Evaluation of the Recent African Gravity Databases V2.x

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Abstract

In the framework of the activities of the IAG Sub-Commission on the gravity and geoid in Africa, a recent set of gravity databases has been established. They are namely: AFRGDB_V2.0 and AFRGDB_V2.2. The AFRGDB_V2.0 has been created using the window remove-restore technique employing EGM2008 as geopotential Earth model complete to degree and order 1800. The AFRGDB_V2.2 has been established using the Residual Terrain Model (RTM) reduction technique employing GOCE DIR_R5 complete to degree and order 280, using the best RTM reference surface. The available gravity data set for Africa, used to establish the above mentioned two independently derived databases, consists of shipborne, altimetry derived gravity anomalies and of land point gravity data. In particular, the data set of point gravity values shows clear deficits with regard to a homogeneous data coverage over the completely African continent. The establishment of the gravity databases has been carried-out using the weighted least-squares prediction technique, in which the point gravity data on land has got the highest precision, while the shipborne and altimetry gravity data got a moderate precision. In this paper a new gravity data set on land and on sea, which became recently available for the IAG Sub-Commission on the gravity and geoid in Africa, located partly in the gap areas of the data set used for generating the gravity databases, has been employed to evaluate the accuracy of the previously created gravity databases. The results show reasonable accuracy of the established gravity databases considering the large data gaps in Africa.

Keywords

Africa · Geoid determination · Gravity field · Gravity interpolation · Window technique

1 Introduction

The International Association of Geodesy (IAG) has established, some years ago, the Sub-Commission on the gravity and geoid in Africa. The main task of that Sub-Commission is to determine a precise regional geoid for the continent. In order to achieve its main goal, the IAG Sub-Commission on the gravity and geoid in Africa has established a recent set of gravity databases. This set comprises the AFRGDB_V2.0 (Abd-Elmotaal et al. 2018) and AFRGDB_V2.2 (Abd-Elmotaal et al. 2020). The aim of this investigation is to perform an external validation of

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the above mentioned gravity databases employing a recently available gravity data set. This data set was not used in creating V2.x gravity databases and is located partly in the gap areas of the data.

In the following, the gravity data sets used to establish the recent gravity databases AFRGDB_V2.x will be presented. The different methodologies applied for establishing the AFRGDB_V2.x gravity databases will be described. The recently available gravity data set used for the validation process will be presented. The validation of the AFRGDB_V2.x will be performed and discussed.

2 Data Used for Establishing the AFRGDB_V2.x Gravity Databases

The basis for the creation of a gravity anomaly database across the entire African continent in a homogeneous and comprehensive manner is formed by three complementary data sets. There are three types of data available.

The available land point gravity data, is the most important data set for determining the geoid at the continent. Before they enter the merging scheme, they have to pass a laborious gross-error detection process. This data screening step was developed by Abd-Elmotaal and Kühtreiber (2014) using the least-squares prediction technique (Moritz 1980). During this gross-error detection the gravity anomaly at the computational point is predicted using the neighbouring points and then compared to the measured gravity anomaly value. A possible erroneous measurement is removed from the data if the difference between the measurement and the predicted value exceeds a certain threshold. Afterwards, a grid-filtering scheme (Abd-Elmotaal and Kühtreiber 2014) on a grid of $1' \times 1'$ is applied to the screened land data to improve the behaviour of the empirical covariance function especially near the origin (Kraiger 1988). The statistics of the land free-air gravity anomalies, after the gross-error detection and the grid-filtering, are illustrated in Table 1. The distribution of the available land gravity data set, with its obvious large data gaps, is shown in Fig. 1a.

