Journal of Manufacturing Systems 78 (2025) 109–123

Contents lists available at ScienceDirect

Journal of Manufacturing Systems

journal homepage: www.elsevier.com/locate/jmansys

Technical paper

Assisted production system planning by means of complex robotic assembly line balancing

Louis Schäfer *, Stefan Tse, Marvin Carl May, Gisela Lanza

wbk Institute of Production Science, Karlsruhe Institute of Technology (KIT), Kaiserstr. 12, Karlsruhe, 76137, Baden-Wuerttemberg, Germany

ARTICLE INFO

Keywords:
Production system planning
Assembly line balancing
Mathematical optimization

ABSTRACT

Today, manufacturers and suppliers are challenged to deliver customized products at the lowest possible cost and in increasingly shorter time frames, due to the increasing number of variants. Achieving this demands efficient production system planning. However, current planning in the manufacturing industry is heavily reliant on manual processes and individual expertise. Prior research tackles this issue by aiming to develop a comprehensive approach for assisted, model-based rough planning of production systems. This article focuses the optimization of variant-specific production systems. The basis for this is a process precedence graph that restricts the optimization of the assignment of process steps to stations. In the mathematical modeling of the *Assembly Line Balancing Problem* (ALBP), this work addresses complex constraints, including the selection of station equipment, the utilization of multiple robots per station and a non-discrete assignment of tasks. The approach developed is applied to the example of a Tier 1 automotive supplier, where the multi-criteria solution of the ALBP allows an evaluation of the planning result. To this end, this work compares the algorithmically generated solution both qualitatively and quantitatively with an example of manual expert planning. Thereby it demonstrates the broad, industrial applicability of the approach. Consequently, this research contributes to enhancing efficiency in production system planning, leading to sustainable reductions in both costs and time.

1. Introduction

Nowadays, production companies face several challenges in launching innovative products quickly and cost-effectively in the global market [1]. In the automotive and automotive supplier industry three trends can be observed:

In recent years, there has been strong growth in the automotive industry worldwide. At the same time, the overall systems developed are becoming increasingly complex [2] e.g. with Industrial Internet of Things [3], leading to the emergence of extensive, personalized knowledge [4].

A second complexity driver is the constantly increasing number of variants for companies to meet customer demands for individual products [6]. In response to this, the automotive supplier industry has to adapt to this diversity of variants from original equipment manufacturers (OEMs) and produces a complex variety of customer variants under high cost pressure.

Lastly, 39% of all vehicles are being produced in China. Not only this cost pressure from the Chinese automotive industry on the rest of the world, but the overall rapid development in China is influencing the average car life cycle, which has fallen from eight to less than six years [7].

As a result, manufacturing companies are confronted with **high-frequency and ever shorter development and planning cycles**. From the perspective of production system planning, these developments ultimately result in the need for **efficient planning processes** in order to meet the high demand for **customer-specific variants** with the shortest possible *time to market* (TTM). [8]

1.1. Problem description

This study focuses on variant-specific production system planning as it occurs at Tier 1 suppliers in the automotive industry. They mostly produce specifically requested customer variants from a very broad product portfolio. To initiate an order, the supplier first carries out an initial evaluation of the manufacturability of the desired customer variant and the manufacturing costs of an ideal production system by means of rough planning. This process can be characterized as follows: Due to high quantities of the requested variant, the **task** considers the customer-specific rough planning of new production systems rather

* Corresponding author.
E-mail address: louis.schaefer@kit.edu (L. Schäfer).
URL: https://www.wbk.kit.edu/21_4125.php (L. Schäfer).

https://doi.org/10.1016/j.jmsy.2024.11.008
Received 2 September 2024; Received in revised form 22 October 2024; Accepted 12 November 2024
Available online 27 November 2024

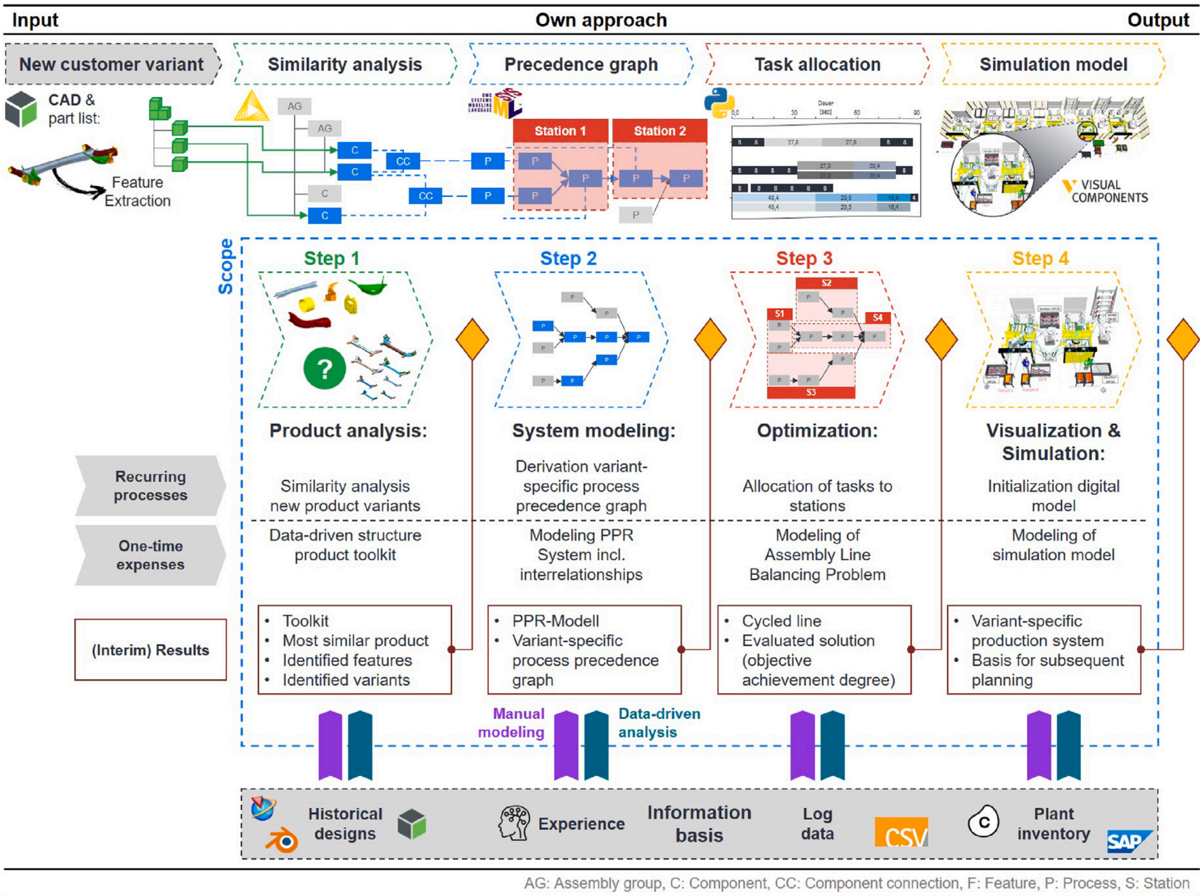

Fig. 1. Method of assisted production system planning according to Schäfer et al. [5].

than scheduling orders in existing systems. Regarding its **scope**, the planning focuses on variant-specific sections, where highly automated lines for the production and assembly of individual customer variants are planned, procured and put into operation. Lastly, the planning is **based** on product and order information available at the time of rough planning [9].

Within this industry process several problems arise, that need to be addressed with a novel research approach: In the first step of product analysis, the available product information for the requested variant is often limited and subject to future changes. The variants produced are functionally similar, but the differences in design and structure have a huge impact on process and production system planning. Secondly, process planning from product characteristics to production processes is based on experience and is often dependent on personal knowledge. Most importantly, the current document-based optimization of the production system (e.g. in Excel) is time-consuming, manual, user-bound and does not consider multiple target criteria. Lastly, planning results are not integrated into digital (simulation) models for further use in detailed planning.

1.2. Prior research

A holistic approach was developed to address these four problems (see Fig. 1). In the first step, a product analysis identifies features and connecting elements through similarity analyses according to the principle that similar products are produced similarly [10]. The second step models relationships between product, process and production system including metadata and empirical knowledge to derive the process precedence graph [11]. In the third step, the process steps need to be assigned to stations considering their precedence relationships [12]. In the last step, the optimized production system is visualized as a digital (simulation) model, integrating the solution into a holistic digital planning process. This paper only focuses on the third step by means of Assembly Line Balancing (ALB).

1.3. Goal

This article proposes an approach for using multi-criteria optimization in rough planning of variant-specific production systems. In the literature, this problem is referred to as *Assembly Line Balancing Problem* (ALBP) and maps the real problem in mathematical detail to allocate process steps to stations whilst optimizing specific objective functions [13]. In reference to a real-world industrial use case, the mathematical modeling should be as realistic as possible and should guarantee that solvers can find an optimal solution.

1.4. Structure of work

Section 2 contains the relevant foundations for this work and evaluates the current state of research regarding ALB. Section 3 presents the own approach. Subsequently, in Section 4 the approach is applied using the example of an automotive supplier. A discussion of the developed methodology and the findings obtained, as well as a summary and possible extensions are provided in Sections 5 and 6 respectively.

2. Literature review

This chapter summarizes the basic knowledge of the optimization in production system planning. This is followed by the state of the art and a final conclusion.

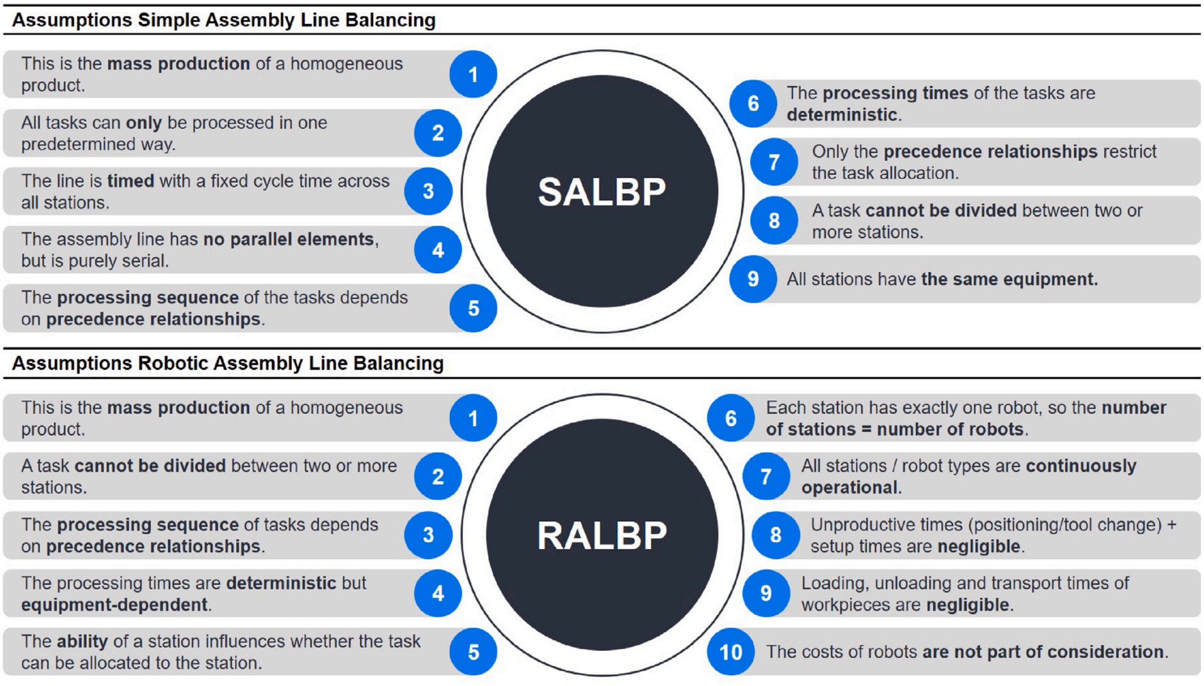

Fig. 2. Top: Assumptions of Simple ALBP according to Boysen & Fliedner et al. [14]. Bottom: Assumptions of Robotic ALBP according to Chutima [15].

