Real-time coordination of integrated transmission and distribution systems: Flexibility modeling and distributed NMPC scheduling

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1. Introduction

With the rapid adoption of distributed energy resources (DERs) in distribution systems, the aggregated flexibility of all these controllable devices can play an important role in dispatching problems in transmission systems. It can improve the operational efficiency of the overall power grid and enhance reliability when integrating increased levels of renewable energy resources [1]. Hence, coordinating integrated transmission and distribution systems (ITD) becomes essential for efficiently operating future power systems [2,3].

Multiperiod dispatch problems for ITD systems usually couple individual steady-state optimal power flow (OPF) optimization problems over multiple time periods [4–6]. The coupling constraints include the generator ramping limits, the model of distributed energy storage systems (ESS), and other time-dependent constraints to consider the controllable devices with time-variant properties. However, it is still a challenge to solve a multiperiod AC optimal power flow (MOPPF) for ITD systems. On the one hand, the AC OPF is generally NP-hard [7], and the complexity of solving an MOPPF is further magnified by the inter-coupling of subsequent time periods [5]. On the other hand, collecting necessary and realistic data from multiple stakeholders (i.e., TSOs and DSOs), including grid topology, load profiles, and other sensitive information regarding consumer behaviors, is either not preferred or restricted by regulations [3]. To address these challenges and achieve an efficient operation of the overall ITD, recent research analyzed the determination of the aggregated flexibility of the controllable devices in distribution networks [8,9]. The flexible dispatch region of a distribution network is summarized in a time-coupled power-energy band, taking into account the network topology [10] and operational constraints. However, the proposed ITD framework does not consider the coordination between multiple TSOs in a data-preserving manner, and the proposed inner approximation is computationally inefficient, requiring solving multiple mixed-integer linear programming (MILP) problems iteratively.

This paper proposes a real-time distributed operational architecture to coordinate integrated transmission and distribution systems (ITD). At the distribution system level, the distribution system operator (DSO) calculates the aggregated flexibility of all controllable devices by power-energy envelopes and provides them to the transmission system operator (TSO). At the transmission system level, a distributed nonlinear model predictive control (DNMPC) approach is proposed to coordinate the economic dispatch of multiple TSOs, considering the aggregated flexibility of all distribution systems. The subproblems of the proposed approach are associated with different TSOs and individual time periods. In addition, the aggregated flexibility of controllable devices in distribution networks is encapsulated, re-calculated, and communicated through the power-energy envelopes, facilitating a reduction in computational complexity and eliminating redundant information exchanges between TSOs and DSOs, thereby enhancing privacy and security. The framework’s effectiveness and applicability in real-world scenarios are validated through simulated operational scenarios on a summer day in Germany, highlighting its robustness in the face of significant prediction mismatches due to severe weather conditions.

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Data preservation
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A B S T R A C T
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To enable privacy preservation and improve computational efficiency, distributed operation frameworks enable TSOs and DSOs to operate independently and collaborate effectively by sharing limited information with a subset of other operators [11–16]. These proposed distributed frameworks can maintain data privacy and decision-making independence and are based on distributed AC OPF [12,17] and MPOPF with receding horizon [18]. In addition to the aforementioned distributed algorithms, aladin is proposed for generic nonconvex optimization problems with convergence guarantees in [19]. aladin-type algorithms have been successfully applied to solve the single period AC OPF for heterogeneous power systems by a single-machine numerical simulation [15,20,21], as well as in a geographically distributed environment [22]. However, these aforementioned studies either lack a convergence guarantee or their scalability is limited by the computational complexity, which so far hinders an application to MPOPF in ITD systems.

In this paper, we propose an economic dispatch problem for ITD systems over multiple periods and utilize an aladin-type distributed NMPF to solve the optimization problem efficiently while preserving data privacy. The major contributions of this paper are summarized as follows:

1. We propose a novel real-time framework that combines the flexibility aggregation method [1] and distributed optimization [3] for coordinating the economic dispatch problem of ITD systems. At the distribution system level, the DSO computes the feasible dispatch region of all controllable devices leveraging the LinDistFlow model [23]. This region is communicated to the TSO. At the transmission system level, considering the aggregated flexibility of distribution systems, the TSOs solve the coordinated economic dispatch problem using a distributed approach. The scheme of the proposed operational architecture is shown in Fig. 1, as inspired by the actual situation in Germany.

2. In contrast to our previous work [3], we develop an aladin-type distributed NMPF approach for the multiperiod coordination of ITD systems within the proposed real-time framework. This approach is capable of decoupling the large-scale dispatch problem associated with different system operators and individual time periods. This particular design distributes the computational complexity of MPOPF over different stakeholders for computational affordability while maintaining the privacy of relevant information.