The gravity data set used to generate AFRGDB_V2.x comprises in addition data over the oceanic region. The goal of the African Geoid Project is the calculation of the geoid on the African continent. Data within the data window which are located on the oceans are used to stabilize the solution at the continental margins to avoid the Gibbs phenomenon. The sea data consists of shipborne point data and altimetry-derived gravity anomalies along tracks. The altimetry-derived data set was derived from the average of 44 repeated cycles of the satellite altimetry mission GEOSAT by the National Geophysical Data Center NGDC (www.ngdc.noaa.gov) (Abd-Elmotaal and Makhloof 2013, 2014). The derived gravity anomalies are given along its ground tracks and have a good spatial coverage as can be realized from Fig. 1c. The distribution of the shipborne data is given in Fig. 1b. The shipborne and altimetry-derived free-air anomalies have passed a gross-error detection scheme developed by Abd-Elmotaal and Makhloof (2013), also based on the least-squares prediction technique. It estimates the gravity anomaly at the computational point utilizing the neighbourhood points, and defines a possible blunder by comparing it to the given data value which is currently being examined for an error. The gross-error technique works in an iterative scheme till it reaches 1.5 mgal or better for the discrepancy between the predicted and data values. A stochastic weighting combination between the shipborne and altimetry data took place (Abd-Elmotaal and Makhloof 2014) in order to merge both data sets into one homogeneous data set. Then a grid-filtering process on a grid of $3' \times 3'$ has been applied to the shipborne and altimetry-derived gravity anomalies to decrease their dominating effect on the gravity data set. The statistics of the shipborne and altimetry-derived free-air anomalies, after the gross-error detection and grid-filtering, are listed in Table 1.

More details about the used data sets can be found in Abd-Elmotaal et al. (2018).

Table 1 Statistics of the gravity anomalies used to generate AFRGDB_V2.x. Units in [mgal]

Data category	No. of points	Statistical parameters			
		Min	Max	Mean	Std.
Land	126,202	−163.20	465.50	9.84	40.93
Shipborne	148,674	−238.30	354.40	−6.21	34.90
Altimetry	70,589	−172.23	156.60	4.09	18.23

3 Methodology for Creating AFRGDB_V2.x

The two gravity databases for the African continent (versions V2.0 and V2.2) which are here evaluated, have been created based on principally different methodologies. In the following subsections the applied methodologies will be shortly described. They mainly differ in the way how the high-frequency part of the gravity anomalies is reduced before a suitable interpolation or prediction technique is applied to get gridded data.

3.1 Methodology for Creating AFRGDB_V2.0

The version V2.0 of AFRGDB relies on the window remove-restore technique which is used to smooth the signal of