2.1. Basic knowledge

In **production system planning**, planning tasks can be structured according to the levels of production: Production network, location, production system and workstation [16], whereby the focus of this work is the ALB on production system level. Here, Hagemann [9] divides planning processes into concept planning, rough planning and detailed planning phases. Starting from rough to detailed, the phases deal with the comparison of different production concepts during product planning, the creation of an optimal production system design and the detailed planning of the layout as well as the individual stations [17]. Due to a lack of data in an early phase, the rough planning follows from the ideal to the real production system, which differ in the consideration of boundary conditions. This paper deals with the ideal rough planning of a new production system. Here, planning tasks differ depending on the DIN 8580 processes [18]. While in part manufacturing (e.g. groups 1–3), product features are extracted and their process steps put into order, in assembly (group 4) an optimal production sequence has to be derived from the bill of materials. This sequence must fulfill all precedence relationships between process steps. Optimization algorithms can be used for this purpose.

Mathematical optimization models describe real-world problem with the aim of making good decisions using mathematical methods. Depending on the characteristics of the objective function and the constraints as well as the decision variables, optimization models are classified as Integer (linear) Programming or *Mixed-Integer Linear Programming* (MILP) [19]. MILP – as the problem considered in this paper – is NP-hard [20,21] and thus belongs to the mathematically most difficult problems to solve [22]. Solution algorithms for solving optimization models can be classified into exact and heuristic methods. This work uses a default solver, which decides on the exact solution algorithm to be used when solving the mathematical problem. In the case of multiple objective criteria, the presented approach applies the scalarization method, which combines the objective functions with weights to form a single substitute objective function [23].

When modeling mathematical models, simplifying assumptions are often made. This also applies to the *Simple ALBP* (SALBP), which was first formulated by Salveson in 1955 [24] (see Fig. 2 top).

Table 1
Classification scheme for ALBP according to Boysen et al. [14].

Precedence graph (α)	
α_1	product-specific precedence graph
α_2	structure of precedence graph
α_3	characteristics of processing time
α_4	sequence-based processing times
α_5	restrictions on task assignment
α_6	process alternatives
Stations & assembly line (β)	
β_1	transportation of products
β_2	layout of assembly line
β_3	parallelization
β_4	allocation of equipment
β_5	station-based processing times
β_6	further configuration possibilities
Objective function (γ)	

Since assumption 9 (all stations are the same) does not adequately represent reality in production system planning, Rubinovitz et al. [25] expand the ALBP to the so-called *Robotic ALBP* (RALBP). This extends the initial problem to include a selection of possible equipment (e.g. robots) that can be used at the stations and have an effect on e.g. processing times of individual tasks [26]. Chutima [15] summarizes common assumptions of literature on RALBP in Fig. 2 bottom. The following section summarizes the relevant state of the art in this context.

2.2. Literature review

In order to compare application-specific formulations of the ALBP, Boysen et al. [14] propose a scheme for classifying existing ALB approaches. The classification scheme shown in Table 1 is based on the properties of the precedence graph (α), the properties of stations and the *Assembly Line* (AL) (β) as well as the objective function (γ).

Further approaches extend this classification scheme [14] including the industrial context of the use case [27] and the selection of exact and heuristic solution methods [28]. Based on this analysis, this results in five criteria for the evaluation and characterization of approaches from the literature and enable a comparison between these:

C1 compares the scope of application and the assumptions made with it and distinguishes between real (R), fictitious (F) and benchmark (B) data sets.

C2 specifies the criteria of the selected objective function.

C3 compares the AL design and deals with the properties of the stations (multi-manned, parallel stations, selection of station type).

C4 evaluates the properties of the process steps, focusing on precedence relationships and the divisibility of tasks.

C5 compares the selected optimal (O) or heuristic (H) solution methods.

The literature on (R)ALB is wide-ranging, as shown by Battaïa & Dolgui, which includes more than 300 papers up to 2013 [27]. This current state of research focuses on more recent developments and refers to Ghosh & Gagnon for the beginnings of ALBP in earlier work [29]. This paper considers similar or relevant articles that treat a single-product line with fixed cycle time as MILP without stochastic components.

For a better understanding of the modeling assumptions, a look at the application area [C1] is crucial. Most studies use fictitious benchmark data sets instead of real use cases. An investigation of ALB on real-world problems is conducted in manual assembly lines [30], automotive assembly lines [31], automotive engine manufacturing [32], car body manufacturing [33] and battery production [34].

Although realistic production system planning considers multiple objectives [C2], several approaches involve a single-objective optimization problems. In this work, the cycle time is given by the customer, which necessitate the minimization of the number of stations. Most of the analyzed literature evaluates the cost of opening a station as well as additional station equipment and minimize a total cost function. A single objective function can be formed based on the number of stations or employees (see [31,34–39]). Abdous & Delorme et al. depicts ergonomics both as a fatigue and recovery model and as ergonomic investment costs [40]. Another factor to be considered is the sustainability, e.g. through energy consumption [35] or CO2-emission [32]. Lopes & Pastre et al. optimize the AL length by an abstract objective function [41]. *Job Shop Scheduling* (JSS) as a related problem takes further optimization criteria into account such as capacity utilization [42] and specific energy consumption [43] due to the higher availability of information in production operations.

The AL design [C3] is varied by, for example, allocating multiple robots or workers to a station and station-internal scheduling to maintain precedence relationships within the station. Yuan & Xu et al. consider station-internal scheduling in two-sided ALBP, where the work piece is processed from both sides and the worker can assume different positions on each side [39]. Michels & Lopes et al. present parallel stations as the only approach using a parallelism factor, whereby the processing time can be halved due to the asynchronously loaded, parallel stations [33]. More than half of the models make decisions about the station equipment, either through free combination [30] or predefined station types [40]. The equipment influences task processing through the specifically required equipment [44] or equipment-dependent processing times [45].

For the mathematical modeling of ALBP, the characteristics of process steps [C4] are relevant. In addition to the precedence relationships at station level, a fundamental property of ALBP, the station-internal scheduling considers the precedence relationships within a station. All multi-manned approaches consider a station-internal scheduling, with one exception [33]. The scheduling is usually modeled with start and end times for each task. Michels & Lopes do without because their application focuses on spot welding in car body construction, where station-internal precedence relationships between individual welding points do not exist [33]. In the literature, process steps generally cannot be divided across several stations, so that a task must always be processed completely at one station. With welding operations, however, a division of tasks must be possible. From the analyzed approaches,

● **Fulfilled**
◐ **Partially fulfilled**
○ **Not fulfilled**
F Fictional data
R Real use case
BM Benchmark data
E Exact
H Heuristic
EH Exact & heuristic

		C1	C2	C3			C4		C5
		Application	Ojective function	*multi-manned*	Parallel stations	Station type	S.-internal sched.	Non-discrete	Solution approach
[1]	Abdous et al. (2022)	*F*	multi-crit.	○	○	●	○	○	*E*
[6]	Albus & Seeber (2021)	*R*	costs	○	○	●	○	○	*E*
[13]	Chen et al. (2018)	*R*	multi-crit.	●	○	●	●	○	*H*
[14]	Chi et al. (2022)	*BM*	multi-crit.	○	○	●	○	◐	*EH*
[19]	Fattahi et al. (2011)	*BM*	multi-crit.	●	○	○	●	○	*EH*
[29]	Li et al. (2022)	*R*	CO_2 emissions	○	○	●	○	◐	*E*
[30]	Lopes et al. (2021)	*BM*	number stations	○	○	○	○	○	*EH*
[31]	Lopes et al. (2020)	*BM*	length of the line	●	○	○	●	○	*EH*
[33]	Michels et al. (2018)	*R*	costs	●	●	●	○	○	*E*
[33]	Navas-Barrios et al. (22)	*R*	number stations	○	○	○	○	○	*E*
[38]	Nugraha et al. (2021)	*R*	costs	○	○	●	○	○	*E*
[40]	Roshani & Giglio (2015)	*BM*	costs	●	○	○	●	○	*E*
[43]	Sahin & Kellegöz (2019)	*BM*	costs	●	○	●	●	○	*EH*
[54]	Weckenborg et al. (20)	*BM*	multi-crit.	●	○	●	●	○	*E*
[56]	Yilmaz & Yilmaz (2020)	*BM*	multi-crit.	●	○	○	●	○	*EH*
[57]	Yuan et al. (2015)	*R*	number stations	●	○	○	●	○	*E*

Fig. 3. Overview of the current state of research on ALB.

only a cross-station design enables the task to be distributed across two adjacent stations [32,35].

Depending on the complexity of the considered problems, which is determined by the type and number of variables as well as the size of the problem, the analyzed studies use solution approaches with heuristic and exact solution algorithms [C5]. Heuristic methods are a proven alternative for finding a good solution within a short time when exact methods such as Branch-&-Bound- or Branch-&-Cut-algorithm require long computing times [46]. Fattahi & Roshani et al. applies a Branch-&-Cut-algorithm to reach an optimal solution [36].

2.3. Summary of the state of the art

In literature, various optimization approaches exist. Objective functions [C2] and constraints, that are heavily influenced by the AL-Design [C3] and the modeled attributes of process steps, characterize the ALBP. Several papers optimize one cost objective function or apply their multi-criteria optimization mostly to fictional or benchmark datasets. Only Chen & Cheng et al. optimize a real use case by multiple criteria [31]. With regard to the AL design [C3], Michels & Lopes et al. enable several robots to work on the work piece at the same time [33]. The stations have equipment-dependent processing and handling times, but a station-internal scheduling regarding the precedence relationships is not used in spot welding. No paper considers a divisibility of process steps [C4] (e.g. weld seams) with a non-discrete task to station assignment. The analyzed approaches use exact and heuristic solution methods [C5] (see Fig. 3).

Table 2
Definition of a Complex ALBP (CALBP).

	Characteristics	S	R
1	This is the mass production of a homogeneous product.	✓	✓
2	Tasks can be processed in more than one predetermined way.	X	✓
3	The line is timed with a fixed cycle time across all stations.	✓	X
4	The assembly line can consist of parallel stations.	X	✓
5	The processing sequence of tasks depends on precedence relationships.	✓	✓
6	The processing times of the tasks are deterministic but equipment-dependent.	X	✓
7	Not only the precedence relationships restrict the task allocation.	X	✓
8	Tasks can be divided between two or more stations.	X	X
9	All stations do not have the same equipment.	X	✓
10	The ability of a station determines whether the task can be allocated to the station.	X	✓
11	Stations can have more than one robot (multi-manned ALB).	X	X
12	All stations are continuously operational.	X	✓
13	Unproductive times (positioning and tool change) and setup times are not negligible.	X	X
14	Load and unload as well as transportation times are not negligible.	X	X
15	The costs of robots are to be considered.	X	X

According to the analysis above, none of the identified approaches can adequately depict the given real-world problem with a combination of multi-criteria objective function, multi-manned equipment-dependent stations and a non-discrete assignment of tasks.

3. Own approach

This chapter describes the characterization of the problem and the applied methodology. Subsequently, the mathematical model of the problem is formulated and explained.

3.1. Problem characterization

The analyzed real-world problem in this paper involves allocating tasks to stations within a given cycle time whilst optimizing several criteria. It therefore falls into category I of the classification scheme proposed by Boysen & Fliedner et al. [14]. More precisely due to an equipment-dependent modeling of the stations, it can be described as a RALBP. Therefore, following the simplifications of general RALBP according to Chutima [15] (see Fig. 2), Table 2 defines a **Complex ALBP (CALBP)** by means of its characteristics. A comparison to simplifying assumptions of Simple (S) and Robotic (R) ALBP by Boysen et al. [14] and Chutima [15] respectively indicates the complexity of the problem at hand. A check mark (✓) indicates common assumptions of the complex ALBP taken from literature (1–7, 9, 10, 12). For example, both SALBP and RALBP as well as the complex ALBP at hand all consider the mass production of a homogeneous product (1) but while SALBP requires all stations to have the same equipment, with RALBP and CALBP stations can differ regarding their equipment (e.g. robots) (9).

Assumptions taken from S- and RALBP generally depict a simplification of real-world optimization problems. However, many assumptions of Boysen & Fliedner et al. [14] and Chutima [15] as seen in Table 2 are not applicable to the problem at hand to model the real-world use case sufficiently. New characteristics, that are contributed for the first time, include the division of tasks across two or more stations (8), considering more than one robot per station (11), their costs (15) as well as a consideration of unproductive (13) and handling times (14). None of these represent simplifying assumptions, but rather a more detailed modeling of the real problem. Especially the non-discrete assignment of tasks to stations (8), that is necessary for balancing long joining tasks (such as welding) adds to the complexity of the mathematical problem. Taking these aspects into account is considered the core innovation of this article. CALBP characteristics can be summarized as follows:

$\boldsymbol{\alpha}$: The precedence relationships between tasks and the type of task restrict the allocation of tasks to stations. Some tasks can only be processed on certain station types (α_5 = type). As a result, there are several possibilities to conduct a task. Furthermore, it is necessary to divide certain tasks between several stations, especially when joining by welding.

