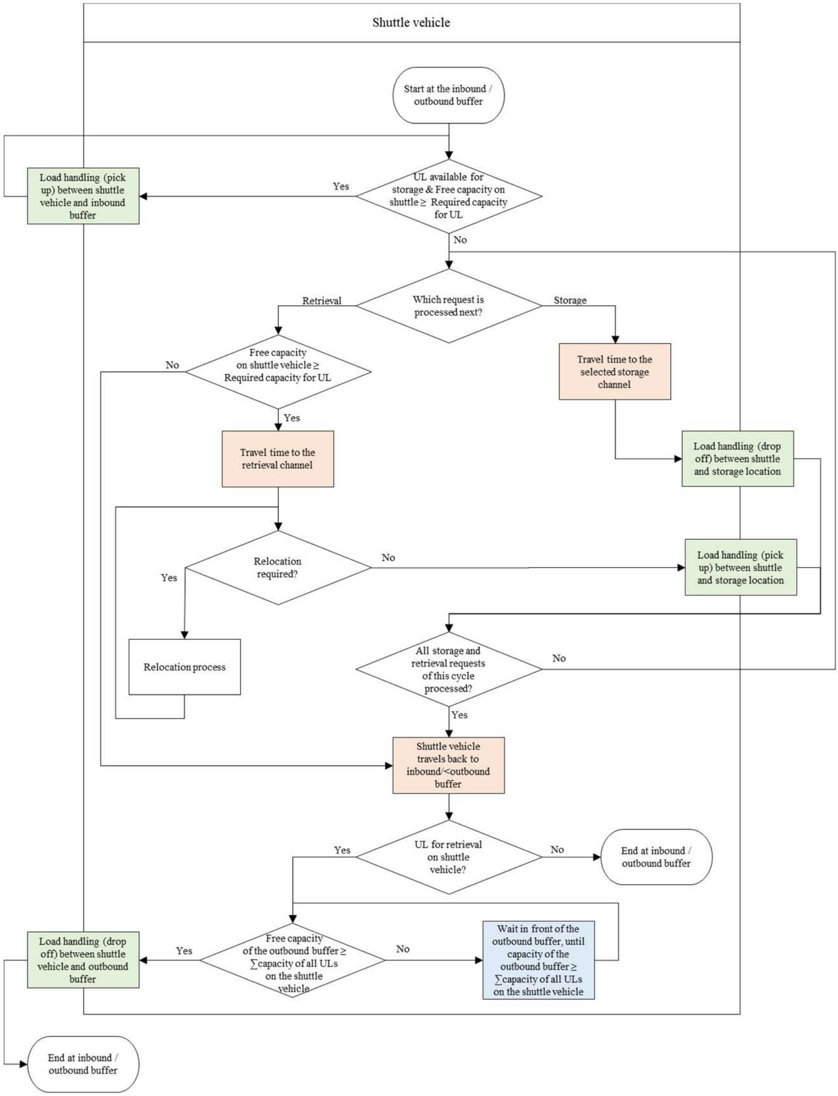


Figure A.4: Control logic of the shuttle vehicle

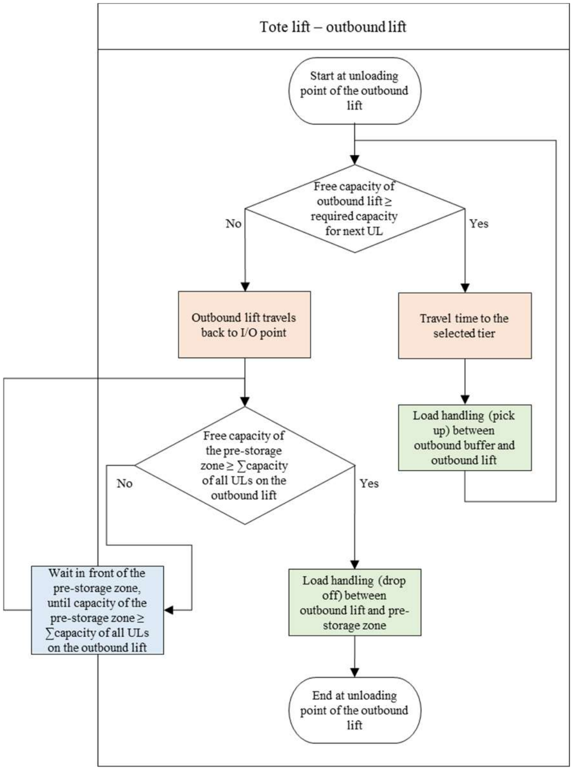


Figure A.5: Control logic of the outbound lift

B Velocity-Time Dependency

The calculation of the throughput is dependent on the travel time of the material handling equipment (lift, shuttle vehicle, or satellite) and the load handling time (pick up and drop off). Whereas the load handling time is dependent on the use case but constant over time, the travel time depends on the distance traveled and the time required. Figure B.1 depicts the velocity-time dependency for lift, shuttle vehicle and satellite. The dotted line shows the real behavior in which acceleration and deceleration are not linear. Also additional time for positioning at the end of the movement is considered. For simplification, acceleration and deceleration are linearized and the time for positing is ignored (solid line).

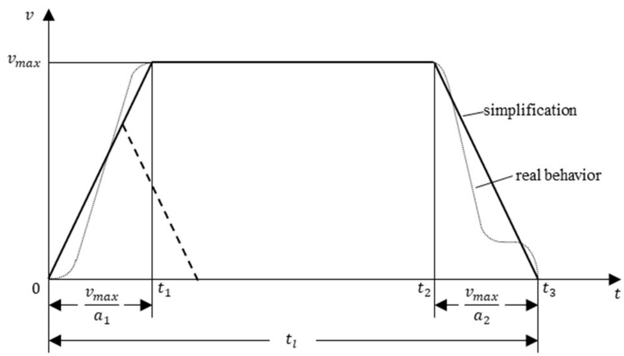


Figure B.1: Velocity-time dependency (Arnold and Furmans 2019, p. 207)

Two types of profiles can be distinguished depending on whether the maximum velocity v_{max} is reached or not. The two different types are:

i) Triangular profile (dashed line): For traveling short distances, the maximum possible velocity v_{max} can not be reached. Immediately after the acceleration phase the deceleration phase starts.

ii) Trapezoid profile (solid line): For traveling long distances, the maximum possible velocity v_{max} can be reached. After acceleration, the material handling equipment travels with constant velocity, before the deceleration phase starts.

The distance l, that the load handling equipment (e.g., lift or shuttle vehicle) travels during t_l can be calculated by integrating over time.

$$l(t_l) = \int_0^{t_l} v(t)dt = \begin{cases} \frac{1}{2}\frac{|a_1 a_2|}{a_1+|a_2|} t_l^2 & for\ t_l < v_{max}\frac{a_1+|a_2|}{|a_1 a_2|} \\ t_l v_{max} - \frac{v_{max}^2}{2}\left(\frac{1}{a_1}+\frac{1}{|a_2|}\right) & for\ t_l \geq v_{max}\frac{a_1+|a_2|}{|a_1 a_2|} \end{cases} \quad \text{(B.1)}$$

To get the total travel time t_l, equation (B.1) is transformed to:

$$t_l = \begin{cases} \sqrt{2l\left(\frac{1}{a_1}+\frac{1}{|a_2|}\right)} & for\ l < \frac{1}{2}v_{max}^2\left(\frac{1}{a_1}+\frac{1}{|a_2|}\right) \\ \frac{v_{max}}{2}\left(\frac{1}{a_1}+\frac{1}{|a_2|}\right)+\frac{l}{v_{max}} & for\ l \geq \frac{1}{2}v_{max}^2\left(\frac{1}{a_1}+\frac{1}{|a_2|}\right) \end{cases} \quad \text{(B.2)}$$

For further simplification, the assumption is made that the acceleration a_1 is equal the deceleration a_2. If the acceleration a_1 and deceleration a_2 are different, the harmonic mean can be calculated:

$$a_1 = a_2 = a = \frac{2|a_1 a_2|}{a_1+|a_2|} \quad \text{(B.3)}$$

For the travel time calculation of the material handling equipment usually all distances and the required travel times are calculated using the equations for $l \geq \frac{1}{2}v_{max}^2\left(\frac{1}{a_1}+\frac{1}{|a_2|}\right)$. However, using this equation, the travel times for short travel distances (triangular profile) are then to high. However, since short travel distances only make up a small proportion of the total number of travels, the overall error for calculating the mean travel time can be neglected (Arnold and Furmans (2019), p. 208).

Therefore, only the simplified equation (B.4) is used, where $\frac{v_{max}}{a}$ is the constant time for the acceleration and deceleration phase and $\frac{l}{v_{max}}$ is the variable time for the distance traveled.

$$t_l = \frac{v_{max}}{a} + \frac{l}{v_{max}} \tag{B.4}$$

The following figure shows the comparison of the exact calculation with the simplified equation. It can be seen that there are deviations in the calculated travel time for short travel distances. In most cases, the use of the simplified equation (see equation (B.4)) is sufficient for the calculation of the average travel time. However, if short distances are frequently traveled or the distance between the individual storage locations or tiers is very small, then a case differentiation should be made.

Table B.1: Input values

Parameter	Value
l	0.5 m - 10 m
v_{max}	4 m/s
a	3 m/s²

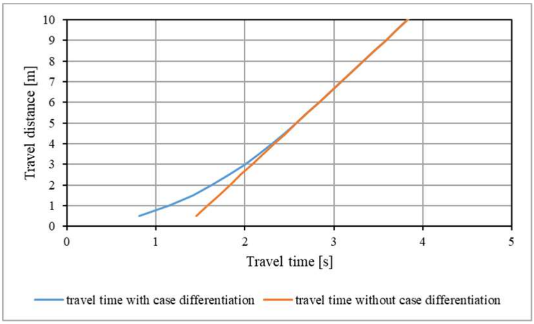


Figure B.2: Comparison of the travel time with case differentiation (equation (B.2)) with the travel time without case differentiation (equation (B.4))

C Summary Single-Command and Dual-Command Cycles for the Lift

This chapter summarizes the individual travel times and indicates the expected, minimum and maximum value depending on the number of tiers.

Case 1

Table C.1: Average travel time and the minimum and maximum travel time for each scenario

	Single-command cycle	**Dual-command cycle**
$\boldsymbol{n_t} = \mathbf{1}$	$E\left(t_{SC\_Lift}\right) = 0$ $t_{SC\_Lift\_Min} = 0$ $t_{SC\_Lift\_Max} = 0$	$E\left(t_{DC\_Lift}\right) = 0$ $t_{DC\_Lift\_Min} = 0$ $t_{DC\_Lift\_Max} = 0$
$\boldsymbol{n_t} > \mathbf{1}$	$E\left(t_{SC\_Lift}\right) = equation$ (3.6) $t_{SC\_Lift\_Min} = 0$ $t_{SC\_Lift\_Max} = 2\frac{(n_t-1)d_{sy}}{v_{lift}} + 2\frac{v_{lift}}{a_{lift}}$	$E\left(t_{DC\_Lift}\right) =$ equation (3.16) $t_{DC\_Lift\_Min} = 0$ $t_{DC\_Lift\_Max} = 2\frac{(n_t-1)d_{sy}}{v_{lift}} + 3\frac{v_{lift}}{a_{lift}}$

Case 2

Table C.2: Average travel time and the minimum and maximum travel time for each scenario

	Single-command cycle	Dual-command cycle
$\boldsymbol{n_t = 1}$	$E(t_{SC\_Lift}) = 2\frac{d_{I/O_y}}{v_{lift}} + 2\frac{v_{lift}}{a_{lift}}$ $t_{SC\_Lift\_Min} = 2\frac{d_{I/O_y}}{v_{lift}} + 2\frac{v_{lift}}{a_{lift}}$ $t_{SC\_Lift\_Max} = 2\frac{d_{I/O_y}}{v_{lift}} + 2\frac{v_{lift}}{a_{lift}}$	$E(t_{DC\_Lift}) = 2\frac{d_{I/O_y}}{v_{lift}} + 2\frac{v_{lift}}{a_{lift}}$ $t_{DC\_Lift\_Min} = 2\frac{d_{I/O_y}}{v_{lift}} + 2\frac{v_{lift}}{a_{lift}}$ $t_{DC\_Lift\_Max} = 2\frac{d_{I/O_y}}{v_{lift}} + 2\frac{v_{lift}}{a_{lift}}$
$\boldsymbol{n_t > 1}$	$E(t_{SC\_Lift}) = equation$ (3.9) $t_{SC\_Lift\_Min} = 2\frac{d_{I/O_y}}{v_{lift}} + 2\frac{v_{lift}}{a_{lift}}$ $t_{SC\_Lift\_Max} = 2\frac{(n_t-1)d_{sy}+d_{I/O_y}}{v_{lift}} + 2\frac{v_{lift}}{a_{lift}}$	$E(t_{DC\_Lift})$ = equation (3.17) $t_{DC\_Lift\_Min} = 2\frac{d_{I/O_y}}{v_{lift}} + 2\frac{v_{lift}}{a_{lift}}$ $t_{DC\_Lift\_Max} = 2\frac{(n_t-1)d_{sy}+d_{I/O_y}}{v_{lift}} + 3\frac{v_{lift}}{a_{lift}}$

Case 3

Table C.3: Average travel time and the minimum and maximum travel time for each scenario

	Single-command cycle	**Dual-command cycle**
$\boldsymbol{n_t = 1}$	$E(t_{SC\_Lift}) = 0$ $t_{SC\_Lift\_Min} = 0$ $t_{SC\_Lift\_Max} = 0$	$E(t_{DC\_Lift}) = 0$ $t_{DC\_Lift\_Min} = 0$ $t_{DC\_Lift\_Max} = 0$
$\boldsymbol{n_t > 1}$	$E(t_{SC\_Lift}) = equation$ (3.11) $t_{SC\_Lift\_Min} = 0$ $t_{SC\_Lift\_Max} = MAX\left[2\frac{(n_{I/O}-1)d_{sy}}{v_{lift}} ; \frac{((n_t-n_{I/O})d_{sy}}{v_{lift}}\right] + 2\frac{v_{lift}}{a_{lift}}$	$E(t_{DC\_Lift}) =$ equation(3.18) $t_{DC\_Lift\_Min} = 0$ $t_{DC\_Lift\_Max} = 2\frac{(n_t-1)d_{sy}}{v_{lift}} + 3\frac{v_{lift}}{a_{lift}}$

Case 4

Table C.4: Average travel time and the minimum and maximum travel time for each scenario

	Single-command cycle	Dual-command cycle
$\boldsymbol{n_t = 1}$	$E(t_{SC\_Lift}) = 0$ $t_{SC\_Lift\_Min} = 0$ $t_{SC\_Lift\_Max} = 0$	$E(t_{DC\_Lift}) = 0$ $t_{DC\_Lift\_Min} = 0$ $t_{DC\_Lift\_Max} = 0$
$\boldsymbol{n_t > 1}$	$E(t_{SC\_Lift}) = equation$ (3.13) $t_{SC\_Lift\_Min} = MIN\left[2\frac{d_{sy}-d_1}{v_{lift}}\,;2\frac{d_1}{v_{lift}}\right] + 2\frac{v_{lift}}{a_{lift}}$ $t_{SC\_Lift\_Max} = MAX\left[2\frac{(n_1-1)d_{sy}+d_1}{v_{lift}}\,;\,2\frac{(n_t-n_1-1)d_{sy}+d_{sy}-d_1}{v_{lift}}\right] + 2\frac{v_{lift}}{a_{lift}}$	$E(t_{DC\_Lift}) =$ equation (3.19) $t_{DC\_Lift\_Min} = MIN\left[2\frac{d_{sy}-d_1}{v_{lift}}\,;2\frac{d_1}{v_{lift}}\right] + 2\frac{v_{lift}}{a_{lift}}$ $t_{DC\_Lift\_Max} = 2\frac{(n_t-1)d_{sy}}{v_{lift}} + 3\frac{v_{lift}}{a_{lift}}$

D Results of Chapter 3

In this chapter, the results of the performed calculations and simulations are listed.

