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A choice-based approach to dynamic capacitated multi-item lot sizing with demand uncertainty

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ABSTRACT

With the purpose of planning and implementing pricing decisions on a tactical level as well as production decisions on an operational level, we consider – in an integrated form – the capacitated multi-item lot sizing problem with uncertain item demands and price-dependent discrete choice demand. The model is embedded into an overarching rolling horizon procedure allowing for adaptations to changes in demand and cost parameters. We first formulate the static problem version as a nonlinear mathematical program with underlying multinomial logit demand and subsequently linearize it to make it viable for mathematical programming solvers. Uncertainty of demands is taken into account by Monte Carlo simulation. More specifically, we generate random demand scenarios and utilize them as input data for the sample average approximation problem version. We further endow the problem setting with possibilities to incorporate pricing policy requirements such as restricting the number of price adaptations or defining periods without price adaptations. Overall, the developed approach yields a powerful tool for balancing item demands via pricing in a way favorable for adhering to available production capacities and thereby striking a balance between revenues and costs. Computational results confirm that adapting prices to time-dependent demand and cost parameters is exploited effectively to maintain a deliberately controlled production environment. Moreover, the integrated pricing and production setting allows to study the effect of pricing policy restrictions and demand uncertainties upon attainable profits.

1. Introduction

Striking a favorable balance between supply and demand is an essential prerequisite for utilizing available production resources efficiently in accordance with market conditions as represented by customer demand characteristics. Traditionally, matching inventories and order requests is accomplished hierarchically in advanced planning and scheduling systems with sales information serving as input for lot sizing, inventory management, and production scheduling [1]. This approach is mainly due to the computational complexity of optimization problems arising in the two disciplines of product pricing on the tactical level and production planning on the operational level, respectively. Nonetheless, there is agreement that intertwining these disciplines yields the opportunity of exploiting synergies [2,3]. From an organizational perspective, one possibility fostering the disciplines to coalesce would be to introduce feedback loops between pricing and lot sizing reporting whether previously received information turned out beneficial [4]. Disadvantages of this approach originate from its sequential character: First, pricing policy recommendations may be incompatible

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with available production capacities; vice versa, production quantity proposals may be conflicting with actual product demands. Secondly, recommendations from either side are likely to be dismissed as pricing decisions are primarily revenue-driven, whereas production decisions are primarily cost-driven. A more objective approach, hence, is found in a simultaneous consideration of both the revenue side and the cost side with the goal of serving the entire organization [5,6]. In this paper, for the underlying setting of multi-item capacitated lot sizing considered over a tactical planning horizon of several months, we take on this perspective through a model-based methodology interrelating operational production decisions with tactical pricing decisions. Hence, interaction between supply and demand is facilitated leading to a higher degree of responsiveness with respect to matching sales to production capacities over time.

Over the last years, an increase in the interest for operations research settings with integrated discrete choice demand models has emerged as shown by works on product assortment [7,8], product line selection and design [9–11], facility location [12,13], transportation networks [14,15]. In choice-based optimization, the modeling of demand is integrated into the model, allowing for incorporating the effect of demand-related decisions such as pricing. Hence, demand no longer serves as an external input, but can be influenced proactively through setting the price decision variables. Apparently, models become more complex in this way. The work at hand examines pricing-dependent production planning and, in doing so, pays considerable attention to dealing with demand uncertainty and time dynamics. On top of the complexity inherent to the static problem setting of capacitated multi-item lot sizing with explicit demand choice modeling, nonlinearity and nonconvexity further complicate matters in terms of impeding computational tractability. From an organizational perspective, directly relating revenues and costs has historically suffered from the association of both disciplines with different planning horizons: While pricing is located on the tactical to strategic level, production planning and scheduling occurs at the operational level. However, with the emergence of variable and dynamic pricing opportunities, both planning levels are increasingly interrelated, which suggests to replace static with dynamic pricing schemes. Example 1 illustrates the possible advantages of price-based demand control in terms of overall achievable profit, namely 1) avoidance of shortages, 2) utilization of regular lot sizing trade-offs, and 3) exploitation of revenue potentials.

Example 1. We consider a single product over two periods with fixed production capacity of $C = 100$ units per period, fixed setup costs of $A = 50$ monetary units, fixed unit inventory holding costs of $H = 0.5$, and fixed unit shortage costs of $S = 3$ monetary units. Shortage costs serve as a penalty as we assume that units whose demand cannot be met from production or inventory must be sourced from external suppliers at a larger cost.

a) *Avoidance of shortages and utilization of regular lot sizing trade-offs:*
In the base scenario, assume that a constant price of $r = 1$ for both periods leads to demands of $d_1 = 110, d_2 = 50$ which could be, e.g., due to changing seasonality. As a consequence of $C = 100$, a shortage of $d_1 - C = 10$ results in the first period, i.e., we produce 100 units in the first and 50 units in the second period. This leads to a revenue of $rd_1 + rd_2 = 110 + 50 = 160$, shortage costs of $S(d_1 - C) = 30$, and setup costs in both periods of $A + A = 100$. The overall profit is $160 - 30 - 100 = 30$. In particular, shortages occur in the first period and setups in both periods.
Contrarily, when we apply price control with prices $r_1 = 1.25$ and $r_2 = 1$, demand of the first period – depending on the customers' price elasticity – may drop to $d_1 = 50$ while $d_2 = 50$ remains. This leads to a revenue of $r_1 d_1 + r_2 d_2 = 62.5 + 50 = 112.5$, setup costs only in the first period of $A = 50$, and inventory holding costs for the second period's units of $H d_2 = 25$. The overall profit is $112.5 - 50 - 25 = 37.5$. In particular, no shortages occur, inventory is utilized, and through price control the capacity becomes capable of fully covering both periods' demand.

b) *Exploitation of revenue potentials and utilization of regular lot sizing trade-offs:*
In the base scenario, assume that a constant price of $r = 1$ for both time periods leads to demands of $d_1 = d_2 = 50$ which could be, e.g., due to unchanged seasonality. As a consequence of $C = 100$, both demands are produced in the first period. This leads to a revenue of $rd_1 + rd_2 = 50 + 50 = 100$, setup costs only in the first period of $A = 50$, and inventory holding costs for the second period's units of $H d_2 = 25$. The overall profit is $100 - 50 - 25 = 25$. In particular, inventory is utilized.
Contrarily, when we apply price control with prices $r_1 = r_2 = 0.75$, demand of both – depending on the customers' price elasticity – may rise to $d_1 = d_2 = 100$. This leads to a revenue of $r_1 d_1 + r_2 d_2 = 75 + 75 = 150$, and setup costs in both periods of $A + A = 100$. The overall profit is $150 - 100 = 50$. In particular, setups occur in both periods, but the difference between additional setup costs and original inventory holding costs is outweighed by additional revenues. △

In this paper, we address the integration of pricing and lot sizing as follows: We first formulate the static version of the capacitated multi-item lot sizing problem with demand choice as a nonlinear mathematical program. We then derive a linearized version of the problem when a discrete number of possible price levels for each product is assumed. Since product pricing in practice typically is not entirely arbitrary, this represents a closer approximation of reality than unconstrained prices would do. To obtain a sustainable pricing policy over time and to prevent customers from being upset due to frequent erratic price changes, we further enrich the model with the possibility of incorporating pricing restrictions depending on the pricing trajectory. As demand predictions become increasingly volatile the farther one sees into the future, we take into account uncertainty of demands through sampling of demand scenarios. Since demand predictions for the close future exhibit less volatility, embedding pricing and lot sizing into a rolling horizon procedure yields a viable way of addressing uncertainty dynamically and adapting decisions upon scenario realizations. The developed methodology supports choice-based optimization to be employed in a sequence of decision making steps with the goal of supporting pricing and production decisions over an extended period of time. For instance, in this paper, the overseen planning horizon can be thought of as a year with weekly production decisions and monthly pricing decisions. Fig. 1 summarizes the developed rolling horizon outline:

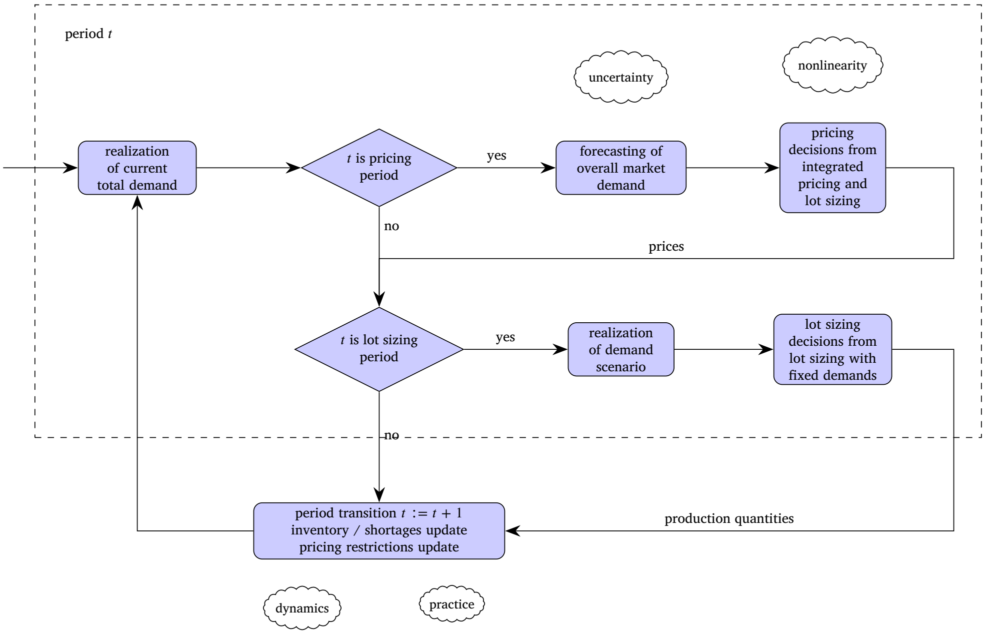


Fig. 1. Rolling horizon outline and related complexity drivers for the integrated pricing and lot sizing problem over time.

Depending on user-selected execution times for making pricing decisions and lot sizing decisions, the respective problem settings are tackled with appropriate data on uncertain demands associated to them. In particular, pricing decisions first lead to a disaggregation of the overall market demand into single item demands; this is followed by operational lot sizing decisions ensuring to serve these single item demands. Further, practical aspects can be fed into the pricing task in the form of a-priori restrictions.

The remainder of the paper is organized as follows: In section 2, we discuss relevant literature from the realms of discrete choice demand modeling in mathematical optimization as well as capacitated multi-item lot sizing. In section 3, we introduce the formal setting and solution outline for the capacitated multi-item lot sizing problem with demand choice, uncertain customer demands, and its embedding in a rolling time horizon. The numerical experiments in section 4 demonstrate the methodology's applicability in practice as seen from several instances covering long planning horizons of one year. Finally, section 5 yields an outlook on future research with respect to the specific lot sizing application and to generalization possibilities of the rolling horizon framework for choice-based optimization.

