Letter

Model-independent description of $B \to D\pi\ell\nu$ decays

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We introduce a new parametrization of $B \to D\pi\ell\nu$ form factors using a partial-wave expansion and derive bounds on the series coefficients using analyticity and unitarity. This is the first generalization of the model-independent formalism developed by Boyd, Grinstein, and Lebed for $B \to D\ell\nu$ to semileptonic decays with multihadron final states, and enables data-driven form-factor determinations with robust, systematically improvable uncertainties. Using this formalism, we extract the form-factor parameters for $B \to D_2^*(\to D\pi)\ell\nu$ decays in a model-independent way from fits of data from the Belle Experiment. We find that the semileptonic data are compatible with the presence of two poles in the $D\pi$ S-wave channel, which is the scenario preferred by nonleptonic decays and unitarized chiral perturbation theory.

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Motivation. Experimental measurements of tree-level semileptonic B-meson decays enable theoretically clean determinations of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $|V_{ub}|$ and $|V_{cb}|$, allowing for sensitive tests of the Standard Model by overconstraining the CKM unitarity triangle [1–3]. Further, $|V_{ub}|$ and $|V_{cb}|$ are parametric inputs to predictions for loop-level flavor-changing processes that are sensitive to new high-scale physics beyond the reach directly detectable by the LHC [4,5].

A major challenge for both inclusive and exclusive determinations of $|V_{ub}|$ is suppressing the CKM-favored $B \to X_c\ell\nu$ background, which exhibits a similar experimental signature and is $\mathcal{O}(100)$ times more abundant than $B \to X_u\ell\nu$ decays. The background subtraction process is further complicated by the orbitally excited states, collectively referred to as D^{**}, whose kinematic distributions remain poorly understood and branching fractions exhibit uncertainties of approximately 20% [6]. In measurements performed by the Belle and Belle II collaborations, the remaining "gap" between the sum of all considered exclusive modes and the inclusive $B \to X\ell\nu$ branching fraction, comprising unmeasured nonresonant $B \to X_c\ell\nu$ decays, is generally treated in simulation by assuming a composition of equal parts of $B \to D^{(*)}\eta\ell\nu$ decays, as prescribed in Ref. [7]. Because neither experimental evidence nor theoretical predictions exist for $B \to D^{(*)}\eta\ell\nu$ decays, a 100% uncertainty is assumed for the corresponding branching fractions. For these reasons, the $X_c\ell\nu$ modeling uncertainty is hard to quantify and becomes dominant for studies of inclusive $B \to X_{c/u}\ell\nu$ decays [7–11].

Exclusive measurements relying on tagged methods, in which machine learning algorithms are employed to fully reconstruct the companion B meson through exclusive decay modes [12,13], do not rely as directly on $X_c\ell\nu$ modeling as inclusive analyses. However, significant differences in these reconstruction algorithms' performances between data and simulation is accounted for by performing a calibration using a decay with a well known branching fraction: inclusive $B \to X\ell\nu$ [14]. This calibration, in turn, becomes a leading source of systematic error for tagged analyses [15–17]. In addition, the limited knowledge of $B \to D^{**}\ell/\tau\nu$ branching fractions and form factors are large systematic uncertainties in studies of rare processes or lepton flavor universality tests such as $B \to K\nu\nu$ and $R(X_{\tau/\ell})$ at Belle II [18,19] or $R(D^*)$ at the LHCb experiment [20,21].