D.1 Values of Figure 3.2 and Figure 3.3

Table D.1: Values for Figure 3.2 and Figure 3.3

Number of tiers	Cycle time analytical exact [s]	Cycle time analytical approximation [s]	Cycle time simulation [s]	EBM - 95 % confidence interval simulation [s]	Relative error exact / simulation [%]	Relative error approximation / simulation [%]
1	8.00	8.00	8.00	0.0000	0.00	0.00
2	9.46	10.79	8.81	0.0075	-7.35	-22.49
3	10.03	10.92	9.32	0.0090	-7.60	-17.14
4	10.38	11.04	9.69	0.0099	-7.06	-13.94
5	10.63	11.17	10.01	0.0105	-6.19	-11.52
6	10.85	11.29	10.28	0.0112	-5.54	-9.87
7	11.04	11.42	10.54	0.0117	-4.72	-8.34
8	11.21	11.54	10.76	0.0123	-4.18	-7.28
9	11.37	11.67	10.96	0.0128	-3.75	-6.45
10	11.53	11.79	11.16	0.0132	-3.28	-5.67
11	11.67	11.92	11.34	0.0138	-2.97	-5.10
12	11.82	12.04	11.51	0.0142	-2.70	-4.63
13	11.96	12.17	11.68	0.0146	-2.41	-4.17
14	12.10	12.29	11.84	0.0151	-2.16	-3.77
15	12.24	12.42	11.99	0.0155	-2.04	-3.52
16	12.38	12.54	12.16	0.0159	-1.75	-3.12
17	12.51	12.67	12.30	0.0164	-1.68	-2.96
18	12.64	12.79	12.45	0.0169	-1.58	-2.77
19	12.78	12.92	12.59	0.0174	-1.48	-2.59

20	12.91	13.04	12.74	0.0179	-1.34	-2.39
21	13.04	13.17	12.87	0.0184	-1.34	-2.32
22	13.17	13.29	13.01	0.0189	-1.20	-2.13
23	13.30	13.42	13.15	0.0194	-1.15	-2.03
24	13.43	13.54	13.29	0.0199	-1.04	-1.87
25	13.56	13.67	13.43	0.0205	-1.00	-1.80
26	13.69	13.79	13.57	0.0210	-0.91	-1.66
27	13.82	13.92	13.69	0.0215	-0.93	-1.65
28	13.95	14.04	13.83	0.0221	-0.83	-1.52
29	14.07	14.17	13.96	0.0226	-0.79	-1.45
30	14.20	14.29	14.09	0.0232	-0.78	-1.41
31	14.33	14.42	14.23	0.0238	-0.68	-1.29
32	14.46	14.54	14.36	0.0244	-0.72	-1.30
33	14.59	14.67	14.49	0.0249	-0.66	-1.22
34	14.71	14.79	14.62	0.0255	-0.64	-1.17
35	14.84	14.92	14.75	0.0261	-0.59	-1.10
36	14.97	15.04	14.89	0.0267	-0.55	-1.05
37	15.09	15.17	15.02	0.0273	-0.53	-1.01
38	15.22	15.29	15.14	0.0280	-0.55	-1.02
39	15.35	15.42	15.27	0.0286	-0.50	-0.95
40	15.48	15.54	15.40	0.0292	-0.47	-0.91
41	15.60	15.67	15.53	0.0298	-0.47	-0.89
42	15.73	15.79	15.66	0.0304	-0.42	-0.83
43	15.85	15.92	15.79	0.0310	-0.40	-0.79
44	15.98	16.04	15.92	0.0317	-0.41	-0.79
45	16.11	16.17	16.04	0.0323	-0.40	-0.77
46	16.23	16.29	16.17	0.0329	-0.40	-0.76
47	16.36	16.42	16.31	0.0335	-0.33	-0.67
48	16.49	16.54	16.43	0.0341	-0.32	-0.65
49	16.61	16.67	16.55	0.0347	-0.35	-0.68
50	16.74	16.79	16.68	0.0354	-0.34	-0.66

D.2 Values of Figure 3.5 and Figure 3.6

Table D.2: Values for Figure 3.5 and Figure 3.6

Number of tiers	**Cycle time analytical exact for $d_{I/Oy} = 1\,m$ [s]**	**Cycle time analytical exact for $d_{I/Oy} = 2\,m$ [s]**	**Cycle time analytical exact for $d_{I/Oy} = 6\,m$ [s]**	**Cycle time simulation for $d_{I/Oy} = 1\,m$ [s]**	**EBM - 95 % confidence interval simulation for $d_{I/Oy} = 1\,m$ [s]**	**Cycle time simulation for $d_{I/Oy} = 2\,m$ [s]**	**EBM - 95 % confidence interval simulation for $d_{I/Oy} = 2\,m$ [s]**	**Cycle time simulation for $d_{I/Oy} = 6\,m$ [s]**	**EBM - 95 % confidence interval simulation for $d_{I/Oy} = 6\,m$ [s]**	**Relative error exact / simulation for $d_{I/Oy} = 1\,m$ [%]**	**Relative error exact / simulation for $d_{I/Oy} = 2\,m$ [%]**	**Relative error exact / simulation for $d_{I/Oy} = 6\,m$ [%]**
1	11.17	11.67	13.67	10.31	0.0000	11.27	0.0000	13.67	0.0000	-8.32	-3.56	0.00
2	11.29	11.79	13.79	10.57	0.0023	11.46	0.0017	13.79	0.0011	-6.85	-2.91	-0.01
3	11.42	11.92	13.92	10.80	0.0034	11.64	0.0026	13.92	0.0018	-5.73	-2.42	0.00
4	11.54	12.04	14.04	11.01	0.0044	11.81	0.0034	14.04	0.0024	-4.82	-1.99	-0.01
5	11.67	12.17	14.17	11.21	0.0052	11.97	0.0042	14.16	0.0031	-4.11	-1.67	-0.01
6	11.79	12.29	14.29	11.39	0.0060	12.12	0.0049	14.29	0.0037	-3.50	-1.39	-0.01
7	11.92	12.42	14.42	11.57	0.0067	12.27	0.0055	14.41	0.0044	-3.01	-1.17	-0.02
8	12.04	12.54	14.54	11.73	0.0073	12.41	0.0061	14.54	0.0050	-2.62	-1.02	-0.02
9	12.17	12.67	14.67	11.89	0.0080	12.55	0.0067	14.66	0.0056	-2.33	-0.91	-0.02
10	12.29	12.79	14.79	12.04	0.0085	12.69	0.0073	14.79	0.0063	-2.07	-0.81	-0.02
11	12.42	12.92	14.92	12.19	0.0091	12.82	0.0079	14.91	0.0069	-1.84	-0.74	-0.02
12	12.54	13.04	15.04	12.33	0.0096	12.95	0.0085	15.04	0.0075	-1.70	-0.68	-0.03
13	12.67	13.17	15.17	12.47	0.0102	13.08	0.0090	15.16	0.0082	-1.56	-0.63	-0.03
14	12.79	13.29	15.29	12.61	0.0108	13.22	0.0096	15.29	0.0088	-1.43	-0.58	-0.02
15	12.92	13.42	15.42	12.75	0.0113	13.35	0.0102	15.41	0.0094	-1.33	-0.54	-0.03
16	13.04	13.54	15.54	12.88	0.0119	13.47	0.0108	15.54	0.0101	-1.24	-0.50	-0.02
17	13.17	13.67	15.67	13.01	0.0124	13.60	0.0114	15.66	0.0107	-1.17	-0.48	-0.03
18	13.29	13.79	15.79	13.15	0.0130	13.73	0.0120	15.79	0.0113	-1.07	-0.44	-0.02
19	13.42	13.92	15.92	13.28	0.0136	13.86	0.0126	15.91	0.0120	-1.02	-0.42	-0.03
20	13.54	14.04	16.04	13.41	0.0141	13.99	0.0132	16.04	0.0126	-0.96	-0.40	-0.03
21	13.67	14.17	16.17	13.54	0.0147	14.11	0.0138	16.16	0.0132	-0.91	-0.39	-0.04
22	13.79	14.29	16.29	13.67	0.0153	14.24	0.0144	16.29	0.0138	-0.86	-0.37	-0.03
23	13.92	14.42	16.42	13.80	0.0159	14.37	0.0150	16.41	0.0145	-0.82	-0.35	-0.04
24	14.04	14.54	16.54	13.93	0.0164	14.49	0.0157	16.54	0.0151	-0.79	-0.34	-0.04
25	14.17	14.67	16.67	14.06	0.0170	14.62	0.0163	16.66	0.0157	-0.75	-0.32	-0.03
26	14.29	14.79	16.79	14.19	0.0176	14.75	0.0169	16.78	0.0164	-0.72	-0.32	-0.04
27	14.42	14.92	16.92	14.32	0.0182	14.87	0.0175	16.91	0.0170	-0.67	-0.29	-0.03
28	14.54	15.04	17.04	14.45	0.0188	15.00	0.0181	17.03	0.0176	-0.66	-0.30	-0.06

29	14.67	15.17	17.17	14.57	0.0194	15.12	0.0187	17.16	0.0183	-0.64	-0.29	-0.01
30	14.79	15.29	17.29	14.70	0.0200	15.25	0.0194	17.28	0.0189	-0.61	-0.28	-0.04
31	14.92	15.42	17.42	14.83	0.0206	15.38	0.0200	17.41	0.0195	-0.59	-0.26	-0.03
32	15.04	15.54	17.54	14.96	0.0212	15.50	0.0206	17.53	0.0202	-0.57	-0.25	-0.06
33	15.17	15.67	17.67	15.08	0.0218	15.63	0.0212	17.66	0.0208	-0.54	-0.26	-0.06
34	15.29	15.79	17.79	15.21	0.0224	15.75	0.0218	17.77	0.0214	-0.54	-0.25	-0.11
35	15.42	15.92	17.92	15.34	0.0230	15.88	0.0224	17.92	0.0221	-0.50	-0.26	0.02
36	15.54	16.04	18.04	15.46	0.0236	16.01	0.0231	18.03	0.0227	-0.50	-0.21	-0.07
37	15.67	16.17	18.17	15.58	0.0242	16.13	0.0237	18.16	0.0233	-0.54	-0.24	-0.06
38	15.79	16.29	18.29	15.72	0.0248	16.25	0.0243	18.28	0.0239	-0.46	-0.26	-0.05
39	15.92	16.42	18.42	15.85	0.0255	16.37	0.0249	18.40	0.0246	-0.44	-0.27	-0.10
40	16.04	16.54	18.54	15.97	0.0260	16.51	0.0256	18.52	0.0252	-0.44	-0.20	-0.10
41	16.17	16.67	18.67	16.09	0.0267	16.63	0.0262	18.65	0.0258	-0.47	-0.21	-0.09
42	16.29	16.79	18.79	16.22	0.0273	16.76	0.0268	18.77	0.0265	-0.46	-0.18	-0.10
43	16.42	16.92	18.92	16.35	0.0279	16.88	0.0274	18.90	0.0271	-0.40	-0.24	-0.10
44	16.54	17.04	19.04	16.47	0.0285	17.01	0.0280	19.01	0.0278	-0.43	-0.18	-0.16
45	16.67	17.17	19.17	16.60	0.0291	17.13	0.0287	19.14	0.0284	-0.39	-0.19	-0.15
46	16.79	17.29	19.29	16.74	0.0297	17.26	0.0293	19.26	0.0290	-0.34	-0.20	-0.19
47	16.92	17.42	19.42	16.85	0.0304	17.39	0.0300	19.39	0.0296	-0.41	-0.16	-0.14
48	17.04	17.54	19.54	16.98	0.0310	17.50	0.0306	19.53	0.0302	-0.39	-0.21	-0.08
49	17.17	17.67	19.67	17.11	0.0316	17.63	0.0312	19.64	0.0309	-0.35	-0.24	-0.12
50	17.29	17.79	19.79	17.23	0.0322	17.76	0.0318	19.76	0.0315	-0.35	-0.20	-0.18

D.3 Values of Figure 3.8 and Figure 3.9

Table D.3: Values for Figure 3.8 and Figure 3.9

Number of tiers	Cycle time analytical exact [s]	Throughput [ULs/hour]	Cycle time simulation [s]	EBM - 95 % confidence interval simulation [s]	Relative error exact / simulation [%]
1	16.74	215.08	16.66	0.0335	-0.47
2	16.50	218.20	16.38	0.0337	-0.72
3	16.27	221.29	16.13	0.0336	-0.85
4	16.05	224.32	15.90	0.0334	-0.92
5	15.84	227.30	15.69	0.0327	-0.96
6	15.64	230.20	15.46	0.0322	-1.18
7	15.45	233.03	15.28	0.0316	-1.10
8	15.27	235.78	15.09	0.0307	-1.17
9	15.10	238.44	14.92	0.0300	-1.19
10	14.94	240.99	14.79	0.0291	-0.99
11	14.79	243.44	14.60	0.0282	-1.31
12	14.65	245.76	14.48	0.0273	-1.19
13	14.52	247.96	14.36	0.0264	-1.09
14	14.40	250.03	14.24	0.0254	-1.13
15	14.29	251.95	14.11	0.0245	-1.29
16	14.19	253.73	14.04	0.0237	-1.09
17	14.10	255.35	13.94	0.0228	-1.17
18	14.02	256.81	13.85	0.0220	-1.20
19	13.95	258.10	13.79	0.0213	-1.15
20	13.89	259.21	13.72	0.0206	-1.21
21	13.84	260.15	13.67	0.0200	-1.21
22	13.80	260.90	13.64	0.0195	-1.17
23	13.77	261.47	13.61	0.0192	-1.15
24	13.75	261.85	13.60	0.0188	-1.08
25	13.74	262.04	13.58	0.0187	-1.13
26	13.74	262.04	13.58	0.0187	-1.16
27	13.75	261.85	13.60	0.0189	-1.10
28	13.77	261.47	13.61	0.0192	-1.15

29	13.80	260.90	13.64	0.0196	-1.15
30	13.84	260.15	13.68	0.0202	-1.19
31	13.89	259.21	13.73	0.0207	-1.18
32	13.95	258.10	13.80	0.0214	-1.06
33	14.02	256.81	13.87	0.0222	-1.08
34	14.10	255.35	13.94	0.0229	-1.12
35	14.19	253.73	14.04	0.0238	-1.05
36	14.29	251.95	14.15	0.0246	-1.01
37	14.40	250.03	14.25	0.0255	-1.01
38	14.52	247.96	14.39	0.0265	-0.90
39	14.65	245.76	14.53	0.0274	-0.84
40	14.79	243.44	14.65	0.0282	-0.94
41	14.94	240.99	14.78	0.0292	-1.05
42	15.10	238.44	14.97	0.0298	-0.89
43	15.27	235.78	15.14	0.0307	-0.84
44	15.45	233.03	15.32	0.0315	-0.81
45	15.64	230.20	15.53	0.0322	-0.72
46	15.84	227.30	15.71	0.0328	-0.85
47	16.05	224.32	15.94	0.0332	-0.70
48	16.27	221.29	16.16	0.0336	-0.67
49	16.50	218.20	16.43	0.0336	-0.44
50	16.74	215.08	16.67	0.0335	-0.42

D.4 Values of Figure 3.11 and Figure 3.12

Table D.4: Values for Figure 3.11 and Figure 3.12

Number of tiers	Cycle time analytical exact for $d_{sy} = 0.5\,m$ [s]	Cycle time analytical exact for $d_{sy} = 1.0\,m$ [s]	Cycle time simulation for $d_{sy} = 0.5\,m$ [s]	EBM - 95 % confidence interval simulation for $d_{sy} = 0.5\,m$ [s]	Cycle time simulation for $d_{sy} = 1.0\,m$ [s]	EBM - 95 % confidence interval simulation for $d_{sy} = 1.0\,m$ [s]	Relative error exact / simulation for $d_{sy} = 0.5\,m$ [%]	Relative error approximation / simulation for $d_{sy} = 1.0\,m$ [%]	Relative error exact / simulation for $d_{sy} = 0.5\,m$ [%]	Relative error approximation / simulation for $d_{sy} = 1.0\,m$ [%]
1	8.00	8.00	8.00	0.0000	8.00	0.0000	0.00	0.00	0.00	0.00
2	12.19	12.38	11.20	0.0306	11.74	0.0335	-8.83	-26.51	-5.40	-22.08
3	13.69	14.06	12.59	0.0301	13.40	0.0332	-8.74	-19.10	-4.88	-14.42
4	14.53	15.06	13.47	0.0286	14.47	0.0323	-7.84	-15.03	-4.09	-10.56
5	15.10	15.80	14.11	0.0273	15.30	0.0313	-7.02	-12.45	-3.30	-8.10
6	15.53	16.40	14.60	0.0265	15.96	0.0305	-6.38	-10.70	-2.78	-6.52
7	15.89	16.93	15.05	0.0256	16.55	0.0302	-5.58	-9.14	-2.30	-5.32
8	16.20	17.41	15.46	0.0248	17.05	0.0300	-4.81	-7.81	-2.09	-4.60
9	16.48	17.85	15.80	0.0245	17.56	0.0298	-4.31	-6.89	-1.64	-3.75
10	16.74	18.28	16.08	0.0245	18.00	0.0300	-4.06	-6.31	-1.51	-3.32
11	16.98	18.68	16.38	0.0239	18.42	0.0301	-3.65	-5.64	-1.42	-2.98
12	17.20	19.08	16.65	0.0239	18.83	0.0304	-3.31	-5.08	-1.28	-2.65
13	17.42	19.46	16.91	0.0239	19.27	0.0310	-3.01	-4.60	-1.01	-2.21
14	17.63	19.84	17.14	0.0240	19.65	0.0316	-2.86	-4.30	-0.98	-2.04
15	17.84	20.21	17.37	0.0241	20.03	0.0320	-2.70	-4.02	-0.90	-1.84
16	18.04	20.58	17.59	0.0244	20.37	0.0334	-2.56	-3.76	-1.04	-1.89
17	18.24	20.94	17.83	0.0239	20.75	0.0340	-2.27	-3.37	-0.90	-1.66
18	18.43	21.30	18.05	0.0245	21.15	0.0346	-2.11	-3.12	-0.70	-1.38
19	18.62	21.66	18.27	0.0242	21.50	0.0359	-1.89	-2.83	-0.72	-1.33
20	18.81	22.01	18.45	0.0246	21.88	0.0366	-1.94	-2.81	-0.62	-1.17
21	18.99	22.37	18.65	0.0249	22.21	0.0380	-1.84	-2.65	-0.70	-1.20
22	19.18	22.72	18.85	0.0255	22.60	0.0387	-1.72	-2.47	-0.50	-0.95
23	19.36	23.07	19.05	0.0260	22.93	0.0403	-1.65	-2.35	-0.58	-0.99
24	19.54	23.41	19.25	0.0259	23.28	0.0415	-1.49	-2.14	-0.59	-0.97
25	19.72	23.76	19.42	0.0267	23.66	0.0423	-1.56	-2.18	-0.44	-0.78
26	19.90	24.11	19.60	0.0270	23.97	0.0438	-1.50	-2.08	-0.55	-0.85
27	20.08	24.45	19.82	0.0272	24.35	0.0447	-1.32	-1.86	-0.42	-0.70
28	20.25	24.79	20.00	0.0276	24.70	0.0459	-1.25	-1.77	-0.37	-0.62

