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The 2024 phononic crystals roadmap

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Roadmap

The 2024 phononic crystals roadmap

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Abstract

Over the past 3 decades, phononic crystals experienced revolutionary development for understanding and utilizing mechanical waves by exploring interaction between mechanical waves and structures. With the significant advances in manufacture technologies from nanoscale to macroscale, phononic crystals attract researchers from diverse disciplines to study abundant directions such as bandgaps, dispersion engineering, novel modes, reconfigurable control, efficient design algorithms and so on. The aim of this roadmap is to present the current state of the art, an overview of properties, functions and applications of phononic crystals, opinions on the challenges and opportunities. The various perspectives cover wide topics on basic property, homogenization, machine learning assisted design, topological, non-Hermitian, nonreciprocal, nanoscale, chiral, nonlocal, active, spatiotemporal, hyperuniform properties of phononic crystals, and applications in underwater acoustics, seismic wave protection, vibration and noise control, thermal transport, sensing, acoustic tweezers, written by over 40 renown experts. It is also intended to guide researchers, funding agencies and industry in identifying new prospects for phononic crystals in the upcoming years.

Keywords: phononic crystals, bandgap, homogenization, topological phononic crystals, nonreciprocal phononic crystals, noise and vibration control, nanophononic crystals

Contents

1. Introduction 4
2. Basic properties of phononic crystals 7
3. Homogenization of acoustic and elastic metamaterials 10
4. Phononic crystals with machine learning 12
5. Topological and non-Hermitian phononic crystals 15
6. Nonreciprocal phononic crystals 18
7. Nanophononic crystals 21
8. Chirality and acoustic activity 24
9. Willis phononic crystals and metamaterials 26
10. Active and tunable phononic crystals 29
11. Spatiotemporal phononic crystals 32
12. Functions and properties—hyperuniformity and correlated disorder in phononic structures to control acoustic waves 35
13. Pentamode materials for underwater acoustics control 38
14. Seismic phononic crystals 40
15. Piezoelectric phononic crystals for vibro-acoustic control 43
16. Damped phononic crystals and the concept of metadamping 46
17. Thermal transport in phononic crystals 49
18. Phononic crystals for sensing applications 51
19. Phononic crystals for acoustic tweezers 54
20. Lattice structures for sound absorption 56

Data availability statement 58

Reference 59

1. Introduction

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Status

In the past 3 decades, phononic crystals have been experiencing revolutionary development for providing a new strategy to manipulate mechanical waves by artificially designed structures [1, 2]. As seen figure 1(a), the number of publications in this topic shows dramatical increasing tendence from 1993 to 2023. In fact, many publications related with phononic crystals may also use other terms, so that the practical number of publication is higher. For instance, resonant sonic or phononic crystals are widely recognized as acoustic or elastic metamaterials since the pioneer work in 2000 [3]. Focusing on the fundamental scientific problem, interaction between mechanical waves and structures, phononic crystals enable us to discover on-demand and even abnormal wave behaviors in artificial structures. Although phononic crystals normally refer to periodic structures, thanks to the homogenization theory, gradient phononic crystals are designed to steer wavefront pattern efficiently [4]. When the thickness of the entire gradient structure is subwavelength, it is also named as acoustic or elastic metasurfaces [5, 6]. Latterly, multiple functions such as excellent mechanical and wave properties by the same structure are also performed, which is widely recognized as metastructures [7]. From phononic crystals to metamaterials, metasurfaces and metastructures, the frontiers of phononic crystals become broader.

According to fundamental properties, the studies of phononic crystals can be casted into numerous categories, such as linear or nonlinear, reciprocal or nonreciprocal, topological or trivial, Hermitian or non-Hermitian, passive or active, ordered or random, symmetric or asymmetric, local or nonlocal, time-independent or time-dependent, lossless or damped, which attract researchers from diverse disciplines. Different from other classic waves, due to the effect of boundary conditions mechanical waves have multiple polarizations and can be further coupled into new kinds of waves, especially wave modes in half-space surface, plates and beams, which are the basic structures for many research and application objects. Consequently, phononic crystals can contribute to various demands in different communities and boost the interdisciplinary research. From figure 1(b), engineering, physics and materials science possess the top three in the number of publications among the branches of research area. Despite the importance of the fundamental research, phononic crystals exhibit great potentials in applications, such as vibration and noise control, sensing, structural health monitoring, heat control, as possible revolutionary solutions comparing to traditional ways. Driving from both fundamental and applicable aspects, funding agency from different countries/regions like China, US, Spain, Germany, EU, Japan, continuously support the development of phononic crystals as shown in figure 1(c).

Current and future challenges

The development of phononic crystals represents an integrative process as 'design-fabrication-verification-application': (1) In the design step, the forward design employs wave dynamics mechanisms to calculate output wave properties for considered geometric structures; the backward design employs various optimization algorithms and wave physics to obtain suitable geometric configurations according to the demanded functions. However, most studies focus on geometric configuration to generate desired functions without considering the rich properties of materials that may promote the overall performance of phononic crystals beyond wave functions; in addition, how to properly design phononic crystals considering the fabricating constraint and application environment is challenging. (2) In the fabrication step, designed phononic structures with excellent wave functions may require very high geometric precision and complex configuration, resulting in the difficulty in fabrication. The development of additive manufacturing helps to the fabrication of phononic crystals significantly in the past decade. However, there are limited material choice and printing precision generally according to the fabricating technologies. How to fabricate phononic crystals cheaply, efficiently for quantity production remains challenging. It strongly requires to combine the geometric and fabricating variates to address the challenges. (3) In the verification step, in principle the targeted wave functions from the design can be validated by numerical tools and experiments with existing technologies from nanoscale to macroscale. However, since multiple fabricating errors appear, the performance of wave functions may decrease or even be distorted. It is also necessary to diagnose the quality of phononic crystals and evaluate the corresponding wave functions. (4) In the application step, it is witnessed that the applicable functions of phononic crystals are explored largely, especially for noise and vibration control in different industries such as vehicles transportation, aerospace engineering, civil engineering, *et al.* The possible applicable environment of phononic crystals can be complex and dynamic. For example, the wave sources in practice may be multiple, high-intensity and random; the material properties will change during times; the designed structures may bear different loads; defects may

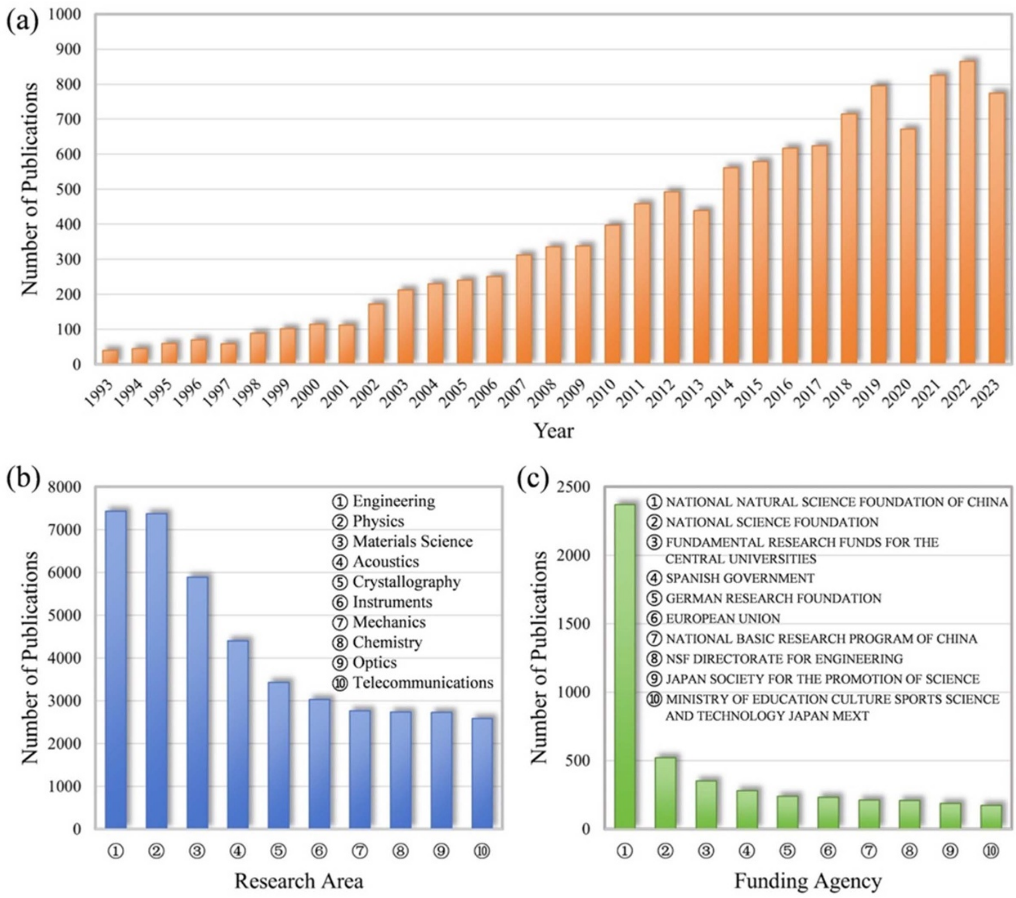

Figure 1. From 1993 to 2023, the number of publications with (a) year, (b) research area and (c) funding agency by searching from the web of science refined by topic 'phononic crystals' or 'sonic crystals' on 18 March 2024.

be brought into the structures. How to improve the reliability of phononic crystals for applications or even commercialization is challenging.

Advances in science and technology to meet challenges

The advances in intelligent algorithms such as machine learning and topological optimization, and new physical theories such as topological insulators and non-Hermitian physics will help to explore more powerful and robust functions of phononic crystals and inverse design phononic crystals according to the fabrication and application demands [8]. The essential properties of mechanical waves in different media, such as loss, multiphysics coupling, dispersive, multimode and mode conversation, can help to explore new physical behaviors in mechanical waves system. In addition, phononic crystals also provide a platform to realize new ideas of topological and non-Hermitian physics, such as high-order topological insulators, which are not easy to validate in other physical systems [9]. The technological upgrade in fabrication in additive manufacturing, injection molding, computer numerical control lathe process and so on will enable to promote the fabricating efficiency and precision and decrease cost. The development of NDT and structural health monitoring including linear and nonlinear ultrasonic testing can help to diagnose the quality of phononic crystals even during the fabricating process [10]. Considering the application demands from different industries, the reliability and durability properties are suggested to be analyzed. More communications between academic and industrial communities are encouraged to bring the phononic crystals from laboratory to society.