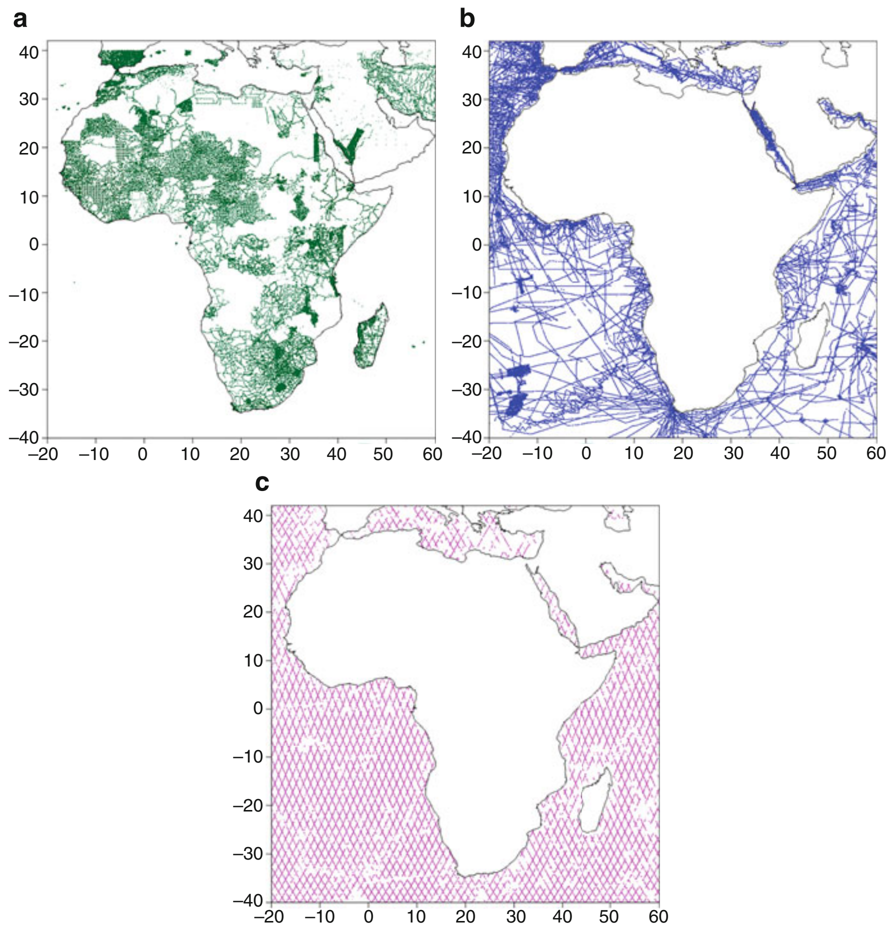


Fig. 1 Distribution of the (**a**) land, (**b**) shipborne and (**c**) altimetry gravity data points for Africa used to generate AFRGDB_V2.x

the gravity attraction and avoids the double consideration of topographical masses. This leads to un-biased reduced anomalies with minimum variance. The window technique, which was introduced by Abd-Elmotaal and Kühtreiber (1999, 2003), consists of a remove and a restore step. When performing the remove step, the measured free-air gravity anomalies Δg_F are decomposed into the contribution of the topographic-isostatic masses for the fixed data window ($\Delta g_{TI\,win}$), the long wavelength component modelled by a global geopotential model (GPM) (Δg_{GPM}), the contribution of the topographic-isostatic masses in terms of spherical harmonics up to d/o n_{max} of the same data window (Δg_{wincof}). The synthesis of Δg_{GPM} and Δg_{wincof} is performed to the maximum degree $n_{max} = 1800$. Furthermore, the EGM2008 geopotential model (Pavlis et al. 2012) is used as the GPM. From this spectral decomposition the window-reduced gravity anomalies can be expressed by Abd-Elmotaal and Kühtreiber (1999, 2003) (cf. Fig. 2)

$$\Delta g_{win\text{-}red} = \Delta g_F - \Delta g_{TI\,win} - \Delta g_{GPM}\Big|_{n=2}^{n_{max}} + \\ + \Delta g_{wincof}\Big|_{n=2}^{n_{max}} . \qquad (1)$$

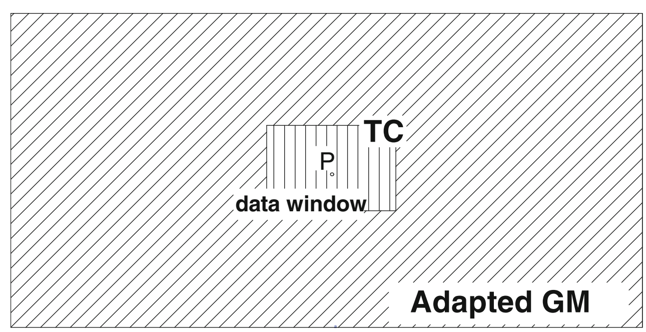


Fig. 2 The window remove-restore technique

The reduced and smoothed gravity anomalies represented in Eq. (1) are point values. They are interpolated on the $5' \times 5'$ target grid covering the geographical window ($-40° \le \phi \le 42°; -20° \le \lambda \le 60°$) of the African continent. The technique used to get the $\Delta g^G_{win\text{-}red}$ is an unequal weight least-squares interpolation technique (Moritz 1980). A smart fitting technique of the empirically determined covariance function by employing a least-squares regression algorithm (Abd-Elmotaal and Kühtreiber 2016) has been implemented in the interpolation process.