29	20.43	25.14	20.20	0.0283	25.06	0.0472	-1.16	-1.65	-0.32	-0.55
30	20.61	25.48	20.37	0.0287	25.36	0.0486	-1.16	-1.61	-0.47	-0.68
31	20.78	25.82	20.56	0.0290	25.72	0.0499	-1.08	-1.52	-0.41	-0.60
32	20.96	26.16	20.77	0.0294	26.06	0.0511	-0.92	-1.32	-0.40	-0.57
33	21.13	26.51	20.91	0.0304	26.42	0.0525	-1.07	-1.46	-0.31	-0.47
34	21.31	26.85	21.12	0.0305	26.76	0.0536	-0.85	-1.22	-0.32	-0.46
35	21.48	27.19	21.29	0.0313	27.12	0.0551	-0.90	-1.25	-0.26	-0.38
36	21.65	27.53	21.46	0.0317	27.41	0.0564	-0.91	-1.24	-0.43	-0.54
37	21.82	27.86	21.64	0.0324	27.78	0.0574	-0.85	-1.16	-0.31	-0.41
38	22.00	28.20	21.81	0.0329	28.13	0.0592	-0.84	-1.13	-0.27	-0.36
39	22.17	28.54	21.98	0.0338	28.39	0.0606	-0.87	-1.15	-0.54	-0.62
40	22.34	28.88	22.14	0.0344	28.81	0.0619	-0.92	-1.19	-0.26	-0.33
41	22.51	29.22	22.32	0.0345	29.11	0.0633	-0.85	-1.10	-0.36	-0.42
42	22.68	29.56	22.52	0.0349	29.52	0.0643	-0.72	-0.96	-0.13	-0.18
43	22.85	29.90	22.70	0.0356	29.79	0.0662	-0.69	-0.92	-0.35	-0.39
44	23.03	30.23	22.85	0.0367	30.17	0.0669	-0.78	-1.00	-0.20	-0.23
45	23.20	30.57	23.03	0.0372	30.41	0.0691	-0.73	-0.94	-0.54	-0.56
46	23.37	30.91	23.20	0.0378	30.78	0.0701	-0.73	-0.93	-0.42	-0.43
47	23.54	31.24	23.38	0.0383	31.22	0.0716	-0.66	-0.85	-0.09	-0.10
48	23.71	31.58	23.56	0.0388	31.49	0.0731	-0.62	-0.80	-0.29	-0.29
49	23.88	31.92	23.71	0.0396	31.84	0.0744	-0.69	-0.86	-0.23	-0.23
50	24.05	32.26	23.90	0.0405	32.13	0.0758	-0.63	-0.80	-0.38	-0.37

D.5 Values of Figure 3.14 and Figure 3.15

Table D.5: Values for Figure 3.14 and Figure 3.15

Capacity of lift [capacity/lift]	Cycle time analytical exact optimized sequencing [s]	Cycle time analytical approximation optimized sequencing [s]	Cycle time analytical approximation FCFS sequencing [s]	Cycle time analytical Strategy 1 [s]	Cycle time simulation optimized sequencing [s]	EBM - 95 % confidence interval simulation optimized sequencing [s]	Cycle time simulation FCFS sequencing [s]	EBM - 95 % confidence interval simulation FCFS sequencing [s]	Cycle time simulation Strategy 1 [s]	EBM - 95 % confidence interval simulation Strategy 1 [s]	Relative error exact optimized sequencing / simulation [%]	Relative error approximation optimized sequencing / simulation [%]	Relative error approximation FCFS sequencing / simulation [%]	Relative error Strategy 1 / simulation [%]
1	16.74	16.79	16.92	16.79	16.63	0.0336	16.63	0.0337	16.65	0.0336	-0.67	-0.99	-1.72	-0.83
2	24.05	24.17	24.33	16.79	23.93	0.0400	23.90	0.0400	16.65	0.0475	-0.49	-0.99	-1.82	-0.84
3	34.19	34.52	35.75	28.17	34.02	0.0427	35.19	0.0558	27.89	0.0492	-0.49	-1.47	-1.58	-0.98
4	39.92	40.47	43.17	28.17	39.45	0.0487	42.47	0.0733	27.96	0.0565	-1.19	-2.57	-1.65	-0.75
5	49.13	50.21	54.58	38.52	48.71	0.0526	53.80	0.0889	37.88	0.0586	-0.87	-3.08	-1.46	-1.69
6	54.24	55.83	62.00	38.52	53.62	0.0611	61.05	0.1072	37.91	0.0641	-1.16	-4.13	-1.56	-1.62
7	63.19	65.39	73.42	48.47	62.58	0.0652	72.39	0.1228	47.34	0.0688	-0.97	-4.48	-1.42	-2.37

D.6 Values of Figure 3.16

Table D.6: Values for Figure 3.16

Capacity of lift [capacity/lift]	Cycle time analytical exact optimized sequencing [s]	Cycle time analytical approximation optimized sequencing [s]	Cycle time analytical approximation FCFS sequencing [s]	Cycle time analytical Strategy 1 [s]	Throughput exact optimized sequencing [ULs/hour]	Throughput approximation optimized sequencing [ULs/hour]	Throughput FCFS sequencing [ULs/hour]	Throughput Strategy 1 [ULs/hour]
1	16.74	16.79	16.92	16.79	215	214	212	214
2	24.05	24.17	24.33	16.79	299	297	295	428
3	34.19	34.52	35.75	28.17	316	312	302	383
4	39.92	40.47	43.17	28.17	362	355	333	511
5	49.13	50.21	54.58	38.52	366	358	329	467
6	54.24	55.83	62.00	38.52	398	386	348	560
7	63.19	65.39	73.42	48.47	399	385	343	519

D.7 Values of Figure 3.17, Figure 3.18, Figure 3.19 and Figure 3.20

Table D.7: Values for Figure 3.17, Figure 3.18, Figure 3.19 and Figure 3.20

Percentage of small ULs [%]	Cycle time analytical approximation random [s]	Throughput random small ULs [ULs/hour]	Throughput random large ULs [ULs/hour]	Throughput random all ULs [ULs/hour]	Cycle time analytical Strategy 1 [s]	Throughput Strategy 1 small ULs [ULs/hour]	Throughput Strategy 1 large ULs [ULs/hour]	Throughput Strategy 1 all ULs [ULs/hour]	Cycle time simulation random [s]	EBM - 95 % confidence interval simulation random [s]	Relative error approximation / simulation [%]
0	16.74	0.00	215.08	215.08	16.74	0.00	215.08	215.08	16.64	0.0318	-0.57
5	16.93	10.91	207.24	218.15	16.74	11.03	209.56	220.59	16.79	0.0339	-0.83
10	17.12	22.13	199.18	221.31	16.74	22.64	203.76	226.39	17.06	0.0354	-0.38
15	17.33	33.68	190.88	224.56	16.74	34.88	197.64	232.51	17.30	0.0371	-0.20
20	17.55	45.58	182.33	227.91	16.74	47.79	191.18	238.97	17.52	0.0388	-0.20
25	17.78	57.84	173.53	231.37	16.74	61.45	184.35	245.80	17.74	0.0406	-0.23
30	18.03	70.48	164.45	234.93	16.74	75.91	177.12	253.03	18.00	0.0423	-0.14
35	18.29	83.51	155.09	238.60	16.74	91.24	169.45	260.70	18.17	0.0442	-0.65
40	18.57	96.95	145.43	242.38	16.74	107.54	161.31	268.84	18.51	0.0454	-0.31
45	18.86	110.83	135.46	246.29	16.74	124.88	152.63	277.52	18.80	0.0477	-0.34
50	19.17	125.16	125.16	250.33	16.74	143.38	143.38	286.77	19.12	0.0489	-0.28
55	19.51	139.98	114.53	254.50	16.74	163.16	133.49	296.66	19.48	0.0502	-0.17
60	19.87	155.29	103.53	258.81	16.74	184.35	122.90	307.25	19.79	0.0514	-0.39
65	20.26	171.13	92.15	263.28	16.74	207.11	111.52	318.63	20.17	0.0525	-0.42
70	20.67	187.53	80.37	267.89	16.74	231.62	99.27	330.88	20.56	0.0534	-0.58
75	21.12	204.51	68.17	272.68	16.74	258.09	86.03	344.12	20.98	0.0538	-0.70
80	21.61	222.11	55.53	277.63	16.74	286.77	71.69	358.46	21.53	0.0533	-0.36
85	22.14	240.36	42.42	282.78	16.74	317.94	56.11	374.04	22.03	0.0522	-0.49
90	22.72	259.30	28.81	288.11	16.74	351.94	39.10	391.05	22.59	0.0502	-0.57
95	23.35	278.97	14.68	293.65	16.74	389.18	20.48	409.67	23.21	0.0463	-0.59
100	24.05	299.41	0.00	299.41	16.74	430.15	0.00	430.15	23.91	0.0385	-0.59

D.8 Values of Figure 3.22 and Figure 3.23

Table D.8: Values for Figure 3.22 and Figure 3.23

Percentage of small ULs [%]	Cycle time analytical approximation [s]	Cycle time simulation [s]	EBM - 95 % confidence interval simulation [s]	Relative error approximation / simulation [%]
0	28.17	28.01	0.0387	-0.57
5	28.49	28.25	0.0429	-0.87
10	28.83	28.63	0.0459	-0.73
15	29.19	28.96	0.0489	-0.82
20	29.57	29.31	0.0517	-0.92
25	29.97	29.69	0.0552	-0.95
30	30.40	30.10	0.0576	-0.97
35	30.84	30.52	0.0599	-1.06
40	31.32	31.02	0.0619	-0.97
45	31.82	31.47	0.0649	-1.12
50	32.36	31.97	0.0664	-1.20
55	32.93	32.49	0.0689	-1.33
60	33.54	33.06	0.0702	-1.45
65	34.19	33.63	0.0710	-1.66
70	34.89	34.27	0.0720	-1.81
75	35.64	34.96	0.0724	-1.95
80	36.46	35.70	0.0728	-2.12
85	37.34	36.52	0.0712	-2.25
90	38.29	37.32	0.0682	-2.60
95	39.33	38.30	0.0621	-2.70
100	40.47	39.28	0.0515	-3.01

D.9 Values of Figure 3.24 and Figure 3.25

Table D.9: Values for Figure 3.24 and Figure 3.25

Number of channels	**Cycle time analytical exact for $d_{bs} = 0.5\ m$ [s]**	**Cycle time analytical exact for $d_{bs} = 1.0\ m$ [s]**	**Cycle time analytical exact for $d_{bs} = 2.0\ m$ [s]**	**Cycle time simulation for $d_{by} = 0.5\ m$ [s]**	**EBM - 95 % confidence interval simulation for $d_{by} = 0.5\ m$ [s]**	**Cycle time simulation for $d_{by} = 1.0\ m$ [s]**	**EBM - 95 % confidence interval simulation for $d_{by} = 1.0\ m$ [s]**	**Cycle time simulation for $d_{by} = 2.0\ m$ [s]**	**EBM - 95 % confidence interval simulation for $d_{by} = 2.0\ m$ [s]**	**Relative error exact / simulation for $d_{bs} = 0.5m$ [%]**	**Relative error exact / simulation for $d_{bs} = 1.0\ m$ [%]**	**Relative error exact / simulation for $d_{bs} = 2.0\ m$ [%]**
1	19.73	20.13	20.93	18.31	0.0000	19.27	0.0000	20.62	0.0000	-7.77	-4.50	-1.52
5	22.08	22.48	23.28	21.27	0.0079	21.95	0.0073	23.02	0.0067	-3.78	-2.44	-1.11
10	23.57	23.97	24.77	23.18	0.0115	23.72	0.0109	24.66	0.0102	-1.70	-1.07	-0.46
15	24.96	25.36	26.16	24.74	0.0148	25.24	0.0144	26.12	0.0139	-0.89	-0.49	-0.15
20	26.32	26.72	27.52	26.19	0.0184	26.67	0.0181	27.52	0.0177	-0.49	-0.18	0.01
25	27.67	28.07	28.87	27.58	0.0222	28.05	0.0219	28.90	0.0216	-0.31	-0.06	0.09
30	29.01	29.41	30.21	28.99	0.0261	29.45	0.0258	30.26	0.0256	-0.09	0.12	0.15
35	30.35	30.75	31.55	30.36	0.0300	30.76	0.0300	31.61	0.0296	0.02	0.02	0.18
40	31.69	32.09	32.89	31.73	0.0340	32.16	0.0338	32.96	0.0337	0.12	0.20	0.19
45	33.03	33.43	34.23	33.08	0.0380	33.49	0.0379	34.30	0.0378	0.16	0.17	0.20
50	34.37	34.77	35.57	34.41	0.0421	34.82	0.0421	35.64	0.0418	0.11	0.15	0.21
55	35.70	36.10	36.90	35.74	0.0459	36.20	0.0459	36.98	0.0460	0.11	0.26	0.21
60	37.04	37.44	38.24	37.10	0.0501	37.56	0.0499	38.32	0.0500	0.16	0.33	0.21
65	38.38	38.78	39.58	38.39	0.0543	38.89	0.0540	39.66	0.0541	0.05	0.29	0.20
70	39.71	40.11	40.91	39.82	0.0582	40.28	0.0579	40.99	0.0582	0.28	0.43	0.20
75	41.05	41.45	42.25	41.10	0.0626	41.46	0.0623	42.33	0.0624	0.13	0.03	0.19
80	42.38	42.78	43.58	42.51	0.0664	42.88	0.0661	43.66	0.0665	0.30	0.24	0.19
85	43.71	44.11	44.91	43.82	0.0707	44.25	0.0707	44.99	0.0706	0.24	0.30	0.18
90	45.05	45.45	46.25	45.16	0.0747	45.56	0.0743	46.33	0.0748	0.24	0.25	0.17
95	46.38	46.78	47.58	46.44	0.0786	46.89	0.0789	47.66	0.0789	0.13	0.23	0.17
100	47.72	48.12	48.92	47.81	0.0829	48.26	0.0827	49.00	0.0830	0.19	0.30	0.16

