A Simplified Approach for Calculating Heat Transfer Coefficients for Fluid Guiding Elements with Alternating Redirections of Flow

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Fluid guiding elements (FGEs), additive manufactured inserts that enhance heat transfer, have shown the potential to reduce the size of pipe-in-pipe heat exchangers up to a factor of 20. Due to their unique design, the calculation of the geometrical parameters for a specific application remains challenging and was initially solved by computationally expensive computational fluid dynamics simulations. This work presents a simplified approach, which treats the fluid and the FGE as pseudo-homogeneous to enable the fast calculation of effective heat transfer coefficients. The approach is developed based on water in laminar flow state and subsequently tested with different fluids.

Keywords: Fluid guiding elements, Heat exchanger, Heat transfer coefficients, Modeling

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1 Introduction

Temperature control of fluids is a critical aspect of many industrial processes, as it can have a significant impact on the product quality and yield in many applications ranging from bulk chemicals and polymer processing to food and beverage production. Having control over process temperatures is especially important in chemical reaction engineering as thermal runaways can have serious consequences and may even cause safety hazards. This is a challenge in reactor design, as for tubular reactors, the reaction volume scales with the second power of the diameter ($V \propto d^2$), while the area available for heat transfer only has a linear correlation with the diameter ($A \propto d$). When scaling up a reactor technology, this often leads to challenges as unwanted radial temperature gradients can arise.

Our additive manufactured fluid guiding elements (FGEs) (Fig. 1) tackle this challenge by dividing the fluid flow into multiple partial flows and alternately guiding these partial flows to the wall, where the fluid can be heated or cooled [1]. These thin-walled inserts can be retroactively installed in existing tube-and-shell heat exchangers to improve their thermal efficiency or filled or coated with catalyst for reaction applications [2–4]. By altering their geometry, the heat flux into the fluid can be modified deliberately. This means that their heat transport properties can be adjusted for a given heat source to optimize the temperature profile to match the ideal trajectory as close as possible.

Until now, computationally expensive computational fluid dynamics (CFD) calculations were necessary to benchmark the thermal performance for a given geometry. This makes optimizing the geometry for a specific scenario a time-consuming task. In this work, we present a simplified approach to calculate an effective heat transport coefficient to enable the transition from iterative to knowledge-based design.

2 Methodology

2.1 Functional Principle of Fluid Guiding Elements

Fluid guiding elements are additive manufactured pipe inserts, which divide a fluid into multiple partial flows. At each redirection, two partial flows change their position as the upper fluid is guided below the separating metal wall, and the lower fluid emerges from underneath. These redirections are arranged alternately so that each partial flow travels from the center of the pipe to its outer wall and back again as illustrated in Fig. 2. By guiding the flow this way, heat can be transported into the fluid not only by conduction but also by convection via mass transport. As the surface-to-volume ratio of empty pipes diminishes with increasing diameters, the heat transfer into the fluid

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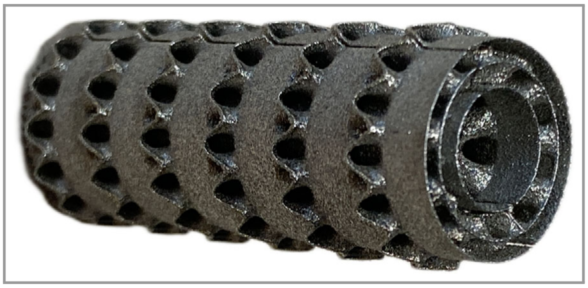

Figure 1. Fluid guiding element manufactured via selective laser melting.

becomes more and more limited by the internal heat conduction of the fluid itself. FGEs can help to overcome this issue, but as a trade-off, the pressure drop increases. To minimize the pressure drop as much as possible, an effort was taken to reduce the wall thickness to about 180 µm while maintaining sufficient mechanical stability [1].

2.2 Approach

Due to the swapping of the different partial flows, any two-dimensional model of a FGE cannot be continuous, since swapping the position of two streams without overlapping pathways would always require a third dimension. With an initial value problem, this is not a problem since an explicit term for each derivative is available (see Eq. (3)) and no iteration is needed. When modeling a counter-flow heat exchanger however, the problem becomes a boundary value problem (BVP), which needs to be solved iteratively. Both explicit and implicit methods were tested to solve the system of differential equations, but it was not possible to find a solution for the BVP. Therefore, a simpler model was needed which treats the FGE as pseudo-homogeneous, so that a one-dimensional approach could be used. When modeling in one dimension only, correlations are needed for an effective heat transfer coefficient which approximates the real three-dimensional behavior of the heat transfer in the FGE. Correlations for such heat transfer coefficients can be found in the literature for different heat transfer problems, e.g., empty pipes, annular gaps, fixed beds, or monoliths. Since our additive manufactured FGEs are not yet widely in use, currently no such correlation exists for FGE.