$\boldsymbol{\beta}$: The AL is timed by a fixed, cross-station cycle time, which is given by the customer based on the planned sales volume and the available production time. Parallel stations in a line can halve the station-internal cycle time through an asynchronous loading. The *process ability* of stations determines the possible allocation of tasks and is influenced by the station equipment. Each station has one or two robots ($\beta_3 = pwork^2$). According to some underlying combination rules, the equipment is explicitly modeled and not selected from predefined alternatives ($\beta_4 = res^{\circ}$), which influences the costs of the stations. Finally, unproductive times like loading, unloading and positioning of robots must also be taken into account as a significant portion of the station time ($\beta_5 = \Delta t_{unp}$).

$\boldsymbol{\gamma}$: Production system planning optimizes multiple, partly competing objective criteria. Therefore, the objective function requires a weighting of criteria (e.g. main objective cost minimization $\gamma = Co$).

3.2. Applied methodology

An iterative approach from rough to fine is appropriate for the development of a complex mathematical model, as described in the method by Schäfer & Kochendörfer et al. [47]. This method follows data-driven development according to Bach [48], in which a minimum executable example is first created and then iteratively expanded. The result of this iterative development is the model presented in the following section.

3.3. Formulation and explanation of the model

An optimization model consists of objective functions and constraints. First, the objective functions are introduced, followed by the constraints. To consider multiple objective criteria, a substitute objective function (see equality (28) in appendix) is introduced by means of scalarization using a weighting vector λ with λ_c for cost, λ_a for area, λ_f for flexibility and λ_d for tolerance deviations as well as the single-criteria objective functions (24)–(27). To ensure comparability of the different objective values (costs G_c, area G_a, flexibility G_f and tolerance deviations G_d), they are each normalized. Thereby, the optimal values of the single-criteria optimization (G_c^*, G_a^*, G_f^* and G_d^* respectively) are used as normalization coefficients in the multi-criteria optimization G_m. The overall objective function G_m aims to maximize the degree of objective achievement. For this purpose, objectives originally formulated as minimization problems (costs, area requirement, tolerance deviation) are each multiplied by a factor of −1. In the following, cost minimization, maximization of flexibility and quality as well as optimization of ergonomics are examined.

The prioritized cost function in (24) relies on the number of stations and includes labor and capital costs [49]. Labor costs for manual handling c_{handlH} are incurred not only during the active time of the worker, but during the entire cycle time. Capital costs include interest and depreciation for the stations. Each station is based on basic equipment for basic cost c_{base} and additional equipment for further costs like $c_{addRobot}$ for a second processing robot, c_{handlR} for a handling robot and c_{addTC} for a turntable cell. Besides the investment costs, the operating costs of the line are also included. To maintain the linearity of the model, a known operating period is assumed, whereby time-related cost rate can be converted into time-independent costs and the objective function can be linearized.

Two objective functions regard the flexibility by minimization of used area as factory or variant flexibility and maximization of cycle time reserve as volume flexibility. Minimization of used area is similarly structured as the cost function, where all cost coefficients c can be replaced through equipment-based area coefficients a (see equality (25) in appendix). Maximizing the cycle time reserve competes with minimizing costs, especially the number of stations (see equality (26) in appendix). A high cycle time reserve enables demand peaks to be covered, for example. Due to its gradient, which shows optimal values with a maximum number of stations, this target criterion is introduced exclusively as part of the multi-criteria optimization.

A distinction between tolerance-critical and normal tasks enables to model the quality as the degree of fulfillment of tolerance requirements. The time at which a task is completed in relation to the position of a station in the line is seen as an indicator of the degree of fulfillment, since tolerance-critical tasks should ideally be finished at the end of the line. This avoids, for example, deformation due to subsequent heat and tolerance deviations that occurred earlier can be compensated by this. Therefore, the binary variable f_{ij} takes the value 1 if the task i is completed on station j and depict the tolerance deviations through the difference between the last station n and the completing station $j * f_{ij}$ (see equality (27) in appendix).

In the literature, the ergonomic is usually considered as exhaustion of workers depending on their workload or as minimum requirement for the workplace [14]. Because of the early phases of production system planning, a detailed assessment of work processes is unrealistic. Since human activity within the RALBP is limited to handling processes, ergonomics is introduced here as a constraint regarding the maximum handling weight.

The model is characterized by these constraints, hence the constraints are presented below.

Constraint (1) ensures that the allocated tasks $i \in I$ can only be processed on stations $j \in J$ with the required abilities $k \in K$ and must be completed over all stations. For this purpose, the entries of the matrix F_{ik} assume the value 1 if the task i must be processed on station type k. The variable x_{ijk} takes a value between 0 and 1, which represents the allocated non-discrete proportion of tasks.

$$\sum_{j\in J} x_{ijk} = F_{ik} \quad \forall i \in I, \forall k \in K \tag{1}$$

Tasks can only be allocated to opened stations and opening a station is only possible with at least one allocated task, which guarantee constraint (2) and (3). Also, each station only has one station type (see constraint (4)). The variable v_{jk} assumes the value 1 if the station j is open. While, x_{ijk} describes the assigned proportion, w_{ijk} indicates whether a task is being processed on a station at all. M represents a large number for the Big-M-Constraint.

$$\sum_{i\in I} x_{ijk} \le v_{jk} \cdot M \quad \forall j \in J, \forall k \in K \tag{2}$$

$$v_{jk} \le \sum_{i\in I} w_{ijk} \quad \forall j \in J, \forall k \in K \tag{3}$$

$$\sum_{k\in K} v_{jk} \le 1 \quad \forall j \in J \tag{4}$$

Precedence graphs define technical or organizational sequence restrictions between tasks [14]. This work assumes, based on existing literature, that tasks with a precedence relationship must be completed at separate, sequential stations. The constraints (5)–(9) realize the compliance of precedence relationships. If a task i is already being processed between station h and station j, then $y_{ij} = 1$ applies to j and all subsequent stations as noted by the lower bound constraint (5) and the upper bound constraint (6). Similarly, the variable z_{ij} indicates, whether the task is already finished at station by constraints (7) and (8). The precedence relationship is finally implemented in constraint (9): If task m has to be completed before task n, then the precedence matrix P_{mn} takes the value 1. Task n can only be started, respectively y_{nj} can only assume the value 1 if $z_{mj} = 1$.

$$\sum_{h=1}^{j} \sum_{k\in K} x_{ihk} \le y_{ij} \quad \forall i \in I, \forall j \in J \tag{5}$$

$$y_{ij} \le M \cdot \sum_{h=1}^{j} \sum_{k\in K} x_{ihk} \quad \forall i \in I, \forall j \in J \tag{6}$$

$$z_{ij} \le \sum_{h=1}^{j-1} \sum_{k\in K} x_{ihk} \quad \forall i \in I, \forall j \in J \tag{7}$$

$$1 - z_{ij} \le M \cdot (1 - \sum_{h=1}^{j-1} \sum_{k\in K} x_{ihk}) \quad \forall i \in I, \forall j \in J \tag{8}$$

$$y_{nj} \cdot P_{mn} \le z_{mj} \quad \forall m, n \in I, \forall j \in J \tag{9}$$

A special feature of the model is the implementation of the divisibility of tasks, which has not yet been considered in the state of the art (see Section 2.2), but is relevant for joining tasks such as welding. The non-discrete assignment takes place with the variable x_{ijk}, which is assigned a value in the range of [0,1] for proportional processing of a task at a station of type. For a realistic modeling of continuous assignment, the divisibility is limited by a share minimum x_i^{min} (see constraint (10)) and the overall processing time is extended overall stations (see constraint (11)–(14)). For example, if a weld seam is split, an overlap must be welded.

Constraint (10) ensures that all allocated tasks exceed the share minimum. To represent an overlap of task i on station j and the resulting extend of the overall processing time, the model introduces the variable o_{ij}. constraint (11) sets $o_{ij} = 1$ when station j processes task i but not yet completed. Similarly, constraints (12) and (13) set $o_{ij} = 0$ if the task i is either not processed on station j or has been fully completed. Constraint (12) forces $o_{ij} = 0$ for all task types that do not allow splitting.

$$x_{ijk} \ge x_i^{\min} \cdot w_{ijk} \quad \forall i \in I, \forall j \in J, \forall k \in K \tag{10}$$

$$o_{ij} \ge w_{ij1} - \sum_{h=1}^{j} x_{ih1} \quad \forall i \in I, \forall j \in J \tag{11}$$

$$o_{ij} \le x_{ij1} \cdot M \quad \forall i \in I, \forall j \in J \tag{12}$$

$$o_{ij} \le (1 - \sum_{h=1}^{j} x_{ih1}) \cdot M \quad \forall i \in I, \forall j \in J \tag{13}$$

$$o_{ij} \le 1 - w_{ijk} \quad \forall i \in I, \forall j \in J, \forall k \in K \setminus \{1\} \tag{14}$$

The resulting cycle time of a station depends on its equipment. The model presented focuses on the three exemplary station types (welding, laser cutting, testing) with robots, but can be extended as required. Either one ($\alpha_j = 0$) or two ($\alpha_j = 1$) robots are used at each station. A turntable enables parallel processing of the tasks on the back of the table with simultaneous loading and unloading on the front. Despite the rotation time of the turntable, more processing time can be allocated to it. The algorithm assigns either a *Turntable Cell* (TC) with turntable ($\delta_j = 1$) or an *Orbit Cell* (OC) without turntable ($\delta_j = 0$) to a station.

Although the literature mostly does not consider unproductive times (see Section 2.2), the selected equipment influences the productive processing time and the unproductive time of stations. Since in reality handling processes on highly automated, synchronized robotic lines are crucial, this model examines handling processes in three aspects: First, each station can be loaded by human ($\beta_j^H = 1$) or by robots ($\beta_j^R = 1$). Similarly, the unloading and transfer to the next station can be carried out by human ($\gamma_j^H = 1$) or by robots ($\gamma_j^R = 1$). Finally, during joining processes ($k = 1$), additional parts that have not yet been joined with other parts are loaded in addition to the transfer from the previous station, modeled in constraints (15)–(17).

Constraint (15) guarantees the loading of parts to the first station ($l_{p1} = 1$), if part $p \in P$ ($N_{ip} = 1$) is required for the allocated task w_{i11}. The loading of parts to the subsequence stations is realized in constraint (16) by considering loading of the same part on previous stations.

$$l_{p1} \geq w_{i11} \cdot N_{ip} \forall i \in I, \forall p \in P \tag{15}$$

$$l_{pj} \geq w_{ij1} \cdot N_{ip} - \sum_{h=1}^{j-1} l_{ph} \quad \forall i \in I, \forall j \in J \setminus \{1\}, \forall p \in P \tag{16}$$

$$l_{pj} \leq \sum_{i \in I} (w_{ij1} \cdot N_{ip}) \forall p \in P, \forall j \in J \tag{17}$$

A more general approach of constraint (15) represents constraint (18), which checks the necessity of the loaded parts for the allocated tasks on each station and avoids a loading of all parts on the first station. If the algorithm decides for a station with automatic loading ($\beta_j^R = 1$), automatic transfer of the semi-finished products to the next station ($\gamma_j^R = 1$), a turntable ($\delta_j = 1$) and a processing robot ($\alpha_j = 0$), then both constraints apply and the handling time in constraint (18) and the processing time in constraint (19) are limited by the cycle time c_t. Other options of a station are manual loading ($\beta_j^H = 1$) and manual transfer of the semi-finished products to the next station ($\gamma_j^H = 1$). The handling time in constraint (18) consists of the time for removing the parts from the previous cycle $t^{out,R}$ by robot or $t^{out,H}$ by human, the time for loading the parts (per robot $t_p^{in,R}$ or human $t_p^{in,H}$ across all parts p) and the time for turning the table t^{turn}. In the processing time in constraint (19), the total processing time t_i of a task i is multiplied by the proportion x_{ijk} processed at station j in addition to the non-productive time for positioning the robots t^{pos} for tasks i processed on station j with type q. A penalty term for overlaps is optionally taken into account by time for overlap the welded seam t^{o} multiply with o_{ij}. It is assumed that a second robot halves the processing time. Analog to the handling side, t^{turn} and t^{res} are added. The variable t^{res} determines the cycle time reserve for optimizing volume flexibility. Stations without turntables only have one constraint for maintaining the cycle time, which combines the handling time and processing time according to the same pattern (see constraint (20)). A challenge in modeling is the transfer between the stations. It is assumed that the transfer of the semi-finished products takes place synchronously at the end of a cycle. To avoid collisions between humans and robots, the same transfer time is assumed for both.