Number of channels	**Cycle time analytical approximation for $d_{bs} = 0.5\,m$ [s]**	**Cycle time analytical approximation for $d_{bs} = 1.0\,m$ [s]**	**Cycle time analytical approximation for $d_{bs} = 2.0\,m$ [s]**	**Cycle time simulation for $d_{by} = 0.5\,m$ [s]**	**EBM - 95 % confidence interval simulation for $d_{by} = 0.5\,m$ [s]**	**Cycle time simulation for $d_{by} = 1.0\,m$ [s]**	**EBM - 95 % confidence interval simulation for $d_{by} = 1.0\,m$ [s]**	**Cycle time simulation for $d_{by} = 2.0\,m$ [s]**	**EBM - 95 % confidence interval simulation for $d_{by} = 2.0\,m$ [s]**	**Relative error approximation / simulation for $d_{bs} = 0.5m$ [%]**	**Relative error approximation / simulation for $d_{bs} = 1.0\,m$ [%]**	**Relative error approximation / simulation for $d_{bs} = 2.0\,m$ [%]**
1	21.53	21.80	22.33	18.31	0.0000	19.27	0.0000	20.62	0.0000	-17.60	-13.15	-8.31
5	22.60	22.87	23.40	21.27	0.0079	21.95	0.0073	23.02	0.0067	-6.23	-4.20	-1.63
10	23.93	24.20	24.73	23.18	0.0115	23.72	0.0109	24.66	0.0102	-3.25	-2.02	-0.30
15	25.27	25.53	26.07	24.74	0.0148	25.24	0.0144	26.12	0.0139	-2.13	-1.18	0.21
20	26.60	26.87	27.40	26.19	0.0184	26.67	0.0181	27.52	0.0177	-1.55	-0.73	0.44
25	27.93	28.20	28.73	27.58	0.0222	28.05	0.0219	28.90	0.0216	-1.27	-0.52	0.57
30	29.27	29.53	30.07	28.99	0.0261	29.45	0.0258	30.26	0.0256	-0.96	-0.28	0.64
35	30.60	30.87	31.40	30.36	0.0300	30.76	0.0300	31.61	0.0296	-0.79	-0.35	0.66
40	31.93	32.20	32.73	31.73	0.0340	32.16	0.0338	32.96	0.0337	-0.63	-0.13	0.68
45	33.27	33.53	34.07	33.08	0.0380	33.49	0.0379	34.30	0.0378	-0.55	-0.14	0.68
50	34.60	34.87	35.40	34.41	0.0421	34.82	0.0421	35.64	0.0418	-0.56	-0.13	0.68
55	35.93	36.20	36.73	35.74	0.0459	36.20	0.0459	36.98	0.0460	-0.54	0.00	0.67
60	37.27	37.53	38.07	37.10	0.0501	37.56	0.0499	38.32	0.0500	-0.45	0.08	0.66
65	38.60	38.87	39.40	38.39	0.0543	38.89	0.0540	39.66	0.0541	-0.54	0.05	0.64
70	39.93	40.20	40.73	39.82	0.0582	40.28	0.0579	40.99	0.0582	-0.27	0.20	0.63
75	41.27	41.53	42.07	41.10	0.0626	41.46	0.0623	42.33	0.0624	-0.40	-0.18	0.61
80	42.60	42.87	43.40	42.51	0.0664	42.88	0.0661	43.66	0.0665	-0.22	0.04	0.60
85	43.93	44.20	44.73	43.82	0.0707	44.25	0.0707	44.99	0.0706	-0.25	0.11	0.58
90	45.27	45.53	46.07	45.16	0.0747	45.56	0.0743	46.33	0.0748	-0.25	0.06	0.56
95	46.60	46.87	47.40	46.44	0.0786	46.89	0.0789	47.66	0.0789	-0.34	0.05	0.55
100	47.93	48.20	48.73	47.81	0.0829	48.26	0.0827	49.00	0.0830	-0.26	0.13	0.53

D.10 Values of Figure 3.26 and Figure 3.27

Table D.10: Values for Figure 3.26 and Figure 3.27

Capacity of the shuttle vehicle	Cycle time analytical optimized sequencing [s]	Cycle time analytical FCFS sequencing [s]	Cycle time simulation optimized sequencing [s]	EBM - 95 % confidence interval simulation optimized sequencing [s]	Cycle time simulation FCFS sequencing [s]	EBM - 95 % confidence interval simulation FCFS sequencing [s]	Relative error analytical / simulation optimized sequencing [%]	Relative error analytical / simulation FCFS sequencing [%]
1	47.67	47.67	47.83	0.0829	47.86	0.0826	0.34	0.41
2	64.33	72.33	64.43	0.0810	72.41	0.1480	0.15	0.10
3	77.95	97.00	77.77	0.0757	97.32	0.2143	-0.24	0.33
4	90.56	121.67	90.13	0.0709	121.82	0.2770	-0.48	0.13
5	102.70	146.33	101.90	0.0665	146.41	0.3399	-0.78	0.05
6	114.59	171.00	113.35	0.0648	171.14	0.4071	-1.09	0.08
7	126.33	195.67	124.57	0.0639	195.85	0.4678	-1.41	0.09
8	137.98	220.33	135.65	0.0633	220.19	0.5253	-1.72	-0.06
9	149.56	245.00	146.59	0.0634	244.61	0.5982	-2.03	-0.16
10	161.10	269.67	157.44	0.0636	269.62	0.6707	-2.32	-0.02

D.11 Values of Figure 3.31 and Figure 3.32

Table D.11: Values for Figure 3.31 and Figure 3.32

Filling degree [%]	Cycle time analytical relocation one side [s]	Cycle time analytical relocation both sides [s]	Cycle time analytical relocation random [s]	Cycle time simulation relocation one side [s]	EBM - 95 % confidence interval simulation relocation one side [s]	Cycle time simulation relocation both sides [s]	EBM - 95 % confidence interval simulation relocation both sides [s]	Cycle time simulation relocation random [s]	EBM - 95 % confidence interval simulation relocation random [s]	Relative error analytical / simulation relocation one side [%]	Relative error analytical / simulation relocation both sides [%]	Relative error analytical / simulation relocation random [%]
5	52.13	51.95	52.75	52.29	0.0899	52.22	0.0888	52.96	0.1041	0.30	0.52	0.40
10	52.55	52.22	53.73	52.58	0.0924	52.56	0.0910	54.03	0.1149	0.06	0.64	0.56
15	52.94	52.47	54.62	52.93	0.0937	52.80	0.0927	54.89	0.1234	-0.01	0.62	0.48
20	53.29	52.71	55.44	53.29	0.0953	53.08	0.0938	55.67	0.1301	0.00	0.70	0.41
25	53.61	52.94	56.20	53.52	0.0964	53.30	0.0953	56.32	0.1360	-0.17	0.67	0.22
30	53.91	53.17	56.90	53.81	0.0981	53.47	0.0958	57.01	0.1387	-0.19	0.56	0.20
35	54.19	53.40	57.54	54.03	0.0979	53.73	0.0966	57.78	0.1443	-0.30	0.62	0.40
40	54.45	53.62	58.14	54.30	0.0995	53.87	0.0973	58.36	0.1461	-0.28	0.46	0.37
45	54.70	53.85	58.70	54.46	0.0996	54.10	0.0978	58.88	0.1495	-0.43	0.45	0.30
50	54.93	54.08	59.22	54.68	0.1009	54.19	0.0982	59.45	0.1508	-0.45	0.19	0.39
55	55.15	54.31	59.71	54.84	0.1008	54.41	0.0993	59.85	0.1534	-0.55	0.18	0.24
60	55.35	54.55	60.17	55.03	0.1014	54.62	0.0994	60.40	0.1558	-0.58	0.12	0.39
65	55.56	54.79	60.60	55.14	0.1018	54.64	0.1004	60.74	0.1562	-0.75	-0.27	0.24
70	55.76	55.04	61.00	55.33	0.1026	54.94	0.1002	61.33	0.1582	-0.77	-0.17	0.54
75	55.96	55.29	61.38	55.46	0.1044	55.08	0.1009	61.45	0.1589	-0.91	-0.38	0.12
80	56.17	55.56	61.74	55.77	0.1040	55.32	0.1012	62.16	0.1601	-0.72	-0.43	0.67
85	56.43	55.85	62.08	56.06	0.1051	55.65	0.1022	62.21	0.1613	-0.65	-0.36	0.21
90	56.78	56.20	62.40	56.53	0.1081	55.95	0.1038	62.83	0.1608	-0.44	-0.44	0.68
95	57.57	56.77	62.71	57.63	0.1140	56.64	0.1072	62.85	0.1618	0.10	-0.23	0.22
98	59.59	57.92	62.89	59.81	0.1266	57.93	0.1108	63.06	0.1537	0.37	0.02	0.28
99	61.70	59.58	62.94	61.92	0.1459	59.83	0.1261	63.15	0.1559	0.34	0.43	0.33

D.12 Values of Figure 3.33, Figure 3.34 and Figure 3.35

Table D.12: Values for Figure 3.33, Figure 3.34 and Figure 3.35

Filling degree [%]	**Cycle time analytical c_s = 1 [s]**	**Cycle time analytical c_s = 2 [s]**	**Cycle time analytical c_s = 3 [s]**	**Cycle time analytical c_s = 4 [s]**	**Cycle time simulation c_s = 1 [s]**	**EBM - 95 % confidence interval simulation c_s = 1 [s]**	**Cycle time simulation c_s = 2 [s]**	**EBM - 95 % confidence interval simulation c_s = 2 [s]**	**Cycle time simulation c_s = 3 [s]**	**EBM - 95 % confidence interval simulation c_s = 3 [s]**	**Cycle time simulation c_s = 4 [s]**	**EBM - 95 % confidence interval simulation c_s = 4 [s]**
5	51.95	70.91	86.81	101.70	52.22	0.0888	71.06	0.0955	86.93	0.1030	101.71	0.1163
10	52.22	71.44	87.61	102.76	52.56	0.0910	71.70	0.1026	87.79	0.1170	102.90	0.1365
15	52.47	71.94	88.36	103.76	52.80	0.0927	72.26	0.1075	88.66	0.1267	104.02	0.1518
20	52.71	72.42	89.08	104.72	53.08	0.0938	72.83	0.1111	89.55	0.1334	105.00	0.1609
25	52.94	72.88	89.78	105.65	53.30	0.0953	73.27	0.1146	90.22	0.1405	105.90	0.1709
30	53.17	73.34	90.46	106.57	53.47	0.0958	73.67	0.1179	90.76	0.1450	106.86	0.1788
35	53.40	73.80	91.15	107.48	53.73	0.0966	74.24	0.1200	91.45	0.1503	107.75	0.1826
40	53.62	74.25	91.83	108.39	53.87	0.0973	74.55	0.1229	91.86	0.1538	108.27	0.1913
45	53.85	74.71	92.51	109.30	54.10	0.0978	74.93	0.1233	92.48	0.1570	109.29	0.1941
50	54.08	75.16	93.20	110.22	54.19	0.0982	75.14	0.1255	93.07	0.1597	110.00	0.1998
55	54.31	75.63	93.89	111.15	54.41	0.0993	75.49	0.1270	93.54	0.1610	110.65	0.2005
60	54.55	76.10	94.60	112.09	54.62	0.0994	75.86	0.1292	93.96	0.1641	110.99	0.2089
65	54.79	76.58	95.32	113.05	54.64	0.1004	76.29	0.1293	94.56	0.1665	111.94	0.2085
70	55.04	77.07	96.06	114.03	54.94	0.1002	76.69	0.1312	95.29	0.1706	112.53	0.2134
75	55.29	77.58	96.82	115.05	55.08	0.1009	76.86	0.1324	95.59	0.1710	113.30	0.2182
80	55.56	78.11	97.62	116.12	55.32	0.1012	77.30	0.1350	96.57	0.1771	114.14	0.2223
85	55.85	78.70	98.50	117.28	55.65	0.1022	77.87	0.1364	97.01	0.1788	115.24	0.2296
90	56.20	79.40	99.55	118.68	55.95	0.1038	78.45	0.1424	98.01	0.1877	116.79	0.2416
95	56.77	80.54	101.27	120.97	56.64	0.1072	80.08	0.1520	100.33	0.2061	119.60	0.2672
98	57.92	82.83	104.70	125.55	57.93	0.1108	82.82	0.1670	104.58	0.2417	125.83	0.3244
99	59.58	86.15	109.68	132.20	59.83	0.1261	87.51	0.2273	109.70	0.3096	-	-

Filling degree [%]	Throughput $c_s = 1$ [ULs/hour]	Throughput $c_s = 2$ [ULs/hour]	Throughput $c_s = 3$ [ULs/hour]	Throughput $c_s = 4$ [ULs/hour]	Relative error analytical/simulation $c_s = 1$ [%]	Relative error analytical / simulation $c_s = 2$ [%]	Relative error analytical / simulation $c_s = 3$ [%]	Relative error analytical / simulation $c_s = 4$ [%]
5	138.59	203.08	248.81	283.18	0.39	0.21	0.14	0.01
10	137.88	201.58	246.55	280.26	0.57	0.37	0.21	0.13
15	137.23	200.18	244.46	277.56	0.56	0.44	0.34	0.25
20	136.60	198.85	242.49	275.02	0.68	0.57	0.53	0.27
25	136.00	197.58	240.60	272.59	0.52	0.53	0.49	0.23
30	135.41	196.34	238.77	270.24	0.68	0.45	0.33	0.27
35	134.84	195.13	236.98	267.96	0.61	0.60	0.33	0.25
40	134.27	193.94	235.23	265.71	0.47	0.40	0.03	-0.11
45	133.70	192.76	233.49	263.50	0.52	0.30	-0.03	-0.01
50	133.13	191.58	231.76	261.30	0.14	-0.03	-0.14	-0.20
55	132.56	190.41	230.05	259.12	0.14	-0.19	-0.38	-0.45
60	131.99	189.23	228.33	256.94	0.03	-0.31	-0.68	-0.98
65	131.41	188.04	226.60	254.76	-0.12	-0.38	-0.80	-0.99
70	130.83	186.84	224.86	252.57	-0.20	-0.49	-0.81	-1.34
75	130.22	185.62	223.09	250.33	-0.57	-0.94	-1.28	-1.54
80	129.60	184.35	221.26	248.03	-0.56	-1.06	-1.09	-1.73
85	128.92	182.98	219.30	245.56	-0.32	-1.06	-1.53	-1.77
90	128.12	181.37	216.99	242.67	-0.39	-1.20	-1.57	-1.62
95	126.82	178.79	213.30	238.07	-0.33	-0.58	-0.93	-1.15
98	124.32	173.84	206.30	229.38	0.02	-0.01	-0.12	0.21
99	120.85	167.14	196.93	217.86	0.43	1.55	0.02	-

D.13 Values of Figure 3.36

Table D.13: Values for Figure 3.36

	Side-by-side				One-behind-the-other							
Filling degree [%]	**Cycle time analytical c_s = 1 [s]**	**Cycle time analytical c_s = 2 [s]**	**Cycle time analytical c_s = 3 [s]**	**Cycle time analytical c_s = 4 [s]**	**Cycle time analytical c_s = 1 [s]**	**Cycle time analytical c_s = 2 [s]**	**Cycle time analytical c_s = 3 [s]**	**Cycle time analytical c_s = 4 [s]**	**Deviation c_s = 1 [%]**	**Deviation c_s = 2 [%]**	**Deviation c_s = 3 [%]**	**Deviation c_s = 4 [%]**
5	51.95	70.91	86.81	101.70	51.95	71.08	86.90	102.06	0.00	0.25	0.10	0.35
10	52.22	71.44	87.61	102.76	52.22	71.77	87.77	103.43	0.00	0.46	0.19	0.64
15	52.47	71.94	88.36	103.76	52.47	72.40	88.59	104.70	0.00	0.65	0.26	0.89
20	52.71	72.42	89.08	104.72	52.71	73.00	89.37	105.88	0.00	0.80	0.33	1.10
25	52.94	72.88	89.78	105.65	52.94	73.56	90.11	107.00	0.00	0.91	0.37	1.26
30	53.17	73.34	90.46	106.57	53.17	74.09	90.84	108.06	0.00	1.00	0.41	1.38
35	53.40	73.80	91.15	107.48	53.40	74.59	91.54	109.07	0.00	1.07	0.43	1.46
40	53.62	74.25	91.83	108.39	53.62	75.08	92.24	110.05	0.00	1.10	0.45	1.51
45	53.85	74.71	92.51	109.30	53.85	75.55	92.93	110.99	0.00	1.12	0.45	1.52
50	54.08	75.16	93.20	110.22	54.08	76.01	93.62	111.91	0.00	1.11	0.45	1.51
55	54.31	75.63	93.89	111.15	54.31	76.46	94.31	112.81	0.00	1.09	0.44	1.47
60	54.55	76.10	94.60	112.09	54.55	76.90	95.00	113.70	0.00	1.05	0.42	1.42
65	54.79	76.58	95.32	113.05	54.79	77.35	95.70	114.58	0.00	0.99	0.40	1.34
70	55.04	77.07	96.06	114.03	55.04	77.79	96.42	115.47	0.00	0.93	0.37	1.25
75	55.29	77.58	96.82	115.05	55.29	78.25	97.16	116.39	0.00	0.86	0.34	1.15
80	55.56	78.11	97.62	116.12	55.56	78.73	97.93	117.35	0.00	0.78	0.32	1.05
85	55.85	78.70	98.50	117.28	55.85	79.27	98.79	118.44	0.00	0.73	0.29	0.98
90	56.20	79.40	99.55	118.68	56.20	79.98	99.84	119.85	0.00	0.73	0.29	0.97
95	56.77	80.54	101.27	120.97	56.77	81.35	101.67	122.58	0.00	0.99	0.40	1.31
98	57.92	82.83	104.70	125.55	57.92	84.51	107.22	128.91	0.00	1.98	2.35	2.60
99	59.58	86.15	109.68	132.20	59.58	88.28	112.87	136.45	0.00	2.41	2.83	3.12