2. Literature review

We discuss literature on discrete choice (DC) demand models, their use in general frameworks and specific applications of choice-based (especially attraction-based) optimization, and lot sizing with integrated pricing. Game-theoretic and revenue management works are omitted as these disciplines do not encompass complex choice-based settings with periodical demand realizations. Concerning the treatment of uncertainty via sample average approximation (SAA), we refer to the seminal paper [16] on solving stochastic discrete optimization problems using Monte Carlo sampling [17].

2.1. DC demand modeling

A comprehensive survey on demand function modeling is given by [18]. The discussion structures demand models depending upon the factors of price, rebate, lead time, space, quality, and advertising. Further, it encompasses a separate consideration of single and multi-firm settings, and it is established that the multinomial logit (MNL) model is among the most widely used demand models. Likewise, [19] surveys various forms of demand models, how they interact on an individual level, and how they add up to aggregate demand. In addition, further aspects are discussed such as demand estimation, forecasting, competitive demand situations, and behavioral influences. [20] provides a comprehensive overview on DC demand modeling in the form of a compilation of various models' properties.

Historically, [21] provides the distributional foundation for utility-based decision making and focuses on the random utility as the residue of the overall utility and its deterministic component. In [22], McFadden develops the MNL choice model where random utility follows an identical and independent extreme value distribution leading to constant choice probability ratios even when alternatives are removed (independence of irrelevant alternatives (IIA)). Over the years, MNL has become one of the standard approaches for modeling customer choice in case of alternative products as it allows for analytically tractable expressions and an accessible interpretation of resulting choice probabilities [20]. McFadden's work has been the starting point for many further choice models. In particular, [23] shows that any choice model relying on random utility maximization can be approximated by mixed

MNL which derives from MNL through random parameters. To further close the gap between behavioral models and DC theory, [24] enriches MNL models with realistic facets such as factor disturbances, combining different preference types, or latent variables. We note that the basic attraction model (BAM) – of which the MNL choice model is a special case – has been first formulated by [25] on an axiomatic basis considering basic consumer behavior assumptions. Finally, [26] provides a generalization of the BAM to alleviate the overoptimism and to improve the modeling opportunities of spill and recapture. [27] addresses DC from the perspective of product differentiation since quality, packaging, design, color, and style, are seen as major drivers of consumer choice. The combination of product differentiation and DC has led to a plethora of applications of integrated pricing and product line selection.

We next discuss how DC is accounted for within optimization problems. To this end, we first examine general approaches as they address several features encountered in the paper at hand; we then continue with a discussion of research on applications of integrated pricing and optimization where the methodological outline is related to our paper.

2.2. General approaches to choice-based optimization

Even though most research on integrated pricing and optimization addresses specific application domains (cf. subsection 2.3), some recent initiatives have promoted the development of a generalizing perspective with respect to several realms also encountered in the paper at hand, namely structure of coupled pricing and optimization, use of randomness, and linearization of model components: [2] examines the general architecture of integrating disaggregated demand into optimization models. Their discussion entails utility theory, its translation to choice models, and practical aspects such as parameter estimation. [3] follows a similar line of research and demonstrates how choice-based control of the demand side is technically embedded into mixed-integer linear program (MILPs) for operational systems. A general optimization framework is obtained where individual choice is incorporated through a simulative approach. This facilitates the embedding of arbitrary error term distributions in the random utility part. Concerning the linearization of mixed-integer nonlinear programs (MINLPs) encompassing choice-based demand, [13] distills two generic ideas which are employed time and again in applications (assortment optimization, product line selection, location planning) which they classify as method-based (resulting from dealing with fractions in formulations) and property-based (resulting from dealing with independence of irrelevant alternatives).

2.3. Applications of integrated pricing and optimization

Existing research on applications of integrated pricing and optimization illustrates that many of the aspects arising in the paper at hand – such as, e.g., problem integration, fractional and nonlinear model formulations, model reformulations, rolling horizon procedures – have practically played a major role in the field of choice-based optimization. By far the most prominent application area is product line selection (PLS). As an early work, [28] assumes given choice preferences and develops an integer program (IP) as well as heuristics. [29] traces the computational complexity of PLS with integrated pricing to nonconcave profits, which is also characteristic for choice-based optimization in general. Consequently, they advocate for the use of approximation procedures. In case of probabilistic choice, [30] displays the typical fractional character of IP formulations encountered in choice-based PLS. The aforementioned complexity drivers (customer choice, concavity, computational complexity) are tackled by [9,10] for the case of attraction choice models (including MNL) via reformulations of MINLPs as models with concave objective and linear constraints allowing for improved computational tractability. More recent contributions in PLS elaborate on specific extensions such as market segmentation [31] or compromise alternatives [11]. A problem related to choice-based PLS is integrated assortment and pricing, where retailers must establish product offerings and prices. [7] considers demand to be learned dynamically via Bayesian updating, i.e., demand is explored online in a time-dynamic setting with similarities to the rolling horizon outline. However, the setting is then tackled by stochastic dynamic programming. [8] finds that choice-based assortment nowadays must be placed within a multichannel setting encompassing digital and on-site customer choices. The problem is approached by a MILP formulation as well as a heuristic procedure. Another application area of choice-based optimization falls into the category of managing transportation systems and networks. In [32], nonlinear models are yielded for urban transportation incorporating demands depending on prices, transport supply levels, road infrastructure, and speed. The research stream [14,33,34] addresses problem blendings (operator profit maximization with traveler cost minimization), two-phase approaches to cope with computational complexity, and multiple commuter classes, respectively. [15] devises a MINLP formulation and tailored solution algorithm for a carsharing network with demands depending on offered services. Service network design with attraction choice is discussed by [35] where choice encompasses several realistic influences such as distance, congestion, and price. For the resulting nonlinear model, reformulations are derived as well as decomposition and heuristic approaches. In the realms of airline management, [36] integrates scheduling, fleeting, and pricing subject to an itinerary-specific supply-demand model. However, the resulting nonlinear model runs into computational difficulties when going to instances beyond the ones presented. [37] blends flight scheduling, itinerary pricing, and aircraft fleeting so as to optimize profit. MNL demand leads to an MINLP with linear constraints and concave objective allowing for computational experimentation. Finally, we mention the rather strategic discipline of competitive location where choice-based model formulations are found for the maximum capture problem [38,39], classical facility location [40], preventive health care networks [41], and school location [42].

2.4. Capacitated multi-item lot sizing

Since we discuss works related to integrated pricing and lot sizing, we first refer to [43,44] for reviews on dynamic capacitated multi-item lot sizing. Assuming linear demand, [45] devises an MILP formulation and establishes convexity results. Coordination through pricing, supplier selection, and lot sizing along a serial supply chain is explored by [46] resulting in a MINLP. Due to the computational complexity, also a heuristic approach based on power-of-two policies is developed. A multi-objective MINLP is devised by [47] to coordinate pricing, lot sizing, and supplier selection. Due to the different objectives (profit for pricing, delivery correctness for lot sizing, defective avoidance for supplier selection), multi-objective algorithms (such as NSGA-II and SPEA2) for determining Pareto solutions are employed. The multi-channel setting of integrated pricing and lot sizing with pricing each channel individually and each channel having its own attraction-based demand model is scrutinized by [48]. The problem is formulated as a nonconvex MINLP which is subsequently transformed into a convex problem by exploiting demand properties. This formulation is then tackled by outer approximation. [49] discusses capacitated dynamic joint lot-sizing and pricing with uncertain linear demand. The problem is formulated as a multi-stage stochastic program and solved by decomposition consisting of first-stage pricing, second-stage lot sizing, and third-stage delayed pricing. [50] extends the discussion by accounting for new products to be factored in. The analysis is based on the integration of a demand diffusion model and results in an analysis of different pricing strategies which are dependent on a specific price change rate parameter. The discussion yields an analysis of specific strategies such as constant, dynamic, and sequential pricing and lot sizing. Even though not directly related to discrete choice demand models, [51] considers iso-elastic demand functions in joint lot-sizing and dynamic pricing such that complementarity and substitution between products can be considered in metaheuristic and matheuristic approaches.

Overall, we conclude that several works consider parts of the paper's aspects independently from each other. However, a substantial research gap is identified for the combination and integration of choice-based lot sizing with demand uncertainty in a time-dynamic setting.

3. Models and algorithms

Upon providing required terminology and notation for the integrated dynamic pricing and lot sizing with attraction-based demand choice and demand uncertainty in subsection 3.1, we introduce the associated MINLP formulation in the deterministic setting and transform it to a MILP version in subsection 3.2. Further, we account for practical pricing scheme restrictions in subsection 3.3. In subsection 3.4, we incorporate demand uncertainty as part of a stochastic solution outline. Finally, the rolling horizon procedure in subsection 3.5 arranges the sequential and repetitive retrieval of pricing schemes and production plans over time.

3.1. Setting, terminology and notation

We consider the integrated pricing and lot sizing problem for the capacitated multi-item case with demand uncertainty. It consists on the one hand of determining for all products in the set of products P their selling price for each period of the planning horizon $\overline{T}$; on the other hand, we have to determine the production quantities in each period of $\overline{T}$ such that customer demands are served either directly from production or by stored units from stock. In addition, there are competing products available on the market which are subsumed in the set P_{ext} of external products. Hence, the set of product alternatives available to the customers' discretion is $P \cup P_{ext}$. While the overall market demand is received as part of the external input data, individual product demands are assumed to follow an attraction choice model, i.e., demand for a product is proportional to its relative attraction in comparison to the total attraction of all products. Attraction models allow for demand representation in terms of product market shares, and thereby admit a probabilistic approach to demand. In fact, attraction models represent a frequently used class of demand models in economics, marketing, and operations management [18]. Hence, when attraction is measured by utility, the market share of product p is defined as $\frac{U_p}{\sum_{p' \in P \cup P_{ext}} U_{p'}}$. With a price of r_p monetary units for product $p \in P \cup P_{ext}$, utility is assumed to follow the MNL model with $U_p := U_p(r_p) = e^{\alpha_p + \beta_p r_p}$ with price-independent sensitivity parameter $\alpha_p \in \mathbb{R}$ and price sensitivity parameter $\beta_p < 0$. Therefore, we subsequently assume that the assumptions of the MNL model, in particular IIA, are fulfilled. Clearly, if this assumption is known to be violated in a practical setting, other choice models such as nested logit or multinomial probit must be initiated instead. Nonetheless, as we will ultimately use utility values as model input data, the core of the following analysis remains valid also for other discrete choice models.