$$t^{\text{out,R}} + \sum_{p \in P} l_{pj} \cdot t_p^{\text{in,R}} + t^{\text{turn}} + t^{\text{res}} \leq c_t + (3 - \beta_j^R - \gamma_j^R - \delta_j) \cdot M \quad \forall j \in J \tag{18}$$

$$\sum_{i \in I} \Big(\sum_{q \in \{1,2\}} (t^{\text{pos}} \cdot w_{ijq}) + \frac{1}{2} \cdot t_i \cdot x_{ijk} + t^{\text{o}} \cdot o_{ij} \Big) + t^{\text{turn}} + t^{\text{res}} \leq c_t + (2 - \alpha_j - \delta_j) \cdot M \forall j \in J, \forall k \in K \tag{19}$$

$$t^{\text{out,R}} + \sum_{p \in P} l_{pj} \cdot t_p^{\text{in,R}} + \sum_{i \in I} \Big(\sum_{q \in \{1,2\}} (t^{\text{pos}} \cdot w_{ijq}) + t_i \cdot x_{ijk} + t^{\text{o}} \cdot o_{ij} \Big) + t^{\text{res}} \leq c_t + (2 + \alpha_j - \beta_j^R - \gamma_j^R + \delta_j) \cdot M \forall j \in J, \forall k \in K \tag{20}$$

The consideration of an ergonomic handling of parts and assemblies is modeled as constraints (21)–(23). Constraint (21) restricts the weights of the loaded parts on a station, unless the station uses a robot for loading ($\beta_j^R = 1$). Since the maximum weight m_{limit} also applies to the transfer between stations, the weight of subassemblies m_j^{product} is calculated by the sum of product weight m_p used at the station j (see constraint (22)). Then constraint (23) limits this weight to the maximum weight limit (except for $\gamma_j^R = 1$), analog to constraint (21).

$$m_p \cdot l_{pj} \leq m_{\text{limit}} + M \cdot \beta_j^R \quad \forall j \in J, \forall p \in P \tag{21}$$

$$m_j^{\text{product}} = \sum_{h=1}^{j} \sum_{p \in P} m_p \cdot l_{pj} \quad \forall j \in J \tag{22}$$

$$m_j^{\text{product}} \leq m_{\text{limit}} + M \cdot (1 + \gamma_j^R - v_{jk}) \quad \forall j \in J, \forall k \in K \tag{23}$$

4. Application and results

After a presentation of the real-world industrial use case, the analyzed product as well as the production at hand are described (Section 4.1). Subsequently, the implementation of the ALBP is explained (Section 4.2). Followed by the results (Section 4.3) as well as a comparative evaluation and assessment of various solutions (Section 4.4) and a sensitivity analysis of the key parameters (Section 4.5), the validation of the solution of multi-criteria optimization by means of a comparison with manual planning completes this chapter (Section 4.6).

4.1. Use case and initialization

This chapter pursues the goal of applying the previously presented approach of the present work. To this end, variant-specific production system planning at a German Tier 1 automotive supplier is considered. The globally active company, headquartered in Germany, has a turnover of almost 9 billion euros in 2023 with its approximately 23,000 employees worldwide and operates 73 production sites in 26 countries. In the automotive sector, the supplier is particularly active in the areas of structural parts, chassis and suspension components.

The product considered in this work is the *Rear Twist Beam* (RTB). The supplier in question produces a total of around 8 million axles per year, with an annual peak volume of approximately 300,000 axles per customer.

The production of the specific RTB customer variants is usually carried out according to the so-called build-to-print principle. Here, the customer – in this case the *Original Equipment Manufacturer* (OEM) – is in charge of the product design. The subsequent planning task of the supplier includes the analysis of the requested customer variant, the identification of the necessary process steps and the line balancing (see Section 1.2 overall approach). In general, the production of an RTB includes the manufacturing (e.g. pressing) of the sheet metal parts and the assembly (e.g. welding) as well as the painting and finishing of the assembly. The former and the latter are carried out across variants with centrally available resources (e.g. press shop, paint line, etc.). The focus of the planning task is therefore on optimizing the variant-specific, highly automated assembly line. This primarily includes joining (welding), cutting (laser cutting) and testing processes.

The characterization of the optimization problem can be derived from the assumptions and simplifications made in Table 2. According to the classification of Boysen & Fliedner et al. [14], this is an ALBP of type I. The cross-station cycle time is determined by the required output quantity and the available production time:

$$c_t \leq \frac{248 \frac{d}{y} \cdot 3 \frac{shifts}{d} \cdot 7{,}5 \frac{h}{shift} \cdot 3.600 \frac{s}{h}}{250.000 \frac{orders}{y}} = 80{,}352 \text{ s} \approx 80 \text{ s}$$

The scalarization method is used to combine the partially competing objectives within the multi-criteria optimization using the weighting vector λ. In the use case of the Tier 1 supplier the weights are chosen according to the importance of the respective goal and introduced as follows: costs $\lambda_c = 0.6$, area $\lambda_a = 0.15$, flexibility $\lambda_f = 0.15$ and tolerance deviations $\lambda_d = 0.1$. In Section 4.4 however a multi-criteria

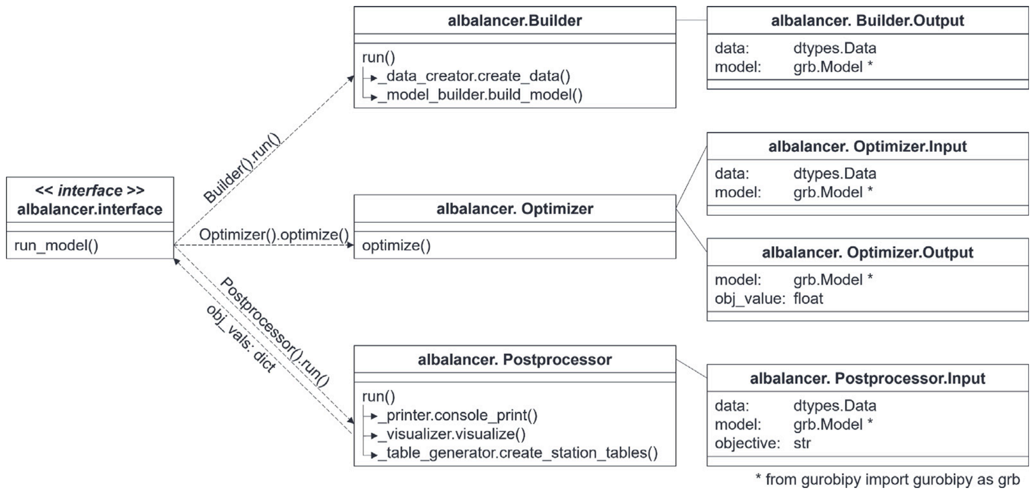

Fig. 4. UML diagram with the structure of the albalancer package including method calls through the interface.

Pareto curve (see Fig. 6) is given for 0.1 weight increments in order to showcase all weight combinations.

Of the three task types k at a station, welding tasks can be distributed non-discretely across multiple stations. Besides the choice between one or two robots at a station, the decision between TC and OC also influences the resulting costs, the required area and the behavior of the station. Handling operations with sub-assemblies or parts weighing less than 10 kg can be carried out by humans, while those weighing more than 10 kg can only be handled by robots. This option also changes the costs, area and handling time required. The customer-specific product, the twist-beam axle, is defined by its components and features such as weight and the process precedence graph. Depending on size, weight and shape, humans and robots require different loading times. The tolerance-critical tasks s_2 (bearing bush) and s_3 (adapter plate) for both sides left and right have an influence on the relative position of the twist-beam axle inside the vehicle.

The divisibility of the tasks is limited to a minimum of 15% ($x_{ij2} \geq 0,15 \quad \forall \quad x_{ij2} > 0$) and an additional process duration of 3 s for the necessary overlap when splitting a task are set for the processes. This non-discrete assignment adds to the complexity of the problem by turning $I = 29$ tasks into a total of $29 * 1/0.15 \approx 29 * 6 \approx 174$ “subtasks”. According to Chutima [15], the computational complexity of ALB problems is computed according to $I!/2^P$ where I is the number of tasks (here 174 “subtasks”) and P is the number of precedence relationships (here $P = 54$). The real-world problem at hand therefore has a computational complexity of $174!/2^{54} \approx 3.6 * 10^{299}$. In addition, Mukund Nilakantan and Ponnambalam [50] distinguish between small-sized, medium-sized and large-sized ALB problems using the number of tasks (here 174 “subtasks”) and introduce a WEST ratio, i.e. the ratio of the number of tasks and stations (here 7 stations — see Section 4.3). Accordingly, the present real-world problem with $174/7 \approx 25$ can be classified as a large-scale problem.

4.2. Implementation

For performance reasons, the proprietary solver Gurobi is selected for the exact solution of the ALBP, which, depending on the problem formulation and instance, uses a variety of pre-implemented solution algorithms. For the implementation with Python, the package “albalancer” is developed, including its own interface for calling the model. The resulting modular code structure is shown in Fig. 4.

It is implemented using the function build_model(), which uses the standardized structure of the Gurobi model object grb.model. This is realized by successively adding variables (grb.addVars()), constraints (grb.addConstr()) and objective functions (grb.setObjective()). To improve the solvability of the model, some adjustments were made. These include the consideration of numerical instabilities caused by floating-point operations, the automatic correction of poor conditioning of the problem using Big-M terms, and the introduction of complexity-reducing constraints. The last of these arise from the combination of existing constraints and do not change the permissible set.

4.3. Results

The optimal results of the single-objective optimizations are introduced using individual objective functions in order to evaluate the quality of the multi-criterial solution by the objective deviation. Table 3 provides an overview of the results.

Except for the optimization of the cycle time reserve, the other solutions have seven stations and a cycle time reserve of 0 s. The cost optimum is 11.19 million euros, where the singe-criterion area minimization with 0.44 million euros and the multi-criteria optimization with 0.27 million euros (2.39%) are close to each other.

Similarly, the results for minimizing the space requirement of a line are comparable, where the single-objective cost-optimal solution is more space-saving than the multi-criteria optimization, but both have less than 10% target deviation.

In contrast, the deviation from the optimal value for the other single-objective results is significantly higher for the quality index, but the optimum is achieved with multi-criteria optimization. In view of the conflicting nature of volume flexibility with the other objective functions, it is not surprising that all solutions have a target deviation of 48 s regarding the cycle time reserve.

The solutions differ in terms of their goal achievement, the selected station equipment and the task assignment.

The station equipment for cost minimization is shown in Table 4: With a total of nine processing robots across all stations, only one of seven stations uses a turntable, as a particularly large number of parts are loaded at station 3, which increases the loading time. Loading is done manually, while the transfer from station 3 is done by robots due to the weight of the parts (over 10 kg). Since no other tasks are possible at station 2 (laser cutting) and station 7 (testing), the low utilization is unavoidable. All other stations are almost fully utilized.

Since neither costs nor space requirements are relevant for single-objective quality optimization, its resulting line is equipped with four TC stations, ten processing robots and only robots for transferring the semi-finished products. As expected, the process allocation shows that the tolerance-critical weld seams (s_2 and s_3, on the right and left) are completed only at the last possible station 6. The configuration of the rest of the line (e.g. TC or OC) is irrelevant.

Maximizing the cycle time reserve is not very meaningful as a single-objective analysis due to its gradient (optimal target achievement

Table 3
Overview of the results.

		Stations [#]	Costs [Mio. €]	Area [m^2]	Quality index [1]	Cycle time reserve [s]	Computing time [s]
Single-objective	Stations	**7**	12,2862	87,91	17	0,00	23,46
	Costs	**7**	**11,1892**	76,13	20	0,00	790,22
	Used area	**7**	11,6314	**73,75**	18	0,00	593,00
	Quality	**7**	13,0758	93,30	**11**	0,00	45,87
	Cycle time reserve	20	40,6362	256,89	37	**48,00**	87,87
	Optimum	**7**	**11,1892**	**73,75**	**11**	**48,00**	–
Multi-criteria	Absolute	**7**	11,4566	80,52	**11**	2,37	387,24
	Objective deviation	**0%**	2,39%	9,19%	**0%**	−95,06%	–

Table 4
Line configuration for single-objective cost minimization.

