Turbulence closure in linearized analyses of non-uniform density flows

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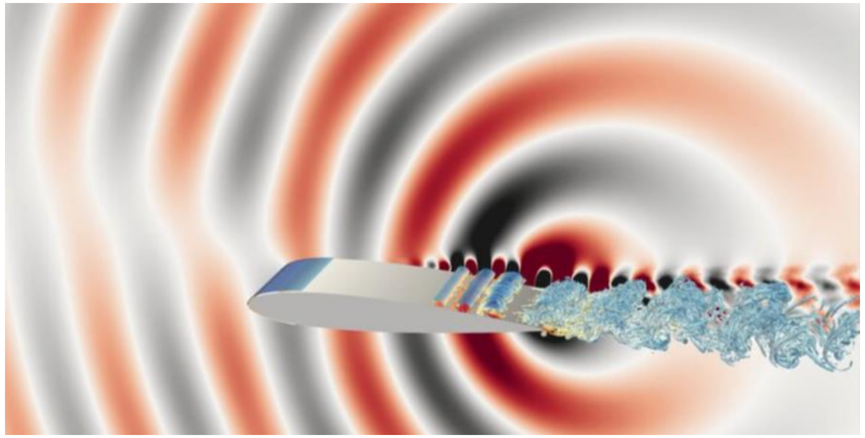
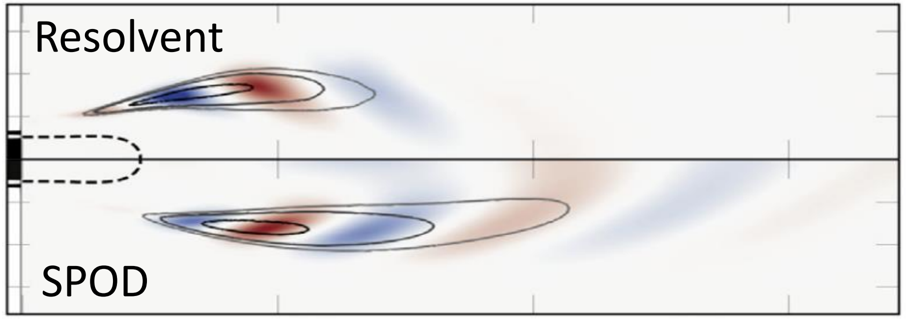
FLOW INSTABILITIES & DYNAMICS TECHNISCHE UNIVERSITÄT BERLIN

Examples of wave packets in density varying flows

Airfoil aeroacoustics (Demange et al. 2024)

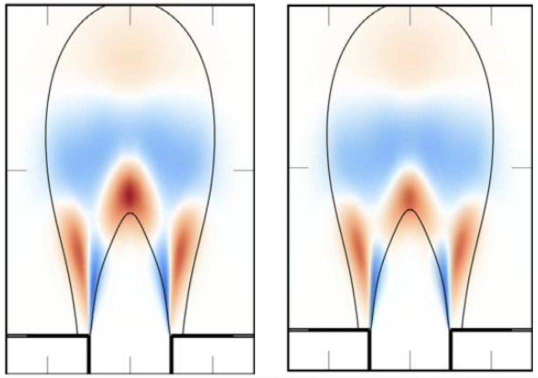


Resolvent analysis of a turbulent flame (Casel et al. 2022)



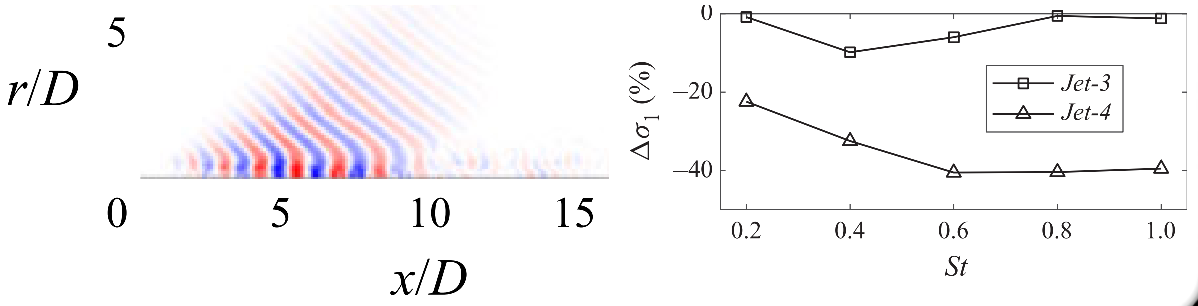
Turbulent flame response to acoustics (Kaiser et al. 2023)