To find an empirical correlation between the geometrical properties of an FGE, the fluid and the effective heat transfer coefficient, many data points with varying geometric parameters and operating conditions are needed. Obtaining this data experimentally takes a long time, which is why a two-dimensional model was developed. This model is continuous in axial direction and discrete in radial direction with one variable for the temperature of each partial flow. The results from calculating different geometric variations were then used to compare them with a simplified one-dimensional model. The difference between both models was minimized using the least-squares approach by introducing a factor f to an existing correlation for the heat transfer in annular gaps. The obtained correction factor f was then compared with a dimensionless number χ, which can be calculated explicitly and includes parameters about the fluid and the geometry of the FGE. A distinct correlation was found, and an empirical correction was fitted to this correlation. The simplified model was then validated using experimental data from an earlier publication [1].

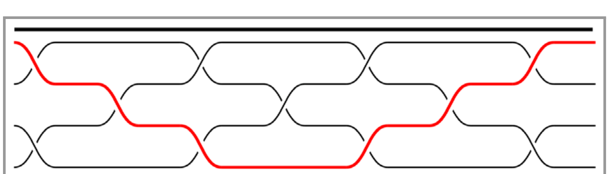

Figure 2. Flow path emerging by alternately swapping partial flows.

2.3 Two-Dimensional Model

To simulate the heat transfer in FGEs in two dimensions, the following assumptions were made:
- steady-state and isobaric operation
- constant wall temperature T_w
- no heat losses
- no radial gradients inside each partial flow
- instant swapping of two partial flows

With the length of the FGE L, the distance between swapping two flows a, and the number of partial flows N, it is possible to determine the radial position of each partial flow after each swap Eq. (1):

$$\mathbf{M} = \begin{bmatrix} 1 & 2 & 2 & 4 & 4 & \dots \\ 2 & 1 & 4 & 2 & 3 & \dots \\ 3 & 4 & 1 & 3 & 2 & \dots \\ 4 & 3 & 3 & 1 & 1 & \dots \end{bmatrix} \tag{1}$$

The axial index i for each axial coordinate z can be calculated using Eq. (2):

$$i(z) = \left\lfloor \frac{z}{a} \right\rfloor \tag{2}$$

The radial position of each partial flow k can be determined by calculating $i(z)$ and then looking up its position in the respective column of the matrix M. The row index of the matching entry equals its radial position index j. Using this technique, it becomes possible to set up the energy balance between the partial flow k with its inner and outer neighbor with the indices $M_{i,j+1}$ and $M_{i,j-1}$, respectively.

$$\frac{dT_k(z)}{dz} = -\frac{(kA)_{\text{inner}}\left(T_k(z) - T_{M_{j+1,i}}(z)\right) + (kA)_{\text{outer}}\left(T_k(z) - T_{M_{j-1,i}}(z)\right)}{\dot{C}_k} \tag{3}$$

If $(j-1)$ is 0, the temperature $T_{M_{j-1,i}}(z)$ is replaced by the constant wall temperature T_w. If $(j+1)$ is larger than the dimension of the matrix, this partial flow is at the center of the tube and therefore has no inner neighbor, so the temperature $T_{M_{j+1,i}}(z)$ is set to the value of $T_k(z)$ to zero the inner heat flux.

The integration of the resulting system of N differential equations is done in MATLAB using the variable order method solver ode15s [5]. Since the heat transfer through the metal separating both streams is several orders of magnitude higher than the heat transfer into the fluid, it can be neglected:

$$kA = \left(\frac{1}{(\alpha_{\mathrm{AG}}A)_{\mathrm{inner}}} + \frac{1}{(\alpha_{\mathrm{AG}}A)_{\mathrm{outer}}} \right)^{-1} \tag{4}$$

