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Event Discrete Decision-Making for Autonomous Vehicles at Non-Signalized Intersections

Daniel LEYER [a,1], Hannes WEINREUTER [a] and Michael HEIZMANN [a]
[a] *IIIT, KIT, Germany*

Abstract. Non-signalized urban intersections are one of the remaining challenging situations when it comes to autonomous driving. Situations may arise that are not clearly regulated by the traffic regulations and therefore require cooperation between vehicles to resolve the situations. This paper introduces a general modeling approach for driving at intersections and decision-making there. The decision process is modeled based on discrete event systems (DES). Within the modeling, limited visibility is considered. Unexpected behavior of other vehicles as well as deadlock situations are also accounted for. The decision-making process is based on explicitly modeled features and events and can thus be explained to and understood by its users.
The algorithm is evaluated with extensive simulations and the results show that the model is able to deal with multiple cooperation vehicles and reliably avoids collisions even during unexpected behavior of the cooperation vehicles and deadlock situations.

Keywords. decision making, autonomous vehicle, DES, intersections

1. Introduction

In recent years, autonomous driving has become a more and more realistic scenario. This is highlighted by Waymo's start of operation of a fleet of autonomous vehicles without a human safety driver on board [1] and the introduction of the *drive pilot* option for some Mercedes vehicles. The latter enables autonomous driving in certain highway scenarios and is the first commercially available SAE level 3 system [2]. Despite those successes, there are still major challenges on the way to driving fully autonomously in any scenario.

One of those challenges are non-signalized intersections. This type of intersection is very common in Germany and other countries and is especially prevalent in residential areas with low traffic volumes. There are no traffic lights or priority signs at these intersections, traffic is instead regulated by the *right before left* (RBL) rule. As stated in the German traffic regulations, drivers are required to yield to traffic participants that

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[1]Corresponding Author: Daniel Leyer, IIIT KIT, Hertzstraße 16, 76187 Karlsruhe, Germany; E-mail: daniel.leyer@kit.edu

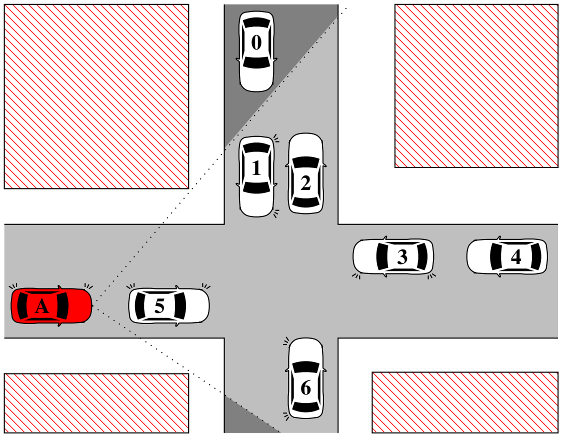


Figure 1. Exemplary situation showing a deadlock situation. As the four vehicles directly at the intersection, vehicles 1, 3, 5 and 6, all intend to turn left, each of them has to yield to the vehicle to its right. This constitutes a deadlock as no vehicle has priority over all other vehicles. This situation further shows the roles the other vehicles assume from the A-V's perspective: Vehicle 5 is in front of the A-V, making it the L-V. Vehicle 2 currently blocks the A-V, it is thus assigned the B-V label. As vehicle 1 approaches from the A-V's left, it takes the role of the Y-V. The driving paths of the A-V and vehicle 3 do not overlap but its existence is relevant, making it a D-V. Vehicles 4 and 6 are both P-Vs as the A-V has to yield to either. Due to the current visibility conditions vehicle 0 is not yet visible.

approach an intersection on the next road to the right of their own; also, oncoming traffic has priority over drivers turning left. This covers most conceivable scenarios at this type of intersections. It is however possible for deadlocks to occur. In that case each vehicle has to yield to at least one other vehicle. An example is provided in figure 1. Further challenges for autonomous vehicles in this scenario are drivers who do not adhere to the priorities or to find the appropriate behavior in edge cases.

In this work we assume no direct communication between vehicles or the infrastructure and thus only the observable state of the other vehicles is available to the autonomous vehicle (A-V) running the proposed algorithm. For ease we assume that all vehicles follow the center of their lanes and the driving direction (*left*, *right* or *straight*) is known when they are close to the intersection, c.f. [3, 4]. To avoid having to model each conceivable combination of arriving vehicles and their interdependent behavior, the model only considers those vehicles that are currently relevant for decision-making. Also, the interactions of the A-V with these relevant vehicles are viewed independently from each other, i.e. interdependencies between the driving behavior of the cooperation vehicles are not modeled. The A-V will only drive if all pairwise interactions are predicted to be safe.

The proposed decision-making procedure is modeled as a discrete event system (DES). At any time during execution the model is in one of its states, a transition to another state is only possible if the corresponding event for that occurs. The DES model is updated periodically; in each time step first events are checked if they are active after which the model transitions to another state if necessary. This approach offers two benefits. First, one is able to explicitly design the driving behavior of the A-V, ideally to mimic a human-like driving style. Second, the behavior can easily be adapted by modifying some parameters. An earlier version of this algorithm exists [5], however, it only supports T-intersections and was tested on fewer maps.

2. Related Work

The problem of decision-making at intersections has been investigated before. The first kind of algorithms relies on communication between the vehicles themselves or with a central controller. The algorithm by Dresner and Stone [6] requires vehicles to reserve a trajectory from a central controller, human driven vehicles are supported by using traffic lights. Alternatively, the optimal passing order can be determined by a central controller [7]. Azimi et al. [8] propose a similar approach, however vehicles communicate directly with each other. Lin et al. [9] centrally determine the driving order through the conflict zones of an intersection based on a graph. Pourmehrab et al. [10] use an optimization based approach running on a central intersection controller. It optimizes the trajectory of A-Vs and the traffic light timing. The latter is used to also allow vehicles driven by humans to drive through the intersection. The decision-making can also be based on Reinforcement Learning [11]. A central controller determines the driving commands for the A-Vs at the intersection, human driven vehicles still follow the signs at the intersection. Chen et al. [12] divide the approach into two phases; first the vehicles change into the right lane, then the driving order is established. All vehicles need to be autonomous and be able to communicate with the controller. The major downside of these approaches is certainly that all vehicles are required to use the same protocol. Also, not all support human driven vehicles which makes their application less likely.