D.14 Values of Figure 3.38

Table D.14: Values for Figure 3.38 – Percentage of empty storage channels [%]

Filling degree [%]	Percentage of empty storage channels [%]																				
	Percentage of small ULs [%]																				
	0	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**
5	95.00	94.87	94.74	94.60	94.45	94.30	94.13	93.96	93.78	93.59	93.39	93.18	92.95	92.71	92.45	92.18	91.89	91.57	91.23	90.87	90.48
10	90.00	89.75	89.48	89.20	88.92	88.62	88.30	87.97	87.63	87.27	86.90	86.50	86.09	85.65	85.19	84.71	84.19	83.65	83.08	82.47	81.82
15	85.00	84.62	84.22	83.82	83.40	82.96	82.51	82.04	81.55	81.05	80.53	79.98	79.42	78.83	78.21	77.57	76.90	76.21	75.48	74.71	73.91
20	80.00	79.49	78.97	78.44	77.89	77.33	76.76	76.16	75.56	74.93	74.29	73.62	72.94	72.24	71.52	70.77	70.00	69.21	68.39	67.54	66.67
25	75.00	74.37	73.73	73.08	72.41	71.74	71.05	70.35	69.64	68.92	68.18	67.43	66.67	65.89	65.09	64.29	63.46	62.62	61.76	60.89	60.00
30	70.00	69.25	68.49	67.73	66.96	66.18	65.40	64.62	63.82	63.03	62.22	61.41	60.60	59.78	58.95	58.11	57.27	56.43	55.57	54.71	53.85
35	65.00	64.13	63.26	62.39	61.53	60.67	59.81	58.96	58.11	57.26	56.42	55.57	54.74	53.90	53.07	52.24	51.42	50.60	49.78	48.96	48.15
40	60.00	59.01	58.03	57.07	56.13	55.20	54.29	53.39	52.50	51.63	50.77	49.92	49.09	48.27	47.46	46.67	45.88	45.11	44.35	43.60	42.86
45	55.00	53.89	52.82	51.78	50.77	49.79	48.84	47.91	47.02	46.14	45.29	44.47	43.66	42.88	42.12	41.38	40.65	39.95	39.26	38.59	37.93
50	50.00	48.78	47.62	46.51	45.45	44.44	43.48	42.55	41.67	40.82	40.00	39.22	38.46	37.74	37.04	36.36	35.71	35.09	34.48	33.90	33.33
55	45.00	43.67	42.44	41.28	40.19	39.18	38.22	37.32	36.47	35.66	34.90	34.17	33.49	32.84	32.21	31.62	31.06	30.52	30.00	29.51	29.03
60	40.00	38.57	37.27	36.09	35.00	34.00	33.08	32.22	31.43	30.69	30.00	29.35	28.75	28.18	27.65	27.14	26.67	26.22	25.79	25.38	25.00
65	35.00	33.48	32.14	30.95	29.89	28.93	28.07	27.29	26.57	25.92	25.32	24.76	24.25	23.78	23.33	22.92	22.53	22.17	21.83	21.51	21.21
70	30.00	28.40	27.04	25.88	24.88	24.00	23.23	22.54	21.92	21.37	20.87	20.41	20.00	19.62	19.27	18.95	18.65	18.37	18.11	17.87	17.65
75	25.00	23.33	22.00	20.91	20.00	19.23	18.57	18.00	17.50	17.06	16.67	16.32	16.00	15.71	15.45	15.22	15.00	14.80	14.62	14.44	14.29
80	20.00	18.30	17.04	16.07	15.29	14.67	14.15	13.71	13.33	13.01	12.73	12.48	12.26	12.06	11.88	11.72	11.58	11.45	11.33	11.21	11.11
85	15.00	13.31	12.20	11.41	10.82	10.36	10.00	9.70	9.46	9.25	9.07	8.92	8.78	8.66	8.56	8.46	8.38	8.30	8.23	8.17	8.11
90	10.00	8.42	7.57	7.03	6.67	6.40	6.20	6.04	5.91	5.80	5.71	5.64	5.57	5.52	5.47	5.42	5.38	5.35	5.32	5.29	5.26
95	5.00	3.77	3.33	3.11	2.98	2.89	2.82	2.78	2.74	2.71	2.68	2.66	2.64	2.63	2.62	2.61	2.59	2.59	2.58	2.57	2.56
96	4.00	2.90	2.57	2.41	2.32	2.25	2.21	2.18	2.15	2.13	2.12	2.10	2.09	2.08	2.07	2.07	2.06	2.05	2.05	2.05	2.04
97	3.00	2.06	1.84	1.74	1.68	1.65	1.62	1.60	1.59	1.58	1.57	1.56	1.55	1.55	1.54	1.54	1.53	1.53	1.53	1.53	1.52
98	2.00	1.28	1.16	1.11	1.08	1.07	1.05	1.05	1.04	1.03	1.03	1.03	1.02	1.02	1.02	1.02	1.02	1.01	1.01	1.01	1.01
99	1.00	0.58	0.54	0.53	0.52	0.52	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.51	0.50	0.50	0.50	0.50	0.50	0.50	0.50

D.15 Values of Figure 3.39, Figure 3.40 and Figure 3.41

Table D.15: Values for Figure 3.39, Figure 3.40 and Figure 3.41 – Cycle time analytical [s]

Filling degree [%]	Cycle time analytical [s]																				
	Percentage of small ULs [%]																				
	0	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**
5	51.93	52.45	52.99	53.56	54.15	54.79	55.45	56.16	56.91	57.70	58.54	59.44	60.40	61.43	62.53	63.71	64.99	66.36	67.86	69.48	71.25
10	51.93	52.45	52.99	53.56	54.16	54.80	55.48	56.19	56.95	57.76	58.62	59.54	60.53	61.58	62.72	63.94	65.26	66.69	68.24	69.94	71.79
15	51.93	52.45	52.99	53.57	54.17	54.82	55.50	56.23	57.00	57.82	58.70	59.64	60.65	61.74	62.91	64.17	65.53	67.01	68.62	70.38	72.31
20	51.93	52.45	53.00	53.57	54.19	54.84	55.53	56.27	57.05	57.89	58.79	59.76	60.79	61.90	63.10	64.40	65.80	67.33	68.99	70.81	72.80
25	51.93	52.45	53.00	53.58	54.20	54.86	55.56	56.31	57.11	57.97	58.89	59.87	60.93	62.07	63.31	64.64	66.08	67.65	69.36	71.23	73.28
30	51.93	52.45	53.00	53.59	54.22	54.88	55.60	56.36	57.18	58.05	58.99	60.00	61.08	62.25	63.52	64.88	66.36	67.97	69.73	71.64	73.74
35	51.93	52.45	53.01	53.60	54.23	54.91	55.64	56.41	57.25	58.14	59.10	60.13	61.24	62.44	63.74	65.14	66.65	68.30	70.10	72.06	74.20
40	51.93	52.45	53.01	53.61	54.25	54.94	55.68	56.47	57.33	58.24	59.22	60.28	61.41	62.64	63.96	65.40	66.95	68.63	70.47	72.47	74.66
45	51.93	52.46	53.02	53.62	54.28	54.98	55.73	56.54	57.41	58.35	59.35	60.43	61.59	62.85	64.20	65.67	67.25	68.97	70.85	72.89	75.12
50	51.93	52.46	53.03	53.64	54.30	55.02	55.79	56.62	57.51	58.46	59.49	60.60	61.79	63.07	64.45	65.95	67.57	69.32	71.23	73.31	75.59
55	51.93	52.46	53.03	53.66	54.34	55.07	55.86	56.70	57.61	58.59	59.64	60.77	61.99	63.30	64.71	66.24	67.89	69.68	71.62	73.74	76.06
60	51.93	52.46	53.05	53.68	54.37	55.12	55.93	56.80	57.73	58.73	59.81	60.97	62.21	63.55	64.99	66.54	68.23	70.05	72.03	74.18	76.54
65	51.93	52.47	53.06	53.71	54.42	55.19	56.02	56.91	57.86	58.89	59.99	61.17	62.44	63.81	65.28	66.86	68.58	70.43	72.44	74.63	77.03
70	51.93	52.47	53.08	53.74	54.47	55.26	56.11	57.03	58.01	59.06	60.19	61.40	62.70	64.09	65.59	67.20	68.94	70.83	72.88	75.10	77.53
75	51.93	52.48	53.10	53.79	54.54	55.35	56.23	57.17	58.18	59.26	60.41	61.65	62.97	64.39	65.92	67.56	69.33	71.25	73.33	75.59	78.06
80	51.93	52.49	53.13	53.84	54.62	55.47	56.37	57.34	58.38	59.48	60.66	61.92	63.28	64.72	66.28	67.95	69.76	71.71	73.82	76.12	78.62
85	51.93	52.50	53.17	53.92	54.73	55.61	56.54	57.54	58.61	59.74	60.95	62.24	63.63	65.10	66.69	68.40	70.23	72.22	74.37	76.71	79.26
90	51.93	52.53	53.24	54.03	54.89	55.80	56.77	57.81	58.91	60.07	61.32	62.65	64.06	65.58	67.20	68.95	70.83	72.86	75.07	77.46	80.07
95	51.93	52.59	53.38	54.24	55.16	56.13	57.15	58.24	59.40	60.63	61.93	63.32	64.80	66.39	68.09	69.92	71.88	74.01	76.31	78.82	81.55
98	51.93	52.71	53.62	54.58	55.60	56.68	57.82	59.02	60.30	61.65	63.09	64.63	66.26	68.01	69.89	71.91	74.09	76.44	78.98	81.76	84.78
99	51.93	52.84	53.85	54.93	56.06	57.26	58.53	59.87	61.28	62.79	64.39	66.10	67.92	69.87	71.96	74.21	76.63	79.25	82.10	85.19	88.58

Table D.16: Values for Figure 3.39, Figure 3.40 and Figure 3.41 – Cycle time simulation [s]

Filling degree [%]	Cycle time simulation [s]																				
	Percentage of small ULs [%]																				
	0	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**
5	51.76	52.38	52.91	53.41	53.99	54.67	55.39	56.10	56.83	57.70	58.54	59.25	60.30	61.27	62.40	63.61	64.78	66.12	67.42	69.13	71.13
10	51.78	52.29	52.89	53.48	54.12	54.66	55.38	56.14	56.89	57.75	58.56	59.47	60.53	61.54	62.49	63.73	64.93	66.37	67.96	69.45	71.80
15	51.75	52.30	52.89	53.48	54.11	54.80	55.39	56.27	56.90	57.71	58.71	59.60	60.62	61.75	62.95	63.93	65.21	66.82	68.04	69.87	72.36
20	51.76	52.34	52.88	53.62	54.23	54.79	55.43	56.30	57.07	57.86	58.74	59.75	60.75	61.64	62.99	64.12	65.73	66.70	68.09	70.13	72.91
25	51.78	52.39	52.82	53.50	54.19	54.80	55.56	56.18	57.10	57.95	58.82	59.87	60.87	61.99	62.86	64.50	66.03	67.32	68.80	70.09	73.43
30	51.74	52.45	52.93	53.53	54.22	54.82	55.65	56.28	57.06	57.98	58.94	60.00	61.04	62.07	63.59	64.70	66.19	67.50	68.60	70.46	73.86
35	51.82	52.43	53.02	53.63	54.10	54.84	55.59	56.38	57.24	58.08	59.11	60.09	61.20	62.23	63.33	65.10	66.04	68.18	69.82	71.59	74.36
40	51.73	52.47	52.96	53.56	54.32	54.95	55.68	56.43	57.21	58.21	59.10	60.28	61.19	62.53	63.85	65.06	66.89	68.03	70.03	71.39	74.69
45	51.83	52.30	52.95	53.50	54.24	54.92	55.67	56.54	57.35	58.24	59.30	60.45	61.50	62.33	63.93	65.61	67.26	68.59	70.00	72.40	75.19
50	51.79	52.31	53.03	53.69	54.19	54.95	55.82	56.61	57.38	58.40	58.89	60.35	61.67	62.65	63.60	65.95	67.34	68.82	70.15	73.22	75.61
55	51.81	52.46	53.08	53.48	54.32	55.05	55.68	56.59	57.59	58.52	59.56	60.31	61.96	63.28	64.62	65.97	67.93	69.55	71.13	73.01	75.95
60	51.74	52.41	52.99	53.63	54.28	55.01	55.75	56.68	57.67	58.46	59.64	60.52	62.11	63.33	64.80	66.13	67.80	69.63	71.87	73.95	76.26
65	51.88	52.38	53.03	53.70	54.32	55.17	55.98	56.92	57.85	58.89	59.55	61.07	62.18	62.88	64.87	66.80	68.51	69.79	71.76	74.39	76.72
70	51.83	52.40	53.03	53.62	54.44	55.20	55.96	56.84	58.00	58.91	59.87	60.47	62.54	63.32	65.26	67.11	68.10	70.23	72.50	74.48	77.12
75	51.82	52.33	53.01	53.54	54.41	55.25	56.22	57.01	57.79	59.01	60.38	61.20	62.79	63.79	65.92	67.02	68.66	70.64	72.72	74.83	77.50
80	51.80	52.44	53.05	53.70	54.50	55.38	56.20	57.32	58.26	59.49	60.08	61.56	63.28	64.49	66.11	67.26	69.20	70.52	72.60	75.19	77.89
85	51.80	52.44	53.10	53.89	54.73	55.62	56.33	57.30	58.28	59.62	60.75	61.94	63.42	64.91	65.62	67.27	69.65	71.83	73.27	76.06	78.31
90	51.88	52.57	53.13	53.91	54.56	55.64	56.66	57.47	58.71	60.09	60.80	62.48	63.49	64.88	66.37	67.82	70.36	72.34	74.57	76.84	79.26
95	51.85	52.51	53.30	54.30	55.07	55.83	57.02	58.09	59.20	60.22	61.78	62.89	64.18	66.21	67.75	69.72	71.57	72.66	75.87	78.33	80.89
98	51.77	52.63	53.55	54.32	55.35	56.48	57.35	58.83	60.22	61.61	62.77	64.24	66.06	67.88	69.45	71.81	73.35	76.46	78.77	81.78	84.57
99	51.85	52.51	53.70	54.69	56.08	57.21	58.51	59.88	61.16	62.72	64.34	66.01	67.87	69.69	71.91	74.20	76.68	79.23	82.13	85.10	87.44

Table D.17: Values for Figure 3.39, Figure 3.40 and Figure 3.41 – EBM - 95 % confidence interval simulation [s]