The heat transfer coefficient α is calculated using correlation from the VDI Wärmeatlas (section G2, [6]) for annular gaps:

$$\mathrm{Nu}_{m,L} = \frac{\alpha_{AG} d_h}{\lambda} \tag{5a}$$

$$\mathrm{Nu}_{m,L} = \left(\mathrm{Nu}_1^3 + \mathrm{Nu}_2^3 + \mathrm{Nu}_3^3 \right)^{\frac{1}{3}} \tag{5b}$$

$$\mathrm{Nu}_1 = 3.66 + \left[4 - \frac{0.102}{\left(\frac{d_i}{d_a}\right) + 0.02} \right] \left(\frac{d_i}{d_a} \right)^{0.04} \tag{5c}$$

$$\mathrm{Nu}_2 = f_g \left(\mathrm{Re}\ \mathrm{Pr}\ \frac{d_h}{l} \right)^{\frac{1}{3}} \tag{5d}$$

$$\mathrm{Nu}_3 = \left\{ \frac{2}{1 + 22\ \mathrm{Pr}} \right\}^{\frac{1}{6}} \left(\mathrm{Re}\ \mathrm{Pr} \frac{d_h}{l} \right)^{\frac{1}{2}} \tag{5e}$$

The calculation of the auxiliary values d_h and f_g as well as the dimensionless numbers Re and Pr are given in the VDI Wärmeatlas [6].

The dataset used for developing the empirical correlation consists of 100 datapoints. For each simulation, a set of parameters was chosen with a Sobol sequence to ensure evenly distributed, quasi-random parameters. The bounds for each parameter are tabulated in Tab. 1 and are chosen such that FGE heat ratios χ in the whole interval [0 1] are represented (Sect. 2.5, Eq. (14)). The resulting Reynolds numbers in the outer annular gap were in a range between 40 and 230 for this dataset, so only the laminar flow state is investigated.

Table 1. Parameter space for dataset.

Partial flows N [–]	2–5
Pipe diameter d [mm]	20–50
Length L [cm]	30–80
Swapping distance a [mm]	5–200
Flow rate $\dot{m}$ [kg h^{-1}]	10–30

2.4 One-Dimensional Model

To reduce the complexity to one dimension, an additional assumption was made:

– The temperature mixing effect of FGEs achieved by alternately swapping and guiding the partial flows to the wall is high enough, so that radial temperature gradients can be neglected.

If this assumption is true, a mean temperature can be assumed over the radial coordinate. Since the only heat transfer then occurs at the outer wall, the heat transfer coefficient can be calculated by using correlations for annular gaps [6]. Since the assumption is most likely not correct for most scenarios, an empirical correction is necessary to compensate for the deviation caused by radial temperature gradients.

Constant Wall Temperature: To enable a comparison between the two-dimensional model, a corresponding one-dimensional model was implemented with the following differential equation:

$$\frac{dT(z)}{dz} = \frac{\alpha_{\mathrm{FGE}} \pi d (T_{\mathrm{wall}} - T(z))}{\dot{C}} \tag{6}$$

We define the heat transfer coefficient for the FGEs α_{FGE} as the heat transfer coefficient for the outer annular gap of the FGE α_{AG} multiplied by a correction factor f:

$$\alpha_{\mathrm{FGE}} = f \alpha_{\mathrm{AG}} \tag{7}$$

The integration is done in MATLAB using the solver ode45 for non-stiff differential equations [5]. The correction factor f was determined for each datapoint so that the mean outlet temperature of the two-dimensional model equals the outlet temperature of the one-dimensional model. This was done using a least-squares approach with lsqnonlin [5].

Counter-flow: Since all datapoints for fitting the empirical correlation are obtained via simulations, it is important to validate the derived correlation with experimental data. For this, we chose data from an earlier work where a tube-in-tube heat exchanger was investigated [1]. To model the tube-in-tube heat exchanger with a one-dimensional model, the following differential equations were used:

$$\frac{dT_{\mathrm{core}}(z)}{dz} = \frac{\dot{q}(z)}{\dot{C}_{\mathrm{core}}} \tag{8}$$

$$\frac{dT_{\mathrm{shell}}(z)}{dz} = \frac{\dot{q}(z)}{\dot{C}_{\mathrm{shell}}} \tag{9}$$

$$\dot{q}(z) = k \pi d (T_{\mathrm{core}}(z) - T_{\mathrm{shell}}(z)) \tag{10}$$

$$k = \left(\frac{1}{\alpha_{\mathrm{shell}}} + \frac{1}{\alpha_{\mathrm{core}}} \right)^{-1} \tag{11}$$

