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Approximate and exact approaches to energy-aware job shop scheduling with dynamic energy tariffs and power purchase agreements

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ABSTRACT

With the goal of strategically analyzing energy tariff structures and operationally establishing energy-aware production schedules, we develop approximate and exact methods for generating energy-efficient Pareto schedules in job shop environments. Energy costs are influenced by dynamic market prices and available fixed tariff contracts. In this context, power purchase agreements (PPAs) have recently emerged to support renewable energy generation, guaranteeing a fixed price for a specified amount of renewable energy sold to industrial customers. To integrate energy-related aspects, we extend the job shop setting by machine states, energy sources, carbon emissions, energy consumption, and time-dynamic energy tariffs. We develop a time-indexed mathematical programming formulation integrated into the ϵ-constraint method to achieve Pareto efficiency concerning production makespan and energy costs minimization. Our research addresses the challenges of integrating energy market characteristics with production scheduling, tackling nonlinearity and time dynamics while managing NP-hardness of energy-aware job shops. Key contributions include developing a model-based methodology for optimizing energy-aware schedules, integrating this approach within an algorithmic framework for determining Pareto schedules, and creating a decision-making workflow for analyzing energy tariffs. In particular, this facilitates an analysis of the largely unexplored PPA tariff. Using 2023 energy price data from the European Network of Transmission System Operators for Electricity (ENTSO-E), we provide extensive numerical experimentation to analyze trade-offs related to schedule determination, energy price data, PPA and tariff specifications, and working time restrictions. This provides insights into the interplay between tariff selection and production scheduling, relevant to the strategic financing and operational management of energy-aware production systems.

1. Introduction

1.1. Motivation

Over the past two decades, a multitude of environmental, societal, and political reasons have led to a substantial rise of energy prices [1]. At the same time, energy suppliers have rolled out dynamic pricing schemes, such as time-of-use (ToU) tariffs that differentiate between off-peak and on-peak periods, and real-time pricing (RTP) tariffs that adjust prices hourly, to alleviate pressure on the main grid by influencing energy demand [2]. This aligns with national governments' initiatives to source nearly all household and industrial energy from renewable sources like wind and solar, with the ultimate aim of achieving ambitious climate protection targets [3]. In this context, a recently introduced instrument promoting renewable energy use while providing price certainty is the power purchase agreement (PPA), which allows companies to source renewable energy from suppliers at a fixed rate over an extended period [4]. In light of increased energy price volatility from renewable sources, manufacturing companies must balance economic efficiency – such as energy costs and production times – with ecological sustainability, including energy consumption and carbon emissions, to remain competitive [5]. Overall, this necessitates coordinated efforts to harmonize production schedules with current energy market conditions and strategic tariff instruments like PPAs. By effectively balancing these measures, companies can produce economically while also supporting the expansion of renewable energy generation. To our knowledge, the integration of these aspects in energy-aware job shop scheduling, particularly regarding the complexity of time dynamics, multiple objectives, and PPA integration, has not been studied before. This paper makes a two-fold contribution: First, we present operational models for energy-aware scheduling that analyze various energy cost tariff structures and scheduling constraints.

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Second, we conduct extensive numerical experiments to develop managerial guidelines for configuring energy-aware production systems and associated tariff structures.

1.2. Problem description

We integrate energy market and production system properties into our decision-making methodology to provide a holistic view of energy-aware production systems, explicitly considering the following problem characteristics:

1. **Fixed vs. dynamic energy prices:** Dynamic pricing and tariffs increasingly reflect renewable energy integration. This approach helps to balance fluctuating energy supply and demand [6]. Consequently, optimizing energy costs has become essential for manufacturers. By adapting to these pricing models, manufacturers not only reduce costs but also support the growth of renewable energy. This optimization is crucial for leveraging energy price volatility at a granular time resolution to maintain economic competitiveness [7]. To effectively coordinate energy supply and demand, the planning methodology must be flexible enough to incorporate various pricing schemes, including ToU, RTP, traditional fixed tariffs for conventional energy sources, and PPA tariffs for renewables. Hence, dynamic pricing is utilized to modulate the temporal patterns of energy consumption [8].
2. **Distributed energy sources:** The transition to distributed energy sources, particularly renewables like photovoltaics and wind turbines, is being driven by national governments as a strategic response to grid stress and climate protection goals [9, 10]. This shift from conventional to renewable energy sourcing requires a more comprehensive description of the energy market configuration, incorporating a company's energy sourcing options as a key component of the existing infrastructure [11]. Specifically, distinguishing sourcing options by primary energy source type (renewable vs. non-renewable) is essential to enhance transparency in manufacturing companies' progress towards achieving sustainability goals [12].
3. **Production system:** While energy awareness can significantly impact various disciplines like master production planning, capacity planning, lot sizing, and machine scheduling [13], this paper specifically focuses on energy-aware job shop machine scheduling. This focus is due to the high computational complexity (strong NP-hardness) of job shop environments [14], making them representative for the most challenging production systems in terms of optimal planning of manufacturing processes. Consequently, the methodology presented can be easily adapted to the aforementioned disciplines. Additionally, job shops represent a significant portion of industrial shop floor configurations [15].
4. **Processing characteristics:** Existing processing characteristics and their associated energy requirements dictate the potential for balancing scheduling performance between productivity and sustainability as shown by the shop-floor classification of Gao et al. [5] in the context of energy-efficient production systems. The execution model of a production system encompasses all machine, job, and organization-related characteristics that must be adhered to during operations, including possible machine states (on, off, idle, standby), execution modes (speed-up, regular), and processing requirements (set-ups). As per Gahm et al. [12], these elements form the basis for describing the operational constraints and energy dynamics of the system. We assume that the job shop production floor exhibits energy dependency in machine states, affecting energy consumption and carbon emissions, as is typical in scheduling-related energy consumption modeling [5].
5. **Multiple performance criteria:** The ultimate goal of achieving green production is to minimize total energy demand from primary energy sources, thereby reducing carbon emissions, while ensuring that production outcomes meet specified quantities [12]. As proposed by the classification of typical production- and energy-related objectives outlined by Fernandes et al. [15], we break this target into interrelated performance criteria across economic and environmental dimensions, coordinated through intelligent resource allocation. Specifically, our approach explores trade-offs between production efficiency (measured by makespan) and various energy-related objectives (emissions, energy consumption, and energy costs) as captured by multi-objective approaches accounting for conflicting objectives [5].
6. **Integration of planning levels:** While production scheduling optimizes resource allocation and timing at the operational level [16], PPAs guide long-term energy procurement decisions at the strategic level [11]. Likewise, supplier selection and work-time restrictions are positioned at the strategic level. In general, enterprise functions are organized hierarchically within a multi-level decision-making structure. This hierarchy typically follows a top-down, unidirectional flow, where higher-level decisions cascade down as constraints or parameters for lower-level processes [17]. Through comparative analysis, we demonstrate how a bi-directional approach – similar to the production decision-making framework by Silver et al. [16] – can facilitate feedback between planning levels through information flows both from strategy to operations and vice versa. This process involves evaluating operational schedule quality under different higher-level decisions, enabling the outcomes to guide potential adaptations at the strategic level.

The ENTSO-E Transparency Platform [18] provides comprehensive day-ahead energy price data with hourly granularity. Analysis of this data reveals significant price volatility, largely attributable to the variable nature of feed-in from renewable sources. This information is crucial for industrial entities participating in the energy market. Specifically, granular insights from energy price data enable these organizations to optimize production schedules to capitalize on lower energy costs and make informed decisions about long-term energy procurement and hedging strategies.

For analyzing the impact of strategic energy tariff decisions on operational energy-aware production schedules, this paper introduces a comprehensive methodology for identifying the Pareto frontier of job shop schedules concerning production makespan and energy costs. Central to this methodology is the development of a mixed-integer programming (MIP) model, termed the energy-aware job shop problem ($eJSP$), which tackles the nonlinearities and time dynamics inherent to the original problem setting. Utilizing the ϵ-constraint method in combination with $eJSP$, the approach can compute either an approximate or the exact Pareto frontier, thereby accounting for both production- and energy-related objectives. The computational complexity of $eJSP$ limits the use of exact algorithms to small job instances. For larger instances, approximate algorithms are proposed to obtain practical solutions. For the approximate case, job sequences are established using the most work remaining heuristic, followed by energy sourcing, machine states determination, and operation timings derived from $eJSP$; the incorporation of this MIP model within our approximate method classifies it as a matheuristic approach. In the exact case, all decisions are based on $eJSP$. The complexity is increased by dynamic energy pricing, energy-dependent machine states, and multi-objective goals, including minimizing makespan, energy costs, energy consumption, and carbon emissions. Overall, the systematic approach presented in this paper, summarized in Fig. 1 and detailed in Section 3, offers a structured method for making balanced, energy-aware production decisions in job shop environments with volatile energy prices.

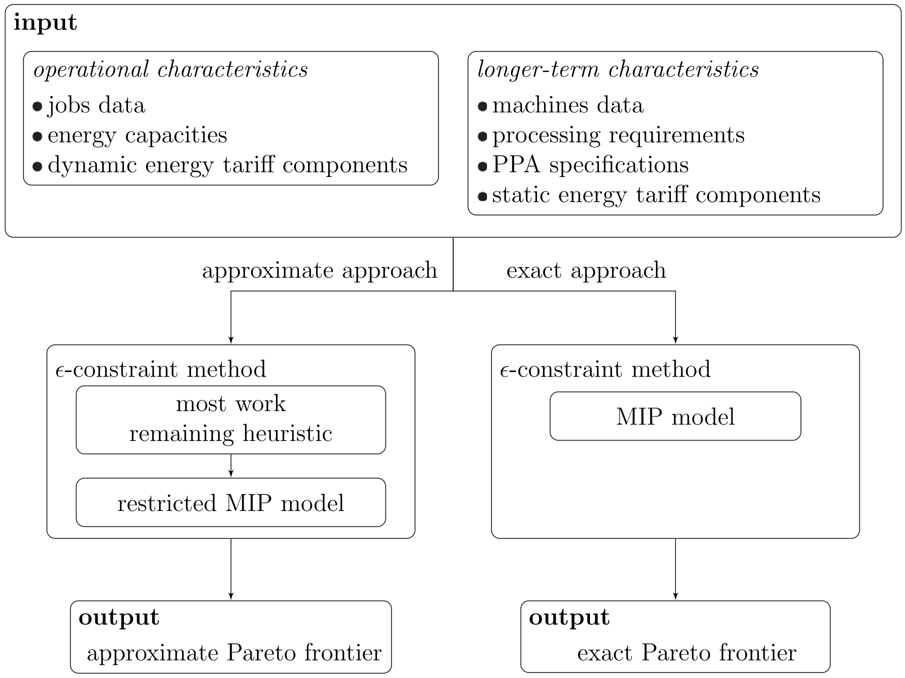

Fig. 1. Overview on MIP-based approximate and exact solution methodology to energy-aware job shop scheduling.

The paper remainder is organized as follows: In Section 2, we discuss literature on machine scheduling under dynamic energy pricing. In Section 3, we present the research methodology developed in this paper, which encompasses the $eJSP$ model, a computationally tractable linearization, and its integration into the ϵ-constraint method. This comprehensive approach facilitates both the approximate and exact determination of Pareto schedules. In Section 4, we illustrate the effectiveness of our developed methodology through a comprehensive case study that employs real-world energy price data. The case study underscores the practical value of the paper's methodology in terms of providing managerial insights for optimizing scheduling operations and selecting energy tariffs. Finally, Section 5 provides a conclusion and points to future research directions.

2. Literature review

We conduct a literature review on energy-aware extensions to machine scheduling, consistently emphasizing the key topics of dynamic energy pricing, machine state-dependent energy consumption, and the integration of renewable energy sources through the PPA pricing scheme. The discussion reflecting the interplay between these topics is twofold: In Section 2.1, we first provide a thorough examination of literature on machine scheduling influenced by dynamic energy pricing and related aspects of energy consumption. In Section 2.2, we then survey PPAs as the most recent mechanism fostering corporate renewable energy sourcing. The concluding Section 2.3 finally puts the review in perspective by identifying a major research gap: the lack of systematic integration between energy-aware scheduling and diverse energy tariff structures.