Concluding remarks

Over the past 30 years, the development of phononic crystals has been driving new frontiers in wave physics from nano to macro scale. This Phononic Crystals Roadmap consists of a serious of short and formalized sections with different directions of phononic crystals representing by leading scientists from different countries, as seen in figure 2. The object of this Roadmap is to provide the state of arts, challenging and

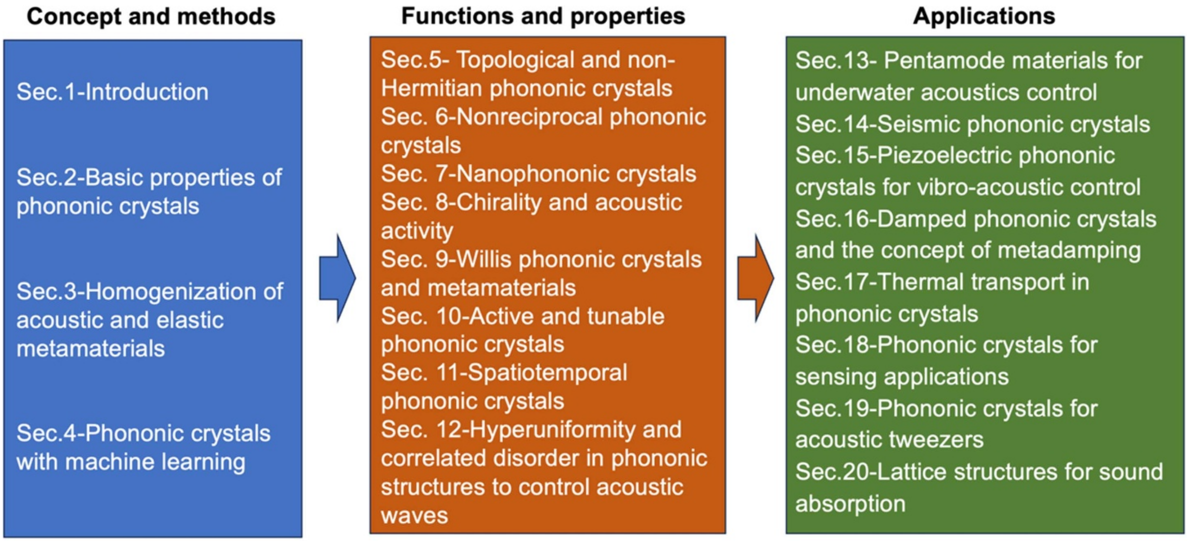

Figure 2. Flow chart presenting the sections of the roadmap article. It consists of three main parts: concept and methods, functions and properties, and applications.

new opportunity of the development of phononic crystals. It should be noted that there are also other aspects of phononic crystals not fully covered in this roadmap, such as aperiodic materials, gradient materials, quasiperiodic/quasicrystalline materials and fractal/hierarchical designs, which leaves opportunities for discussions in future. We call for more collaborations among different subjects and between academic and industrial communities, more support from non-profit and profit organizations, more effective training for new generations of scientists and engineers, to further develop and make good use of phononic crystals in the next decades.

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2. Basic properties of phononic crystals

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Status

Phononic crystals (PCs), proposed thirty years ago, have opened up many new avenues in our ability to control and manipulate acoustic/elastic waves propagation, both through advances in the demonstration and discovery of fundamental physical phenomena, and through the proposal of innovative applications. PCs consist of periodic arrangements of a unit cell composed of one or more scatterers in a host medium [1, 2]. Many PCs functionalities derive from judicious engineering of their dispersion curves and in particular their bandgaps, based on an appropriate choice of geometrical (crystal lattice, inclusion and unit cell shapes) and material (solid or fluid, acoustic and elastic properties) parameters. Due to the periodicity of the structure, the bandgaps can originate from the Bragg scattering mechanism, even though their existence and width can be also affected by the internal resonances of the scatterers that occur in the same frequency range. However, the inclusions can be designed to exhibit resonances at different frequencies and contribute to the opening of bandgaps based on the hybridization mechanism resulting from avoided-crossing of dispersion curves, the so-called locally resonant bandgaps. When the latter occur at very low frequency (sub-wavelength) regime, the whole structure can be described by dynamical effective properties [3] giving rise to the phenomena of negative mass density or stiffness in the context of acoustic metamaterials, and even to Willis type constitutive equations in more complex structures.

Since the wave propagation is prohibited in the bandgaps, their evanescence was proposed very early to be exploited for efficient sound isolation, but also confinement, waveguiding and filtering phenomena in waveguides and cavities which are useful in signal processing and telecom and information technology, acoustic wave devices, optomechanics (OM), sensing or energy harvesting. From the theoretical side, while the early studies mostly focused on two-dimensional (2D) PCs, the computation and demonstration of dispersion curves and bandgaps were extended to 3D, to semi-infinite PCs supporting surface acoustic waves, and to the Lamb modes in structured plates. In particular, let us mention the proposal of the new platform of pillared PCs constituted [11] by an array of pillars on a plate or on a substrate. Indeed, these structures can exhibit both Bragg gaps and sub-wavelength gaps, the latter resulting from the local resonances of individual pillars.

Current and future challenges

From early 2000's, the refractive properties of PCs were also investigated such as negative refraction phenomena and their applications in imaging and sub-wavelength focusing, self-collimation and beam-splitting, design of cloaking, superlens and hyperlens devices. The first experimental designs of PCs were made at macroscopic scale down to sub mm, spanning the kHz and MHz frequency range. However, thanks to the continuous progress in the nanofabrication as well as in self-assembling of colloids, micro and nanostructures PCs were fabricated and the range of frequency was extended to GHz and even sub-THz. These achievements were very beneficial, not only for telecom applications, but also for the development of nanoscale OM where an enhanced interaction between phonons and photons confined in the same cavity or waveguide can be expected. Also, the effect of structuration and designed defects could be investigated for nanoscale thermal transport and potential applications in thermoelectrics and thermal diode. In parallel, many advancements were achieved in the very low frequency regime in the study of mechanical metamaterials including pentamode crystals with liquid-like behavior, auxetic and chiral structures, as well as in the design of sound absorbing structures and seismic metamaterials that are used to prevent the environment from damage or harmful vibrations. Advances in additive manufacturing and the addition of piezoelectric materials or components have also expanded the design space for architected metamaterials and opened up new possibilities for wave excitation, tunability and active devices by the application of external stimuli, such as a stress in elastomeric structures [12], an electric or magnetic field with piezoelectric or magnetoelastic materials, or the change of temperature or matter. Most of the above concepts, properties and phenomena continue their growing in current research [13–16] and have created new challenges including the search of more advanced design of bandgaps and new functionalities in architected materials such as tunability, incorporation of stimuli or active components, non-linear effects such as in granular media or origami structures, gradient index devices, lightweight structures with high mechanical properties, more elaborate sensors and energy harvesting geometries, thermal transport at nanoscale,… Over the past decade, we have also witnessed the emergence of new concepts and phenomena, often stemming from condensed matter physics or photonics, which have given rise to several physical novelties and applications. One example is the opening up to the topic of topological phononics that has led to the conception of various topological phases emulating the analogs of Hall, valley-Hall and spin-Hall effects and the demonstration

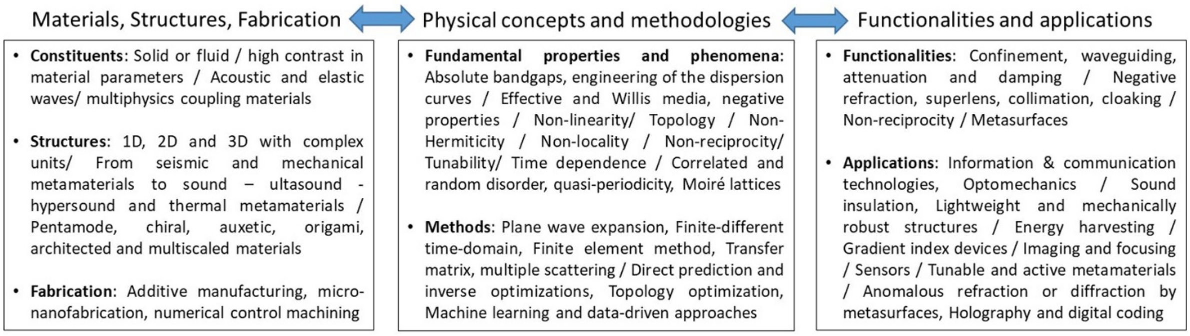

Figure 3. Brief summary of current and challenging topics in phononic crystals and metamaterials, from fundamental concepts to structures and manufacturing, through to functionalities and applications.

of robust non-reciprocal edge states, or corner, vortex and Fano topological modes. Current interests include [9, 17] synthetic spaces, Weyl semi-metal phases, non-Abelian topological states, fragile topology, extension to quasi-periodic and disordered structures and discovery of the analogue of Anderson topological insulators, Moiré lattices, …

Advances in science and technology to meet challenges

The search of new physical concepts or mechanisms and advanced functionalities, as well as advances in manufacturing at different scales, with more complex geometries and using a wider variety of materials, are helping to meet these challenges and pave the way for more practical or industrial applications.

Besides the topological phononics, novel phenomena were evidenced in recent years by the incorporation of non-Hermiticity (NH) in periodic and finite phononic systems [18, 19] which is introduced by means of effective loss and gain, in particular using piezo-components. The complex spectra of the NH systems can exhibit exceptional points in parity-time symmetry structures, with the coalescence of the eigenvalues and eigenvectors and enhanced sensitivity to small perturbations. They also enable the intriguing skin effect which can be exploited for non-reciprocal propagation or geometry dependent localization and may find applications in energy harvesting, sensing or defect detection. The topology of the NH energy spectra has also been investigated while the tuning of NH effect can induce nontrivial topological phases both in periodic and disordered structures. More generally, the incorporation of NH in topological structures will be beneficial to control and manipulate the propagation of topological robust states. Non-reciprocal scatterings and behaviors have also been achieved [20] in activated materials with a modulation of their constitutive properties in space and time or by means of nonlinear media. Let us also mention the rapid growth of sub-wavelength acoustic metasurfaces [5, 21] allowing the realization of anomalous reflection, refraction and diffraction phenomena which can overcome the bulky size of phononic crystals.