For the constant inlet temperature of both core and shell, a Dirichlet boundary condition was used:

$$T_{\text{core}}(z) = T_{\text{core,in}} \quad \text{for } z = 0 \tag{12a}$$

$$T_{\text{shell}}(z) = T_{\text{shell,in}} \quad \text{for } z = L \tag{12b}$$

Both outlets were assumed to be perfectly insulated, so a Neumann boundary condition was chosen:

$$\frac{dT_{\text{core}}(z)}{dz} = 0 \quad \text{for } z = L \tag{13a}$$

$$\frac{dT_{\text{shell}}(z)}{dz} = 0 \quad \text{for } z = 0 \tag{13b}$$

The MATLAB solver bvp4c was used to compute the boundary value problem [5].

2.5 Characteristic Number for Fluid Guiding Elements

The purpose of fitting the correction factor f to the data is to see if a characteristic number χ can be found which correlates with the correction factor. This characteristic number should be easy to calculate and incorporate the properties of the FGE as well as those of the fluid itself. We propose that χ is defined as the ratio between the actual heat transfer in the outer channel between two swaps (distance a) and the maximum heat transfer which would be reached if the fluid temperature equals the wall temperature. This ratio can be calculated using the following equation:

$$\chi = 1 - \exp\left(\frac{a\alpha_{\text{AG}}\pi DN}{\dot{C}}\right) \tag{14}$$

This equation contains the most important parameters of the FGE which are the distance between two swaps a, the number of partial flows N and the outer diameter D as well as the heat capacity stream $\dot{C}$ of the fluid. The heat transfer coefficient α_{AG} can be calculated using established correlations [6].

3 Results and Discussion

3.1 Two-Dimensional Model

An example of the output of the two-dimensional model is shown in Fig. 3. At the inlet on the left side, water enters with a temperature of 95 °C and is cooled by a wall with a constant temperature of 10 °C. Each partial flow is guided from the center of the pipe to the cool outer wall, so that a rising and falling temperature profile emerges for each partial flow.

The mean fluid temperature at the outlet is 23.8 °C. If an empty pipe with the same parameters of the example shown in Fig. 3 would be used, the water would only be cooled