LES

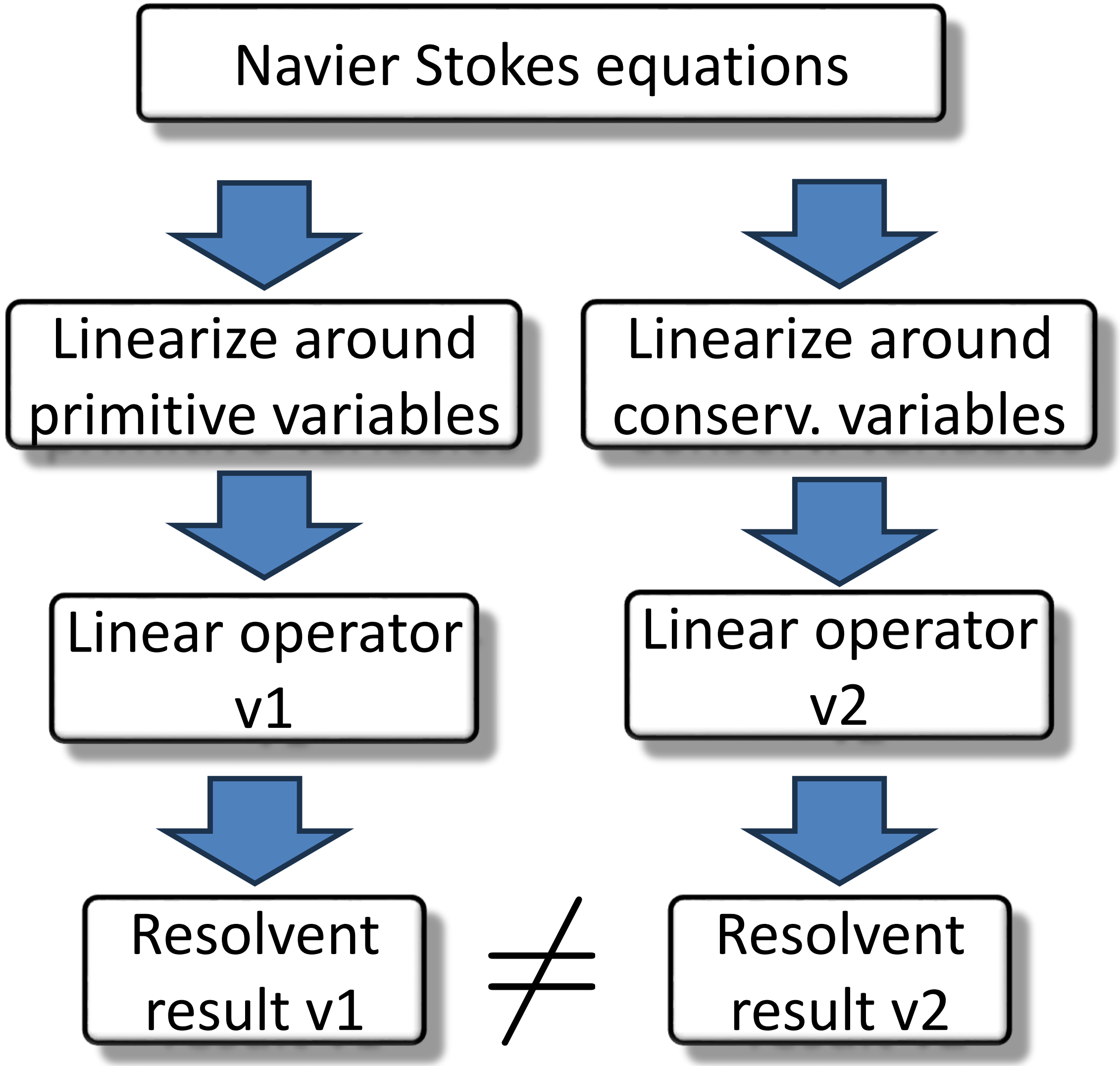


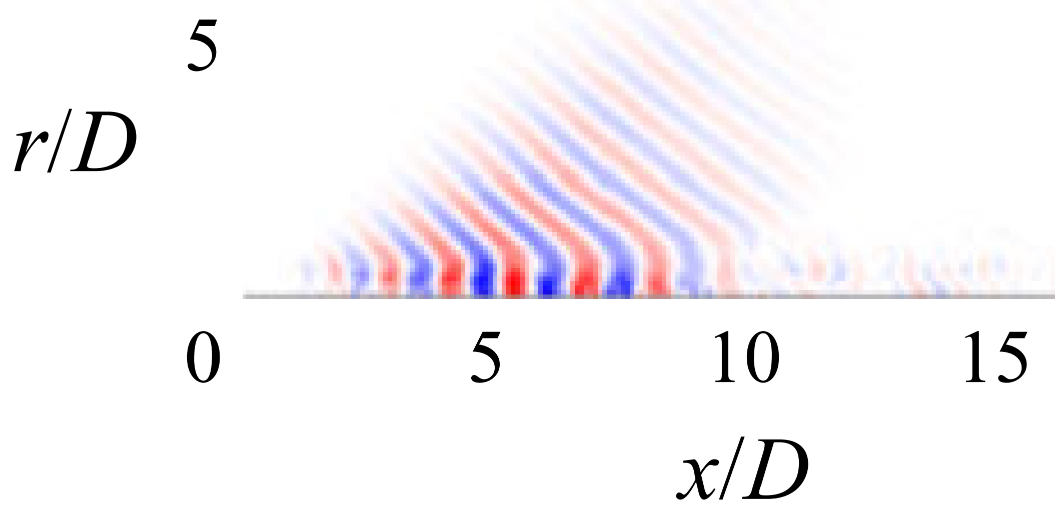
Linear model

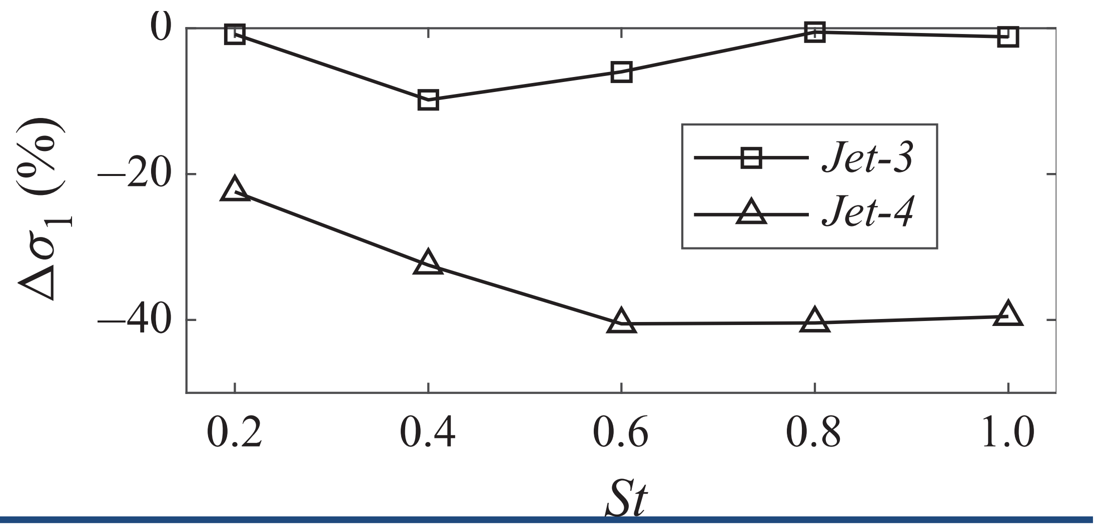
Mean flow ambiguity in transsonic jets (Karban et al. 2020)



Mean flow ambiguity (simplified)







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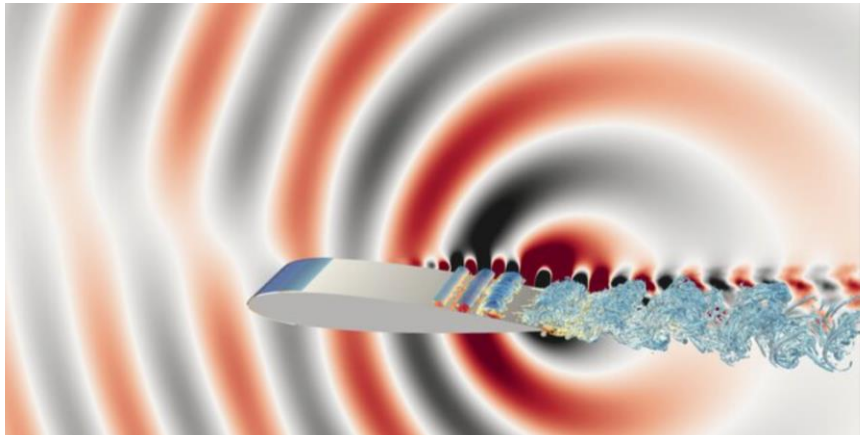

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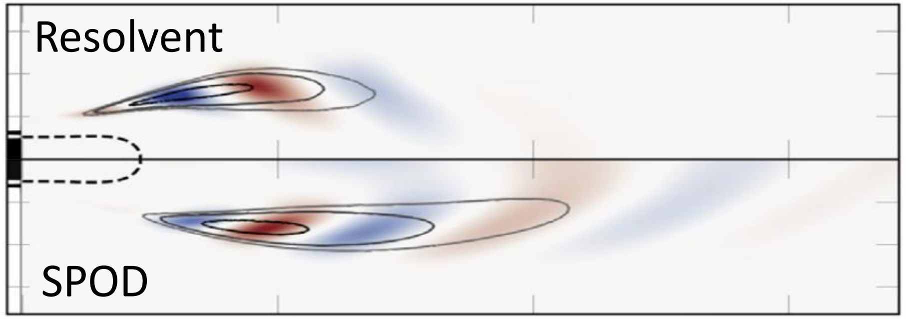

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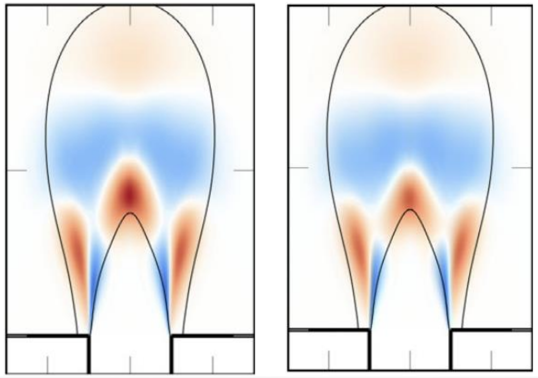

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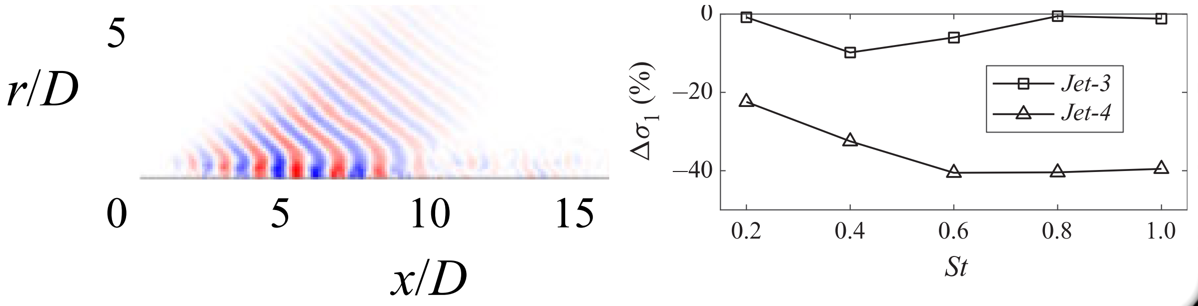
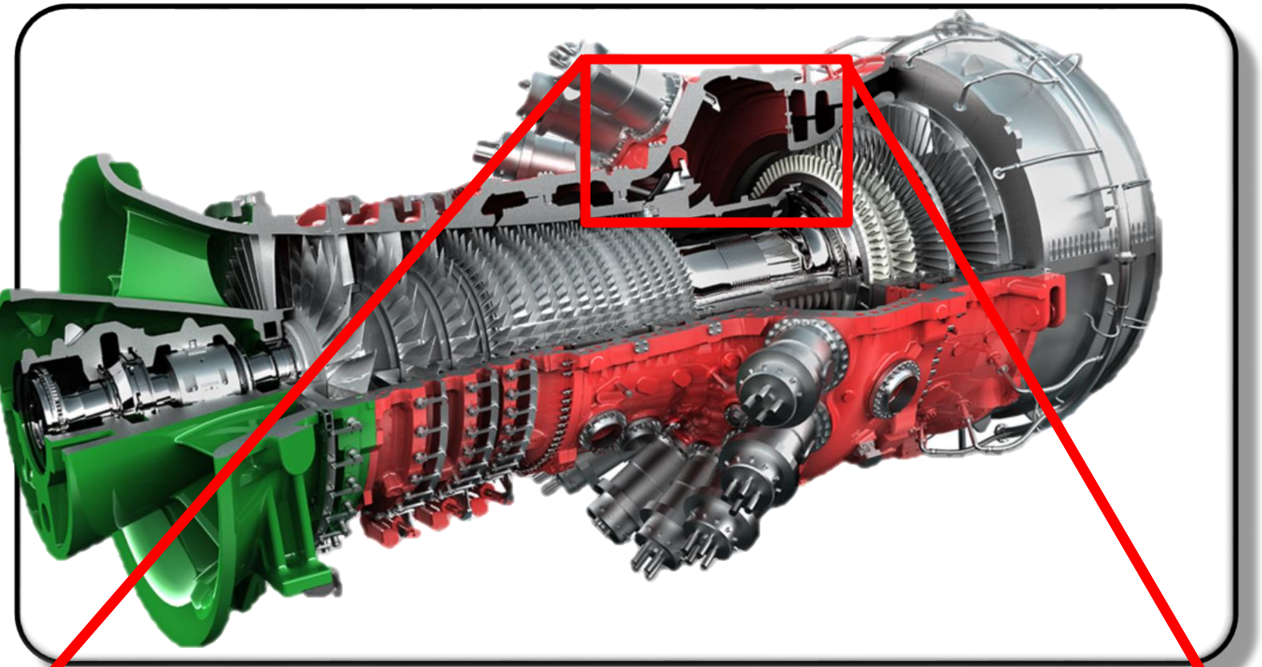
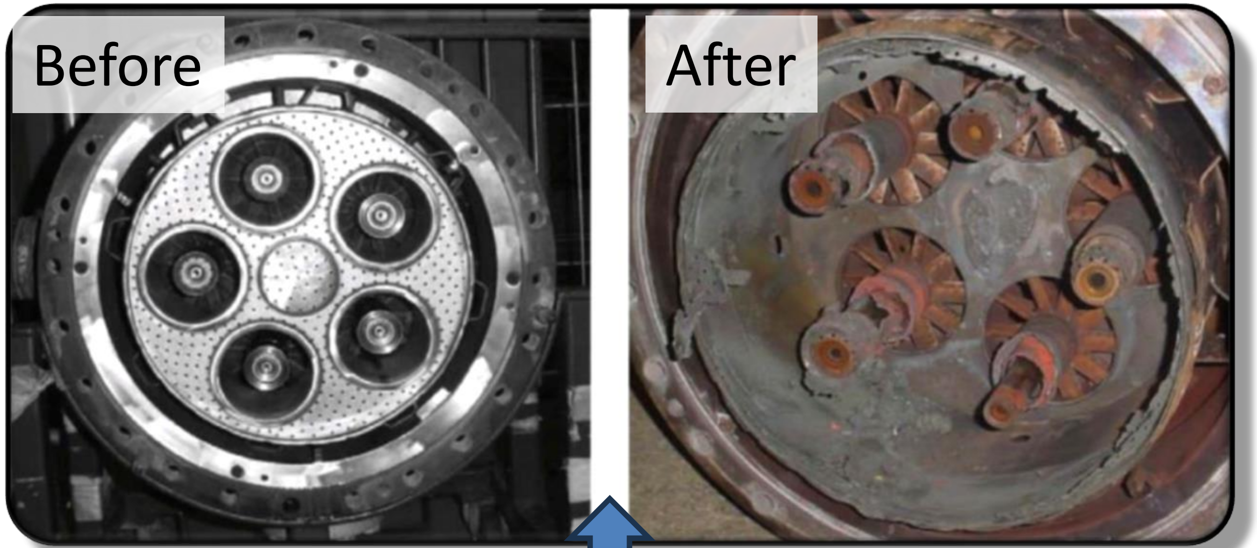