3. We conduct a comprehensive simulation by using real-world measurement data—including load profiles and solar and wind outputs—from a summer day in Germany marked by significant prediction discrepancies due to heavy rainfall, sourced from the ENTSO-E Transparency Platform [24]. This simulation, involving over 100,000 state variables divided into 400 subproblems at the transmission level, underscores the proposed approach’s efficiency, scalability, and practical relevance for TSO–DSO coordination.

The rest of this paper is organized as follows: Section 2 presents the system model and problem formulation. Section 3 introduces the proposed distributed algorithm with implementation details. Section 4 elaborates on numerical results. Section 5 concludes this paper.

2. Problem formulation

This section presents a distributed framework for coordinating ITD systems. As shown in Fig. 1, at the distribution level, each DSO calculates its own feasible region taking into account its controllable devices and provides it to the corresponding TSO. The TSO then solves a coordinated economic dispatch problem over multiple time periods, considering the aggregated flexibility of the DSOs. Throughout this paper, solar and wind generation are considered as negative demands.

Fig. 1. Proposed real-time coordination of integrated transmission and distribution systems.

2.1. Model of flexibility in distribution systems

In this section, we consider a radial distribution system denoted by a directed tree graph $G(\mathcal{N}, \mathcal{L})$, where $\mathcal{N} = \{1, \ldots, N_{\text{bus}}\}$ is the set of buses. The set $\mathcal{L} \subseteq \mathcal{N} \times \mathcal{N}$ collects “links” or “lines” for all $(i, j) \in \mathcal{L}$. The number of links in a distribution network is $N_{\text{line}}$. Bus 1 is the slack (root) bus and is assumed to have a fixed voltage. We also assume that the distribution systems have a pure tree topology, i.e., $N_{\text{bus}} = N_{\text{line}} + 1$ holds. We leverage the definition of connectivity matrices $C^s$, $C^v$ and $C^{\text{pec}}$ with respect to generator, ESS and the point of common coupling (PCC) between transmission and distribution, as defined in [25].

Definition 1 (26]). Let $C^{\text{inc}} \in \mathbb{R}^{N_{\text{line}} \times N_{\text{bus}}}$ be the incidence matrix of a given radial network; we set $[C^{\text{inc}}]_{bi} = +1$ if bus $i$ is the head of branch $b$ and $[C^{\text{inc}}]_{bi} = -1$ if bus $i$ is the tail of branch $b$.

Details about incidence matrices refer to [5,25].

2.1.1. Exact feasible set

We use the LinDistFlow model [10] to describe the relationship between the voltages and net loads in distribution systems by the following linear power flow equation:

$$1 = e_i^T U_k, \quad (1a)$$
$$0 = C^{\text{inc}} U_k - 2 R P_k^l - 2 X Q_k^l; \quad (1b)$$
$$0 = e_i P_k^{\text{pec}} - P_k^d - (C^{\text{inc}})^T P_k^l - C^v P_k^l; \quad (1c)$$
$$0 = e_i Q_k^{\text{pec}} - Q_k^d - (C^{\text{inc}})^T Q_k^l; \quad (1d)$$

$$U_k \leq U_k \leq \bar{U}; \quad (1e)$$
$$P_k^d \leq P_k^l \leq \bar{P}; \quad (1f)$$

where $e_i = [1, 0, \ldots, 0]^T \in \mathbb{R}^{N_{\text{bus}}}, R = \text{diag}(r), X = \text{diag}(x), r, x \in \mathbb{R}^{N_{\text{bus}}}$ denote the resistance and reactance vectors respectively, $U_k$ denotes the vector of squared voltage magnitude at the time instant $k$, $P_k^{\text{pec}}, Q_k^{\text{pec}}$ denote active and reactive power exchanges with the transmission system at the PCC of the distribution system. We use vectors $P_k^d, Q_k^d$ to denote the active and reactive nodal power consumptions, $P_k^l, Q_k^l$ to denote the active and reactive branch flows, and $P_k^l$ to denote the nodal consumptions by distributed energy storage systems (ESS) at time period $k$. Moreover, (1a) fixes the voltage magnitude at the slack bus. Eqs. (1b)–(1d) are the LinDistFlow constraints. Upper and lower bounds (1f) restrict the voltage magnitude at each bus and the charging/discharging power of ESSs. We rewrite the above power flow Eqs. (1a)–(1d) in a compact form:

$$M g_k + B P_k^l + D_k = 0, \quad (2)$$
where

\[
M = \begin{bmatrix}
  e_1^T & 0 & 0 & 0 & 0 \\
  C_{\text{inc}} & -2R & -2X & 0 & 0 \\
  0 & -(C_{\text{inc}})^T & 0 & e_1 & 0 \\
  0 & 0 & -(C_{\text{inc}})^T & 0 & e_1
\end{bmatrix}.
\]