The next class of algorithms does not rely on communication between vehicles or with an intersection controller. The *ballroom intersection protocol* (BRIP, [13]) requires all vehicles to use the algorithm. It specifies a spatial and temporal pattern which vehicles have to follow while driving through an intersection. It, however, leaves very little space between vehicles and may lead to accidents if vehicles do not follow their time slot. This issue is addressed by an extension to the BRIP [14] which adds safety gaps between vehicles. Partially observable Markov decision processes (POMDP) are also commonly used for decision-making at intersections [15–18]. Hubmann et al. [15] assume the remaining vehicles as hidden variables, the resulting behavior is adapted to the most likely behavior of the remaining vehicles. Bouton et al. [16] investigate decision-making at intersections with limited visibility where other vehicles or pedestrians can suddenly appear. Shu et al. [17] define critical turning points at an intersection from which a left turn can be initiated. They select the most efficient one using a POMDP. Xia et al. [18] employ a POMDP solution for driving through non-signalized intersections. A different approach is chosen by Kreutz and Eggert [19], they use an extended version of the intelligent driver model (IDM) for decision-making at intersections. To model the interaction the vehicles from the other street are projected onto the own lane.

Finally, one can use an ontology to describe traffic scenarios, including the traffic rules [20]. An ontology can also be used for decision-making, as is done by Zhao et al. [21]. The ontology-based approaches are similar to the algorithm presented in this work in that a traffic scenario is abstracted and decisions are made based on this abstraction. Our proposed algorithm has two main aspects that set it apart from most works presented above: First, it supports both A-Vs and vehicles driven by humans. Second, due to the utilization of a DES, all decisions can be easily explained. This approach requires explicit modeling of all decision paths and is in contrast to especially probabilistic and learning based solutions.

3. Decision Making at Intersections Using DES

In this section the DES-based decision-making algorithm is introduced. This includes the classification of relevant cooperation vehicles (C-V), the maps, the features and events the DES is based on and the DES itself. Based on the latter the driving behavior is determined by a modified IDM.

3.1. Relevant Vehicles

Although situations at intersections can be very crowded, not all vehicles present there interact with the vehicle running the proposed algorithm (A-V) and are therefore not relevant for the decision-making process. The A-V proceeds into the intersection only if all relevant C-Vs allow it; thus ensuring a safe and collision-free passage. In total, five relations with C-Vs can be identified; an example where all are present is shown in figure 1:

- **Priority vehicle (P-V):** According to the RBL rule, a vehicle has priority over the A-V if it approaches from the next street to the right of the A-V's street while the A-V does not intend to turn right. If a C-V approaches from the direction straight ahead and drives straight or turns right while the A-V turns left it also has priority and is thus assigned the P-V label. In both cases the vehicle closest to the intersection is classified as a P-V.
- **Yielding vehicle (Y-V):** There are two possible constellations for a vehicle to be assigned this label. A C-V takes the role of a Y-V if it either approaches from the next street to the left relative to the A-V's entry direction or if it approaches straight ahead relative to the A-V and is turning left, while the A-V is driving straight through the intersection or turns right. If a C-V is turning right it is not classified as a Y-V no matter the entry direction. This is due to the fact that a vehicle that turns right has priority in any constellation.
- **Leading vehicle (L-V):** The vehicle driving directly in front of the A-V is classified as the leading vehicle. It is relevant as it limits the acceleration and speed of the A-V.
- **Blocking Vehicle (B-V):** The traffic regulations prohibit vehicles to enter an intersection if they are forced to wait inside it. To respect this regulation the C-V closest to the intersection leaving it on same street as the A-V is considered to be the B-V.
- **Deadlock vehicle (D-V):** The D-V stands out in that it does not interact with the A-V directly, i.e. their drive paths do not overlap. It can only exist at X-intersections and is the vehicle from straight ahead if both, the A-V and the D-V, turn left or if both drive straight. It is only relevant in a deadlock as the other C-Vs involved in the deadlock do have to yield to it or have priority over it and it is thus involved in the deadlock.

Based on these definitions, two C-Vs can be assigned the P-V or the Y-V label. The remaining labels are assigned only once. Any vehicle that is not assigned any of the aforementioned labels, like vehicles behind the A-V are not considered in the decision-making process. Additionally, vehicles that are not yet visible have to be omitted. To account for possible hidden vehicles, some relevant C-Vs have reference points on the

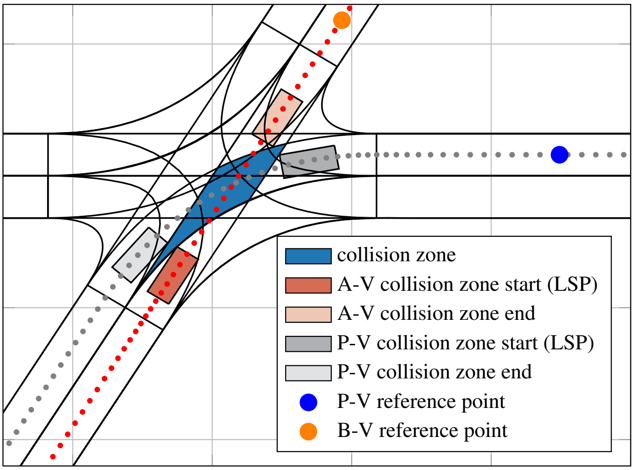


Figure 2. The collision zone of the A-V and its P-V at intersection 17. The dotted lines display the trajectories of the vehicles. The positions where the vehicles are just before and after the collision zone are shown, at these positions the corresponding distances are zero: $d^{x}_{\mathrm{c,p},d} = 0\,\mathrm{m},\ d \in \{\mathrm{b},\mathrm{e}\},\ x \in \{\mathrm{a},\mathrm{p}\}$. The references points of the B-V and P-V from the A-V's perspective are also shown. The lanelet structure of the map is clearly visible. The grid spacing is 10 m.

streets they would enter from if they exist. The corresponding C-V is considered to be non-existent if its reference point is already visible, otherwise its existence is assumed to be unknown. For the P-V the reference point is placed 25 m from the intersection center on the entry lane and for the B-V at the distance of 15m on the lane the A-V will leave the intersection on.