Filling degree [%]	EBM - 95 % confidence interval simulation [s]																				
	Percentage of small ULs [%]																				
	0	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**
5	0.083	0.086	0.088	0.091	0.094	0.097	0.098	0.102	0.104	0.106	0.109	0.111	0.111	0.112	0.114	0.115	0.112	0.110	0.105	0.098	0.091
10	0.083	0.085	0.088	0.092	0.094	0.097	0.101	0.103	0.105	0.108	0.110	0.113	0.114	0.116	0.115	0.116	0.114	0.112	0.108	0.103	0.098
15	0.083	0.086	0.089	0.092	0.095	0.097	0.102	0.105	0.107	0.109	0.112	0.113	0.116	0.118	0.120	0.117	0.119	0.117	0.111	0.106	0.103
20	0.083	0.086	0.089	0.093	0.096	0.099	0.103	0.105	0.108	0.112	0.114	0.115	0.117	0.118	0.121	0.121	0.121	0.116	0.113	0.108	0.108
25	0.083	0.085	0.090	0.092	0.095	0.100	0.101	0.105	0.108	0.111	0.113	0.117	0.119	0.121	0.121	0.124	0.125	0.122	0.118	0.109	0.111
30	0.083	0.086	0.090	0.094	0.098	0.100	0.105	0.106	0.111	0.111	0.116	0.119	0.121	0.122	0.127	0.124	0.128	0.124	0.119	0.115	0.114
35	0.083	0.086	0.091	0.094	0.098	0.099	0.106	0.107	0.111	0.114	0.119	0.121	0.123	0.125	0.127	0.130	0.126	0.131	0.127	0.122	0.117
40	0.084	0.086	0.089	0.092	0.098	0.100	0.103	0.107	0.112	0.115	0.117	0.122	0.123	0.128	0.130	0.131	0.133	0.129	0.129	0.119	0.119
45	0.083	0.086	0.089	0.095	0.099	0.100	0.105	0.110	0.114	0.115	0.121	0.125	0.126	0.126	0.130	0.135	0.138	0.134	0.127	0.127	0.121
50	0.083	0.086	0.090	0.095	0.097	0.103	0.107	0.109	0.112	0.117	0.115	0.123	0.129	0.128	0.128	0.138	0.137	0.135	0.129	0.132	0.123
55	0.083	0.087	0.092	0.094	0.099	0.103	0.105	0.112	0.114	0.120	0.121	0.123	0.133	0.137	0.138	0.139	0.143	0.142	0.137	0.130	0.123
60	0.083	0.087	0.090	0.096	0.100	0.102	0.105	0.111	0.118	0.118	0.125	0.126	0.134	0.137	0.138	0.140	0.143	0.143	0.141	0.135	0.125
65	0.083	0.087	0.090	0.096	0.100	0.105	0.110	0.114	0.118	0.124	0.123	0.132	0.135	0.133	0.140	0.146	0.147	0.143	0.141	0.138	0.127
70	0.083	0.087	0.091	0.094	0.102	0.105	0.108	0.113	0.122	0.125	0.129	0.124	0.139	0.136	0.145	0.147	0.145	0.146	0.144	0.138	0.129
75	0.083	0.087	0.091	0.095	0.101	0.107	0.110	0.115	0.117	0.127	0.134	0.133	0.140	0.142	0.149	0.149	0.150	0.149	0.148	0.141	0.131
80	0.083	0.088	0.092	0.096	0.103	0.109	0.113	0.119	0.126	0.132	0.131	0.137	0.145	0.147	0.151	0.150	0.153	0.150	0.148	0.143	0.132
85	0.083	0.087	0.092	0.097	0.105	0.112	0.114	0.121	0.125	0.134	0.136	0.142	0.147	0.151	0.149	0.151	0.157	0.156	0.152	0.147	0.135
90	0.083	0.089	0.093	0.099	0.104	0.113	0.120	0.123	0.131	0.137	0.140	0.149	0.149	0.153	0.158	0.159	0.163	0.163	0.160	0.153	0.140
95	0.083	0.090	0.097	0.105	0.111	0.116	0.127	0.135	0.141	0.145	0.153	0.157	0.160	0.168	0.172	0.176	0.178	0.173	0.175	0.167	0.154
98	0.083	0.090	0.104	0.109	0.120	0.132	0.134	0.151	0.159	0.166	0.173	0.177	0.188	0.197	0.201	0.207	0.207	0.212	0.210	0.206	0.194
99	0.083	0.092	0.108	0.122	0.138	0.147	0.162	0.174	0.176	0.185	0.195	0.204	0.214	0.223	0.231	0.236	0.240	0.242	0.245	0.241	0.229

Table D.18: Values for Figure 3.39, Figure 3.40 and Figure 3.41 – Relative error analytical/ simulation [%]

Filling degree [%]	**Relative error analytical / simulation [%]**																				
	Percentage of small ULs [%]																				
	0	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**
5	-0.33	-0.13	-0.15	-0.27	-0.30	-0.21	-0.11	-0.10	-0.14	-0.01	-0.01	-0.32	-0.16	-0.27	-0.21	-0.17	-0.31	-0.37	-0.64	-0.50	-0.17
10	-0.29	-0.30	-0.19	-0.16	-0.07	-0.26	-0.16	-0.10	-0.11	-0.01	-0.10	-0.13	0.00	-0.06	-0.37	-0.33	-0.51	-0.48	-0.41	-0.71	0.00
15	-0.36	-0.28	-0.19	-0.17	-0.13	-0.03	-0.21	0.08	-0.17	-0.19	0.02	-0.07	-0.05	0.02	0.07	-0.36	-0.49	-0.29	-0.86	-0.74	0.07
20	-0.34	-0.21	-0.21	0.09	0.09	-0.09	-0.18	0.07	0.03	-0.05	-0.09	0.00	-0.07	-0.42	-0.17	-0.43	-0.11	-0.95	-1.32	-0.97	0.16
25	-0.30	-0.12	-0.33	-0.15	-0.01	-0.10	0.00	-0.23	-0.02	-0.03	-0.12	-0.01	-0.11	-0.14	-0.71	-0.22	-0.08	-0.50	-0.82	-1.62	0.22
30	-0.37	0.01	-0.14	-0.11	0.01	-0.11	0.10	-0.14	-0.21	-0.12	-0.08	0.01	-0.08	-0.30	0.11	-0.29	-0.26	-0.70	-1.65	-1.68	0.16
35	-0.22	-0.03	0.03	0.05	-0.24	-0.13	-0.08	-0.06	-0.02	-0.10	0.02	-0.07	-0.07	-0.34	-0.64	-0.06	-0.93	-0.18	-0.40	-0.65	0.20
40	-0.40	0.03	-0.09	-0.10	0.11	0.01	0.00	-0.08	-0.20	-0.05	-0.21	0.00	-0.37	-0.17	-0.18	-0.52	-0.09	-0.89	-0.63	-1.51	0.04
45	-0.21	-0.30	-0.12	-0.23	-0.07	-0.11	-0.11	-0.01	-0.11	-0.18	-0.09	0.04	-0.16	-0.83	-0.42	-0.09	0.02	-0.56	-1.21	-0.68	0.09
50	-0.27	-0.28	0.01	0.09	-0.20	-0.12	0.05	-0.02	-0.22	-0.11	-1.02	-0.40	-0.19	-0.66	-1.34	0.00	-0.34	-0.73	-1.55	-0.13	0.03
55	-0.24	0.01	0.09	-0.34	-0.04	-0.03	-0.31	-0.19	-0.05	-0.13	-0.14	-0.77	-0.05	-0.03	-0.14	-0.40	0.07	-0.19	-0.70	-1.00	-0.15
60	-0.37	-0.10	-0.11	-0.09	-0.18	-0.21	-0.32	-0.20	-0.10	-0.47	-0.28	-0.74	-0.16	-0.34	-0.29	-0.62	-0.63	-0.59	-0.22	-0.32	-0.36
65	-0.10	-0.16	-0.06	-0.02	-0.18	-0.03	-0.06	0.02	-0.03	-0.01	-0.75	-0.17	-0.42	-1.47	-0.63	-0.09	-0.10	-0.92	-0.95	-0.33	-0.40
70	-0.19	-0.14	-0.10	-0.24	-0.06	-0.12	-0.28	-0.34	-0.02	-0.27	-0.54	-1.53	-0.26	-1.21	-0.50	-0.13	-1.23	-0.85	-0.52	-0.84	-0.54
75	-0.22	-0.28	-0.16	-0.47	-0.23	-0.20	-0.03	-0.28	-0.67	-0.42	-0.06	-0.73	-0.30	-0.94	0.00	-0.81	-0.98	-0.87	-0.84	-1.02	-0.71
80	-0.25	-0.09	-0.15	-0.26	-0.23	-0.16	-0.30	-0.04	-0.20	0.01	-0.96	-0.59	0.00	-0.36	-0.25	-1.03	-0.80	-1.68	-1.68	-1.23	-0.94
85	-0.26	-0.12	-0.15	-0.06	0.00	0.02	-0.38	-0.43	-0.57	-0.20	-0.33	-0.50	-0.33	-0.30	-1.64	-1.67	-0.83	-0.53	-1.50	-0.86	-1.21
90	-0.10	0.08	-0.22	-0.24	-0.60	-0.30	-0.20	-0.58	-0.33	0.02	-0.86	-0.26	-0.90	-1.07	-1.26	-1.67	-0.66	-0.72	-0.66	-0.81	-1.02
95	-0.17	-0.15	-0.15	0.10	-0.15	-0.53	-0.23	-0.26	-0.34	-0.68	-0.25	-0.69	-0.97	-0.27	-0.50	-0.28	-0.45	-1.86	-0.59	-0.62	-0.81
98	-0.31	-0.16	-0.14	-0.49	-0.46	-0.36	-0.81	-0.32	-0.12	-0.08	-0.52	-0.61	-0.31	-0.21	-0.63	-0.14	-1.01	0.03	-0.27	0.03	-0.25
99	-0.15	-0.62	-0.29	-0.44	0.03	-0.10	-0.03	0.02	-0.20	-0.12	-0.07	-0.14	-0.07	-0.26	-0.07	-0.02	0.07	-0.03	0.04	-0.11	-1.30

Table D.19: Values for Figure 3.39, Figure 3.40 and Figure 3.41 – Throughput [ULs/hour]

Filling degree [%]	**Throughput [ULs/hour]**																				
	Percentage of small ULs [%]																				
	0	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**
5	138	140	143	145	147	150	152	155	158	161	163	167	170	173	177	180	184	188	192	197	202
10	138	140	143	145	147	150	152	155	158	160	163	166	169	173	176	180	183	187	191	196	200
15	138	140	143	145	147	150	152	155	157	160	163	166	169	172	176	179	183	186	190	194	199
20	138	140	143	145	147	150	152	155	157	160	163	166	169	172	175	178	182	185	189	193	197
25	138	140	143	145	147	149	152	154	157	160	163	165	168	171	174	178	181	185	188	192	196
30	138	140	142	145	147	149	152	154	157	160	162	165	168	171	174	177	180	184	187	191	195
35	138	140	142	145	147	149	152	154	157	159	162	165	167	170	173	176	180	183	186	190	194
40	138	140	142	145	147	149	152	154	156	159	162	164	167	170	173	176	179	182	185	189	192
45	138	140	142	145	147	149	151	154	156	159	161	164	166	169	172	175	178	181	184	188	191
50	138	140	142	145	147	149	151	154	156	158	161	163	166	169	171	174	177	180	183	187	190
55	138	140	142	145	147	149	151	153	156	158	160	163	165	168	171	173	176	179	182	185	189
60	138	140	142	144	147	149	151	153	155	158	160	162	165	167	170	173	175	178	181	184	188
65	138	140	142	144	147	149	151	153	155	157	160	162	164	167	169	172	174	177	180	183	186
70	138	140	142	144	146	148	150	153	155	157	159	161	164	166	168	171	174	176	179	182	185
75	138	140	142	144	146	148	150	152	154	156	158	161	163	165	168	170	173	175	178	181	184
80	138	140	142	144	146	148	150	152	154	156	158	160	162	164	167	169	172	174	177	180	183
85	138	140	142	144	146	147	149	151	153	155	157	159	161	163	166	168	170	173	176	178	181
90	138	140	142	144	145	147	149	150	152	154	156	158	160	162	164	167	169	171	174	177	179
95	138	140	141	143	145	146	148	149	151	153	155	156	158	160	162	164	166	169	171	174	176
98	138	140	141	142	143	145	146	147	149	150	152	153	155	156	158	160	161	163	165	167	169
99	138	139	140	141	142	143	144	145	146	147	149	150	151	152	153	155	156	157	159	160	162

D.16 Values of Figure 3.42

Table D.20: Values for Figure 3.42

Rack length [m]	Rack height [m]	Inbound Lift or outbound lift throughput [ULs/hour]	Throughput one shuttle vehicle – storage + retrieval [ULs/hour]	Throughput all shuttle vehicles – storage + retrieval [ULs/hour]	Bottleneck	Expected throughput of the SBS/RS [stored ULs/hour], [retrieved ULs/hour]	Relative error analytical / simulation [%]
25	**2.5**	507.84	207.25	1036.27	Lift	507.84	2.0705
25	**5**	468.55	207.25	2072.54	Lift	468.55	3.6216
25	**7.5**	441.22	207.25	3108.80	Lift	441.22	2.3931
25	**10**	418.33	207.25	4145.07	Lift	418.33	1.7357
25	**12.5**	398.23	207.25	5181.34	Lift	398.23	1.3762
25	**15**	380.21	207.25	6217.61	Lift	380.21	1.3253
25	**17.5**	363.87	207.25	7253.88	Lift	363.87	0.7102
25	**20**	348.95	207.25	8290.14	Lift	348.95	0.8848
25	**22.5**	335.25	207.25	9326.41	Lift	335.25	0.9021
25	**25**	322.61	207.25	10362.68	Lift	322.61	0.5974
50	**2.5**	507.84	161.33	806.66	Shuttle	403.33	2.2213
50	**5**	468.55	161.33	1613.30	Lift	468.55	3.3013
50	**7.5**	441.22	161.33	2419.95	Lift	441.22	2.1687
50	**10**	418.33	161.33	3226.61	Lift	418.33	1.3748
50	**12.5**	398.23	161.33	4033.26	Lift	398.23	1.1449
50	**15**	380.21	161.33	4839.91	Lift	380.21	0.9002
50	**17.5**	363.87	161.33	5646.56	Lift	363.87	0.9564
50	**20**	348.95	161.33	6453.21	Lift	348.95	0.5534
50	**22.5**	335.25	161.33	7259.86	Lift	335.25	0.2401
50	**25**	322.61	161.33	8066.52	Lift	322.61	0.4359
100	**2.5**	507.84	111.79	558.95	Shuttle	279.47	2.1119
100	**5**	468.55	111.79	1117.90	Lift	468.55	3.3934
100	**7.5**	441.22	111.79	1676.84	Lift	441.22	2.1612
100	**10**	418.33	111.79	2235.79	Lift	418.33	1.1352
100	**12.5**	398.23	111.79	2794.74	Lift	398.23	0.9211
100	**15**	380.21	111.79	3353.69	Lift	380.21	0.7539
100	**17.5**	363.87	111.79	3912.64	Lift	363.87	0.3526
100	**20**	348.95	111.79	4471.58	Lift	348.95	0.6073

100	**22.5**	335.25	111.79	5030.53	Lift	335.25	0.4993
100	**25**	322.61	111.79	5589.48	Lift	322.61	0.4910
150	**2.5**	507.84	85.53	427.63	Shuttle	213.82	1.3010
150	**5**	468.55	85.53	855.26	Shuttle	427.63	0.9381
150	**7.5**	441.22	85.53	1282.90	Lift	441.22	2.2495
150	**10**	418.33	85.53	1710.53	Lift	418.33	1.4520
150	**12.5**	398.23	85.53	2138.16	Lift	398.23	0.9813
150	**15**	380.21	85.53	2565.79	Lift	380.21	0.9210
150	**17.5**	363.87	85.53	2993.43	Lift	363.87	0.6836
150	**20**	348.95	85.53	3421.06	Lift	348.95	0.5433
150	**22.5**	335.25	85.53	3848.69	Lift	335.25	0.8685
150	**25**	322.61	85.53	4276.32	Lift	322.61	0.2787
200	**2.5**	507.84	69.26	346.28	Shuttle	173.14	-1.2420
200	**5**	468.55	69.26	692.56	Shuttle	346.28	-0.3649
200	**7.5**	441.22	69.26	1038.84	Lift	441.22	1.8501
200	**10**	418.33	69.26	1385.12	Lift	418.33	1.4330
200	**12.5**	398.23	69.26	1731.40	Lift	398.23	0.9754
200	**15**	380.21	69.26	2077.68	Lift	380.21	0.9925
200	**17.5**	363.87	69.26	2423.96	Lift	363.87	0.3223
200	**20**	348.95	69.26	2770.24	Lift	348.95	0.7299
200	**22.5**	335.25	69.26	3116.52	Lift	335.25	0.3050
200	**25**	322.61	69.26	3462.79	Lift	322.61	0.2881

E Equations for Throughput

The throughput per hour in dependency of the lift configuration can be calculated using the following equations (without considering waiting times):

For the use case in chapter 3.3.1

The lift performs storage or retrieval operations:

$$\lambda_{Lift} = \frac{3600}{E(SC_{Lift})} \tag{E.1}$$

For the use case in chapter 3.3.2

The lift performs storage or retrieval operations:

$$\lambda_{Lift} = \frac{3600}{E(DC_{Lift})} \cdot 2 \tag{E.2}$$

For the use case in chapter 3.3.3

For the case that the lift performs combined storage and retrieval operation, the total throughput can be calculated but also the average number of ULs which are stored or retrieved.

The total throughput of the lift is the sum of all ULs which are stored and retrieved.

$$\lambda_{Lift} = \frac{3600}{E(DC_{Lift})} \cdot 2 \tag{E.3}$$

The number of ULs which are stored on average in one hour is only half of the total throughput. The same applies to the number of retrievals per hour.

$$\lambda_{Lift} = \frac{3600}{E(DC_{Lift})} \tag{E.4}$$

For the use case in chapter 3.3.4

The lift performs storage or retrieval operations:

$$\lambda_{Lift} = \frac{3600}{E(MC_{Lift})} \cdot c_{lift} \tag{E.5}$$

For the use case in chapter 3.3.5

The lift performs storage or retrieval operations:

$$\lambda_{Lift} = \frac{3600}{E(MC_{TwoSizesOfUL\_Lift})} \cdot \left(\left(\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}} \right) \cdot 1 + \left(\frac{\frac{P_{small}}{2}}{\frac{P_{small}}{2}+1-P_{small}} \right) \cdot 2 \right) \tag{E.6}$$

In this case, the throughput of the individual command cycles must be weighted with the probability of occurrence. In the case of a single-command cycle, only one UL is processed. In the case of a dual-command cycle, two ULs are processed.

For the use case in chapter 3.3.6

The lift performs storage or retrieval operations.