Note that \( M \in \mathbb{R}^{(2N_{\text{bus}}+N_{\text{line}}+1)(2N_{\text{bus}}+2N_{\text{line}}+2)} \) is a square matrix since for radial distribution grids, \( N_{\text{bus}} = N_{\text{line}} + 1 \). In (2), \( M \) and \( B \) remain time-invariant. All dependent variables \( x_k \) are influenced by controllable power injections \( P_k^l \) from ESSs, as well as the load demands \( P_k^l \) and \( Q_k^l \) at each time period \( k \). Therefore, in this paper, the flexibility in distribution systems primarily arises from the integration of ESSs.

In [27], it is shown that the squared voltage magnitude \( U \) can be explicitly expressed by the active and reactive power injections. However, positive definiteness of resistance and reactance for all the branches is required, a condition not universally met in practical power system datasets, as discussed in [28]. To extend the applicability of the proposed coordination framework to a broader range of power systems, we generalize the result from [27]. With the assistance of graph theory, we rigorously demonstrate the invertibility of matrix \( M \), affirming that all state variables, including squared voltage magnitude \( U \), can be explicitly expressed in terms of controllable power injections for all radial networks. This expansion significantly enhances the robustness and versatility of the proposed framework for practical power systems.

**Lemma 1.** For a given radial network denoted by \( G(N', E) \), let bus \( i \) be a leaf of graph \( G \), let branch \( a \) be an edge connected to leaf bus \( \beta \), then there is only one nonzero element in the \( jth \) column of incidence matrix \( C_{\text{inc}}(G) \), and it is located in the \( a \)th row.

This lemma follows directly from the fact that a leaf has only one parent in a radial network.

**Lemma 2 ([26]).** A radial network with at least two buses has at least two leaves. Deleting a leaf from a radial network with \( N \) buses produces a radial network with \( N - 1 \) buses.

**Proposition 1.** Given a radial network \( G \), matrix \( M \) is invertible.

The detailed proof can be found in Appendix. As a result of the generalized Proposition 1, for a given distribution grid, all the dependent variables in \( x_k \) can be expressed explicitly in terms of the controllable power injections \( P_k^l \), and thus, the exact feasible set is convex and can be written in a convex polytope with respect to \( P_k^l \):

\[
P_k^l = \{ P_k^l \in \mathbb{R}^e \mid A'P_k^l \leq b' \},
\]

where \( e \) denotes the number of ESSs. In the example illustrated in Fig. 2, the blue polytope represents an efficient feasible set constrained by upper and lower voltage bounds along with power limits of ESSs (11).

**2.2.1. Maximum volume inner approximation**

In this paper, the flexibility of distribution systems primarily arises from the integration of ESS in (1). Instead of applying the exact feasible set (3), we replace the complex polytope with a strictly inner hyperbox approximation, enhancing computational efficiency while maintaining safe operation guarantees within the system, i.e.,

\[
B_k^l \subseteq \overline{P}_k^l, \quad \forall k \in \{1, 2, \ldots, N_k\},
\]

where the hyperbox \( B_k^l \) is defined as

\[
B_k^l(P_k^{\text{ppr}}, Q_k^{\text{ppr}}) = \{ P_k^l \in \mathbb{R}^e \mid P_k^{\text{ppr}} \leq P_k^l \leq Q_k^{\text{ppr}} \},
\]

Note that \( P_k^{\text{ppr}} \) and \( Q_k^{\text{ppr}} \) are upper and lower bounds of the inner hyperbox approximation. The 2-dimensional green box in Fig. 2 represents an inner hyperbox approximation to the exact feasible set (blue polytope). To maximize the performance of the resulting ESS system, we adopt the so-called maximum volume inner hyperbox [29]. The hyperbox (5) can be written as \( B_k^l(\zeta, \xi + \zeta) \) and the inner approximation can be obtained by solving the following optimization problem:

\[
\max \sum_{i \in E} \ln(\zeta_i),
\]

\[
s.t. \quad A'\xi + A'\zeta \leq b', \quad (6a)
\]

where \( A' \) is the positive part of \( A \) and \( E = \{1, \ldots, e\} \) is the set of ESSs. However, in practice, it can occur that the standby mode of a ESS is excluded by the inner approximation, i.e.,

\[
3i \in E, \quad [L_k^{\text{ppr}}]_i < 0 \quad \text{or} \quad [U_k^{\text{ppr}}]_i > 0,
\]

i.e., the origin is not included in the resulting hyperbox (green), as shown in Fig. 2(a).