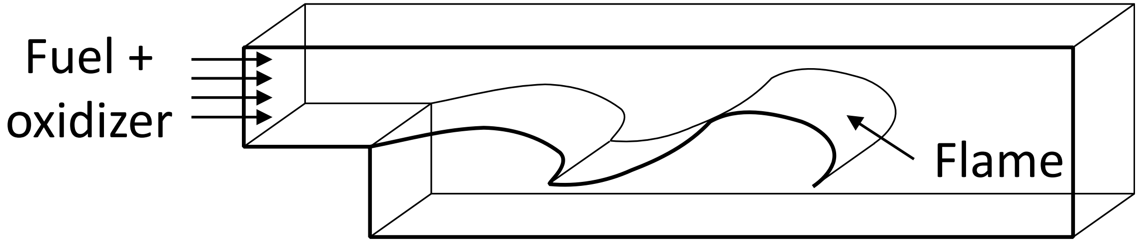
Thermoacoustic instabilities increase emissions, decrease efficiency or even destroy the engine





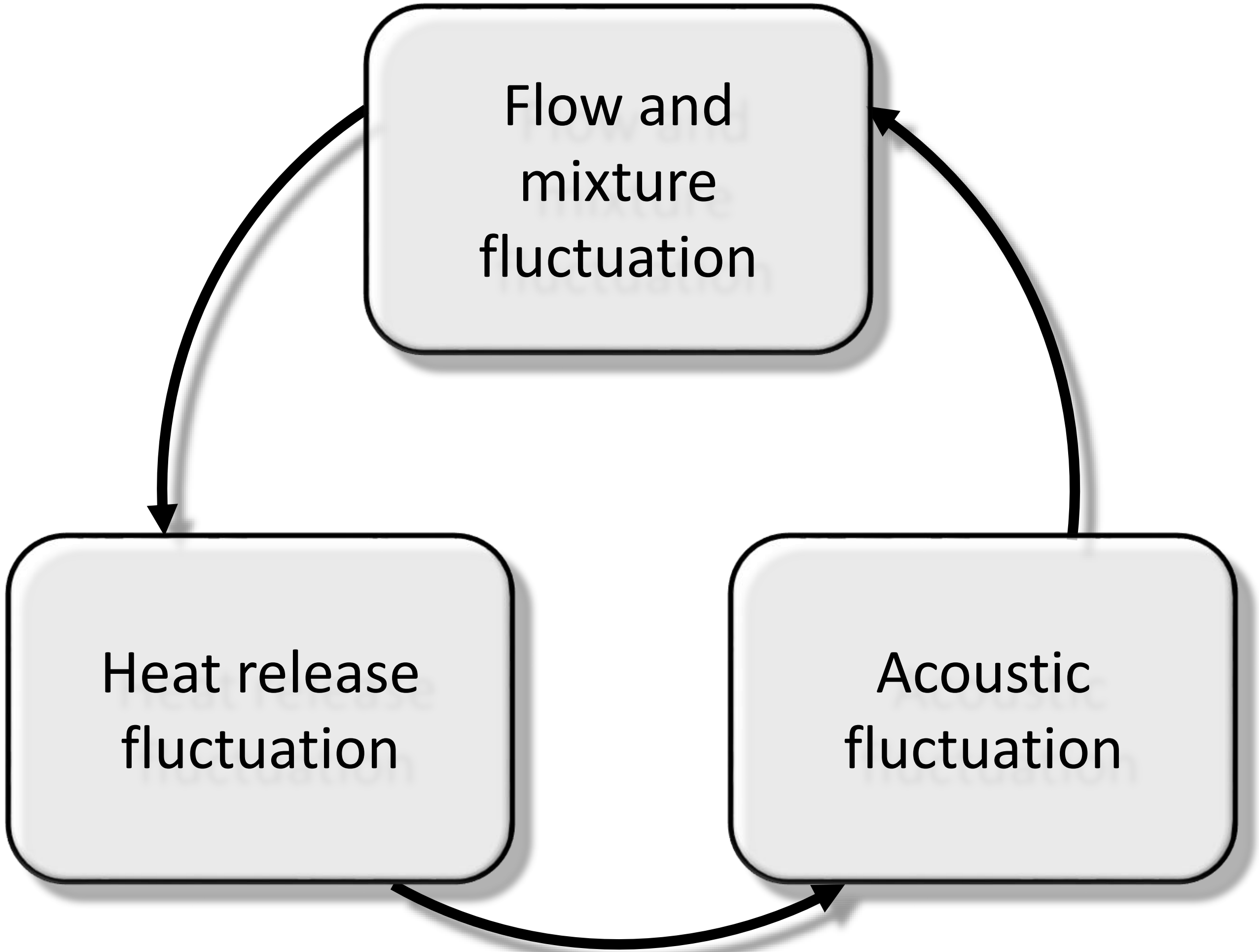
Lieuwen 2005

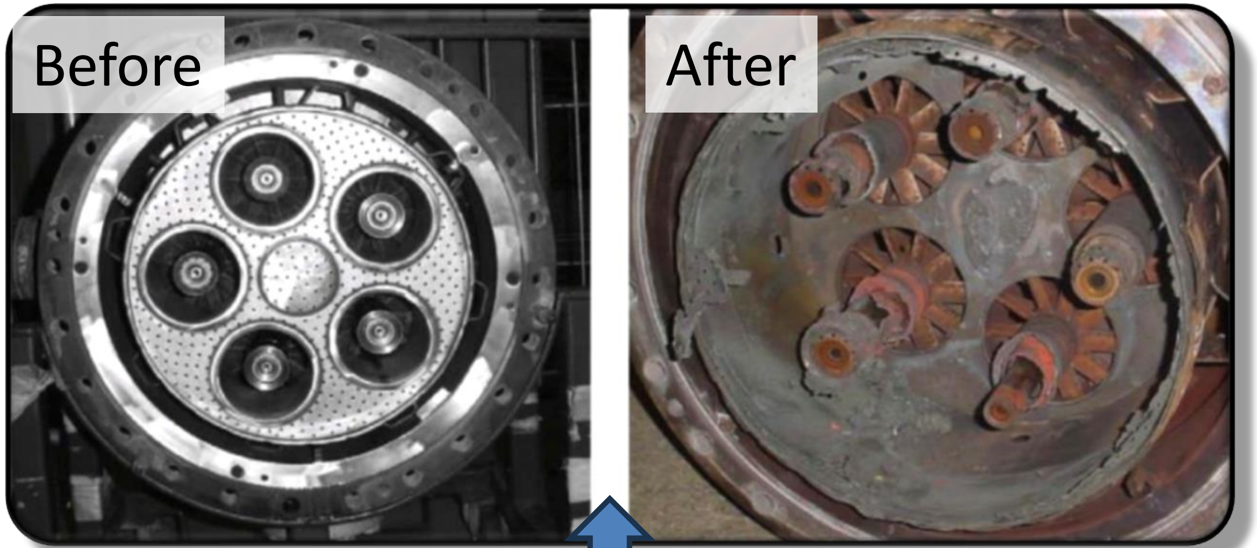
Flame configuration



Unstable operating point

Thermoacoustic instabilities increase emissions, decrease efficiency or even destroy the engine