The progress in micro and nanofabrication has a major influence in developing PC-based OM structures [13, 16, 22, 23] working at high phonon frequencies of several GHz, including synchronization or sensing phenomena in coupled cavities. Recent studies are dealing with the nano-opto-electro-mechanical systems (NOEMS) where the interaction between electrons, photons and phonons and the available degrees of freedom will lead to advancing the potential of low power information processing and transmitting and may enable multiple functions for coherent signal processing in the classical and quantum domains [22, 23]. Also, OM effects based on robust topological edge modes becomes a currently challenging topic. Finally, the incorporation of metallic nano-antennas enables the use of acousto-plasmonic coupling for the excitation and detection of acoustic phonons. At macroscale, additive manufacturing technologies involving multiple materials are developing for the purpose of multi-functionalities such as lightweight, load-bearing, energy absorbing, sound and noise reduction *et al.*

Finally, the development of multiscale theories and the extension of the parameters space from linear, real and static to nonlinear, complex and time dynamic will help to make more efficient and smart phononic crystals. This research direction can take benefit from the use of machine learning and artificial intelligence [24, 25] that greatly enhance the exploration space and make data-driven design an emerging approach for the realization of high-performance phononic metamaterials.

Concluding remarks

Since the first proposal of phononic crystals, a large number of physical concepts have been developed for controlling the propagation of acoustic/elastic waves and innovative designs satisfying various functionalities have been realized. Figure 3 briefly summarizes the general framework of the development of phononic crystals. Progress in the field of phononics parallels that of photonics, although greater freedom can be found in the choice of materials (fluids or solids, contrasting stiffness, density or absorption) and in the ease of manufacturing acoustic and mechanical metamaterials enabling the demonstration of some physical phenomena which are difficult to achieve in other fields. The last decade has seen the emergence of several new concepts such as topology, non-Hermiticity, non-reciprocity, as well as the design of more efficient devices for specific applications such as metasurfaces, optomechanics, non-linear structures, sound insulation and energy harvesting. They have paved the way for many exciting new research topics that will be pursued over the next decade in interdisciplinary work combining fundamental and applied physics, acoustics, mechanics, chemistry, artificial intelligence and mathematics.

3. Homogenization of acoustic and elastic metamaterials

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Status

The propagation of waves through complex heterogeneous media involves numerous multiple scattering processes, making their mathematical description challenging. When the propagation occurs at wavelengths larger than the average distance between particles, the excited fields cannot distinguish between individual inhomogeneities and perceive the medium as a single, locally homogeneous material. This regime is called the homogenization limit, and the objective of homogenization theory is to describe this material with a family of effective constitutive parameters that relate some averaged fields. This way, all complex multiple scattering processes are encapsulated in the effective parameters, making the global propagation of waves akin to that through homogeneous materials [26].

Heterogeneous materials typically consist of a matrix or background containing ordered or disordered scatterers. Quasi-static homogenization theories retrieve the effective parameters of these materials as averaged parameters weighted by the filling fraction or volume fraction of the scatterers. Thus, the effective parameters are similar to those of the background or the inhomogeneities, depending on this parameter and even the spatial distribution of scatterers.

Although simple from the conceptual point of view, for the specific case of acoustic and elastic composites this kind of materials can lead to very interesting effective parameters, where mixing soft and rigid scatterers in a given background it is possible to obtain transparent materials with different propagation velocities than that of the background, which can be used for instance for the design of advanced refractive lenses [4].

Current and future challenges

Quasi-static homogenization theories can also result in materials with effective properties not present in the constituent materials, such as anisotropic acoustic fluid-like materials [27], where the effective composite, given its internal structure, exhibits fluid-like behavior but with anisotropic mass density. These materials are essential for fabricating acoustic cloaking shells and similar devices predicted within the framework of transformation acoustics [28, 29]. Despite their long history, fabricating such materials remains a challenging problem.

The quasi-static homogenization procedure assumes a very long wavelength limit in developing the mathematical derivations of the effective parameters, making them 'broadband' in the sense that the material behaves as an effective medium from (ideally) zero frequency up to a cut-off frequency defining the homogenization limit. Beyond this limit, the wave detects the material's internal structure, necessitating more complex homogenization methods.

In most homogenization methods applied in the literature, three wavelengths are involved: the wavelength of the field in the background, in the effective material, and in the scatterers. Typically, these three wavelengths are assumed to be large, but only those of the background and the effective material are required. If this is satisfied but the wavelength inside the inclusions is very short, such as inclusions with low speed of sound or wave-guide low dimensional materials, we can have a homogeneous material with strongly frequency-dependent parameters. These structures are called metamaterials because their effective parameters, being generally resonant, can present any real value, including negative or infinite [3, 30, 31].

Dynamic homogenization of metamaterials can lead to phenomena like Willis coupling [32], occurring when dynamic homogenization methods are applied to structures with broken spatial symmetries or strongly non-local interactions. This effect, closely related to chirality, can enhance non-reciprocal wave propagation [33].

Despite the long history of studying metamaterials with negative parameters, both theoretically and experimentally, two challenges remain. Near the resonant regime, these materials present high dissipation, hindering most of their extraordinary properties. When the effective velocity collapses and approaches zero, the effective wavelength also collapses, invalidating the initial assumption of the long wavelength limit. This contradiction must be resolved through non-local homogenization theories, and the physics involved in such metamaterials are still in an early stage [34].

Additionally, frequency-dependent metamaterials have important limitations that must be overcome, such as the absence of quickly reconfigurable or tunable metamaterials, and active or non-reciprocal metamaterials required for various devices based on wave control. Although several mechanisms are currently being explored, the future of metamaterials is oriented towards efficient modeling and fabrication of these advanced structures [35].

Advances in science and technology to meet challenges

Consequently, the main challenges of metamaterials are both at the theoretical and practical level, since we still need of advanced homogenization methods to accurately describe them, and new fabrication techniques need to be developed to fabricate the most challenging structures. Besides the methods described above to overcome these limitations, it is worth to mention some advanced concepts of metamaterials like metasurfaces [5], non-linear metamaterials [36] and time-modulated metamaterials [37].

These structures, while not fitting the traditional definition of bulk homogeneous metamaterials, can exhibit unique functionalities that circumvent some of the challenges associated with conventional metamaterials. For instance, metasurfaces can provide advanced control over wavefronts with a much thinner profile, making them suitable for applications where bulk metamaterials are impractical. Non-linear metamaterials introduce the possibility of creating materials with properties that change in response to external stimuli, enabling dynamic and adaptive functionalities. Time-modulated metamaterials, on the other hand, offer the potential for advanced control over wave propagation by dynamically altering the material properties in time, leading to novel effects such as one-way wave transmission and non-reciprocal behavior.

Concluding remarks

The field of metamaterials has seen remarkable advancements, yet significant challenges remain that must be addressed to fully realize their potential in practical applications. Efficient modeling and advanced homogenization methods are crucial for accurately describing these complex materials. Theoretical advancements are needed to refine our understanding of effective medium theories, especially in the context of non-local interactions and dynamic homogenization.

On the practical side, developing innovative fabrication techniques is essential for constructing the most challenging and advanced metamaterial structures. Current limitations in manufacturing, such as achieving the necessary precision and scalability, hinder the practical deployment of metamaterials in real-world devices. Overcoming these limitations will require interdisciplinary collaboration, combining insights from materials science, engineering, and nanotechnology.

In summary, the future of metamaterials passes on overcoming both theoretical and practical challenges through advanced modeling, innovative fabrication techniques, and the exploration of new types of metamaterials such as metasurfaces and time-modulated structures. Continued interdisciplinary research and development will drive the evolution of metamaterials, unlocking their full potential for a wide range of applications in technology and industry. Through these efforts, metamaterials can achieve the extraordinary capabilities envisioned by researchers, transforming fields from telecommunications to medical imaging, and beyond.

Acknowledgments

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4. Phononic crystals with machine learning

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Status

Nowadays, acoustic functional materials represented by phononic crystals have become potential candidates for the control of acoustic and elastic waves due to their excellent acoustic properties [11]. Carefully designed periodic/non-periodic artificial structures can achieve a series of extraordinary properties such as blocking, consuming, enhancing, guiding or localizing waves. However, the increasing complexity of requirements has led to an explosive growth in the degree of nonlinearity between wave characteristics and structures, making traditional structural design methods such as empirical trial and error method unsustainable. As the core of artificial intelligence, machine learning is now becoming an elegant solution for various research field [38]. Naturally, due to the advantages of both efficiency and computational cost, machine learning was introduced into optical artificial structures [39] and subsequently acoustic artificial structures [40], becoming a link between structures and wave characteristics in the design process [25], as show in figure 4. The current research is mainly based on two aspects. On the one hand, for a specific structure, the trained machine learning model is used to speed up the solution and obtain corresponding wave characteristic, which to a certain extent replaces the original physical or numerical calculation process [41, 42]. In addition, it is even expected to discover materials with new physical properties [43]. On the other hand, what is more promising is the so-called inverse design, which obtains the corresponding structures according to the wave characteristics set by the application need [44–46]. The reason is that from the application perspective, we are often demand-driven for the actual structure. However, such inverse physical process is more complex and difficult to solve. Therefore, researchers use the data obtained through the forward process to train the inverse model. So as to replace the inverse physical process is currently one of the most important directions in the field of machine learning empowered phononic crystals. The mainstream machine learning models can be divided into three categories: supervised learning, unsupervised learning and reinforcement learning [47]. There are many specific algorithms in each category. For a specific problem, an appropriate machine learning model should be selected. It is worth mentioning that in order to train a model with better generalization ability using less data, researchers are tending to add physical constraints to the training process [48, 49], even directly simulating neural networks using metamaterial units [50–52]. This kind of research not only makes the model well trained, but also provides a certain interpretability, which is becoming a cutting-edge direction.