To address this issue, instead of focusing on maximizing the volume in \( \mathbb{R}^e \) space, i.e., finding an equilibrium where ESSs have wide ranges of permissible power output intervals, we propose to maximize the volume within the \( \mathbb{R}^{2e} \) space, thereby expanding both charging and discharging power limits of the ESSs according to

\[
\max \sum_{i \in E} \ln(\xi_i + \zeta_i) + \ln(-\xi_i),
\]

\[
s.t. \quad A'\xi + A'\zeta \leq b', \quad (7b)
\]

enabling scenarios where, for instance, both ESSs can charge even during periods of high system load, as illustrated in Fig. 2(b), but more importantly, the origin is included in the hyperbox (green).

**2.2. Coordinated economic dispatch for ITD systems**

**2.2.1. Aggregating distribution systems**

Since LinDistFlow (1) ignores the power losses along branches, power exchanged with the transmission can be expressed with the assistance of conservation of power for all time periods \( k \in \mathcal{K},

\[
P_k^{\text{ppr}} = \mathbf{1}^T P_k^l + \mathbf{1}^T P_k^{\text{g}}, \quad P_k^l \in B_k^l,
\]

\[
Q_k^{\text{ppr}} = \mathbf{1}^T Q_k^l,
\]

where \( B_k^l \) is calculated by (7). As a result, at the transmission level, a distribution system can be modeled as a load and multiple ESSs at the PCC.

**2.2.2. Multiperiod AC optimal power flow**

The bus injection model [30] with complex voltages expressed in polar coordinates is employed at the transmission level. Here, \( \theta_k, V_k \) and the reactive power injection from the ESSs to the transmission network can be expressed as

\[
\theta_k = \text{angle of } V_k, \quad V_k = \text{magnitude of } V_k.
\]

Fig. 2. Comparison of inner approximation methods with 2 ESSs located in the heavily loaded IEEE 33-bus system. The orange and the purple lines show the upper and lower bounds on the squares of voltage magnitudes \( U_k \). The red lines show the limits on ESSs’ power output \( P_k \).
stack nodal voltage angles $\theta_{k,i}$ and magnitudes $v_{k,i}$ for all bus $i$ at time period $k$ respectively. $P_{k,i}^{\text{loc}}$, $Q_{k,i}^{\text{loc}}$ stack active and reactive power exchanges (8) for all distribution systems, respectively. $Y = G + jB$ denote the complex bus admittance matrix, where $j = \sqrt{-1}$ and $G, B \in \mathbb{R}^{N_{\text{bus}} \times N_{\text{bus}}}$.

The resulting multiperiod AC optimal power flow (MPOPF) for coordinating different TSOs can be written as

$$
\min_{\kappa = 1} \sum_{k=1}^{N_{k}} (P_{k,i}^{\text{loc}})^{\top} \text{diag}(a_{k}) P_{k,i}^{\text{loc}} + a_{k}^{i} P_{k,i} + a_{k}^{i} 1, \quad (9a)
$$

subject to \forall k \in K := \{1, 2, \ldots, N_{k}\}

$$
P_{k}^{\text{p}}(\Theta_{k}, V_{k}) = C_{k}^{\text{p}} P_{k}^{\text{p}} + P_{k}^{\text{d}} - C_{k}^{\text{loc}} P_{k}^{\text{loc}} - C_{k}^{i} P_{k}^{i}, \quad (9b)
$$

$$
Q_{k}^{i}(\Theta_{k}, V_{k}) = C_{k}^{i} Q_{k}^{i} - Q_{k}^{d} - C_{k}^{\text{loc}} Q_{k}^{\text{loc}}, \quad (9c)
$$

$$
|S_{k}(\Theta_{k}, V_{k})| \leq 3, \quad (9d)
$$

$$
V \leq V_{\text{pcc}} = \bar{V}, P_{k}^{p} \leq P_{k}^{p}^{\text{p}}, Q_{k}^{i} \leq Q_{k}^{i}^{\text{p}}, \quad (9e)
$$

$$
E_{k} = E_{k-1} + \Delta t \cdot P_{k}^{q}, \quad (9f)
$$

$$
P_{k}^{p} = P_{k}^{p} + \Delta t P_{k}^{q} \text{ with initial state } E_{0} = E(t_{0}). \quad (9g)
$$

$$
E \leq E_{k} \leq \bar{E}, \quad P_{k}^{p} \leq P_{k}^{p}^{\text{p}}, \quad \bar{R} \leq \Delta P_{k}^{p} \leq \bar{R}, \quad (9h)
$$