The most commonly used description of $B \to D^{**}\ell\nu$ decays is the Leibovich-Ligeti-Stewart-Wise parametrization [22,23], extended to include $\mathcal{O}(\alpha_s)$ corrections and relaxing several assumptions with central values from the fit given in Refs. [24,25]. This parametrization includes a single D_0^* resonance. Studies in the context of unitarized chiral perturbation theory, however, have shown that the scalar member of the D^{**} family, the $D_0^*(2300)$, is an overlap of two states with poles near $(2.1 - i0.1)$ and

$(2.45 - i0.13)$ GeV [26–28]. Consequently, the S-wave line shape is not described by a simple Breit-Wigner distribution, but has a more complex structure. This conclusion is supported by lattice quantum chromodynamics (LQCD) calculations of isospin-1/2 $D\pi$ scattering [29–31] and a reinterpretation [32] of the partial-wave analysis of $B^+ \to D^-\pi^+\pi^+$ decays by LHCb [33]. Further, in Ref. [34], Le Yaouanc, Leroy, and Roudeau point out that in the fits to the Leibovich-Ligeti-Stewart-Wise parametrization tail effects from the D^* resonance are omitted and consequently overestimates the D_0^* contribution.

To address these and other limitations of existing parametrizations, in this Letter we develop the first model-independent description of resonant and nonresonant $B \to D\pi\ell\nu$ decays based on analyticity and unitarity. We then apply our formalism, which accommodates arbitrary line shapes, to fit experimental spectrum measurements and draw conclusions about the pole structure of the S-wave channel.

Form-factor parametrization. Semileptonic $B \to D\pi\ell\nu$ decays are characterized by five kinematic variables: the momentum transfer square q^2,[1] the helicity angle of the charged lepton $\cos\theta_l$, the helicity angle of the D meson $\cos\theta$, the azimuthal angle between the $\ell\nu$ and $D\pi$ planes χ, and the invariant mass of the hadronic system $M_{D\pi}$.

Form-factor decompositions for charged-current semileptonic decays involving two final state hadrons have been performed for $B \to \pi\pi\ell\nu$ decays [35,36] and involve a partial-wave decomposition in $\cos\theta$ to disentangle contributions from different hadronic resonances. Following a similar strategy, we express the $B \to D\pi\ell\nu$ hadronic matrix elements as

$$\begin{aligned}
\langle D(p_D)\pi(p_\pi)|V^\mu|B(p_B)\rangle &= i\epsilon^\mu_{\nu\rho\sigma}p^\rho_{D\pi}p^\sigma_B\sum_{l>0}L^{(l),\nu}g_l(q^2,M^2_{D\pi}),\\
\langle D(p_D)\pi(p_\pi)|A^\mu|B(p_B)\rangle &= \frac{1}{2}\sum_{l>0}\left(L^{(l),\mu}+\frac{4}{\lambda_B}[(p_B\cdot p_{D\pi})q^\mu-(p_{D\pi}\cdot q)p^\mu_B]L^{(l),\nu}q_\nu\right)f_l(q^2,M^2_{D\pi})\\
&\quad+\frac{M_{D\pi}(M_B^2-M^2_{D\pi})}{\lambda_B}\left[(p_B+p_{D\pi})^\mu-\frac{M_B^2-M^2_{D\pi}}{q^2}q^\mu\right]\sum_{l>0}L^{(l),\nu}q_\nu\mathcal{F}_{1,l}(q^2,M^2_{D\pi})\\
&\quad+M_{D\pi}\frac{q^\mu}{q^2}\sum_{l>0}L^{(l),\nu}q_\nu\mathcal{F}_{2,l}(q^2,M^2_{D\pi})\\
&\quad+\left[(p_B+p_{D\pi})^\mu-\frac{M_B^2-M^2_{D\pi}}{q^2}q^\mu\right]f_+(q^2,M^2_{D\pi})+\frac{M_B^2-M^2_{D\pi}}{q^2}q^\mu f_0(q^2,M^2_{D\pi}). \qquad (1)
\end{aligned}$$

The vector $L^{(l)}$ is related to the angular momentum of the final-state hadron system in the B-meson rest frame, and is uniquely defined via

$$\begin{aligned}
L^{(l)}_\mu q^\mu &= M_B W^l P_l(\cos\theta),\\
L^{(l)}_\mu p^\mu_{D\pi} &= 0, \qquad (2)
\end{aligned}$$

where $W = |\vec{q}||\vec{p_D}|/(M_B M_{D\pi})$ and P_l are the Legendre polynomials. The threshold factor $\lambda_B = M_B^4 + M_{D\pi}^4 + q^4 - 2(M_B^2M_{D\pi}^2 + M_{D\pi}^2q^2 + q^2M_B^2)$.