For the case of $c_{lift} = 4$:

$$\lambda_{Lift} = \frac{3600}{E(MC_{TwoSizesOfUL\_Lift})} \cdot \left(\left(\frac{\frac{P_{small}}{2}}{\frac{P_{small}}{2}+1-P_{small}} \right) \cdot \left(\frac{\frac{P_{small}}{2}}{\frac{P_{small}}{2}+1-P_{small}} \right) \cdot 4 + 2 \cdot \left(\frac{\frac{P_{small}}{2}}{\frac{P_{small}}{2}+1-P_{small}} \right) \cdot \left(\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}} \right) \cdot 3 + \left(\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}} \right) \cdot \left(\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}} \right) \cdot 2 \right) \tag{E.7}$$

For the use case in chapter 3.4.1

The shuttle vehicle performs storage and retrieval operations. If only storage or retrieval operations are performed, the throughput can be calculated using the given equation.

$$\lambda_{Shuttle} = \frac{3600}{E(SC_{Shuttle})} \tag{E.8}$$

For the use case in chapter 3.4.2

A balanced system is assumed. This means, that storage and retrieval operations are carried out with equal frequency. If the shuttle vehicle performs combined storage and retrieval operations the following equation can be used to calculate the throughput:

$$\lambda_{Shuttle} = \frac{3600}{E(DC_{Shuttle})} \cdot 2 \tag{E.9}$$

For the use case in chapter 3.4.3

The shuttle vehicle performs combined storage and retrieval operations:

$$\lambda_{Shuttle} = \frac{3600}{E(MC_{Shuttle})} \cdot 2c_s \tag{E.10}$$

For the use case in chapter 3.4.4

The shuttle vehicle performs storage or retrieval operations.

Throughput for the case of storage operations:

$$\lambda_{Shuttle} = \frac{3600}{E(SC_{Shuttle\_Storage})} \tag{E.11}$$

Throughput for the case of retrieval operations:

$$\lambda_{Shuttle} = \frac{3600}{E(SC_{Shuttle\_Retrieval})} \tag{E.12}$$

For the use case in chapter 3.4.5

The shuttle vehicle performs combined storage and retrieval operations:

$$\lambda_{Shuttle} = \frac{3600}{E(DC_{Shuttle})} \cdot 2 \tag{E.13}$$

For the use case in chapter 3.4.6

The shuttle vehicle performs combined storage and retrieval operations:

$$\lambda_{Shuttle} = \frac{3600}{E(MC_{Shuttle})} \cdot 2c_s \tag{E.14}$$

For the use case in chapter 3.4.7

The shuttle vehicle performs combined storage and retrieval operations.

In this case, the throughput of the individual command cycles must be weighted with the probability of occurrence.

$$\lambda_{Shuttle} = \frac{3600}{E\left(MC_{TwoSizesOfUL\_Shuttle\_optimizedSequencing}\right)} \cdot \left(\left(\frac{\frac{P_{small}}{2}}{\frac{P_{small}}{2}+1-P_{small}}\right) \cdot \left(\frac{\frac{P_{small}}{2}}{\frac{P_{small}}{2}+1-P_{small}}\right) \cdot 4 + 2 \cdot \left(\frac{\frac{P_{small}}{2}}{\frac{P_{small}}{2}+1-P_{small}}\right) \cdot \left(\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}}\right) \cdot 3 + \left(\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}}\right) \cdot \left(\frac{1-P_{small}}{\frac{P_{small}}{2}+1-P_{small}}\right) \cdot 2\right) \tag{E.15}$$

F Simulation Studies – Preliminary Experiments

To define the required warm-up phase, the number of replications and the impact of different seed values, some experiments using the AnyLogic simulation model are carried out.

F.1 Warm-up Phase and Number of Replications

At the start of the simulation, the SBS/RS is randomly filled based on the expected value of the filling degree and the percentage of small ULs. Thus, the system is already in a steady state after a few storage and retrieval operations. Since the computing time is not very high, the warm-up phase with 10,000 storage and retrieval operations was selected. After 10,000 storage and retrieval operations, the system is considered to be settled, and all preceding measurements are discarded.

The number of replications is determined using the confidence interval. The 95 % confidence interval is calculated based on the Student t-distribution (see Arnold and Furmans 2019, p. 100 ff.). If this interval shows less than 1 % deviation from the mean, the simulation is terminated. Based on preliminary simulation runs with different configurations, it has been determined that the settling phase consists of 10,000 storage and retrieval operations, and an additional 100,000 storage and retrieval operations are required for replications.

F.2 Different Seed Values

The seed value is used to initialize the random number generator in the AnyLogic model. Depending on the seed value, the random number varies. This also changes the sequence in which the storage and retrieval requests are processed. This experiment evaluates the impact of different seed values.

The same simulation was run with 50 different seed values (1 to 50). To determine the deviation between the individual simulation runs, the 95 % confidence interval for the cycle time of the inbound lift, outbound lift and shuttle vehicle was determined (see Table F.1).

Table F.1: Results for the seed value experiment

	Inbound lift	**Outbound lift**	**Shuttle vehicle**
Mean cycle time [s]	16.6655	16.6655	78.5255
EBM - 95 % confidence interval simulation [s]	0.0044	0.0044	0.0294

The results show that the error bound for the mean of the 95 % confidence interval for different seed values is very small. For this reason, the simulation experiment will be performed for one seed value only in this work.

G Results of Simulation Experiment

In this chapter, the waiting times for the performed simulation runs from chapter 5.2 are listed.

G.1 Values of Experiment 1.2

Table G.1: Results of experiment 1.2, waiting times – single-deep, one-behind-the-other lift, side-by-side shuttle vehicle, random tier - random storage location, storage and retrieval requests are processed after FCFS, shuttle vehicle starts the storage process only fully loaded

Input			Waiting times						
Lift capacity [capacity/lift]	**Shuttle vehicle capacity [capacity/shuttle]**	**Buffer capacity [capacity/buffer]**	**Inbound lift waits for storage request [s]**	**Inbound lift waits in front of inbound buffer [s]**	**Shuttle vehicle waits for storage request [s]**	**Shuttle vehicle waits for retrieval request [s]**	**Shuttle vehicle waits in front of outbound buffer [s]**	**Outbound lift waits for retrieval request [s]**	**Outbound lift waits in front of pre-storage zone [s]**
2	**2**	**1**	0.0000	0.1655	497.64	0.0000	0.0063	2.3533	0.0000
2	**2**	**2**	0.0000	0.0056	493.28	0.0000	0.0030	2.1891	0.0000
2	**2**	**3**	0.0000	0.0000	493.63	0.0000	0.0000	2.1802	0.0000
2	**2**	**4**	0.0000	0.0000	493.63	0.0000	0.0000	2.1802	0.0000
2	**2**	**5**	0.0000	0.1655	497.64	0.0000	0.0063	2.3533	0.0000

G.2 Values of Experiment 2.2

Table G.2: Results of experiment 2.2, waiting times – double-deep, side-by-side lift, one-behind-the-other shuttle vehicle, random tier - random storage location, storage and retrieval requests are processed after FCFS, shuttle vehicle starts the storage process not fully loaded

Input			Waiting times						
Lift capacity [capacity/lift]	**Shuttle vehicle capacity [capacity/shuttle]**	**Buffer capacity [capacity/buffer]**	**Inbound lift waits for storage request [s]**	**Inbound lift waits in front of inbound buffer [s]**	**Shuttle vehicle waits for storage request [s]**	**Shuttle vehicle waits for retrieval request [s]**	**Shuttle vehicle waits in front of outbound buffer [s]**	**Outbound lift waits for retrieval request [s]**	**Outbound lift waits in front of pre-storage zone [s]**
2	**2**	**1**	0.0000	3.8258	206.55	0.0000	0.0612	1.6746	0.0000
2	**2**	**2**	0.0000	0.2751	173.37	0.0000	1.2096	0.0934	0.0000
2	**2**	**3**	0.0000	0.0991	163.40	0.0000	12.706	0.0000	0.0000
2	**2**	**4**	0.0000	0.0740	162.65	0.0000	14.930	0.0000	0.0000
2	**2**	**5**	0.0000	0.0661	162.96	0.0000	14.582	0.0000	0.0000

G.3 Values of Experiment 2.3

Table G.3: Results of experiment 2.3, waiting times – double-deep, side-by-side lift, one-behind-the-other shuttle vehicle, random tier - random storage location, storage and retrieval requests are processed after FCFS, shuttle vehicle starts the storage process only fully loaded

Input			**Waiting times**						
Lift capacity [capacity/lift]	**Shuttle vehicle capacity [capacity/shuttle]**	**Buffer capacity [capacity/buffer]**	**Inbound lift waits for storage request [s]**	**Inbound lift waits in front of inbound buffer [s]**	**Shuttle vehicle waits for storage request [s]**	**Shuttle vehicle waits for retrieval request [s]**	**Shuttle vehicle waits in front of outbound buffer [s]**	**Outbound lift waits for retrieval request [s]**	**Outbound lift waits in front of pre-storage zone [s]**
2	**2**	**1**	-	-	-	-	-	-	-
2	**2**	**2**	0.0000	0.4446	381.28	0.0000	0.0040	4.2379	0.0000
2	**2**	**3**	0.0000	0.0258	370.82	0.0000	0.1230	3.8563	0.0000
2	**2**	**4**	0.0000	0.0034	371.53	0.0000	0.0000	3.6620	0.0000
2	**2**	**5**	0.0000	0.0000	370.58	0.0000	0.0000	4.0885	0.0000

G.4 Values of Experiment 2.4

Table G.4: Results of experiment 2.4, waiting times – double-deep, side-by-side lift, one-behind-the-other shuttle vehicle, random tier - random storage location, batch two small ULs and store them together on the same tier, storage and retrieval requests are processed after FCFS, shuttle vehicle starts the storage process only fully loaded

Input			Waiting times						
Lift capacity [capacity/lift]	**Shuttle vehicle capacity [capacity/shuttle]**	**Buffer capacity [capacity/buffer]**	**Inbound lift waits for storage request [s]**	**Inbound lift waits in front of inbound buffer [s]**	**Shuttle vehicle waits for storage request [s]**	**Shuttle vehicle waits for retrieval request [s]**	**Shuttle vehicle waits in front of outbound buffer [s]**	**Outbound lift waits for retrieval request [s]**	**Outbound lift waits in front of pre-storage zone [s]**
2	**2**	**1**	-	-	-	-	-	-	-
2	**2**	**2**	0.0000	2.3685	280.35	0.0000	0.0008	2.3676	0.0000
2	**2**	**3**	0.0000	2.3076	278.82	0.0000	0.0006	2.3079	0.0000
2	**2**	**4**	0.0000	0.3169	229.00	0.0000	0.0000	0.3177	0.0000
2	**2**	**5**	0.0000	0.2726	227.32	0.0000	0.0000	0.2729	0.0000

G.5 Values of Experiment 3.1

Table G.5: Results of experiment 3.1, waiting times – double-deep, small and large ULs, side-by-side lift, one-behind-the-other shuttle vehicle, random storage location, batch two small ULs and store them together on the same tier, storage and retrieval requests are batched, shuttle vehicle starts the storage process only fully loaded

Input			**Waiting times**						
Lift capacity [capacity/lift]	**Shuttle vehicle capacity [capacity/shuttle]**	**Buffer capacity [capacity/buffer]**	**Inbound lift waits for storage request [s]**	**Inbound lift waits in front of inbound buffer [s]**	**Shuttle vehicle waits for storage request [s]**	**Shuttle vehicle waits for retrieval request [s]**	**Shuttle vehicle waits in front of outbound buffer [s]**	**Outbound lift waits for retrieval request [s]**	**Outbound lift waits in front of pre-storage zone [s]**
2	**2**	**1**	0.0000	1.5318	277.12	0.0000	0.0361	1.5313	0.0000
2	**2**	**2**	0.0000	0.1885	243.93	0.0000	0.0000	0.1880	0.0000
2	**2**	**3**	0.0000	0.0153	239.74	0.0000	0.0000	0.0156	0.0000
2	**2**	**4**	0.0000	0.0017	239.58	0.0000	0.0000	0.0022	0.0000
2	**2**	**5**	0.0000	0.0000	239.59	0.0000	0.0000	0.0001	0.0000

G.6 Values of Experiment 3.2

Table G.6: Results of experiment 3.2, waiting times – double-deep, small and large ULs, side-by-side lift, one-behind-the-other shuttle vehicle, random storage location, batch two small ULs and store them together on the same tier, storage and retrieval requests are batched, shuttle vehicle starts the storage process only fully loaded

Input			Waiting times						
Lift capacity [capacity/lift]	Shuttle vehicle capacity [capacity/shuttle]	Buffer capacity [capacity/buffer]	Inbound lift waits for storage request [s]	Inbound lift waits in front of inbound buffer [s]	Shuttle vehicle waits for storage request [s]	Shuttle vehicle waits for retrieval request [s]	Shuttle vehicle waits in front of outbound buffer [s]	Outbound lift waits for retrieval request [s]	Outbound lift waits in front of pre-storage zone [s]
2	**2**	**1**	0.0000	1.5868	277.48	0.0000	0.0353	1.5859	0.0000
2	**2**	**2**	0.0000	0.1521	244.99	0.0000	0.0000	0.1525	0.0000
2	**2**	**3**	0.0000	0.0345	242.51	0.0000	0.0000	0.0348	0.0000
2	**2**	**4**	0.0000	0.0083	241.12	0.0000	0.0000	0.0091	0.0000
2	**2**	**5**	0.0000	0.0000	239.46	0.0000	0.0000	0.0004	0.0000

G.7 Values of Experiment 3.3

Table G.7: Results of experiment 3.3, waiting times – double-deep, small and large ULs, side-by-side lift, one-behind-the-other shuttle vehicle, random tier - random storage location, storage requests are batched, retrieval requests are batched, if possible shuttle vehicle starts the storage process only fully loaded

Input			Waiting times						
Lift capacity [capacity/lift]	**Shuttle vehicle capacity [capacity/shuttle]**	**Buffer capacity [capacity/buffer]**	**Inbound lift waits for storage request [s]**	**Inbound lift waits in front of inbound buffer [s]**	**Shuttle vehicle waits for storage request [s]**	**Shuttle vehicle waits for retrieval request [s]**	**Shuttle vehicle waits in front of outbound buffer [s]**	**Outbound lift waits for retrieval request [s]**	**Outbound lift waits in front of pre-storage zone [s]**
2	**2**	**1**	0.0000	4.0132	372.87	0.0000	0.0428	4.9345	0.0000
2	**2**	**2**	0.0000	0.2469	287.14	0.0000	0.0000	1.5000	0.0000
2	**2**	**3**	0.0000	0.0138	282.00	0.0000	0.0000	1.2875	0.0000
2	**2**	**4**	0.0000	0.0001	281.39	0.0000	0.0000	1.2538	0.0000
2	**2**	**5**	0.0000	0.0000	280.52	0.0000	0.0000	1.2284	0.0000

G.8 Values of Experiment 3.4

Table G.8: Results of experiment 3.4, waiting times – double-deep, small and large ULs, side-by-side lift, one-behind-the-other shuttle vehicle, random tier - random storage location, storage and retrieval requests are processed after FCFS, if possible shuttle vehicle starts the storage process only fully loaded

Input			Waiting times						
Lift capacity [capacity/lift]	**Shuttle vehicle capacity [capacity/shuttle]**	**Buffer capacity [capacity/buffer]**	**Inbound lift waits for storage request [s]**	**Inbound lift waits in front of inbound buffer [s]**	**Shuttle vehicle waits for storage request [s]**	**Shuttle vehicle waits for retrieval request [s]**	**Shuttle vehicle waits in front of outbound buffer [s]**	**Outbound lift waits for retrieval request [s]**	**Outbound lift waits in front of pre-storage zone [s]**
	2	**1**	0.0000	3.3637	382.90	0.0000	0.0341	5.2957	0.0000
2	**2**	**2**	0.0000	0.2255	303.65	0.0000	0.0000	2.1469	0.0000
2	**2**	**3**	0.0000	0.0070	297.82	0.0000	0.0000	1.9254	0.0000
2	**2**	**4**	0.0000	0.0000	298.69	0.0000	0.0000	1.9373	0.0000
2	**2**	**5**	0.0000	0.0000	298.69	0.0000	0.0000	1.9373	0.0000

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Abbreviations

Notation	Description
3D	Three Dimensional
AS/RS	Automated Storage and Retrieval System
CA	Cross Aisle
EBM	Error Bound for the Mean
ER	Entity Relationship
FCFS	First Come First Served
GA	Genetic Algorithm
GH	Greedy Heuristics
H	Heuristic
I/O	Input/Output
IP	Integer Programming
KPI	Key Performance Indicator
LAP	Linear Assignment Problem
LIFO	Last In First Out
MIP	Mixed Integer Programming
MLS	Multi-Level Shuttle
NN	Nearest Neighbor
POSC	Point of Service Completion
r	Relocation
R	Retrieval
RIO	Return to I/O Point
S	Storage
SA	Simulated Annealing
SBS/RS	Shuttle-Based Storage and Retrieval System
SV	Satellite Vehicle
SysML	Systems Modeling Language
TC	Tier-Captive
TS	Tabu Search
TT	Tier-to-Tier
UL	Unit Load
UML	Unified Modelling Language
VNS	Variable Neighbor Search

