Toolbox for an Analytical Determination of a Gearbox-Generator-Combination

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***Abstract*—This paper deals with the concept development of a gearbox design for optimum brake energy recovery of a truck trailer with the use of a synchronous reluctance machine. Planetary and or spur gears are selected on the basis of a utility analysis. Based on this, a computer-aided gearbox design tool is developed, which generates an ideal gearbox concept for brake energy recovery on the basis of a motor/generator characteristic and a drive cycle. The optimal gear ratio as well as possible gearbox topologies and gearing are calculated automatically. The possible gearbox topologies are 1- to 3-stage gears consisting of planetary or spur gear stages, and in the case of multi-stage gearboxes, permutations of them. In addition, the configuration of axle drive (one generator with differential) or wheel drive (two generators) can be specified as a starting condition. A joint integration and design process of the gearbox and generator offers many advantages such as compactness, easier handling and faster calculation.**

An optimization criterion is used to select the appropriate gear unit. As a result, the user receives the optimum gearbox topology, the distribution of the total gear ratio among the stages and the gear geometry, as well as further information on mass, costs, installation space and energy yield. Finally, the gear concept synthesis is validated using analytical equations and commercial gear design software, such as KISSsys and KISSsoft.

For this reason, a combination of real driving data determined in VECTO and the analytical calculation of the gearbox generator system is used.

***Index Terms*—Mechanical Power Transmission, Analytical Model, Drive Train**

I. Introduction

Due to the electrification of the powertrain, the combination of motor/generator and gearbox plays an increasingly important role. The current state of the art is to combine two systems that have been designed and optimized independently of each other, so it is not possible to develop an optimized general solution. Depending on the application, the joint development of a gearbox and generator unit has some essential advantages: on the one hand, the load points that are transmitted during the drive can be transmitted more optimally to the generator map. On the other hand, the general efficiency can be optimized to a specific driving cycle.

To describe the design tool in more detail, the use case of an electrified refrigerated truck trailer is considered. A standard rigid axis is fitted with a gearbox, generator, inverter, and battery to make the system suitable for braking energy recovery. A tool chain has already been developed for the overall system in [1]. The essential parameters such as power, torque, losses, consumption, etc. of the vehicle are provided with the help of "Vehicle Energy Consumption calculation TOol" (VECTO) [2]–[5]. With those parameters of the truck and wheels, it is possible to determine the corresponding power at the wheels of the trailer and thus the possible usable energy for braking energy recovery. As braking energy recovery is the main focus of this research, towing operation is not investigated yet.

II. Start- and Boundary Conditions

The following section describes the start- and boundary conditions that are chosen for the example calculations. As already mentioned, VECTO is used for calculating the occurring braking energy recovery and driving losses. For this, an example truck has to be selected. In all the following calculations, a $26\,\mathrm{t}$ truck with a power of $324\,\mathrm{kW}$ diesel engine and a 12-speed automatic transmission with the drive cycle "Urban-Delivery" are simulated. The driving cycle simulated in this way serves as the basis for calculating the potential braking energy points. Table I provides an overview of the most important key data of the driving cycle. In Fig. 1 the equivalent speed graph is plotted.

The considered generators are synchronous Reluctance Machines (synRM) of different topologies, with peak powers

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TABLE I: Overview of drive cycle Urban-Delivery from VECTO

Track length	100 km
Duration	229 min of which 75 min recuperation is possible
Speed	$v_{\text{max}} = 85\,\text{km}\,\text{h}^{-1}$ $v_{\text{avg}} = 26\,\text{km}\,\text{h}^{-1}$
Theoretically available Energy	89 kW h
Energy required	35 kW h (while operating a 9 kW refrigeration unit)

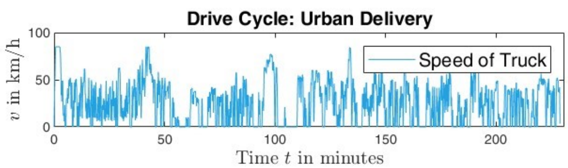


Fig. 1: Truck speed during the drive cycle "Urban Delivery" provided by VECTO

and torques of 60 kW and 185 N m for the system configuration (with axle drive), and 30 kW and 64 N m for the system configuration (with wheel drive).