Station	Station-Type	OC/TC	n Robots	Max. parts weight	Loading	Weight of semi-finished product	Transfer
1	Welding	OC	1	2,62 kg	Human	6,93 kg	Human
2	Laser cutting	OC	1	0,00 kg	–	6,93 kg	Human
3	Welding	TC	2	0,60 kg	Human	9,93 kg	Human
4	Welding	OC	2	7,93 kg	Human	17,86 kg	Robot
5	Welding	OC	1	1,05 kg	Human	19,01 kg	Robot
6	Welding	OC	2	1,05 kg	Human	20,06 kg	Robot
7	EOL Testing	OC	0	0,00 kg	–	20,06 kg	Robot

Table 5
Line configuration for multi-criteria optimization.

Station	Station-Type	OC/TC	n Robots	Max. parts weight	Loading	Weight of semi-finished product	Transfer
1	Welding	OC	1	2,62	Human	6,93	Human
2	Laser cutting	OC	1	0,00	–	6,93	Human
3	Welding	TC	2	7,93	Human	15,66	Robot
4	Welding	OC	1	1,05	Human	16,71	Robot
5	Welding	OC	2	1,05	Human	17,76	Robot
6	Welding	TC	2	0,60	Human	20,06	Robot
7	EOL testing	OC	0	0,00	–	20,06	Robot

with maximum number of stations). The maximum possible cycle time reserve in the application is 48 s. It is calculated from the specified customer cycle time (80 s) and the longest, indivisible processing time (30 s) plus transfer time (2 s).

The multi-criteria optimization aims to achieve the best possible compromise between the single-criteria solutions. A higher weighting of the costs ($\lambda_c = 0.6$) as expected leads to a high similarity to the solution of the single-criteria cost minimization. However, the number of TCs increases to two, which results in a cost increase of 2.39% and an increase in the required area of 5.77%. The line configuration is shown in Table 5 and still includes seven stations, with a cycle time reserve of 2.37 s being achieved.

The assigned station equipment and its tasks are visualized for the multi-criteria optimization problem using a Gantt chart for each opened station. The station occupancy in Fig. 5 shows that the tolerance-critical components are welded on the last possible station, which results in an optimal quality index. This means that the relatively heavy torsion beam is loaded at station 3, which requires an additional transfer robot. Overall, the production of the twist-beam axle is very symmetrical, except at stations 4 and 5, although this is not specified as a constraint.

4.4. Comparative evaluation and assessment

The results of the exact optimization illustrate how the design of the stations depends on the optimization objective. Depending on the labor and robot costs, the total weight of the semi-finished product is kept below 10 kg for a long time in the single-objective minimization of costs and area in order to avoid more expensive robot transfers. The comparison of both solutions shows that a cost-effective line configuration uses TCs to save a processing robot, while the opposite appears to a space-saving line configuration. The single-objective quality optimization identifies a solution in which, despite the sequence restriction by the precedence graph, the tolerance critical components are welded late.

The multi-criteria optimization provides a sufficient solution with the weighting factors defined in the application: There is no or only a small deviation from the objective for quality index and costs, and the area requirement increases by less than 10% compared to the optimal value. However, the maximization of the cycle time reserve is considered a strongly competing objective and is therefore almost ignored. The result is a Pareto efficient solution where no target criterion can be improved without worsening another.

Due to the dependence of the solution on weights in the scalarization of multi-criteria optimization problems, Fig. 6 shows the Pareto curve of the target achievement with different weightings (in 0.1 weight increments) of the three target criteria costs λ_c, space requirement λ_a and tolerance deviation λ_d. The weight of the cycle time reserve λ_f is set to zero. Since the target values are discrete, some solutions overlap, which is why fewer than 66 points (all possible combinations with 0.1 weight increments of the three target criteria) are shown. The saturation or intensity of a point indicates the number of partitions that share the same objective function values.

4.5. Sensitivity analysis

The results are based on real parameters regarding the basic equipment costs of a station, as well as the costs of TCs, robots and workers. To validate the influence of these parameters, we defined various scenarios to conduct a sensitivity analysis and assess the derived results.

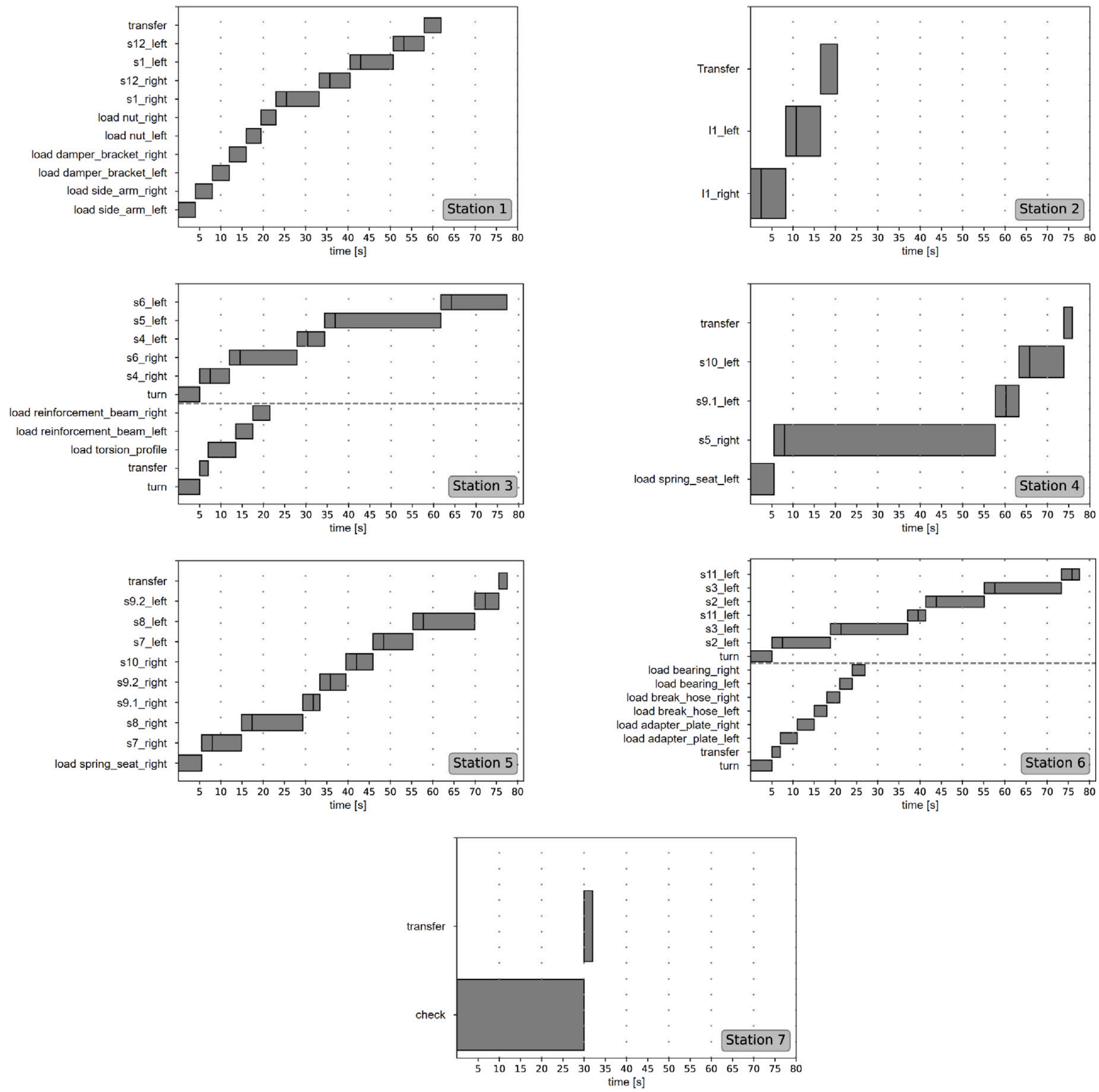

Fig. 5. Task allocation of the resulting 7 stations with multi-criteria optimization.

Since the basic equipment costs of a station only shift the objective values without altering the resulting line configuration, no further analysis of this parameter is required.

A first sensitivity analysis reflects upon labor costs. They affect the deployment of workers for handling processes. Consequently, the algorithm chooses to use robots instead of workers at a certain trade-off point. In this case, only the costs are affected, which is why the cost function was minimized in the subsequent analysis. Table 6 presents the results of different solutions for handling and transfer under varying labor costs, ceteris paribus.

Depending on the labor costs, the algorithm proposes three different solutions, which vary in the use of robots for handling and transfer. For labor costs below 50 euros per hour (see first case in Table 6), the solution deploys as many workers as possible. However, after the third station, the semi-finished product exceeds a weight over 10 kg, which is too heavy for manual handling, and requires transfer by a robot. Table 4 displays this solution. Although robots are more expensive than workers at a labor cost of 60 euros per hour (see second case in Table 6), the algorithm prefers robots for transfer across all stations. This removes the weight constraint imposed on workers, allowing the first station to be used more efficiently with more processes and heavier parts. This process allocation eliminates the need for a turntable at the third station, reducing costs, as a turntable is more expensive than using three robots instead of three workers across station 3 to 5. The process allocation is more balanced and the cycle time reserve increases to 0.12 s compared to 0 s of the previous solution. When the labor costs reach or exceed 70 euros per hour (see third case in Table 6), a break-even point is achieved, making workers more expensive than robots. As a result, the algorithm selects robots for all transfer and handling

Table 6
Results of handling (H) and transfer (T) for different labor costs with human (h), robot (r) or no handling (-).

Labor costs	Type	Stations 1	2	3	4	5	6	7
≤ 50 €/h	H	h	–	h	h	h	h	–
	T	h	h	h	r	r	r	r
60 €/h	H	h	–	h	h	h	h	–
	T	r	r	r	r	r	r	r
≥ 70 €/h	H	r	r	r	–	r	r	–
	T	r	r	r	r	r	r	r

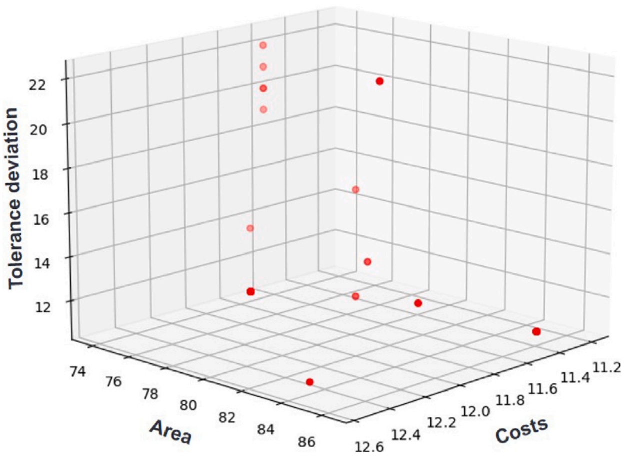

Fig. 6. Representation of Pareto efficient solutions depending on the target criteria of cost, area and tolerance deviation.

operations. Due to the shorter handling and transfer times for robots, more time for processing tasks is available, and the tasks are distributed more evenly, resulting in a higher cycle time reserve of 0.62 s.

A second sensitivity analysis regards resource costs. Here, robots are the alternative to human for handling and transfer tasks, and their usage is influenced by both robot and worker costs. Higher robot costs lead to increased use of human workers, and vice versa. Therefore, similar solutions are produced here in reverse order. The robot costs considered here include the procurement, operation and maintenance costs over three years. At low robot costs (approx. €162,200), no workers are employed, and all transfer and handling tasks are performed by robots. At higher machine costs (approx. €202,200), robots are largely avoided. However, after the third station, transfer by workers is no longer possible due to the weight of the semi-finished product.

The last sensitivity analysis covers the additional cost of turntables, that parallelize handling and processing tasks and enhance station efficiency. The TC at station 3 in Table 4 will be replaced by an OC with two robots instead of one robot if the additional costs exceeds €600,000. An additional robot proves to be more economical than either a turntable or an additional station.