The effects which have to be subtracted in the remove-step in order to smooth the point wise given gravity anomalies to improve the interpolation results are added back, but now in the nodes of the equidistant target grid. The applied technique is described in Abd-Elmotaal and Kühtreiber (1999, 2003) and can be formally expressed by

$$\Delta g_F^G = \Delta g_{win\text{-}red}^G + \Delta g_{TI\;win}^G + \Delta g_{EGM2008}^G\Big|_{n=2}^{n_{max}} - \\ - \Delta g_{wincof}^G\Big|_{n=2}^{n_{max}} \; . \qquad (2)$$

The superscript G which is added to the involved values (compare (1) and (2)) indicates the gridded values. Δg_F^G computed by (2) represent the values for the AFRGDB_V2.0 gravity database for Africa. In Abd-Elmotaal et al. (2018) more details about the establishment of AFRGDB_V2.0 can be found.

It is worth mentioning, that the harmonic analysis (Abd-Elmotaal and Kühtreiber 2021; Abd-Elmotaal et al. 2013) of the topographic-isostatic masses needed to compute the term Δg_{wincof} in Eq. (1) is the most time consuming part in the window remove-restore process employed for the creation of the AFRGDB_V2.0 gravity database.

3.2 Methodology for Creating AFRGDB_V2.2

The creation of version V2.2 of the gravity database for Africa is based on the RTM reduction technique, proposed first by Forsberg (1984). The remove step of the modified RTM technique used in the creation of the AFRGDB_V2.2 gravity database, employing the best smoothed DHM as RTM surface, can mathematically be expressed by

$$\Delta g_{RTM\text{-}red} = \Delta g_F - \Delta g_{RTM\;win} - \Delta g_{Dir\_R5}\Big|_{n=2}^{n_{max}} \; , \qquad (3)$$

where $\Delta g_{RTM\text{-}red}$ refers to the RTM-reduced gravity anomalies, Δg_F refers to the measured free-air gravity anomalies, Δg_{Dir_R5} stands for the contribution of the GOCE Dir_R5 global reference geopotential model (Bruinsma et al. 2014). The RTM effect on gravity $\Delta g_{RTM\;win}$ of the topographic masses is computed from a fixed data window. Here $n_{max} = 280$ is the used upper maximum degree. The reduced anomalies are interpolated on a $5' \times 5'$ grid for the African result window using the same technique described in Sect. 3.1 yielding the interpolated gridded reduced anomalies $\Delta g_{RTM\text{-}red}^G$. The restore step for the modified RTM technique used for creating the AFRGDB_V2.2 gravity database for Africa can mathematically be expressed by

$$\Delta g_F^G = \Delta g_{RTM\text{-}red}^G + \Delta g_{RTM\;win}^G + \Delta g_{Dir\_R5}^G\Big|_{n=2}^{n_{max}} \; , \qquad (4)$$

where the superscript G stands again for values computed at the grid points. Δg_F^G computed by (4) represent the values for the AFRGDB_V2.2 gravity database for Africa. More details about the establishment of the AFRGDB_V2.2 gravity database can be found in Abd-Elmotaal et al. (2020).

It should be mentioned, that the required computations to establish the AFRGDB_V2.2 gravity database for Africa described in Sect. 3.2 are fairly faster than the technique used to create the AFRGDB_V2.0 gravity database described in Sect. 3.1.

4 The New Data Set Used for the Validation

A new gravity data set, covering part of the gaps appearing in the AFRGDB_V2.x gravity data (cf. Fig. 1), became recently available for the IAG Sub-Commission on the gravity and geoid in Africa. This gridded gravity data set comprises 27,121 grid points on land and 16,659 grid points on sea. The distribution of the new gravity data is illustrated in Fig. 3. Table 2 gives the statistics of the new gravity anomaly