III. Model

Fig. 2 shows the simplified structure of the design tool that was developed in this paper, based on MATLAB.

At the top level, a drive cycle (given by VECTO) and the torque-speed-efficiency map (T-n-η map) of one or more generators are required as input. Using the drive cycle and the maps, the ideal overall gearbox ratio is first calculated for each generator, the so-called "Total Ratio Optimization" (green block). Based on this, all possible gear topologies are calculated which are suitable for the total gear ratio and the best topology of these is selected – "Topology Calculation" (blue block). If all possible topologies have been calculated, the selection for the best one is made on the basis of an evaluation criterion, which is further described in subsection III-C.

After an ideal gearbox and the equivalent topology have been calculated for each generator, all results are compared again using the same evaluation scheme in an overall evaluation – "Overall Rating" (orange block) and the best overall concept consisting of drive cycle, generator, and gearbox is output.

A. Total Ratio Optimization

Regardless of the gearbox types considered, the ideal total gear ratio must first be determined. For a better overview, Fig. 3 shows a highly simplified representation of how the design tool works.

At first, it must be determined in which points braking energy can be used at all, as in this code, only braking energy is considered. For this purpose, a separate sub-function was written, which loads the energy at the wheel and the corresponding speed from VECTO, and with a comparison

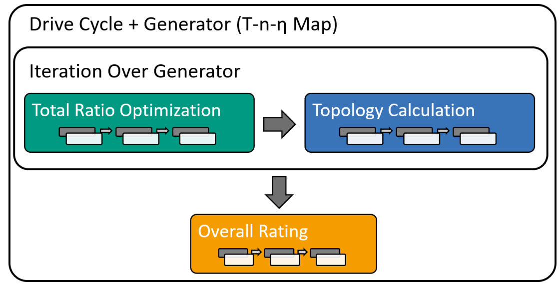


Fig. 2: Explanation of the main script visualized in a process diagram

with the maximum speed, defines the points at which the wheel power is negative (further explanations can be seen in [1]). With these points and the corresponding time step, the potential braking energy can be determined.

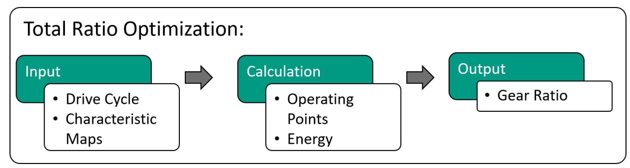


Fig. 3: Total ratio optimization script visualized in a process diagram

The corresponding operating points can be adapted to the power curve via the gear ratio in order to achieve the highest possible peak power/torque [1].

B. Topology Optimization

After executing the ideal total ratio optimization, the structure has to be implemented in the form of a gear topology. Fig. 4 shows a simplified flow chart of the developed code and the structure.

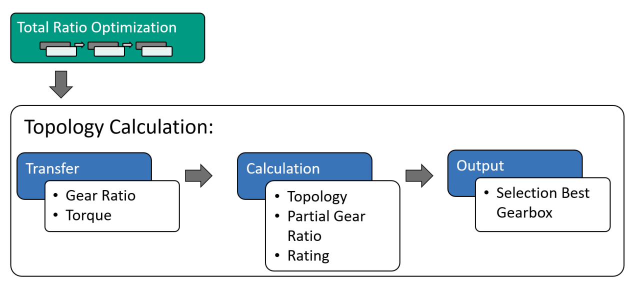


Fig. 4: Topology calculation script visualized in a process diagram

The ideal total ratio $i_{\text{w,ideal}}$ is provided by the optimization, as well as the T-n-η map of the generator. The calculation considers all possible topologies with partial gear ratios which allow the realization of $i_{\text{w,ideal}}$. Once all possible topologies are calculated, they are evaluated according to VDI 2225-3 [6] (see subsection III-C).

This evaluation is then used to select the topology and, from this, the gear unit itself, which meets the criteria of the initial target system best.