where $P_{k}^{p}, Q_{k}^{i} : \mathbb{R}^{N_{\text{bus}} \times N_{\text{bus}}} \rightarrow \mathbb{R}^{N_{\text{bus}}}$ represent the vector functions of active and reactive power injections for all buses at time period $k$, and the corresponding $i$th element can be expressed as

$$
[P_{k}^{p}]_{i} = v_{k,i} \sum_{j \in N} v_{k,j} (G_{k,i,j} \cos \theta_{k,j} + B_{k,i,j} \sin \theta_{k,j}), \quad (10a)
$$

$$
[Q_{k}^{i}]_{i} = v_{k,i} \sum_{j \in N} v_{k,j} (G_{k,i,j} \sin \theta_{k,j} - B_{k,i,j} \cos \theta_{k,j}), \quad (10b)
$$

with angle difference $\theta_{k,j} = \theta_{k,i} - \theta_{k,j}$. Similarly, $S_{k}^{i}$ are non-linear mappings $\mathbb{R}^{N_{\text{bus}} \times N_{\text{bus}}} \rightarrow \mathbb{R}^{N_{\text{bus}}}$ representing apparent branch power flows for all branches at time period $k$ for the detailed formulation of branch power flows, we refer readers to [30]. Evidently, MPOPF (9) constructs a simultaneous formulation of $N_{k}$ AC OPF problems with standard power flow constraints (9b)-(9f), coupled by intertemporal interactions (9g)-(9h) and the corresponding upper and lower bounds (9h). Notice that ESSs possess time-variant power limits in (9h), due to the inner hyperbox approximation (5) utilized for aggregating distribution systems.

Rather than devising intricate mathematical models to precisely represent distribution systems, the flexibility aggregation method offers a substantial reduction in computational complexity of the MOPPF in (9). It enhances the scalability of the proposed framework without sacrificing the active involvement of distribution systems in the dispatch problems.

3. Methodology

This section presents the proposed distributed real-time coordination framework of ITD systems using a receding horizon scheme while considering day-ahead forecast and actual values. Compared to the classical distributed MPC scheme, only solving the structured optimal control problem either in a spatially distributed manner or in a temporally distributed manner [31], our approach decouples the optimization problems across both different system operators and periods, with each subproblem representing an individual single-period AC OPF of a single transmission system.

3.1. Distributed formulation

We describe a coordination problem of ITD systems by a tuple $C = (N, \mathcal{L}, K, \mathcal{R})$ over $N_{t}$ time periods. Thereby, $N$ denotes the set of all buses, $\mathcal{L}$ the set of all branches, and $\mathcal{R} = \{T_{1}, T_{2}, \ldots\}$ denotes the set of coordinated transmission systems.

The objective function (9a) summarizes quadratic generation cost from all regions $\ell \in \mathcal{R}$ over all time periods $k \in K$. This enables a straightforward separation of the objective function across different system operators and time periods:

$$
f(x) = \sum_{k \in K} \sum_{\ell \in \mathcal{R}} f_{k,\ell}(x_{k,\ell}), \quad (10a)
$$

where $x_{k,\ell}$ represents state variables in the transmission system $\ell$ at the time period $k$ and $x$ is a vector stacking all the subvectors $x_{k,\ell}$.

The constraints of MPOPF (9) can be decoupled across time periods, where each of the temporal coupling constraints (9i)-(9g) is associated with only one specific transmission system. Therefore, these temporal coupling constraints can be written in the following standard affinely coupled form

$$
\sum_{k \in K} A_{k,\ell} x_{k,\ell} = 0, \quad \ell \in \mathcal{R}, \quad (10b)
$$

where the sparse matrices $A_{k,\ell}$ contain non-zero elements of $[-1, 1, 0]$, connecting the state variables current $E_{k,\ell}$ and $P_{k,\ell}^{\text{loc}}$ with neighboring time periods $\{k-1, k+1\}$ for each transmission system $\ell$.

Regarding spatial coupling among different TSOs, we follow the idea of sharing components [32], i.e., sharing nodal voltage angles and magnitudes at both sides of connecting tie-lines between neighboring transmission systems. The resulting spatial coupling constraints are linear and can be written in the following affinely coupled form

$$
\sum_{k \in K} \Gamma_{k,\ell} x_{k,\ell} = 0, \quad \ell \in K, \quad (10c)
$$

where the sparse matrices $\Gamma_{k,\ell}$ contain non-zeros elements of $[-1, 1]$ connecting the coupling voltage angles and magnitudes between neighboring transmission systems for each time period $k$.