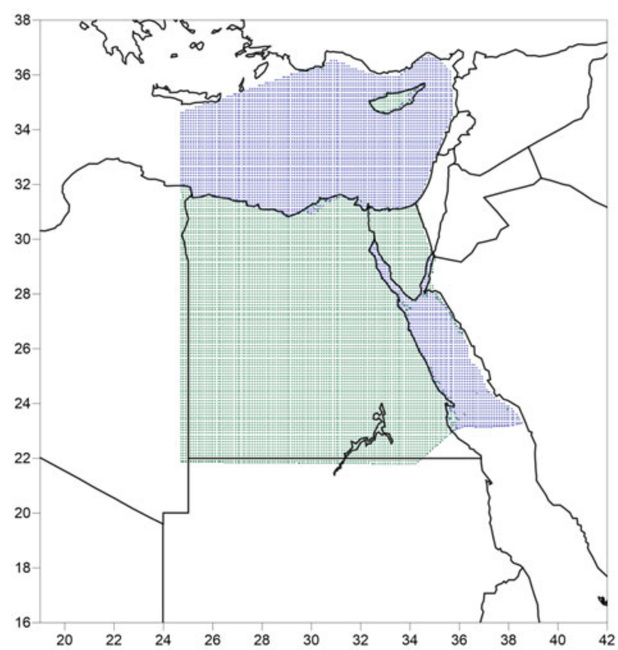


Fig. 3 The distribution of the new gravity data used to evaluate AFRGDB_V2.x (green: land data, blue: sea data)

Table 2 Statistics of the new gravity anomalies used to evaluate AFRGDB_V2.x. Units in [mgal]

Data category	No. of points	Statistical parameters			
		Min	Max	Mean	Std.
Land	27,121	−55.70	350.28	6.70	28.76
Sea	16,659	−210.50	234.49	−37.63	53.52

data. In the validation of AFRGDB_V2.x presented here, the new data is not used for an update of the database. The recent two solutions AFRGDB_V2.0 and AFRGDB_V2.2 are interpolated on the grid of the newly acquired data. The resulting residuals (differences) are used for the validation.

5 Validation of AFRGDB_V2.0 and AFRGDB_V2.2

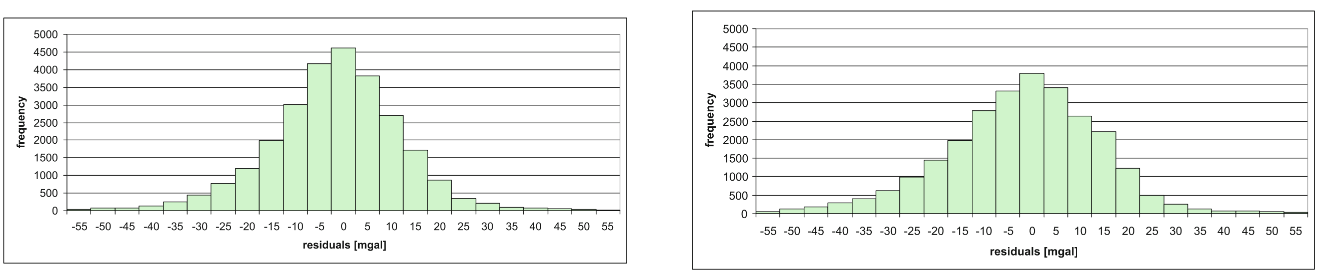
The new gravity data set has been used to evaluate the accuracy of the AFRGDB_V2.x. As can be clearly seen in Fig. 1, the data collected so far show large gaps especially in the north-eastern region of the African continent. With the different methods used to create the AFRGDB_V2.x databases, the influence of this shortcoming should also be reduced. With the new data, a validation can be carried out under unfavorable data conditions.

Figure 4 shows the histogram of the residuals from the difference between the AFRGDB_V2.x and the new land data contained in the new data grid. It can be concluded, that the AFRGDB_V2.0 adjusts better than the AFRGDB_V2.2 because the precision index of the AFRGDB_V2.0 is larger than that of AFRGDB_V2.2.