4.6. Validation

4.6.1. Comparison with metaheuristic algorithm

To benchmark the solution against the state of the art, the results are compared to a metaheuristic algorithm. For this purpose, in prior research [12] we programmed a genetic algorithm based on the NSGA-II algorithm according to Verma et al. [51]. In Schäfer et al. [12] the solution approach and all methods used are described. In summary, the Python library *Distributed Evolutionary Algorithms in Python* (DEAP) according to Fortin et al. [52] has been expanded to include additional functions such as an initial algorithm to create valid solutions, mutations to explore the solution space and repair mechanisms to repair invalid individuals. This algorithm is now applied to the present real-world use case. Results can be taken from Figs. 7–9. Fig. 7 shows that an increase in the number of generations and the population size generally leads to improved solutions (purple) and that the developed algorithm works. Here, Fig. 8 shows the best fitness per generation for a single-objective optimization. Regarding the three selected objective functions cost, area and tolerance deviation, convergence can be observed after about 60 generations. Unsurprisingly, the optimal values (see Table 8, line 2) of the heuristic optimization are worse than those of the exact optimization with *Gurobi* (Grb.) (see Table 8, line 1). Advantages regarding computing times are neglible since the exact optimization only takes a few minutes using a standard industry PC (see Table 3). Nevertheless, to complete the comparison, Fig. 9 shows the Pareto curve of the multi-criteria optimization with DEAP.

4.6.2. Comparison with manual planning

This validation compares the results of the multi-criteria optimization with the manual planning created by planners. Table 7 represents the manual planning. The production line consists of 5 stations for welding, 1 for laser cutting, and 1 for visual inspection. Five workers are required for loading the stations. Only the first two stations have handling robots for transportation. Four of the five welding stations are configured with TC, and all five stations are equipped with two welding robots each. The manual planning processes the RTB symmetrically. In comparison with the multi-criteria optimization, the algorithmic result of the line is quite similar to the symmetrical processing of the manual planning, without this being defined as an explicit requirement. Overall, two-thirds of the tasks are performed in parallel on the right and left sides.

According to the objective criteria of costs, required area, quality, and cycle time reserve, both solutions are comparable, despite the manual planning explicitly considering only cost minimization. The planners implicitly consider the factors of quality and ergonomics through the late processing of tolerance-critical parts and the maximum weight limit for humans. There is no consideration of flexibility. The comparison of the manual planning (line 3) with the algorithmic one (line 1) in Table 8 shows that *Gurobi* (Grb.) provides a better solution. This includes cost savings amounting to 10.6% or 1.3644 Mio € over the course of three years as well as a reduction of the necessary area by 43.4% or 61.622 m^2. In addition, tolerance deviations were reduced and the algorithmic solution includes a cycle time reserve of 2.37 s that represents a maximum possible increase in annual production volumes of 3.06%.

An even better solution can be obtained when adapting the mathematical model regarding the task assignment. In the presented ALB model, constraints (5)–(9) are used to ensure that the precedence graph is adhered to, in that two tasks between which a precedence relation exists cannot be assigned to the same station. Relaxing these constraints can be achieved by allowing the assignment of a successor tasks to the station of the predecessor task, provided that the successor task is fully processed at the station under consideration. This is stated in the newly introduced constraint (9–II). Constraints (5) and (6) are dropped. In constraints (7) and (8), only the summation over h needs to be adjusted.

$$z_{ij} \leq \sum_{h=1}^{j} \sum_{k \in K} x_{ihk} \quad \forall i \in I, j \in J \tag{7–II}$$

$$1 - z_{ij} \leq M \cdot \left(1 - \sum_{h=1}^{j} \sum_{k \in K} x_{ihk}\right) \quad \forall i \in I, j \in J \tag{8–II}$$

$$(1 - P_{mn}) + z_{mj} \geq \sum_{h=1}^{j} \sum_{k \in K} x_{nhk} \quad \forall m, n \in I, j \in J \tag{9–II}$$

These adjustments (ceteris paribus) result in a line configuration with only six stations, further minimizing costs (10.8944 Mio. €), area (76.23 m^2) and tolerance deviations (quality index 10). The cycle time reserve increases to 6.65 s which accounts for a maximum possible increase in annual production volumes of 9.5% (see Table 8 line 4). A discussion of these results with expert planners indicates that this is partly due to simplifying assumptions. However, another main advantage of multi-criteria optimization lies in the comparability of the solutions, as the effects of changes in the line configuration on multiple objective criteria (e.g. cost savings with a simultaneous decrease in quality) can be quantified.

5. Discussion & outlook

5.1. Discussion

Even though there are several approaches for multi-criteria optimization of production lines considering the precedence graph (see Section 2.2), most companies accomplish this task manually, due to

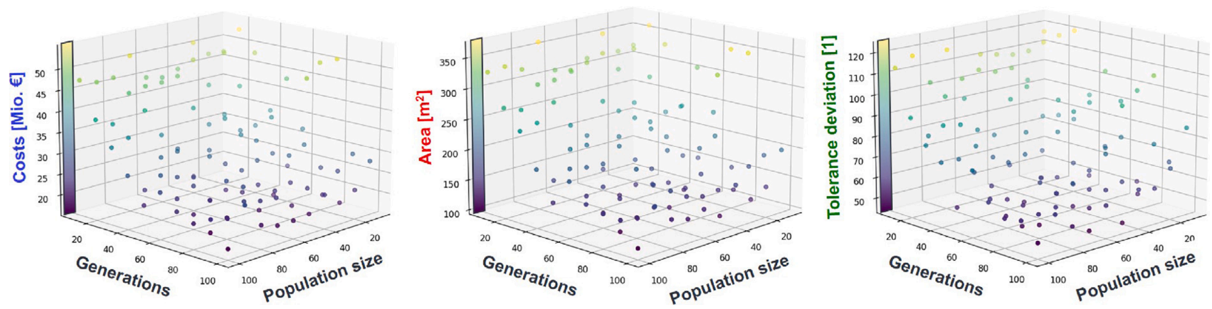

Fig. 7. Representation of Pareto efficient solutions depending on the target criteria of cost, area and tolerance deviation.

Table 7
Details of the manually planned line configuration.

	Station 1	Station 2	Station 3	Station 4	Station 5	Station 6	Station 7
Station type	Welding	Laser-Cutting	Welding	Welding	Welding	Welding	EOL testing
OC/TC	TC	TC	TC	TC	TC	TC	OC
n robots	2	1	2	2	2	2	0
Loading	Human	–	Human	Human	Human	Human	–
Transfer	Robots	Human	Human	Robots	Robots	Robots	Robots
Processing time station[a] [s]	15,40	11,48	62,80	160,06	96,63	85,76	30,00
Processing time robots[b] [s]	7,70	11,48	31,40	80,03	48,32	42,88	30
Cycle time station[c] [s]	12,70	16,48	35,40	85,03	53,32	47,88	30
Overlapping	–	–	–	–	2	2	–

[a] Station processing time = total processing time for all tasks at a station
[b] Processing time per robot = processing time of the station/n robots
[c] Cycle time of the station = processing time per robot + time to turn the turntable (for TC) Line cycle time = maximum station cycle time

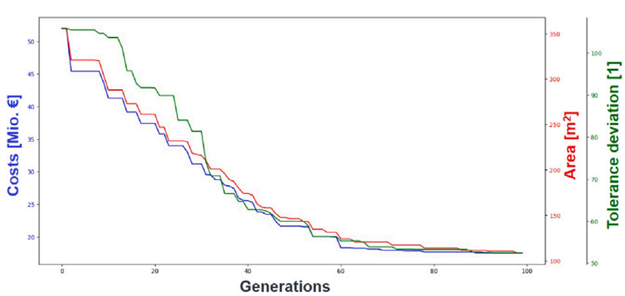

Fig. 8. Fitness over generations of metaheuristic approach.

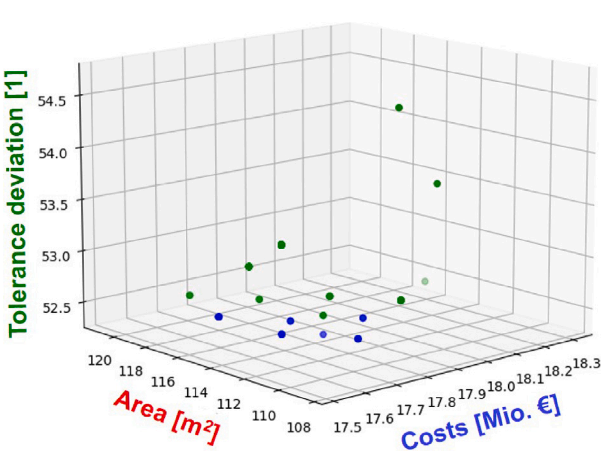

Fig. 9. Representation of Pareto efficient solutions (blue) and other individuals (green) of the last generation. Due to the discrete target values, some of the solutions overlap. A high saturation i.e. a darker point is an indicator for several points overlapping. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 8
Quantitative comparison of planning results.

	Stations [#]	Costs [Mio. €]	Area [m^2]	Quality index [1]	Cycle time reserve [s]
Grb.	7	11.4566	80.52	11	2.37
NSGA-II	8	17.5200	110.87	48	0.00
Manual	7	12.8210	142.142	17	0.00
Grb.–II	**6**	**10.8944**	**76.23**	**10**	**6.65**

the simplification of the real problem and the assumptions of modeling (see Boysen & Fliedner et al. [14]). In contrast, this approach analyzes and optimizes an real-world industrial problem [C1] by employing a mathematical model that is as realistic as possible.

This approach applies the data-driven development [48] by first defining a runtime minimal model and then iteratively extending it [47]. A preliminary study with a heuristic solution algorithm demonstrates the general solvability of the problem [12]. By using the default solver Gurobi, this method can identify the optimal solution of the multi-criteria optimization model with the quantified objective criteria such as cost, quality, time and flexibility. The scalarization method, which combines multiple objective functions with weights into a single substitute objective function, guarantees a weak Pareto efficient solution in mixed-integer optimization [53]. Therefore, one objective function can only be improved at the expense of another objective function. If the Pareto front is non-convex, Pareto efficient points cannot be found in the non-convex part [54].

A comparison with the characterization by Boysen & Fliedner et al. [14] and common assumptions in literature (see [15]) shows the complexity of this ALBP. Considering sequence restrictions by precedence graph, tasks are allocated to specific stations based on their type and the required processing equipment. Since each station can accommodate up to two robots, it is a multi-manned AL design [C3]. Equipment-dependent, unproductive times are explicitly taken into account. As the first research work, the present work considers the divisibility of tasks when assigning them to multiple stations [C4].

The quality of the solution depends on changes in demand, which need to be discussed. While the output can be reduced, an increase in output is limited by the cycle time reserve. The calculated cycle time reserve of 6.65 s represents a maximum possible increase in annual production of 9.5%. In general, multi-criteria optimization allows explicit consideration of multiple objective criteria compared to manual planning.

5.2. Outlook

Despite the very high degree of fulfillment of the requirements by the developed approach to assisted production system planning, the benefits can be increased further through extensions and future research needs, especially with regard to the simplified assumptions in the current work.

An earlier optimization of the production system requires an increased need for information. Further research can thus, for example, integrate accessibility simulations or robot path planning into the early phase of production system planning through an extended optimization model. Likewise, the extension can be made along the variation of the vertical integration, where the current focus is on the variant-specific line balancing. The integration of the associated parts manufacturing and any make-or-buy decisions leads to a global optimization of the production system.

Availability and parameter correlations are currently only estimated statically and on the basis of empirical values. By combining optimization and material flow simulation, an extended approach can consider the dynamic system behavior and various demand scenarios in line balancing and enable an integrated dimensioning of buffer capacities and positions. Based on the exact geometric position of a component or feature, the level of detail in the planning can be further increased by taking into account unproductive times more precisely.

In view of the future obligation for suppliers to carry out *Life Cycle Assessments* (LCA), it also makes sense to expand the multi-criteria optimization problem to include sustainability and opportunistic maintenance aspects [55].

6. Conclusion

6.1. Managerial insights

Using mathematical optimization for ALB provides significant advantages in terms of efficiency, cost savings, and resource utilization. State of the art optimization algorithms such as Gurobi can quickly (computing times < 10 min on standard industry PCs) identify the most effective allocation of tasks across stations, optimizing multiple objectives and ensuring a balanced workload. This reduces production bottlenecks, increases throughput (here up to 9.5%), and optimizes resource usage. However, it could be investigated whether a subsequent improvement of the solutions to Pareto optimality is possible or if their Pareto efficiency can be proven. For a balanced consideration of several objectives, Goal Programming can be considered as an alternative to scalarization.