Thereby, the MOPPF in the transmission level can be decoupled across different system operators and time periods and reformulated in standard affinely distributed form

$$
\min_{\kappa \in K} \sum_{k \in K} \sum_{\ell \in \mathcal{R}} f_{k,\ell}(x_{k,\ell}) \quad \text{DECoupled Objective} \quad (10a)
$$

$$
\text{s.t. } \forall k \in K, \sum_{\ell \in \mathcal{R}} \Gamma_{k,\ell} x_{k,\ell} = 0 \quad \text{SPATIAL COUPLINGS} \quad (10b)
$$

$$
\text{s.t. } \forall \ell \in \mathcal{R}, \sum_{k \in K} A_{k,\ell} x_{k,\ell} = 0 \quad \text{TEMPORAL COUPLINGS} \quad (10c)
$$

$$
\text{s.t. } \left\{ \begin{array}{l}
\forall k \in K, \forall \ell \in \mathcal{R} \quad \left[ \begin{array}{l}
\lambda_{\ell} \quad (x_{k,\ell}) \leq 0 \quad |v_{\ell} = v_{\ell}
\end{array} \right]
\end{array} \right. \quad \text{DECoupled CONSTRAINTS} \quad (10d)
$$

where $\lambda_{\ell}$, $v_{\ell}$, and $\nu_{\ell}$ denote Lagrangian multipliers associated with the corresponding constraints. Constraints (10d) correspond to the standard AC OPF constraints (9b)-(9e) with power and energy limits on the ESSs (9h) for each transmission system $\ell \in \mathcal{R}$ over all time periods $k \in K$.

3.2. Real-time distributed coordination scheme

As (9) is reformulated in an affine-coupled distributed form (10), it can be solved efficiently by using distributed optimization algorithms. In this paper, we tailor the ALADIN algorithm [19] to deal with (10) in a closed loop. The resulting distributed coordination scheme in receding horizon fashion is outlined in Algorithm 1.

Based on the local measurements collected in Step (1), Step (2) of Algorithm 1 outlines the tailored ALADIN algorithm to solve (10). Step (2.a) solves $N_{k} \cdot |R|$ subproblems, in which the regional TSO deals with the $N_k$ temporal subproblems in parallel locally. These problems are constructed using the Lagrangian of (10) by dualizing the spatial coupling (10b) and temporal coupling (10c). Based on the decoupled solutions $v_{\ell}, \nu_{\ell}$, Step (2.b) computes sensitivities of objective and constraints with respect to the current iteration of ALADIN. Here, in order to improve the numerical robustness of the algorithm, a small
Algorithm 1 Distributed Real-Time Coordination of ITD Systems

Offline:
- Choose initial guess \((x^0, \lambda^0, k^0)\) for closed loop

Repeat:

1. The local operator of the regional transmission systems measures the current states \((E_{\ell}^{\theta}(t), P_{\ell}^{\theta}(t))\) for all \(\ell \in R\).
2. Solve (10) cooperatively to obtain solution \((x^*, \lambda^*, k^*)\) by repeating
   a. Solve decoupled NLPs for all \(k \in K\) and \(\ell \in R\)
      \[
      \min_{y_{\ell k}} f_{\ell k}(y_{\ell k}) \quad \text{s.t.} \quad h_{\ell k}(y_{\ell k}) \leq 0 \quad |v_{\ell k}|
      \]
   b. Compute the Jacobian matrix \(J_{\ell k}\) of active constraints \(h_{\ell k}\) at the local solution \(y_{\ell k}\) by
      \[
      J_{\ell k} = \begin{cases} \partial [h_{\ell k}(y_{\ell k})] & \text{if } [h_{\ell k}(y_{\ell k})] = 0, \\ 0 & \text{otherwise} \end{cases}
      \]
   with \([\cdot]\) denotes the i-th row, and gradient
      \[
      g_{\ell k} = \nabla f_{\ell k}(y_{\ell k}).
      \]
      and choose Hessian approximation
      \[
      0 < H_{\ell k} \approx \nabla^2 \left\{ f_{\ell k}(y_{\ell k}) + v_{\ell k}^T h_{\ell k}(y_{\ell k}) \right\}.
      \]
   c. Update \((x \leftarrow y + \Delta y, \lambda \leftarrow \lambda^Q, k \leftarrow k^Q)\) by solving
      \[
      \min_{\Delta y, k} \sum_{k \in K} \left\{ \frac{1}{k} \lambda_k \right\} + \sum_{\ell \in R} \left[ k_{\ell}^T s_{\ell k} + \frac{\mu_k}{2} \| s_{\ell k} \|^2_2 \right]
      \]
      \[
      \left. + \sum_{k \in K} \sum_{\ell \in R} \left\{ \frac{1}{2} \lambda_k \right\} \text{ s.t. }\right. \quad \sum_{\ell \in R} (I_{\ell k}(y_{\ell} + \Delta y_{\ell}) = s_{\ell k} \quad |v_{\ell k}|, \quad k \in K, \ell \in R,
      \]
      \[
      \sum_{k \in K} A_{\ell k}(y_{\ell} + \Delta y_{\ell}) = s_{\ell k} \quad |k^Q|, \quad \ell \in R, \quad J_{\ell k} = 0, \quad k \in K, \ell \in R.
      \]
5. Reinitialize for all \(\ell \in R\)
   \[
   x_{\ell}^0 \leftarrow (x^{\ell}_{\ell 1}, \ldots, x^{\ell}_{\ell 2}, x^{\ell}_{\ell 3}, x^{\ell}_{\ell 4})^T, \quad k_{\ell}^0 \leftarrow (k^{\ell}_{\ell 1}, \ldots, k^{\ell}_{\ell 2})^T, \quad 0
   \]
   with \([-i]\) the elements w.r.t kth time coupling, and
   \[
   \lambda^0 \leftarrow (\lambda^{\ell}_{\ell 1}, \ldots, \lambda^{\ell}_{\ell 2})^T.
   \]
   Then, set \(i \leftarrow i + 1\) and go to Step 1).