2.1. Machine scheduling subject to dynamic energy pricing

Energy-efficient machine scheduling under dynamic energy pricing has become increasingly important for balancing productivity and sustainability, as evidenced by the growing body of research. The discussion is subdivided into two sections: In Section 2.1.1, we begin by examining the single-objective setting, where many works employ modeling techniques similar to ours. In Section 2.1.2, we then explore bi- and multi-objective settings, which demonstrate diverse goal systems and inspire our combination of production- and energy-related objectives. For each reference, we highlight the optimized objective, note energy-related modeling specifics, and briefly discuss the solution approach used. Our discussion implies that while dynamic pricing is a common theme, other energy-related factors – such as machine states, processing speeds, and alternative energy sources – are only addressed in select works.

2.1.1. Single objective setting

Table 1 summarizes contributions where only one objective function is presumed. Note that works addressing the sum of several criteria as a single objective function are also included.

Single machine. Shrouf et al. [19] minimize energy costs originating from consumption and machine state switching by deciding upon processing start times and machine statuses. They employ a genetic algorithm (GA) and conduct a numerical analysis. Che et al. [20] devise a greedy insertion heuristic for minimizing total electricity costs. They derive proofs for schedule properties and employ the outline in a manufacturing process case study. Aghelinejad et al. [21] improve the MIP formulation by Shrouf et al. [19] and provide a more detailed formulation taking into account state-dependent energy costs. A heuristic is presented as well as a GA, and the methods are compared to each other in a numerical study. Aghelinejad et al. [22] augment their setting to speed scalability for which computational complexity is analyzed. As a result, they suggest a dynamic programming (DP) scheme. Wichmann et al. [23] combine scheduling and lot sizing to optimize all production-related costs including energy costs. The model accounts for machine states and a time resolution combining micro- and macro-periods. In an MIP-based solution approach, the model confirms its utility in case of high energy price volatility. Kim et al. [10] propose a hybrid metaheuristic approach that integrates energy generation, storage, and consumption into production scheduling. This method optimizes energy costs by treating energy as a logistical component, incorporating strategies for initial solutions, job sequencing, idle time management, and energy supply planning.

Table 1
Classification of literature on single-objective energy-aware machine scheduling subject to dynamic energy pricing.

	[19]	[20]	[21]	[22]	[23]	[10]	[24]	[25]	[26]	[27]	[28]	[29]	[30]	[31]	[32]	[33]	[34]	[35]	[36]	[37]	[9]	[38]	[39]
Machine environment																							
Single machine	x	x	x	x	x	x																	
Parallel machines							x	x	x	x	x	x	x	x									
Flow shop															x	x	x	x	x				
Flexible flow shop																							
Job shop																				x			
Flexible job shop																					x	x	x
Objective system																							
Single objective (scalar)	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x
Bi-objective																							
Multi-objective																							
Objective(s)																							
Energy costs	x	x	x	x		x	x	x		x	x	x	x		x	x	x	x	x	x	x	x	x
Energy consumption									x														
Carbon emissions																							
Carbon costs																							
Production costs					x	x														x	x		
Other costs							x													x			
Regret for costs														x									
Makespan												x	x									x	
Completion time																							
Tardiness																							
Sojourn time																							
Machine unavailability																							
Number of machines																							
Price coordination																							
ToU	x	x	x	x	x	x	x	x	x	x	x	x	x	x		x			x	x	x	x	x
RTP															x		x	x					
Processing features																							
Machine speeds				x						x													
Machine states	x		x	x	x	x							x			x						x	
Energy sources																							
Renewables integration															x		x		x				
Different sources																					x		
Generation/storage						x											x			x	x		
MIP formulation	x	x	x		x	x	x	x		x	x			x	x	x	x	x	x	x	x	x	x
Solution approach																							
MIP-based					x			x							x		x			x		x	
Heuristic		x	x					x			x					x		x	x		x		x
Metaheuristic	x		x			x	x		x	x				x		x							
Other (CP, DP, ...)				x								x	x									x	

Parallel machines. Moon et al. [24] pioneered energy-aware scheduling, using a hybrid GA to minimize combined production and variable electricity costs through operation shifting and insertion techniques. Ding et al. [25] propose a column generation heuristic using a time-interval-based MIP formulation to minimize energy costs, achieving optimality gaps below 0.05% with low energy price volatility and below 1% with high volatility. Kong et al. [26] propose a variable neighborhood search for rescheduling during disruption events with the goal of minimizing energy consumption of both original and new jobs. The metaheuristic leverages general properties for the rescheduling setting which are first proven. Experimentation illustrates the effect of different, specifically tailored neighborhood types. Zhang et al. [27] analyze a two-stage problem with identical parallel machines at stage 1 and unrelated parallel machines at stage 2. They propose a hybrid metaheuristic combining tabu search and greedy insertion to minimize electricity costs by optimizing machine speeds, job assignments, and processing intervals. Wu et al. [28] devise a combined assignment and insertion heuristic for minimizing electricity costs. The model is provided as a continuous-time MIP along with tightening and relaxation possibilities. Pei et al. [29] minimize normalized makespan and electricity costs via a quadratic MIP, reformulated with second-order-cone constraints and tightened cuts. They propose a rounding-based heuristic resembling local search, benchmarked against a lower bounding technique. Heydar et al. [30] address the online problem with machine states. Jobs arrive sequentially and processing times are revealed only upon order placement. The goal is to minimize the sum of makespan and energy costs where the latter depend on machine states. Due to the online character, the problem is tackled through simulation-based approximate dynamic programming (ADP). Feng and Peng [31] address uncertainty using a min–max regret model with interval processing times, minimizing electricity and machine launching costs. They develop an iterative exact algorithm based on scenario-specific properties and a memetic differential evolution heuristic for larger instances.

Flow shop. Zhai et al. [32] address on-site wind energy generation for RTP. They develop a time-indexed MIP that optimizes wind energy use for production or grid sales using a rolling horizon approach with updated wind speed and electricity price forecasts. Wang et al. [33] analyze permutation flow shops with electricity costs objective, leading to a Johnson-inspired heuristic, a DP-inspired heuristic, and an iterated local search. Fazli Khalaf and Wang [34] use a two-stage stochastic approach to integrate renewables and energy storage for minimizing total electricity costs under RTP. The first stage schedules based on day-ahead forecasts, while the second reacts to RTP. Busse and Rieck [35] scrutinize RTP to minimize energy costs, introducing a heuristic and rule-based sampling approach for larger instances. They evaluate the forecast error of a mid-term approach with the reference being electricity prices known at planning time. Ghorbanzadeh and Ranjbar [36] develop MIPs to minimize energy costs, incorporating renewable energy constraints. They propose a decomposition heuristic and conduct sensitivity analyses on parameters like time horizon length.

Table 2
Classification of literature on multiobjective energy-aware machine scheduling subject to dynamic energy pricing.

	[40]	[41]	[42]	[43]	[44]	[45]	[46]	[47]	[48]	[49]	[50]	[7]	[51]	[52]	[53]	[54]	[55]	[56]	[57]	[58]	[59]	[60]	[61]	[62]	[63]
Machine environment																									
Single machine	x	x	x	x	x																				
Parallel machines						x	x	x	x	x	x	x													
Flow shop													x	x											
Flexible flow shop															x	x	x	x					x	x	
Job shop																			x	x					
Flexible job shop																					x	x			x
Objective system																									
Single objective (scalar)																									
Bi-objective	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x			
Multi-objective																							x	x	x
Objective(s)																									
Energy costs		x	x	x	x	x	x			x	x	x	x	x	x	x	x		x	x	x	x	x	x	x
Energy consumption								x	x			x						x							
Carbon emissions	x												x										x		
Carbon costs																								x	
Production costs																									
Other costs																									x
Regret for costs																									
Makespan		x				x		x	x	x	x					x		x			x	x	x		x
Completion time			x																						
Tardiness				x											x		x		x					x	
Sojourn time	x													x						x					
Machine unavailability					x																				x
Number of machines							x																		
Price coordination																									
ToU	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x		x	x	x	x
RTP												x									x				x
Processing features																									
Machine speeds							x	x		x		x			x	x	x							x	
Machine states		x			x							x				x		x					x	x	
Energy sources																									
Renewables integration	x													x											
Different sources																							x		
Generation/storage												x											x		
MIP formulation	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	x	
Solution approach																									
MIP-based	x	x	x	x		x				x	x	x	x	x			x	x			x				
Heuristic							x		x		x														
Metaheuristic			x	x	x	x		x		x					x	x		x	x		x	x	x	x	x
Other (CP, DP, ...)																				x					

Job shop. Golpîra et al. [37] develop a robust nonlinear MIP model to address wind speed and demand uncertainties, minimizing the total of costs (production, electricity, discharge) and penalties (power restraints, missed robustness). The model is linearized to enable an MIP-based solution procedure, hedging against peak demand charges. Moon and Park [9] address a flexible job shop with distributed energy sources and storage, minimizing the total of production and electricity costs. They propose a hybrid algorithm intertwining production and energy scheduling using two MIP formulations solved by MIP or constraint programming (CP) solvers. Park and Ham [38] address flexible job shops minimizing the total of makespan and energy costs, considering machine states due to scheduled downtime. They solve the problem using available solvers for integer programming (IP) and CP formulations. Shen et al. [39] address flexible job shops, minimizing energy costs under a maximum makespan constraint. They analyze the fixed-sequence case for optimality, develop two heuristics for this setting, and propose an iterative tabu search hybridized with these heuristics to tackle the complexity of the original problem.

2.1.2. Bi- and multi-objective settings

Table 2 summarizes contributions where two or more objective functions are presumed and handled by multi-objective optimization approaches.

Single machine. Liu [40] minimizes flow time and carbon emissions under uncertain renewable energy availability, using a lexicographic-weighted Tchebycheff method for Pareto solutions. They handle uncertainty with interval numbers and arithmetic. Cheng et al. [41] address minimizing electricity costs and makespan, providing complexity analysis and optimality rules for machine-state-dependent energy costs. They employ the ϵ-constraint method to heuristically generate Pareto solutions for large instances. Rubaiee and Yildirim [42] and Rubaiee et al. [43] develop MIP-based metaheuristics, combining weighted sum methods with ant colony and genetic algorithms to tackle multi-objective problems balancing completion time, tardiness, and energy costs. Sin and Chung [44] introduce machine failures and aim to minimize energy costs and machine unavailability. They model machine states and formulate the problem as a non-linear MIP, solving it with a hybrid GA.

Parallel machines. Wang et al. [45] address makespan and energy cost objectives, proposing an augmented ϵ-constraint method, a constructive heuristic inspired by local search, and an implementation of the non-dominated sorting genetic algorithm II (NSGA-II). Zeng et al. [46] develop insertion and iterative search heuristics for the objectives of minimizing the electricity costs and number of machines used. Experimentation exhibits the applicability for larger problem sizes. Cota et al. [47] minimize makespan and energy consumption under speed scalability, formulating the problem as an MIP and solving it with enhanced adaptive large neighborhood search embedding a learning automaton. Similarly, Anghinolfi et al. [48] address the same objectives, devising a split-greedy heuristic with exchange search. Rego et al. [49] consider the speed-scaling version for makespan and energy cost minimization and specify an NSGA-II version which is compared to the ϵ-constraint method for instances of various sizes. Gaggero et al. [50] minimize makespan and energy costs using symmetry-breaking properties to create a compact MIP formulation. They enhance Anghinolfi et al. [48]'s split-greedy heuristic and demonstrate that their compact MIP reduces computation time and outperforms both the original heuristic and an NSGA-II implementation. Hilbert et al. [7] examine ToU, RTP, and PPA tariffs in a speed-scalable scheduling and lot-sizing problem with machine states and energy storage. They analyze electricity cost-consumption tradeoffs under price volatility, formulating a nonlinear MIP that is linearized and solved heuristically using fix-relax-and-optimize methods.

Flow shop. Zhang et al. [51] aim to minimize electricity costs and carbon emissions by establishing a time-indexed IP model to approximate the Pareto frontier, illustrating the trade-off between these objectives. Biel et al. [52] employ a two-stage stochastic approach for integrating uncertain wind power. Stage 1 minimizes flowtime and expected energy cost, while stage 2 allows schedule adaptations based on actual wind data. The IP model is solved using the weighted sum method with numerous intermittent wind power scenarios. Ding et al. [53] address flexible flow shops, aiming to minimize tardiness and power costs with speed scaling. They develop a hybrid discrete particle swarm optimization enhanced by tabu-search-based local search for the nonlinear MIP. Geng et al. [54] consider a hybrid flow shop with reentrance for minimizing makespan and energy costs, accounting for different machine states and speeds. They propose an improved multiobjective ant lion optimization algorithm. Schulz et al. [55] discuss a flexible flow shop with discrete machine speeds to minimize energy costs and tardiness. They compare a time-indexed and a sequence-dependent model against the ϵ-constraint method, focusing on the impact of speed options and dynamic energy pricing. Chen et al. [56] examine energy consumption and makespan minimization in a flexible flow shop by providing an enhanced NSGA-II version that utilizes an improved construction heuristic and additional genetic operators. Dong and Ye [61] tackle makespan, energy costs, and carbon emissions minimization in a two-stage flexible flow shop. The first stage focuses on makespan, while the second addresses energy costs and carbon emissions with distributed energy sources. An enhanced hybrid salp swarm algorithm and NSGA-III are used, showing distributed sources' positive effect on reducing emissions. Li et al. [62] examine a flexible flow shop optimizing energy costs, carbon trading costs, and tardiness, considering machine state and speed dependencies. They propose a mixed-integer nonlinear programming (MINLP) model solved via a hybrid approach combining Q-learning and NSGA-II. Hereby, diverse Pareto solutions are generated more rapidly than with conventional multiobjective algorithms.