Current and future challenges

Machine learning has powerful data processing capabilities and relatively low research thresholds. As high-quality open-source frameworks continue to emerge and the learning community improves, its development and integration with many disciplines are very rapid. Although we are witnessing machine learning bringing some new ideas and preliminary results to phononic crystals research, we still face many challenges with existing research, which casts a shadow over the entire research direction. These challenges mainly come from the many shortcomings that still exist in the machine learning models themselves, and it would make people suspicious the applicability of combining machine learning with phononic crystals. Specifically, we have the following three main challenges on this research field that deserve in-depth thinking at present and in the future:

(1) The data availability and quality. Both supervised and unsupervised learning models heavily rely on high-quality training data. However, acquiring comprehensive datasets for phononic crystals, especially for complex applications, can be challenging. Current research on phononic crystals is faced with a series of complex situations such as dynamic nonlinearity, multi-physics fusion, and noise defects. When there is an actual new demand without a physical basis, it is difficult to obtain data. Although we may collect some data in production experiments, data cleaning and annotation to improve quality is also a challenging task.
(2) The interpretability and trustworthiness. The interpretability and trustworthiness of machine learning models remain long-maligned issues. The most essential problem is that although the training data comes from physical reality, the flow of data in the model replaces physical calculations, which makes the machine learning models like black boxes. In the neural network algorithm, we know how data is transferred through the connections between neurons, but we do not know what the physical significance of each step of data transfer is to the relationship between input and output. Meanwhile, the selection of hyperparameters has certain human factors. Even if the final model meets

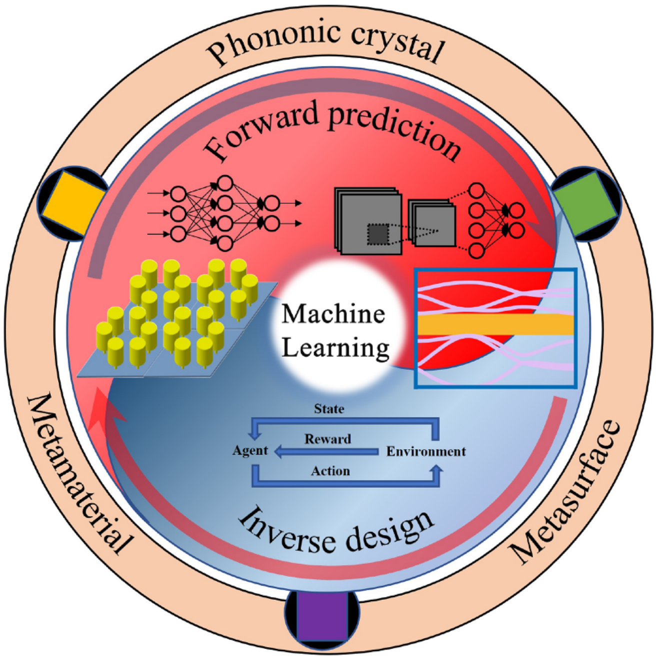

Figure 4. Diagrammatic sketch of machine learning for properties characterization and structural design of phononic crystals.

the requirements, there is still insufficient confidence in the credibility of the model.

(3) The scalability and generalization. As phononic crystal applications diversify and evolve, machine learning models must demonstrate scalability and generalization across different material systems, geometries, and operating conditions. Ensuring robust performance beyond training data domains is crucial for practical deployment in realistic applications. Meanwhile, Bridging the gap between computational predictions and experimental outcomes is essential for final validating the quality of models.

Advances in science and technology to meet challenges

The development of science and technology is the key to breaking through bottlenecks in various fields. Regarding the field of machine learning with phononic crystals, we have the following thoughts to try to analyze the science and technology we lack:

(1) Deepening of basic research on acoustics and mechanics. As we mentioned above, the data availability and quality is one of the main challenges. Generally speaking, we can use Monte Carlo sampling, SOBOL sequence and Latin hypercube methods to selecting optimal sample size [53]. In addition, we can apply transfer learning to reduce data requirements [54]. However, all these methods fail to solve the underlying problem, that is the training of machine learning models requires humans to first master the actual physical laws and train with the data or equations obtained thereby. To a large extent, the difficulty in obtaining data is due to the complexity of the requirements. So, in order to apply machine learning in more complex phononic crystals research, we must rely on the development of the basic disciplines of acoustics and mechanics.

(2) Develop interpretable machine learning models. Visualizing changes of physical meaning in data flow is an urgent problem to be solved. One possible approach is to incorporate physical laws into the model. Many researchers are currently making attempts in this direction and have proposed applicable physics-informed neural network for different research problems. Moreover, compared with purely data-driven neural network learning, physics-informed neural network imposes physical information constraints during the training process, so it can learn a model with more generalization capabilities using fewer data samples.

(3) Improve generalization ability and applicability. In order to improve the generalization quality of the model, researchers have proposed a series of methods such as data augmentation, model compression and acceleration, ensemble learning etc. These methods can make the model more adaptable when migrating to new data. However, we still look forward to better technical development to fundamentally solve the problem of generalization ability and applicability. The concept of meta-learning may become an important direction in solving this challenge in the future [55]. Currently, meta-learning technology has become a hot topic at the forefront of research fields such as healthcare [56] and engineering [57] where data is sparse and labeling is difficult. The basic idea is to dynamically select an appropriate model or dynamically adjust hyperparameters according to different tasks. By learning on multiple tasks, it helps the model discover common learning rules, thereby improving the model's generalization ability on new tasks.

Concluding remarks

The development of artificial intelligence is affecting all aspects of the world, and the extent is constantly deepening. There is no doubt that it is also an irreversible trend in the field of phononic crystals research. The intersection of phononic crystals and machine learning represents a frontier of innovation with profound implications. However, the higher demand drives will bring challenges to the application of machine learning models. We believe that the main challenges lie in three aspects: data acquisition, model interpretability and generalization ability. In order to fundamentally seek solutions to overcome them, we believe that firstly, it needs to rely on the further development of the basic disciplines of acoustics and mechanics to solve some physical problems before it can be used for model training; secondly, enhancing the interpretable and the possible way is to add physical constraints to the model; last, through possible development of meta-learning technology, comprehensively innovate the application of machine learning and further improve the generalization ability of the model. All in all, the research field of phononic crystals with machine learning is full of opportunities and challenges coexist. We will witness the future of the integration of technological revolution and basic research.

Acknowledgments

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5. Topological and non-Hermitian phononic crystals

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Status

The steering of acoustic waves plays a significant role in many fields of modern technologies, including sound and noise control, structure health monitoring, medical imaging and therapy. In decades, acoustic metamaterials, holding fascinating properties that cannot be expected in natural materials, have extremely enriched the capabilities of acoustic manipulation and inspired many novel functional devices. However, robust wave propagation through complex medium remains a challenge. Very recently, by transplanting the concept of band topology in condensed matter physics, the topological and non-Hermitian phononic crystals have been proposed and become a hot topic in the research community since they offer the ability to manipulate and direct the propagation of waves while providing robust topological protection [19, 58, 59]. One of the most intriguing properties is that they are robust against backscattering even in the presence of some exaggerated defects, such as path bending, scatterers missing and disorders, which endows topological and non-Hermitian phononic crystals with promising potential for the development of novel devices and technologies. These properties can be attributed to the time-reversal symmetry or other specific symmetries of the systems.

Topological phononic crystals. The first wave of verifying topological phase in acoustics includes a theoretical work [60] about constructing quantum Hall phase by introducing moving medium to break time-reversal symmetry (see figure 5(a)). After that, a series of intriguing acoustic topological phases have been experimentally demonstrated, including quantum spin Hall phases and valley Hall phases where the pseudo-spin-momentum locked transports are guaranteed, higher-order topological phases [61] with quantized dipole or multipole moments (see figure 5(b)) and topological semimetals where two or more topological bands degenerate in momentum space.

Non-Hermitian phononic crystals. Non-Hermitian physics describes a non-conservative system where energy exchanges with the outside environment. The introduction of non-Hermiticity, i.e. gain and loss, may lead to exceptional points where two or more eigenstates degenerate, as well as some unexpected phenomena without Hermitian counterpart. On the one hand, the interplay between non-Hermiticity and acoustic topological insulators can create unusual topological states with amplified or attenuated properties (see figure 5(c)) [62]. On the other hand, non-Hermiticity itself can induce topological phase transition [63] since it effectively acts as coupling strength (see figure 5(d)). Moreover, a new topological phase, known as the non-Hermitian skin effect, is unique to non-Hermitian systems. It is related to a non-trivial point-gap signatured by a non-zero winding number and describes that all modes are localized to the boundaries of the system [58].

The rapid growth of topological and non-Hermitian phononic crystals comes from the cutting-edge achievements in condensed matter physics and in turn serves as a powerful tool for acoustic manipulation, allowing for more possibilities in designing novel devices. It can also contribute to the studies of elastic waves, where the system can support more modes. Besides the similar phenomena observed in acoustic system, the all-polarized topological elastic insulators have been verified.

Current and future challenges

The main expectation of introducing topological and non-Hermitian concepts to phononic crystals is to seek robust methods to manipulate sound waves. Thus, constructing various topological phases is an urgent task. However, we can hardly find versatile approaches to realizing gain effect and nonlinear effect in acoustics due to the characteristics of acoustic waves. These limitations extremely hinder the explorations of new topological phases and restrict the potential applications in broader scenarios.

Non-Hermitian phenomena with gain effect. Loss effect is a nonnegligible factor in most acoustic systems since it may come from the intrinsic thermal-viscous effect or the leakage caused by the scattering. In elastic system, loss comes from the damping effect of the materials. Therefore, the previous works on non-Hermitian phononic crystals mainly rely on the specific distributions of sound-absorptive materials, where the amplified modes cannot be excited and the created fields are confined to the source. A few works employed complex implementation of external circuits to introduce gain effect, which needs enormous efforts on the consistency and stability of the hardware [19]. Thus, constructing gain effect is a key to unveiling the exact picture of non-Hermitian phononic crystals.