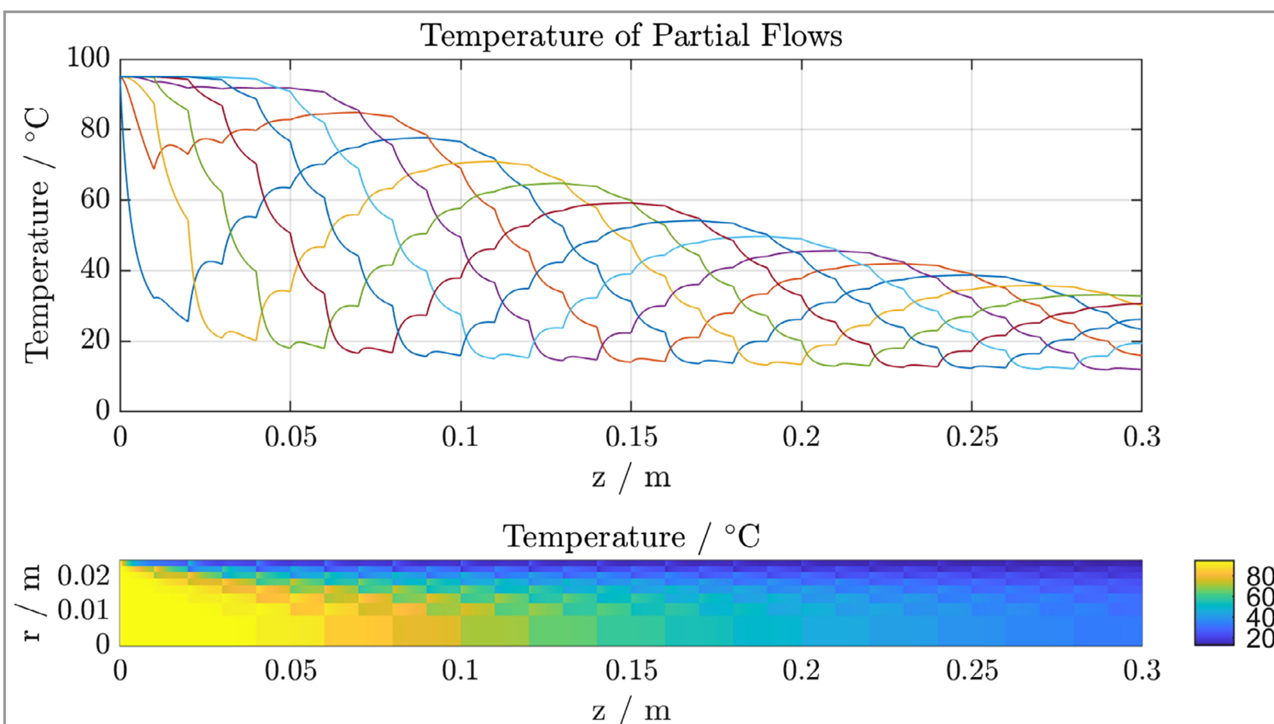

Figure 3. Temperature distribution in pipe with fluid guiding elements. Calculated using the two-dimensional model. Parameter: d_i = 5 cm, N = 8, a = 1 cm, L = 30 cm.

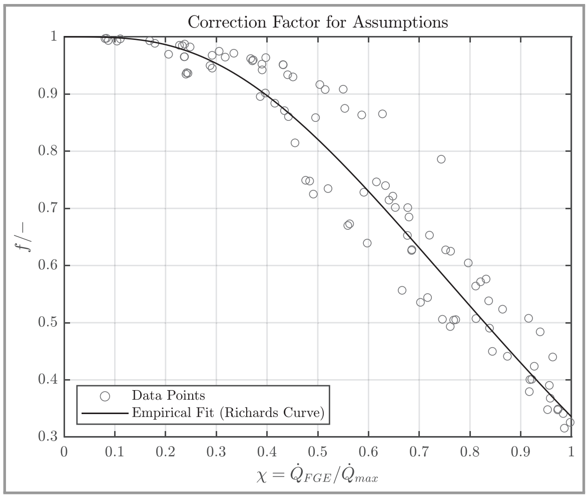

Figure 4. Correction factor f as a function of characteristic number χ.

to a temperature of 61.5 °C. To reach the same outlet temperature which FGE can achieve in 30 cm, the length of an empty pipe would need to be about 108 cm long. These comparisons have been calculated by replacing α_{FGE} with the heat transfer coefficient for empty pipes with the same dimensions as described in [6].

3.2 Comparison and Empirical Correlation

After determining the necessary correction factor to match the outlet temperature of both models, it is possible to plot the correction factor of each datapoint $f(\chi)$ as a function of the characteristic number χ. It is noticeable that f is close to 1 when χ is close to 0. This is expected, since $\chi = 0$ means that the swapping of the partial flows is extremely quick in relation to the heat transfer into the fluid. This means that the assumption formulated in Sect. 2.4 is nearly true because the frequent swapping achieves a good temperature mixing effect, and therefore little to no correction should be necessary. If χ approaches 1 however, the heat transfer between the outer channel and the wall is much higher than the internal heat transfer of the FGE, and therefore radial temperature gradients arise, which results in a larger deviation from the assumption (Fig. 4).

The scattered data follows a trend which resembles a generalized logistic function, also called a Richards curve. After eliminating parameters which are statistically irrelevant for this scenario, the function is defined as follows:

$$f(\chi) = 1 + \frac{A-1}{\left(1-\exp(-\chi B)\right)^{1/C}} \tag{15}$$

The parameters obtained by fitting the curve to the datapoints are presented in Tab. 2.

In Fig. 5, the differences between the inlet and outlet temperature for all 100 datapoints calculated by the one- and two-dimensional models are compared after applying the empirical correction. Nearly all points are within ±10 % deviation of the two-dimensional model which is a promising result considering the simple correction in combination with the wide range of parameters in Tab. 1.

To check if the empirical correlation which was derived based on calculations with water leads to reasonable results with different fluids, the simulation was repeated with 20 datapoints and 2 other fluids with different properties (Tab. 3). The results are shown in Fig. 6, and again the differences between the two-dimensional and simplified models were below ±10 % deviation in most simulations.

Table 2. Parameter for empirical function.