The standard expressions for $B \to D^*\ell\nu$ and $B \to D_2^*\ell\nu$ decays [37,38] are recovered from the $l = 1$ and $l = 2$ terms by replacing $M_{D\pi}$ and $L^{(l)}$ with the corresponding masses and polarization vectors.

Using Eqs. (1) and (2), it is straightforward to derive the $B \to D\pi\ell\nu$ differential decay rate. After performing the angular integration and dropping terms that are helicity suppressed, we obtain the double differential decay rate for massless leptons,

$$\begin{aligned}
\frac{\mathrm{d}^2\Gamma}{\mathrm{d}M^2_{D\pi}\mathrm{d}q^2} &= \frac{G_F^2|V_{cb}|^2}{(4\pi)^5}M_B\Bigg(W\frac{\lambda_B}{M_B^2}\frac{4|f_+|^2}{3}\\
&\quad+M^2_{D\pi}\sum_{l>0}\frac{4W^{2l+1}}{3(2l+1)}\Bigg[(M_B^2-M^2_{D\pi})^2\frac{|\mathcal{F}_{1,l}|^2}{\lambda_B}\\
&\quad+\frac{(l+1)}{l}q^2\left(|g_l|^2+\frac{|f_l|^2}{\lambda_B}\right)\Bigg]\Bigg), \qquad (3)
\end{aligned}$$

which will be used later in our analysis. The fully general fivefold differential decay rate allowing for interference effects between different partial waves is provided in the Supplemental Material [39].

Unitarity bounds. Model-independent constraints on the $B \to D\pi\ell\nu$ form factors arise from analyticity and unitarity. We begin with the two-point functions

[1] It is sometimes useful to instead consider the dependence on the recoil parameter $w = (M_B^2 + M_{D\pi}^2 - q^2)/(2M_BM_{D\pi})$.

$$\Pi^{L/T}_{(J)}(q) \equiv i \int d^4x\, e^{iq\cdot x} \langle 0|J^{L/T}(x) J^{L/T}(0)|0\rangle, \quad (4)$$

where J^μ denotes a $b \to c$ flavor-changing vector or axial-vector current and L/T denotes the component longitudinal or transverse to q^μ. Susceptibilities $\chi^{L/T}_{(J)}$ are defined from derivatives of $\Pi^{L/T}_{(J)}(q)$ as

$$\chi^L_{(J)}(Q^2) \equiv \frac{\partial \Pi^L_{(J)}}{\partial q^2}\bigg|_{q^2=Q^2} = \frac{1}{\pi}\int_0^\infty \mathrm{d}q^2 \frac{\mathrm{Im}\Pi^L_{(J)}(q^2)}{(q^2-Q^2)^2},$$
$$\chi^T_{(J)}(Q^2) \equiv \frac{1}{2}\frac{\partial^2 \Pi^T_{(J)}}{\partial (q^2)^2}\bigg|_{q^2=Q^2} = \frac{1}{\pi}\int_0^\infty \mathrm{d}q^2 \frac{\mathrm{Im}\Pi^T_{(J)}(q^2)}{(q^2-Q^2)^3}, \quad (5)$$