Nonlinear acoustic topological phases. The unprecedented phenomena generated in nonlinear topological photonic systems, such as topological solitons and nonlinear topological edge states, have been widely studied, while the counterpart in topological phononics is less explored [58]. The reason lies in the fact that the nonlinearity for airborne acoustics is negligible. However, the nonlinear phenomena may bring many benefits to some aspects of acoustic engineering. For

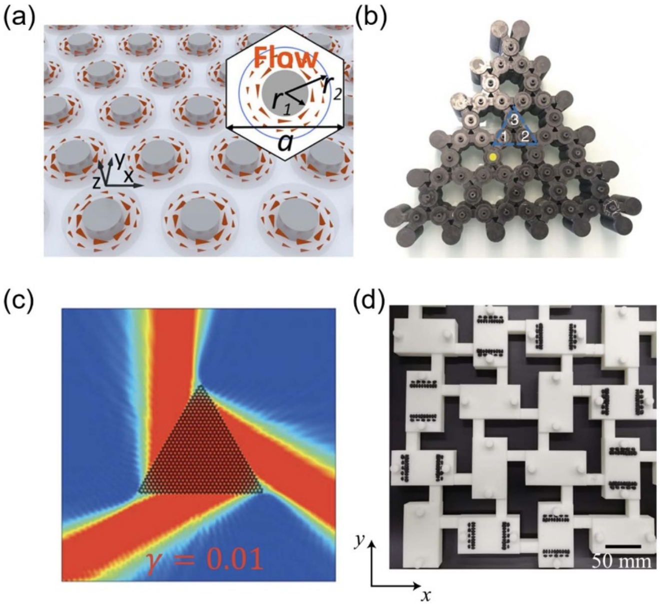

Figure 5. (a) The acoustic analogue to the topological insulator occurring in the quantum hall effect, where the flow plays the role of pseudo-magnetic field to break the time-reversal symmetry. Reprinted (figure) with permission from [60], Copyright (2015) by the American Physical Society. (b) The 2D kagome lattice of acoustic topological insulator for supporting topological corner states. Reproduced from [61], with permission from Springer Nature. (c) The amplified mode excited by the interplay between the topologically protected valley transport and non-Hermitian modulation. Reprinted (figure) with permission from [62], Copyright (2018) by the American Physical Society. (d) The acoustic implementation of higher-order topological insulator induced by non-Hermitian modulations solely. Reproduced from [63] CC BY 4.0.

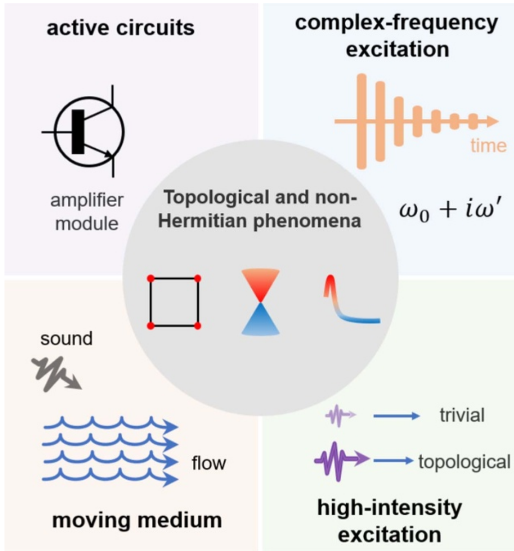

Figure 6. For constructing more fruitful topological phases, i.e. higher-order topological states, topological semimetals and non-Hermitian skin effect, in topological and non-Hermitian phononic crystals with gain and nonlinear effects, several schemes should be considered to overcome the challenges.

example, in medical imaging and ultrasound non-destructive tests, researchers usually want to introduce nonlinear signals to increase the resolution of imaging. Besides, acoustic levitation is another important phenomenon that uses sound waves with nonlinear characteristics. Thus, further study of nonlinear acoustic topological phases offers intriguing possibilities for understanding and controlling the behavior of acoustic waves in complex systems, paving the way for innovative technologies and devices.

Advances in science and technology to meet challenges

To bring topological and non-Hermitian phononic crystals into real-world applications, a lot of effort should be devoted to overcoming the challenges mentioned above (see figure 6). Here, we discuss some possible options that may address the questions.

Complex-frequency excitation. Typically, we attribute the gain and loss effect to the material properties. However, for a propagating wave, the intensity profile along the propagation direction is determined by the frequency and wave number together. Thus, we can utilize a time-varying signal to compensate the loss induced by the materials, and even mimic the gain effect. Here, the operator of time-varying means adding an imaginary part to the frequency mathematically. By normalizing the created field to the excited signal at every moment, the interplay between the acoustic wave and virtual gain/loss medium can be revealed vividly [64].

Moving medium. The introduction of moving medium to acoustic systems can break the time-reversal symmetry since sound waves traveling in upstream and downstream directions are different. Meanwhile, moving medium can also provide energy to acoustic waves to construct gain effect [65]. The energy exchange is bridged by the vortex. For the gain part, the flow inhomogeneous caused by the scatterer will generate the vortex, leading to the vortex-sound interaction that amplifies the sound waves. For the loss part, the addition of metal mesh will avoid the generation of vortex, the sound wave only experiences the damping effect.

High-intensity sound excitation. The linearization of the acoustic governing equation is under the assumption of low-intensity sound waves, which means the equation itself is nonlinear. Thus, when high-intensity sound wave is enforced to an acoustic resonator, nonlinearities in the resonator can affect the wave behaviors [66]. Here, nonlinear effects can manifest in several ways, including frequency shift, amplitude-dependent response, harmonic generation and nonlinear absorption. By considering these nonlinear effects, we can redesign the non-Hermitian phononic crystals to observe the nonlinearity-induced topological phase transition and topological solitons.

Concluding remarks

Thanks to the convenient fabrication and flexible measurement, acoustic system is an ideal platform to verify topological and non-Hermitian notions. Meanwhile, the intriguing topological phases may inspire more wave phenomena. For now, topological and non-Hermitian phononic crystals represent a new frontier in acoustics, allowing for robust manipulation of sound waves. Considering the huge significance and existing challenges, we envision the researches on this topic will attract much attention for a long time. Once the challenges have been met, the achievements may bring possibilities to revolutionize the current technologies.

Acknowledgments

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6. Nonreciprocal phononic crystals

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Status

The principle of reciprocity is crucial for managing waves and vibrations in linear, time-invariant systems, ensuring symmetrical energy transport between two points in space. However, in systems requiring asymmetric wave motion, such as one-way acoustic circulators and diodes, this principle poses a challenge. To tackle this, significant effort has been devoted to developing nonreciprocal phononic crystals. Broadly, there are two approaches to achieving nonreciprocal dynamics in phononic crystals: intrinsic and extrinsic. The intrinsic approach primarily utilizes nonlinearity, structural chirality, or dissipation to achieve nonreciprocity in passive, non-gyroscopic phononic crystals with time-invariant material properties and boundaries. Nonlinearity-based nonreciprocal systems often rely on generating higher harmonics, nonlinear resonances, and bifurcation in the presence of spatial asymmetry. However, they often face fundamental limitations. First, they cannot ensure isolation when excited simultaneously from both sides, as the superposition principle does not apply to nonlinear systems. Consequently, these devices cannot function as conventional isolators to protect a source from back reflections. Additionally, there is a trade-off between the degree of nonreciprocity achievable in passive, nonlinear resonators, and the magnitude of forward transmission. To address these limitations, the extrinsic approach focuses on exploiting space-time modulated material properties and boundaries to achieve nonreciprocal wave propagation in active, linear phononic crystals. This approach uses active pump-wave-like external stimuli to generate a bias of energy transmit to break time-reversal symmetry, resulting in nonreciprocal propagation of acoustic and elastic waves. Various mechanisms shown in figure 7, such as those realized in piezoelectric materials [67], magnetic [68] and mechanical elements [69, 70], enable the active control necessary to modulate material properties, making active phononic crystals a practical and promising candidate for applications in acoustic and elastic wave control that require nonreciprocal signal transmission.

While the absence of reciprocity may increase the complexity of analysis and realization, it also leads to novel wave guiding phenomena, as demonstrated in the examples mentioned above. Nonreciprocity finds applications across various acoustic wave domains, including air, water, biological tissues, and solids. One area of application that stands out as potential game-changers in the acoustics community is surface acoustic wave devices, which offer a promising platform for nonreciprocal effects due to their extensive industrial and research use. These devices have the potential to significantly impact fields ranging from telecommunications to sensing [71].

Current and future challenges

Understanding the principles behind violating reciprocity in phononic crystals is well-established, but developing effective nonreciprocity strategies for various applications remains a significant challenge.

Incorporating nonlinearity into phononic crystals, for example, to achieve nonreciprocal behaviors can be complex. While combining nonlinearity and spatial asymmetry can lead to nonreciprocal effects, there are fundamental limitations, as discussed previously. Passive nonreciprocal devices can exhibit vastly different transmissions for waves propagating in opposite directions but fail to provide isolation when the system is excited simultaneously from both sides. This limitation prevents them from functioning as conventional isolators, which protect a source from back reflections, due to the fact that nonlinear systems do not satisfy the superposition principle. Additionally, there is a trade-off between the degree of nonreciprocity achievable in passive nonlinear resonators and the magnitude of forward transmission. Therefore, managing the trade-off between nonlinearity and transmission efficiency is critical.

Creating active nonreciprocal phononic crystals that can stably and accurately tune space-time modulation also poses a significant challenge. Achieving stable dynamic control and adaptability in these active phononic crystals, where the direction of nonreciprocity can be switched or modulated in real-time, requires innovative, meticulous designs and materials, such as stable, precise, and fast responsive electronic micro-controller systems. The electronic control can easily become unstable as the device size increases in realistic scenarios where multiple control pairs are present and interact strongly. Common ways to suppress the mutual interactions includes constructing fewer active unit cells or geometric separation. However, they are not optimal in large-scale manufacturing for future applications.

Furthermore, many nonreciprocal phononic crystals only exhibit nonreciprocal effects within a limited frequency range, such as the inherently narrow-band nature of topological wave phenomena. Expanding this bandwidth while maintaining efficient nonreciprocal behaviors is crucial. This is particularly important in various applications, such as airborne acoustics in the range of human hearing, biomedical ultrasound, and underwater sonar, which utilize and require a wide range of frequencies. The diverse range of length scales and material properties involved in applied acoustics indicates the need for a variety of strategies for efficient nonreciprocal phenomena.