In the discretized version of the problem, we assume that the price r_p comes from a pre-specified set L_p of possible prices, i.e., $r_p \in L_p$. Due to the rolling horizon procedure which proceeds period-wise until the overall planning horizon $\overline{T}$ is covered, we also introduce the set $T \subseteq \overline{T}$ of consecutive periods considered in the current planning step. As our models employ pricing on a tactical time horizon T, data for the operational lot sizing problem is incorporated into the integrated model through demand forecasts over T as well. As a result of tactical pricing and operational lot sizing, T typically covers a large number of production periods such that the influence of the end-of-horizon effect in the rolling horizon procedure (as known from pure lot sizing [52]) is negligible in the integrated setting. This is also confirmed by manual experimentation. The notation used for the integrated pricing and lot sizing problem (IPCLSP) and its variants is summarized in Table 1. We further introduce the following terminology:

selected price The selected price r_{pt} is the amount of monetary units obtained for selling product $p \in P$ in period $t \in T$. For the output of the final model IPCLSP in subsubsection 3.2.3, it holds that $r_{pt} := \sum_{l \in L_p} R_{pl} z_{ptl}$.

pricing scheme A pricing scheme r subsumes the selected prices for all products $p \in P$ in all periods $t \in T$ in a matrix, i.e., we have $r := (r_{pt})_{p \in P, t \in T}$ and r contains all pricing information over planning horizon T.

pricing snapshot The pricing snapshot r_t is composed of the selected prices for all products $p \in P$ in a specific period $t \in T$, i.e., we have $r_t := (r_{pt})_{p \in P}$ and r_t contains all pricing information for period t.

Table 1
Notation for model IPCLSP and its variants.

sets and indices	
t	time period index
$\overline{T}$	set of all time periods
T	set of specific time periods currently considered
p	product index
P	set of own products
P_{ext}	set of external products
l	price level index
L_p	set of price levels for product $p \in P$
ξ	demand scenario index
Ξ	set of demand scenarios
parameters	
R_{pl}	price for product $p \in P$ at level $l \in L_p$
D_t	overall demand in period $t \in T$
β_p	price sensitivity parameter of discrete choice model for product $p \in P$
α_p	price-independent sensitivity parameter of discrete choice model for product $p \in P$
U_p	utility of product $p \in P \cup P_{ext}$
U_{pl}	utility of product $p \in P$ if price level $l \in L_p$ is selected
H_{pt}	unit inventory holding cost for product $p \in P$ in period $t \in T$
S_{pt}	unit shortage cost for product $p \in P$ in period $t \in T$
A_{pt}	production setup cost for product $p \in P$ in period $t \in T$
C_t	total production capacity in period $t \in T$
$\bar{i}_p$	minimum ending inventory for product $p \in P$
n_{max}	maximum allowable number of price changes per product over time horizon $\overline{T}$
$n_p^{<T}$	number of price changes for product p that have occurred until period $t \in T$
$Pr(\xi)$	probability for demand scenario $\xi \in \Xi$
decision variables	
r_{pt}	selected price for product $p \in P$ in period $t \in T$
d_{pt}	demand resulting from selected price for product $p \in P$ in period $t \in T$
x_{pt}	production quantity of product $p \in P$ in period $t \in T$
y_{pt}	production indicator for product $p \in P$ in period $t \in T$
i_{pt}	inventory of product $p \in P$ at the end of period $t \in T$
s_{pt}	shortage of product $p \in P$ in period $t \in T$
δ_{pt}	market share of product $p \in P$ in period $t \in T$
γ_t	inverse of total utilities over all products $p \in P \cup P_{ext}$ in period $t \in T$
z_{ptl}	selection indicator for price level $l \in L_p$ of product $p \in P$ in period $t \in T$
ϕ_{ptl}	inverse of total utilities over all products $p \in P \cup P_{ext}$ in period $t \in T$ in case that $z_{ptl} = 1$, zero otherwise
Δ_{ptl}	price change indicator for product $p \in P$ in period $t \in T$ involving price level $l \in L_p$

demand The (market) demand D_t gives the total number of requested units over all products $p \in P$ in period $t \in T$; D_t is an exogenous model input subject to uncertainty concerning the market potential in period $t \in T$.

product demand The product demand d_{pt} gives the number of requested units for product $p \in P$ in period $t \in T$. It depends on the prices $r_{p't}$ selected for all products $p' \in P$ and the utility of all external products $p' \in P_{ext}$. For the output of the final model IPCLSP in subsubsection 3.2.3, it holds that $d_{pt} := D_t \sum_{l \in L_p} U_{pl}\phi_{ptl}$.

production schedule A production schedule x subsumes the production quantities for all products $p \in P$ in all periods $t \in T$ in a matrix, i.e., we have $x := (x_{pt})_{p \in P, t \in T}$ and x contains all production quantity information over planning horizon T.

production snapshot The production snapshot x_t is composed of the production quantities for all products $p \in P$ in a specific period $t \in T$, i.e., we have $x_t := (x_{pt})_{p \in P}$ and x_t contains all production quantity information for period t.

3.2. Mathematical programming models

To tackle pricing and lot sizing in an integrated fashion with available mathematical programming solvers, we successively derive a linear model formulation to resolve the problem-inherent issue of nonlinearity and to account for practical price setting limitations.

3.2.1. Nonlinear model formulation with continuous price variables

We provide a first MINLP for the integrated lot sizing and pricing problem. In the following model, the prices r_{pt} are continuous decision variables. Moreover, we have a price-dependent utility function $U_p(r_{pt})$ for products $p \in P$, and no uncertainty is considered yet. Concerning pricing, the model decides about prices r_{pt}; demands d_{pt}, product market shares δ_{pt}, and the inverse γ_t of total utilities over all products serve as dependent decision variables. Concerning lot sizing, the model decides about production quantities x_{pt} and production indicators y_{pt}; inventory levels i_{pt} and shortages s_{pt} serve as dependent decision variables.

$$\max \quad \sum_{p \in P} \sum_{t \in T} r_{pt} d_{pt} - \sum_{p \in P} \sum_{t \in T} (H_{pt} i_{pt} + A_{pt} y_{pt} + S_{pt} s_{pt}) \tag{1}$$

$$\text{s.t.} \quad \sum_{p \in P_{ext}} U_p \gamma_t + \sum_{p \in P} \delta_{pt} = 1 \qquad t \in T \tag{2}$$

$$d_{pt} = \delta_{pt} D_t \qquad p \in P, t \in T \tag{3}$$

$$\delta_{pt} = U_p(r_{pt})\gamma_t \qquad p \in P, t \in T \tag{4}$$

$$i_{p,t-1} + x_{pt} + s_{pt} - d_{pt} = i_{pt} \qquad p \in P, t \in T \tag{5}$$

$$i_{p,\max\{t \mid t \in T\}} \geq \bar{i}_p \qquad p \in P \tag{6}$$

$$\sum_{p \in P} x_{pt} \leq C_t \qquad t \in T \tag{7}$$

$$x_{pt} \leq M y_{pt} \qquad p \in P, t \in T \tag{8}$$

$$r_{pt}, d_{pt}, x_{pt}, i_{pt}, s_{pt} \geq 0 \qquad p \in P, t \in T \tag{9}$$

$$y_{pt} \in \{0, 1\} \qquad p \in P, t \in T \tag{10}$$

$$\delta_{pt} \in [0, 1] \qquad p \in P, t \in T \tag{11}$$

$$\gamma_t \geq 0 \qquad t \in T \tag{12}$$

The objective (1) gives the profit as the difference between revenues from sales and inventory holding costs, production setup costs, and shortage costs. Observe that the revenue is a nonlinear function since both r_{pt} and d_{pt} are decision variables where the latter is additionally dependent on δ_{pt} which in turn depends on γ_t as discussed below. Moreover, we allow for shortages (inducing shortage costs) in order to facilitate external sourcing of product units whose demands cannot be met by production or inventories as could be the case under random overall market demand (cf. subsection 3.4). Constraint (2) states that all market shares, i.e., those of external and own products, must add up to 1. We note that – fitting to the use of utility-related attraction-based demand choice – variable γ_t amounts to the inverse of the sum of all product utilities such that constraints (3)-(4) define the market share based on attraction choice. Constraints (5)-(8) are related to lot sizing and account for inventory balance, minimum ending inventories, production capacity, and linkage between production quantities and indicators, respectively. Variable domains are prescribed by constraints (9)-(12). While there are several ways of resolving shortages (e.g., backlogging, lost sales, shortages), we incorporate them exemplary through shortages.

We observe that due to $\delta_{pt} = U_p(r_{pt})\gamma_t \Leftrightarrow U_p(r_{pt}) = \frac{\delta_{pt}}{\gamma_t}$ and the equivalences

$$U_p(r_{pt}) = e^{\alpha_p + \beta_p r_{pt}} \Leftrightarrow \frac{\delta_{pt}}{\gamma_t} = e^{\alpha_p + \beta_p r_{pt}} \Leftrightarrow r_{pt} = \frac{1}{\beta_p}(\ln \frac{\delta_{pt}}{\gamma_t} - \alpha_p),$$

the first term of the objective function

$$\max \sum_{p \in P} \sum_{t \in T} r_{pt} d_{pt} = \max \sum_{p \in P} \sum_{t \in T} r_{pt} \delta_{pt} D_t = \max \sum_{p \in P} \sum_{t \in T} \frac{1}{\beta_p}(\ln \frac{\delta_{pt}}{\gamma_t} - \alpha_p)\delta_{pt} D_t$$

is a nonlinear function in the variables δ_{pt} and γ_t.

3.2.2. Nonlinear model formulation with discrete price variables

As a byproduct of the practical requirement that prices shall not be arbitrary, but rather come from a preselected range of options, the issue of nonlinearity is alleviated as follows: We impose the restriction that only specific price levels $l \in L_p$ with associated prices R_{pl} can be charged for product $p \in P$. As a consequence, we are able to eliminate the δ_{pt}-variables due to the relation $\delta_{pt} = \sum_{l \in L_p} U_{pl} z_{ptl} \gamma_t$ where U_{pl} is the utility for product p when priced at level l. This will be helpful for obtaining a linearized model version. The utility values for the specific price levels can be precomputed as $U_{pl} = e^{\alpha_p + \beta_p R_{pl}}$, i.e., we can get rid of the MNL function as an internal part of the model. Concerning pricing, the model decides about the price level selection indicators z_{ptl}; the inverse γ_t of total utilities over all products serves as dependent decision variable. The lot sizing decision variables remain unchanged.