Job shop. Kurniawan et al. [57] aim to minimize weighted tardiness and energy costs using a genetic algorithm enhanced by distributed elite local search, effectively balancing exploration and exploitation. The problem is framed as operation sequencing and start time determination, with chromosome encoding addressing these aspects. Pan et al. [58] utilize deep reinforcement learning to minimize sojourn time and energy costs in flexible job shops, framing the problem as a Markov decision process suitable for deep Q-learning. Burmeister et al. [59] study RTP tariffs in flexible job shops, minimizing energy costs and makespan. Their memetic algorithm enhances NSGA-II with local refinement, outperforming standard NSGA-II and ϵ-constraint methods by producing a richer approximated Pareto front. Jia et al. [60] focus on flexible job shops, minimizing energy costs and makespan via a learning-enhanced evolutionary algorithm. Their method, leveraging elite individual properties, outperforms other multiobjective approaches in diversity and convergence. Gong et al. [63] address the potential increase in nighttime labor costs from dynamic energy tariffs as seen from optimizing makespan, energy cost, labor cost, maximal workload, and total workload in flexible job shops. NSGA-III is used for both ToU and RTP scenarios.

2.2. Power purchase agreements (PPAs)

This section reviews literature on PPAs, focusing on their role in renewable energy integration. It also highlights key studies related to PPA scheduling and optimization contributions. The discussion indicates that research on PPA integration in operational production decisions is currently rather limited. PPAs are a relatively new mechanism aimed at enhancing the integration of renewable energy into the energy mix through mandatory corporate commitments, as outlined by the World Business Council for Sustainable Development (WBCSD) [64]. A PPA is a long-term contract between companies and energy suppliers that guarantees the procurement of renewable energy at a fixed rate. These contracts not only enhance corporate sustainability by promoting renewable energy generation but also serve as a hedge against price fluctuations. Technically, PPAs are often structured as contracts for difference, where cash settlements occur if the PPA rates differ from market rates. PPAs have recently gained significance due to price volatility in energy markets and companies' increasing interest in investing in renewable energy. As a result, the future use of PPAs is anticipated to rise further Busch et al. [4]. Literature on scheduling related to PPAs is in its infancy, with Hilbert et al. [7] being the only study to address the energy-efficient allocation of energy to production processes under ToU, RTP, and PPA tariffs. They formulate a bicriteria lot-sizing and scheduling problem aimed at minimizing energy costs and consumption, modeling it as an MINLP, which is then linearized to an MIP and addressed using heuristic methods. Biggins and Brown [65] develop an optimization model for the utilization of battery storage and hydrogen electrolysis, tailored to the specifications of various PPAs and fluctuating green hydrogen market prices. Lei and Sandborn [66] derive an optimal scheduling strategy for predictive maintenance operations in wind farms managed through PPAs. Their approach considers forecasts of turbine remaining useful life and PPA specifications, such as energy delivery targets, pricing, and penalties for under-delivery. The optimal timing for maintenance operations is determined to maximize the value of predictive maintenance options.

2.3. Literature review conclusion

As implied by the literature review in Sections 2.1 and 2.2, most research on energy-efficient scheduling focuses on specific contexts, leading to the development of specialized solution methodologies. Remarkably, even crucial aspects within this domain, such as energy-related machine states and the integration of renewable or distributed energy sources, receive only intermittent and unsystematic attention. A notable imbalance exists in the research on dynamic energy tariffs, with a disproportionate focus on ToU models at the expense of RTP and PPA models. This skewed emphasis has led to a significant knowledge gap in understanding how these diverse tariff models interact and impact energy-aware production systems. Addressing this gap is pivotal for developing strategies that effectively balance productivity with sustainability, ultimately contributing to the broader goal of achieving carbon-neutral manufacturing.

3. Methodology

We develop a model-based methodology optimizing energy-aware schedules that integrate production planning with energy considerations. Our algorithmic approach accounts for various energy tariff models, including the understudied PPA tariff, and offers decision-makers a structured method to evaluate renewable energy integration instruments. After introducing the problem characteristics and modeling assumptions in Section 3.1, we develop a MINLP model called $eJSP$

(energy-aware job shop problem) in Section 3.2. This model optimizes job shop production scheduling, accounting for machine-specific energy use, diverse energy sources (conventional and renewable), and varied tariff structures (dynamic and fixed, including PPAs). In Section 3.3, $eJSP$ is linearized into an MIP model. Sections 3.4 and 3.5 introduce additional constraints to adapt the model for approximate approaches and for embedding it into the ϵ-constraint method, respectively. Finally, in Section 3.6, we outline an algorithm for determining the energy-efficient production frontier, both approximately and exactly, aimed at minimizing makespan and energy costs by utilizing the developed $eJSP$ extensions.

3.1. Problem characteristics and modeling assumptions

We next address the underlying modeling assumptions and relate them to the objectives, constraints, and decisions of $eJSP$.

3.1.1. Objective system

Our daily production scheduling aims to balance operational efficiency with sustainability. While we consider four key factors – makespan, energy costs, carbon emissions, and energy consumption – we employ a bi-objective optimization approach primarily focusing on minimizing makespan and energy costs. We then evaluate the resulting feasible schedules for their energy consumption and carbon emissions. This approach ensures economically efficient and environmentally sustainable production schedules, effectively addressing both operational and ecological concerns. Our approach aligns with industrial best practices for two key reasons. First, organizational efficiency is achieved by minimizing makespan, which is crucial for effective planning, particularly in scheduling worker shifts. Second, economic competitiveness is maintained by reducing energy costs, essential for financial viability in today's competitive market. By prioritizing makespan and energy costs, we effectively address both operational and financial aspects of production, ensuring a balanced and practical approach to scheduling. We further emphasize that minimizing makespan often correlates strongly with reducing carbon emissions and energy consumption, making their separate inclusion in the objective system largely redundant. By focusing on makespan, we inherently drive improvements in these sustainability metrics. Long-term sustainability goals, such as reducing energy consumption and carbon emissions, are primarily addressed through energy market design and dynamic energy tariffs. Government regulations, including the introduction of PPAs, play a crucial role by incentivizing production shifts to periods of higher renewable energy availability through adjusted energy prices. This approach effectively aligns production scheduling with broader sustainability objectives.

3.1.2. Data and constraints

$eJSP$ is a time-indexed model that divides working days into discrete time slots, e.g., a 24-h working day with 15-min time slots results in 96 time slots. We refer to an operation as the processing of a specific job on a specific machine; operations start at slot beginnings, with durations as integer multiples of slot length., e.g., multiples of 15 min. As we are examining an energy-aware job shop, we extend the traditional job shop model by energy-related factors. Machines consume energy and produce carbon emissions; energy consumption and carbon emissions vary based on machine states. Machine states include off, idle, and on; the idle state is defined as the period between two on-states. Energy consumption and carbon emissions occur only in the on- and idle-states, with values dependent on the specific state. Consequently, the times before the first on state and after the last on state are considered the first and second off states, respectively.

Machines receive the energy required for their operations from the energy market on which we consider several energy sources in the form of conventionals (e.g., gas and coal) and renewables (e.g., wind and solar). Options for the energy pricing structure is twofold: Dynamic pricing is derived from energy market auctions; these prices are provided to the decision-maker in advance for the upcoming working day. This reflects day-ahead spot markets with hourly price adjustments based on renewable energy market penetration. Fixed pricing is established through two main mechanisms: For conventional energy sources, fixed tariffs are typically set on a monthly basis. For renewable energy sources, long-term PPAs guarantee renewable energy delivery at a negotiated cost rate. Dynamic pricing enables decision makers to capitalize on price fluctuations throughout the day, while fixed pricing offers stability and predictability in energy costs. Thus, model $eJSP$ employs a fine-grained temporal resolution to capture energy price volatility. This approach ensures a high level of detail across all related aspects, including machine statuses, operations timings, and energy sourcing.

3.1.3. Decisions

By integrating energy-related considerations into the job shop environment, decision-makers are tasked with both scheduling jobs on machines and selecting appropriate energy sources. Therefore, within the constraints of the energy-aware job shop setting, the optimization model must primarily address the following two critical decisions for each operation: First, the temporal allocation, defined as the time interval during which the operation is to be executed, and second, the energy source allocation, specifying the selected power supply (conventional vs. renewable) for the operation's execution. These two critical choices are represented by independent decision variables, directly controlled by the decision-maker and fundamental to the optimization process. All remaining decisions and objective value outcomes result from and are influenced by these independent decision variables. Consequently, the dependent decision variables – machine states, effective makespan, energy costs, carbon emissions, and energy consumption – derive their values from the independent ones.

3.2. Mathematical model for energy-aware job shop scheduling

We formulate $eJSP$ for a set of jobs J subject to an energy market E on the respective working day, with the goal of minimizing an objective function $\pi \in \{makespan, costs\}$. Here, $makespan$ denotes the effective makespan and $costs$ denotes the total energy costs. We denote the resulting MINLP model by $eJSP(J, E, \pi)$. Table 3 summarizes the notation used in this model (see the Eqs. (1)–(33) in Box I) and its subsequent derivatives. The MINLP formulation presents three main challenges, addressed sequentially in the following subsections to ensure computational tractability:

Nonlinearity Arising from the interaction of energy price tariffs and machine utilization. In Section 3.3, we derive a linearized version of model $eJSP(J, E, \pi)$.

Complexity status Resulting from the energy-aware extension of the job shop setting and the NP-hardness of job shop scheduling. In Section 3.4, we introduce job sequence fixation for practical computational tractability. In Section 3.6, we provide an approximate solution approach for large job sets.

Multiple objectives Stemming from the need to balance economic efficiency and environmental sustainability. In Sections 3.5 and 3.6, we contribute an algorithm for determining Pareto schedules, taking into account both makespan and energy costs.

Objective (1) specifies the function to be minimized, which can be either the effective makespan or the total energy costs. Constraints (2) define the on-state indicator for each machine in each period by relating it to all operations performed on the respective machine. Constraints (3) state that each machine is in exactly one state during each time slot. Constraints (4) ensure that no infeasible change in machine state occurs between successive time slots for all machines. Constraints (5) and (6) specify that no machine begins in an idle-state

Table 3
Notation for $eJSP(J,E,\pi)$ and derivatives.