Glossary

Notation	Description
a	Acceleration and deceleration
A	Set of all storage locations of the considered storage system
$\|A\|$	Total number of storage locations
a_{lift}	Acceleration and deceleration of the lift
$a_{l_{in}}$	Acceleration and deceleration of the inbound lift
$a_{l_{out}}$	Acceleration and deceleration of the outbound lift
a_s	Acceleration and deceleration of the shuttle vehicle
a_1	Acceleration and deceleration
a_2	Acceleration and deceleration
$Category\_a$	The category of the storage location $a \in A$
$Category\_r$	The category of retrieval request $r \in R$
$Category\_s$	The category of storage request $s \in S$
$Column\_a$	The channel of the storage location $a \in A$
$Column\_r$	The channel of retrieval request $r \in R$
$c_{b_{in}}$	Capacity of the inbound buffer
$c_{b_{out}}$	Capacity of the outbound buffer
c_{lift}	Capacity of the lift
$c_{l_{in}}$	Capacity of the inbound lift
$c_{l_{out}}$	Capacity of the outbound lift
c_s	Capacity of the shuttle vehicle
d_{ab}	Travel distance between storage location a and storage location b
d_{ax}	Dimension of one block of the aisle in x-direction
d_{ay}	Dimension of one block of the aisle in y-direction
d_{az}	Width of the aisle in z-direction
d_{bs}	Distance between the I/O point of the buffer to the middle of the first storage channel in x-direction
d_{bx}	Dimension of the buffer in x-direction
d_{bz}	Dimension of the buffer in z-direction
d_{b0}	Travel distance between storage location b and the I/O point
$d_{I/Oy}$	Distance from the ground floor to the first tier in y-direction

$d_{l_{in}}$	In y-direction, the distance between the lowest tier and the loading point between the pre-storage zone and the inbound lift
$d_{l_{out}}$	In y-direction, the distance between the lowest tier and the loading point between the pre-storage zone and the outbound lift
d_{lx}	Dimension of the lift in x-direction
d_{lz}	Dimension of the lift in z-direction
d_{sx}	Dimension of one storage location in x-direction
d_{sy}	Height of one tier
d_{sz}	Dimension of one storage location in z-direction
d_{0a}	Travel distance between the I/O point and storage location a
d_1	Distance between the I/O point level and the tier below
$E(DC_{Lift})$	Average cycle time of a dual-command cycle for the lift
$E(DC_{Shuttle})$	Average cycle time of a dual-command cycle for the shuttle vehicle
$E(H_{liftMax})$	Average maximum travel distance for multi-command cycles for the lift
$E(L_{relocation})$	Expected relocation channel distance
$E\left(L_{\substack{relocation\\ OneSideNew}}\right)$	Expected relocation channel distance for the relocation process on one side, for the case that small and large ULs are stored
$E\left(L_{\substack{relocation\\ BothSidesNew}}\right)$	Expected relocation channel distance for the relocation process on both sides, for the case that small and large ULs are stored
$E(MC_{Lift})$	Average cycle time of a multi-command cycle for the lift
$E\left(MC_{\substack{Lift\_FCFS\_\\ sequencing}}\right)$	Average cycle time of a multi-command cycle for the lift with FCFS sequencing
$E\left(MC_{\substack{Lift\_optimized\_\\ sequencing\_\\ approximation}}\right)$	Average cycle time of a multi-command cycle for the lift with optimized sequencing - approximation
$E\left(MC_{\substack{Lift\_optimized\_\\ sequencing\_\\ exact}}\right)$	Average cycle time of a multi-command cycle for the lift with optimized sequencing – exact approximation

$E(MC_{Lift\_Strategy1})$	Average cycle time of a multi-command cycle for the lift for Strategy 1
$E(MC_{Shuttle})$	Average cycle time of a multi-command cycle for the shuttle vehicle
$E(MC_{TwoSizesOfUL\_Lift})$	Average cycle time of a multi-command cycle with two different sizes of ULs for the lift
$E(MC_{TwoSizesOfUL\_Lift\_Random})$	Average cycle time of a multi-command cycle with two different sizes of ULs for the lift under random control policy
$E(MC_{TwoSizesOfUL\_Lift\_Strategy1})$	Average cycle time of a multi-command cycle with two different sizes of ULs for the lift for Strategy 1
$E(MC_{TwoSizesOfUL\_Lift\_Strategy1\_New})$	Average cycle time of a multi-command cycle with two different sizes of ULs for the lift for Strategy 1 – extended equation
$E(MC_{TwoSizesOfUL\_Lift\_OptimizedSequencing})$	Average cycle time of a multi-command cycle with two different sizes of ULs for the lift with optimized sequencing
$E(MC_{TwoSizesOfUL\_Shuttle\_OptimizedSequencing})$	Average cycle time of a multi-command cycle with two different sizes of ULs for the shuttle vehicle with optimized sequencing
$E(n_{acceleration\_kt})$	Average number of acceleration operations
$E(n_{load\_k})$	Average number of loading operations
$E(n_{tMax\_kt})$	Average value of the maximum of each combination – Average maximum tier
$E(n_{unload\_kt})$	Average number of unloading operations
$E(SC_{Lift})$	Average cycle time of a single-command cycle for the lift
$E(SC_{Shuttle})$	Average cycle time of a single-command cycle for the shuttle vehicle
$E(SC_{Shuttle\_Retrieval})$	Average cycle time of a single-command cycle for a retrieval operation of the shuttle vehicle in a double-deep storage system
$E(SC_{Shuttle\_Storage})$	Average cycle time of a single-command cycle for a storage operation of the shuttle vehicle in a double-deep storage system

$e_{s,k}$	Binary decision variable to determine storage sequence order
$E(t)$	Average travel time
$E(t_{DC\_Lift})$	Average travel time of a dual-command cycle for the lift
$E(t_{DC\_Shuttle})$	Average travel time of a dual-command cycle for the shuttle vehicle
$E(t_{InboundLiftBusy})$	Expected time in which the inbound lift is busy
$E\left(t_{\substack{InboundLift\\CycleTime}}\right)$	Expected cycle time of the inbound lift
$E(t_{InboundLiftIdle})$	Expected time in which the inbound lift is idle
$E\left(t_{\substack{InboundLiftWait\\ForStorageRequest}}\right)$	Waiting time of the inbound lift for a storage request
$E\left(t_{\substack{InboundLiftWait\\InfrontOf\\InboundBuffer}}\right)$	Waiting time of the inbound lift in front of the inbound buffer
$E(t_{OutboundLiftBusy})$	Expected time in which in outbound lift is busy
$E\left(t_{\substack{OutboundLift\\CycleTime}}\right)$	Expected cycle time of the outbound lift
$E(t_{OutboundLiftIdle})$	Expected time in which the outbound lift is idle
$E\left(t_{\substack{OutboundLiftWait\\ForRetrievalRequest}}\right)$	Waiting time of the outbound lift for retrieval requests
$E\left(t_{\substack{OutboundLiftWait\\InfrontOf\\PreStorageZone}}\right)$	Waiting time of the outbound lift in front of the pre-storage zone
$E(t_{relocation})$	Average cycle time for one relocation operation
$E(t_{relocationBothSides})$	Average cycle time for one relocation operation for the relocation process on both sides, for the case that small and large ULs are stored
$E\left(t_{\substack{relocationBothSides\\New}}\right)$	Average cycle time for one relocation operation for the relocation process on both sides, for the case that small and large ULs are stored
$E(t_{relocationOneSide})$	Average cycle time for one relocation operation for the relocation process on one side, for the case that small and large ULs are stored

$E(t_{relocationOneSideNew})$	Average cycle time for one relocation operation for the relocation process on one side, for the case that small and large ULs are stored
$E(t_{SC\_Lift})$	Average travel time of a single-command cycle for the lift
$E(t_{SC\_Shuttle})$	Average travel time of a single-command cycle for the shuttle vehicle
$E(t_{ShuttleVehicleBusy})$	Expected time in which the shuttle vehicle is busy
$E\left(t_{\substack{ShuttleVehicle\\CycleTime}}\right)$	Expected cycle time of the shuttle vehicle
$E(t_{ShuttleVehicleIdle})$	Expected time in which the shuttle vehicle is idle
$E\left(t_{\substack{ShuttleVehicleWaits\\ForRetrievalRequest}}\right)$	Waiting time of the shuttle vehicle for retrieval requests
$E\left(t_{\substack{ShuttleVehicleWaits\\ForStorageRequest}}\right)$	Waiting time of the shuttle vehicle for storage requests
$E\left(t_{\substack{ShuttleVehicleWaits\\InFrontOf\\OutboundBuffer}}\right)$	Waiting time of the shuttle vehicle in front of the outbound buffer
$E(\#InboundLiftULs)$	Number of ULs that are transported per cycle by the inbound lift
$E(\#Outbound\ LiftULs)$	Number of ULs that are transported per cycle by the outbound lift
$E(\#ShuttleVehicleULs)$	Number of ULs that are transported per cycle by the shuttle vehicle
H	Total height
H_{lift}	Total height of the lift – maximum travel distance of the lift
ID_a	The ID of the storage location $a \in A$
ID_r	The ID of retrieval request $r \in R$
ID_s	The ID of storage request $s \in S$
k	Size of the command cycles
$\lambda_{AllShuttleVehicles}$	Expected throughput of the all shuttle vehicles in one aisle
$\lambda_{InboundLift}$	Expected throughput of the inbound lift
λ_{Lift}	Expected throughput of the inbound lift
$\lambda_{OutboundLift}$	Expected throughput of the outbound lift
$\lambda_{SBS/RS}$	Expected throughput of the SBS/RS

$\lambda_{ShuttleVehicle}$	Expected throughput of the shuttle vehicle
$\lambda_{ShuttleVehicleRetrieval}$	Expected retrieval throughput of the shuttle vehicle
$\lambda_{ShuttleVehicleStorage}$	Expected storage throughput of the shuttle vehicle
l	Travel distance
L	Total length
L	Set of processed positions in one command cycle
L_{Rack}	Total length of the rack
$l_x \subset L$	Storage requests are processed on position {1, 2} of each command cycle, l_x is a proper subset of L
$l_y \subset L$	Retrieval requests are processed on position {4, 6} of each command cycle, l_y is a proper subset of L
$l_r \subset L$	Relocation operation are processed on position {0, 3, 5} of each command cycle, l_r is a proper subset of L
N	Set of UL categories
n_{aisles}	Total number of aisles
$n_{buffers}$	Total number of buffers
n_c	Total number of storage channels (one tier, one side)
$n_{channels}$	Total number of storage channels
$n_{I/O}$	The tier number on which level the I/O point between the pre-storage zone and the lift is
n_{lifts}	Total number of lifts
n_{load}	Number of load handling times
n_r	Total number of rows in one storage channel
n_{rows}	Total number of rows in one storage channel
$n_{shuttles}$	Total number of shuttles
n_t	Total number of tiers
n_{tiers}	Total number of tiers
n_1	Number of tiers below the I/O point
m	Number of operation cycles (number of command cycles) for each block
$P_{empty}(z)$	Probability of an empty storage channel
$P_{empty}(z_{forSmallULs})$	Probability of an empty storage channel, for the case that small and large ULs are stored
$P_{emptyNew}(z)$	Probability of an empty storage channel, for calculating the number of relocation operations before storage

$P_{full}(z)$	Probability of a full storage channel, for the case that small and large ULs are stored
$P_{fullNew}(z)$	Probability of a full storage channel
$P_{full}(z_{forSmallULs})$	Probability of a full storage channel, for the case that small and large ULs are stored
$P_{halffull}(z)$	Probability of a half-full storage channel
$P_{halffull}(z_{forSmallULs})$	Probability of a half-full storage channel, for the case that small and large ULs are stored
$P_{R1}(z)$	Probability of retrieving an UL from a full storage channel – front row
$P_{R2}(z)$	Probability of retrieving an UL from a half-full storage channel
$P_{R2}(z_{forSmallULs})$	Probability of retrieving an UL from a half-full storage channel, for the case that small and large ULs are stored
$P_{R3}(z)$	Probability of retrieving an UL from the back row of a full storage channel and relocating of the blocking UL into the front row of a half-full storage channel
$P_{R4}(z)$	Probability of retrieving an UL from the back row of a full storage channel and relocating of the blocking UL into the back row of an empty storage channel
$P_{relocation}(z)$	Probability of a required relocation operation
$P_{relocation}(z_{forSmall\,ULs})$	Probability of a required relocation operation, for the case that small and large ULs are stored
$P_{S1}(z)$	Probability of storing an UL in an empty storage channel
$P_{S1}(z_{forSmallULs})$	Probability of storing an UL in an empty storage channel, for the case that small and large ULs are stored.
$P_{S2}(z)$	Probability of storing an UL in a half-full storage channel
$P_{S2}(z_{forSmallULs})$	Probability of storing an UL in a half-full storage channel, for the case that small and large ULs are stored.
P_{small}	Percentage of small ULs
R	Set of retrieval requests

$r_{a(p,w),l,k,1}$	Binary decision variable to determine, if a relocation is executed and to get the relocation sequence order – relocation retrieval operation
$r_{a(p,w),l,k,2}$	Binary decision variable to determine if a relocation is executed and to get the relocation sequence order – relocation storage operation
Row_a	The row of the storage location $a \in A$
Row_r	The row of retrieval request $r \in R$
S	Set of storage requests
$SCategory\_n$	Number of ULs of category $n \in N$ in the current set of storage requests
$Size\_a$	The size of the storage location $a \in A$
$Size\_r$	The size of the UL of retrieval request $r \in R$
$Size\_s$	The size of the UL of storage request $s \in S$
$State\_a$	The state of the storage location (1 for occupied and 0 for empty)
$t_{DC\_Lift\_Min}$	Minimum travel time of the lift for a dual-command cycle
$t_{DC\_Lift\_Max}$	Maximum travel time of the lift for a dual-command cycle
$t_{d_{ij}}$	Travel time between tier i and tier j
$t_{d_{0i}}$	Travel time between the I/O point and tier i
$t_{d_{j0}}$	Travel distance between tier j and the I/O point
t_l	Total travel time
t_{load}	Load handling time, transfer time, pick up time between pre-storage zone – lift and lift – buffer
t_n	Load handling time number n
$t_{SC\_Lift\_Min}$	Minimum travel time of the lift for a single-command cycle
$t_{SC\_Lift\_Max}$	Maximum travel time of the lift for a single-command cycle
t_{total}	Total load handling time
t_{unload}	Load handling time, transfer time, drop off time between pre-storage zone – lift and lift – buffer
$t_{(un)load}$	Load handling between shuttle vehicle and storage location
$t_{(un)loadSB}$	Load handling time between shuttle vehicle and buffer

$t_{(un)loadback}$	Load handling time, transfer time, drop off time, pick up time between shuttle vehicle – storage location (back row)
$t_{(un)loadfront}$	Load handling time, transfer time, drop off time, pick up time between shuttle vehicle – storage location (front row)
t_0	Dead time
t_1	Load handling time, transfer time, drop off time, pick up time between pre-storage zone – lift and lift – buffer
t_2	Load handling time, transfer time, drop off time, pick up time between shuttle vehicle – storage location (front row)
t_3	Load handling time, transfer time, drop off time, pick up time between shuttle vehicle – storage location (back row)
v	Maximum velocity
v_{lift}	Maximum velocity of the lift
$v_{l_{in}}$	Maximum velocity of the inbound lift
$v_{l_{out}}$	Maximum velocity of the outbound lift
v_{lift}	Maximum velocity
v_{max}	Maximum velocity of the shuttle vehicle
$x_{a(p,w),l,k}$	Binary decision variable to determine storage sequence order
$x_{bothside}$	Factor for the number of relocations on both sides
$x_{oneside}$	Factor for the number of relocations on one side
$y_{a(p,w)l,k}$	Binary decision variable to determine retrieval sequence order
z	Filling degree
$z_{a(p,w),l,k}$	Binary decision variable to get the state of every storage location (occupied or empty)
$z_{forSmallULs}$	Filling degree for small ULs, for the case that small and large ULs are stored

List of Publications

Azka, A., Milushev, V., Fischer, G., Mittwollen, M. and Padhy, M. (2021). Shuttle-Systeme: Dynamik und Wechselwirkung auf den Regalbau. Logistics Journal, Proceedings (ISSN: 2192-9084), Volume 2021, Issue 17, 1–8.

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