On the importance of turbulence modelling in a linear mean field analysis of turbulent flame configurations

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Thomas Ludwig Kaiser, Thorsten Zirwes, Feichi Zhang, Kilian Oberleithner

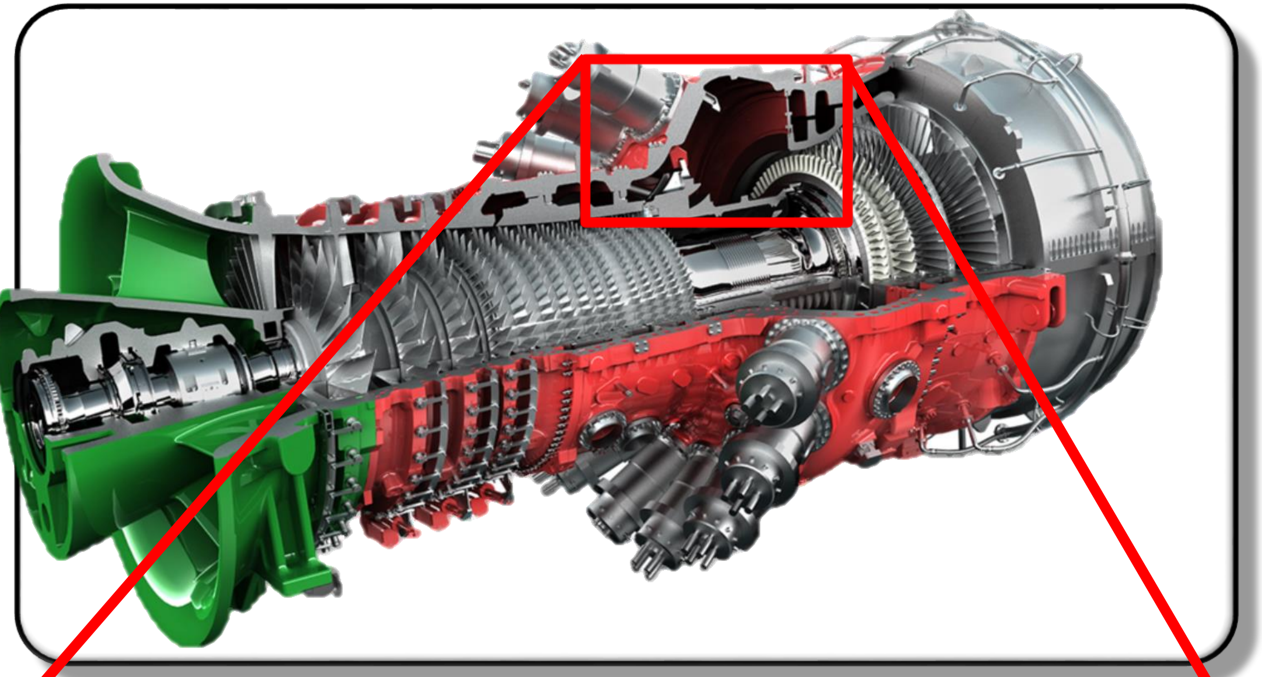
Technische Universität Berlin

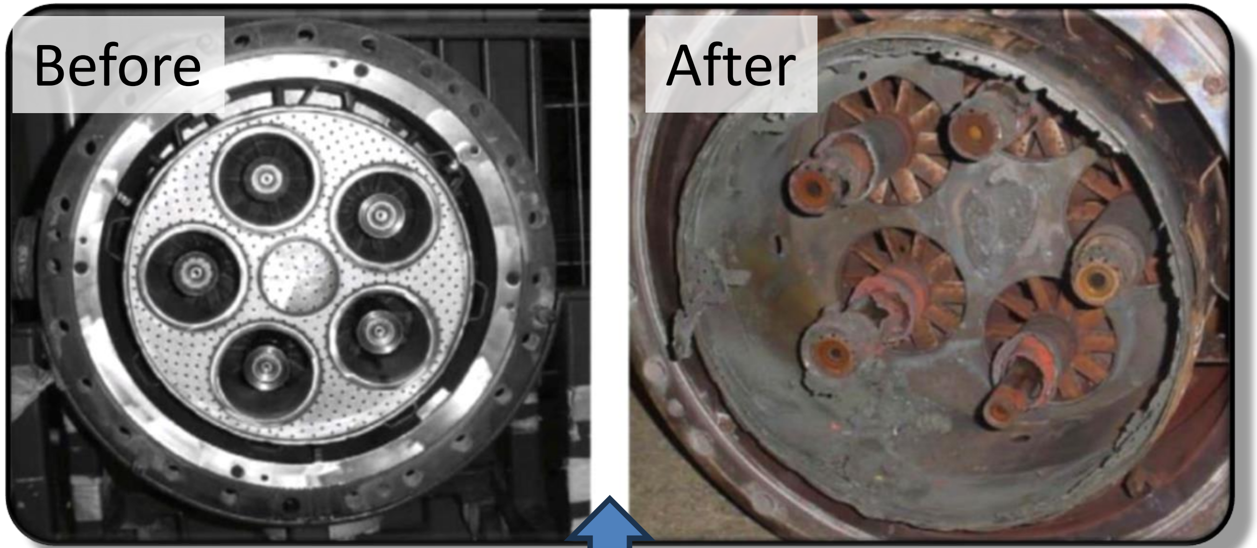
Karlsruher Institut für Technologie

University of Stuttgart

FLOW INSTABILITIES & DYNAMICS TECHNISCHE UNIVERSITÄT BERLIN

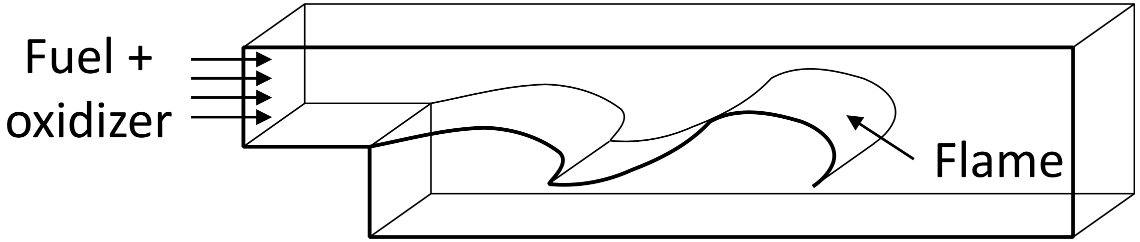
Thermoacoustic instabilities increase emissions, decrease efficiency or even destroy the engine