A	−0.3361
B	1.7913
C	−0.2613

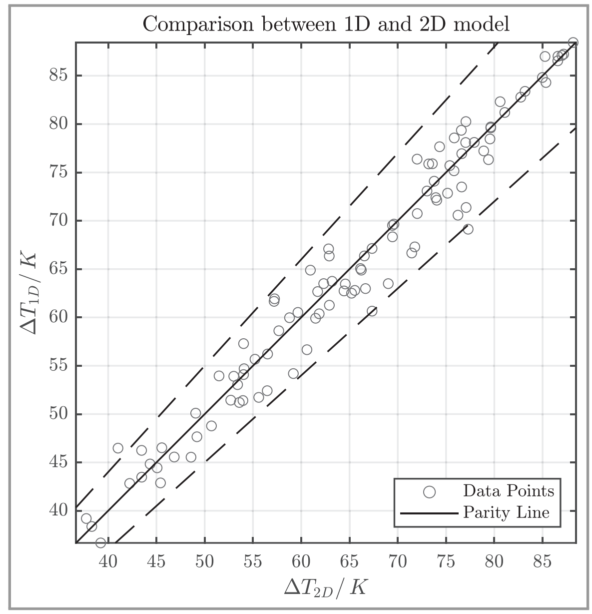

Figure 5. Comparison between the inlet and outlet temperature calculated by the two-dimensional model and simplified pseudo-homogeneous approach.

Table 3. Properties of additional fluids at inlet temperature of 5 °C.

	Water	n-Pentane	Ethylene glycol
Viscosity ν [mPa s]	1.51	0.25	44.09
Heat capacity c_p [kJ kg^{-1} K^{-1}]	4.19	2.23	2.31
Thermal conductivity λ [W m^{-1} K^{-1}]	0.58	0.12	0.25

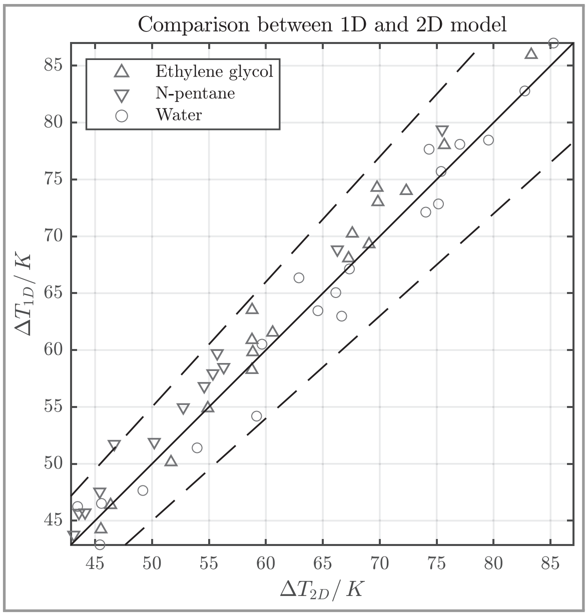

Figure 6. Comparison between the inlet and outlet temperature calculated by the two-dimensional model and simplified pseudo-homogeneous approach with additional fluids.

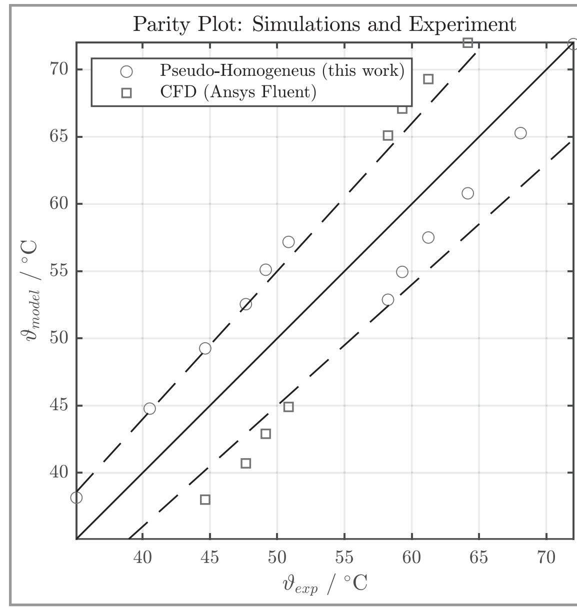

Figure 7. Validation of simplified approach via comparison with experimental data. Additional CFD data taken from previous works [1].

3.3 Validation

Up until this part, we used more complex simulations to find an empirical correlation for simpler simulations with our pseudo-homogeneous approach. After obtaining such correlation, it is important to validate the model by comparing it with experimental data to ensure it is usable for real applications. For this, we used data from an earlier publication [1] and calculated both outlet temperatures using the model described in Sect. 2.4. The geometric dimensions for the experimental setup are given in the appendix (Tab. A1). In total, six mass flow rates between 5 and 30 kg h^{-1} were investigated (Fig. 7).