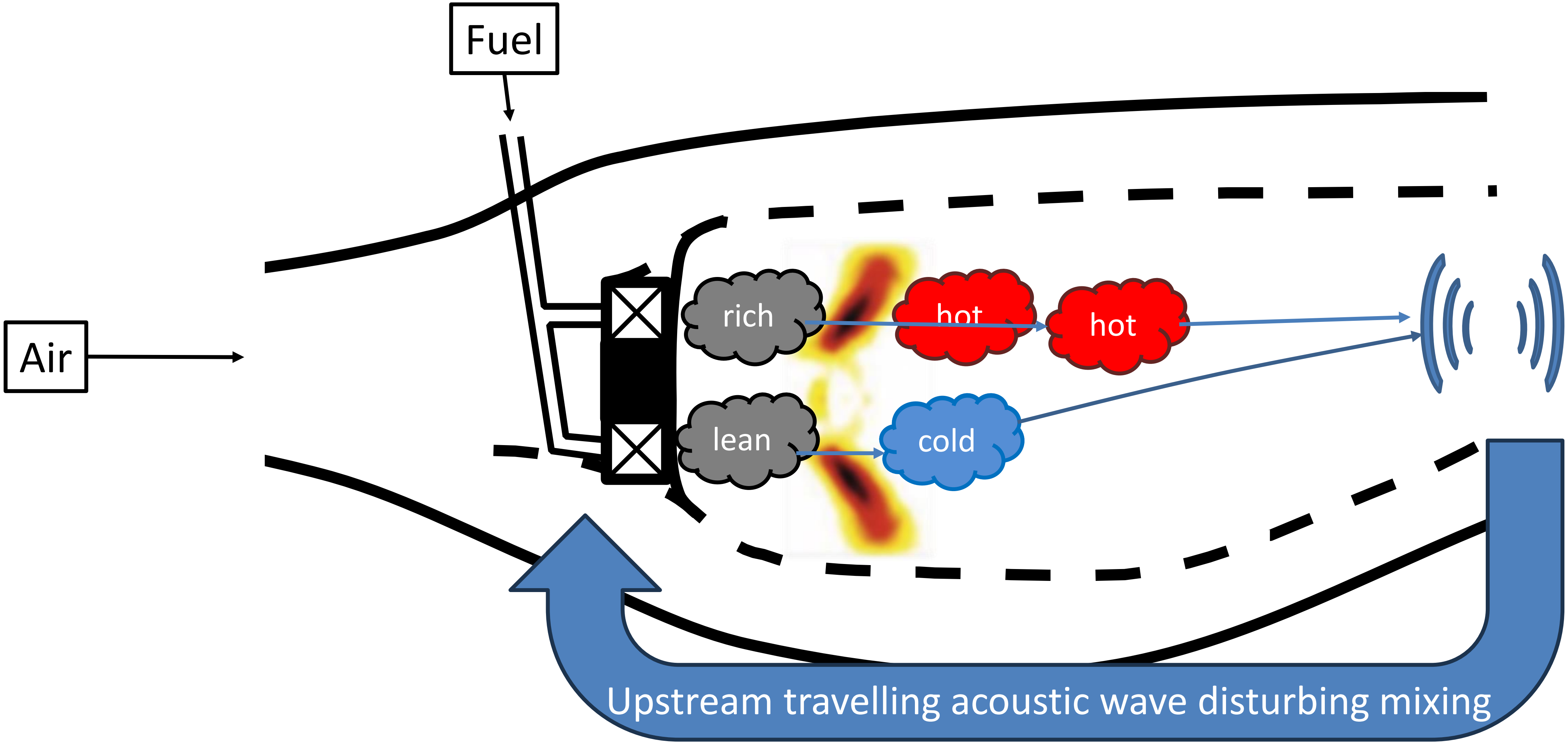




Lieuwen 2005

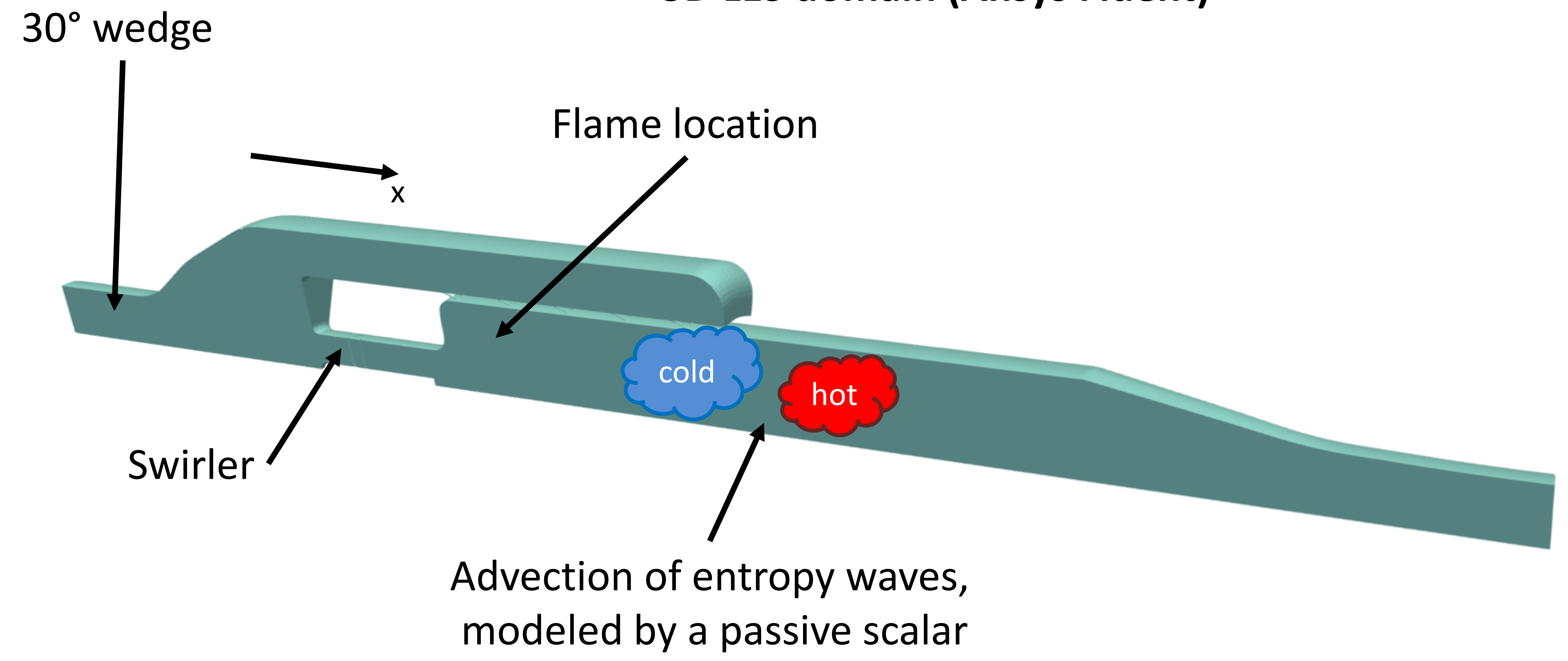
Unstable operating point

Entropy waves are one mechanism that can cause thermoacoustic instabilities



LES of a ANSYS industrial model burner

3D LES domain (Ansys Fluent)



Non-linear governing equations (Example: passive scalar transport)

Conservative form

$$\frac{\partial \rho c}{\partial t} + \nabla \cdot (\rho \mathbf{u} c) = \nabla \cdot (\rho D \nabla c)$$

$$\Phi = \overline{\Phi} + \widehat{\Phi} + \Phi'$$

Linear framework

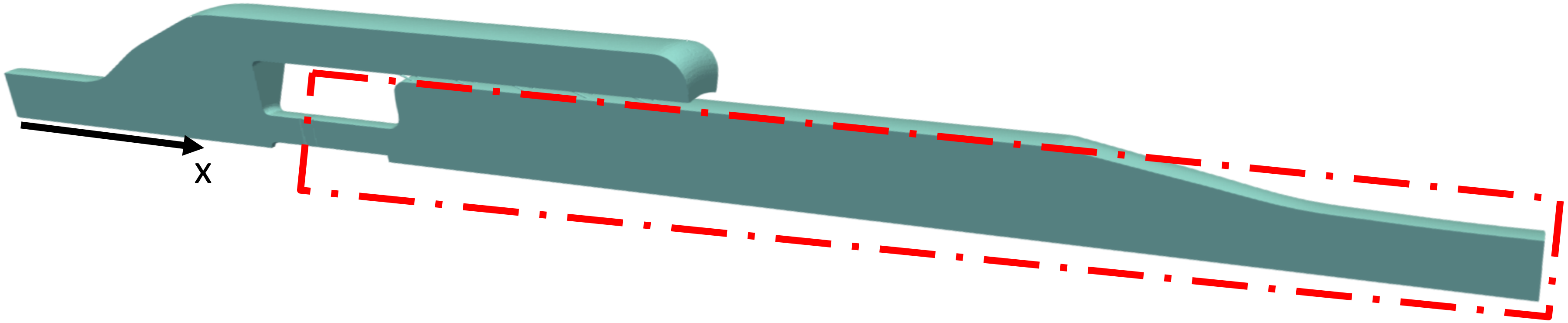
$$\widehat{\rho} = 0 \qquad \widehat{\mathbf{u}} = \mathbf{0}$$