With this, the first term of the objective function can be written as

$$\max \sum_{p \in P} \sum_{t \in T} r_{pt} d_{pt} = \max \sum_{p \in P} \sum_{t \in T} r_{pt} \delta_{pt} D_t = \max \sum_{p \in P} \sum_{t \in T} \sum_{l \in L_p} R_{pl} U_{pl} z_{ptl} \gamma_t D_t,$$

leading to

$$\max \quad \sum_{p \in P} \sum_{t \in T} \sum_{l \in L_p} R_{pl} U_{pl} z_{ptl} \gamma_t D_t - \sum_{p \in P} \sum_{t \in T} (H_{pt} i_{pt} + A_{pt} y_{pt} + S_{pt} s_{pt}) \tag{13}$$

$$\text{s.t.} \quad \sum_{p \in P_{ext}} U_p \gamma_t + \sum_{p \in P} \sum_{l \in L_p} U_{pl} z_{ptl} \gamma_t = 1 \qquad t \in T \tag{14}$$

$$\sum_{l \in L_p} z_{ptl} = 1 \qquad p \in P, t \in T \tag{15}$$

$$i_{p,t-1} + x_{pt} + s_{pt} - D_t \sum_{l \in L_p} U_{pl} z_{ptl} \gamma_t = i_{pt} \qquad p \in P, t \in T \tag{16}$$

$$i_{p,\max\{t \mid t \in T\}} \geq \bar{i}_p \qquad p \in P \tag{17}$$

$$\sum_{p \in P} x_{pt} \leq C_t \qquad t \in T \tag{18}$$

$$x_{pt} \leq M y_{pt} \qquad p \in P, t \in T \tag{19}$$

$$x_{pt}, i_{pt}, s_{pt} \geq 0 \qquad p \in P, t \in T \tag{20}$$

$$y_{pt} \in \{0,1\} \qquad p \in P, t \in T \tag{21}$$

$$\gamma_t \geq 0 \qquad t \in T \tag{22}$$

$$z_{ptl} \in \{0,1\} \qquad p \in P, t \in T, l \in L_p \tag{23}$$

Thus, objective (13) represents the discretized version of the former objective (1); constraint (14) combines the former constraints (2) and (4) for the purpose of discretization; constraint (15) prescribes that for each product and period exactly one price level is selected.

3.2.3. Linear model formulation with discrete price variables

Finally, to arrive at a linearized formulation, we introduce an auxiliary variable ϕ_{ptl} for substituting the bilinear term $\phi_{ptl} = z_{ptl}\gamma_t$ which leads – together with the three constraints $\phi_{ptl} \leq M z_{ptl}$ (ensuring that $\phi_{ptl} = 0$ if $z_{ptl} = 0$), $\phi_{ptl} \leq \gamma_t$ and $\phi_{ptl} \geq \gamma_t + M(z_{ptl} - 1)$ (ensuring that $\phi_{ptl} = \gamma_t$ if $z_{ptl} = 1$) – to the following MILP IPCLSP(T) for the integrated pricing and capacitated lot sizing problem over period set T. Concerning pricing, the model decides about the price level selection indicators z_{ptl}; the inverse γ_t of total utilities over all products serves as dependent decision variable; the inverse ϕ_{ptl} of total utilities over all products in its conditioned form on $z_{ptl} = 1$ serves as auxiliary variable. The lot sizing decision variables remain unchanged.

$$\max \quad \sum_{p \in P} \sum_{t \in T} \sum_{l \in L_p} R_{pl} U_{pl} \phi_{ptl} D_t - \sum_{p \in P} \sum_{t \in T} (H_{pt} i_{pt} + A_{pt} y_{pt} + S_{pt} s_{pt}) \tag{24}$$

$$\text{s.t.} \quad \sum_{p \in P_{ext}} U_p \gamma_t + \sum_{p \in P} \sum_{l \in L_p} U_{pl} \phi_{ptl} = 1 \qquad t \in T \tag{25}$$

$$\phi_{ptl} - M z_{ptl} \leq 0 \qquad p \in P, t \in T, l \in L_p \tag{26}$$

$$\phi_{ptl} \leq \gamma_t \qquad p \in P, t \in T, l \in L_p \tag{27}$$

$$\phi_{ptl} - M(z_{ptl} - 1) \geq \gamma_t \qquad p \in P, t \in T, l \in L_p \tag{28}$$

$$\sum_{l \in L_p} z_{ptl} = 1 \qquad p \in P, t \in T \tag{29}$$

$$i_{p,t-1} + x_{pt} + s_{pt} - D_t \sum_{l \in L_p} U_{pl} \phi_{ptl} = i_{pt} \qquad p \in P, t \in T \tag{30}$$

$$i_{p,\max\{t \mid t \in T\}} \geq \bar{i}_p \qquad p \in P \tag{31}$$

$$\sum_{p \in P} x_{pt} \leq C_t \qquad t \in T \tag{32}$$

$$x_{pt} \leq M y_{pt} \qquad p \in P, t \in T \tag{33}$$

$$x_{pt}, i_{pt}, s_{pt} \geq 0 \qquad p \in P, t \in T \tag{34}$$

$$y_{pt} \in \{0,1\} \qquad p \in P, t \in T \tag{35}$$

$$\gamma_t \geq 0 \qquad t \in T \tag{36}$$

$$z_{ptl} \in \{0,1\} \qquad p \in P, t \in T, l \in L_p \tag{37}$$

$$\phi_{ptl} \geq 0 \qquad p \in P, t \in T, l \in L_p \tag{38}$$

Using the auxiliary variables ϕ_{ptl}, objective (24) represents the linearized version of the former objective (13); constraint (25) represents the linearized version of the former constraint (14); constraints (26) to (28) are a linear representation of the bilinear term $\phi_{ptl} = z_{ptl}\gamma_t$.

3.3. Pricing restrictions in practice

Since it is undesirable from a practical perspective to have price changes too frequently, we restrict their occurrences. To this end, we introduce the following decision variables indicating a change in the selected price level:

Table 2
Setting of Δ_{pt} depending on z_{ptl} and $z_{p,t-1,l}$.

z_{ptl}	$z_{p,t-1,l}$	Δ_{ptl}
1	0	1
0	1	1
1	1	0
0	0	0

$$\Delta_{ptl} = \begin{cases} 1, & \text{if } z_{ptl} \neq z_{p,t-1,l} \\ 0, & \text{else} \end{cases} \qquad p \in P, t \in T, t > 1, l \in L_p$$

For the different combinations of z_{ptl} and $z_{p,t-1,l}$, the setting of Δ_{pt} must be asserted according to Table 2.

Observe that each line can be viewed as an implication with the antecedent depending on z_{ptl}, $z_{p,t-1,l}$ and the consequent depending on Δ_{ptl}. With $z = 1 \Leftrightarrow \mathbf{z}$ and $z = 0 \Leftrightarrow \neg\mathbf{z}$, we transform the implications to disjunctive normal form using the material conditional ($\mathbf{z} \Rightarrow \boldsymbol{\Delta} \Leftrightarrow \neg\mathbf{z} \vee \boldsymbol{\Delta}$) and De Morgan's second rule ($\neg(\mathbf{z} \wedge \boldsymbol{\Delta}) \Leftrightarrow \neg\mathbf{z} \vee \neg\boldsymbol{\Delta}$). This means the following for the first line of Table 2:

$$\begin{aligned} & \mathbf{z}_{ptl} \wedge \neg\mathbf{z}_{p,t-1,l} \Rightarrow \boldsymbol{\Delta}_{ptl} && \Leftrightarrow \quad \neg(\mathbf{z}_{ptl} \wedge \neg\mathbf{z}_{p,t-1,l}) \vee \boldsymbol{\Delta}_{ptl} \\ \Leftrightarrow \quad & \neg\mathbf{z}_{ptl} \vee \mathbf{z}_{p,t-1,l} \vee \boldsymbol{\Delta}_{ptl} && \Leftrightarrow \quad 1 - z_{ptl} + z_{p,t-1,l} + \Delta_{ptl} \geq 1 \\ \Leftrightarrow \quad & \Delta_{ptl} \geq z_{ptl} - z_{p,t-1,l} \end{aligned}$$

In the same manner, it follows for the second, third, and fourth line of Table 2 that

$$\begin{aligned} \Delta_{ptl} &\geq -z_{ptl} + z_{p,t-1,l} && p \in P, t \in T, t > 1, l \in L_p \\ \Delta_{ptl} &\leq 2 - z_{ptl} - z_{p,t-1,l} && p \in P, t \in T, t > 1, l \in L_p \\ \Delta_{ptl} &\leq z_{ptl} + z_{p,t-1,l} && p \in P, t \in T, t > 1, l \in L_p \end{aligned}$$

Since mathematical programs are formulated in a conjunctive manner, all four constraints have to hold simultaneously. Finally, with I denoting the index set for the disjoint intervals T_i of time periods with $i = 1, 2, \ldots, |I|$ (for instance, each T_i can be a set of periods covering a tactical horizon such as one to three months) where

$$\begin{aligned} T &= T_1 \cup T_2 \cup \ldots \cup T_{|I|} \\ &= \{1, \ldots, i_1\} \cup \{i_1 + 1, \ldots, i_2\} \cup \ldots \cup \{i_{|I|-1} + 1, \ldots, i_{|I|}\}, \end{aligned}$$

we can impose for each product $p \in P$ in each interval T_i that at most n_{max} price changes are allowed from the very beginning until the end of interval T_i through the constraint

$$\sum_{t \in T_i} \sum_{l \in L_p} \Delta_{ptl} \leq 2(n_{max} - n_p^{<T_i}) \qquad p \in P, i \in I \tag{39}$$

where $n_p^{<T_i}$ gives the number of changes already registered for product $p \in P$ until the beginning of interval T_i. Note that the factor on the right hand side of the constraint is 2 since a price change implies that one price level is deactivated while another one is activated.

Finally, it is a frequent practical requirement that a certain number n_{fix} of periods must elapse after a price level change until the next price level change is allowed to take place. With

$$\mathcal{T}_{=n_{fix}} = \left\{ \{t, t+1, \ldots, t + n_{fix} - 1\} \mid t \in T, t + n_{fix} - 1 \leq |T| \right\}$$

as the set of all intervals of exactly n_{fix} consecutive time periods, we can add the constraints

$$\sum_{t \in T_{=n_{fix}}} \sum_{l \in L_p} \Delta_{ptl} \leq 2 \qquad p \in P, T_{=n_{fix}} \in \mathcal{T}_{=n_{fix}} \tag{40}$$

$$\sum_{\substack{t \in T_{=n_{fix}} \\ last_p \leq t \leq last_p + n_{fix}}} \sum_{l \in L_p} \Delta_{ptl} = 0 \qquad p \in P \tag{41}$$

which imply that once a price change (activation or deactivation) occurs with respect to price level l in time period t, no further price change (deactivation or activation) can occur with respect to l in the n_{fix}-period interval starting in time period t. Likewise, a change is prohibited until at least n_{fix} periods have elapsed since the last period when the price of product $p \in P$ was changed.

We collect pricing restrictions in a restriction set $\mathcal{R}$. For instance, the discussed pricing restrictions can be stored as $\mathcal{R} := \{n_{max}, T_1, T_2, \ldots, T_{|I|}\}$ or as $\mathcal{R} := \{n_{fix}\}$ holding all information required for constraints (39)-(41). The discussed restrictions are chosen exemplary, and the user is free to define $\mathcal{R}$; $\mathcal{R} = \emptyset$ indicates the setting without any pricing restrictions.

Different business environments are obtained from the chosen settings of n_{max} and/or n_{fix} according to the company's business model. For instance, for a B2C business where dynamic pricing is a standard approach, large values for n_{max} and small values for n_{fix} are typical; conversely, for a B2B business where dynamic pricing is only beginning to be implemented, values for n_{max} and/or n_{fix} have to be selected specifically to reflect the available degrees of freedom in price setting.