Lieuwen 2005

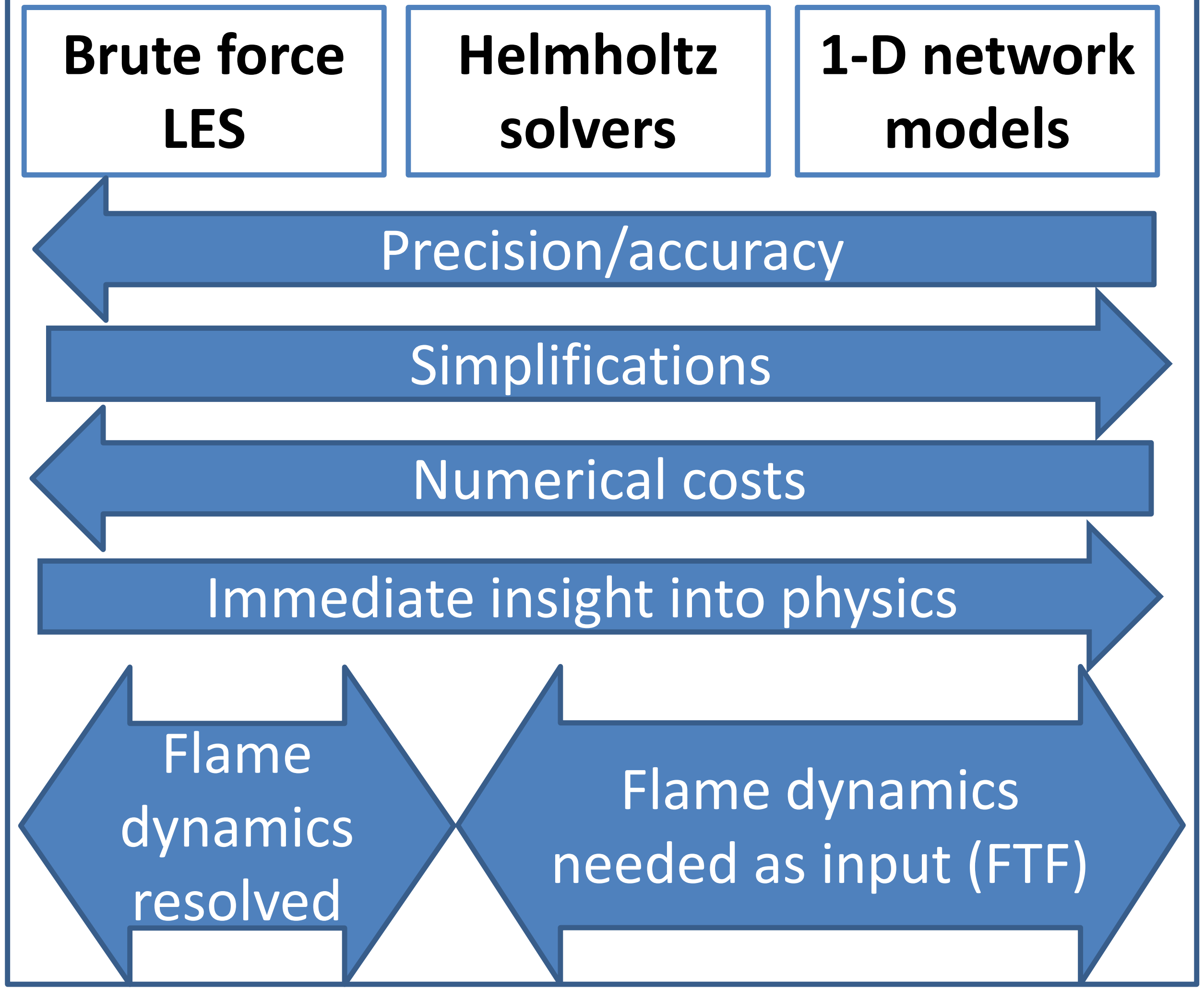
Flame configuration



Unstable operating point

Motivation: Prospect on linearized turbulent reacting flows on thermoacoustics

Current methods to study thermoacoustics



Brute force LES

Helmholtz solvers

1-D network models

Precision/accuracy

Simplifications

Numerical costs

Immediate insight into physics

Flame dynamics resolved

Flame dynamics needed as input (FTF)

Turbulent flame modelling in linearized equations of a turbulent reacting flow

- Holistic model including flame dynamics
- Fewer simplifications
- Immediate insight into physics (optimization)
- Shape/adjoint optimization

Talks in MS04: Linearized Modelling of Flame Dynamics

1 Thomas Ludwig Kaiser
TU Berlin, Germany

Challenges in modelling turbulent transport of density varying flows in the linear framework