3.2. Maps

The topology of an intersection is reflected in the map. In this work the lanelets concept [22] is utilized in a simplified version. It divides the intersection into lane segments. A segment starts where several lanes merge into one and ends where a lane splits into multiple lanes. Using the lanelets, the start and end of an intersection can be defined: An intersection begins where the entry lanelet ends, i.e. the position where the entry lane starts to diverge. It ends where the lanelet exiting the intersection starts, i.e. the position where all lanes within the intersection are merged. Within the intersection area, many driving paths overlap each other. For each pair of overlapping paths, a collision zone (CZ) exists. The algorithm has to ensure that there is only a single vehicle in it at any time. Related to that are the latest stopping points (LSP). These describe the last point a vehicle could stop at while not blocking any other driving path that does not originate from its own entry direction. If a vehicle passes its LSP it blocks – at least parts of – the intersection. To limit the duration during which the intersection is occupied, the A-V does not stop after passing its LSP. In figure 2 the CZ, the LSPs and the reference points are showcased.

The final important aspect of the maps is the limited visibility. In a real urban environment there are typically many obstacles that block the view into the other streets at an intersection. Only after one is relatively close to the intersection center one is oftentimes able to overview an intersection completely. To implement this limitation the hatched areas in figure 1 next to the roads block the visibility and vehicles inside these occluded areas are not visible to the A-V. The visibility at a map is defined by the distance of the most exposed corner of the hatched obstacle to the curb. The distance is measured along the bisecting line between the directions of the two adjacent streets and can be set individually for each obstacle.

3.3. Features

The model for decision-making is based on events to transition between states. The events themselves are based on features that describe the current situation. The superscript of a feature indicates for which C-V it is calculated: $(\cdot)^x, \quad x \in \{\mathrm{a}, \mathrm{p}_1, \mathrm{p}_2, \mathrm{y}_1, \mathrm{y}_2, \mathrm{l}, \mathrm{b}, \mathrm{d}\}$. These are the A-V, P-V$_{1/2}$, Y-V$_{1/2}$, L-V, B-V and D-V, respectively. All vehicles are modelled as rectangles with a fixed geometric dimension of a length of $l_v = 4.4$ m and a width of $w_v = 1.8$ m. All distances are not Euclidean but are measured along the lane center of the driving path. All necessary features are calculated in each time step. For the sake of readability the time index is omitted.

The core feature used in the algorithm is the *distance to scenario* d_{s}^x. It describes the position of the vehicle with respect to the intersection area and is positive during the approach, zero inside the intersection area and negative after the vehicle has left the intersection.

The collision zones, created by overlapping driving paths of the vehicle x and its C-V x_{c}, may only be occupied by one vehicle at a time. They are therefore important aspects for the algorithm and thus the *distance to collision zone* is defined for the beginning $d_{\mathrm{c},x_{\mathrm{c}},\mathrm{b}}^x$ and the end $d_{\mathrm{c},x_{\mathrm{c}},\mathrm{e}}^x$ of the collision zone. Besides the spatial distance also the temporal one is calculated by assuming a constant velocity for vehicle x:

$$t_{\mathrm{c},x_{\mathrm{c}},\cdot}^x = \frac{d_{\mathrm{c},x_{\mathrm{c}},\cdot}^x}{v^x} \tag{1}$$

The *distance to stop* d_{b}^x describes the distance required to stop completely given the velocity v^x and acceleration a^x:

$$d_{\mathrm{b}}^x(v^x, a^x) = \begin{cases} -\frac{(v^x)^2}{2a^x}, & a^x < 0\,\mathrm{m\,s^{-2}} \\ 0\,\mathrm{m}, & a^x = 0\,\mathrm{m\,s^{-2}} \wedge v^x = 0\,\mathrm{m\,s^{-1}} \\ \infty, & \text{otherwise} \end{cases} \tag{2}$$

The *free distance behind the B-V* $d_{\mathrm{f}}^{\mathrm{b}}$ describes the length of free street behind the B-V that can be used by the A-V if it were to pass the intersection:

$$d_{\mathrm{f}}^{\mathrm{b}} = d_{\mathrm{i}}^{\mathrm{b}} - l_{\mathrm{v}} + d_{\mathrm{b}}^{\mathrm{b}}\left(v^{\mathrm{b}}, a_{\mathrm{e}}\right), \tag{3}$$

where $d_{\mathrm{i}}^{\mathrm{b}}$ is the current distance from the intersection end to the B-V. To enable the A-V to follow the B-V sooner, the distance the B-V will travel in any case even if it stops with an emergency deceleration of $a_{\mathrm{e}} = -7.5\,\mathrm{m\,s^{-2}}$ is accounted for. The final feature is the *distance to the last stopping point* d_{l}^x.

Table 1. Base events (BE) for the DES for decision-making.