3.4. Stochastic solution approach

We employ a stochastic solution approach based on a two-stage stochastic program to account for uncertainties in the overall demand values D_t arising over all periods $t \in T$. This solution approach, which tackles the snapshot problem posed at the current time, will be embedded into a rolling horizon procedure in subsection 3.5. Thereby, it will be possible to adjust prices and lot sizes dynamically as prescribed by the user-selected control parameters of the rolling horizon procedure discussed in subsection 3.5. In order to provide pricing information on a rather binding basis (as typical for B2B businesses), the snapshot problem reflects the situation where we have to decide about the pricing scheme in advance (design stage) in order to communicate prices for the periods in T prescribed by decision variables z_{ptl}, ϕ_{pt}, γ_t. Lot sizing actions, contrarily, are obtained from the scenario-dependent lot sizing decisions (recourse stage) prescribed by decision variables $x^{\xi}_{pt}, y^{\xi}_{pt}, i^{\xi}_{pt}, s^{\xi}_{pt}$ for demand scenario $\xi \in \Xi$. Hence, the lot sizing decisions allow for a scenario-dependent evaluation of the costs associated to the scenario-independent pricing decisions. In terminology of two-stage stochastic programming, first-stage decisions on pricing exhibit validity in the form of invariance for every scenario $\xi \in \Xi$, whereas second-stage decisions on lot sizing allow for commencement of the production process as prescribed for any actually observed scenario $\xi \in \Xi$ once it will have been observed. This stochastic solution approach is then repeated for the purpose of replanning as part of the rolling horizon scheme discussed in subsection 3.5. We remark that in this rolling horizon scheme, there is no need to replan lot sizing decisions as long as the realized demand scenario ξ^* fulfills $\xi^* \in \Xi$ since the solution to the integrated pricing and lot sizing problem implicitly encompasses action prescriptions on lot sizing for all $\xi \in \Xi$, i.e., also for $\xi^* \in \Xi$. Problem IPCLSP(T) considered for period set T under uncertainty with a set of demand scenarios Ξ is subsequently denoted by IPCLSP(T,Ξ).

$$
\begin{aligned}
&\text{IPCLSP}(T,\Xi):\\
&\max \quad \Pi(T,\Xi) := \sum_{\xi\in\Xi} Pr^{\xi}\Big(\sum_{p\in P}\sum_{t\in T}\sum_{l\in L_p} R_{pl} U_{pl} \phi_{ptl} D^{\xi}_t \\
&\qquad\qquad - \sum_{p\in P}\sum_{t\in T} (H_{pt} i^{\xi}_{pt} + A_{pt} y^{\xi}_{pt} + S_{pt} s^{\xi}_{pt})\Big)\\
&\text{s.t.} \quad \sum_{p\in P_{ext}} U_p \gamma_t + \sum_{p\in P}\sum_{l\in L_p} U_{pl}\phi_{ptl} = 1 && t\in T\\
&\phi_{ptl} \le M z_{ptl} && p\in P, t\in T, l\in L_p\\
&\phi_{ptl} \le \gamma_t && p\in P, t\in T, l\in L_p\\
&\phi_{ptl} \ge \gamma_t + M(z_{ptl}-1) && p\in P, t\in T, l\in L_p\\
&\sum_{l\in L_p} z_{ptl} = 1 && p\in P, t\in T\\
&i^{\xi}_{p,t-1} + x^{\xi}_{pt} + s^{\xi}_{pt} - D^{\xi}_t \sum_{l\in L_p} U_{pl}\phi_{ptl} = i^{\xi}_{pt} && p\in P, t\in T, \xi\in\Xi\\
&i^{\xi}_{p,\max\{t\,|\,t\in T\}} \ge \bar{i}^{\xi}_p && p\in P, \xi\in\Xi\\
&\sum_{p\in P} x^{\xi}_{pt} \le C_t && t\in T, \xi\in\Xi\\
&x^{\xi}_{pt} \le M y^{\xi}_{pt} && p\in P, t\in T, \xi\in\Xi\\
&x^{\xi}_{pt}, i^{\xi}_{pt}, s^{\xi}_{pt} \ge 0 && p\in P, t\in T, \xi\in\Xi\\
&y^{\xi}_{pt} \in \{0,1\} && p\in P, t\in T, \xi\in\Xi\\
&\gamma_t \ge 0 && t\in T\\
&z_{ptl} \in \{0,1\} && p\in P, t\in T, l\in L_p\\
&\phi_{ptl} \ge 0 && p\in P, t\in T, l\in L_p
\end{aligned}
$$

To make IPCLSP(T,Ξ) computationally tractable, we employ Monte Carlo sampling as a standard approach used for two-stage stochastic programming [17]. Hence, we restrict the set of considered demand scenarios to a number of randomly drawn scenarios. To this end, we draw N demand scenario realizations and store them in set Ξ_N. IPCLSP(T,Ξ) is then solved with $\Xi := \Xi_N$ where $Pr(\xi) := \frac{1}{N}$ for all $\xi \in \Xi_N$, i.e., we solve IPCLSP(T,Ξ_N) as a sample-based problem version of IPCLSP(T,Ξ) providing an approximation $\Pi(T,\Xi_N)$ of $\Pi(T,\Xi)$.

This yields prices $r_{pt} := \sum_{l\in L_p} R_{pl} z_{ptl}$ for product $p \in P$ in period $t \in T$ which are implemented regardless of the realized demand scenario. Contrarily, lot sizing decisions depend on the scenario and cannot be implemented for all demand scenarios. Concerning the lot sizing task, from its integration into the stochastic model we recognize its role as a forward-oriented evaluation function of the pricing scheme in terms of costs which are incurred depending on the pricing scheme. Clearly, in reality, lot sizing is done according to a short lookahead time window for which there is certainty (or almost certainty) about the upcoming demands. In particular, it may be that this realized demand scenario ξ^* has not been part of the sample-based determination of the pricing decisions through

Table 3
Decisions and controls occurring in integrated pricing and lot sizing systems.

	pricing	lot sizing
utilized model	IPCLSP(T,Ξ)	CLSP(T,ξ^*)
decision character	virtual	physical
decisions (independent variables)	price r_{pt}	production quantity x_{pt}
decisions (dependent variables)	demand d_{pt}	inventory level i_{pt}, shortage s_{pt}
replanning periods	T^P_{plan}	T^{LS}_{plan}
planning horizon	h^P	h^{LS}

solving IPCLSP(T,Ξ_n) as a proxy for IPCLSP(T,Ξ), i.e., $\xi^* \notin \Xi$. Therefore, in the rolling horizon procedure discussed in subsection 3.5, the lot sizing problem will be solved for a specific demand scenario ξ^* which is considered as most appropriate for determining the concrete production quantities. Therefore, we compute the demands in demand scenario ξ^* which result from the market shares stored in $\delta_{pt} := \sum_{l\in L_p} U_{pl}\phi_{ptl}$ as

$$d^{\xi^*}_{pt} := D^{\xi^*}_t \sum_{l\in L_p} U_{pl}\phi_{ptl},$$

and we transfer them to the lot sizing problem CLSP(T,ξ^*) in order to determine the concrete production quantities:

CLSP(T,ξ^*):

$$\min \quad C(T,\xi^*) := \sum_{p\in P}\sum_{t\in T}(H_{pt}i_{pt} + A_{pt}y_{pt} + S_{pt}s_{pt})$$

$$\begin{aligned}
\text{s.t.} \quad & i_{p,t-1} + x_{pt} + s_{pt} - d^{\xi^*}_{pt} = i_{pt} && p\in P, t\in T\\
& i_{p,\max\{t\,|\,t\in T\}} \geq \bar{i}_p && p\in P\\
& \sum_{p\in P} x_{pt} \leq C_t && t\in T\\
& x_{pt} \leq M y_{pt} && p\in P, t\in T\\
& x_{pt}, i_{pt}, s_{pt} \geq 0 && p\in P, t\in T\\
& y_{pt} \in \{0,1\} && p\in P, t\in T
\end{aligned}$$

3.5. Rolling horizon scheme

To allow model users to specify the validity duration of a pricing scheme and to react to demand realizations with production schedule adaptations, we embed the stochastic solution approach into a rolling horizon procedure providing the opportunity for reoptimization. The advantage of the rolling horizon procedure lies in its ability to react to previously unanticipated changes concerning the demand process from a certain period onwards. With pricing and lot sizing decisions to be made repetitively, we must account for both types of decisions which, however, typically are evoked in frequencies differing from each other. Hence, to control a lot sizing application with integrated pricing, a rolling horizon framework addressing both decisions must yield the required amount of flexibility for both subproblems. To this end, we denote the sets of upcoming replanning periods by T^P_{plan} and T^{LS}_{plan}, indicating the periods at which the next replanning for pricing and lot sizing will be carried out, respectively. We note that between two successive planning steps of either pricing or lot sizing, the previously computed pricing or lot sizing plan is put into practice as planned without any recomputation other than the update of physical inventory levels. While pricing is carried out for a tactical horizon of $h^P \in \mathbb{N}_0$ periods, lot sizing is carried out for an operational horizon of $h^{LS} \in \mathbb{N}_0$ periods with $h^{LS} \leq h^P$. We further observe that the decisions are of a different nature: While pricing relates to the virtual quantities of price and demand, lot sizing refers to the physical quantities of produced units and inventory levels. As such, lot sizing decisions must account for the limited physical size of inventories and production capacities. For this reason, the consequences of uncertainty must be reviewed more frequently by adjusting lot sizing decisions on a finer temporal scale as compared to pricing decisions. Whilst this would already be achieved by updating inventory statuses once demands are realized and production quantities are implemented, the availability of lot sizing model CLSP(T,ξ^*) also suggests a lot sizing planning adaptation before the integrated pricing and lot sizing model IPCLSP(T,Ξ) is executed for the next time. Table 3 summarizes the most important decisions and controls which are obtained from the usage of models IPCLSP(T,Ξ) and CLSP(T,ξ^*) and utilized for controlling an integrated pricing and lot sizing application, respectively.

With these definitions, we are in a position to formulate the workflow of a rolling horizon scheme for the integrated pricing and lot sizing application in Algorithm 1. We remark that problem IPCLSP(T,Ξ) is solved approximately through problem IPCLSP(T,Ξ_N) as a proxy. As explained previously, in practice – due to the physical restriction of inventory and production capacities, the stochastic influence on actual demand realizations, and the undesirability of having price changes too often – lot sizing decisions typically must be adapted and replanned more frequently than pricing decisions, especially in a B2B context.

Algorithm 1 rollingHorizonProcedure.