Figure 5 shows the histogram of the validation of the AFRGDB_V2.x gravity database in respect to the new grid data on sea. Here also, Fig. 5 shows, using the precision index as decision parameter, that the accuracy of AFRGDB_V2.0 is better than that of AFRGDB_V2.2, at least in this region under consideration.

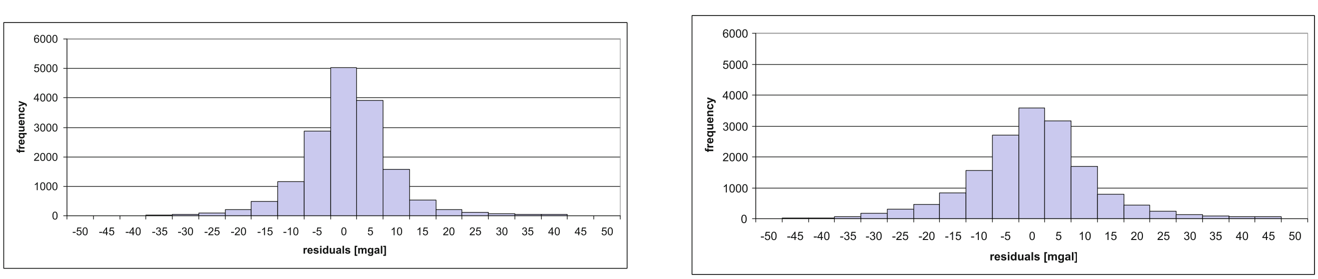
Figure 6 shows the histogram of the validation of the full data for the AFRGDB_V2.x gravity databases. This figure also confirms the previous conclusion that the AFRGDB_V2.0 fits better than the AFRGDB_V2.2 to the new data.

While 68.03% of the new grid points have differences less than 10 mgal for the AFRGDB_V2.0, this holds for 57.66% of the AFRGDB_V2.2. The respective residuals are shown in Fig. 7.



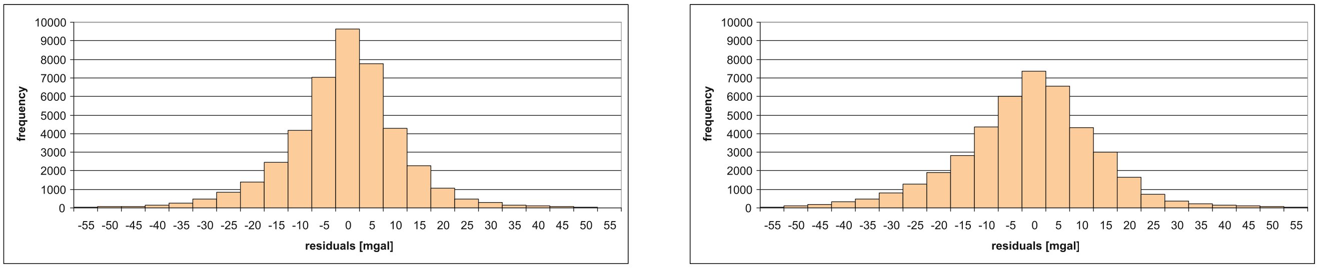
(a) (b)

Fig. 4 Histogram of the validation on land for the (**a**) AFRGDB_V2.0 and (**b**) AFRGDB_V2.2 gravity database for Africa