While utilizing nonlinearity optimally for nonreciprocal phononic devices has seen significant development, combining nonlinearity with other reciprocity-breaking

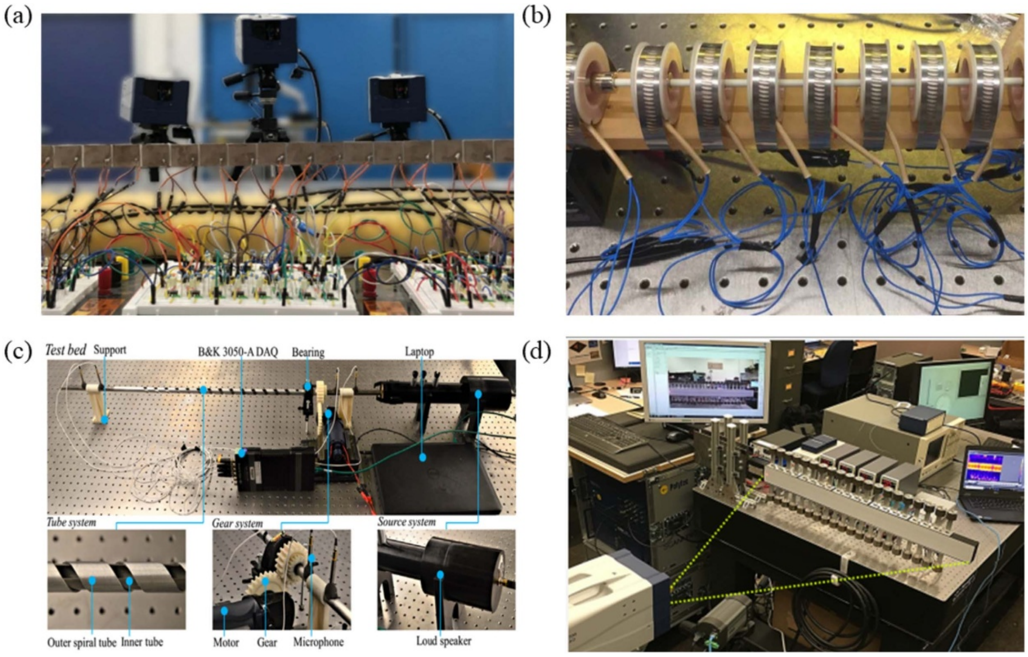

Figure 7. Various types of linear space-time modulated phononic crystal systems. (a), (b) Nonreciprocal phononic crystal enabled by sliding magnets controlled by electrical coil. Reprinted (figure) with permission from [68], Copyright (2018) by the American Physical Society. (c) Space-time modulated phononic crystal driven mechanically by a motor system. Reprinted (figure) with permission from [69], Copyright (2020) by the American Physical Society. (d) Nonreciprocal surface acoustic wave system via space-time modulated spring-mass oscillators. Reprinted (figure) with permission from [70], Copyright (2020) by the American Physical Society.

mechanisms shows promise for even more remarkable outcomes. For example, recent studies have explored the nonlinear dynamics of topologically protected edge states, where topological phase transitions are triggered by the intensity of the input signal. However, much of the potential of nonlinear topological acoustics remains untapped. In addition, the combined effects of nonlinearity and space-time modulations have not yet been extensively studied in the literature. With recent advancements in modeling complex nonlinear and spatiotemporally varying elastic materials, as well as experimental realizations of dynamic elastic media, the study of nonreciprocity is expected to expand to modulated nonlinear media for future applications featuring large deformations, which may involve mathematical complications.

Advances in science and technology to meet challenges

Nonlinear and space-time modulated media have long been known to exhibit nonreciprocal dynamics, such as nonreciprocal harmonic generation. However, a significant limitation of these systems is their amplitude-dependent nature, which makes individual and independent manipulation of wave conversion challenging, thus limiting their flexibility in wave control applications. To overcome this limitation, a recent proposal introduced a linear mechanism-based wave-conversion metalayer that is independent of incident amplitude [72]. This metalayer enables the generation of harmonics with freely controlled frequencies, phases, and amplitudes. Experimental demonstrations using a piezoelectric-based sensor-actuator control loop showed the generation of harmonics of arbitrary characteristics upon arbitrary flexural incidence (figure 8(a)). This prototype offers more freedom in wave manipulation compared to traditional nonlinear and space-time modulated systems.

Another promising direction to facilitate topological phenomena supported by active pump-wave-like space-time modulation is to utilize synthetic dimensions. This approach treats time modulation as a synthetic spatial dimension, enabling the realization of higher-dimensional topological effects in lower spatial dimensionalities. By employing synthetic dimensions, the overall complexity and instability required for the realization of pump-wave space-time modulation can be greatly reduced. Recent experimental work has demonstrated 2D classical-wave quantum Hall effects with 1D topological pumping in 2D passive elastic plates, without breaking time-reversal symmetry through space-time modulation [76].

The study of nonreciprocity can also extend beyond traditional nonlinear and pump-wave approaches to intersect with research in out-of-equilibrium systems, such as active matter, where the constituent particles of wave-bearing materials have internal energy sources. This includes investigating nonreciprocal wave dynamics in systems featuring nonreciprocal interactions. Continued research in these nonreciprocal systems promises to demonstrate novel physics and provide insight into complex emergent behaviors. Efforts in realizing various types of nonreciprocal systems have led to significant achievements, including robotic metamaterials [73], odd elasticity in a micropolar beam [74], and odd mass

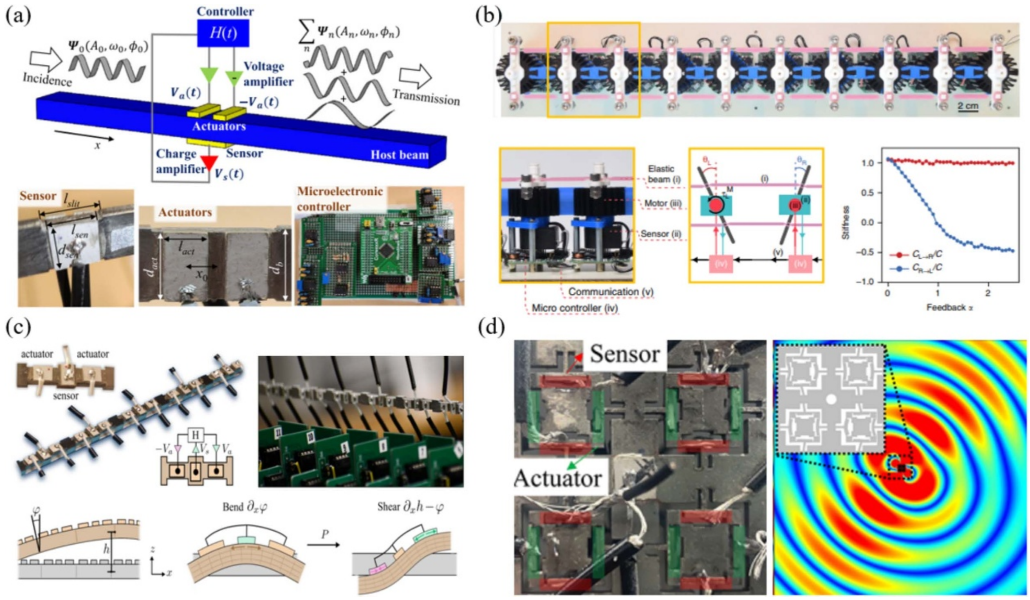

Figure 8. Illustration of different active nonreciprocal mechanical systems with nonreciprocal coupling. (a) A linear flexural metalayer capable of arbitrary wave frequency conversion. Reprinted (figure) with permission from [72], Copyright (2022) by the American Physical Society. (b) Robotic metamaterials exhibiting nonreciprocal behaviors. Reproduced from [73]. CC BY 4.0. (c) One-dimensional micropolar metabeam with odd elasticity. Reproduced from [74]. CC BY 4.0. (d) Two-dimensional elastic metamaterials featuring odd mass density. Reproduced with permission from [75].

density in 2D elastic metamaterials [75], as shown in figure 8. These systems leverage nonreciprocal interactions, or in other words sensing-actuating loops, among their constituent building blocks and demonstrate nonreciprocal dynamics, often termed the non-Hermitian skin effect, over a broad frequency spectrum. Although still bored with system stability when multiple sensing-actuating loops interact strongly, this characteristic effectively mitigates the narrow bandwidth limitation commonly encountered in other conventional nonreciprocal systems, such as topological wave systems.

Concluding remarks

In conclusion, nonreciprocal phononic crystals represent a dynamic and rapidly advancing field with significant potential for groundbreaking applications in acoustics and beyond. While progress has been made in understanding the underlying principles and developing novel structures, several challenges remain. These include the need for materials with adaptable properties, the advancement of efficient fabrication techniques, and the integration of nonreciprocal phononic crystals into practical devices. As the technical challenges outlined in this roadmap are addressed, we anticipate substantial progress in nonreciprocal phononic crystals and metamaterials. The pursuit of these active nonreciprocal systems capable of dynamic tuning and the exploration of novel reciprocity-breaking mechanisms offer exciting prospects for future research in active topological insulators and one-way wave-based devices. By overcoming these challenges and fully harnessing the potential of nonreciprocal phononic crystals, we can anticipate transformative advancements in acoustics, signal processing, and beyond.

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7. Nanophononic crystals

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Status

In 1905, Albert Einstein published four manuscripts that certainly had an impact in Physics. His second work [77] deals with Brownian motion, a source of dissipation and dephasing for many processes. Despite its random nature, the spectral properties of Brownian motion can be engineered beyond cooling down. In the generic case of phonons, i.e. mater vibrations, phononic crystals offer a paradigmatic way to tailor wave transport and energy exchange among different excitations, the future prospects of which we want to explore here. The periodic constitutive elastic properties of phononic crystals, e.g. the material density and stiffness tensor, tailor the spectrum and displacement profile of any mechanical vibration provided that its wavelength is commensurable with the lattice spacing. Although phononic crystals span over a huge frequency from few Hz (infra-sounds) to few THz (heat-carrying vibrations at room temperature), we focus on phononic crystals in the hyper-sonic and thermal frequency regions, i.e. frequencies from few GHz to few THz. In our view, the most promising prospects of phononic crystals are the control of nanoscale thermal transport, their use as transducers for quantum information processing and to enable phononic-bath engineering. While these directions are not exhaustive, we think they offer exciting prospects in the coming years. The exploration of new materials to enable new functionalities, interfaces, and frequency control is of transversal future interest. Examples of it are gallium phosphide, whose piezoelectricity is used for optomechanics-enabled microwave-to-optics conversion [78], and diamond, which can host magnetic-field-tuneable and strain-sensitive GHz spin-qubits [79]. Finally, the push to increase the working frequency [80] of phononic crystals is guided by the technological prospects of the field, for example in ultra-high-frequency RF signal processing for 5G communication devices. 2D materials also offer the potential of layered materials as active components for thermal management [81]. An extensive review of the field spanning a much broader energy range and applications can be read elsewhere [82].