Require: overall planning horizon $\overline{T}$, length h^P of planning horizon for each pricing optimization, length h^{LS} of planning horizon for each lot sizing optimization, set T^P_{plan} of pricing reoptimization periods, set T^{LS}_{plan} of lot sizing reoptimization periods, initial inventory levels $\bar{i}_0$ and shortage $\bar{s}_0$, pricing restriction set $\mathcal{R}$, number N of demand scenario samples for uncertainty consideration

$t := 1$
while $t \leq \overline{T}$ **do**
    obtain D_t
    if $t \in T^P_{plan}$ **then**
        $T^P := \{t, t+1, \dots, t+h^P-1\}$
        update demand scenario set Ξ, i.e., update $D^\xi_{t'}$ for $t' \in T^P$ with $t' > t$, $\xi \in \Xi$
        generate demand scenario samples Ξ_N by sampling from Ξ
        obtain $r_{t'}$ for $t' \in T^P$ by solving IPCLSP(T^P, Ξ_N)
    end if
    if $t \in T^{LS}_{plan}$ **then**
        $T^{LS} := \{t, t+1, \dots, t+h^{LS}-1\}$
        fix demand scenario ξ^* over T^{LS}, e.g., according to most probable demand scenario
        obtain $x_{t'}$ for $t' \in T^{LS}$ by solving CLSP(T^{LS}, ξ^*)
    end if
    implement r_t, x_t, compute $\bar{i}_t$ and $\bar{s}_t$
    $t := t + 1$
end while

Ensure: implementation of overall pricing scheme and production schedule

This double horizon approach is illustrated in Fig. 2. It shows how demand scenarios are used in the rolling horizon procedure for the two repetitively occurring subproblems of integrated pricing and capacitated lot sizing (IPCLSP(T, Ξ), to be solved at times t^P) and capacitated lot sizing (CLSP(T, ξ^*), to be solved at times t^{LS}). Whereas in the former, a set of demand scenarios Ξ is considered at time t^P, the latter makes use of an anticipated demand scenario ξ^* which is to be expected at time t^{LS} for the near future.

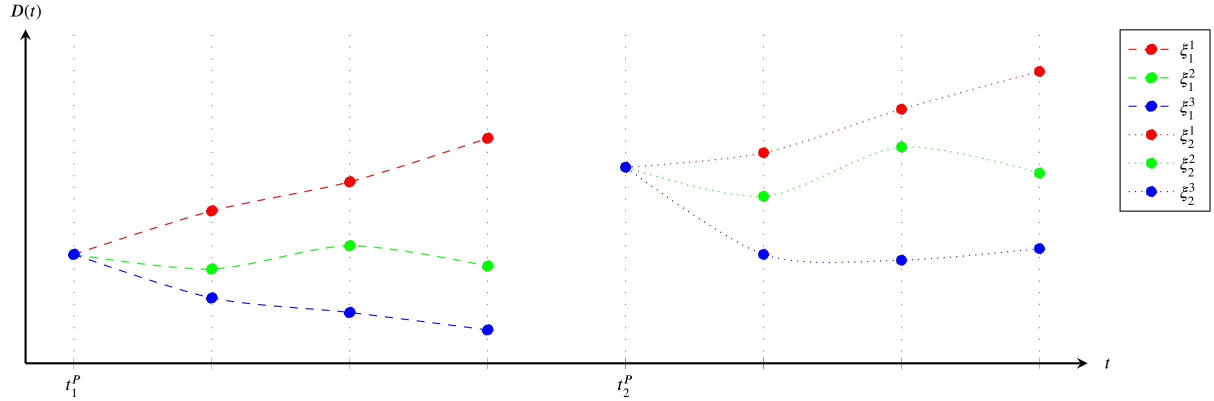
4. Computational experiments

We examine four computational research questions in order to assess the developed methodology's suitability for practical purposes:

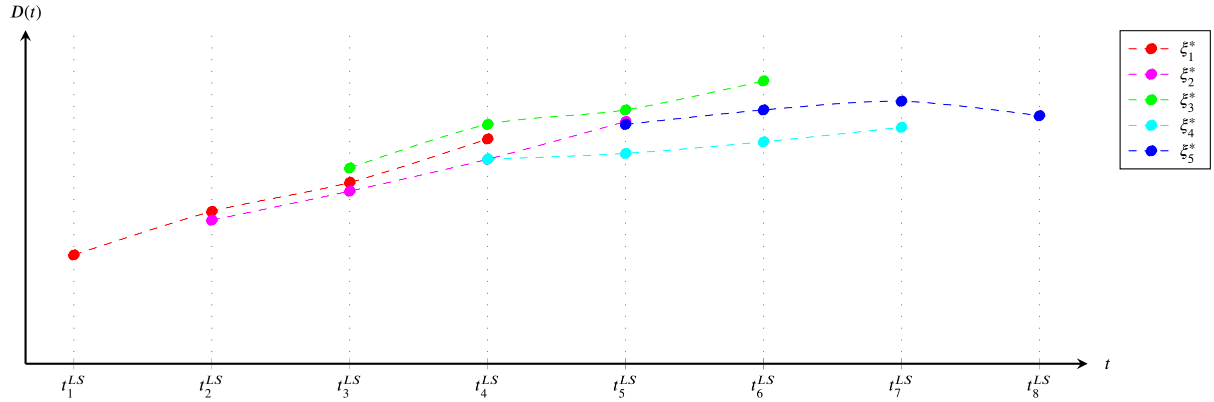
- Influence of problem size on computational time
- Influence of pricing restrictions on number of price changes and profit
- Influence of demand uncertainty on profit
- Benchmarking of dynamic pricing with static pricing

Experiments are executed on a personal desktop computer with Intel Core 3.2 GHz processor and 16 GB RAM under Microsoft Windows 10 (64-bit). Algorithms are coded in Python 3.8; IP models are coded in Python using the docplex modeling library and solved via IBM ILOG CPLEX 20.1.0 solver.

Experiments with an overall planning horizon of one year (52 weeks, $\overline{T} = 52$) are based on an instance generator producing instances with a pricing time horizon of 52 weeks for each solve ($h^P = 52$) and a lot sizing time horizon of 26 weeks for each solve ($h^{LS} = 26$). Pricing reoptimization is executed monthly ($T^P_{plan} = \{1, 5, 9, 13, 17, 22, 26, 31, 35, 39, 44, 48\}$); lot sizing reoptimization is executed weekly ($T^{LS}_{plan} = \{1, 2, 3, \dots, 52\}$). Thus, to ensure that every pricing reoptimization (also the last one in week 48) utilizes forecasted demands of 52 weeks, we draw base demand scenarios for $48 + 52 = 100$ weeks. Subsequently, we discuss the random parameters of the computational experiments. Cost parameters and pricing model parameters are generated through random drawings in a similar fashion as explained by [53]. However, in contrast to existing literature, our data generation explicitly allows for multiple planning horizons to be covered in the rolling horizon procedure, time-dependency of cost parameters, and demand scenarios originating from a stochastic process. With $\#P^{all}$ denoting the sum of the number of own and external products, the overall demand for all products is drawn from a discrete uniform distribution over $\{50 \cdot \#P^{all}, \dots, 250 \cdot \#P^{all}\}$; the same proportionality applies for the weekly overall production capacity for own products drawn from $\{50 \cdot |P|, \dots, 250 \cdot |P|\}$. Price levels originate from $\{1, 2, \dots, 15\}$ and price sensitivities are drawn uniformly from $[-0.75, -0.25]$. Holding cost and fixed cost parameters are drawn from the uniform distribution over $[0.1, 0.5]$ and $[25, 75]$, respectively. The shortage cost parameter is 10 yielding different trade-offs between all cost parameters depending on the drawings of the holding cost and fixed cost parameters. Since both planning time horizons ($h^P = 52$, $h^{LS} = 26$) cover rather large portions of the overall planning horizon ($\overline{T} = 52$), we do not restrict ending inventories in any of the model formulations, i.e., $\bar{i}_p = 0$ for $p \in P$. Demand scenarios are chosen with reference to the base demand scenarios, respectively, by randomly drawing in each period from $[0.75, 1.25]$ in order to obtain the related fraction of the base demand scenario. In 20% of all periods, demand values in each scenario are further perturbed by randomly drawing up- and down-phases in which demand values are reduced or increased by 25%, respectively. Throughout all experiments, we assume three external products. Observe that the number of external products does not contribute to the problem complexity as all external product utilities are computed upfront.



(a) Possible demand scenarios $\xi_1^1, \xi_1^2, \xi_1^3$ for IPCLSP(T_1^P, Ξ) with $T_1^P := \{t_1^P, t_1^P + 1, \ldots, t_1^P + 3\}$ and $\xi_2^1, \xi_2^2, \xi_2^3$ for IPCLSP(T_2^P, Ξ) with $T_2^P := \{t_2^P, t_2^P + 1, \ldots, t_2^P + 3\}$.



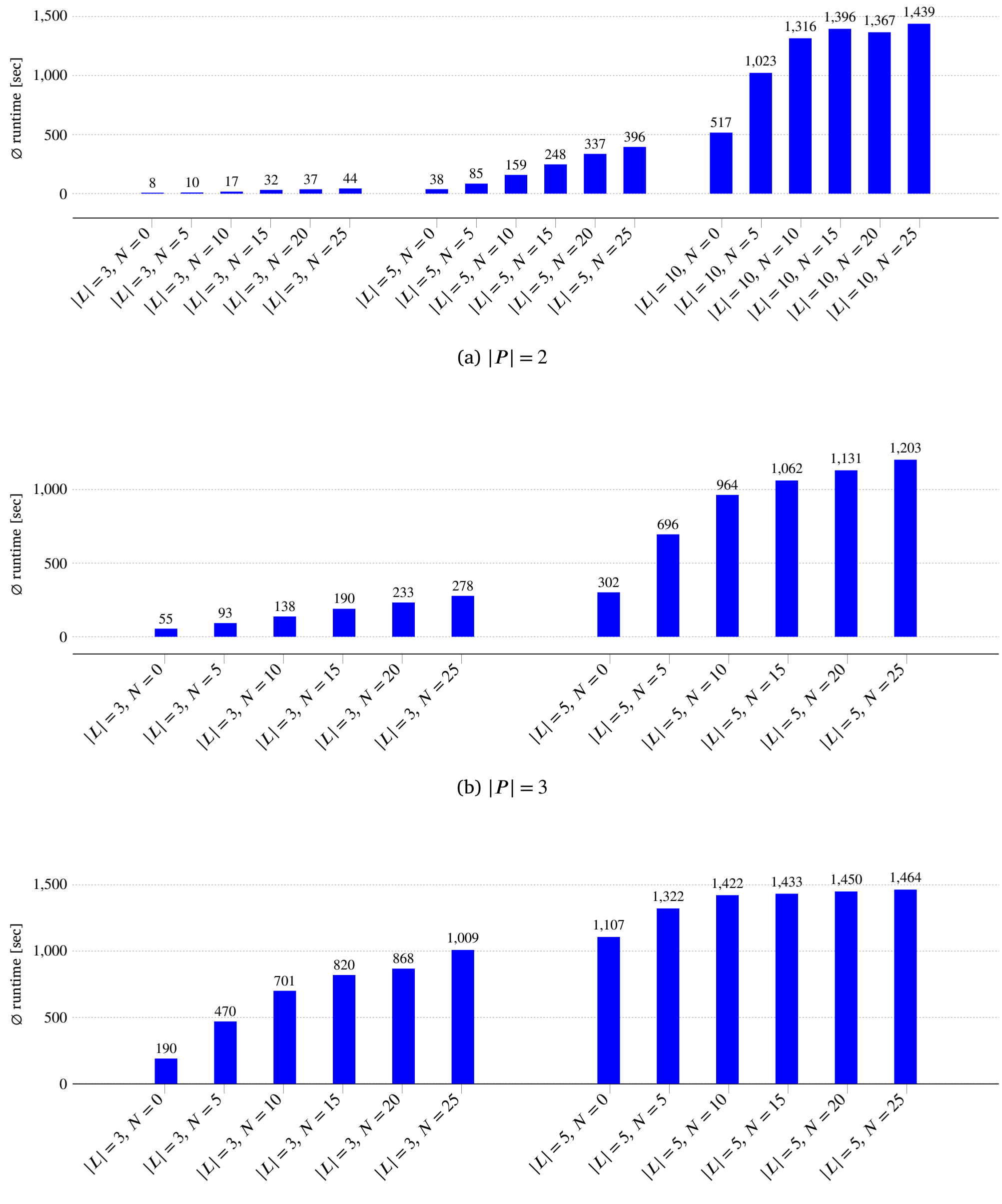
(b) Possible demand scenario ξ_1^* for CLSP(T_1^{LS}, ξ_1^*) with $T_1^{LS} := \{t_1^{LS}, t_2^{LS} + 1, \ldots, t_1^{LS} + 3\}$, ..., ξ_5^* for CLSP(T_5^{LS}, ξ_5^*) with $T_5^{LS} := \{t_5^{LS}, t_5^{LS} + 1, \ldots, t_5^{LS} + 3\}$.