Sets	
M	Set of machines
J	Set of jobs
M_j	Set of machines which must be visited by job $j \in J$
J_i	Set of jobs which must visit machine $i \in M$
O	Set of operations with $O := \{(i,j) \mid i \in M, j \in J_i\}$
T	Set of time slots on a single working day
R	Set of energy sources with $R := \{renewable, conventional\}$
S	Set of machine states with $S := \{on, off_1, off_2, idle\}$
$\bar{S}^{seq}$	Set of successive infeasible machine state pairs with $\bar{S}^{seq} := \{(on, off_1), (idle, off_1), (off_2, off_1), (off_2, on), (off_2, idle), (idle, off_2), (off_1, idle)\}$
Parameters	
p_{ij}	Processing time of operation $(i,j) \in O$
ms_j	Machine sequence of job $j \in J$ with $ms_j = [i^j_{[1]}, i^j_{[2]}, \dots, i^j_{[\lvert M_j \rvert]}]$
js_i	Job sequence on machine $i \in M$ with $js_i = [j^i_{[1]}, j^i_{[2]}, \dots, j^i_{[\lvert J_i \rvert]}]$
$\overline{ec}^s_i$	Energy consumption of machine i in state $s \in S$ per unit time
$\overline{em}^s_i$	Carbon emissions of machine i in state $s \in S$ per unit time
$\overline{costs}_{rt}$	Energy costs per energy unit for energy source $r \in R$ at time slot $t \in T$
E	Energy market comprising energy costs with $E = (costs_{rt})_{r\in R, t\in T}$
Decision variables	
x_{ijt}	Indicator for operation $(i,j) \in O$ being started at time slot $t \in T$
z_{ijt}	Indicator for operation $(i,j) \in O$ being processed at time slot $t \in T$
z^s_{it}	Indicator for machine $i \in M$ being in state $s \in S$ at time slot $t \in T$
α_{rt}	Indicator for use of energy source $r \in R$ at time slot $t \in T$
$C_{\max}$	Maximum completion time
t_{start}	Starting time of first operation
$makespan$	Effective makespan
ec	Total energy consumption over all machines and all time slots
ec_{it}	Energy consumption of machine $i \in M$ at time slot $t \in T$
ec^s_{irt}	Energy consumption of machine $i \in M$ with respect to Energy source $r \in R$ at time slot $t \in T$ when in state $s \in S$
em	Total carbon emissions over all machines and all time slots
em_{it}	Carbon emissions of machine $i \in M$ at time slot $t \in T$
em^s_{it}	Carbon emissions of machine $i \in M$ at time slot $t \in T$ when in state $s \in S$
$costs$	Total energy costs over all machines and all time slots
$costs^s_{it}$	Energy costs of machine $i \in M$ at time slot $t \in T$ when in state $s \in S$

(but hereby allowing them to start in the first off-state) and that every machine ends in the second off-state, respectively. Constraints (7) set the processing indicator of an operation to one for all time slots from the start of processing through the duration of the operation. More specifically, $x_{ijt} = 1$ enforces $z_{ijt'} = 1$ for $t' \in \{t, \dots, t+p_{ij}-1\}$. Constraints (8) set the processing indicator of an operation to zero at a specific time if the operation has not started within the time frame that allows for processing at that time. More specifically, $x_{ijt'} = 0$ for $t' \in \{t-p_{ij}+1, \dots, t\}$ enforces $z_{ijt} = 0$. Constraints (9) ensure that the start of each operation is assigned to exactly one time slot. Constraint (10), in conjunction with objective $\pi = makespan$, defines the production makespan. We note that when $\pi = costs$, it is possible to achieve an effective makespan minimization under minimal costs by minimizing $\pi' = costs + \delta \cdot makespan$ with sufficiently small $\delta > 0$. Constraints (11) ensure that once an operation begins on a specific machine, that machine cannot transition to another job while the operation is still running. More specifically, when considering machine i at time t, then at most one job j can be running by having been started at some time $t' \in \{t-p_{ij}+1, \dots, t\}$. Constraints (12) establish the precedence for each job. More specifically, this is achieved by utilizing the third index t – the timing index – of the x-variables in the factors of all summands. This trick ensures that the right-hand side coincides with the starting time of job j on machine $i^j_{[h]}$, whereas the left-hand side ensures that j must be finished on the previous machine $i^j_{[h-1]}$ by that time. Constraint (13), in conjunction with $\pi = makespan$ or $\pi' = costs + \delta \cdot makespan$, defines the earliest starting time for any operation. Constraint (14) defines the effective makespan. Constraints (15) represent the single energy sourcing constraints in each time slot. Constraint (16) yields the total energy consumption. Constraints (17) provide the total energy consumption of a specific machine during a specific time slot. Constraints (18) and (19) compute the energy consumption of a specific machine in both the on- and idle-state during a time slot, based on a given energy source, respectively. More specifically, consumption for energy source r on machine i at time t is only charged when that energy source r is selected for time t ($\alpha_{rt} = 1$) and machine i is in the on- and idle-state at time t ($z^{on}_{it} = 1$, $z^{idle}_{it} = 1$), respectively. The dependency on the z- and α-variables renders the model a quadratic MINLP. Constraint (20) yields the total carbon emissions. Constraints (21) to (23) provide the carbon emissions incurred by a specific machine in the on- and idle-state during a specific time slot, respectively. Constraint (24) yields the total energy costs. Constraints (25) and (26) provide the energy costs incurred by a specific machine in the on- and idle-state during a specific time slot, respectively. Constraints (27) to (33) are the variable domain constraints.

3.3. Linearization

We perform a linearization on the MINLP model $eJSP(J,E,\pi)$ to transform it into an MIP formulation. This transformation enables the application of efficient linear programming-based solvers, specifically designed for MIPs, thereby enhancing computational tractability and solution efficiency. To achieve this, we need to replace constraints (18) and (19), as they are the only nonlinear elements in the model.

We replace constraints (18) with constraints (34) to (36). Constraints (34) and (35) enforce that $ec^{on}_{irt} = 0$ if $z^{on}_{it} = 0$ or $\alpha_{rt} = 0$, while constraints (36) ensure that $ec^{on}_{irt} = \overline{ec}^{on}_i$ if both $z^{on}_{it} = 1$ and $\alpha_{rt} = 1$:

$$ec^{on}_{irt} \le \overline{ec}^{on}_i z^{on}_{it} \qquad i \in M, r \in R, t \in T \tag{34}$$

$$ec^{on}_{irt} \le \overline{ec}^{on}_i \alpha_{rt} \qquad i \in M, r \in R, t \in T \tag{35}$$

$$ec^{on}_{irt} \ge \overline{ec}^{on}_i (z^{on}_{it} + \alpha_{rt} - 1) \qquad i \in M, r \in R, t \in T \tag{36}$$

$eJSP(J,E,\pi)$:

$$\begin{aligned}
&\min && \pi && && (1)\\
&\text{s.t.} && z_{it}^{on} = \sum_{(i,j)\in O} z_{ijt} \leq 1 && i\in M, t\in T && (2)\\
& && \sum_{s\in S} z_{it}^{s} = 1 && i\in M, t\in T && (3)\\
& && z_{it}^{s_1} + z_{i,t+1}^{s_2} \leq 1 && i\in M, t\in T, (s_1,s_2)\in \bar{S}^{seq} && (4)\\
& && z_{i0}^{idle} = 0 && i\in M, t\in T && (5)\\
& && z_{iT}^{off2} = 1 && i\in M, t\in T && (6)\\
& && \sum_{t'=t}^{t+p_{ij}-1} z_{ijt'} \geq p_{ij}x_{ijt} && (i,j)\in O, t\in T && (7)\\
& && z_{ijt} \leq \sum_{t'\in\{t-p_{ij}+1,\dots,t\}} x_{ijt'} && (i,j)\in O, t\in T && (8)\\
& && \sum_{t\in T} x_{ijt} = 1 && (i,j)\in O && (9)\\
& && \sum_{t\in T}(t+p_{ij})x_{ijt} \leq C_{\max} && (i,j)\in O && (10)\\
& && \sum_{j\in J}\sum_{t'\in\{t-p_{ij}+1,\dots,t\}} x_{ijt'} \leq 1 && i\in M, t\in T && (11)\\
& && \sum_{t\in T}(t+p_{i^j_{[h-1]}j})x_{i^j_{[h-1]}jt} \leq \sum_{t\in T} t x_{i^j_{[h]}jt} && j\in J, h\in\{2,\dots,|M_j|\} && (12)\\
& && t_{start} \leq \sum_{t\in T} t x_{i^j_{[1]}jt} && j\in J && (13)\\
& && makespan = C_{\max} - t_{start} && j\in J && (14)\\
& && \sum_{r\in R}\alpha_{rt} = 1 && t\in T && (15)\\
& && ec = \sum_{i\in M}\sum_{t\in T} ec_{it} && && (16)\\
& && ec_{it} = \sum_{r\in R}(ec_{irt}^{on} + ec_{irt}^{idle}) && i\in M, t\in T && (17)\\
& && ec_{irt}^{on} = \overline{ec}_i^{on} z_{it}^{on}\alpha_{rt} && i\in M, r\in R, t\in T && (18)\\
& && ec_{irt}^{idle} = \overline{ec}_i^{idle} z_{it}^{idle}\alpha_{rt} && i\in M, r\in R, t\in T && (19)\\
& && em = \sum_{i\in M}\sum_{t\in T} em_{it} && && (20)\\
& && em_{it} = em_{it}^{on} + em_{it}^{idle} && i\in M, t\in T && (21)\\
& && em_{it}^{on} = \overline{em}_i^{on} z_{it}^{on} && i\in M, t\in T && (22)\\
& && em_{it}^{idle} = \overline{em}_i^{idle} z_{it}^{idle} && i\in M, t\in T && (23)\\
& && costs = \sum_{i\in M}\sum_{t\in T}(costs_{it}^{on} + costs_{it}^{idle}) && && (24)\\
& && costs_{it}^{on} = \sum_{r\in R}\overline{costs}_{rt} ec_{irt}^{on} && i\in M, t\in T && (25)\\
& && costs_{it}^{idle} = \sum_{r\in R}\overline{costs}_{rt} ec_{irt}^{idle} && i\in M, t\in T && (26)\\
& && x_{ijt}, z_{ijt} \in \{0,1\} && (i,j)\in O, t\in T && (27)\\
& && z_{it}^{s} \in \{0,1\} && i\in M, t\in T, s\in S && (28)\\
& && \alpha_{rt} \in \{0,1\} && r\in R, t\in T && (29)\\
& && C_{\max}, t_{start}, makespan \geq 0 && && (30)\\
& && ec, ec_{it}, ec_{irt}^{on}, ec_{irt}^{idle} \geq 0 && i\in M, r\in R, t\in T && (31)\\
& && em, em_{it}, em_{it}^{on}, em_{it}^{idle} \geq 0 && i\in M, r\in R, t\in T && (32)\\
& && costs_{it}^{on}, costs_{it}^{idle} \in \mathbb{R} && i\in M, t\in T && (33)
\end{aligned}$$

Box I. Mathematical programming formulation for model $eJSP(J,E,\pi)$.

We replace constraints (19) with constraints (37) to (39). Constraints (37) and (38) enforce that $ec_{irt}^{idle} = 0$ if $z_{it}^{idle} = 0$ or $\alpha_{rt} = 0$, while constraints (39) ensure that $ec_{irt}^{idle} = \overline{ec}_i^{idle}$ if both $z_{it}^{idle} = 1$ and $\alpha_{rt} = 1$:

$$ec_{irt}^{idle} \leq \overline{ec}_i^{idle} z_{it}^{idle} \qquad i \in M, r \in R, t \in T \tag{37}$$

$$ec_{irt}^{idle} \leq \overline{ec}_i^{idle} \alpha_{rt} \qquad i \in M, r \in R, t \in T \tag{38}$$

$$ec_{irt}^{idle} \geq \overline{ec}_i^{idle} (z_{it}^{idle} + \alpha_{rt} - 1) \qquad i \in M, r \in R, t \in T \tag{39}$$

We denote the resulting MIP model by $eJSP^{lin}(J, E, \pi)$.

$eJSP^{lin}(J, E, \pi)$: min (1) s.t. (2)–(17), (20)–(33), (34)–(39)

3.4. Job sequence fixation

The basic job shop scheduling problem is strongly NP-hard [14]. As an extension that includes machine states and energy-related factors, $eJSP$ inherits this complexity status, potentially exacerbating computational challenges in practice. This is evidenced by experiments on computational times discussed in Section 4.1. Thus, to enhance computational tractability for larger instances, we also implement a complexity reduction strategy that enables the approximate determination of Pareto schedule frontiers (cf. Algorithm 2 in Section 3.6). This approach involves the imposition of a predetermined job sequence for each machine. This simplification is particularly advantageous when the model's application is the approximation of the Pareto frontier, rather than its exact determination, which would incur prohibitive computational costs. For this purpose, we need to extend the formulation of $eJSP^{lin}(J, E, \pi)$ to ensure that the job sequences $js_i = [j^i_{[1]}, j^i_{[2]}, \dots, j^i_{[|J_i|]}]$ are fixated for each machine $i \in M$ with js_i. Precedence for each machine is established as follows:

$$\sum_{t \in T} t x_{ij^i_{[k-1]}t} \leq \sum_{t \in T} t x_{ij^i_{[k]}t} \qquad i \in M, k \in \{2, \dots, |J_i|\} \tag{40}$$

More specifically, this is achieved by utilizing the third index t – the timing index – of the x-variables in the factors of all summands. This trick ensures that the left- and right-hand sides coincide with the starting time of jobs $j^i_{[k-1]}$ and $j^i_{[k]}$, respectively. Thus, the starting time of job $j^i_{[k]}$ cannot be before that of job $j^i_{[k-1]}$. Recall that constraints (11) ensure that at most one job can be running on a machine i at every time t.

We denote the extension of model $eJSP^{lin}(J, E, \pi)$ by constraints (40) as $eJSP^{lin}_{seq}(J, E, \pi, js)$.