In total, there are 14 different possible topologies (see (1) where k is equal to the number of gear stages). A maximum of three stages are calculated for the total gear ratio, since a higher ratio exceeds the practical speed range. It is also possible to distinguish between two system configurations. Either two machines can be used, i.e. one for each wheel (wheel drive), or a central electric generator connected to both wheels via a differential in the final stage (axle drive).

$$n_{\text{topo}} = \sum_{k=1}^{3} 2^k = 14 \tag{1}$$

For the calculation of the gear ratio distribution, for each of the 14 possible topologies all partial gear ratios are considered which lead to total gear ratios which can deviate from $i_{\text{w,ideal}}$ by a threshold value of

If a combination of partial gear ratio is found, the calculation of the corresponding topology is started for this combination.

First, the input torque is split into the individual stages across all possible topologies. Once the torques for each stage are known, the individual stage sizes (tooth geometry, number of teeth etc.) are calculated. For this purpose, standard dimensioning from the literature is used. Once all stages have been calculated, they are combined to form a complete gear unit.

In addition to the topology and gear ratio optimization, the load carrying capacity of the gear must also be checked. For this purpose, the two most common types of failure, surface durability (pitting) and tooth bending strength, are investigated. Here, the fundamentals of [7] and [8] are used, which provide the corresponding basics. The calculation procedure for spur and planetary gears is very similar. For simplification, only the calculation of spur gears is explained in the following. In both cases, the nominal circumferential force F_{t} (2) is required for the design on the basis of the surface durability (pitting) and tooth bending strength. This can be expressed by the pitch diameter d_1 and the torque at the pinion M_1.

$$F_{\text{t}} = \frac{2 \cdot M_1}{d_1} \tag{2}$$

With the aid of the nominal circumferential force, the minimum pitch diameter of the pinion (3) can now be determined on the basis of the tooth bending strength. With a minimum safety factor against tooth bending strength ($S_{\text{F,min}}$), fatigue strength (σ_{FE}) and tooth width (b).

$$d_1 \geq \sqrt[3]{\frac{12 \cdot M_1 \cdot S_{\text{F,min}} \cdot Y_1 \cdot K_1}{\sigma_{\text{FE}} \cdot Y_2 \cdot (\frac{b}{d_1})^2}} \tag{3}$$

Equivalently, for the surface durability (pitting) (4) is given with minimum safety factor against pitting ($S_{\text{H,min}}$), number of teeth ratio (u) and pitting endurance strength ($\sigma_{\text{H,lim}}$).

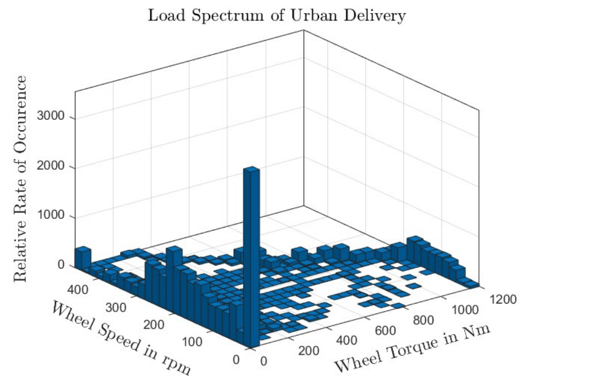


Fig. 5: Visualization of the load spectrum calculated with the Urban-Delivery drive cycle with a $30\,\text{kW}$-map and a total ratio of $i_{\text{w,ideal}} = 26.3$, or approximately $13\,000\,\text{min}^{-1}$ at the generator side.

$$d_1 \geq \sqrt[3]{\frac{(S_{\text{H,min}} \cdot Z_1)^2 \cdot K_2 \cdot 2 \cdot M_1 \cdot (u+1)}{(\sigma_{\text{H,lim}} \cdot Z_2)^2 \cdot b/d_1 \cdot u}} \tag{4}$$

The individual factors and the entire calculation can be seen in [7], [8].

For validation, but also for the further development of a gearbox concept, a load spectrum is required, see Fig. 5. Therefore, as a final step in the design tool, a load spectrum is calculated based on the underlying driving cycle and the map of the synRM. For the calculation, the braking energy recovery points are divided into classes based on their torque and speed. All other operating points of the driving cycle at which recuperation is not possible are included in the classification with their speed, but without any torque load (see white fields in Fig. 5). The values for speed and torque in the respective classes are then averaged. This average value then represents this class. In addition, the time steps of all recuperation points are added up in the respective classes and compared with the duration of the entire driving cycle. This yields the relative frequency with which the respective classes occur. As Fig. 5 shows, most of the points are located at small speeds and torques. Only very rarely, high torque and speed levels are required for the gear unit.