and are related to integrals over the imaginary part of $\Pi^{L/T}_{(J)}(q)$ via dispersion relations. The susceptibilities $\chi^{L/T}_{(V/A)}$ at $Q^2 = 0$ have been computed in perturbation theory to $\mathcal{O}(\alpha_S^2)$ in Ref. [40] and with nonperturbative lattice QCD in Refs. [41,42]. Separately, the optical theorem relates $\mathrm{Im}\Pi^T_{(J)}(q^2)$ to a sum of squared amplitudes for all intermediate states that can appear between the currents in Eq. (4). This sum includes terms with the matrix element $\langle \overline{B} D\pi|J^{L/T}|0\rangle$, which is related to $\langle D\pi|J^{L/T}|B\rangle$ by crossing symmetry and can therefore be parametrized by the form-factor decomposition in Eq. (1). The contribution of the $B \to D\pi\ell\nu$ channel to the dispersion relations in Eq. (5) is then given by evaluating the phase space integrals arising in the sum over states,

$$\mathrm{Im}\Pi^L_A|_{D\pi} = \frac{1}{64\pi^3}\frac{M_B^4}{q^4}\int_{(M_D+m_\pi)^2}^{(\sqrt{q^2}-M_B)^2} \mathrm{d}M^2_{D\pi}\left(M^2_{D\pi}\sum_{l>0}\frac{W^{2l+1}}{2l+1}|\mathcal{F}_{2,l}|^2 + W\frac{(M_B^2-M^2_{D\pi})^2}{M_B^2}|f_0|^2\right),$$
$$\mathrm{Im}\Pi^T_A|_{D\pi} = \frac{1}{192\pi^3}\frac{M_B^4}{q^2}\int_{(M_D+m_\pi)^2}^{(\sqrt{q^2}-M_B)^2} \mathrm{d}M^2_{D\pi}\left(\frac{M^2_{D\pi}}{\lambda_B}\sum_{l>0}\frac{W^{2l+1}}{2l+1}\left(\frac{|\mathcal{F}_{1,l}|^2}{q^2}+\frac{l+1}{l}|f_l|^2\right) + W\lambda_B\frac{|f_+|^2}{q^2M_B^2}\right),$$
$$\mathrm{Im}\Pi^T_V|_{D\pi} = \frac{1}{192\pi^3}\frac{M_B^4}{q^2}\int_{(M_D+m_\pi)^2}^{(\sqrt{q^2}-M_B)^2} \mathrm{d}M^2_{D\pi} M^2_{D\pi}\sum_{l>0} W^{2l+1}\frac{l+1}{l(2l+1)}|g_l|^2. \quad (6)$$

The positivity of the squared amplitudes in the sum over states implies $\mathrm{Im}\Pi^{L/T}_J|_{D\pi} \le \mathrm{Im}\Pi^{L/T}_J$. Inequalities for the $B \to D\pi\ell\nu$ form factors can then be derived from this inequality by inserting Eq. (6) and the perturbative expression for $\mathrm{Im}\Pi^{L/T}_J$. These so-called unitarity bounds provide q^2-dependent constraints that should be incorporated in determinations of the $B \to D\pi\ell\nu$ form factors. Because each form factor only couples to one polarization state of the weak current in Eq. (6), the unitarity bounds are diagonal and apply only to the groups of form factors $\{f_0, \mathcal{F}_{2,l}\}$ and $\{f_+, f_l, \mathcal{F}_{1,l}\}$ and to the single form factor g_l, rather than more general linear combinations. A parametrization of the q^2-dependence of the form factors is required to concretely specify how the bounds are imposed; we turn to this next.

z-expansion and scattering constraints. The model-independent parametrization and bounds presented in previous sections make no assumptions about the number, energies, or line shapes of possible resonances. To render fitting the measured $B \to D\pi\ell\nu$ decay spectra to our parametrization more tractable, it is helpful to include additional theoretical information and make some plausible assumptions.

The semileptonic B-decay form factors can be factorized into a part describing the short-distance weak decay and a part encoding the long-ranged final-state interactions between the hadrons [43,44]:

$$f_l(q^2, M^2_{D\pi}) = \hat{f}_l(q^2, M^2_{D\pi}) h_l(M^2_{D\pi}). \quad (7)$$

The weak-interaction contribution to the form factors of QCD resonances is approximately independent of $M_{D\pi}$ [45]:

$$\hat{f}_l(q^2, M^2_{D\pi}) \approx \tilde{f}_l(q^2) + \mathcal{O}((M_R^2 - M^2_{D\pi})/M_B^2), \quad (8)$$

Indeed, studies of $B \to \pi\pi(K)$ in the context of light cone sum rules [46] and recent LQCD studies of the $B \to \rho$ form factors [47] point towards the smallness of the neglected contributions.