Fig. 2. Consideration of demand scenarios by IPCLSP(T, Ξ) and CLSP(T, ξ^*).

After initial manual experimentation, we have decided – as a result of the incurred computational times – that the design of the systematic experimentation relies on variation of the following parameters which from now on we will refer to as controlled parameters:

- For the number of own products, we consider $|P| \in \{2, 3, 4\}$.
- For the number of price levels, we consider $|L| \in \{3, 5, 10\}$ in the case of $|P| = 2$ and – due to excessive runtimes – $|L| \in \{3, 5\}$ in the case of $|P| \in \{3, 4\}$.
- For the number of samples used in the stochastic procedure, we consider $N \in \{5, 10, 15, 20, 25\}$ and an additional setting (symbolically indicated by $N = 0$) for the deterministic case exhibiting clairvoyance with respect to the realized demand scenario. The latter situation can be thought of as customers having to sign up for their demands in respective periods whilst not allowing for spontaneous buying.
- For the pricing restriction types, we consider $restriction \in \{none, distance, number\}$ where *none* refers to no pricing restrictions, $distance$ refers to a minimum number of periods between price changes of a specific product, and *number* refers to a maximum number of price changes for each product over the entire planning horizon. Specifically, we assume a minimum number of periods between consecutive price changes for the same product of $n_{fix} = 12$ and a maximum number of price changes per product of $n_{max} = 3$.

Subsequently, we refer to a given set of controlled parameters by a controlled parameter configuration (CPC). For each CPC, we obtain the entire parameterization including the random parameters by specifying a seed value for the random instance generator. For each CPC, this procedure is initiated with seed values $\{0, 1, \ldots, 19\}$, i.e., we consider 20 different instances for each CPC. Overall, depending on $|P|, |L|, N, restriction$ and the seed values, we hence analyze $1 \cdot 3 \cdot (5+1) \cdot 3 \cdot 20 = 1080$ for $|P| = 2$ and $1 \cdot 2 \cdot (5+1) \cdot 3 \cdot 20 = 720$ for $|P| \in \{3, 4\}$, i.e., a total of $1080 + 720 + 720 = 2520$ problem instances. Recalling that within each instance, we carry out 12



(a) $|P| = 2$

(b) $|P| = 3$

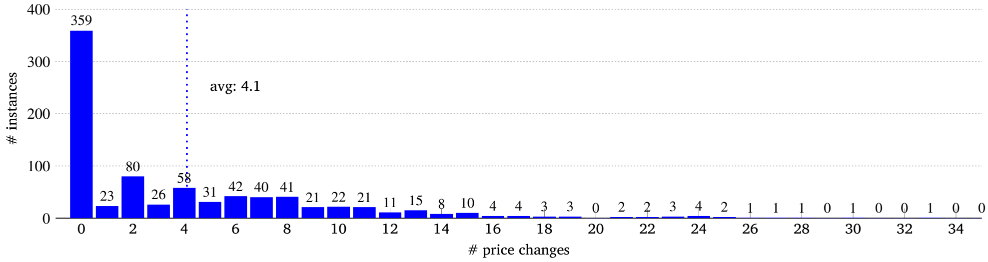
(c) $|P| = 4$

Fig. 3. Average runtime per instance over all seeds and restriction types over all instances with prescribed CPC.

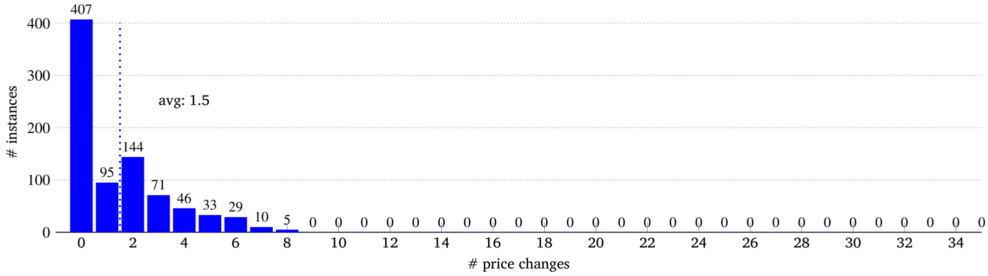
pricing reoptimizations and 52 lot sizing reoptimizations, we recognize the computational burden of this experimentation design. As a result, for each pricing reoptimization we limit the maximum computational time to 120 seconds and employ an optimality gap of 0.01. Manual experimentation has shown that extending the maximum computational time or decreasing the optimality gap further yields no substantial additional benefit as a result of the overall methodology's rolling horizon character and the randomness of demand values.

4.1. Influence of problem size on computational time

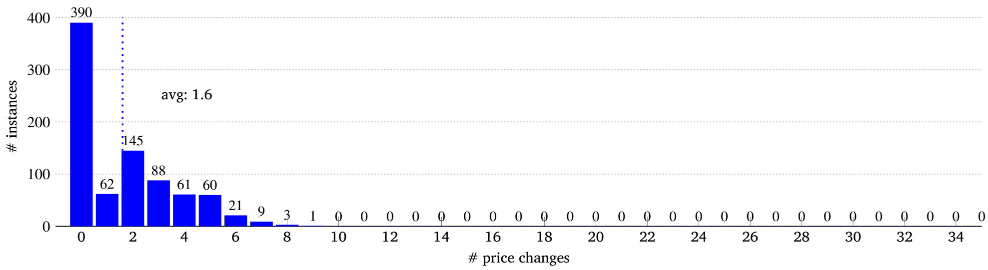
For all examined CPCs, Fig. 3 summarizes the computational effort for running the entire rolling horizon approach from Algorithm 1 in the form of the average computational time required over all restriction types and seed values. We first remark that the upper bound of approximately 1500 seconds results from delimiting the solve time for each instance of the pricing problem to 120 seconds. As seen in Fig. 3a, the computational effort becomes excessively high in case of ten potential price levels already for two products. Therefore, we restrict the further analysis to three and five price levels as illustrated in Fig. 3b and Fig. 3c, respectively. Over all examined numbers of products, we find that increasing the sample size N involved in the uncertainty set Ξ_N of model for-



(a) $restriction = none$



(b) $restriction = distance$



(c) $restriction = number$

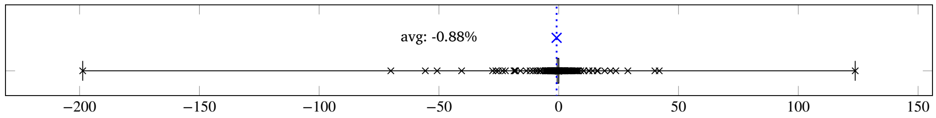
Fig. 4. Distribution of number of price changes depending on pricing restriction type over all instances with prescribed CPC.

mulation IPCLSP(T, Ξ_N) leads to a moderate increase of computational times. Nonetheless, this can lead to a substantial consumption of computational resources as seen to the right of each subfigure. However, the number of considered own products contributes more significantly to the rise of computational times as found from comparing Fig. 3a to Fig. 3c relative to each other. In fact, for $|P| = 4$ and $N \geq 10$ this nearly always leads to utilizing the full 120 seconds granted for solving IPCLSP(T, Ξ_N) which is conducted for each month of the considered overall time horizon of one year.

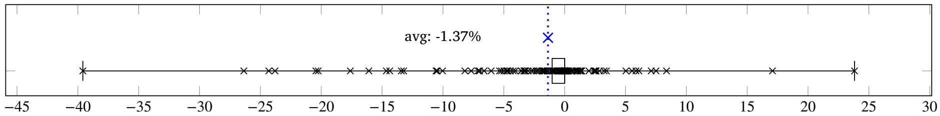
Managerial insight: Computational requirements of an ordinary desktop computer are not capable of handling realistic settings of integrated pricing and lot sizing with five or more products in comparable settings. This result points towards the necessity of developing heuristics both for the specific setting of integrated pricing and lot sizing and for the general task of integrated pricing and production operations when applied to larger product programs.