2 Parth Patki
Georgia Tech, USA

Tri-global linear stability analysis in combustion systems

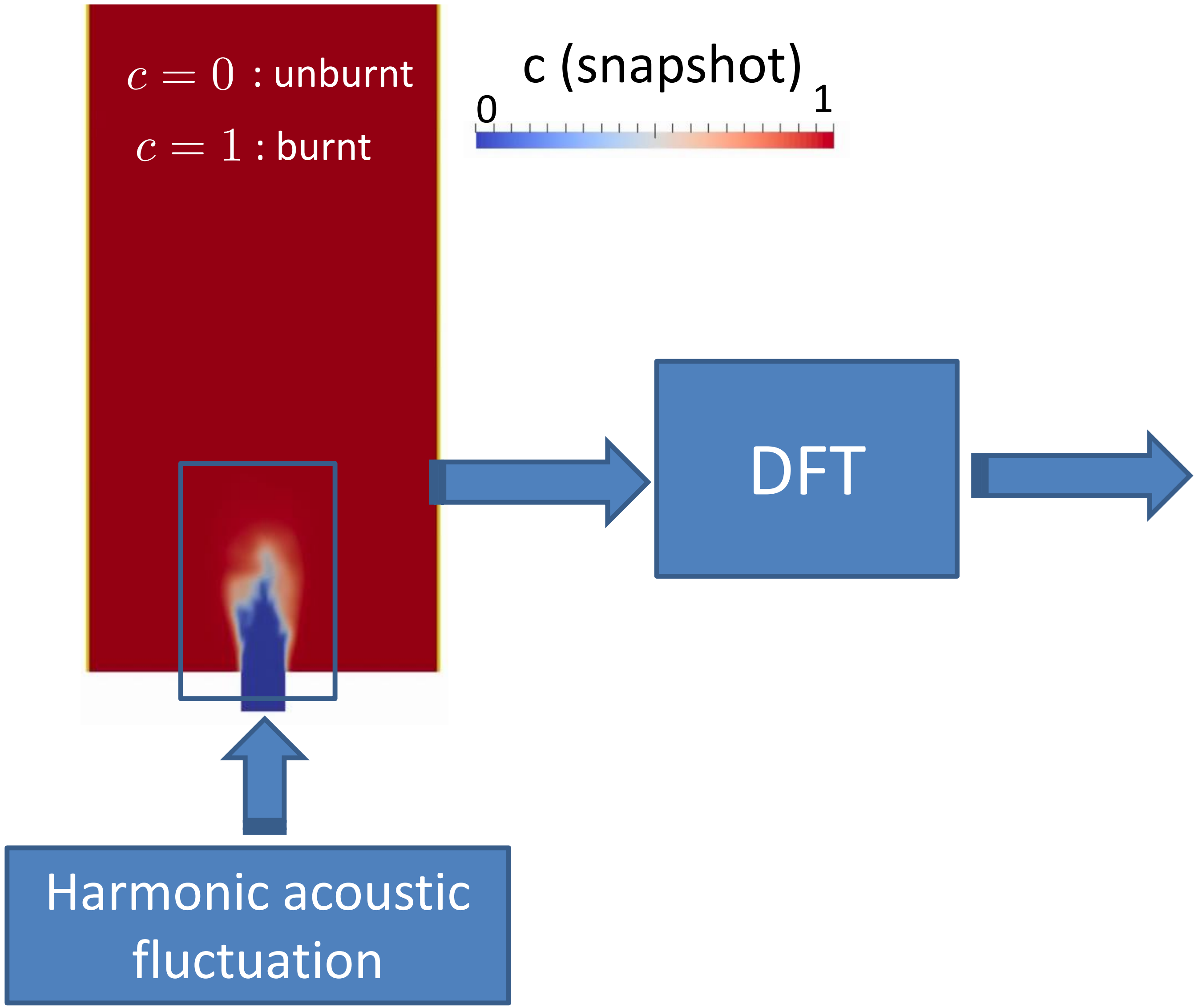
3 Philipp Brokof
TU Munich, Germany

Investigations on the feedback mechanism of intrinsic thermoacoustic modes

4 Chuhan Wang
Tsinghua University, China

Linear modelling of thermoacoustic feedback in a laminar flame combustor

Turbulent flame is acoustically forced and its response measured



$c = 0$: unburnt

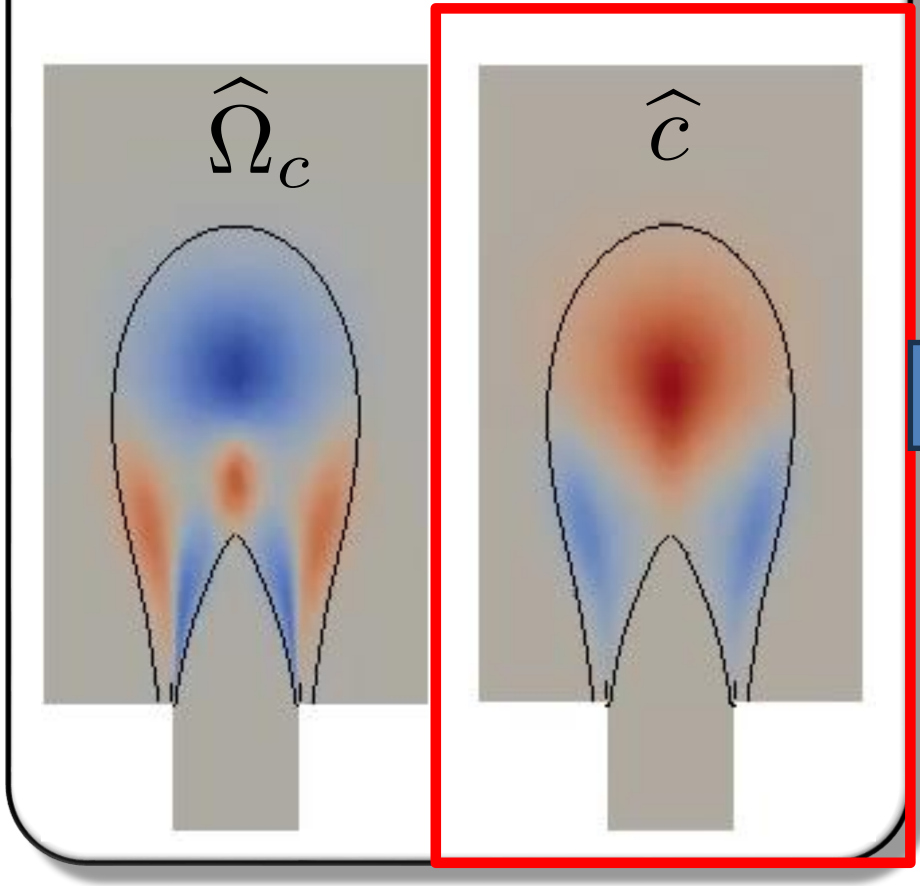
$c = 1$: burnt

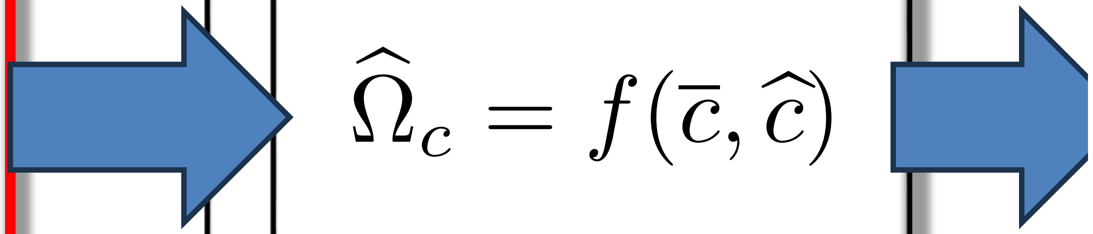
DFT

Harmonic acoustic fluctuation