The q^2-dependent function in Eq. (8) can be expanded as a power series [48–50]

$$\tilde{f}_l(q^2) = \frac{1}{\phi_l^{(f)}(q^2) B_f(q^2)}\sum_{i=0}^\infty a_{li}^{(f)} z^i, \quad (9)$$

where the Blaschke product B_f removes the poles of all subthreshold B_c resonances for a given channel and the change of variables

$$z(q^2, q_0^2) = \frac{q_0^2 - q^2}{(\sqrt{q_+^2 - q^2} + \sqrt{q_+^2 - q_0^2})^2} \quad (10)$$

with $q_+^2 = (M_B + M_D + m_\pi)^2$ maps the kinematically allowed q^2 range onto $|z| < 1$. With a suitable choice of outer functions $\phi_l^{(f)}$, the unitarity bounds on the z-expansion coefficients in Eq. (6) take an especially simple form

$$\sum_{i,l} |a_{li}^{(f)}|^2 < 1, \qquad (11)$$

allowing for an easy integration of the form factors in a fit, including priors on the coefficients. Further, for $q_0^2 = 0\ \mathrm{GeV}^2$, z ranges from 0 to 0.06, such that only a few terms in the expansion are needed to describe the form factors with high precision. The truncated set of coefficients $a_{li}^{(f)}$ is then determined using fits to experimental data, fixing both shape and normalization of the form factors.

Recently discussed problems associated to lower-lying branch cut at $q^2 = (M_B + M_D)^2$ can be incorporated as outlined in Refs. [51–54]. Additional details on the derivation and numerical calculation of the outer functions can be found in the Supplemental Material [39] and Ref. [55].

The semileptonic-decay form factors can also be connected to the S matrix via a dispersion relation [56]

$$\begin{aligned}&\mathrm{Im}\vec{f}(q^2, M_{D\pi}^2 + i\epsilon)\\ &\quad = T^*(M_{D\pi}^2 + i\epsilon)\Sigma(M_{D\pi}^2)\vec{f}(q^2, M_{D\pi}^2 + i\epsilon),\end{aligned} \qquad (12)$$

where the form-factor f is a vector in channel space, T is the T matrix, and Σ contains the relevant phase-space factors and is defined in Ref. [56]. The solution of Eq. (12) is given by the Muskhelishvili-Omnès (MO) matrix Ω [56,57]

$$\begin{aligned}\vec{f}(q^2, M_{D\pi}^2) &= \Omega(M_{D\pi}^2)\vec{P}(q^2, M_{D\pi}^2),\\ \mathrm{Im}\Omega(s + i\epsilon) &= \frac{1}{\pi}\int_{s_{\mathrm{thr}}}^{\infty} \frac{T^*(s')\Sigma(s')\Omega(s')}{s' - s - i\epsilon}\mathrm{d}s',\end{aligned} \qquad (13)$$

where $\vec{P}$ are boundary functions. A numerical algorithm to solve the above integral equation is outlined in Ref. [58].

Following the same arguments as for $\tilde{f}_l$ in Eq. (8), we can neglect the mild dependence of the boundary functions $\vec{P}$ on $M_{D\pi}$ and express them as a power series in z with the same Blaschke factors as the form factors but different outer functions.