C. *Overall Rating — Evaluation according to VDI 2225-3*

This chapter takes a closer look at the calculation of the technical valence according to the design methodology as given by the VDI 2225-3 [6], which serves as a decision criterion for the selection of the gearbox concepts. The technical valence is a function of the weighted evaluation criteria, scaled according to [6].

The evaluation concept is run twice. In the first run, an ideal gearbox concept is determined separately for each generator ("Topology-Calculation"). Here, the evaluation takes place at the topology level. The aim of the second run is to determine the best combination of generator topology and gearbox concept on the overall system level ("Overall-Rating"). The only difference between the topology level and the overall system level is that the first determines whether the overall gear ratio can be implemented as a single-, two- or three-stage gearbox, while the second is used to choose the optimum system solution.

1) Criteria and Weightings: The following criteria are selected for evaluation in descending order of importance. On the basis of the preceding considerations regarding the individual criteria, a pairwise comparison is carried out, and the resulted weights are also added to each single criteria. The criteria are briefly described in the following:

Energy yield (weight = 13) — The recuperation points have to be optimally mapped to the map of the generator. Every watt-hour that is not recuperated may have to be replaced by towing or recharging. This leads to higher investment costs with regard to the required size of the battery and running costs during operation.

Efficiency (weight = 10) — Likewise, the efficiency can be associated with running costs (power loss), but also with investment costs (cooling circuit).

Mass (weight = 8.5) — A large gearbox mass results in a higher total weight of the truck and thus greater energy consumption. This can also lead to a restriction of the payload.

Volume — Although the volume is related to the mass, it only has an effect on the operating costs if the cargo space is restricted.

Cost (weight = 8) — Complex parts (such as ring gear, planet carrier, differential cage) are usually more expensive to manufacture and are considered separately, since VDI 2225-1 [9] primarily takes into account the masses and the material of the components. Large and small parts in great numbers only lead to increased assembly costs and are therefore considered less important.

2) Rating Scale and Technical Valence: Based on the criteria and weightings g_k presented, the technical valence of the gear units can be determined by the ratio of the achieved score p_k to the maximum possible (ideal) score, p_{max} as shown in (5). The technical valence x_w (subscript w = weighted) ranges from 0 to 1.

$$x_w = \frac{\sum_{k=1}^{j} g_k \cdot p_k}{\sum_{k=1}^{j} g_k \cdot p_{max}} \tag{5}$$

In order for an evaluation of a criterion to be carried out, sensible upper and lower limits must first be found or defined. Normally, in the evaluation according to VDI 2225-3 [6], a category is evaluated on a discrete scale from 0 (unsatisfied) to 4 (ideal), but since numerous gear unit concepts are compared here, a continuous scale is more appropriate.

The total braking energy of the driving cycle can be defined as the upper limit for the energy yield from recuperation and corresponds to the energy that can be drawn from the potential recuperation points [1].

Evaluating the efficiency between the theoretical maximum and minimum is not impossible, but it does not make sense, since the topologies only differ in efficiency by a few percentage points. This would therefore lead to a quasi equal evaluation of all gearboxes. In the case of the topologies considered here, it can be seen that the associated efficiencies must be between 95% and 85% on the basis of the rough estimates and calculations used. Therefore, 95% is given the value 4 and 85% is given the value with 1. For all the other criteria. For the other two criteria (mass and cost), the combinations with the greatest mass or the highest cost were selected as the lower limit (and therefore a value of 1) and a value of 4 was selected for the best combination (cheapest and lightest).

IV. Comparison with the Theoretical Principles and Commercial Software

In order to provide a conclusive assessment of the tool's performance, it is compared to two distinct variants in the subsequent section. Once with the currently known theoretical approach and with a typical (commercial) simulation program.