BE	condition
$e_{\mathrm{b1},i}$	P-V$_i$ cannot exist or is not detected while ref. point is visible
$e_{\mathrm{b2},i}$	$t^{\mathrm{a}}_{\mathrm{c,p}i,\mathrm{e}} + \Delta t^{\mathrm{p}} < t^{\mathrm{p}i}_{\mathrm{c,p}i,\mathrm{b}} \Delta d^{\mathrm{a}}_{\mathrm{c,pi,e}} + \wedge d^{\mathrm{p}} < d^{\mathrm{p}i}_{\mathrm{c,pi,b}}$
$e_{\mathrm{b3},i}$	$v^{\mathrm{p}i} < v_{\mathrm{s}} \wedge a_{\mathrm{p}i} \leq 0\,\mathrm{m\,s^{-2}} \wedge d^{\mathrm{p}i}_{\mathrm{s}} < d_{\mathrm{n}} \wedge d^{\mathrm{p}i}_{\mathrm{c,p}i,\mathrm{b}} > 0\,\mathrm{m}$
$e_{\mathrm{b4},i}$	$t^{\mathrm{p}i}_{\mathrm{w}} > t_{\mathrm{y}}$
$e_{\mathrm{b5},i}$	$d^{\mathrm{d}}_{\mathrm{s}} < d^{\mathrm{p}i}_{\mathrm{s}} \wedge e^{\mathrm{d}}_{\mathrm{d}} = e^{\mathrm{p}i}_{\mathrm{d}}$
$e_{\mathrm{b6},i}$	$t^{\mathrm{a}}_{\mathrm{c,y}i,\mathrm{e}} < t^{\mathrm{y}i}_{\mathrm{c,yi,b}}$
$e_{\mathrm{b7},i}$	$d^{\mathrm{a}}_{\mathrm{l}} > d^{\mathrm{a}}_{\mathrm{b}}(v_{\mathrm{a}}, a_{\mathrm{h}}) + d_0 \wedge v^{\mathrm{y}i} < v_{\mathrm{sl}}$
	$\wedge a^{\mathrm{y}i} < 0\,\mathrm{m\,s^{-2}} \wedge d^{\mathrm{y}i}_{\mathrm{c,y}i,\mathrm{b}} > d^{\mathrm{y}i}_{\mathrm{b}}(v^{\mathrm{y}i}, a^{\mathrm{y}i})$
$e_{\mathrm{b8},i}$	$v^{\mathrm{y}i} < v_{\mathrm{s}} \wedge a_{\mathrm{y}i} \leq 0\,\mathrm{m\,s^{-2}} \wedge d^{\mathrm{y}i}_{\mathrm{s}} < d_{\mathrm{n}} \wedge d^{\mathrm{y}i}_{\mathrm{c,y,b}} > 0\,\mathrm{m}$
e_{b9}	reference point visible and no B-V detected
e_{b10}	$d^{\mathrm{b}}_{\mathrm{f}} > l_{\mathrm{v}} + d_{\mathrm{min}}$
e_{b11}	no L-V detected
e_{b12}	$d^{\mathrm{l}}_{\mathrm{s}} < 0\,\mathrm{m}$
e_{b13}	D-V detected
e_{b14}	$v^{\mathrm{d}} < v_{\mathrm{s}} \wedge a_{\mathrm{d}} \leq 0\,\mathrm{m\,s^{-2}} \wedge d^{\mathrm{d}}_{\mathrm{s}} < d_{\mathrm{n}} \wedge d^{\mathrm{d}}_{\mathrm{c,p,b}} > 0\,\mathrm{m}$
e_{b15}	trajectories of A-V and its relevant C-Vs can cause a deadlock
e_{b16}	$d^{\mathrm{a}}_{\mathrm{l}} > d^{\mathrm{a}}_{\mathrm{b}}(v_{\mathrm{a}}, a_{\mathrm{c}})$
e_{b17}	$d^{\mathrm{a}}_{\mathrm{l}} > d^{\mathrm{a}}_{\mathrm{b}}(v^{\mathrm{a}}, a_{\mathrm{e}})$
e_{b18}	$v^{\mathrm{a}} < v_{\mathrm{s}} \wedge a^{\mathrm{a}} \leq 0\,\mathrm{m\,s^{-2}} \wedge d^{\mathrm{a}}_{\mathrm{s}} < d_{\mathrm{n}}$

3.4. Events of the DES

Using these features, the events of the DES for decision-making can be defined. The events are a combination of base events, the latter are listed in table 1 and the events are given in table 2. All base events and events that are related to the P-Vs and Y-Vs exist twice as there might be two instances of these C-Vs.[2]

The process of crossing the intersection is split into six zones, some events and especially the states in the following section are dependent on them. The current zone is determined by the distance to scenario $d^{\mathrm{a}}_{\mathrm{s}}$. While the A-V is still far from the intersection ($d^{\mathrm{a}}_{\mathrm{s}} > 40\,\mathrm{m}$) and after it has passed the intersection ($d^{\mathrm{a}}_{\mathrm{s}} < 0\,\mathrm{m}$) it is in the neutral zones 1 and 6, respectively. In them the driving behavior is only determined by the traffic ahead and not the proposed algorithm. Zones 2 ($40\,\mathrm{m} \geq d^{\mathrm{a}}_{\mathrm{s}} > 25\,\mathrm{m}$) and 3 ($25\,\mathrm{m} \geq d^{\mathrm{a}}_{\mathrm{s}} > 10\,\mathrm{m}$) constitute the prediction phase. In this phase the behavior is adapted early to communicate the A-V's intention in time. In zones 4 ($10\,\mathrm{m} \geq d^{\mathrm{a}}_{\mathrm{s}} > 1\,\mathrm{m}$) and 5 ($1\,\mathrm{m} \geq d^{\mathrm{a}}_{\mathrm{s}} \geq 0\,\mathrm{m}$) the A-V enters and crosses the intersection, in them the focus is on the interaction with the remaining C-Vs and they thus make up the decision phase.

A traffic light analogy is used in this work to signal if the A-V can drive relative to a C-V. Each C-V is therefore assigned a traffic light and the A-V only drives offensively if

[2]In very wide intersections a vehicle turning right might be able to drive despite the vehicle in front of it, which turns left or drives straight, has not passed the intersection. In this case more than two P-Vs can exist. The handling of these additional P-Vs would be the same as for the other two. In this work, however, the positions of the LSPs are adapted so that this case cannot occur.

Table 2. Events for DES based on logic combinations of BEs.