For the S-wave contribution, we compute the Muskhelishvili-Omnès matrix Ω from the S matrix provided in Ref. [26], which was obtained using next-to-leading order unitarized chiral perturbation theory interaction potentials for coupled-channel $D\pi$, $D\eta$, and D_sK scattering from Refs. [59,60]. Consequently, the S-wave two-pole structure is treated in a parametrization-independent manner, solely relying on scattering phase shifts. This allows us to constrain the $B \to D\eta\ell\nu$ and $B \to D_sK\ell\nu$ decay rates from a fit of the $B \to D\pi\ell\nu$ invariant-mass spectrum, as further discussed in the Supplemental Material [39]. A similar procedure could be used to constrain higher-order partial-wave contributions—albeit with additional complications due to contributions from $D^*\pi$ and similar channels. LQCD calculations of coupled-channel $D^*\pi - D\pi$ scattering amplitudes could help determine the missing ingredients.

Experimental fits. To test our new $B \to D\pi\ell\nu$ form-factor description and extract the coefficients of the z expansion from data, we proceed in two steps. First, we fit the measured w and $\cos\theta$ dependence of the $B \to D_2^*\ell\nu$ differential decay width [61] and $B^0 \to D_2^{*-}\pi^+$ branching fraction [6] constraining the $a_{li}^{(f)}$ with Gaussian priors centered at zero with unit width. We employ the least-squares fitting package LSQFIT and use the augmented χ^2_{aug} defined in Refs. [62,63] to assess the goodness of fit. Additional numerical inputs are taken from Refs. [64–67] as discussed in the Supplemental Material [39]. The loose constraints help the fit converge more quickly but have little impact on the final results since the magnitudes of the resulting z coefficients are all of order a tenth or smaller.

As shown in Fig. 1, we find a harder D_2^* w spectrum than Refs. [24,25], i.e., enhanced for low values of w or high values of q^2, and also better describe the data. Possible reasons for this difference are the use of $B \to D_1\pi$ decay data in Refs. [24,25] and the greater flexibility in the model independent approach employed here in comparison to the heavy-quark-effective-theory-based approach of these works.

Next, we fit the $B \to D\pi\ell\nu$ $M_{D\pi}$ spectrum measured recently by Belle [68] using the z-expansion coefficients from the first fit as priors to constrain the shape the D_2^* form factors. Following Refs. [34,69], we parametrize the D^* and D_2^* line shapes by a Breit-Wigner distribution and with Blatt-Weisskopf damping factors [70,71]. In contrast to Ref. [34] we allow the Blatt-Weisskopf radius to be determined in the fit. As shown in Fig. 2, our form-factor

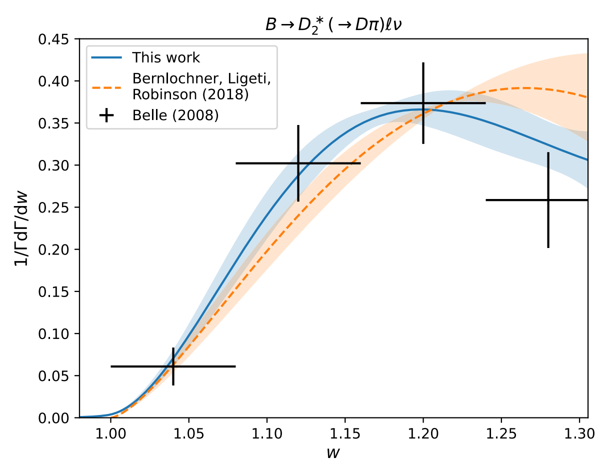

FIG. 1. Normalized $B \to D_2^*\ell\nu$ w spectrum. The black data points are from Ref. [61]. The blue solid curve with error band is our fit result, while the orange dashed curve and band are from Refs. [24,25]. The $\chi^2_{\mathrm{aug}}/\mathrm{d.o.f.} = 6.4/12$ and $Q = 0.9$.