$eJSP^{lin}_{seq}(J, E, \pi, js)$: min (1) s.t. (2)–(17), (20)–(33), (34)–(40)

In Section 3.6, we explain how a viable job sequence js is obtained from the most work remaining heuristic. In this case, the determination of the approximate Pareto frontier is classified as a matheuristic as it combines a problem-specific heuristic procedure with an MIP formulation.

3.5. Makespan enforcement

For the determination of the Pareto frontier using the ϵ-constraint method as presented in Section 3.6, we need a model that minimizes energy costs while ensuring that the effective makespan does not exceed a specified upper bound M^u. To this end, the following upper bound constraint is introduced:

$$C_{\max} - t_{start} \leq M^u \tag{41}$$

We denote the extensions of models $eJSP^{lin}(J, E, \pi)$ and $eJSP^{lin}_{seq}(J, E, \pi, js)$ by constraint (41) as $eJSP^{lin}_{\leq M^u}(J, E, \pi)$ and $eJSP^{lin}_{seq,\leq M^u}(J, E, \pi, js)$, respectively.

$eJSP^{lin}_{\leq M^u}(J, E, \pi)$: min (1) s.t. (2)–(17), (20)–(33), (34)–(39), (41)

$eJSP^{lin}_{seq,\leq M^u}(J, E, \pi, js)$: min (1) s.t. (2)–(17), (20)–(33), (34)–(40), (41)

We need to ensure that the value of M^u is set high enough to permit a feasible solution, as addressed in the algorithmic framework presented in Section 3.6.

3.6. Algorithm for pareto frontier determination

Building on the MIP models introduced in the previous sections, we consolidate our findings in a model-based algorithmic outline for determining the Pareto frontier with respect to minimizing makespan and energy costs using the ϵ-constraint method. To account for the NP-hardness of energy-aware job shops, our procedure allows for the determination of either an approximate or exact version of the Pareto frontier, depending on the user's preferences and available computing time budget. The choice between exact and approximate approaches depends primarily on the size of the job set under consideration. Our computational experiments in Section 4 explore the limits of the exact approach and offer guidance on when to favor the approximate method. Methodologically, both approaches begin by establishing the feasible range $\{M^u_{\min}, \dots, M^u_{\max}\}$ of effective makespans, bounded by the minimum makespan (which prioritizes completion time) and the maximum makespan (which prioritizes energy cost). In the former case, we utilize objective $\pi = makespan$, whereas in the latter case, we utilize objective $\pi = costs + \delta \cdot makespan$ with sufficiently small $\delta > 0$, effectively eliminating any avoidable idle time and thereby minimizing energy costs without extending the makespan unnecessarily. The exact and approximate determinations of the Pareto frontier differ in their outlines as follows:

Exact determination of pareto schedules.

1. Establish $M^u_{\min}$ by solving model $eJSP^{lin}(J, E, \pi)$ for $\pi = makespan$.
2. Establish $M^u_{\max}$ by solving model $eJSP^{lin}(J, E, \pi)$ for $\pi = costs + \delta \cdot makespan$.
3. Determine the optimal job sequences for each machine and the corresponding optimal operation timings by solving model $eJSP^{lin}_{\leq M^u}(J, E, \pi)$ for $M^u \in \{M^u_{\min}, \dots, M^u_{\max}\}$ and $\pi = costs + \delta \cdot makespan$.

Approximate determination of pareto schedules.

1. Apply the most work remaining heuristic (displayed in Algorithm 1) to obtain job sequences js_i on each machine $i \in M$.
2. Establish $M^u_{\min}$ by solving model $eJSP^{lin}_{seq}(J, E, \pi, js)$ for $\pi = makespan$.
3. Establish $M^u_{\max}$ by solving model $eJSP^{lin}_{seq}(J, E, \pi, js)$ for $\pi = costs + \delta \cdot makespan$.
4. Determine the optimal operation timings based on the fixed job sequences js_i for $i \in M$ by solving model $eJSP^{lin}_{seq,\leq M^u}(J, E, \pi, js)$ for $M \in \{M^u_{\min}, \dots, M^u_{\max}\}$ and $\pi = costs + \delta \cdot makespan$.

In summary, Algorithm 2 illustrates the resulting overall procedure comprising both the exact and approximate approach.

The proposed methodology offers considerable flexibility in accounting for tariff models, including dynamic energy pricing, specifying job set characteristics, and comparing algorithmic approaches. This versatility enables extensive numerical experimentation for practical applicability, as will be demonstrated in the following section.

Algorithm 1 `mostWorkRemaining`

Require: job set J

$O := operations(J)$, $M := machines(J)$
$O^{open} := O$, $O^{scheduled} := \emptyset$, $P := precedences(O)$
$js_i := [\,]$ for $i \in M$
repeat
  for all $i \in M$ **do**
    compute $WR(i) := \sum_{o=(i_o,j_o)\in O^{open} \mid i_o=i} p_{i_o j_o}$
  end for
  $i^{MWR} := \text{argmax}_{i\in M}\{WR(i)\}$
  determine $o^{next} = (i^{MWR}, j) \in O^{open}$ as an operation with $(o, o^{next}) \in P$ and $o \in O^{scheduled}$
  append j to $js_{i^{MWR}}$
  $O^{scheduled} := O^{scheduled} \cup \{o^{next}\}$, $O^{open} := O^{open} \setminus \{o^{next}\}$
until $O^{open} = \emptyset$

Ensure: job sequences js_i for $i \in M$

Algorithm 2 `determineParetoFrontier`

Require: job set J, energy market E, $method \in \{exact, approximate\}$, $\delta > 0$

$F := \emptyset$
if $method = exact$ **then**
  $(makespan_{\min}, costs_{\max}) := eJSP^{lin}(J, E, makespan)$
  $(makespan_{\max}, costs_{\min}) := eJSP^{lin}(J, E, costs + \delta \cdot makespan)$
else
  $js :=$ `mostWorkRemaining`(J)
  $(makespan_{\min}, costs_{\max}) := eJSP^{lin}_{seq}(J, E, js, makespan)$
  $(makespan_{\max}, costs_{\min}) := eJSP^{lin}_{seq}(J, E, js, costs + \delta \cdot makespan)$
end if
for $M^u := makespan_{\min}$ **to** $makespan_{\max}$ **do**
  if $method = exact$ **then**
    $(makespan_{M^u}, costs_{M^u}) := eJSP^{lin}_{\leq M^u}(J, E, costs + \delta \cdot makespan)$
  else
    $(makespan_{M^u}, costs_{M^u}) := eJSP^{lin}_{seq,\leq M^u}(J, E, js, costs + \delta \cdot makespan)$
  end if
  $F := F \cup \{(makespan_{M^u}, costs_{M^u})\}$
end for

Ensure: Pareto frontier F

4. Computational experimentation

We perform comprehensive numerical experiments employing Algorithm 2 and associated $eJSP$ models to evaluate the sensitivity of key performance indicators – energy costs, makespan, energy consumption, and carbon emissions – to variations in production system parameters and energy market conditions. In this case study, we utilize hourly energy price data, sourced from the day-ahead auction data provided by the ENTSO-E (European Network of Transmission System Operators for Electricity) initiative, as published on their transparency platform website [18]. To capture various seasonal patterns and weekday variations, as well as the diverse energy price developments throughout the year, we select the 15th day of each month in 2023 as representative. Table 4 provides a summary of the key figures from the energy price data for the selected calendar days.

We analyze each day as a working day in a production company, divided into 15-min intervals, resulting in 96 time slots per day. Jobs are processed on three machines. Regarding the number of jobs in a day, we examine three different scenarios, namely three, five, and ten jobs. To ensure that all operations can be completed within the available time slots, we limit the processing time of each operation to a maximum value $p_{max} = 20$ for three jobs, $p_{max} = 15$ for five jobs, and $p_{max} = 10$ for ten jobs. Operation processing times are randomly selected from $\{2, \ldots, p_{max}\}$. The number of machines a job must visit is randomly chosen from $\{2, 3\}$, and the sequence of these machines is randomly determined from all possible permutations of the machines to be visited by the respective job. To evaluate the cost-effectiveness of early morning production, we examine both the traditional start time of 08:00 (starting slot 32) and an alternative start time of 00:00 (starting slot 0). Energy consumption and emissions of each operation are randomly drawn, without loss of generality, from $\{1, 2, \ldots, 5\}$. Additionally, it is assumed that when a machine is in idle mode, its consumption and emissions are 25% of those in the active state. The long-term PPA tariff for renewable energy is set at the average energy price for the entire year 2023, which is 95.18€ per MWh. The fixed tariff for conventional energy is determined based on the average price for the considered day, simulating an ideal forecasting mechanism.

To guide our numerical analysis, we vary several factors pertaining to both the energy-aware job shop instance and the solution approach. These include the number of jobs, the type of algorithm used, the permissible starting slot for operations on a day, and the tariff types for both conventional and renewable energy costs. We refer to the combination of an instance and solution approach in this context as an energy-aware scheduling configuration (EASC). Table 5 summarizes the various specifications of the EASC elements examined subsequently. This leads to a positional scheme $jobs - alg - start - conv - renew$ with number $jobs$ of jobs, algorithm type alg, starting slot $start$, conventional energy tariff $conv$, and renewable energy tariff $renew$. For example, $5 - ex - 0 - dyn - ppa$ specifies an EASC with five jobs, to be solved by the exact approach, starting times from 00:00, conventional energy priced dynamically, and renewable energy charged at the fixed PPA tariff.

Given the excessive computational times for ten jobs, the exact approach is limited to scenarios with three and five jobs. Consequently, we examine a total of $2 \cdot 2^4 + 1 \cdot 2^3 = 32 + 8 = 40$ different EASCs. We apply Algorithm 2 to each possible combination of day and EASC, which we refer to as a day-specific EASC (dEASC) from now on. This results in $12 \cdot 40 = 480$ dEASCs. Examining each such dEASC requires executing the ϵ-constraint method, which involves multiple solves of $eJSP$ using either the approximate or the exact version, i.e., $eJSP^{lin}_{seq}(J, E, \pi, js)$ and $eJSP^{lin}(J, E, \pi)$, respectively. Computational experiments are performed on a personal desktop computer with Intel Core 3.2 GHz processor and 16 GB RAM under Microsoft Windows 10 (64-bit). Algorithms are coded in Python 3.8; MIP models are coded in Python using the docplex modeling library and solved using the IBM ILOG CPLEX 20.1.0 solver with default tolerances 10^{-4} for the relative MIP gap.

We explore several research questions to derive managerial implications for effectively managing energy tariff options and addressing scheduling constraints. Specifically, we investigate

- the computational times for determining the Pareto frontier,
- the number of distinct Pareto schedules for a specific EASC, and
- the impact of algorithm type, starting slot, and tariff types for both conventional and renewable energy on computational times and solution quality.

4.1. Computational times

Table 6 presents the CPU time needed to determine a single Pareto schedule. The upper table part indicates that the approximate approach (solving $eJSP^{lin}_{seq}(J, E, \pi, js)$) yields acceptable CPU times, typically within few minutes, even for ten jobs. The exact approach (solving $eJSP^{lin}(J, E, \pi)$) demonstrates acceptable CPU times for up to five jobs. However, determining only the first Pareto schedule of the first dEASC with ten jobs using the exact approach takes nearly 16 h (57 292 s). Thus, the exact approach cannot be employed for ten jobs due to excessive CPU times exceeding several hours per instance. The lower table part demonstrates that the number of time slots available for scheduling jobs affects computational effort. Specifically, for three and

Table 4
Price statistics [€ per MWh] by day (small avg and large std dev values printed in bold).

Day	Day#	Min	Avg	Median	Max	Std dev	Min price hour
Overall		−500.0	95.18	98.02	524.27	47.58	Jul 02, 13:00
Jan 15, 2023	01	0.29	**9.26**	4.06	37.47	10.67	14:00
Feb 15, 2023	02	119.1	143.52	134.39	188.76	20.52	23:00
Mar 15, 2023	03	90.84	127.72	119.21	199.28	29.79	13:00
Apr 15, 2023	04	82.17	106.28	105.35	134.51	14.6	13:00
May 15, 2023	05	78.96	112.17	107.82	168.79	21.65	23:00
Jun 15, 2023	06	90	121.10	109.43	179.28	28.42	12:00
Jul 15, 2023	07	−1.03	**32.68**	19.92	88.36	**31.65**	13:00
Aug 15, 2023	08	23.54	93.86	93.70	142.61	**30.78**	13:00
Sep 15, 2023	09	62.89	107.99	98.42	189.87	28.63	12:00
Oct 15, 2023	10	−1.76	**43.32**	8.35	139.9	**52.37**	13:00
Nov 15, 2023	11	70.69	102.41	102.08	129.9	19.24	03:00
Dec 15, 2023	12	75.16	93.24	91.52	121.5	12.39	04:00

Table 5
Elements of examined EASCs.