Current and future challenges

At room temperature, thermal phonons exhibit wavelengths of only a few nanometers, and dephase quickly due to surface scattering with nanoscale roughness. Coherence of thermal phonons at these high temperatures has been controlled so far only in superlattices [83] of atomically-smooth interfaces and nanometer-scale periodicity. However, it is very challenging to confine and route phonon transport in these materials. Implementing components like cavities and waveguides is much more suitable in two-dimensional (2D) semiconductor structures. These 2D-like systems have the advantage of being the ones used for optomechanics and quantum photonics and photonics in general to manipulate different light-matter interactions in the form of photonic and phononic crystals. Thermal transport management using phononic crystals in these structures is potentially disruptive in order to, e.g. route heat along precisely defined paths by blocking and guiding its transport and combine it with other functionalities (optomechanical, quantum, etc). Controlling thermal phonons using this type of structures would have an important impact in the field of quantum photonics where the dephasing of single-photon sources is basically still governed by high-frequency phonons. However, the field of coherent thermal transport [84] is not restricted to the coherent nature of thermal phonons in nanophononic crystals, as it also deals with the transition from incoherent to coherent behavior of thermal phonons. In this sense, the control of ballistic transport is possible with a phononic pattern (periodic or not) and nanophononics has an important role to play here.

Advances in science and technology to meet challenges

Microwave-frequency acoustic devices have been intensively studied for certain classical information processing tasks because of the advantages they offer over their electromagnetic counterparts: smaller wavelengths, and hence reduced sizes, lower crosstalk, and lower losses. These properties also make them good candidates for applications in quantum science and technology as, e.g. interfaces for read-out and manipulation of solid-state qubits. Figure 9 describes very recent approaches of these applications. The most widespread approaches use bulk, surface or thin-film acoustic wave resonators, for which quantum interfaces with color centers, quantum dots and superconducting qubits have been demonstrated. Nevertheless, integration of the former with electronics is not straightforward and the latter typically require large in-plane footprints as they use end facet Bragg-mirrors for resonant enhancement. Phononic crystal cavities can, however, effectively suppress all radiation losses while confining mechanical energy to a wavelength-scale volume, thus drastically suppressing spurious mechanical modes detrimental for quantum acoustodynamics. Another

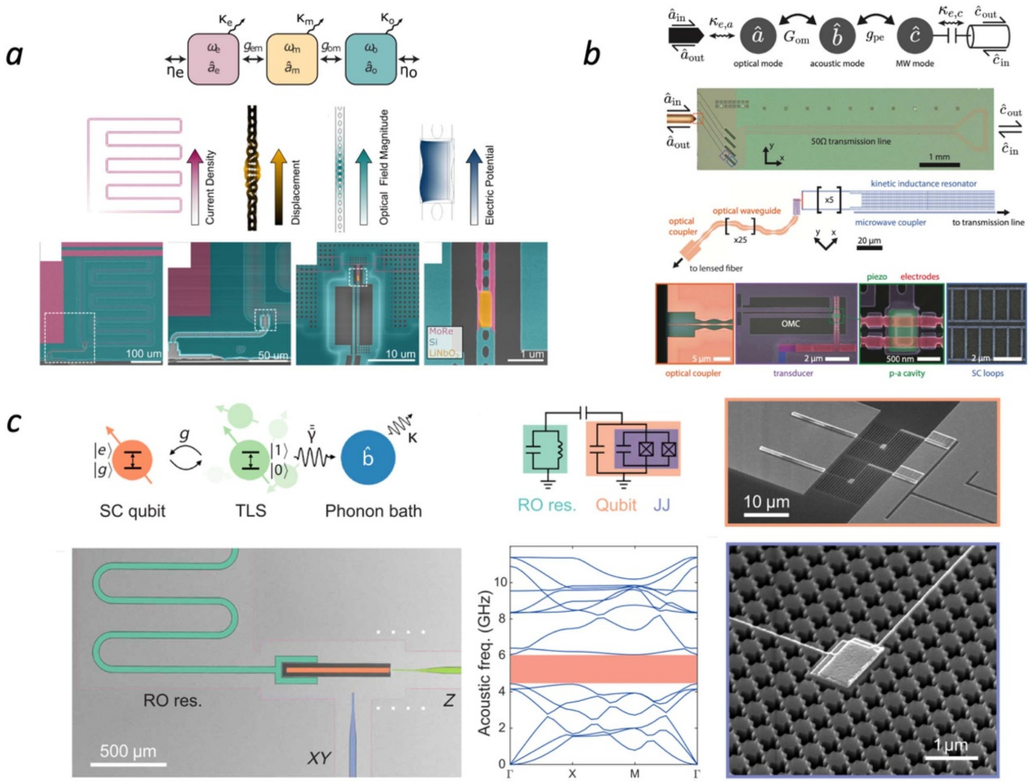

Figure 9. Phononic crystals to implement quantum interfaces, transducers and non-reciprocal phonon and photon transport.} (a) Optomechanical transducer consisting of three coupled bosonic modes: microwave (magenta), mechanical (yellow) and optical (cyan). Reproduced from [85], with permission from Springer Nature. (b) Microwave–optical photon pair generation with a chip-scale transducer. Scheme of a three-mode optomechanical system consisting of two linearly coupled optical modes a1 and a2 and a mechanical mode q coupled to one of the optical modes via an optomechanical interaction to generate optical nonreciprocity. Reproduced from [86], with permission from Springer Nature. (c) A hybrid platform for phonon engineering in superconducting quantum circuits. Reproduced with permission from [88].

promising application of hypersonic phononic crystals in quantum technologies is the development of ultra-coherent gigahertz nanomechanical resonators to mediate the quantum transduction between otherwise decoupled excitations, notably between microwave photons, which are central to stationary superconducting logic circuits, and optical photons, the paradigmatic room-temperature flying qubit in distributed quantum networks [85]. Long-lived phononic-shielded acoustic resonances of piezo-optomechanical [86] or electro-optomechanical resonators [87] can couple to both photon frequencies, with the main difference of the two being the respectively resonant and parametric microwave-mechanics coupling. These have already been successfully employed for bidirectional microwave-to-optics conversion and recently to the generation of microwave–optical photon pairs exhibiting non-classical correlations. In this context, phononic-crystal-based structures (mirrors, cavities, or waveguides) are crucial not only to mediate the cascaded coupling, but also to minimize potential crosstalk-induced losses, especially relevant due to the vulnerability of superconducting devices to absorption of optical photons.

Concluding remarks

The ability of phonons to couple with a wide variety of other excitations (like electrons, photons, spins, other phonons, etc) is vital for the aforementioned applications, yet it poses a significant challenge for the coherence of (non-mechanical) solid-state quantum systems. To address this, these systems are usually cooled to millikelvin temperatures to minimize thermal phonon populations, but it only partially limits phonon-assisted decoherence when the local density of states (LDOS) of the unintended and deleterious phonon bath is large. Recently, approaches to extend the relaxation time of such quantum systems by suppressing the LDOS and spontaneous single-phonon processes using phononic-crystal band gap structures have been identified. An important example is the recent 18-fold reduction (compared to the bulk) of the phonon-induced orbital relaxation rate of a single silicon-vacancy in diamond by embedding it in a free-standing fishbone phononic-crystal nanobeam with a band gap in the 50–70 GHz range [89]. Interestingly, the study of the coherence of superconducting microwave quantum circuits evidence that phononic engineering will likely be an enabling aspect even in the absence of direct coupling between the quantum system of interest and the phonon bath. Bulk or surface material defects, typically described by a spectrally broad bath of tunneling two-level systems (TLS), are omnipresent and generally have both electric and elastic dipoles, which induces TLS-mediated phonon emission. Pioneering works have demonstrated a two-order of magnitude improvement in the TLS lifetimes inside a phononic metamaterial [88], but the associated improvements to the qubit lifetime resulting from phononic engineering are still very moderate [90]. We believe that increasing attention in the quantum computing community will be drawn towards the co-fabrication of phononic band gap structures with even larger band gaps and solid-state qubits.

Acknowledgments

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8. Chirality and acoustic activity

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Status

Chirality, often associated with asymmetry, refers to objects whose mirror image is not superimposable with itself, has garnered significant attention in the field of mechanical metamaterials due to its potential to influence internal rotations and enhance large deformations. This property is well-known in certain molecules, such as sugar, and macroscopic objects like spirals. In 1920, Lindman [91] demonstrated a fundamental link between geometry (chirality) and optical activity, which involves the rotation of the polarization of a transversely linearly polarized wave. Optical activity later paved the way for acoustical activity in non-centrosymmetric crystals such as quartz [92] in the 1970s. Similar to optics, in acoustics, a linearly polarized transverse wave rotates its polarization direction along the propagation axis.

The crucial question arises: *What contributions can chirality make to static and dynamic elasticity?*

Initially, researchers explored the static elastic properties of chiral structures and their effective description by using enriched media such as (chiral) micropolar elasticity [93–95]. Up to now, chirality has been one of the major key players in achieving push-to-twist conversions in metamaterials [94]. In 2017, it was demonstrated that on overall millimeter-sized samples, one can get twists per axial strain exceeding 2°. In addition, a characteristic length scale directly connected to such behaviors can be partially mapped onto a (chiral) micropolar effective-medium description [95].

Subsequently, dynamic, and notably isotropic, acoustical activity have also been studied [96–99]. Chirality is naturally linked to chiral phonons, which play a crucial role in modifying the dispersion relations of acoustic waves, mirroring similar physics as optical activity. In 2019, Frenzel *et al* [96] demonstrated a prominent rotation of the polarization direction of transverse waves in chiral metamaterial samples. This rotation is proportional to the propagation distance and is distinct from nonreciprocal Faraday rotation, as it does not rely on broken time-inversion symmetry but only on chirality.

However, these findings are extremely sensitive to the sample orientation and are only possible upon a very small angle along the main cubic directions.

To solve the directionality aspect and to get isotropic acoustical activity, researchers first looked at 3D periodic approximants of 3D icosahedral quasicrystalline [97] metamaterials consisting of uniaxial chiral meta-rods. Nearly isotropic effective speeds of sound and isotropic acoustical activity are achieved by increasing the order of the approximants. However, the geometrical complexity is rather demanding fabrication. Further effort has been put into obtaining a simpler design, mainly based on geometry-tuned ‘accidental’ isotropy [97, 99]. Finally, Chen *et al* [99] designed a simple-cubic chiral metamaterial that exhibits nearly isotropic acoustical activity (see figure 10(b)). The resulting blueprint is radically simpler than previous proposals, making it amenable to state-of-the-art 3D additive manufacturing (see figure 10(c)).