A priori analysis of reaction rate fluctuations

LES: Response to acoustic actuation





A priori analysis

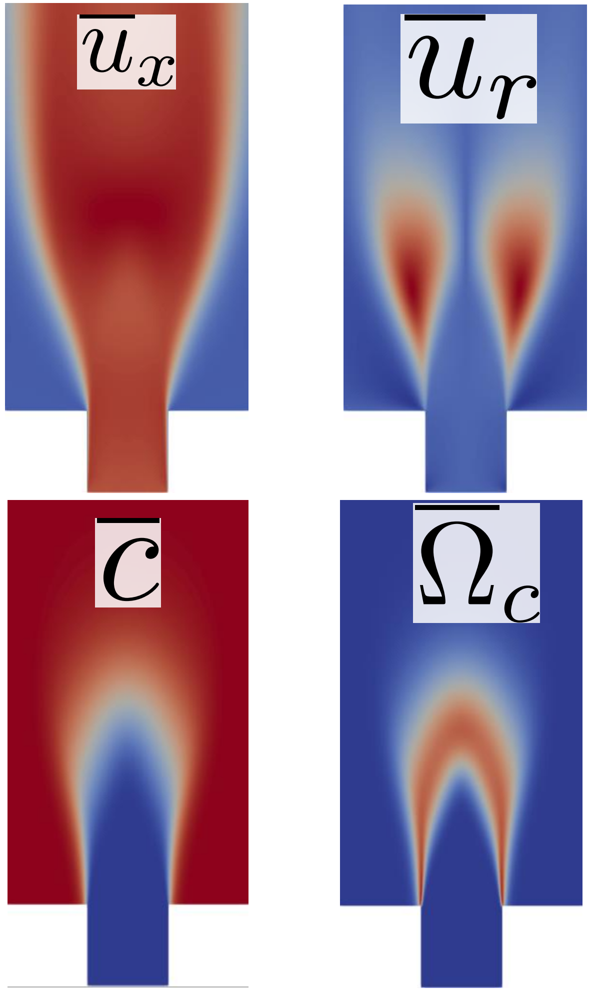
Linearized turbulent flame model

$$\widehat{\Omega}_c = f(\overline{c}, \widehat{c})$$

Kaiser et al., Comb. Flame, 2023

A posteriori analysis of the turbulent flame linearization using the linearized flow solver FELiCS

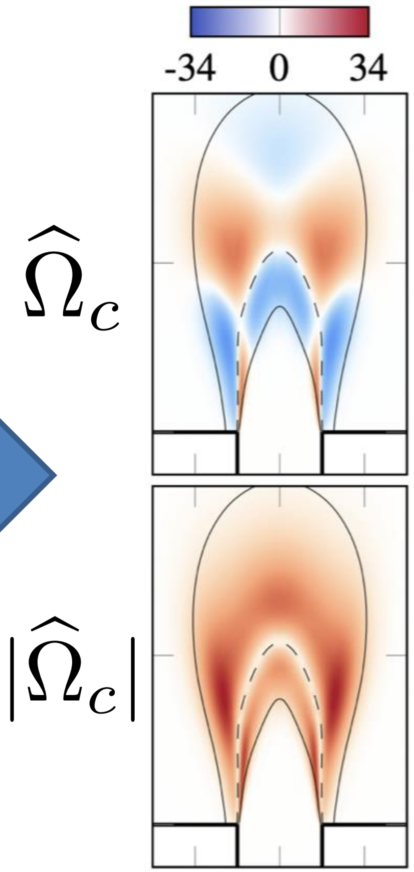
Temporal mean flow



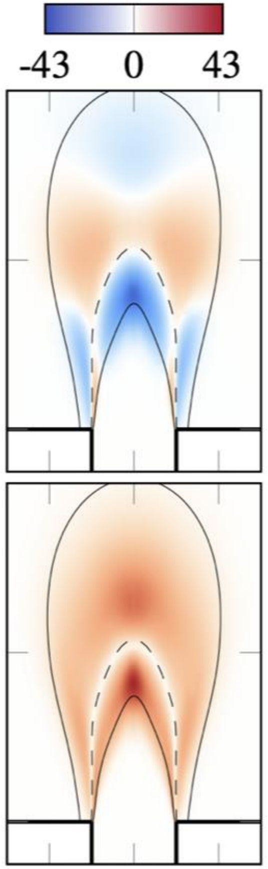
A posteriori analysis of entire set of equations (including linearized reaction model)

FELiCS

Response to acoustic forcing



Validation (LES)