Generally, it is crucial to model the real-world problem as realistically as possible. Inaccurate representations of real-world constraints, such as resource costs, positioning or handling times, can lead to suboptimal solutions that are infeasible or inefficient in practice. Incorporating real-world complexities into the model ensures that the proposed solutions are practical, scalable, and aligned with the actual production environment. On one hand, this leads to more robust decision-making and sustainable operational improvements. On the other hand, it also means more complexity that needs to be mastered. In this article, above all the non-discrete assignment of tasks to stations adds immensely to the complexity of the mathematical problem, resulting in an computational complexity of over $3.6 * 10^{299}$.

However, algorithmic optimization is especially necessary because humans cannot handle such complexity. For Tier 1 suppliers, who must frequently balance assembly lines for every new product request from their customers, the efficiency and speed of mathematical optimization become even more critical. Here, the combination of mathematical optimization and realistic modeling is particularly recommended and delivers significant competitive advantages: It allows them to rapidly adapt to new customer requests, reduce the time and resources needed for ALB, and ensure that the production line is optimized for efficiency, cost-effectiveness and other. By balancing assembly lines faster and more accurately, suppliers can not only meet but exceed customer expectations, offering faster lead times, more flexibility, and higher quality, all while maintaining profitability in a demanding market environment.

For a successful industrial introduction, specific recommendations for companies are briefly discussed: First of all, it is essential for a company to provide the necessary input data in digital form, sufficient data quality and at the right time. Tihlarik [56] shows that the acceptance of users represents a major hurdle to industrialization. Although this article aims to provide computer-assistance for humans and not to replace human planning skills, there is a general mistrust of algorithmically designed solutions. For this reason, it is advisable for companies to present the calculation logic of their approach in a transparent manner, thereby enhancing the clarity and understanding of the solution. Additionally, allowing the planner to influence the calculation logic is also encouraged. Companies should also offer an intuitive user interface to help users navigate the often complex algorithmic planning. From a technical standpoint, it is important to emphasize the integration of the approach into the current system landscape. Rather than introducing yet another tool into an already diverse software environment, the planning assistant should seamlessly integrate with the existing processes, methods, and tools.

6.2. Summary

Automotive manufacturers and their suppliers are confronted with more frequent and shorter development and planning cycles. These can be attributed to three general trends: a growing sales volume of increasingly complex complete systems, the constantly increasing variety of variants and increasing cost pressure from the Chinese automotive industry. This leads to the need to maximize the efficiency of repetitive planning of production systems for the manufacturing of customer-specific product variants. The current state of research in the field of ALBP does not adequately model the real-world problem. This study fills this research gap with a combination of multiple objective functions, multi-manned stations, the option for parallel stations, different station types and task divisibility.

In consideration of a process precedence graph, a complex ALB model is developed that adequately represents the given real-world problem and enables an optimal allocation of the process steps to the stations. With the help of the default solver Gurobi, the defined model can solve an optimal solution in an acceptable computing time, taking into account several objective criteria.

The results of the application and validation based on the example of the rough planning of production systems for the production of customized RTB at an automotive supplier show the general industrial applicability of the developed methodology. The innovative approach generates a line configuration that directly reduces manufacturing costs, space usage, and tolerance deviations while increasing the cycle time reserve compared to the manual alternative. As a result, annual output can rise by as much as 9.5%. Moreover, this planning method can indirectly reduce personnel costs by decreasing the time needed for the planning task. The developed approach for assisted, model-based rough planning of production systems can increase the efficiency of planning and contribute to the reduction of costs and time in a future-oriented planning process.

$$
\begin{array}{llll}
\text{s.t.} & t^{\text{out,H}} + \sum_{p\in P} l_{pj} \cdot t_p^{\text{in,H}} + t^{\text{turn}} + t^{\text{res}} & \leq c_t + (1+\beta_j^R+\gamma_j^R-\delta_j)\cdot M & \forall j \in J \\
& t^{\text{out,H}} + \sum_{p\in P} l_{pj} \cdot t_p^{\text{in,R}} + t^{\text{turn}} + t^{\text{res}} & \leq c_t + (2-\beta_j^R+\gamma_j^R-\delta_j)\cdot M & \forall j \in J \\
& t^{\text{out,R}} + \sum_{p\in P} l_{pj} \cdot t_p^{\text{in,H}} + t^{\text{turn}} + t^{\text{res}} & \leq c_t + (2+\beta_j^R-\gamma_j^R-\delta_j)\cdot M & \forall j \in J \\
& \sum_{i\in I}\Big(\sum_{q\in\{1,2\}} (t^{\text{pos}} \cdot w_{ijq}) + t_i \cdot x_{ijk} + t^{\text{o}} \cdot o_{ij}\Big) + t^{\text{turn}} + t^{\text{res}} & \leq c_t + (1+\alpha_j-\delta_j)\cdot M & \forall j \in J, \forall k \in K \\
& t^{\text{out,H}} + \sum_{p\in P} l_{pj} \cdot t_p^{\text{in,R}} + \sum_{i\in I}\Big(\sum_{q\in\{1,2\}} (t^{\text{pos}} \cdot w_{ijq}) + t_i \cdot x_{ijk} + t^{\text{o}} \cdot o_{ij}\Big) + t^{\text{res}} & \leq c_t + (1+\alpha_j-\beta_j^R+\gamma_j^R+\delta_j)\cdot M & \forall j \in J, \forall k \in K \\
& t^{\text{out,R}} + \sum_{p\in P} l_{pj} \cdot t_p^{\text{in,R}} + \sum_{i\in I}\Big(\sum_{q\in\{1,2\}} (t^{\text{pos}} \cdot w_{ijq}) + \frac{1}{2}\cdot t_i \cdot x_{ijk} + t^{\text{o}} \cdot o_{ij}\Big) + t^{\text{res}} & \leq c_t + (3-\alpha_j-\beta_j^R-\gamma_j^R+\delta_j)\cdot M & \forall j \in J, \forall k \in K \\
& t^{\text{out,H}} + \sum_{p\in P} l_{pj} \cdot t_p^{\text{in,R}} + \sum_{i\in I}\Big(\sum_{q\in\{1,2\}} (t^{\text{pos}} \cdot w_{ijq}) + \frac{1}{2}\cdot t_i \cdot x_{ijk} + t^{\text{o}} \cdot o_{ij}\Big) + t^{\text{res}} & \leq c_t + (2-\alpha_j-\beta_j^R+\gamma_j^R+\delta_j)\cdot M & \forall j \in J, \forall k \in K \\
& t^{\text{out,R}} + \sum_{p\in P} l_{pj} \cdot t_p^{\text{in,H}} + \sum_{i\in I}\Big(\sum_{q\in\{1,2\}} (t^{\text{pos}} \cdot w_{ijq}) + t_i \cdot x_{ijk} + t^{\text{o}} \cdot o_{ij}\Big) + t^{\text{res}} & \leq c_t + (1+\alpha_j+\beta_j^R-\gamma_j^R+\delta_j)\cdot M & \forall j \in J, \forall k \in K \\
& t^{\text{out,H}} + \sum_{p\in P} l_{pj} \cdot t_p^{\text{in,H}} + \sum_{i\in I}\Big(\sum_{q\in\{1,2\}} (t^{\text{pos}} \cdot w_{ijq}) + t_i \cdot x_{ijk} + t^{\text{o}} \cdot o_{ij}\Big) + t^{\text{res}} & \leq c_t + (\alpha_j+\beta_j^R+\gamma_j^R+\delta_j)\cdot M & \forall j \in J, \forall k \in K \\
& t^{\text{out,R}} + \sum_{p\in P} l_{pj} \cdot t_p^{\text{in,H}} + \sum_{i\in I}\Big(\sum_{q\in\{1,2\}} (t^{\text{pos}} \cdot w_{ijq}) + \frac{1}{2}\cdot t_i \cdot x_{ijk} + t^{\text{o}} \cdot o_{ij}\Big) + t^{\text{res}} & \leq c_t + (2-\alpha_j+\beta_j^R-\gamma_j^R+\delta_j)\cdot M & \forall j \in J, \forall k \in K \\
& t^{\text{out,H}} + \sum_{p\in P} l_{pj} \cdot t_p^{\text{in,H}} + \sum_{i\in I}\Big(\sum_{q\in\{1,2\}} (t^{\text{pos}} \cdot w_{ijq}) + \frac{1}{2}\cdot t_i \cdot x_{ijk} + t^{\text{o}} \cdot o_{ij}\Big) + t^{\text{res}} & \leq c_t + (1-\alpha_j+\beta_j^R+\gamma_j^R+\delta_j)\cdot M & \forall j \in J, \forall k \in K
\end{array}
$$

Box I.

CRediT authorship contribution statement

Louis Schäfer: Writing – original draft, Visualization, Validation, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Stefan Tse:** Writing – original draft. **Marvin Carl May:** Writing – review & editing, Writing – original draft, Supervision. **Gisela Lanza:** Writing – review & editing, Supervision, Project administration, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The authors gratefully acknowledge financial funding from the German Federal Ministry of Economic Affairs and Climate Action and the European Union (grant no. 13IK001ZF) as well as organizational support by the VDI Technologiezentrum GmbH.

Appendix

Eqs. (24)–(27) describe the four single-objective functions, and Eq. (28) summarizes these into one multi-criteria substitution function. Box I consists of all equipment-dependant constraints. These inequalities present all the possibilities for calculating the cycle time of a station based on the productive and unproductive time in relation to the allocated equipment.

$$
\begin{aligned}
\text{min cost} \quad G_c = & \sum_{j\in J}\sum_{k\in K}\big(c_{\text{base}} \cdot v_{jk}\big) \\
& + \sum_{j\in J}\big(c_{\text{addRobot}} \cdot \alpha_j + c_{\text{handlR}} \cdot (\beta_j^R+\gamma_j^R) + c_{\text{handlH}} \cdot (\beta_j^H+\gamma_j^H) + c_{\text{addTC}} \cdot \delta_j\big)
\end{aligned} \tag{24}
$$

$$
\begin{aligned}
\text{min area} \quad G_a = & \sum_{j\in J}\sum_{k\in K}\big(a_{\text{base}} \cdot v_{jk}\big) \\
& + \sum_{j\in J}\big(a_{\text{addRobot}} \cdot \alpha_j + a_{\text{handlR}} \cdot (\beta_j^R+\gamma_j^R) + a_{\text{handlH}} \cdot (\beta_j^H+\gamma_j^H) + a_{\text{addTC}} \cdot \delta_j\big)
\end{aligned} \tag{25}
$$

$$\text{max flexibility} \quad G_f = t^{\text{res}} \tag{26}$$

$$\text{min deviations} \quad G_d = n + \sum_{i\in I^{\text{crit}}}\Big(n - \sum_{j\in J} j \cdot f_{ij}\Big) \tag{27}$$

$$\text{max multi-criteria} \quad G_m = -\lambda_c \cdot \frac{G_c}{G_c^*} - \lambda_a \cdot \frac{G_a}{G_a^*} + \lambda_f \cdot \frac{G_f}{G_f^*} - \lambda_d \cdot \frac{G_d}{G_d^*} \tag{28}$$

The following inequalities present all the possibilities for calculating the cycle time of a station based on the productive and unproductive time in relation to the allocated equipment (see Box I).

References

[1] Albers A, Lanza G, Klippert M, Schäfer L, Frey A, Hellweg F, et al. Product-production-codesign: An approach on integrated product and production engineering across generations and life cycles. Procedia CIRP 2022;109(3):167–72. http://dx.doi.org/10.1016/j.procir.2022.05.231.

[2] Albers A, editor. Engineering neu denken und gestalten. Herausforderungen, Anwendungsszenarien und das neue Leitbild Advanced Systems Engineering. Acatech IMPULS, München; 2023, http://dx.doi.org/10.48669/aca_2023-7.

[3] May MC, Glatter D, Arnold D, Pfeffer D, Lanza G. IIoT system canvas — From architecture patterns towards an IIoT development framework. J Manuf Syst 2024;72:437–59. http://dx.doi.org/10.1016/j.jmsy.2023.12.001.

[4] Albers A, Rapp S, Klippert M, Lanza G, Schäfer L. Produkt-Produktions-CoDesign: Ein Ansatz zur integrierten Produkt- und Produktionssystementwicklung über Generationen und Lebenszyklen hinweg. WiGeP News 2022;3(1).