EASC element	Possible specifications
# jobs	3, 5, 10
Algorithm type	approximate (ap), exact (ex)
Starting slot	00:00 (slot 0), 08:00 (slot 32)
Tariff for conventional energy	fixed (fix), dynamic (dyn)
Tariff for renewable energy	PPA (ppa), dynamic (dyn)

five jobs, using 96 time slots (starting slot 0) results in CPU times approximately four times higher than using 64 time slots (starting slot 32). This difference arises from the reduced number of combinatorial options when starting at 08:00. In summary, as the number of jobs increases, CPU times for the exact approach become prohibitive. Similarly, the approximate approach also faces challenges with larger job counts. Thus, we recommend exploring a fully heuristic alternative to the matheuristic outline for the approximate approach in future research.

4.2. Number of pareto solutions

Table 7 provides insights into the number of Pareto solutions related to energy costs and makespan objectives. Overall, we observe that a significant portion of instances yields exactly one Pareto schedule, indicating that in these cases, the two objectives are not in competition but rather complement each other. Specifically, this means that minimizing makespan aligns with minimizing energy costs. The underlying reason is that fluctuations in energy prices throughout the day are insufficient to warrant time intervals without production. Comparing the results for starting slots 0 and 32, we conclude that extending the time horizon significantly increases the number of Pareto schedules. This increase is attributed to additional price fluctuations occurring during the first hours of the day. In summary, the analysis reveals a complementary relationship between minimizing makespan and energy costs, with an extended time horizon increasing the likelihood of identifying more Pareto schedules.

4.3. Influence of algorithm type

Table 8 illustrates the relative improvements achieved by the exact approach compared to the approximate approach in minimizing energy costs for each dEASC. For three jobs, improvements range from 0 to 59.65%, while for five jobs, they range from 0 to 46.09%, with average improvements of 6.17 and 4.98%, respectively. However, median improvements are only 0.66 and 1.12%, indicating that in many dEASCs, the advantage of exact algorithms over approximate algorithms is slight. A closer examination of Figs. 2 and 3 reveals that significant cost improvements are achieved on days 01, 07, and 10. Analyzing the price data in Table 4, we find that these days demonstrate considerable optimization potential due to average energy prices that deviate significantly from the agreed PPA tariff specification of 95.18€ per MWh, along with notable price volatility throughout the day. Consequently, there is significant potential to leverage optimization opportunities through the exact procedure, such as adjusting operational timings or switching between conventional and renewable energy sources. This is evidenced by substantial improvements across all EASCs, except for the EASCs that employ a fixed tariff for conventional energy and a PPA for renewable energy ($ex/ap - 0 - fix - ppa$, $ex/ap - 32 - fix - ppa$). In this case, both fixed tariffs lead to an inability to effectively utilize price fluctuations, thereby severely limiting optimization potential. Regarding the other objectives, we observe that minimizing energy costs positively impacts makespan minimization, consistent with the significant number of instances showing exactly one Pareto schedule (see Section 4.2). In contrast, the effects of energy cost minimization on carbon emissions and energy consumption appear arbitrary, as both improvements and deteriorations occur. This suggests that most energy costs improvements stem from adjusting job orders on machines and altering timings, rather than from creating idleness. In summary, significant optimization potential for energy costs and makespan reduction exists primarily on specific days with notable price volatility, while the impact on carbon emissions and energy consumption is mixed.

4.4. Influence of starting slot

Table 9 illustrates the relative improvements achieved by allowing a start time of 00:00 (slot 0) compared to the conventional working day start at 08:00 (slot 32). For three jobs, improvements range from 0 to 38.92%, for five jobs, they range from 0 to 66.83%, and for ten jobs, they range from 0 to 89.62%, with average improvements of 4.08, 10.18, and 14.39%, respectively. However, median improvements are only 0, 0, and 1.22%, indicating that in many dEASCs, the advantage of start time 00:00 over start time 08:00 is slight. A closer examination (cf. Figure A.1, Figure A.2, Figure A.3 in the supplementary material) reveals that significant cost improvements are achieved on days 07, 11, and 12. Analyzing the price data in Table 4, we observe that these days present significant optimization potential due to notable price volatility, characterized by a sharp price increase after 17:00 on day 07 and relatively low energy tariffs early in the day (03:00 on day 11 and 04:00 on day 12). For these three days, it is crucial to begin production early to maximize coverage of the periods with low energy tariffs. In contrast, on the other days, where energy costs are minimal around noon, the low tariff valley can be effectively covered whether starting at 00:00 or 08:00. Similarly as in the analysis from Section 4.3, these improvements occur across all EASCs, except for the EASCs that employ a fixed tariff for conventional energy and a PPA for renewable energy ($ex - 0/32 - fix - ppa$, $ap - 0/32 - fix - ppa$). In this case, both fixed tariffs lead to an inability to effectively utilize price fluctuations, thereby severely limiting optimization potential. Regarding the other objectives, we observe that minimum attainable makespans are

Table 6
Influence of number of jobs, algorithm type, and starting slot on CPU times.

CPU time	3 jobs			5 jobs			10 jobs		
[sec]	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max
Approximate	1.0	8.92	64.24	0.89	17.87	358.58	0.89	43.02	689.06
Exact	1.38	14.93	142.54	4.23	68.18	689.06	–	–	–
CPU time	3 jobs			5 jobs			10 jobs		
[sec]	Min	Avg	Max	Min	Avg	Max	Min	Avg	Max
Start 0	4.75	19.23	142.54	7.06	68.66	689.06	31.87	340.96	1044.91
Start 32	1.0	4.62	27.53	0.89	17.38	124.31	1.81	3.11	7.42

Table 7
Influence of number of starting slot on number of Pareto solutions.

#	3 jobs				5 jobs				10 jobs			
Pareto	Min	Avg	Max	=1 [%]	Min	Avg	Max	=1 [%]	Min	Avg	Max	=1 [%]
Overall	1	3.19	21	63.55	1	2.65	17	59.90	1	2.79	19	76.04
Start 0	1	3.7	21	60.42	1	3.5	17	56.25	1	4.1	19	68.75
Start 32	1	2.67	13	66.67	1	1.79	8	63.54	1	1.48	8	83.33

Table 8
Improvement through exact algorithm over approximate algorithm.

3 jobs							
Improvement [%]	Min	Avg	Max	$q_{0.25}$	$q_{0.5}$	$q_{0.75}$	$\in [-1, 1]\%$ [%]
Energy costs	0.0	6.17	59.65	0.0	0.66	4.73	55.21
Makespan	0.0	4.86	23.53	0.0	3.04	6.75	41.67
Emissions	−5.05	0.57	3.63	0.0	0.0	0.84	76.04
Consumption	−8.94	1.13	8.18	0.0	0.0	0.88	76.04
5 jobs							
Improvement [%]	Min	Avg	Max	$q_{0.25}$	$q_{0.5}$	$q_{0.75}$	$\in [-1, 1]\%$ [%]
Energy costs	0.0	4.98	46.09	0.06	1.12	3.63	47.92
Makespan	0.0	12.66	29.31	5.55	11.74	21.26	16.67
Emissions	−0.81	1.12	4.74	0.0	1.2	1.85	44.79
Consumption	−1.13	0.9	3.99	0.0	0.44	1.82	62.5

Table 9
Improvement through starting slot 0 over starting slot 32.

3 jobs							
Improvement [%]	Min	Avg	Max	$q_{0.25}$	$q_{0.5}$	$q_{0.75}$	$\in [-1, 1]\%$ [%]
Energy costs	0.0	4.08	38.92	0.0	0.0	4.66	65.62
Makespan	0.0	0.0	0.0	0.0	0.0	0.0	100.0
Emissions	−1.65	0.18	4.81	0.0	0.0	0.0	86.46
Consumption	−1.75	0.16	8.21	0.0	0.0	0.0	88.54
5 jobs							
Improvement [%]	Min	Avg	Max	$q_{0.25}$	$q_{0.5}$	$q_{0.75}$	$\in [-1, 1]\%$ [%]
Energy costs	0.0	10.18	66.83	0.0	0.0	7.7	65.62
Makespan	0.0	0.0	0.0	0.0	0.0	0.0	100.0
Emissions	−4.98	−0.31	0.46	0.0	0.0	0.0	92.71
Consumption	−5.1	−0.33	0.41	0.0	0.0	0.0	90.62
10 jobs							
Improvement [%]	Min	Avg	Max	$q_{0.25}$	$q_{0.5}$	$q_{0.75}$	$\in [-1, 1]\%$ [%]
Energy costs	0.0	14.39	89.62	0.0	1.22	10.04	50.0
Makespan	0.0	0.0	0.0	0.0	0.0	0.0	100.0
Emissions	−1.5	−0.07	2.66	0.0	0.0	0.0	89.58
Consumption	−0.93	0.0	3.12	0.0	0.0	0.0	95.83

unaffected by different starting times. In contrast, the effects of energy cost minimization on carbon emissions and energy consumption appear arbitrary, as both improvements and deteriorations occur. This suggests that most energy costs improvements stem from shifting large portions of entire schedules to earlier times of the day when processing can begin at 00:00, rather than from creating idleness. Another observed effect is that improvements increase with a larger number of jobs. This can be attributed to the longer makespan required to complete all job, making it advantageous to cover the energy tariff valleys between 00:00 and 08:00. In summary, significant cost reduction opportunities exist on specific days with notable price volatility, emphasizing the importance of early production to capitalize on low energy tariffs.

4.5. Influence of dynamic tariff for conventional energy costs

Table 10 illustrates the relative improvements and deteriorations achieved by utilizing the dynamic tariff for conventional energy instead of relying on a fixed tariff. Unlike the analyses on algorithm type and starting slot (see Sections 4.3 and 4.4), but similar to the analysis on PPA availability for renewables (see Section 4.6), it is evident that

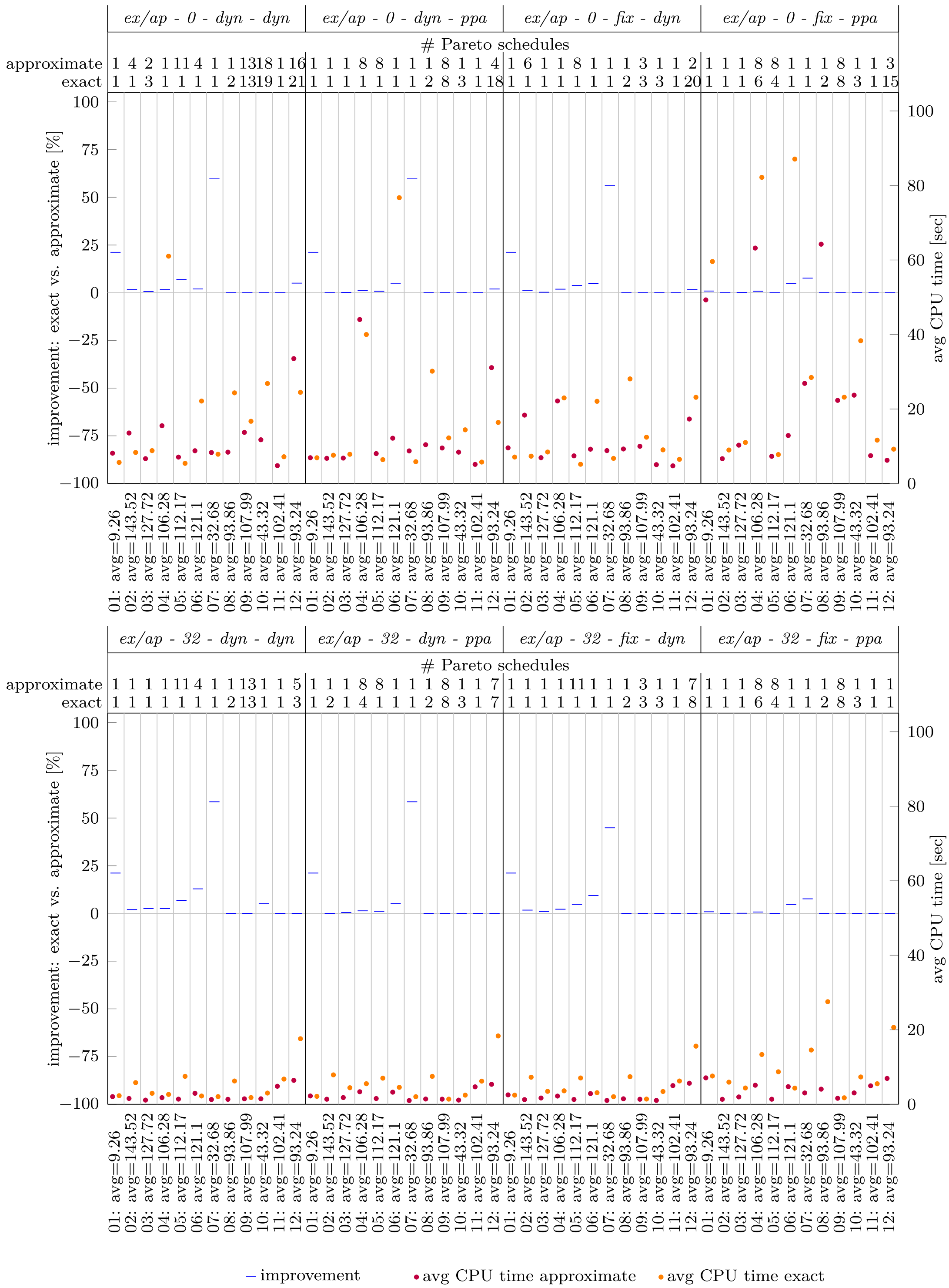

Fig. 2. Relative energy costs improvements, CPU times, and number of Pareto schedules depending on algorithm type for 3 jobs.