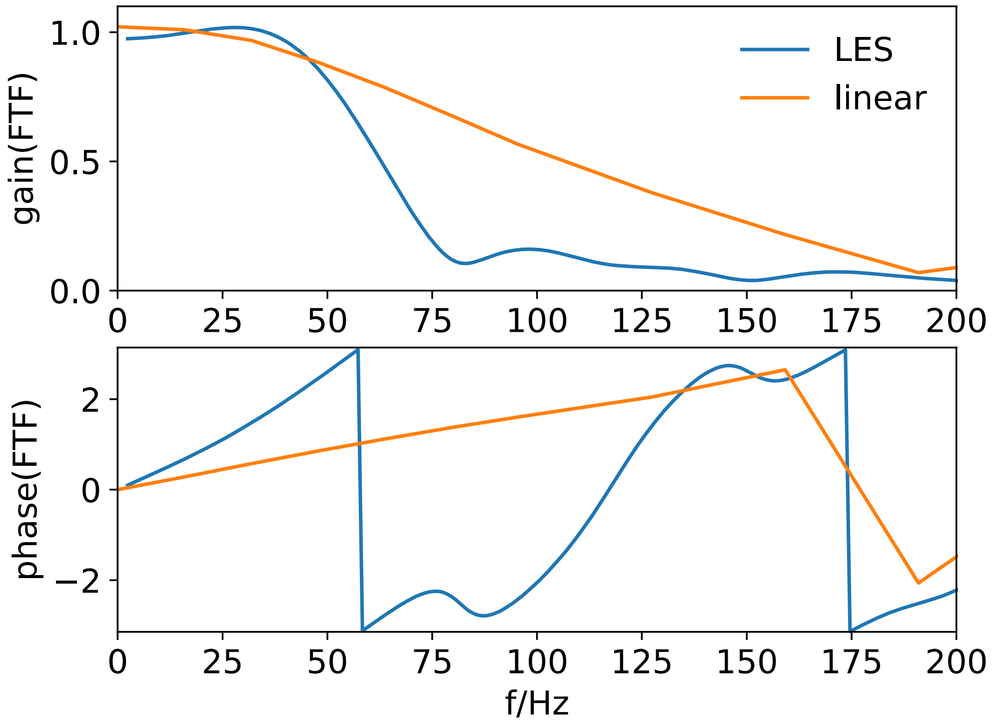


Flame transfer function, currently, incorrect. Probable cause: Insufficient turbulence modelling

Heat release fluctuation

$$\mathrm{FTF} = \frac{\widehat{q}/\overline{q}}{\widehat{u}_{\mathrm{ref}}/\overline{u}_{\mathrm{ref}}}$$

Acoustic velocity fluctuation



State of the Art for Linearization of Turbulent Reacting Flow

✓

✗

In the RANS framework, Reynolds averaging leads to unclosed terms

Non-linear governing equations (Example: mass transport)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\Phi = \overline{\Phi} + \Phi'$$

RANS

$$\nabla \cdot (\overline{\rho}\,\overline{\mathbf{u}}) + \boxed{\nabla \cdot \overline{(\rho \mathbf{u}')}} = 0$$

unclosed term

In the RANS framework, Favre averaging reduces the closure problem

Non-linear governing equations (Example: mass transport)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\Phi = \widetilde{\Phi} + \Phi''$$

RANS

$$\widetilde{\Phi} = \frac{\overline{\rho \Phi}}{\bar{\rho}}$$

$$\nabla \cdot (\bar{\rho}\, \widetilde{\mathbf{u}}) + \nabla \cdot \cancelto{0}{\overline{(\rho \mathbf{u}'')}} = 0$$

No additional closure problem in continuity equation

Inserting the tripple decomposition leads to unclosed linearized continuity equation

Non-linear governing equations (Example: mass transport)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

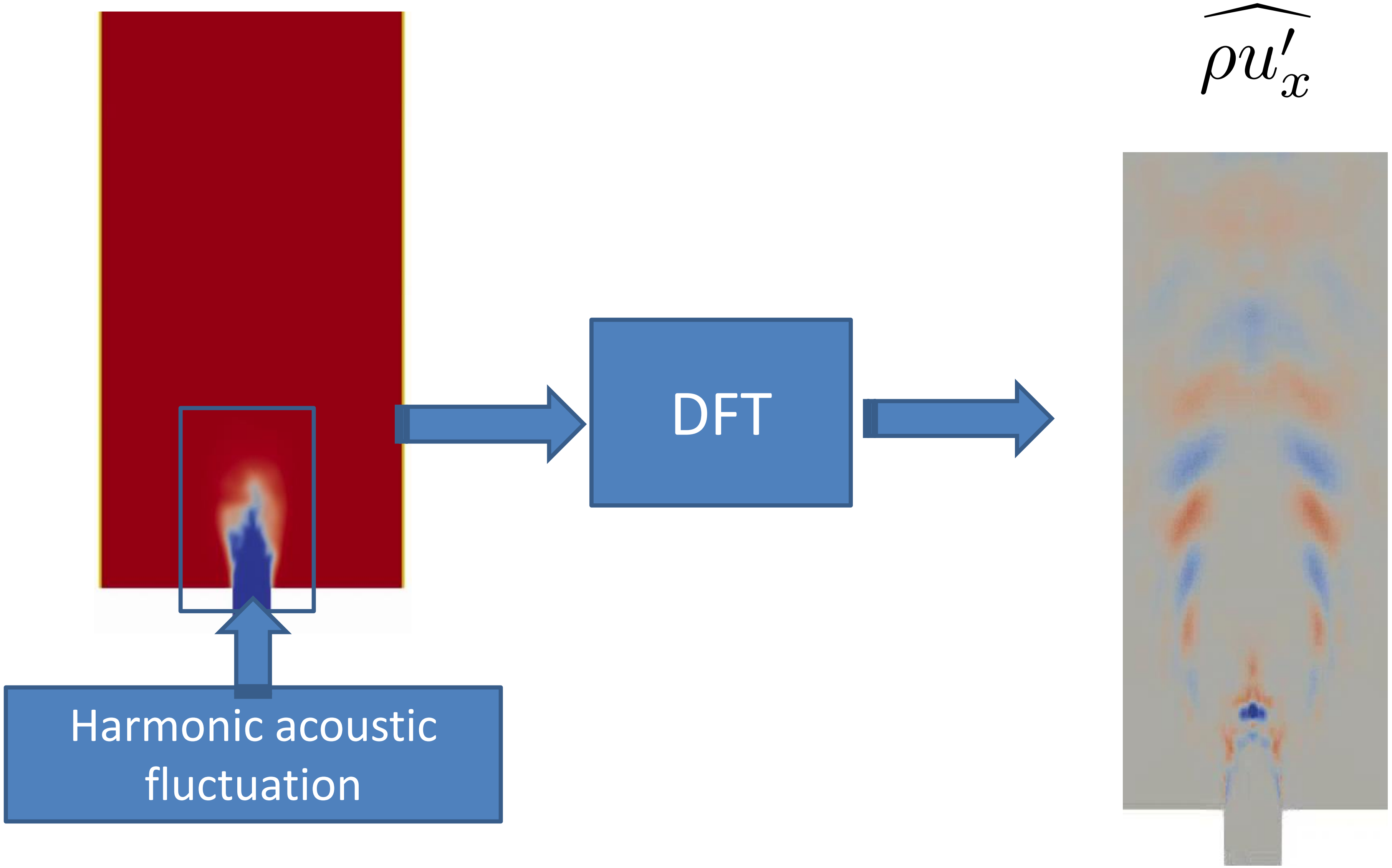
$$\Phi = \overline{\Phi} + \widehat{\Phi} + \Phi'$$