4.2. Influence of pricing restrictions

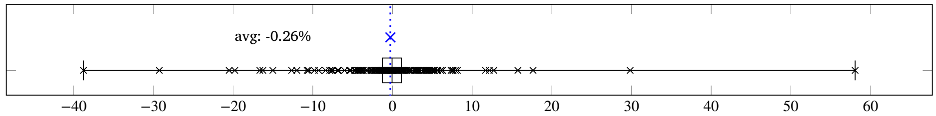
We first analyze the influence of employing pricing restrictions on the difference in the number of price changes compared to the case where no restrictions are imposed upon price setting. Fig. 4 illustrates the effectiveness of the pricing restriction requirements imposed over the entire planning horizon of one year. Regardless of the restriction type, there are plenty of instances without any price changes, i.e., for these instances no price sensitivity is observed and the methodology serves for finding the optimal fixed selection of prices. Over all instances, concerning the reduction of price changes, we see that their average number drops from 4.1 per year in case of $restriction = none$ (cf. Fig. 4a) to 1.5 for $restriction = distance$ (cf. Fig. 4b) and to 1.6 for $restriction = number$ (cf. Fig. 4c). Moreover, the maximum number of price changes drops significantly from 34 to 8 and 9, respectively. Likewise, both distributions on



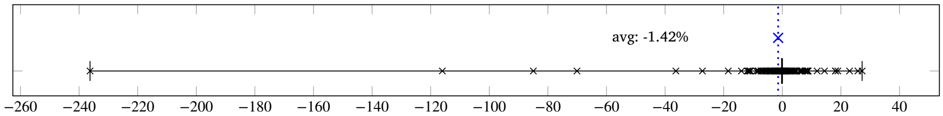
(a) $|P| = 2$, $restriction = distance$



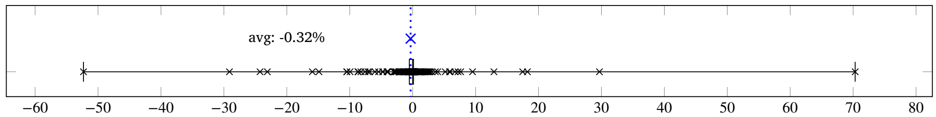
(b) $|P| = 3$, $restriction = distance$



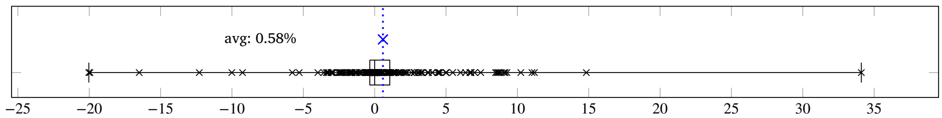
(c) $|P| = 4$, $restriction = distance$



(d) $|P| = 2$, $restriction = number$



(e) $|P| = 3$, $restriction = number$



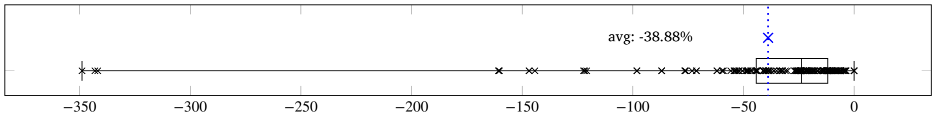
(f) $|P| = 4$, $restriction = number$

Fig. 5. Boxplot for relative profit deterioration (in %) due to pricing restrictions over all instances with prescribed CPC.

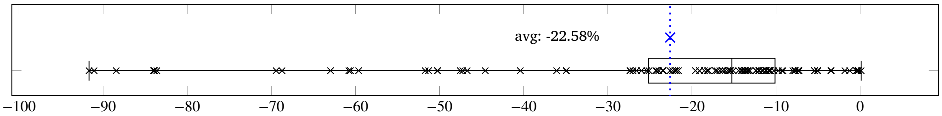
the number of price changes associated with $restriction \in \{distance, number\}$ are shifted to smaller values and appear in a compressed form. Hence, both $restriction = distance$ and $restriction = number$ lead to comparable results.

As a further investigation, we scrutinize the effect of the pricing restrictions on the achievable profits. Surprisingly, on average the impact of the pricing restrictions turns out to be of minor magnitude. Fig. 5 reveals that average deterioration is below 2% regardless of the number of products and restriction type with the largest fraction of deterioration close to negligible. Hence, solving the model formulation IPCLSP(T, Ξ_N) even after incorporating the pricing restrictions prescribed by $restriction \in \{distance, number\}$ leads only to a minor change in the achievable profit. Thus, there are possibilities resulting from the combinatorial nature of the problem setting which allow to shift to other solutions of comparable objective while adhering to the pricing restrictions.

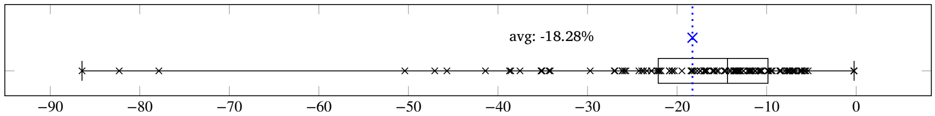
Managerial insight: Pricing restrictions shall be employed only if it is known that those will have a positive effect upon customer perception of the company's pricing behavior. We remark that on the instance level, however, both deterioration and improvement of substantial magnitude can be observed. This is due to the rolling horizon procedure and the non-clairvoyant nature of demand knowledge. Hence, pricing decisions carried out at an early stage of the planning horizon, may severely trim the pricing options in later periods due to pricing restrictions being delimited. Consequently, resulting from the inability to foresee future demands, it cannot be avoided that initial pricing decisions are regretted later on.



(a) $|P| = 2$



(b) $|P| = 3$



(c) $|P| = 4$

Fig. 6. Boxplot for relative profit deterioration (in %) due to demand uncertainty in case of $N = 25$ demand scenario samples compared to case of perfect information over all instances with prescribed CPC.

4.3. Influence of demand uncertainty

To analyze the effect of uncertainty, we compare the achievable profits from the setting with uncertainty to those from the setting where the demand information of the upcoming planning horizons (both concerning pricing and lot sizing) is known to the decision maker deterministically in a clairvoyant fashion ahead of each planning step. In other words, we assume that there exists one realized demand scenario for each problem instance, and we compute the value of perfect information (in the operations research community sometimes also referred to as value of clairvoyance or lookahead) with respect to weekly overall product demands. These demands remain uncertain to the decision maker in the non-clairvoyant setting, and the uncertainty is translated into the randomly drawn scenarios (N demand samples representing possible demand realizations over the planning horizon). Since computational results are carried out with sample sizes $N \in \{5, 10, 15, 20, 25\}$, the case of $N = 25$ demand scenario samples represents the most fine grained resolution of uncertain demand information. Therefore, we restrict the presentation to this case. Fig. 6 displays the deterioration that results from not knowing the upcoming demands precisely while having to rely on the distributional information obtained from sampling as reflected by the uncertainty set Ξ_{25} in model IPCLSP(T, Ξ_{25}). Irrespective of the specific CPC, Fig. 6a to Fig. 6c with average performance deterioration between 18.28% and 38.88% demonstrate that achievable profits are rather sensitive to the type of knowledge of demand information. This stands in stark contrast to the price restriction aspect investigated in the previous subsection 4.2.

Managerial insight: In terms of impact on achievable profits, strengthening the reliability of available demand information would represent a significant step towards eliciting the profitability potential which could then be realized through the developed integrated pricing and lot sizing methodology. Hence, integrating pricing and lot sizing yields the potential of playing a significant role in an effective enterprise management.

4.4. Benchmarking with static pricing

We compare the profits obtained from dynamic pricing with those obtained under static pricing. To obtain a worst-case analysis, results from our developed approach compete against those of the best possible static pricing scheme. Clearly, when no price changes are employed (cf. subsection 4.2), then there is no difference between dynamic and static pricing. Exemplary for a comparison between static and dynamic pricing in case of two own products in the deterministic case, Fig. 7 shows the improvements from dynamic pricing for problem instances with at least one price change. Each mark corresponds to an instance where a price change occurs and gives the relative improvement in profit due to this price change when compared to the best static pricing scheme. We observe a substantial increase in attainable profits once price changes are employed in an instance. In this setting, the overall average of relative profit improvement over all instances with at least one price change amounts to 4.1%; and depending on the number of price changes, the average improvement lies between 0.99% and 8.21%. A similar analysis can be applied to settings with a different number of products and/or demand scenarios. We conclude that dynamic pricing leads to substantial improvement when price-sensitivity can be attributed to the problem instance under consideration. We remark that the utilization of static pricing schemes may also serve as a construction heuristic for generating feasible pricing schemes, upon which related production schedules can be derived.

Managerial insight: Integrating pricing and lot sizing as well as allowing for dynamic pricing is particularly worthwhile when the problem instance data leads to an optimal pricing scheme with several price changes. Decision makers are encouraged to inspect

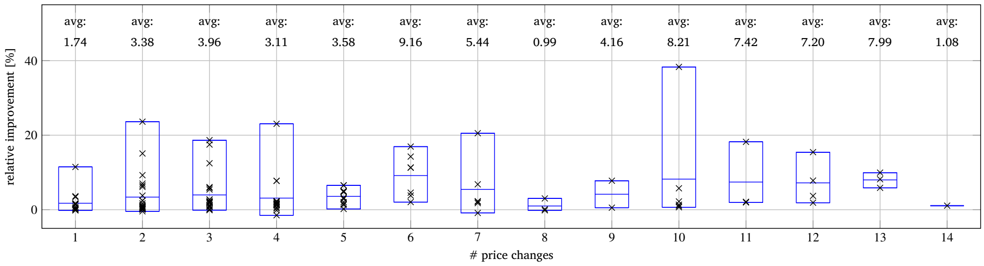


Fig. 7. Relative improvement in attained profits of dynamic pricing compared to static pricing in deterministic case for $|P| = 2$.

instance data beforehand for implicit evidence pointing to potential price adaptations throughout the planning horizon. Typical root causes can be attributed to the interplay between production capacities, seasonality, price sensitivities, and cost rates.

5. Conclusion and outlook

With the integrated pricing and capacitated lot sizing problem, this paper examines a fundamental setting at the interface between marketing activities and production operations. The devised methodology allows for the repetitive and adaptive determination of product prices to control the demand side in a business environment coined by uncertain future demands. Such environments require a flexible determination of production schedules in order to match product supply and demand. The driving factors of complexity, namely nonlinearity and uncertainty, are resolved methodologically through discretization and linearization in the former case and through sampling-based Monte Carlo simulation in the latter case. Time dynamics are tackled through the adaptive consideration of changes in demand forecasts as part of a rolling horizon procedure which accounts for pricing (as design decisions) and lot sizing (as recourse decisions) alike. Likewise, the outline incorporates practical pricing restrictions which have to be asserted in an environment where customer preference for price consistency must be acknowledged.

In terms of managerial insights, we show that integrating pricing and production planning yields an effective approach of proactively managing demand via price setting and matching it with supply as required by the operations of a capacitated production system. At the same time, the developed rolling horizon procedure facilitates reactive decision making with respect to lot sizing activities, factoring in observed realizations of demand scenario paths. In particular, the exploitation of revenue and cost potentials becomes possible which would remain untapped in case of separate considerations of pricing and lot sizing, respectively. Likewise, pricing restrictions implement the idea of pricing consistency over time, thereby ensuring long-term customer loyalty.

For future research, we recommend a generalization to the setting of integrated pricing and capacitated resource planning with several resources to be managed simultaneously. Moreover, as concluded from the computational times for problem instances of even modest size, (meta-) heuristics need to be developed to cope with a larger number of products to be priced over time. Another extension of the devised methodology concerns the incorporation of several customer segments allowing for a further fine-grained optimization of prices on the one hand and customer-dedicated production on the other hand. This extension would be particularly fitting to exclusive goods industries with orders built in a custom shop environment. Finally, we believe that the sensitivity with respect to uncertainty in other model components deserves research attention in order to promote robustness in pricing schemes, e.g., with respect to different models of customer choice.

CRediT authorship contribution statement

Fabian Dunke: Conceptualization, Formal analysis, Investigation, Methodology, Validation, Writing – original draft, Writing – review & editing. **Stefan Nickel:** Conceptualization, Investigation, Methodology, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The Python source code related to this research paper is available online at https://gitlab.kit.edu/fabian.dunke/integratedPricingLotSizing.

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