It was possible to achieve a mean deviation of 4.1 K, while computationally more expensive calculations done in Ansys Fluent reached a mean deviation 6.8 K. In the original paper, the authors attributed these deviations to heat losses in the experimental setup. By calculating the heat flux which should have been transferred from one fluid to the other, it is possible to confirm this hypothesis, as for the lowest mass flow rate, a difference of about 5 % between the experimental and calculated heat flux is observed, which can be accounted to heat losses. As more experimental data becomes available, further refinement of the model will be possible.

Other than heat losses, two other assumptions might lead to inconsistencies between the simulation and experiments. In Sect. 2.3, it was assumed that two partial flows change their position instantly which is not the case in the experiments where this happens over a length of 2–3 mm, which is not insignificant given the total length between two swaps of 6.5 mm. Furthermore, the model neglects possible radial temperature gradients in the partial flows, which would affect the heat transfer between two partial flows. To investigate these effects, a more detailed CFD simulation becomes necessary.

On this account, it must be noted that the simulations done in Ansys Fluent provide greater insight into the heat and mass transport phenomena inside the FGE structures. This simplified model therefore does not aim to replace the CFD model but rather to try out different geometrical parameters quickly and easily without having to model, mesh and calculate numerous FGEs. This way, it becomes possible to only select the most promising configurations for a more detailed simulation and therefore greatly reduce the effort needed to design FGEs for novel applications.

4 Conclusions

By utilizing a pseudo-homogeneous approach, the design of heat exchangers with FGEs can be accelerated significantly. Possible practical applications are, e.g., retroactively installing them in existing shell and tube heat exchangers to improve their thermal performance, integrating them into flow reactors to even out the radial temperature profile and lower the hotspot temperature or to increase the diameter of the tubes of shell-and-tube heat exchangers to reduce the number of tubes needed. Since this method only calculates a mean temperature over the radius, it is important to verify the design of the FGEs with locally resolved CFD calculations if a minimum or maximum temperature must not be exceeded. This simplified approach also enables the simulation of numerically challenging counter-flow reaction–diffusion systems. This application is particularly interesting because the speed of chemical reactions is temperature dependent. By altering the geometry of the FGEs to achieve a certain desired heat transfer coefficient, the ratio of reaction heat versus heat dissipation can be set precisely to optimize the reaction trajectory.

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Appendix

The parameters of the experimental setup used to validate the approach developed in this work are given in Tab. A1.

Table A1. Parameters of experimental setup [1].

	Length L [mm]	288
Core	Diameter d [mm]	10
	Distance between swaps a [mm]	6.5
	Number of partial flows N [–]	4
	Inlet temperature ϑ [°C]	90
Shell	Inner diameter d_i [mm]	12
	Outer diameter d_o [mm]	15.6
	Distance between swaps a [mm]	6.5
	Number of partial flows N [–]	3
	Inlet temperature ϑ [°C]	20

Symbols used

A	[m^2]	Area
a	[m]	Distance between swapping of partial flows
$\dot{C}$	[W K^{-1}]	Heat capacity stream
c_p	[J kg^{-1} K^{-1}]	Specific heat capacity
d	[m]	Diameter
f	[–]	Empirical correction factor
k	[W m^{-2} K^{-1}]	Overall heat transfer coefficient
L	[m]	Length
$\dot{m}$	[kg s^{-1}]	Mass flow rate
N	[–]	Number of partial flows
$\dot{Q}$	[W]	Heat duty
$\dot{q}$	[W m^{-1}]	Heat duty per length
r	[m]	Radial coordinate
T	[K]	Temperature
V	[m^3]	Volume
z	[m]	Axial coordinate

Greek letters

α	[W m^{-2} K^{-1}]	Heat transfer coefficient
λ	[W m^{-1} K^{-1}]	Thermal conductivity
υ	[Pa s]	Viscosity
ϑ	[°C]	Temperature
χ	[–]	FGE characteristic number

Sub- and superscripts

i	Axial position index
j	Partial flow index
k	Radial position index

Abbreviations

AG	Annular gap
BVP	Boundary value problem
CFD	Computational fluid dynamics
FGE	Fluid guiding element

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