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Simulation Of Fuzzy Probability Based Random Fields With Non-Gaussian Marginal Distributions By Efficient Nataf Transformation Method

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If spatially varying uncertain quantities are significant for the results of structural analyses, random fields can be incorporated as uncertainty models in uncertainty quantifications to realistically assess structural safety. A random field is an uncertainty model that is based on probability theory and defined by marginal distributions as well as by a correlation structure based on an autocorrelation function. According to (Liu and Der Kiureghian, 1986; Vořechovský, 2008), if the marginal distributions are not Gaussian normal distributed, the Nataf transformation is required to simulate correlated random variates. A modified correlation coefficient has to be calculated based on the marginal distributions and on the target correlation coefficient. The modified correlation coefficient is then applied for the standard scheme of simulating correlated Gaussian normal random variates, which are finally transformed to the required marginal distributions. Then, the resulting correlation of the random samples fulfills the prescribed correlation structure defined by the target correlation coefficient.

Three major aspects are often neglected in traditional engineering approaches. First, the distribution functions of random variates are assumed to be Gaussian normal distributed. Hence, experimentally measured data is reduced to statistical moments, which are then applied in a normal distribution and many information about the original distribution is lost. Second, if non-Gaussian distributions are considered, the Nataf transformation is often neglected for the simulation of correlated random variates, which results in incorrect correlation relationships. Both simplifications influence the results of uncertainty quantifications and, hence, should instead be considered comprehensively in order to ensure a structural safety assessment close to reality.

The third aspect is the neglection of non-stochastic uncertainties. Uncertainty characteristics can be distinguished in aleatoric and epistemic uncertainties. Aleatoric uncertainties can be modeled by stochastic models, whereas epistemic uncertainties require to be modeled by non-stochastic approaches, such as interval or fuzzy quantities. Uncertainty models with both uncertainty characteristics are applied, e.g., in (Schietzold et al. 2019, 2021; Fina et al., 2023), where epistemic and aleatoric uncertainty is combined, which is referred to as polymorphic uncertainty. Therein, epistemic uncertainties in parameter estimation of random fields are modelled by fuzzy quantities. This fuzzy parametrization leads to random fields with fuzzy valued probabilities, which are referred to as fuzzy probability-based random fields.

The Nataf transformation allows the estimation of the modified correlation coefficient. This estimation involves computational effort, since it requires an optimization procedure based on an integral equation whose complexity depends on the marginal distributions, which are included in the integral. This numerical cost is negligible for uncertainty quantification if only correlated random variates are involved, since the modified correlation coefficient is only required to be calculated once for each correlation relation. Nataf transformation is also required for the simulation of random fields, see (Vořechovský, 2008), but the target correlation coefficients spatially change, due to the autocorrelation structure. Hence, the Nataf transformation would be required to be re-evaluated for each observation point in the simulated random field. The numerical effort is even more increased in case of fuzzy probability-based random fields, where the relation between target and modified correlation coefficient must be computed dependent on the fuzzy parameters of the marginal distributions.

In (Xiao, 2014), an interpolation for the relation between target and modified correlation coefficients is proposed. In this contribution, the interpolation approach is extended to be used for random fields and for fuzzy probability-based random fields, as shown in Figure 1. The presented method allows the incorporation of arbitrary marginal distribution functions, including any theoretical distribution and empirical distribution.

The applicability of the proposed methods for real world engineering problems is demonstrated in numerical examples. Therefore, applications in the simulation of demanding fuzzy probability-based random fields and in the simulation of random fields for the structural safety assessment of shells with geometric imperfections, based on (Fina et al., 2023), are shown. Finally, the limits of the method, with respect to numerical precision and to the limits of the achievable target correlation coefficient between the investigated marginals, are presented.

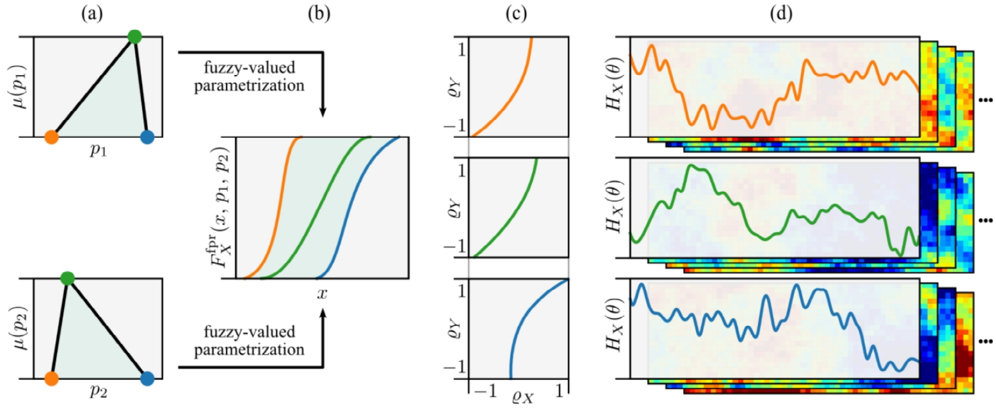

Fig. 1. Scheme for simulation of fuzzy probability-based random fields incorporating the Nataf-Transformation.
(a) Membership functions $\mu(p_1)$ and $\mu(p_2)$ of fuzzy-valued distribution parameters p_1 and p_2;
(b) fuzzy-valued cumulative distribution function F_X^{fpr} of fuzzy probability-based random variate X;
(c) target and modified correlation coefficients, ϱ_X and ϱ_Y, dependent on fuzzy-valued distribution;
(d) realizations of fuzzy probability-based random field $H_X(\theta)$.

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