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On the energetics of dynamic cohesive crack formation

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Dynamic fracture under static loading, as it takes place in the absence of a perfect pre-crack, is studied from simple 1D and 2D examples. Special emphasis is placed on the cohesive fracture energy which gives rise to a finite duration of the fracture process and – in conjunction with the material's tensile strength – introduces a material specific length scale.

In case of the 2D problem (plate with hole under remote tension/compression), a (quasi-)spontaneous formation of the finite crack increment, as predicted by FFM, can only be identified for compressive loading (stable crack growth). For tensile loading (unstable crack growth), a non-monotonic dependence of the crack velocity on the hole radius was observed, attributed to a competition between the effects of crack initiation under higher overall stress for smaller holes and a comparatively larger process zone w.r.t. the hole radius.

1 Introduction

Linear elastic fracture mechanics presupposes the existence of a perfect pre-crack. However, in many situations loaded structures fail from stress concentrations weaker than $r^{-1/2}$. Remedy may be given by considering the strength σ_c and the toughness G_c as two independent material parameters as it is, for instance, done in the framework of Finite Fracture Mechanics (FFM) [1] or Cohesive Zone Models (CZM). FFM assumes the spontaneous formation of a finite crack increment in a quasi-static setting, raising questions on the role of the kinetic energy in such processes. CZMs, commonly implemented within the framework of the Finite Element Method (FEM), may account for inertia effects and hence allow to investigate their influence on spontaneous crack formation.

2 Decohesion of 1D tensile bar

As an illustrative example, a pre-stressed tensile bar undergoing decohesion at its support is considered first (Fig. 1). Due to an excess of released energy compared to the fracture surface energy G_c, the initially quasi-static problem becomes highly dynamic.

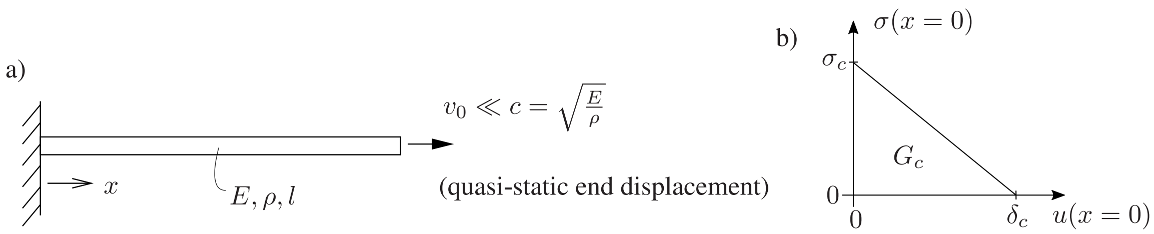

Fig. 1: (a) Initial boundary value problem with (b) cohesive boundary condition at $x = 0$.

Solving the 1D wave equation $u_{,tt}(x,t) - c^2 u_{,xx}(x,t) = 0$ with the indicated initial and boundary conditions, the displacement along the bar is given by

$$u(x,t) = \left(\varepsilon_0 + \frac{v_0}{l}\,t\right)x + \frac{v_0\delta_c}{c\,l\varepsilon_0}\left(\frac{\delta_c}{\varepsilon_0}\left[\exp\left(\frac{\varepsilon_0}{\delta_c}(ct-x)\right) - 1\right] - ct + x\right)H(ct-x)$$

where E is the Young's modulus, ρ the mass density, l the length of the bar, c the wave speed, v_0 the boundary velocity, $\varepsilon_0 := \sigma_c/E$ the initial strain in the pre-stressed bar, δ_c the critical displacement at complete fracture, $H(.)$ the Heaviside function and t the time. The above solution holds as long as the left end of the bar undergoes decohesion according to Fig. 1b and no wave reflection from the right end occurs.

With quasi-static loading ($v_0 \ll c$), the time until complete decohesion may be computed as $t_c \approx t_0 \ln(l\varepsilon_0/v_0 t_0)$ where $t_0 := \delta_c/\varepsilon_0 c$. Evaluating now the energies in the bar at the instant of complete fracture (i.e. $t = t_c$) shows that the kinetic energy equals approximately 50% of the fracture energy. That is, the necessary released energy has to be 50% higher than the fracture energy in order to cause complete rupture, with the fracture process taking place in a highly dynamic manner.

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3 Infinite plate with hole under remote tension / compression

In the following, brittle fracture of an infinite plate with a circular hole subjected to remote compression (Fig. 2a) and tension (Fig. 3a) is analysed by means of FFM and CZM. For both loading cases, FFM predicts the spontaneous formation of a finite crack increment Δa from the quasi-static energy balance being satisfied in the mean over the fracture process. Further crack propagation is then identified as stable in case of compression and unstable in case on tension [2]. The notion of "spontaneous finite crack formation", inherent to FFM, is to be analysed here by solving the same boundary value problem by means of finite elements accounting for inertia effects as well as cohesive fracture. A linear traction separation law is used for the CZM, analogous to that in Fig. 1b. In the following, length parameters are normalised utilising Irwin's characteristic material length $l_c := (K_{Ic}/\sigma_c)^2$ with $K_{Ic} = \sqrt{G_c E}$ as suggested in [2].