both significant improvements (as indicated in the max column) and substantial deteriorations (as indicated in the min column) are possible. For three jobs, improvements range from −186.14 to 87.6%, for five jobs, they range from −104.66 to 90.55%, and for ten jobs, they range from −153.46 to 88.55%, with average improvements of 3.04, 5.08, and −2.43%, respectively. However, median improvements are 0, 0, and 0%, respectively, showing that over all dEASCs, improvements and deteriorations essentially level out. A closer examination (cf. Figure A.7, Figure A.8, Figure A.9 in the supplementary material) reveals that whether improvements or deteriorations occur strongly depends on the EASC, in particular on the complementary tariff for renewable energy. If it is dynamic (as shown on the table bottom for EASC $ap{-}32{-}fix/dyn{-}dyn$), the same dynamic tariff for conventional energy provides no additional benefit and results in a relative disadvantage compared to a fixed tariff which would cap cost rates at the day's average price, i.e., significantly lower than in the dynamic tariff at certain hours. Therefore, when renewable energy tariffs are dynamic, opting for a fixed conventional tariff is advisable. Conversely, if the

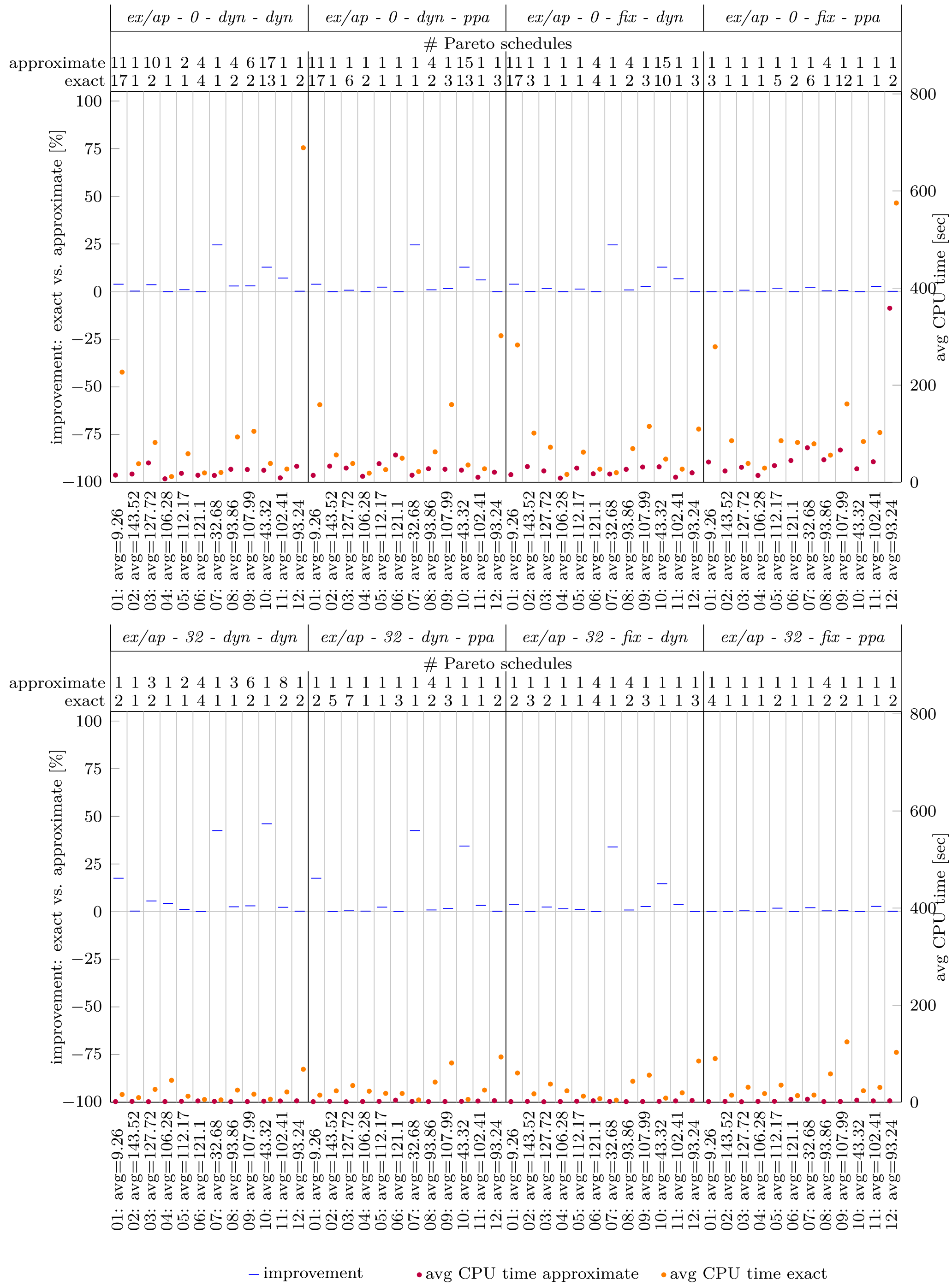

Fig. 3. Relative energy costs improvements, CPU times, and number of Pareto schedules depending on algorithm type for 5 jobs.

renewable tariff is a fixed PPA (as shown on the table bottom for EASC $ap - 32 - fix/dyn - ppa$), a dynamic tariff for conventional energy introduces new opportunities and offers a relative advantage over a fixed one. Although the PPA renewables tariff has a cap of 95.18€ per MWh, which can often be outperformed by the fixed conventional tariff, the ability to access significantly lower rates through the dynamic conventional tariff leads to substantial improvements compared to the case without any dynamic option. Analyzing the price data in Table 4, we observe that significant cost improvements or deteriorations – depending on the EASC and the complementary renewable energy tariff – are achieved on days 01, 07, and 10. These days exhibit considerable optimization or deterioration potential due to average energy prices deviating significantly from the agreed PPA specification of 95.18€ per MWh. Certainly, this makes it worthwhile (harmful) to having booked (not having booked) the dynamic conventional energy tariff. Regarding the other objectives, we observe that minimum attainable makespans are unaffected by different energy tariffs. The impact of minimizing energy costs on carbon emissions and energy consumption is minimal but slightly negative. Under a dynamic tariff, schedules may adversely affect emissions and consumption reduction, as the likelihood of using conventional energy increases compared to a fixed-rate scenario. In summary, the effectiveness of a dynamic tariff for conventional energy depends on the renewable energy tariff type of the EASC: It is beneficial when paired with a fixed PPA tariff for renewables, while a fixed tariff

Table 10
Improvement through dynamic tariff over fixed tariff.

3 jobs							
Improvement [%]	Min	Avg	Max	$q_{0.25}$	$q_{0.5}$	$q_{0.75}$	$\in [-1, 1]\%$ [%]
Energy costs	−186.14	3.04	87.6	−5.43	0.0	5.58	13.54
Makespan	0.0	0.0	0.0	0.0	0.0	0.0	100.0
Emissions	−5.05	−0.21	0.0	0.0	0.0	0.0	92.71
Consumption	−8.94	−0.23	0.0	0.0	0.0	0.0	94.79
5 jobs							
Improvement [%]	Min	Avg	Max	$q_{0.25}$	$q_{0.5}$	$q_{0.75}$	$\in [-1, 1]\%$ [%]
Energy costs	−104.66	5.08	90.55	−4.42	0.0	6.12	22.92
Makespan	0.0	0.0	0.0	0.0	0.0	0.0	100.0
Emissions	−5.43	−0.26	0.44	0.0	0.0	0.0	91.67
Consumption	−5.1	−0.22	0.16	0.0	0.0	0.0	94.79
10 jobs							
Improvement [%]	Min	Avg	Max	$q_{0.25}$	$q_{0.5}$	$q_{0.75}$	$\in [-1, 1]\%$ [%]
Energy costs	−153.46	−2.43	88.55	−5.12	0.0	3.96	18.75
Makespan	0.0	0.0	0.0	0.0	0.0	0.0	100.0
Emissions	−2.74	−0.24	0.31	−0.17	0.0	0.0	89.58
Consumption	−3.22	−0.22	0.63	−0.19	0.0	0.0	95.83

3 jobs: ap - 32 - fix/dyn - dyn			
Improvement [%]	Min	Avg	Max
Energy costs	−186.14	−28.41	0.0
Makespan	0.0	0.0	0.0
Emissions	−0.37	−0.08	0.0
Consumption	−0.35	−0.05	0.0

3 jobs: ap - 32 - fix/dyn - ppa			
Improvement [%]	Min	Avg	Max
Energy costs	0.0	23.63	85.16
Makespan	0.0	0.0	0.0
Emissions	−1.42	−0.18	0.0
Consumption	−1.83	−0.19	0.0

for conventional energy is preferable when a dynamic renewables tariff is in place.

4.6. Influence of PPA for renewable energy costs

Table 11 illustrates the relative improvements and deteriorations achieved by utilizing the fixed PPA tariff for renewable energy instead of relying on a dynamic tariff. Unlike the analyses on algorithm type and starting slot (see Sections 4.3 and 4.4), but similar to the analysis on the dynamic tariff for conventional energy (see Section 4.5), it is evident that both significant improvements (as indicated in the max column) and substantial deteriorations (as indicated in the min column) are possible. For three jobs, improvements range from −1346.11 to 30.72%, for five jobs, they range from −958.25 to 32.2%, and for ten jobs, they range from −773.6 to 32.0%, with average improvements of −99.1, −61.89, and −25.8%, respectively. However, median improvements are 3.5, 2.36, and 5.12%, respectively, showing that over all dEASCs, both improvements and deteriorations are possible. A closer examination (cf. Figure A.1, Figure A.2, Figure A.3 in the supplementary material) reveals that whether improvements or deteriorations occur strongly depends on the EASC, in particular on the complementary tariff for conventional energy. If it is dynamic (as shown on the table bottom for EASC *ap - 32 - dyn - ppa/dyn*), a fixed PPA tariff for renewables introduces new opportunities and offers a relative advantage over the same dynamic tariff for renewables. We also note that the improvements achieved are not limited to specific days but accumulate throughout the entire year. This is primarily due to the significant opportunity presented by the fixed PPA tariff, which is utilized whenever the dynamic tariff exceeds 95.18€ per MWh. Unlike the dynamic tariff for conventional energy discussed in Section 4.5, this scenario occurs infrequently on days with low average dynamic tariffs (e.g., days 01, 07, 10), but more often on days with higher average dynamic tariffs (e.g., days 02, 03, 04). Conversely, if the conventional tariff is fixed (as shown on the table bottom for EASC *ap - 32 - fix - ppa/dyn*), a similar fixed PPA tariff for renewables provides no additional benefit, but rather results in a tremendous relative disadvantage compared to a dynamic tariff for renewables. More specifically, the dynamic tariff for renewable energy would enable the use of cost rates that are significantly lower than both the fixed conventional rate (which reflects the day's average energy price) and the PPA rate (set at 95.18€ per MWh). This is particularly evident on days 01, 07, and 10, where the minimum dynamic cost rates are 0.29, −1.03, and −1.76€ per MWh, respectively. On these days, the vast majority of hourly rates falls below the PPA rate. Consequently, the fixed PPA rate results in a significant deterioration on these days when compared to the dynamic rate. Therefore, when the conventional energy tariff is fixed, choosing a dynamic renewable tariff instead of the PPA is advisable. Regarding the other objectives, we observe that minimum attainable makespans are unaffected by different energy tariffs. The impact of minimizing energy costs on carbon emissions and energy consumption is negligible, with all quartiles showing improvements of less than 1%. In summary, the effectiveness of a fixed PPA tariff for renewable energy depends on the conventional energy tariff type of the EASC: It is beneficial when paired with a dynamic tariff for conventionals, while a dynamic tariff for renewable energy is preferable when a fixed conventionals tariff is in place.