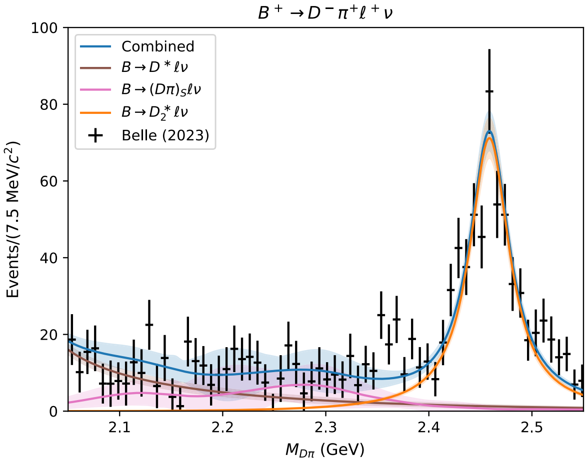

FIG. 2. Fit of the measured $M_{D\pi}$ spectrum [68] using the z expansion to parametrize D_2^* and S-wave form factors. The $\chi^2_{\mathrm{aug}}/\mathrm{d.o.f.} = 124.4/133$ and $Q = 0.69$. Only data for the more precise B^+ mode is shown.

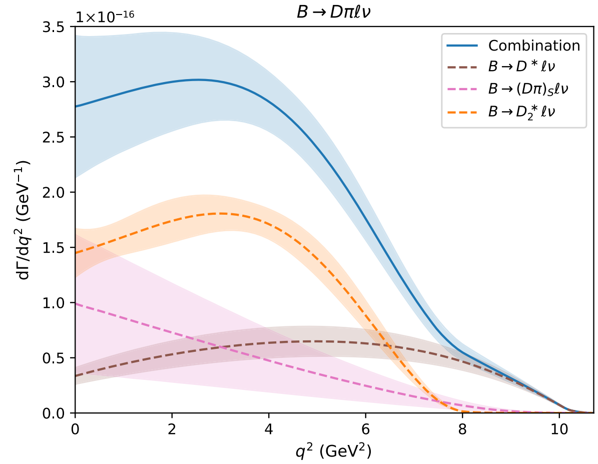

FIG. 3. Predicted partial-wave decomposition of the $B \to D\pi\ell\nu$ q^2 spectrum (dashed and dotted curves with error bands) and their total (solid curve with error band) from the fit in Fig. 2.

parametrization provides a good description of the data over the entire invariant-mass range.

Our fit to the $B \to D\pi\ell\nu$ invariant-mass spectrum can be used to make predictions for related quantities. Figure 3 shows the predicted partial-wave contributions to the q^2 spectrum. After integrating over the momentum transfer, we obtain for the D-wave channel $\mathrm{Br}(B \to D_2^*(\to D\pi^\pm)\ell\nu) = (1.90 \pm 0.11) \times 10^{-3}$, which is larger than Belle's determination in Ref. [68]. This is because the smooth falling function employed by Belle to describe the seemingly nonresonant contributions overlaps with the D_2^* resonance, whereas in our description, the S-wave and D^* components are negligible near the resonance. For the S-wave contribution, we obtain $\mathrm{Br}(B \to (D\pi)_S\ell\nu) = (1.03 \pm 0.27) \times 10^{-3}$, which agrees with the arguments made in Ref. [34] but is smaller than the branching fraction usually assigned in experimental analyses. Finally, the P-wave contribution in $B^+ \to D^-\pi^+\ell^+\nu_\ell$ decays, to which on-shell D^* decays can not contribute, amounts to a branching ratio of $(9.2 \pm 0.9) \times 10^{-4}$ for $M_{D\pi} \leq 2.5$ GeV.

Implications and outlook. We present the first model-independent description of $B \to D\pi\ell\nu$ decays based on unitarity and analyticity of the relevant form factors and the factorization of final-state interactions. This constitutes the first generalization of the BGL parametrization to multi-hadron final states and provides the first step towards a model-independent study of semileptonic B-meson decays into higher resonances and nonresonant final states. Our framework does not include any assumptions about line shapes of resonances and is extendable to other decay processes with charmed mesons in the final state such as $B \to D^*\pi\ell\nu$ or $B_s \to DK\ell\nu$. Further, it is also valid for final states with more than two hadrons, and can be combined with other known $b \to c$ form factors in a global fit to obtain constraints on less well-known form factors (see, e.g., Ref. [72]). By replacing the D meson by a pion, the unitarity bounds can be applied to $B \to \pi\pi\ell\nu$ decays, including the phenomenologically interesting $B \to \rho\ell\nu$ channel, which is the target of first LQCD calculations beyond the narrow-width limit [47], as well as non-resonant backgrounds, which constitute the dominant systematic uncertainty [73].