Lin. transport equation, **conservative form**

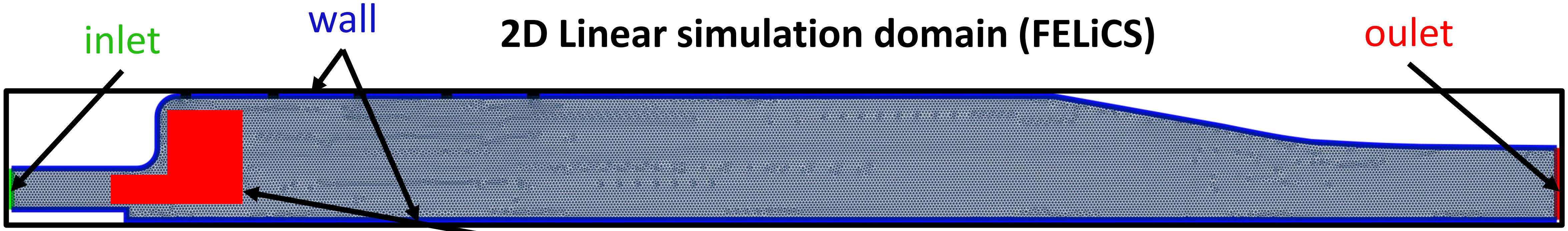
$$-i\omega\overline{\rho}\widehat{c} + \nabla \cdot (\overline{\rho}\,\overline{\mathbf{u}}\widehat{c}) = \nabla \cdot (\overline{\rho}\,(D + D_{\mathrm{t}})\,\nabla\widehat{c})$$

LES and linear simulation domain

3D LES domain (Ansys Fluent)



2D Linear simulation domain (FELiCS)



$$-i\omega\overline{\rho}\widehat{c} + \nabla\cdot\left(\overline{\rho}\,\overline{\mathbf{u}}\widehat{c}\right) = \nabla\cdot\left(\overline{\rho}\left(D + D_{\mathrm{t}}\right)\nabla\widehat{c}\right) + \widehat{\Omega}_c$$