perturbation is added to the second-order derivatives (13) approximated by positive definite \(H_{\ell k}\). Notice that under a mild assumption for the perturbation as outlined in [3, Theorem 2], the local quadratic convergence can be guaranteed. Step (2.c) solves the coupled QP (14) with only equality constraints. Taking the temporal coupling (14c) as local equality constraints for the \(\ell\)th region, one can solve (14) in a
decentralized manner that only requires neighbor-to-neighbor communications. For more details, the reader is referred to [33]. Algorithm 1 terminates if the primal conditions
\[
\max_{\ell \in R} \sum_{k \in K} I_{\ell k}(x_{\ell k}) \leq \epsilon, \quad \max_{\ell \in R} \sum_{k \in K} A_{\ell k}(x_{\ell k}) \leq \epsilon,
\]
and dual condition
\[
\max_{\ell \in R} \| y_{\ell k} - x_{\ell k} \| \leq \epsilon
\]
hold. Practically, the dual condition (15b) is sufficient to ensure a small violation of the condition (15), when the predefined tolerance \(\epsilon\) is small enough [34]. Under some regularity assumptions, Algorithm 1 has local quadratic convergence guarantees for both primal and dual iterations. One can construct the proof of this result by following that the coupled QP (14) is equivalent to the Newton-type method while the local solutions maps are Lipschitz continuous. A detailed analysis can be found in [19,20].
tasks at the transmission level involve 187,776 state variables divided into 4 transmission systems, each across 96 time periods, resulting in a total of 384 subproblems.

Note that, in Figs. 3 to 6, the data are arranged in multiple columns to enable a detailed comparative analysis. Specifically, the first four columns in each figure correspond to data from four distinct control areas, i.e., four TSOs and their respective DSOs, respectively. The final column integrates this data, offering a synthesized overview of these four control areas. This configuration facilitates a straightforward comparison across the spatial decomposition to ensure a structured and clear presentation of the simulation results.

4.2. Isolated vs. Coordinated operation mode

Three distinct operating strategies are explored in the case study: isolated operation, centralized coordination, and distributed coordination. In all these strategies, the flexibility of distribution systems is aggregated to the transmission level as proposed in Section 2, and the dispatch problems at the transmission level are optimized with a receding horizon. The primary differences between these strategies lie in their operational methodologies and how they address the economic dispatch problems at the transmission level.
In isolated operation mode, each TSO operates in an islanded manner without any communication or power exchange with other transmission systems. The results of the economic dispatches per time period are comprehensively visualized in Fig. 4. The net power generations—calculated as the positive stacked bars minus the negative stacked bars—marginally exceed the actual demands (red lines) over a 24-h period, across all instances in Fig. 4. This indicates that the balance between supply and demand is maintained, with minimal power line losses.

Contrary to isolated operation, both the centralized and the distributed coordinations utilize the combined system model (9) to facilitate autonomous power exchange (purple bars) between TSOs, aiming to minimize overall generation costs, as depicted in Fig. 5. Fig. 6 demonstrates the state of charge (Soc) of ESSs, highlighting the effective autonomous management in supporting dispatch tasks while adhering to the energy constraints of the ESSs.

A noteworthy instance occurs at 13:00, highlighted as vertical dotted lines, where transmission system $T_1$ encounters a significant prediction mismatch. In this time period, $T_1$ experiences higher actual demand and reduced solar generation, as shown in Fig. 3, coinciding with lower SOC of ESSs in $T_1$, as shown in Fig. 6. In response to this prediction error, power export to other systems (purple bar) is intentionally curtailed as a compensatory measure, demonstrating the system’s capacity to adapt to unexpected operational dynamics.