definition	description
$e_{1,\mathrm{p}i,\mathrm{I}} = e_{\mathrm{b}1,i} \vee e_{\mathrm{b}2,i}$	P-V$_i$: green light (zone 2/3)
$e_{1,\mathrm{p},\mathrm{I}} = \wedge_{i=1}^{n_\mathrm{p}} e_{1,\mathrm{p}i,\mathrm{I}}$	all P-Vs: green light (zone 2/3)
$e_{1,\mathrm{p}i,\mathrm{II}} = e_{\mathrm{b}1,i} \vee (e_{\mathrm{b}3,i} \wedge e_{\mathrm{b}4,i}) \vee e_{\mathrm{b}2,i} \vee (e_{\mathrm{b}5,i} \wedge e_{\mathrm{b}16})$	P-V$_i$: green light (zone 4/5)
$e_{1,\mathrm{p},\mathrm{II}} = \wedge_{i=1}^{n_\mathrm{p}} e_{1,\mathrm{p}i,\mathrm{II}}$	all P-Vs: green light (zone 4/5)
$e_{1,\mathrm{y}i} = e_{\mathrm{b}6,i} \vee e_{\mathrm{b}7,i} \vee e_{\mathrm{b}8,i} \vee e_{\mathrm{b}16}$	Y-V$_1$: green light
$e_{1,\mathrm{y}} = \wedge_{i=1}^{n_\mathrm{y}} e_{1,\mathrm{y}i}$	Y-V: green light
$e_{1,\mathrm{b}} = e_{\mathrm{b}9} \vee e_{\mathrm{b}10}$	B-V: green light
$e_{1,\mathrm{l}} = e_{\mathrm{b}11} \vee e_{\mathrm{b}12}$	L-V: green light
e_2	entered next zone
$e_3 = e_{\mathrm{b}17}$	emergency stop possible
$e_4 = e_{\mathrm{b}15}$	deadlock possible
$e_5 = \bigwedge_{i \in v_{\mathrm{p,dl}}} e_{\mathrm{b}3,i} \bigwedge_{i \in v_{\mathrm{y,dl}}} e_{\mathrm{b}8,i} \wedge e_{\mathrm{b}18} \wedge (e_{\mathrm{b}13} \vee e_{\mathrm{b}14})$	deadlock detected
$e_6 = \bigwedge_{i \in v_{\mathrm{p,dl}}} e_{\mathrm{b}3,i} \bigwedge_{i \in v_{\mathrm{y,dl}}} e_{\mathrm{b}8,i} \wedge (e_{\mathrm{b}13} \vee e_{\mathrm{b}14})$	deadlock of C-Vs detected
e_7	exceeded deadlock wait time
$e_\mathrm{g} = e_{1,\mathrm{p},\mathrm{II}} \wedge e_{1,\mathrm{y}i} \wedge e_{1,\mathrm{b}} \wedge e_{1,\mathrm{l}}$	green light by all C-Vs
$e_\mathrm{dl} = e_4 \wedge e_5 \wedge e_7 \wedge e_{1,\mathrm{l}} \wedge e_{1,\mathrm{b}}$	deadlock can be resolved

all lights show green light. For the two possible P-V instances, P-V$_1$ and P-V$_2$, the exact same conditions apply. The corresponding events are therefore only introduced once. The same is true for Y-V$_1$ and Y-V$_2$.

In order to receive green light by a P-V in zones 2 and 3 ($e_{1,\mathrm{p}i,\mathrm{I}}$) the P-V can either not exist ($e_{\mathrm{b}1,i}$) or the A-V has to be predicted to leave the common collision zone before the P-V even enters it. In order to not obstruct the P-V the A-V further has to have a lead in time of at least $\Delta t^\mathrm{p} = 2.5\,\mathrm{s}$ and distance of no less than $\Delta d^p = 10\,\mathrm{m}$ ($e_{\mathrm{b}2,i}$).

In zones 4 and 5 the P-Vs additionally give green light if the P-V in question is standing close before the intersection (it drives slower than the threshold of $v_\mathrm{s} = 0.15\,\mathrm{m\,s^{-1}}$, $e_{\mathrm{b}3,i}$) and has been stationary for longer than the wait time threshold of $t_\mathrm{y} = 2\,\mathrm{s}$ ($e_{\mathrm{b}4,i}$). In this case it is assumed that the P-V waives it right of way. In addition, it also shows a green light if a D-V is standing ($e_{\mathrm{b}14}$) in front of one the P-Vs ($e_{\mathrm{b}5,i}$).

The A-V gets a green light from the Y-Vs ($e_{1,\mathrm{y}i}$) if any of the following four conditions is met:

- The A-V is predicted to leaves the collision zone before the Y-V does enter it ($e_{\mathrm{b}6,i}$).
- The Y-V is moving slowly ($v^\mathrm{y} < v_\mathrm{sl} = 2\,\mathrm{m}$) and is braking in a way that its remaining distance to the beginning of the collision zone is larger than the distance required to stop. To be able to stop in case of a false prediction, the A-V needs to be able to stop before its LSP, including a safety distance of $d_\mathrm{o} = 0.2\,\mathrm{m}$, with a hard deceleration of $a_\mathrm{h} = 4.5\,\mathrm{m\,s^{-2}}$ ($e_{\mathrm{b}7,i}$).
- The Y-V is stopped close to the intersection ($e_{\mathrm{b}8,i}$).

- Based on the target velocity of the respective driving situation and a comfort deceleration of $a_c = 2.5\,\mathrm{m\,s^{-2}}$ the A-V can still come to a stop before the LSP (e_{b16}).

The B-V can hinder the A-V from leaving the intersection by blocking its exit lane, causing a stop inside the intersection and blocking the traffic flow. Therefore, the A-V only enters the intersection ($e_{1,b}$) if either the intersection can be viewed fully and no B-V is to be seen (e_{b9}) or the distance behind the B-V is larger than a vehicle length l_v plus the minimum headway d_{min} (e_{b10}).

Like the B-V the L-V does not have a priority relation with the A-V but it can hinder the A-V from passing the intersection. Two possible events allow the A-V to pass the intersection ($e_{1,l}$): The L-V does not exist (e_{b11}) or it exists but has passed the intersection already, i.e. $d_s^l < 0\,\mathrm{m}$ (e_{b12}).

Beside the events related to the C-Vs there are also events that cover the A-V itself. These include e_2 which is active if it entered the next zone in the current time step and the check if an emergency brake before the LSP is still possible ($e_3 = e_{b17}$). To handle deadlock situations four events are required: Event $e_4 = e_{b15}$ is true if a deadlock is possible. A deadlock has occurred (e_5) if all vehicles involved in it are stationary, i.e. the A-V (e_{b18}), the sets of P-Vs and C-Vs that are involved in the deadlock ($v_{p,dl}$, $v_{y,dl}$ and $e_{b3,i}$, $e_{b8,i}$, respectively) and the D-V (e_{b14}) if it exists (e_{b13}). The similar event of a deadlock between the C-Vs (e_6) is necessary because if the A-V attempts to resolve a deadlock, after the deadlock wait time is exceeded (e_7), it is no longer part of the deadlock after it starts moving.