Temporal means of state vartiables on the 2D FELiCS mesh

FLOW INSTABILITIES AND DYNAMICS

Density

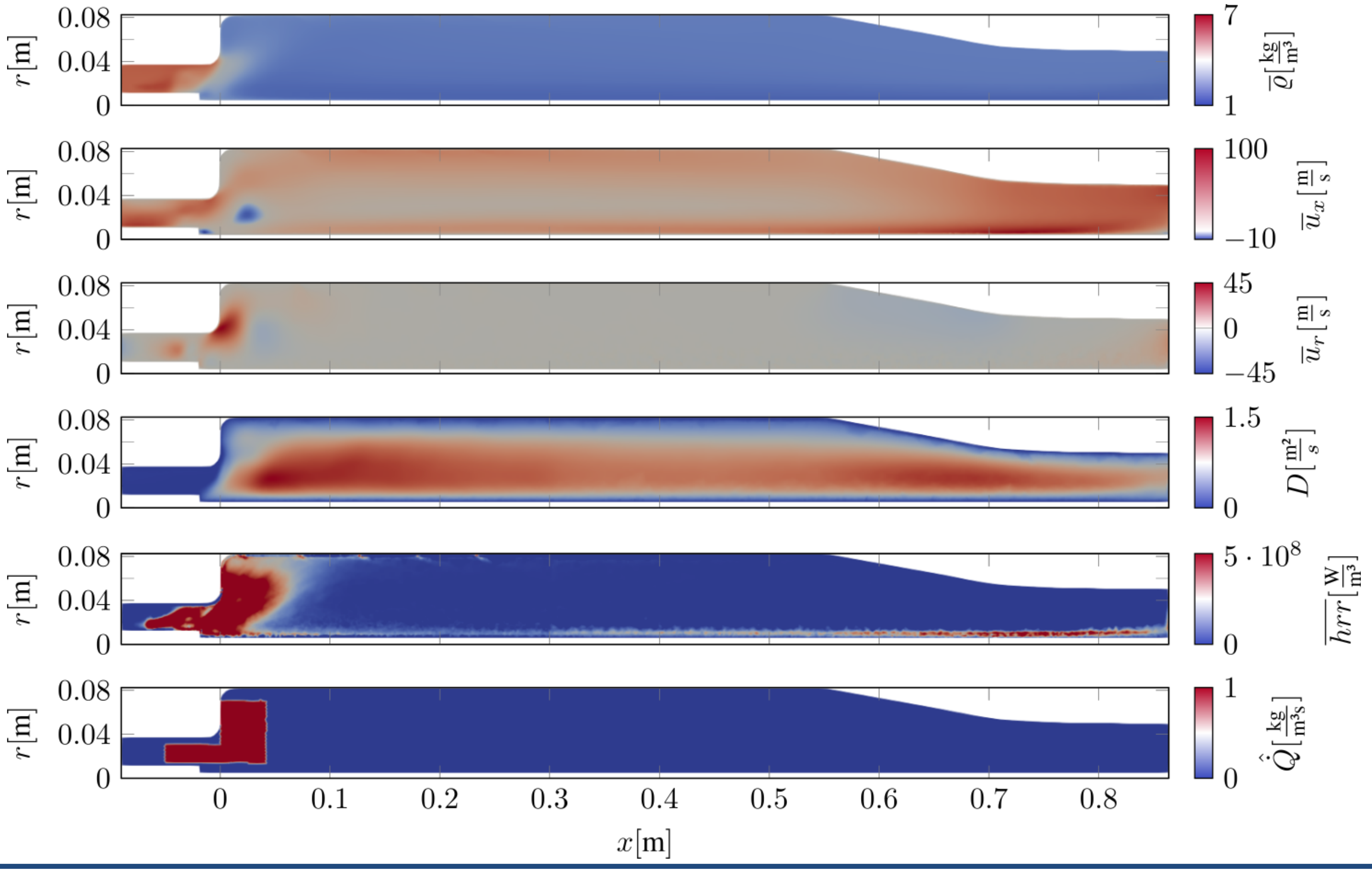
Axial velocity

Radial velocity

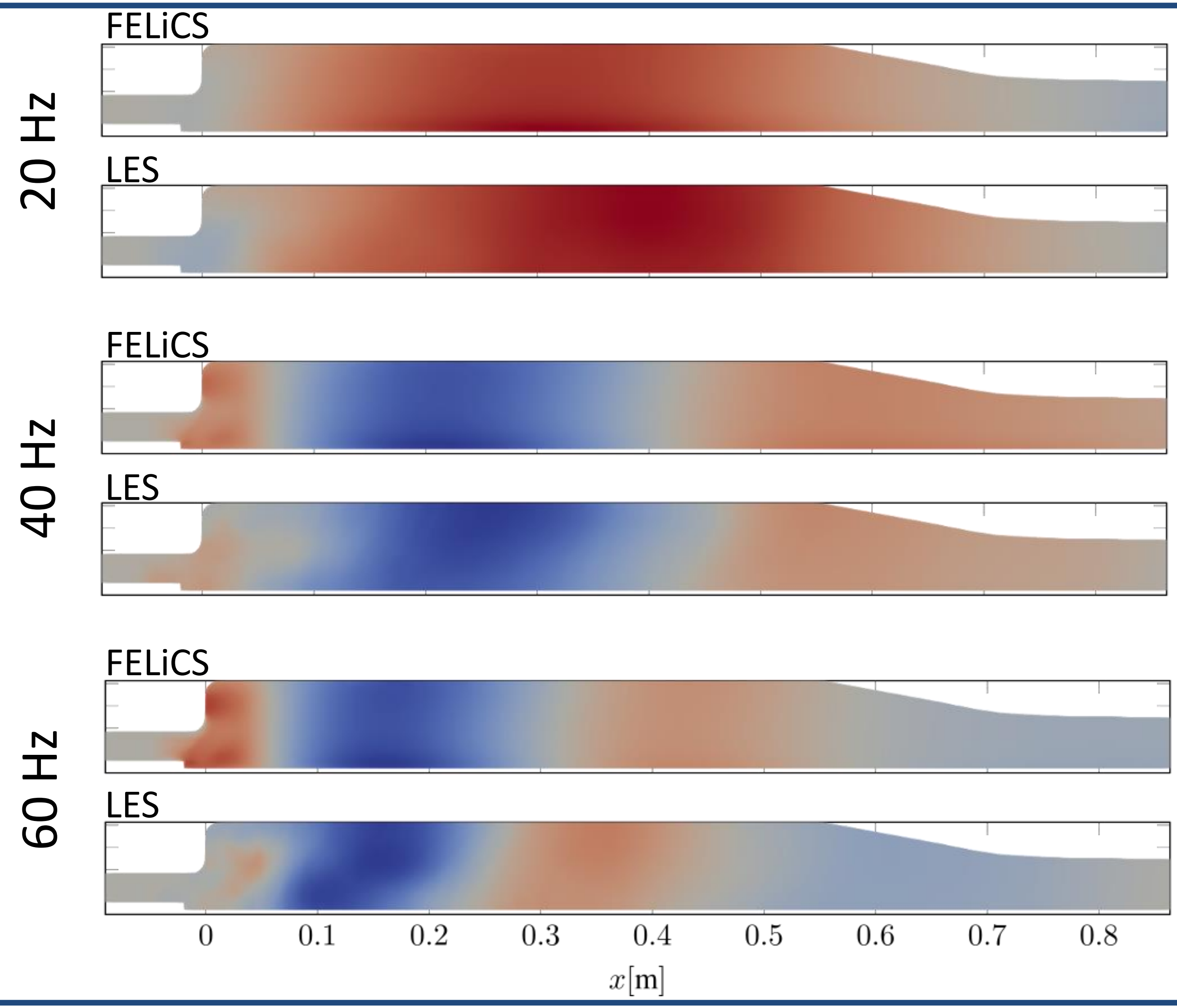
Resulting Diffusivity

Heat release

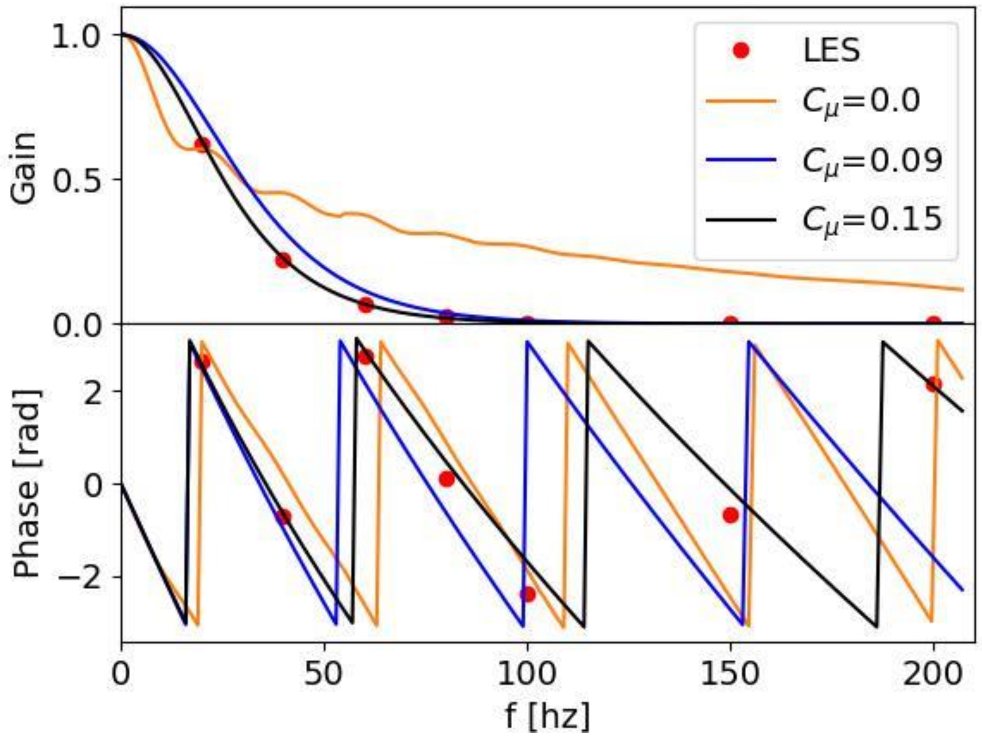
Source term



Linearized results agree excellent with LES.



$$\mathrm{TF}\left(f\right) = \frac{\iint \hat{c}\rho\mathbf{u}\cdot\mathbf{n}\,dA\Big|_{x_{\mathrm{outlet}}}}{\iiint \hat{\dot{Q}}\,dV}$$



Closure problem is even more significant in remaining transport equations

Non-linear governing equations (Example: passive scalar transport)

Conservative form

$$\frac{\partial \rho c}{\partial t} + \nabla \cdot (\rho \mathbf{u} c) = \nabla \cdot (\rho D \nabla c)$$

Advective form

$$\rho \frac{\partial c}{\partial t} + \rho \mathbf{u} \cdot \nabla c = \nabla \cdot (\rho D \nabla c)$$

$$\Phi = \overline{\Phi} + \widehat{\Phi} + \Phi'$$

Linear framework

$$\widehat{\rho} = 0 \qquad \widehat{\mathbf{u}} = 0$$

Lin. transport equation, **conservative form**

$$-i\omega\widehat{\overline{\rho} c} + \nabla \cdot (\overline{\rho}\, \overline{\mathbf{u}} \widehat{c}) = \nabla \cdot (\overline{\rho}\,(D + D_{\mathrm{t}})\,\nabla \widehat{c})$$

Lin. transport equation, **advective form**

$$-i\omega\widehat{\overline{\rho} c} + \overline{\rho}\, \overline{\mathbf{u}} \cdot \nabla \widehat{c} = \nabla \cdot (\overline{\rho}\,(D + D_{\mathrm{t}})\,\nabla \widehat{c})$$

Two versions of the linearized transport equation lead to different results.

FLOW INSTABILITIES AND DYNAMICS

Lin. transport equation, **conservative form**

$$-i\omega\overline{\rho}\widehat{c} + \nabla\cdot\left(\overline{\rho}\,\widehat{\overline{\mathbf{u}}c}\right) = \nabla\cdot\left(\overline{\rho}\left(D + D_{\mathrm{t}}\right)\nabla\widehat{c}\right)$$

RANS continuity equation

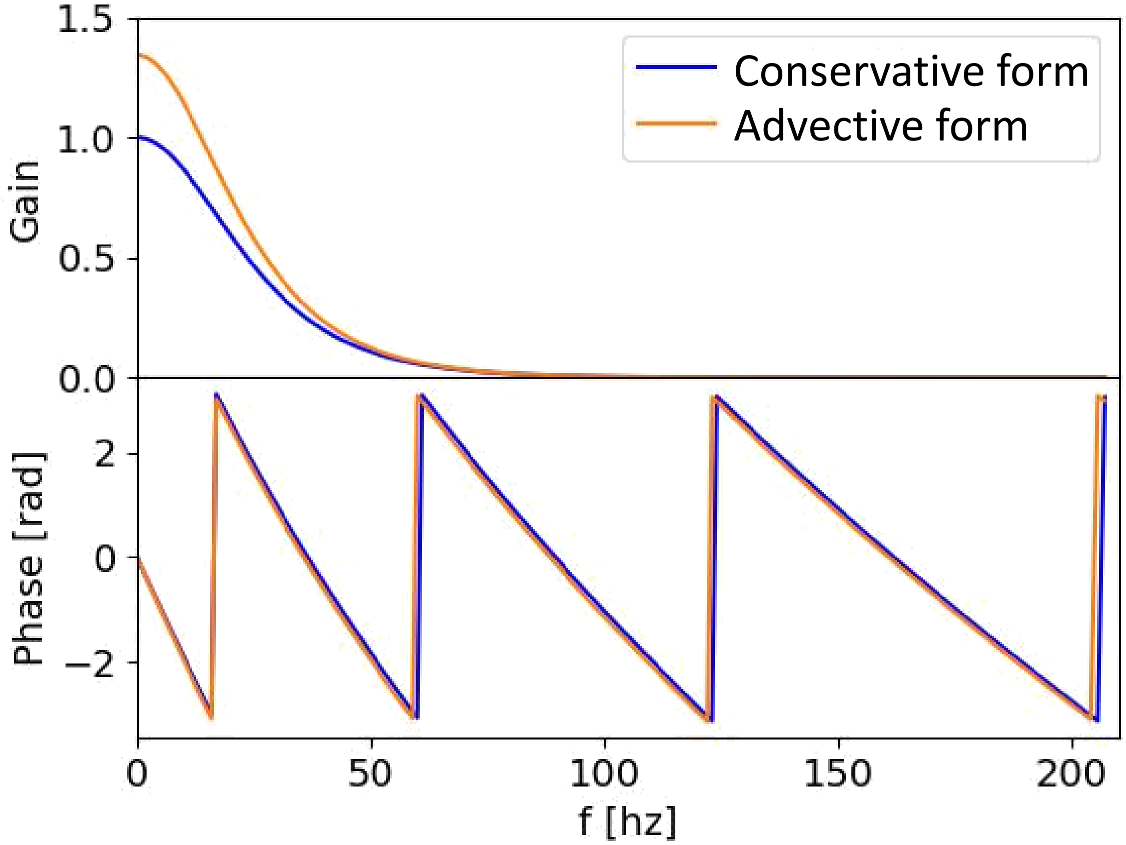
$$\nabla\cdot\left(\overline{\rho}\,\overline{\mathbf{u}}\right) \approx 0$$

$$\overline{\rho'\mathbf{u}'} \neq 0$$

Lin. transport equation, **advective form**

$$-i\omega\overline{\rho}\widehat{c} + \overline{\rho}\,\overline{\mathbf{u}}\cdot\nabla\widehat{c} = \nabla\cdot\left(\overline{\rho}\left(D + D_{\mathrm{t}}\right)\nabla\widehat{c}\right)$$

$$\mathrm{TF}\left(f\right) = \frac{\iint \hat{c}\rho\mathbf{u}\cdot\mathbf{n}\,dA\Big|_{x_{\mathrm{outlet}}}}{\iiint \hat{\dot{Q}}\,dV}$$