A. Comparison ratio of gearbox with literature

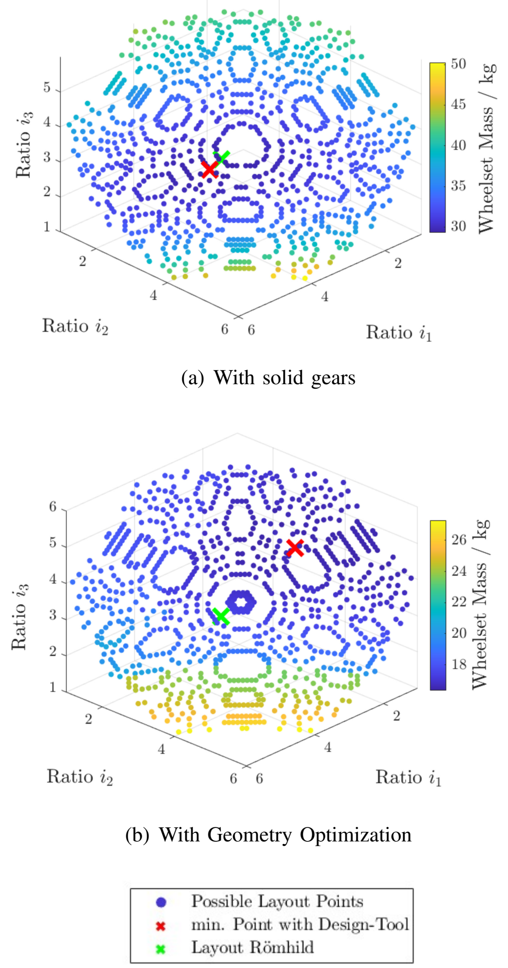
In the publication by Römhild [10], equations are presented which can be used to calculate the minimum wheelset mass analytically. The gear ratio distribution according to Römhild is now compared with the gear ratio distribution, which with the aid of the developed design tool leads to the minimum wheelset mass. For this purpose, the equations, according to Römhild [10], for two and three-stage spur gears, as well as the start and boundary conditions from section II are used as the calculation basis.

Fig. 6(a) shows the comparison of the ideal gear ratio distribution with the calculation results for three-stage spur gears with solid gears. All the calculated points with the design tool are shown in the colormap. The minimum mass calculated with the design tool is marked by a red X, and the calculated mass from [10] is marked by a green X. Since the equations and calculations from [10] are calculated differently, the result points are not on the calculated points of the design tool. [10] uses only solid gears in her calculation. It can be seen that although the results without the geometry optimization are close, they do not match perfectly. The deviations between the values according to Römhild [10] and the calculation results according to the design tool can be explained by the consideration of integer numbers of teeth. The design tool adjusts the total gear ratio i_w compared to the ideal total gear ratio $i_\mathrm{w,ideal}$ in order to obtain integer tooth numbers.

In Fig. 6(b) geometry optimization according to Wittel [11] is implemented, which provides a weight reduction, leads to a significant reduction in the gear set masses. However, in order to maximize the mass reduction, a gear ratio distribution significantly different from Römhild [10] is required, which is found with the help of the developed design tool.

B. Comparison with commercial software

In order to validate the calculated rough design of the gears, the ideal gear concept presented in section II is simulated using commercial software. For this purpose, a load spectrum



(a) With solid gears

(b) With Geometry Optimization

Fig. 6: Comparison of the ideal gear ratio distribution according to the developed design tool and Römhild [10] for three-stage spur gears for minimum wheelset mass, with solid gears and geometry optimization according to Wittel [11].

is calculated (Fig. 5), which is used to simulate the loads on the gearbox over a service life of 600 000 km, which is according to Naunheimer [12] the usual service life of a truck used in urban traffic (stop and go). The commercial software used is KISSsys and KISSsoft (version 2020 D) from KISSsoft AG [13]. KISSsoft is a leading software developer for gearbox calculations and development. For comparison, KISSsoft is used to obtain a suggestion for the coarse layout of the gear geometry. This suggestion is based on the gear ratio distribution of the developed design tool. In addition, KISSsoft can vary the width to diameter ratio $\frac{b}{d_1}$, the ratio of center distance a to tooth width b and the number of teeth of the sun z_s for the rough design. KISSoft also performs a rough sizing, like the developed design tool, using the reference profile according to DIN 867 [14] and the formulas according to ISO 6336-2 [7] and ISO 6336-3 [8]. On this basis, KISSsoft calculates several suggestions for possible gears of the individual stages based on the torque and speed that are entered into the gearbox in KISSsys.