3.5. Decision-Making Model

Based on the events and the zones defined in section 3.4 the decision-making process is modeled as shown in figure 3. The A-V can only be in one of the states associated with the current zone. In the neural zones 1 and 6 there is only one state, s_{10} and s_{60}, respectively. If the vehicle is inside the zones 2 to 5 an offensive state (s_{21}, s_{31}, s_{41}, s_{51} and s_{53}) and a defensive one (s_{22}, s_{32}, s_{42} and s_{52}) exist for each zone. The offensive states of the decision making model are linked to offensive driving, i.e. the A-V drives in a way to pass the interaction without stopping. In the defensive states it decelerates and prepares to brake (zones 2 and 3) or it actually stops before its LSP (zones 4 and 5).

During the prediction phase a change of state can only occur when entering the next zone (e_2). Then, the green light for the P-V relation ($e_{1,p,I}$) is taken into account. This

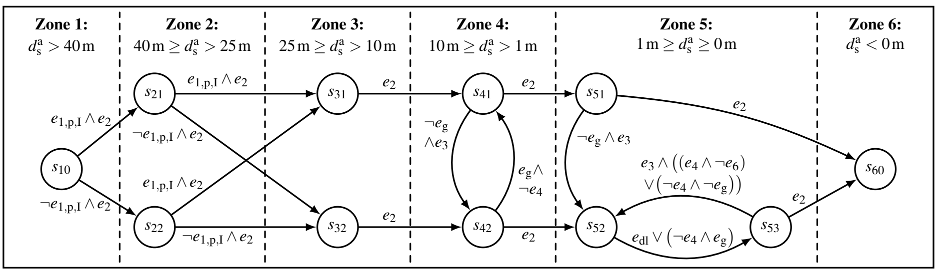


Figure 3. Event discrete decision-making model for autonomous vehicles at non-signalized intersections. The zone of the vehicle depends on the current distance to scenario d_s^a. If no events for a state transition occur, the vehicle remains in its current state. The events are introduced in table 2.

Table 3. Target velocities v_t for different states in $\mathrm{m\,s^{-1}}$; initial velocity: v_0.

state	s_{10}	s_{21}	s_{22}	s_{31}	s_{32}	$s_{4X/5X}$	s_{60}
straight	v_0	8.33	6	7.5	5	6.5	8.33
turning	v_0	8.33	6	5.5	5	4	8.33

measure is intended to avoid changing the behavior too often and to signal to the C-Vs that the A-V is aware of the current priorities and that it intends to adhere to them. Only if the P-Vs permits it, the A-V remains in or changes into an offensive state (s_{21} and s_{31}), otherwise it is being forced into a defensive one (s_{22} and s_{32}). In zones 4 and 5 a change between offensive and defensive states can happen in any time step if the required events are fulfilled. This is necessary as the interaction at the intersection is now imminent and might require quick responses. In contrast to zones 2 and 3 now all C-Vs are taken into account, allowing an offensive behavior only if all vehicles give green light or if the special situation of a deadlock has occurred and needs to be resolved.

In case offensive driving is possible in zones 4 and 5 the vehicles passes through the offensive states s_{41} and s_{51} to finally change into the neutral state s_{60}. If any of the C-Vs does not allow the A-V to pass ($\neg e_{\mathrm{g}}$) and an emergency stop is still possible (e_3) it changes into the defensive state s_{42} or s_{52}, respectively. In zone 4 the transition back to the offensive state s_{41} is possible if all C-Vs show a green light e_{g} and no deadlock is possible ($\neg e_4$). Once the vehicle has reached the state s_{52} the only way to pass the intersection is the transition to state s_{53}, which represents the attempt to drive after driving defensively in zone 5. This is either an attempt to resolve a deadlock situation (e_{dl}) or it occurs after all C-Vs (e_{g}) give a green light if simultaneously no deadlock is possible ($\neg e_4$). Both states s_{51} and s_{53} have a transition to the neutral state s_{60} which is reached after the A-V leaves zone 5. If the A-V is unable to continue driving in state s_{53} it reverts back to state s_{52} in case an emergency stop is still possible (e_3) and if either not all relevant vehicles show green light (e_{g}) while no deadlock is possible ($\neg e_4$) or if not all C-V involved in a deadlock are still stationary (e_6) if a deadlock is possible (e_4).

If a deadlock has occurred, this ambiguous situation can only be resolved if one of the vehicles involved in it starts driving. After a random deadlock wait time has exceeded (e_7) the A-V does so if the deadlock is still active ($e_4 \wedge e_5$) and it can pass the intersection ($e_{1,\mathrm{l}} \wedge e_{1,\mathrm{b}}$); otherwise it gives way according to the RBL rule. If another vehicle attempts a resolution at the same time as the A-V ($e_4 \wedge \neg e_6$) it terminates its attempt in order to avoid a collision. If this happens and a stop before the LSP is still possible (e_3) the wait time is reset and restarted again.

3.6. Behavior generation

The A-V is always in one of the decision-making model's eleven states. For each state a target velocity v_{t} is specified (see table 3), that a vehicle tries to reach if it can drive freely. Besides v_{t} also the L-V needs to be taken into consideration when determining the current acceleration and thus the driving behavior. For this purpose the intelligent driver model (IDM) [23] is used in this work:

$$a = a_{\max}\left(1 - \left(\frac{v^{\mathrm{a}}}{v_{\mathrm{t}}}\right)^4 - \left(\frac{d^*}{\Delta d}\right)^2\right) \tag{4}$$

$$\text{with } d^* = d_{\min} + v^{\mathrm{a}} t_{\min} + \frac{v^{\mathrm{a}}\Delta v}{2\sqrt{a_{\max} a_{\mathrm{b}}}}. \tag{5}$$

The acceleration depends on two term: The first determines the behavior in free flow conditions and the second one if the A-V follows its L-V. In case of no interaction with any other vehicle only the current velocity v^{a}, the target velocity v_{t} and the maximum possible acceleration $a_{\max} = 2.5\,\mathrm{m\,s^{-2}}$ are taken into consideration. If there is a L-V, it has to be considered as well. This is done with Δd, which describes the current distance between the A-V and its L-V, and d^*. The latter represents the desired distance between the two vehicles and depends on the current velocity difference Δd, the minimum headway in distance $d_{\min}$ and time $t_{\min}$ and the braking acceleration a_{b}. In case there is no vehicle ahead Δd is set to infinity, which results in a vanishing posterior term. To force the vehicle to stop at the defined LSPs, virtual vehicles are inserted. This is only done in states s_{42} and s_{52} where the A-V supposed to stop and only if there is no L-V to stop behind. The A-V is set to stop shortly before the LSP in order to have a safety margin. Additionally, the resulting acceleration is limited to a maximum braking acceleration of $a_{\mathrm{em}} = 7.5\,\mathrm{m\,s^{-2}}$ to ensure realistic driving behavior as the negative acceleration of the original IDM in (4) is not limited.