[5] Schäfer L, Frank A, May MC, Lanza G. Automated derivation of optimal production sequences from product data. Procedia CIRP 2022;107(04):469–74. http://dx.doi.org/10.1016/j.procir.2022.05.010.

[6] Kampker A, Kawollek S, Fluchs S, Marquardt F. Einfluss der Variantenvielfalt auf die automobile Endmontage. ZWF 2019;114(7–8):474–9. http://dx.doi.org/10.3139/104.112097.

[7] Deloitte. Komplexitätsmanagement in der automobilindustrie. In: Proff H, Ullerich S, Pottebaum T, Bitterich C, editors. Deloitte operations complexity management. Deloitte & Touche GmbH; 2016.

[8] May MC, Schäfer L, Frey A, Krahe C, Lanza G. Towards product-production-CoDesign for the production of the future. Procedia CIRP 2023;119:944–9. http://dx.doi.org/10.1016/j.procir.2023.02.172.

[9] Hagemann S. Algorithmische Konzeption von hochautomatisierten Fließmontagesystemen am Beispiel des automobilen Karosseriebaus. [Disseration], Berlin: Technische Universität Berlin; 2022, http://dx.doi.org/10.14279/depositonce-16131.

[10] Schäfer L, Treml N, May MC, Lanza G. Classifying parts using feature extraction and similarity assessment. Procedia CIRP 2023;119:822–7. http://dx.doi.org/10.1016/j.procir.2023.03.127.

[11] Schäfer L, Günther M, Martin A, Lüpfert M, Mandel C, Rapp S, et al. Systematics for an integrative modelling of product and production system. Procedia CIRP 2023;118(04):104–9. http://dx.doi.org/10.1016/j.procir.2023.06.019.

[12] Schäfer L, Tse S, May MC, Lanza G. Multi-objective mathematical optimization in assisted production planning. In: EurOMA - transforming people and processes for a better world. 2024.

[13] Fisel J, Exner Y, Stricker N, Lanza G. Changeability and flexibility of assembly line balancing as a multi-objective optimization problem. J Manuf Syst 2019;53:150–8. http://dx.doi.org/10.1016/j.jmsy.2019.09.012.

[14] Boysen N, Fliedner M, Scholl A. A classification of assembly line balancing problems. European J Oper Res 2007;183(2):674–93. http://dx.doi.org/10.1016/j.ejor.2006.10.010.

[15] Chutima P. A comprehensive review of robotic assembly line balancing problem. J Intell Manuf 2022;33(1):1–34. http://dx.doi.org/10.1007/s10845-020-01641-7.

[16] Wiendahl HP, Reichardt J, Nyhuis P. Handbuch Fabrikplanung: Konzept, Gestaltung und Umsetzung wandlungsfähiger Produktionsstätten. überarb. und erw. Aufl., 2 edn. München: Hanser; 2014, http://dx.doi.org/10.3139/9783446437029.

[17] Steimer C, Cadet M, Aurich JC, Stephan N. Approach for an integrated planning of manufacturing systems based on early phases of product development. Procedia CIRP 2016;57:467–72. http://dx.doi.org/10.1016/j.procir.2016.11.081.

[18] DIN 8580. DIN 8580 manufacturing processes. Beuth; 2003.

[19] Nickel S, Rebennack S, Stein O, Waldmann KH. Operations research. Berlin, Heidelberg: Springer Berlin Heidelberg; 2022, http://dx.doi.org/10.1007/978-3-662-65346-3.

[20] Nourmohammadi A, Fathi M, Ng AH. Balancing and scheduling assembly lines with human-robot collaboration tasks. Comput Oper Res 2022;140:105674. http://dx.doi.org/10.1016/j.cor.2021.105674.

[21] Pearce BW, Antani K, Mears L, Funk K, Mayorga ME, Kurz ME. An effective integer program for a general assembly line balancing problem with parallel workers and additional assignment restrictions. J Manuf Syst 2019;50:180–92. http://dx.doi.org/10.1016/j.jmsy.2018.12.011.

[22] Huang Y. Computing quantum discord is NP-complete. New J Phys 2014;16(3):033027. http://dx.doi.org/10.1088/1367-2630/16/3/033027.

[23] Gunantara N. A review of multi-objective optimization: Methods and its applications. In: Ai Q, editor. Cogent Eng 2018;5(1):1502242. http://dx.doi.org/10.1080/23311916.2018.1502242.

[24] Salveson ME. The assembly-line balancing problem. J Fluids Eng 1955;77(6):939–47. http://dx.doi.org/10.1115/1.4014559.

[25] Rubinovitz J, Bukchin J, Lenz E. RALB – A heuristic algorithm for design and balancing of robotic assembly lines. CIRP Ann 1993;42(1):497–500. http://dx.doi.org/10.1016/S0007-8506(07)62494-9.

[26] Albus M, Huber MF. Resource reconfiguration and optimization in brownfield constrained Robotic Assembly Line Balancing Problems. J Manuf Syst 2023;67:132–42. http://dx.doi.org/10.1016/j.jmsy.2023.01.001.

[27] Battaïa O, Dolgui A. A taxonomy of line balancing problems and their solutionapproaches. Int J Prod Econ 2013;142(2):259–77. http://dx.doi.org/10.1016/j.ijpe.2012.10.020.

[28] Boysen N, Schulze P, Scholl A. Assembly line balancing: What happened in the last fifteen years? European J Oper Res 2022;301(3):797–814. http://dx.doi.org/10.1016/j.ejor.2021.11.043.

[29] Ghosh S, Gagnon RJ. A comprehensive literature review and analysis of the design, balancing and scheduling of assembly systems. Int J Prod Res 1989;27(4):637–70. http://dx.doi.org/10.1080/00207548908942574.

[30] Albus M, Seeber C. Linear optimization for dynamic selection of resources in constrained assembly line balancing problems. Procedia CIRP 2021;104(1):134–9. http://dx.doi.org/10.1016/j.procir.2021.11.023.

[31] Chen YY, Cheng CY, Li JY. Resource-constrained assembly line balancing problems with multi-manned workstations. J Manuf Syst 2018;48:107–19. http://dx.doi.org/10.1016/j.jmsy.2018.07.001.

[32] Li Y, Qiao Z, Li M, Zou Y. Mixed-integer programming for robotic assembly line balancing considering cross-station task and carbon footprint. IFAC-PapersOnLine 2022;55(10):448–51. http://dx.doi.org/10.1016/j.ifacol.2022.09.434.

[33] Michels AS, Lopes TC, Sikora CGS, Magatão L. The Robotic Assembly Line Design (RALD) problem: Model and case studies with practical extensions. Comput Ind Eng 2018;120:320–33. http://dx.doi.org/10.1016/j.cie.2018.04.010.

[34] Navas-Barrios E, Riquett-Rodríguez A, Macías-Jiménez MA, Romero-Conrado AR. An assembling line balancing problem: Lead-acid batteries case study. Procedia Comput Sci 2022;203:525–30. http://dx.doi.org/10.1016/j.procs.2022.07.074.

[35] Chi Y, Qiao Z, Li Y, Li M, Zou Y. Type-1 robotic assembly line balancing problem that considers energy consumption and cross-station design. Systems 2022;10(6):218. http://dx.doi.org/10.3390/systems10060218.

[36] Fattahi P, Roshani A, Roshani A. A mathematical model and ant colony algorithm for multi-manned assembly line balancing problem. Int J Adv Manuf Technol 2011;53(1–4):363–78. http://dx.doi.org/10.1007/s00170-010-2832-y.

[37] Lopes TC, Michels AS, Sikora CGS, Brauner N, Magatão L. Assembly line balancing for two cycle times: Anticipating demand fluctuations. Comput Ind Eng 2021;162:107685. http://dx.doi.org/10.1016/j.cie.2021.107685.

[38] Yilmaz H, Yilmaz M. A mathematical model and tabu search algorithm for multi-manned assembly line balancing problems with assignment restrictions. Eng Optim 2020;52(5):856–74. http://dx.doi.org/10.1080/0305215X.2019.1618288.

[39] Yuan P, Xu Q, Song H. Two-sided with multi-parallel stations assembly line balancing problem and algorithm. In: 2015 8th international symposium on computational intelligence and design. IEEE; 2015, p. 602–5. http://dx.doi.org/10.1109/ISCID.2015.299.

[40] Abdous MA, Delorme X, Battini D, Berger-Douce S. Multi-objective collaborative assembly line design problem with the optimisation of ergonomics and economics. Int J Prod Res 2022;1–16. http://dx.doi.org/10.1080/00207543.2022.2153185.

[41] Lopes TC, Pastre GV, Michels AS, Magatão L. Flexible multi-manned assembly line balancing problem: Model, heuristic procedure, and lower bounds for line length minimization. Omega 2020;95:102063. http://dx.doi.org/10.1016/j.omega.2019.04.006.

[42] Huang X, Guan Z, Yang L. An effective hybrid algorithm for multi-objective flexible job-shop scheduling problem. Adv Mech Eng 2018;10(9). http://dx.doi.org/10.1177/1687814018801442.

[43] Roth S, Kalchschmid V, Reinhart G. Development and evaluation of risk treatment paths within energy-oriented production planning and control. Prod Eng 2021;15(3–4):413–30. http://dx.doi.org/10.1007/s11740-021-01043-5.

[44] Şahin M, Kellegöz T. A new mixed-integer linear programming formulation and particle swarm optimization based hybrid heuristic for the problem of resource investment and balancing of the assembly line with multi-manned workstations. Comput Ind Eng 2019;133:107–20. http://dx.doi.org/10.1016/j.cie.2019.04.056.

[45] Nugraha RC, Ma'ruf A, Nugraha AC, Halim AH. A mixed-integer linear programming formulation for assembly line balancing problem with human-robot shared tasks. J Phys Conf Ser 2021. http://dx.doi.org/10.1088/1742-6596/1858/1/012021.

[46] El-Sherbeny NA. Vehicle routing with time windows: An overview of exact, heuristic and metaheuristic methods. J King Saud Univ Sci 2010;22(3):123–31. http://dx.doi.org/10.1016/j.jksus.2010.03.002.

[47] Schäfer L, Kochendörfer P, May MC, Lanza G. Planning and multi-objective optimization of production systems by means of assembly line balancing. Procedia CIRP 2023;120(3):1125–30. http://dx.doi.org/10.1016/j.procir.2023.09.136.

[48] Bach J, Langner J, Otten S, Holzapfel M, Sax E. Data-driven development, a complementing approach for automotive systems engineering. In: Proceedings of the 2017 IEEE international systems engineering symposium. 2017, p. 1–6. http://dx.doi.org/10.1109/SysEng.2017.8088295.

[49] Amen M. Cost-oriented assembly line balancing: Model formulations, solution difficulty, upper and lower bounds. European J Oper Res 2006;168(3):747–70. http://dx.doi.org/10.1016/j.ejor.2004.07.026.

[50] Mukund Nilakantan J, Ponnambalam SG. Robotic U-shaped assembly line balancing using particle swarm optimization. Eng Optim 2016;48(2):231–52. http://dx.doi.org/10.1080/0305215X.2014.998664.

[51] Verma S, Pant M, Snasel V. A comprehensive review on NSGA-II for multi-objective combinatorial optimization problems. IEEE Access 2021;(9):57757–91. http://dx.doi.org/10.1109/ACCESS.2021.3070634.

[52] Fortin F-a, de Rainville FM, Gardner MA, Parizeau M, Gagne C. DEAP: evolutionary algorithms made easy: The Journal of Machine Learning Research. J Mach Learn Res 2012;(13):2171–5.

[53] Hillier FS, Miettinen K. Nonlinear multiobjective optimization. Boston, MA: Springer US; 1998, http://dx.doi.org/10.1007/978-1-4615-5563-6.

[54] Burachik RS, Kaya CY, Rizvi MM. Algorithms for generating Pareto fronts of multi-objective integer and mixed-integer programming problems. Eng Optim 2022;54(8):1413–25. http://dx.doi.org/10.1080/0305215X.2021.1939695.

[55] Valet A, Altenmüller T, Waschneck B, May MC, Kuhnle A, Lanza G. Opportunistic maintenance scheduling with deep reinforcement learning. J Manuf Syst 2022;64:518–34. http://dx.doi.org/10.1016/j.jmsy.2022.07.016.

[56] Tihlarik A. Evaluation of the use of AI technologies in german engineering: insights from the employee perspective. Discov Glob Soc 2024;2(1):254. http://dx.doi.org/10.1007/s44282-024-00051-x.