Closure problem is even more significant in remaining transport equations

Non-linear governing equations (Example: passive scalar transport)

$$\frac{\partial \rho c}{\partial t} + \boxed{\nabla \cdot (\rho \mathbf{u} c)} - \nabla \cdot (D \nabla c) = 0$$

$$\Phi = \overline{\Phi} + \widehat{\Phi} + \Phi'$$

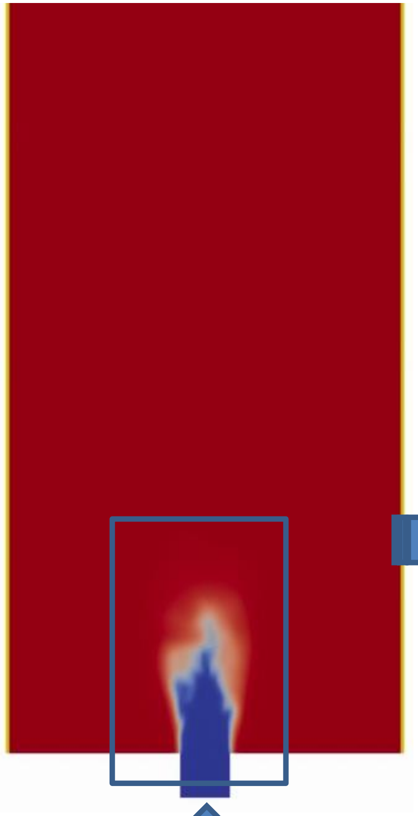
Linear framework

$$\nabla \cdot \widehat{(\rho \overline{\mathbf{u}} \overline{c})} + \nabla \cdot \widehat{(\rho \overline{\mathbf{u}} \widehat{c})} + \nabla \cdot \widehat{(\rho \overline{\mathbf{u}} c')}$$

$$+ \nabla \cdot \widehat{(\rho \widehat{\mathbf{u}} \overline{c})} + \nabla \cdot \widehat{(\rho \widehat{\mathbf{u}} \widehat{c})} + \nabla \cdot \widehat{(\rho \widehat{\mathbf{u}} c')}$$

$$+ \nabla \cdot \boxed{\widehat{(\rho \mathbf{u}' \overline{c})}} + \nabla \cdot \widehat{(\rho \mathbf{u}' \widehat{c})} + \nabla \cdot \widehat{(\rho \mathbf{u}' c')}$$

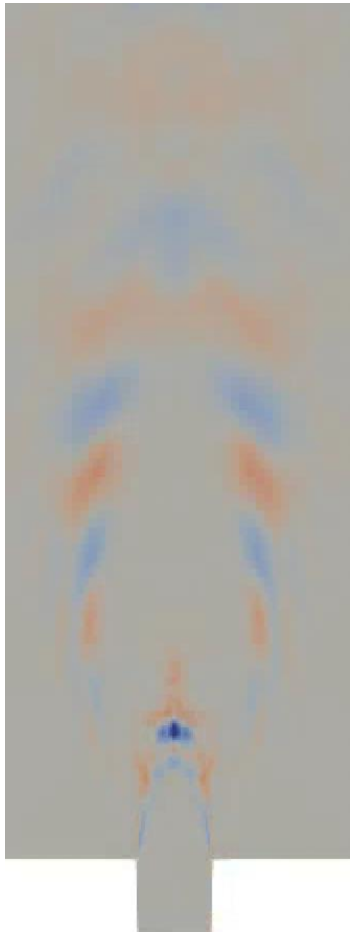
Applying the new tripple decomposition technique reduces closure problem in the remaining transport equations



Harmonic fluctuation (acoustic+passive scalar)

DFT

$$\widehat{(\rho \mathbf{u}'\overline{c})}$$



Solution: New tripple decomposition technique

RANS framework

$$\overline{\rho\Phi''} \overset{!}{=} 0$$

$$\Phi = \widetilde{\Phi} + \Phi''$$

$$\widetilde{\Phi} = \frac{\overline{\rho\Phi}}{\overline{\rho}}$$

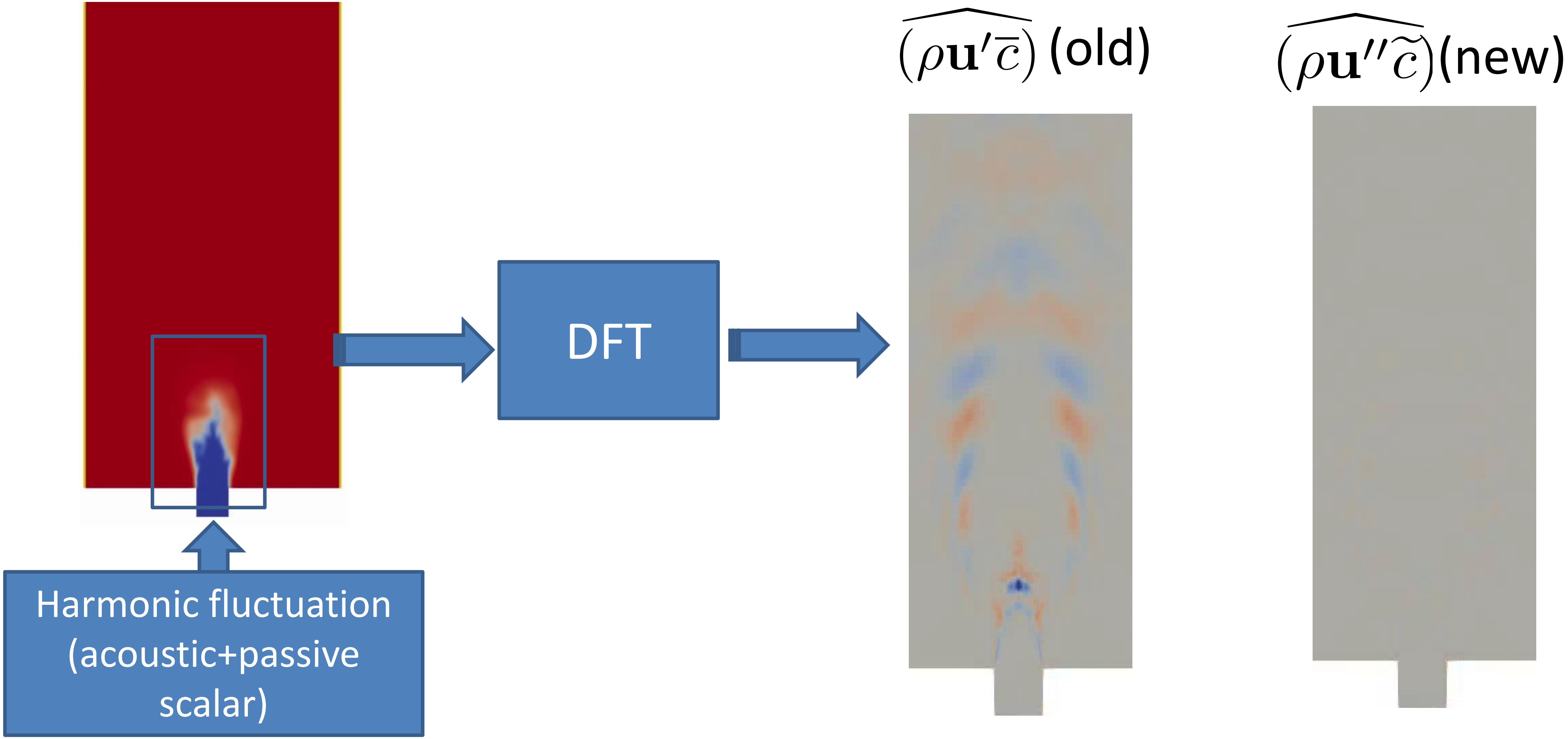
Linear framework

$$\widehat{\rho\Phi''} \overset{!}{=} 0$$

$$\Phi = \widetilde{\Phi} + \mathring{\Phi} + \Phi''$$

$$\mathring{\Phi} = \frac{\widehat{\rho\Phi} - \widehat{\rho}\widetilde{\Phi}}{\overline{\rho}}$$

Applying the new tripple decomposition technique reduces closure problem in the remaining transport equations



New tripple decomposition technique significantly reduces the closure problem

Non-linear governing equations (Example: passive scalar transport)

$$\frac{\partial \rho c}{\partial t} + \nabla \cdot (\rho \mathbf{u} c) - \nabla \cdot (D \nabla c) = 0$$

$$\Phi = \widetilde{\Phi} + \mathring{\Phi} + \Phi''$$

Linear framework

$$\nabla \cdot \widehat{(\rho \tilde{\mathbf{u}} \tilde{c})} + \nabla \cdot \widehat{(\rho \tilde{\mathbf{u}} \mathring{c})} + \nabla \cdot \widehat{(\rho \tilde{\mathbf{u}} c'')}$$
$$+ \nabla \cdot \widehat{(\rho \mathring{\mathbf{u}} \tilde{c})} + \nabla \cdot \widehat{(\rho \mathring{\mathbf{u}} \mathring{c})} + \nabla \cdot \widehat{(\rho \mathring{\mathbf{u}} c'')}$$
$$+ \nabla \cdot \widehat{(\rho \mathbf{u}'' \tilde{c})} + \nabla \cdot \widehat{(\rho \mathbf{u}'' \mathring{c})} + \nabla \cdot \widehat{(\rho \mathbf{u}'' c'')}$$

Inserting the tripple decomposition leads to unclosed linearized continuity equation

Non-linear governing equations (Example: mass transport)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