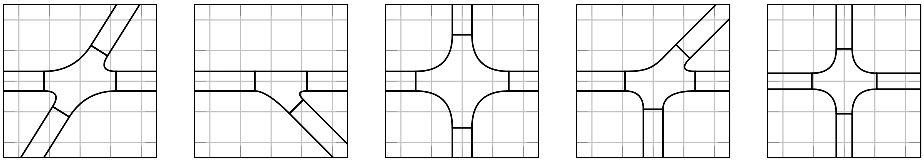
4. Simulations

Testing a decision-making model for autonomous driving requires the interaction with further vehicles. In this work, the validation is carried out via simulations. This solution allows to closely control and monitor the test parameters, e.g. the number of vehicles at an intersection or their turning directions. The vehicles the A-V interacts with at the intersection run a simplified version of the algorithm presented here. The most important aspect of the simplification is that the decision to drive is not reconsidered, i.e. it will not stop after it has decided to drive once. Also, all other vehicles are fully visible at any time. A further modification of the C-V's algorithm lets these vehicles break certain driving regulations. Some vehicles waive their right of way at the intersection and wait for the A-V to drive before them or they have a target speed that is below the speed limit. These can be seen as defensive deviations; but also offensive ones are implemented. C-Vs can drive before the A-V despite having to yield to it or they can initially delay their approach but speed up again shortly after to pass the intersection first. All these modifications of the algorithm are done to test the main algorithm, i.e. that of the A-V, as best as possible.

4.1. Simulation framework

To test the behavior generated by the proposed decision-making model different non-signalized, urban T- and X-intersections are used for the simulations. Those maps consist of synthetic maps (IDs 0-28) with predefined topology variations as well as 14 T- and 3 X-intersections (IDs 29-45) from the City of Karlsruhe.

The synthetic maps (4 T- and 25 X-intersections) are variations of a generic map with symmetric entry lanes and perpendicular roads of equal length (see figure 4). In



(a) (b) (c) (d) (e)

Figure 4. Selection of maps used in the simulation. Variation of (a) road angle of opposite entry direction (ID 17), (b) single angle in t-intersection (ID 2), (c) junction radius (ID 21), (d) angle of single entry road (ID 13) and (e) road width (ID 24). The grid spacing is 10 m for all five plots.

contrast to the real world maps they allow to systematically investigate how a change in certain parameters influences the time it takes to pass an intersection. In case of the T-intersections two of the roads are in line and the angle between these two roads and the third one is varied: $\alpha = \{45°, 90°, 135°\}$ (IDs 0-2) as well as a change of the road length $l = \{92\,\mathrm{m}, 135\,\mathrm{m}\}$ of the opposite entry roads for fixed angles are examined (ID 3,4). At X-intersections two variants for angled streets are investigated. First, only one of the four roads is at an angle, $\alpha = \{90°, 83°, 76°, 69°, 64°, 58°, 51°, 49°, 45°, 42°, 38°\}$ (IDs 5-15), the remaining streets are at an angle of 90° to each other. Second, some maps contain the case that the two street that intersect with each other are straight themselves but meet at an angle, $\alpha = \{83°, 58°, 45°\}$ (IDs 16-18). Furthermore the influence of the size of the intersection areas is investigated. It depends on the turning radius $r_\mathrm{t} = \{8, 10, 12\}$ of the intersection (IDs 19-21). Also, the road widths $w_\mathrm{r} = \{2.6\,\mathrm{m}, 2.8\,\mathrm{m}, 3.0\,\mathrm{m}, 3.2\,\mathrm{m}\}$ (IDs 24-27) and combinations of those parameters (ID 22, 23 and 28) are investigated.

For each simulation run between 5 and 8 vehicles are initialized and their starting positions, initial velocity v_0, driving directions and target streets are assigned randomly. One of these vehicles is assigned to be the A-V at random and runs the decision-making algorithm presented above. The rest of the vehicles is controlled by the simplified algorithm for the C-Vs. Special behavior, i.e. the deviation from the traffic regulations, is assigned randomly to some C-Vs.

The simulations are run using the traffic simulation package *Simulation of Urban Mobility* (SUMO) [24] with a time resolution of $\Delta t = 0.05\,\mathrm{s}$. Following the steps described in section 3 the features, base events and events are computed; the DES is then run based on these. The result of this decision-making process, the acceleration for the following time step, is fed back into the simulation. This is done for each vehicle using the algorithm that is assigned to it.

4.2. Simulation Results

To validate the decision-making model each of the maps was simulated 1200 times with varying conditions. This number is chosen to include 100 runs for each pair of entry and driving directions at an X-intersection. This results in a total number of 55 200 simulations. There were no collisions that involve the A-V and only 11 between the C-Vs. These are considered to be irrelevant as the algorithm from section 3 is being evaluated and not the simplified one for the C-Vs. Also, all collisions can be attributed to the simplified algorithm of the C-Vs which does not reconsider the decision to drive. Especially during deadlock situations this can lead to collisions if two C-Vs decide to start driving at the same time.