4.7. Managerial implications

The previous analysis indicates that energy cost-efficiency is best achieved by concurrent availability of dynamic and fixed tariffs. This dual approach offers two key advantages: Dynamic tariff participation enables engagement with fluctuating electricity prices throughout the day, allowing for potential savings during lower-priced periods; and price protection from fixed tariffs (such as PPAs) provides a safeguard against hourly energy prices exceeding the available fixed price option. As summarized in Table 12, it is beneficial to select either a dynamic conventionals tariff paired with a fixed PPA renewable tariff or a fixed conventionals tariff combined with a dynamic renewable tariff. Thus, we now compare these two tariff configurations, aiming to establish a criterion for identifying the most favorable one.

Table 13 reveals that both improvements and deteriorations are possible, as indicated by the minimum values of −114.63, −83.23, and −125.03% for three, five, and ten jobs, respectively, and maximum values of 27.72, 29.04, and 29.12%. The average and median values suggest that no definitive conclusion can be drawn about the superiority of either tariff configuration. Consequently, we examine the relationship between the fixed tariff for conventional energy and the

Table 11
Improvement through PPA over dynamic tariff.

3 jobs							
Improvement [%]	Min	Avg	Max	$q_{0.25}$	$q_{0.5}$	$q_{0.75}$	$\in [-1, 1]\%$ [%]
Energy costs	−1346.11	−99.1	30.72	−3.94	3.5	11.24	9.38
Makespan	0.0	0.0	0.0	0.0	0.0	0.0	100.0
Emissions	0.0	0.45	4.81	0.0	0.0	0.72	80.21
Consumption	0.0	0.35	8.21	0.0	0.0	0.43	88.54
5 jobs							
Improvement [%]	Min	Avg	Max	$q_{0.25}$	$q_{0.5}$	$q_{0.75}$	$\in [-1, 1]\%$ [%]
Energy costs	−958.25	−61.89	32.2	−5.32	2.36	9.2	15.62
Makespan	0.0	0.0	0.0	0.0	0.0	0.0	100.0
Emissions	−0.44	0.27	5.15	0.0	0.0	0.0	90.62
Consumption	−0.16	0.21	4.86	0.0	0.0	0.0	94.79
10 jobs							
Improvement [%]	Min	Avg	Max	$q_{0.25}$	$q_{0.5}$	$q_{0.75}$	$\in [-1, 1]\%$ [%]
Energy costs	−773.6	−25.8	32.0	−2.05	5.12	12.49	14.58
Makespan	0.0	0.0	0.0	0.0	0.0	0.0	100.0
Emissions	−0.18	0.26	2.66	0.0	0.0	0.16	87.5
Consumption	−0.2	0.25	3.12	0.0	0.0	0.08	91.67

3 jobs: $ap-32-dyn-ppa/dyn$			
Improvement [%]	Min	Avg	Max
Energy costs	0.0	12.41	30.72
Makespan	0.0	0.0	0.0
Emissions	0.0	0.69	3.1
Consumption	0.0	0.35	1.22

3 jobs: $ap-32-fix-ppa/dyn$			
Improvement [%]	Min	Avg	Max
Energy costs	−1346.11	−183.7	27.72
Makespan	0.0	0.0	0.0
Emissions	0.0	0.79	2.85
Consumption	0.0	0.49	1.8

Table 12
Suitability of combinations of conventional and renewable tariff structures.

		Renewable energy tariff	
		fix (PPA option)	dyn
Conventional energy tariff	fix (avg option)	Unfavorable	Favorable
	dyn	Favorable	Unfavorable

fixed PPA rate of 95.18€ per MWh. Based on the price data in Table 4, the average dynamic energy price falls below (above) the PPA rate on days 01, 07, 08, 10, 12 (02, 03, 04, 05, 06, 09, 11). Thus, on the latter days, a dynamic conventionals tariff with a fixed PPA renewables tariff is most advantageous, while on the former days, a fixed conventionals tariff with a dynamic renewables tariff is preferable. The tables at the bottom of Table 13 illustrate this for day 02 (average energy price of 143.52€ per MWh) and day 07 (average energy price of 32.68€ per MWh). This pattern of categorization holds for all days except day 01. On this day, neither tariff option uses fixed tariffs, relying solely on the dynamic tariff, as all operations are completed by 18:00, the first time the hourly energy tariff exceeds the day's average.

Building on the computational experimentation with the methodology presented in Section 3, we derive the workflow shown in Fig. 4, guiding decision-makers in negotiating a company's energy tariff structure for energy-aware production. The workflow connects strategic and operational planning by relying on the performance of operational schedules as an evaluation function for selecting strategic tariffs. Consider a planning horizon D consisting of a set of consecutive days (e.g., a year, $D = \{1, 2, \dots, 365\}$ with avg_d as the forecasted average energy price on day $d \in D$, and c_{PPA} as the fixed PPA tariff for renewable energy over D. The workflow suggests opting for a dynamic conventionals tariff with a fixed PPA renewables tariff if, for most days, the average dynamic price exceeds the PPA rate. Conversely, if this is true for only a few days, a fixed conventionals tariff with a dynamic renewables tariff is recommended. We finally note that forecasting average dynamic prices and negotiating the PPA rate are complex topics beyond this paper's scope, but given the challenges of forecasting hourly prices a year in advance, planning based on daily average prices is a practical approach.

5. Conclusion and outlook

For companies in energy-intensive industries, achieving energy efficiency is crucial for both economic competitiveness and the attainment of sustainability objectives. While research on energy-efficient control for specific shop floor types is growing, there is a significant gap in holistic methodologies that integrate energy-market aspects like dynamic pricing and fixed PPA tariffs. Thus, developing a combined approach to production scheduling and energy cost pricing is essential in order to foster a comprehensive modeling of the interplay between production systems and energy tariff structures within a multi-criteria framework, ultimately promoting efficient practical implementation and understanding of the driving forces in corporate environments.

In this paper, we introduce an energy-aware production planning approach that incorporates dynamic pricing, energy-dependent processing characteristics, and various energy tariff structures, including PPAs. Utilizing real-world energy pricing data from 2023, we apply our model to a synthesized job shop setting, aiming to minimize makespan and energy costs while accounting for energy consumption and carbon emissions. At the operational level, we find that integrating energy-dependent factors into decision-making facilitates the energy-efficient scheduling of job operations. On a broader scale, strategic decisions are informed by extensive numerical experiments in a case study, exploring different model aspects such as energy cost tariffs for conventional and renewable sources, the role of PPAs, flexible processing start times, and a comparison of exact versus approximate solution approaches. Overall, the findings underscore the critical importance of a flexible tariff structure that enables participation in dynamic energy markets while providing hedging opportunities against price spikes.

We identify several promising areas for future research. First, we aim to deepen the understanding of how different tariff structures, such as PPAs, affect system performance. This involves extending the approach to other production systems like lot sizing, batch scheduling, or capacity planning, which currently lack a stronger energy focus. Second, we suggest replacing the matheuristic nature of our approximate approach with a heuristic that operates independently

Table 13
Improvement through $dyn - ppa$ over $fix - dyn$.

3 jobs							
Improvement [%]	Min	Avg	Max	$q_{0.25}$	$q_{0.5}$	$q_{0.75}$	$\in [-1, 1]\%$ [%]
Energy costs	−114.63	−7.8	27.72	−0.41	3.18	7.66	25.0
Makespan	0.0	0.0	0.0	0.0	0.0	0.0	100.0
Emissions	−1.65	0.25	2.85	0.0	0.0	0.0	87.5
Consumption	−0.58	0.13	1.15	0.0	0.0	0.0	93.75
5 jobs							
Improvement [%]	Min	Avg	Max	$q_{0.25}$	$q_{0.5}$	$q_{0.75}$	$\in [-1, 1]\%$ [%]
Energy costs	−83.23	−2.29	29.04	−0.74	2.45	5.88	25.0
Makespan	0.0	0.0	0.0	0.0	0.0	0.0	100.0
Emissions	−1.4	0.01	1.73	0.0	0.0	0.0	93.75
Consumption	−0.99	0.0	0.66	0.0	0.0	0.0	100.0
10 jobs							
Improvement [%]	Min	Avg	Max	$q_{0.25}$	$q_{0.5}$	$q_{0.75}$	$\in [-1, 1]\%$ [%]
Energy costs	−125.03	−5.67	29.12	−0.87	3.98	7.37	20.83
Makespan	0.0	0.0	0.0	0.0	0.0	0.0	100.0
Emissions	−0.35	0.02	0.59	0.0	0.0	0.0	100.0
Consumption	−0.43	0.04	0.68	0.0	0.0	0.0	100.0
Feb: 3 jobs: $ex/ap - 0/32 - dyn - ppa$				Jul: 3 jobs: $ex/ap - 0/32 - fix - dyn$			
Improvement [%]	Min	Avg	Max	Improvement [%]	Min	Avg	Max
Energy costs	26.43	26.95	27.72	Energy costs	−95.06	−44.19	−12.41
Makespan	0.0	0.0	0.0	Makespan	0.0	0.0	0.0
Emissions	0.0	0.71	2.85	Emissions	−1.65	−0.41	0.0
Consumption	0.0	0.28	1.12	Consumption	−0.58	−0.14	0.0

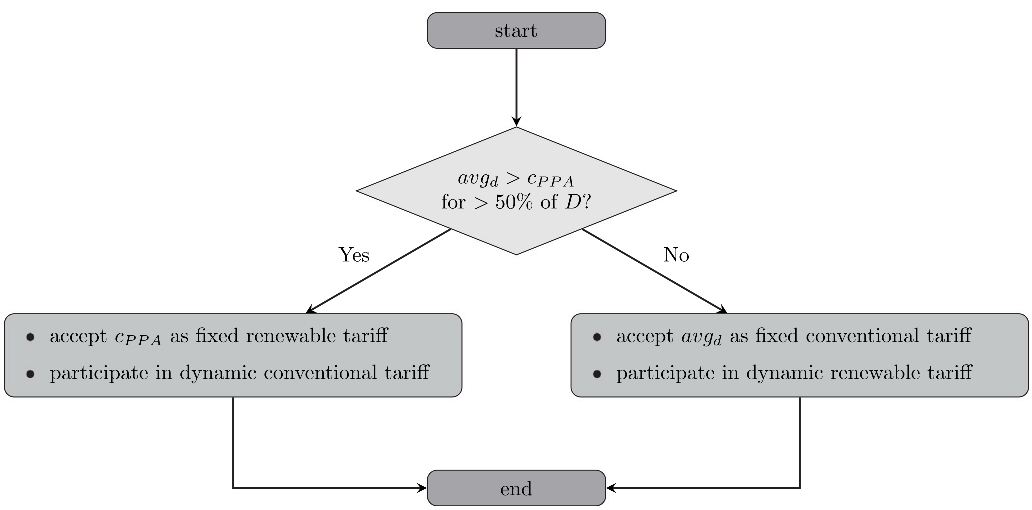

Fig. 4. Summary of decision making steps in tariff selection for conventional and renewable energy.

of mathematical programming. Such a tailored heuristic adaptable to energy-dependent decisions could be valuable for industrial applications with limited computing budget. Lastly, we propose embedding our operational energy-aware scheduling approach into a cross-level planning framework for long-term PPA negotiation. This framework would simultaneously address PPA specifications and energy-aware scheduling, bridging strategic and operational levels. In this context, we also emphasize the importance of reliable forecasting methods to accurately project energy prices throughout the planning horizon.

CRediT authorship contribution statement

Fabian Dunke: Writing – review & editing, Writing – original draft, Validation, Methodology, Investigation, Formal analysis, Conceptualization. **Stefan Nickel:** Validation, Supervision, Investigation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.apenergy.2024.125065.

Data availability

The Python source code related to this research paper is available online at https://gitlab.kit.edu/fabian.dunke/energyAwareJobShopScheduling.

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