The size effect of the fracture initiation load σ_0, being a function of the hole radius, is known to be well captured by FFM [2]. Fig. 2b and Fig. 3b show that this is also the case for CZMs.

However of greater interest here shall be the subsequent fracture process which is represented in Fig. 2c and Fig. 3c in terms of the temporal evolution of the crack length a. Clearly, the spontaneous formation of a finite crack increment Δa, as considered in FFM, is only a model approximation since it violates the upper bound of the mode I crack growth rate given by the Rayleigh wave speed c_R. In case of compressive loading, the dynamic cohesive FE analyses for different hole radii predict a continuous crack evolution (Fig. 2c, solid curves) wherein a high velocity is only observed in the initial stage, correlating with the spontaneous crack formation of FFM (indicated by dashed lines). Afterwards, the simulated crack growth rapidly decelerates, corresponding to stable crack propagation. Note, that for the largest hole radius ($R/l_c = 4.66$), which is significantly larger than the characteristic length, the (continuous) onset of crack advance is not resolved, giving the appearance of spontaneous crack formation on the highly stretched time scale in Fig. 2c.

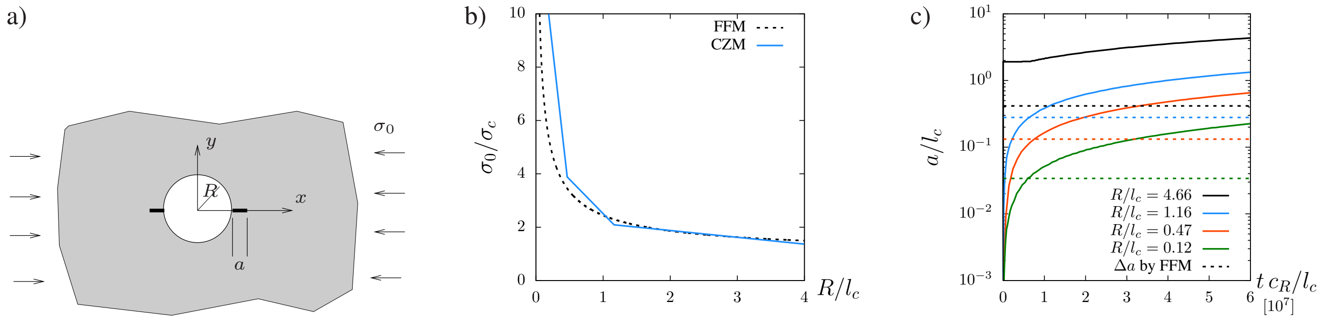

Fig. 2: (a) Plate with hole under remote uniaxial compression, corresponding (b) critical load vs. hole radius and (c) crack evolution.

Under tensile loading (Fig. 3) where crack growth after initiation is unstable [2], the CZM simulations predict a continuous crack acceleration towards the Rayleigh wave speed (Fig. 3c). In this case, a finite increment of "spontaneous" crack formation, as considered in FFM, cannot be identified. Moreover, a non-monotonic dependence of the initial crack speed on the hole radius is observed. This may be attributed to two opposing trends: on the one hand, an enhanced acceleration due to a higher overall stress σ_0 for smaller holes (size effect, see Fig. 3b) and, on the other hand, a decrease as result of the comparatively larger process zone, i.e. smaller R/l_c values.

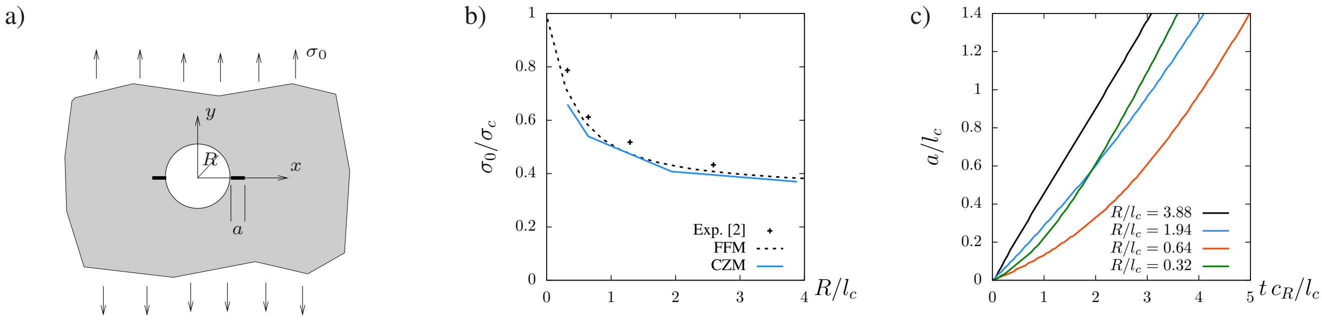

Fig. 3: (a) Plate with hole under remote uniaxial tension, corresponding (b) critical load vs. hole radius and (c) crack evolution (c).

References

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