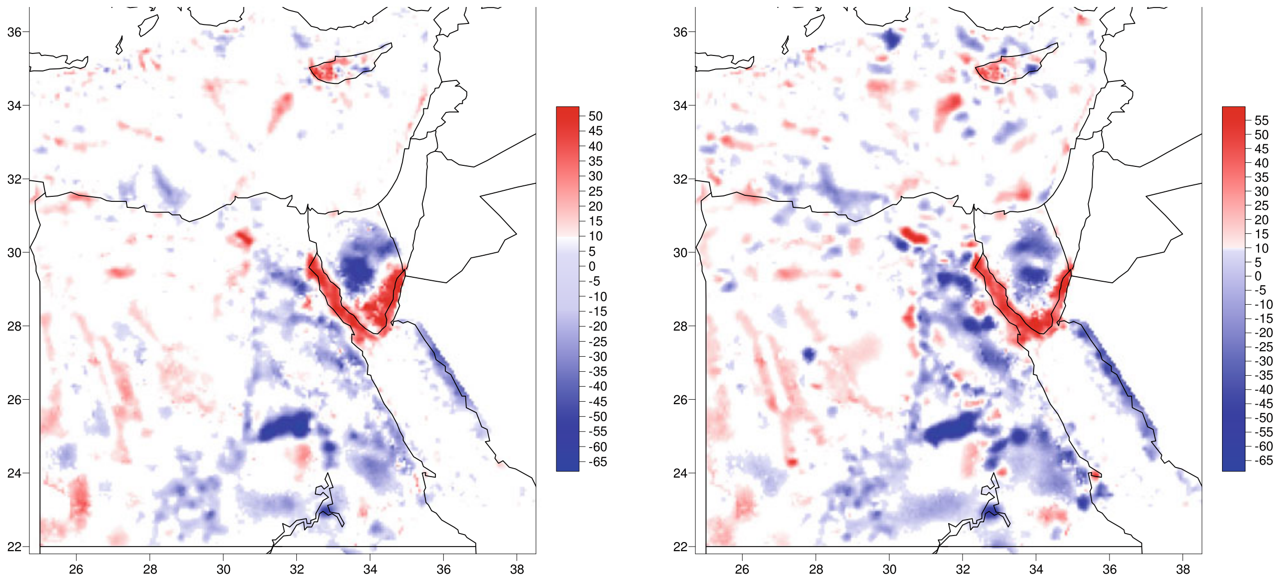
(a) (b)

Fig. 5 Histogram of the validation on sea for the (**a**) AFRGDB_V2.0 and (**b**) AFRGDB_V2.2 gravity database for Africa



(a) (b)

Fig. 6 Histogram of the validation of the full data for the (**a**) AFRGDB_V2.0 and (**b**) AFRGDB_V2.2 gravity database for Africa



(a) (b)

Fig. 7 Validation of the (**a**) AFRGDB_V2.0 and (**b**) AFRGDB_V2.2 gravity database for Africa. Units in [mgal]

6 Conclusion

A validation of the recently established AFRGDB_V2.0 and AFRGDB_V2.2 gravity databases for Africa has been successfully carried out. The new data which are used for validation, covers the north-eastern region of the African continent. In this region occur large data gaps in the previous database, particularly in the point values on land. The performed validation shows that the AFRGDB_V2.0 gravity database is more precise in this region than the AFRGDB_V2.2 gravity database. This becomes obvious from the residuals between the new data used for validation and the respective model (cf. Fig. 7). While 68.03% of the data points have differences less than 10 mgal for the AFRGDB_V2.0, for the AFRGDB_V2.2 this holds only for 57.66% of the data points. This statement is also supported by the statistical parameters in Table 3. They show that the AFRGDB_V2.0 fits better than the AFRGDB_V2.2 to the new data. However, the computation efforts and CPU-time for the AFRGDB_V2.2 gravity database are much less compared to those of the AFRGDB_V2.0 gravity database. The validation, as an external check of the quality of the gravity databases AFRGDB_V2.x for Africa, shows reasonable accuracy of the established gravity databases considering the large data gaps in Africa. The performed validation of the so far used data for establishing the AFRGDB_V2.x databases shows significant discrepancy concerning the new data set for Sinai, which deserves deeper investigation.

Table 3 Statistics of the validation of the AFRGDB_V2.0 and AFRGDB_V2.2 gravity data bases. Units in [mgal]

Gravity database	Statistical parameters			
	Min	Max	Mean	Std.
AFRGDB_V2.0	−54.82	54.53	−0.69	12.06
AFRGDB_V2.2	−55.98	56.04	−1.13	14.55

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