Taking into account recent theoretical considerations and measurements of $B \to D\pi\ell\nu$ decays by Belle, we provide precise predictions for semileptonic decays into the broad two-pole structure in the S-wave and determine the form-factor parameters for $B \to D_2^*\ell\nu$ decays from data. This marks the first time in which a three-component hypothesis consisting of S-wave contributions, D^* virtual contributions and D_2^* contributions, is compared to the measured $M_{D\pi}$-spectrum. Previous works either do not include all three components simultaneously [24,61,74] or do not compare to the measured $M_{D\pi}$ spectra [34]. We demonstrate, in contrast to existing literature, that our treatment of the S-wave is compatible with the $M_{D\pi}$ spectrum measured by Belle and, since it is the clearly favored description of $B^+ \to D^-\pi^+\pi^+$ decays [32], should replace models that assume a single, broad S-wave state, the $D_0^*(2300)$. While more careful studies need to be conducted, the change in the shape for $B \to D_2^*\ell\nu$ decays, as well as the inclusion of the virtual D^* contribution lead to an overall harder q^2 spectrum, potentially resolving some of the discrepancies seen in inclusive analyses at high q^2 [7,10,19].

The coupled-channel nature of the S-wave contribution enables us to obtain predictions for $B \to D\eta\ell\nu$ and $B \to D_sK\ell\nu$ decays purely based on measurements of $B \to D\pi\ell\nu$ decays. For the $D\eta$ S-wave contribution we obtain $\mathrm{Br}(B \to (D\eta)_S\ell\nu) = (1.9 \pm 1.7) \times 10^{-5}$, two orders of magnitude too small to constitute a sizeable portion of the semileptonic gap. Since heavy-quark spin symmetry relates the S-wave scattering matrix to the $D^*\pi - D^*\eta - D_s^*K$ $J^P = 1^+$ S-wave scattering matrix, the same conclusion holds for the $B \to (D^*\eta)_S \ell\nu$ channel. Consequently, both approaches utilized by the Belle and Belle II collaborations in recent measurements to fill the semileptonic gap in terms of $B \to D^{(*)}\eta\ell\nu$ decays, either via a broad S-wave resonance or equidistributed in phase space, are ruled out. While our analysis does not provide alternative candidates to fill the gap, the harder q^2 spectrum of $B \to D\pi\ell\nu$ decays obtained here shift the gap to lower values of q^2, thus opening up the possibility of heavier states accounting for it.

Additional theoretical work, such as a more precise determination of the scattering potentials along the lines of Ref. [60], LQCD determinations of the form factors and light cone sum rules computations of the S-wave form factors [75] would greatly improve the results presented in this letter.

Future experimental measurements of the q^2 and $\cos\theta_l$ spectra of $B \to D_2^*\ell\nu$ decays by Belle II with the already available data set, as well as updated angular analyses of $B^0 \to D^0\pi^-\pi^+$ and $B^0 \to D^0\pi^-K^+$ decays by LHCb, would improve the precision of the form factors presented in this Letter. In the long term, a full partial-wave analysis of $B \to D\pi\ell\nu$ decays is required to ultimately determine the exact composition of the $D\pi$ spectrum in semileptonic decays. Additionally, the final state interactions between D mesons and light hadrons can be tested by measuring femtoscopic correlation functions at the ALICE experiment [76]. This result could provide a direct, orthogonal test of the S-wave two-pole structure in heavy ion collisions [77].

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