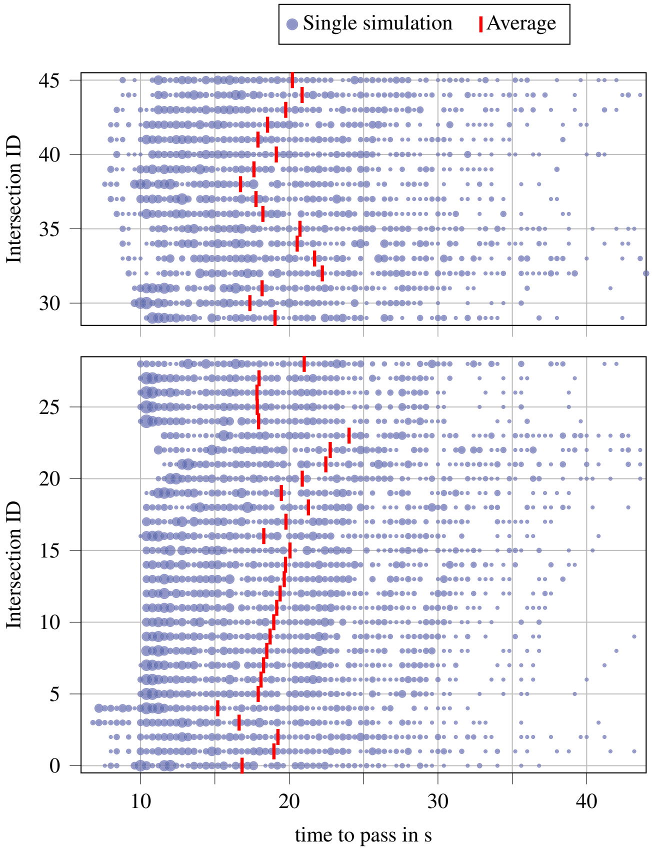


Figure 5. Time to pass the intersection ($-10\,\mathrm{m} \leq d_{\mathrm{s}}^{\mathrm{a}} \leq 30\,\mathrm{m}$) of the A-V for different intersections with a fixed visibility of 10 m for all cases. The top plot (ID 29-45) represents real world T- and X-intersections , while the bottom plot (ID 0-28) shows synthetic variations of generic intersections. Each row represents a single intersection with the same 1200 random simulation configurations on this specific map.

Besides ensuring that the algorithm is able to safely drive through many different intersection geometries, also the duration for the crossing maneuver reveals important conclusions. To that end, figure 5 shows the time it takes the A-V to pass the intersection (t_{p}). This duration is defined as the time it takes to drive from $d_{\mathrm{s}}^{\mathrm{a}} = 30\,\mathrm{m}$ before the intersection to $d_{\mathrm{s}}^{\mathrm{a}} = -10\,\mathrm{m}$ behind it. For easier interpretation the exact times are sampled into bins of length 0.4 s. If multiple runs end up in the same bin the marker size is enlarged.

To compare the influence of the aforementioned variations t_{p} for the generic T-intersection $\mu_{t_p,0} = 16.82\,\mathrm{s}$ ($\sigma_{t_p} = 6.38\,\mathrm{s}$) and X-intersection $\mu_{t_p,5} = 17.91\,\mathrm{s}$ ($\sigma_{t_p} = 6.91\,\mathrm{s}$) are used as references. The simulation results when using the synthetic maps (ID 0-28) reveal the influence of different aspects of the map geometry. Deviations from the perpendicular road angle at the T-intersection result in an increase of μ_{t_p} and σ_{t_p}. This increase is larger for obtuse angles ($\alpha = 135^\circ : \mu_{t_p,2} = 19.24\,\mathrm{s}$), which matches the expected outcome.

Reducing the road length results in a reduction of t_{p}. A shorter road in combination with multiple vehicles with the same entry direction results in vehicles being placed closer to the intersection, not starting the algorithm in state s_{10}. This bias leads to a different approach behavior, resulting in different constellations close to the intersection area. On average these constellations, which resulted form the placement closer to the intersection, allow the A-V to pass the intersection faster. Increasing the deviation of a single or of the two opposite road angles leads to an increase of t_p, being the highest for the most extreme road geometries $\mu_{t_p,15} = 20.05\,\mathrm{s}$ ($\alpha = 52°$) and $\mu_{t_p,18} = 21.28\,\mathrm{s}$ ($\alpha = 45°$) respectively. This behavior is realistic because at these angles the entire intersection is visible only at a small distance to it. As a result, the defensive state is maintained for a longer part of the approach, which leads to higher t_p values. Although such extreme angles are unlikely to appear in real world intersections the algorithm works there as well with the expected results.

For higher turning radii r_t, which cause larger intersection areas, t_p is increased considerably e.g. $\mu_{t_p,21} = 22.46\,\mathrm{s}$ ($r_\mathrm{t} = 12\,\mathrm{m}$). This can be explained by the time spent inside the intersection area. Due to the set target velocities for each state (see table 3) and the last state being reached only when the A-V leaves the intersection, it has to cover a longer distance with a lower velocity. Variations of the road width w_r influence t_p in a minor way, which is expected, as all vehicles follow the center of their lanes and all road widths used are wide enough to pass each other. The aforementioned variations can also be combined. Their effects on t_p are superimposed which results in larger deviations for those cases (e.g. $r_\mathrm{t} = 10\,\mathrm{m}$ and $\alpha = 45° : \mu_{t_p,23} = 24.02\,\mathrm{s}$; $w_\mathrm{r} = 2.6\,\mathrm{m}$ and $\alpha = 45° : \mu_{t_p,28} = 21.00\,\mathrm{s}$). Those combination of multiple variations at once also apply for the real world maps (ID 29-45).

5. Conclusion

The proposed decision-making model for non-signalized intersections is capable of making reasonable, understandable decision in urban scenarios. The algorithm does only rely on properties that can be observed directly and no explicit communication is used. Nonetheless, the model is able to reliably interpret the behavior of its cooperation partners, resulting in a safe crossing of the intersection. This is evident from the absence of collisions, even if deadlocks occur or in cases where the C-Vs do not respect the regulations and force the A-V to react to this behavior. The model for decision-making has been extensively tested in a simulation environment.

This modeling concept of traffic situations allows for a simplification of complex scenarios. In future work, we intend to expand the model to cover further traffic situations like roundabouts or signposted intersections and to check whether the simulated results are also reflected in complex, real world scenarios. Also, simulating entire road networks is a promising future endeavor.

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