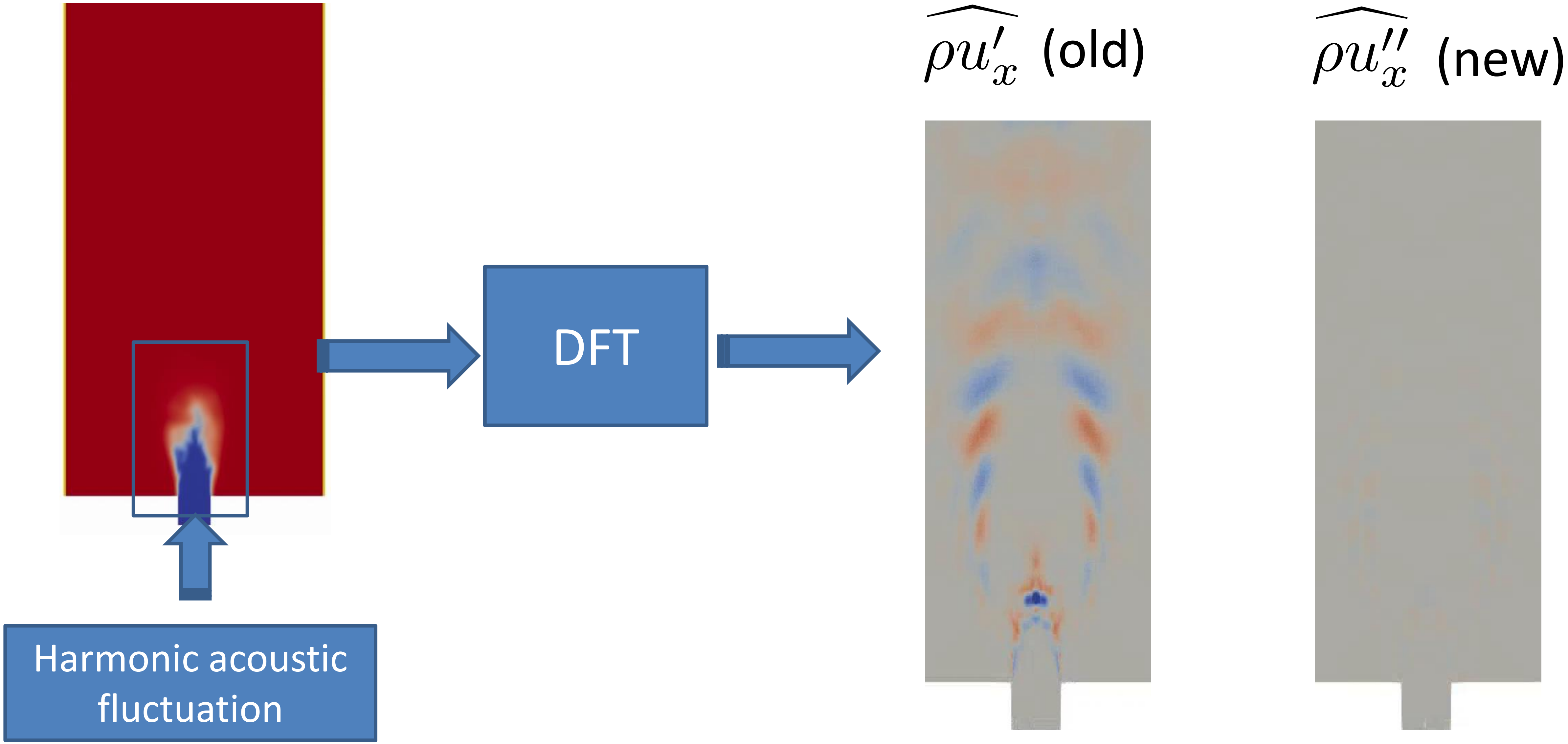
$$\Phi = \overline{\Phi} + \widehat{\Phi} + \Phi'$$

Linear framework

unclosed term

$$\frac{\partial \widehat{\rho}}{\partial t} + \nabla \cdot (\widehat{\rho}\overline{\mathbf{u}}) + \nabla \cdot (\overline{\rho}\widehat{\mathbf{u}}) + \nabla \cdot \widehat{(\rho \mathbf{u}')} = 0$$

Applying the new tripple decomposition technique leads to a closed continuity equation



Conclusion

In the case of linear analysis of turbulent non-uniform density flows:

- **Reynolds averaging leads to non-negligible additional closure problems in linear transport equations (analogous to RANS)**

$$\cancel{\Phi = \overline{\Phi} + \widehat{\Phi} + \Phi'}$$

- **Closure problem can be simplified by performing new decomposition technique**

$$\Phi = \widetilde{\Phi} + \mathring{\Phi} + \Phi''$$

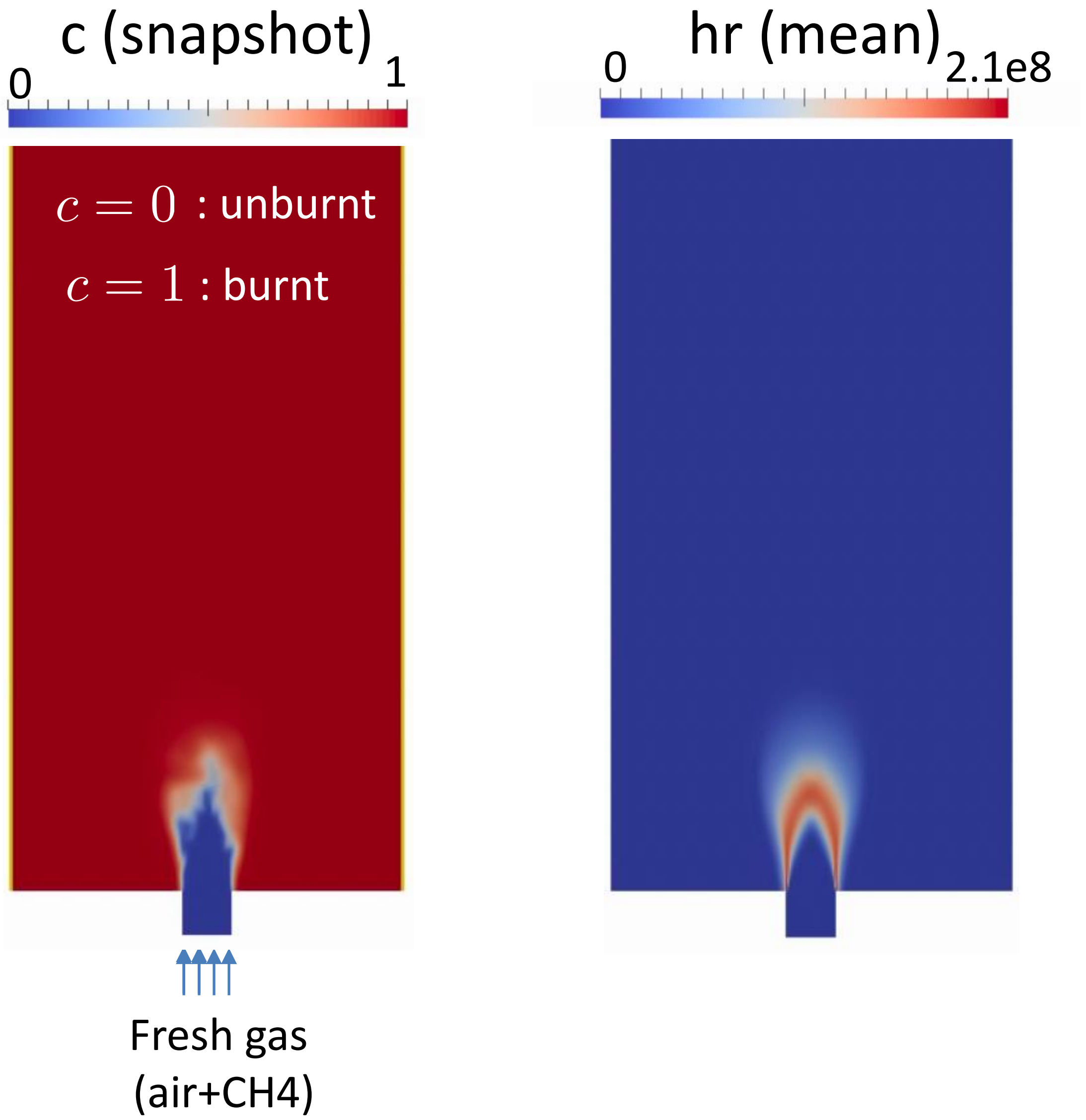
$$\widetilde{\Phi} = \frac{\overline{\rho\Phi}}{\overline{\rho}} \qquad \mathring{\Phi} = \frac{\widehat{\rho\Phi} - \widehat{\rho}\widetilde{\Phi}}{\overline{\rho}}$$

Hypotheses:

- **Different variations of turbulence closure are (part of) the cause of mean flow ambiguity (?)**
- **Addressing turbulence closure solves mean flow ambiguity (?)**

Case of choice: LES of Bunsen flame in cylindrical confinement (Kobayashi configuration)

FLOW INSTABILITIES AND DYNAMICS



Reference solution is a LES conducted at KIT

- Very low Mach number (Ma<0.015)
- Re = 8000

- openFoam
- Smagorinsky turbulence model
- Progress variable, c, in combination with a look-up table
- Turbulence Flamespeed Closure

$$\overline{\Omega}_c \propto \rho_0 \frac{S_t^2}{D_l + D_t} \tilde{c}(1 - \tilde{c})$$

$$\frac{S_t}{S_l} = 1 + \frac{u'}{S_l} \left(1 + Da^{-2}\right)^{-1/4}$$