Table II shows the comparison of the results from the rough design of the gearing using KISSsoft and the results of the ideal gear concept from the developed design tool.

It can be seen that both the gear ratio and the number of teeth of both planetary stages match. It also shows that the results differ in the module and the tooth width. The gearing of the KISSsoft rough design, in both stages, has a slightly smaller module and a larger tooth width. This implies that the tooth according to KISSsoft are slightly smaller in diameter, but wider.

In the next step, both rough designs are evaluated with the load spectrum to assess the quantity and quality of the proposed gear parameters. In this way, KISSsys can be used to simulate the load over the life cycle of the gear unit, once with the gearing from the results of the design tool and once according to KISSsoft. This made it possible to determine the safety factors of the gear teeth, in terms of tooth bending strength S_F and pitting S_H. It can be seen that the design of the gears, which was calculated with the help of the developed design tool is of similar quality to the gears designed by KISSsoft. This could also be observed for other gears. The safety factors of both the gears roughly designed by KISSsoft and the gears found using the design tool are lower than the initially required minimum safeties of S_F, min = 1.2 and S_H, min = 1.3. The safety factors determined in this way are summarized in Table III.

This shows that the quality of the rough gear design, which is carried out with the help of the design tool developed in this paper, can be put on a similar level as that of a commercial gear design software and thus provides reliable results. The essential advantage, however, is the speed of the calculation and the possible varieties, since everything is based on analytical equations it's also easy to adapt to other driving cycles, trucks, gearboxes, generator and many more.

V. Conclusion and Future Work

In the course of this work, a tool was developed to investigate the overall concept of a synchronous reluctance machine and gearbox for an electric axle of a truck trailer. The most important point of the target system was to allow a flexible selection of an ideal gearbox concept, based on given synRM-maps and driving cycles. For the simulation, planetary, and spur gear types commonly used in the automotive sector were selected.

Based on a drive cycle from VECTO and several synRM maps, the tool can determine an ideal gear concept for each machine and select the best combination of machine and gear concept from these.

For this purpose, the ISO-6336 [7], [8], [15] was used as a basis for the automatic design of the gears. The manufacturing costs were calculated using VDI 2225-1 [18] and all other values of the gearbox were calculated estimated. An automated evaluation based on VDI 2225-3 [6], [9] was developed for the evaluation of the gearbox concepts. Finally, a validation of the

TABLE II: Comparison of the gear data from the rough design with the developed design tool and KISSsoft

	1. Stage: Developed Design-Tool:	1. Stage: KISSsoft:	2. Stage: Developed Design-Tool:	2. Stage: KISSsoft:
Gear Ratio	$i_1 = 5.6$		$i_2 = 4.5$	
Standard Module	1.68 mm	1.44 mm	2.98 mm	2.60 mm
Tooth Width	17.50 mm	18.00 mm	31.00 mm	32.48 mm
Number of Teeth Sun Gear	16		16	
Number of Teeth Planetary gear	29		20	
Number of Teeth Ring Gear	74		56	

TABLE III: Tooth root and tooth flank safety of the 1st and 2nd stage. Comparison of the developed design tool and KISSsoft, based on the safeties of sun, planet, and ring gear and the minimum over both stages.

		Developed Design-Tool: Sun	Planet	Ring Gear	min.	KISSsoft: Sun	Planet	Ring Gear	min.
1. Stage	S_H	2.84	2.15	4.14	1.06	2.18	1.64	2.17	0.93
	S_F	1.07	1.13	2.53		0.93	1.02	1.50	
2. Stage	S_H	3.52	2.31	3.50		2.68	1.77	4.26	
	S_F	1.06	1.10	2.14		0.98	1.04	2.18	

developed calculation and evaluation methods was carried out using an ideal overall concept found with the design tool, a two-stage planetary gearbox. The validation was based on the comparison with literature values as well as with commercial gear design software. It became apparent that the developed design tool provides reliable results and is significantly more targeted and flexible in its application than the literature values and software considered.

In future research, a possible drag mode will be investigated in addition to pure braking energy recovery. In addition, thermal models of the gearbox can also be studied analytically to obtain more precise results. Furthermore, a simplified calculation of the gearbox efficiency map is an additional and useful function.

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