Linear framework

unclosed term

$$\frac{\partial \widehat{\rho}}{\partial t} + \nabla \cdot (\widehat{\rho}\overline{\mathbf{u}}) + \nabla \cdot (\overline{\rho}\widehat{\mathbf{u}}) + \boxed{\nabla \cdot \widehat{(\rho \mathbf{u}')}} = 0$$

Unclosed term in continuity equation



Harmonic acoustic fluctuation

DFT

$$\widehat{\rho u'_x}$$

Solution: New tripple decomposition technique

RANS framework

$$\overline{\rho\Phi''} \overset{!}{=} 0$$

↓

$$\Phi = \widetilde{\Phi} + \Phi''$$

↓

$$\widetilde{\Phi} = \frac{\overline{\rho\Phi}}{\overline{\rho}}$$

Linear framework

$$\widehat{\rho\Phi''} \overset{!}{=} 0$$

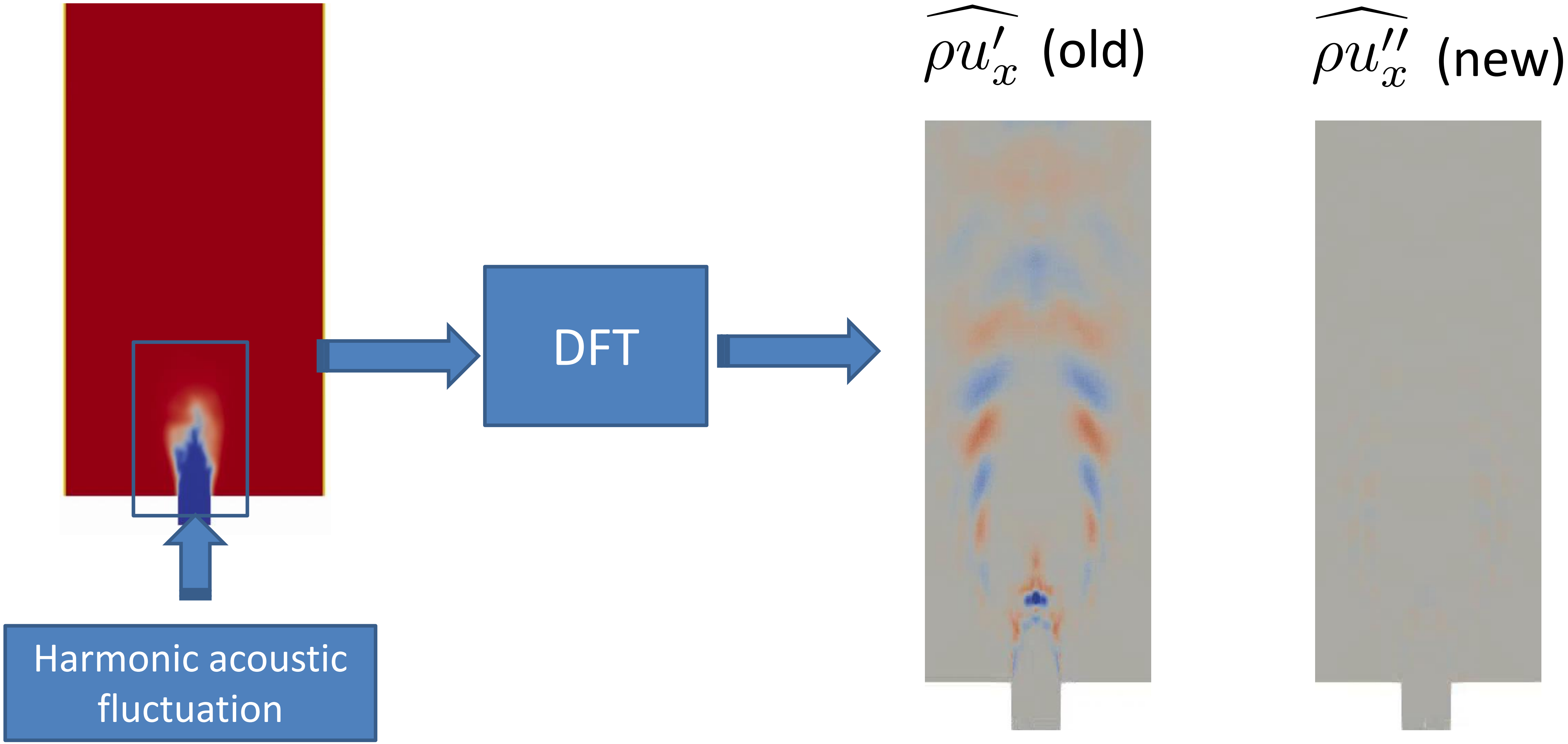
↓

$$\Phi = \widetilde{\Phi} + \mathring{\Phi} + \Phi''$$

↓

$$\mathring{\Phi} = \frac{\widehat{\rho\Phi} - \widehat{\rho}\widetilde{\Phi}}{\overline{\rho}}$$

Applying the new tripple decomposition technique leads to a closed continuity equation



Harmonic acoustic fluctuation

DFT

$\widehat{\rho u'_x}$ (old)

$\widehat{\rho u''_x}$ (new)