### 4.3. Centralized vs. Distributed coordination approaches

The key difference between these two coordination strategies lies in the optimization approaches. Centralized coordination communicates all private data to a centralized entity and employs a centralized algorithm to solve the optimization problem. In contrast, distributed coordination solves the optimization problem based on the proposed algorithm in a distributed fashion with limited information exchanged between TSOs.

Given that both the centralized and the distributed coordination adopt the same system model (9) with 187,776 state variables divided into 384 subproblems, we use centralized solutions as reference solutions to evaluate the effectiveness of the proposed distributed approach in solving the economic dispatch problems at the transmission level.

The convergence performance of the proposed distributed approach across 24 h is demonstrated in Fig. 7, representing a number of iterations to converge, total computing time for solving one economic dispatch problem, primal and dual residual deviations of control power injections and optimality gap for each TSOs, expressed as $\frac{f(x) - f(x^*)}{f(x^*)}$. Notably, all the 96 optimization tasks during the daily operation demonstrate fast convergence in a dozen iterations, under 500 s, with both the primal and dual residuals reaching tolerable values. Compared with centralized coordination, the proposed distributed approach showcases remarkable accuracy in terms of controllable power injections and total optimality gap over all 96 time periods. These results highlight the scalability and numerical robustness for real-world applications in large-scale ITD systems.

The economic efficiency comparison among the three operational strategies, as shown in Table 1, indicates that operating in isolation leads to the highest total costs, whereas centralized coordination results in the lowest. Distributed coordination presents a viable alternative, balancing data privacy and competitive costs, approximately 0.0006% higher than centralized methods. Both coordination strategies effectively find local minimizers of the system model (9), with negligible differences in total costs.

### 5. Conclusion

This paper proposes a novel real-time distributed operational framework for efficient coordination of ITD systems. It employs a flexibility aggregation method at the distribution level, leveraging controllable devices through power-energy envelopes provided by DSOs, thereby avoiding additional computational complexity of economic dispatch problems at the transmission level. Furthermore, the framework’s receding horizon strategy enhances its robustness against prediction mismatches, especially under severe weather conditions, highlighted by a case study of a summer day in Germany. By utilizing real operational data with significant prediction mismatches, this study confirms the framework’s practical relevance and applicability in real-world scenarios. Future work includes further exploring flexibility aggregation methods, utilizing more detailed transmission grid data, and strengthening cyber-physical security.

### CRediT authorship contribution statement

**Xinliang Dai**: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Resources, Software, Visualization, Writing – original draft, Writing – review & editing.

**Yi Guo**: Conceptualization, Formal analysis, Methodology, Project administration, Validation, Writing – original draft, Writing – review & editing.

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**Veit Hagenmeyer**: Supervision, Validation, Writing – review & editing.

**Gabriela Hug**: Methodology, Supervision, Validation, Writing – review & editing.

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**CRediT authorship contribution statement**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
Data availability

The data used in the present paper are available online, and the source is also provided in the paper. The code for mathematical modeling will be made available online once the paper is accepted.

Appendix

Considering a non-slack leaf bus $\alpha$ ($\alpha, \beta$) is the only nonzero element in $\beta$th column in matrix $C^{inc}$ due to the incidence matrix property in Lemma 1. Hence, $(\alpha + 1, \beta)$ is the only nonzero element in $\beta$th column in matrix $M$. Similarly, $(\beta + N^{bus}, \beta + N^{bus})$ and $(\beta + 2N^{bus}, \beta + 2N^{bus})$ are the only nonzero elements in the $(\beta + N^{bus})$th and the $(\beta + 2N^{bus})$th row respectively.

By eliminating the leaf bus $\beta$ of the network $G$, we obtain a reduced radial network $G'$. The resulting matrix $M^{(1)}$ can be viewed as a submatrix of $M$ by removing the set of row $(\alpha + 1, \beta + N^{bus}, \beta + 2N^{bus})$ and the set of column $[\beta, \alpha + N^{bus}, \alpha + 2N^{bus}]$.

Since the nonzero elements in the incidence matrix $C^{inc}(G)$ is $\{-1, 1\}$, the determinant of matrix $M$ can be written as

$$\det(M) = \det(M^{(1)})$$

with the assistance of cofactor expansions.

By further removing non-slack leaves of the resulting reduced radial networks, we have

$$\det(M) = \det(M^{(1)}) = \cdots = \det(M^{(N^{bus}-1)}) = 1.$$  (17)

Therefore, $M$ is invertible for the given radial network.

References