Closure problem is even more significant in remaining transport equations

Non-linear governing equations (Example: passive scalar transport)

$$\frac{\partial \rho c}{\partial t} + \boxed{\nabla\cdot(\rho\mathbf{u}c)} - \nabla\cdot(D\nabla c) = 0$$

$$\Phi = \overline{\Phi} + \widehat{\Phi} + \Phi'$$

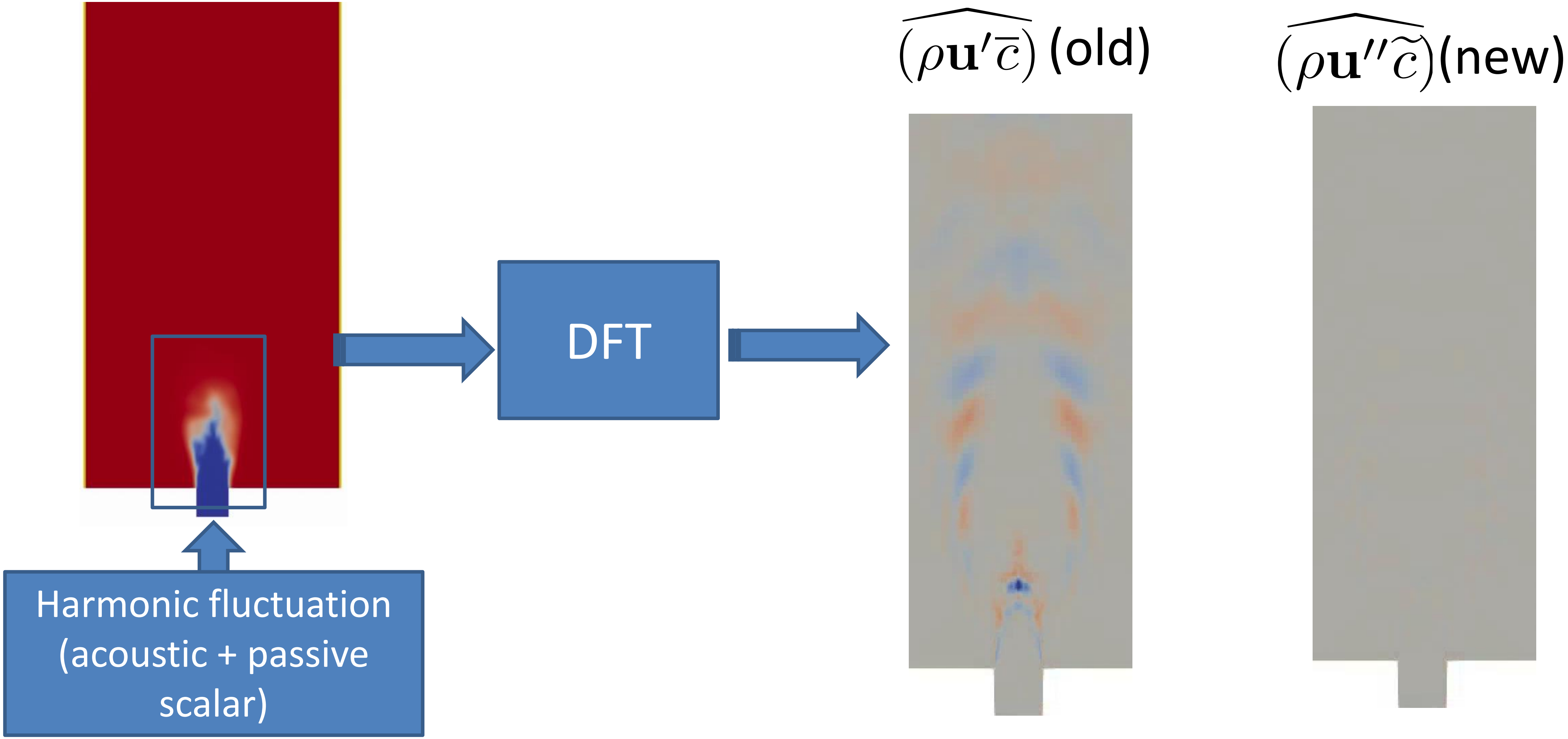
Linear framework

$$\nabla\cdot\boxed{\widehat{(\rho\overline{\mathbf{u}}\overline{c})}} + \nabla\cdot\boxed{\widehat{(\rho\overline{\mathbf{u}}\widehat{c})}} + \nabla\cdot\boxed{\widehat{(\rho\overline{\mathbf{u}}c')}}$$

$$+\nabla\cdot\boxed{\widehat{(\rho\widehat{\mathbf{u}}\overline{c})}} + \nabla\cdot\cancel{\widehat{(\rho\widehat{\mathbf{u}}\widehat{c})}} + \nabla\cdot\boxed{\widehat{(\rho\widehat{\mathbf{u}}c')}}$$

$$+\nabla\cdot\boxed{\widehat{(\rho\mathbf{u}'\overline{c})}} + \nabla\cdot\boxed{\widehat{(\rho\mathbf{u}'\widehat{c})}} + \nabla\cdot\boxed{\widehat{(\rho\mathbf{u}'c')}}$$

Applying the new tripple decomposition technique reduces closure problem in the remaining transport equations



$\widehat{(\rho \mathbf{u}'\overline{c})}$ (old)

$\widehat{(\rho \mathbf{u}''\widetilde{c})}$(new)

DFT

Harmonic fluctuation (acoustic + passive scalar)

New tripple decomposition technique significantly reduces the closure problem

Non-linear governing equations (Example: passive scalar transport)

$$\frac{\partial \rho c}{\partial t} + \boxed{\nabla \cdot (\rho \mathbf{u} c)} - \nabla \cdot (D \nabla c) = 0$$

$$\Phi = \widetilde{\Phi} + \mathring{\Phi} + \Phi''$$

Linear framework

$$\begin{aligned}
&\nabla \cdot \boxed{\widehat{(\rho \tilde{\mathbf{u}} \tilde{c})}} + \nabla \cdot \boxed{\widehat{(\rho \tilde{\mathbf{u}} \mathring{c})}} + \nabla \cdot \cancel{\widehat{(\rho \tilde{\mathbf{u}} c'')}} \\
&+ \nabla \cdot \boxed{\widehat{(\rho \mathring{\mathbf{u}} \tilde{c})}} + \nabla \cdot \cancel{\widehat{(\rho \mathring{\mathbf{u}} \mathring{c})}} + \nabla \cdot \cancel{\widehat{(\rho \mathring{\mathbf{u}} c'')}} \\
&+ \nabla \cdot \cancel{\widehat{(\rho \mathbf{u}'' \tilde{c})}} + \nabla \cdot \cancel{\widehat{(\rho \mathbf{u}'' \mathring{c})}} + \nabla \cdot \boxed{\widehat{(\rho \mathbf{u}'' c'')}}
\end{aligned}$$

Conclusion

- **State of the Art for Linearization of Turbulent Reacting Flow**

✓

✗

- **Reynolds averaging leads to non-negligible additional closure problems in linear transport equations (analogous to RANS)**

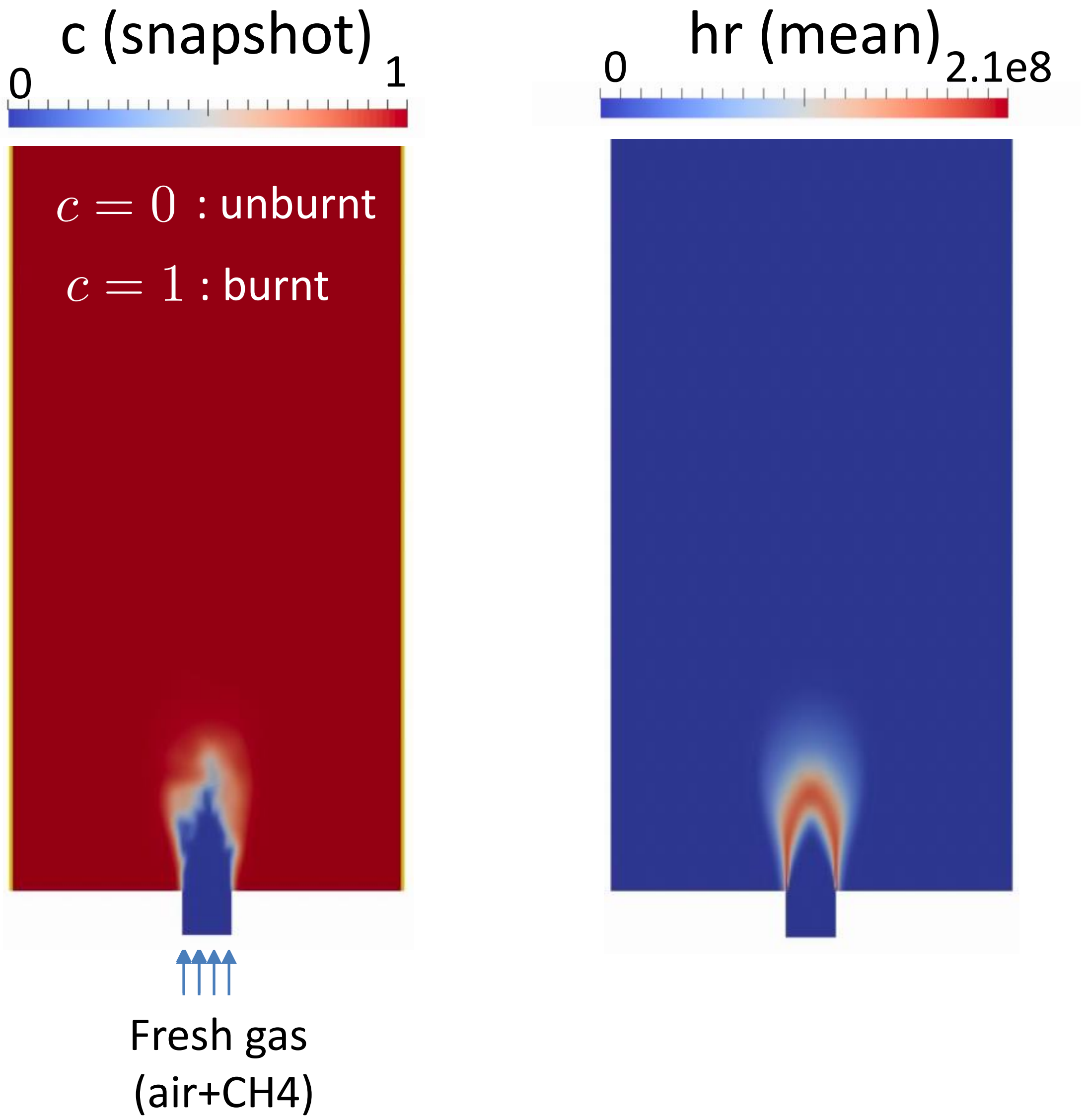
$$\xcancel{\Phi = \overline{\Phi} + \widehat{\Phi} + \Phi'}$$

- **Closure problem can be simplified by performing new decomposition technique i.e. linearize around Favre mean state**

$$\Phi = \widetilde{\Phi} + \mathring{\Phi} + \Phi''$$

$$\widetilde{\Phi} = \frac{\overline{\rho\Phi}}{\overline{\rho}} \qquad \mathring{\Phi} = \frac{\widehat{\rho\Phi} - \widehat{\rho}\widetilde{\Phi}}{\overline{\rho}}$$

Case of choice: LES of Bunsen flame in cylindrical confinement (Kobayashi configuration)



Reference solution is a LES conducted at KIT

- Very low Mach number (Ma<0.015)
- Re = 8000

- openFoam
- Smagorinsky turbulence model
- Progress variable, c, in combination with a look-up table
- Turbulence Flamespeed Closure

$$\overline{\Omega}_c \propto \rho_0 \frac{S_t^2}{D_l + D_t} \tilde{c}(1 - \tilde{c})$$

$$\frac{S_t}{S_l} = 1 + \frac{u'}{S_l}\left(1 + Da^{-2}\right)^{-1/4}$$