Current and future challenges

On the theory side, it is still not known whether there is any upper bound to the push-to-twist effect and the acoustical activity in isotropic chiral metamaterials. If so, then it is either not known how to achieve the upper bound in metamaterials. Theory study on this aspect will be guidelines for metamaterial design. Another research challenge is to propose more versions of chiral metamaterials with isotropic acoustical activity. Currently, the above-mentioned simple-cubic geometry [99] is the only one that is amenable to manufacturing and exhibits isotropic chiral behavior. Other designs would be helpful if chiral metamaterials are to be used for applications.

On the experiment side, the demonstration of isotropic acoustical activity in metamaterials is still missing. Furthermore, both the method for characterizing the polarization direction of elastic waves and the on-demand polarization excitation strategy need further investigation. Albeit the isotropic acoustical activity has already been verified numerically with the proposed simple-cubic metamaterial design [99], transferring the design into an experiment is not as straightforward as one might expect at first sight. Importantly, a finite cross-section of fabricated metamaterial beams can easily break the symmetry required for chiral phonons and destroy the acoustical activity if the metamaterial beam cross-section is not carefully designed. Unless direct measurement of chiral phonons in bulk metamaterials instead of beams can be proposed.

Specific applications of acoustic activity are presently elusive. However, it is conceivable that such metamaterials could find application as an additional layer on transducers, enabling the rotation of the polarization of transverse waves if such manipulation should be necessary.

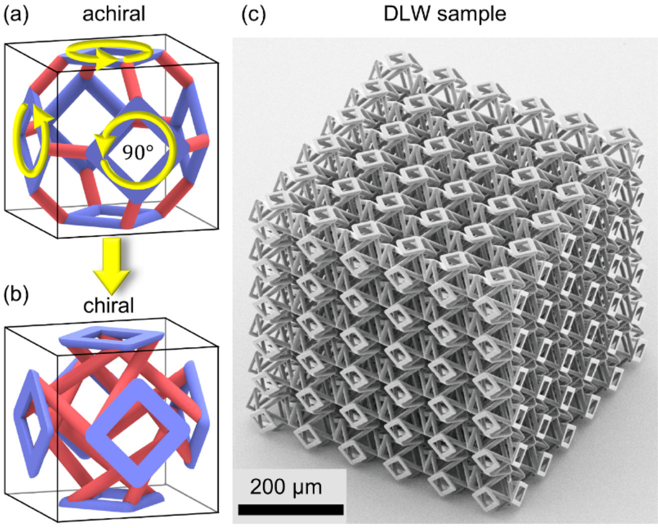

Figure 10. Illustration of the blueprint for a 3D achiral and chiral mechanical metamaterial supporting isotropic chiral phonons. The colors are for illustration only. Both red and blue rods are made from the same elastic material. The chiral structure is obtained by twisting parts of the achiral structure. SEM image of a sample fabricated by direct laser writing (DLW). From [94]. Reprinted with permission from AAAS.

Advances in science and technology to meet challenges

In order to address the upper bounds on static and dynamic mechanical chiral behaviors, continuum effective medium theory, such as micropolar continuum, might be a suitable choice. A previous study has shown that the effective-medium theory is capable of describing static and dynamic chiral responses, in particular the prominent chirality-dependent size effects [95]. Effective-medium theory can avoid complexities of metamaterial microstructures and provide a universal bound irrespective of specific microstructures. As for designing a specific microstructure that can achieve the bound, topology optimization could be a potential choice to find out potential chiral structures, and in fact has been successfully used to design metamaterials with desired achiral static and dynamic properties [95–98].

To completely characterize the polarization direction of elastic waves in chiral metamaterial samples, one promising way is using optical imaging techniques, with the advantages of exhibiting high spatial resolution, high accuracy, and being contact-free. More specifically, digital-image-cross-correlation algorithms have been used to determine two in-plane displacement components for targeted positions on sub-millimetric samples, and the remaining out-of-plane component has been be obtained based on Doppler effect with laser. A combination of both techniques has demonstrated success in obtaining phonon dispersion bands, derived from tracked displacement fields, of metamaterials [100], albeit these samples were not chiral.

Concluding remarks

In recent years, there has been a noteworthy commitment to designing and fabricating chiral elastic metamaterials, driven by the goal of controlling chiral phonons. The design process has seen significant advancements, showcasing a robust methodology, and the fabrication techniques, particularly through direct laser writing, have demonstrated notable efficiency. However, despite these strides in material development and fabrication, the practical applications of these chiral elastic metamaterials remain largely unexplored at this stage.

The focus has primarily been on establishing the feasibility and reliability of the designs and manufacturing processes, with researchers investing considerable effort in effective behavior description and isotropy. The success in constructing these metamaterials capable of manipulating chiral phonons holds promise for groundbreaking applications in various domains, yet the exploration of practical use cases is an area that has received less attention.

As we navigate through this evolving field, the next frontier lies in uncovering and implementing applications that leverage the unique properties of chiral elastic metamaterials. This could potentially include advancements in acoustics, mechanical wave manipulation, or even the development of innovative devices and technologies.

Acknowledgments

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9. Willis phononic crystals and metamaterials

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Status

Metamaterials have been offered a new paradigm to design materials that shows properties beyond what nature can give us. One frontier is Willis metamaterials in elastodynamics, which is originated in the 1980s with the work of physicist John Willis [101]. He proposed an additional term for the wave equations to describe the coupling effect between stress and velocity as well as between momentum and strain, in analogy to bianisotropy in optics. It modifies both Hook's law and Newton's second law, opening interesting directions for exotic wave phenomena. However, the additional Willis coupling term always appears as a high-order small quantity in most instances, until recently researchers have largely enhanced it by tailoring asymmetries combined with resonances [102, 103].

While the research on Willis metamaterials in acoustics is relatively in-depth and extensive, the elastic counterpart is still having huge room to explore. This is because the Willis coupling tensor for elastic waves are much more complicated than those in acoustics [104, 105]. As illustrated in figures 11(a) and (b), we reported the first experimental realization of a Willis metamaterial for flexural wave in a structured beam [103], then extended it to the torsional mode [106]. Recently, Chen *et al* [107] proposed an active Willis meta-layer with asymmetric polarizabilities by utilizing piezoelectric sensor–actuator pairs.

Due to the extra degree of freedom induced by the Willis coupling, some exotic wave properties were exhibited. The most widely explored one is the control of transmission and reflection behaviors. When waves are incident on a Willis metamaterial from different directions, asymmetric reflection can be observed [102]. To the extreme, one can achieve unidirectional zero reflection (UZR) at the exceptional point, if the Willis material is interpreted as a non-Hermitian system [103, 105]. Recent work has also demonstrated that the incorporation of Willis coupling into metasurfaces, which can improve their efficiency when refracting at steep angles [109], or can lead to asymmetric transmission spectrums (figure 11(c)) [108].

Current and future challenges

The most general form of the constitutive relations in Willis metamaterials are presented as

$$\begin{aligned} \sigma_{ij} &= C_{ijkl}\varepsilon_{kl} - \omega^2 S_{ijk} u_k \\ p_j &= -i\omega S_{klj}\varepsilon_{kl} - i\omega \rho_{ij} u_i \end{aligned} \tag{1}$$

where $\boldsymbol{C}$ and $\boldsymbol{\rho}$ are the stiffness and density tensors, $\boldsymbol{\omega}$ is the radial frequency and $\boldsymbol{i,j,k}$ are indices iterating the spatial coordinates. It can be seen that the Willis coupling term $\boldsymbol{S}$ is a complicated third-order tensor. One of the current challenges in Willis metamaterials is to control each element by man-made structural design. On the other hand, is it feasible to develop a framework to extract the whole complicated Willis coupling tensor (as well as other constitutive coefficients) of a certain metamaterial? It is a fundamental problem that cannot be avoided, and is also a difficult task.

Moving forward, considering that the Willis coupling term is imperceptible in most solids, the search on how to make the Willis coupling terms large at will is a particularly crucial topic. Existing attempts present that symmetry breaking together with local resonance are feasible solutions. This strategy also leads to obvious drawbacks of narrow working frequency range. Then how to achieve significant Willis coupling terms in a broadband range is an open question. This capability is particularly crucial when we reduce the operating frequency to low frequency or even static conditions.

Moreover, there remains nontrivial in searching for more promising application scenarios beyond UZR. In optics and electromagnetism, bianisotropic materials have been used to realize anomalous refraction, polarization transformation, isolation and other intriguing phenomena [110]. As counterparts of bianisotropic materials (in electromagnetics), Willis metamaterials need to be further explored on their application potentials. Considering that extreme media correspond to extreme wave phenomena, it is also fruitful to introduce active components or other fields to break the physical constraint of Willis coupling terms, such as the constraint of reciprocity [111].

Advances in science and technology to meet challenges

Let us first set the problem in a relatively simple situation than the full 3D version. In a 1D beam-like Willis metamaterial, the most general form of the constitutive relations for flexural waves [103] is presented as

$$\begin{pmatrix} \frac{bM_{xx}}{k_0E_0I_0} \\ \frac{bQ_{xz}h_0}{k_0E_0I_0} \\ \frac{k_0bN_x}{-i\omega\rho_0b_0h_0^2} \\ \frac{k_0bP_z}{-\omega\rho_0b_0h_0} \end{pmatrix} = \mathcal{K} \begin{pmatrix} \frac{\partial_z\in_{xx}}{k_0} \\ \frac{2\in_{xz}}{k_0h_0} \\ k_0h_0\partial_z u \\ k_0 w \end{pmatrix} \tag{2}$$

where

$$\mathcal{K} = \begin{pmatrix} \frac{EI}{E_0I_0} & \frac{EI'}{E_0I_0} & -\frac{k_0^2h_0^2}{12}\kappa & \frac{k_0^2h_0^2}{12}\kappa'' \\ \frac{EI'}{E_0I_0} & \frac{12\mu A}{E_0A_0} & -\frac{k_0^2h_0^2}{12}\kappa' & -\frac{k_0^2h_0^2}{12}\tau \\ \frac{k_0^2h_0^2}{12}\kappa & \frac{k_0^2h_0^2}{12}\kappa' & \frac{1}{12}\frac{\rho_{xx}I}{\rho_0I_0} & \frac{1}{12}\frac{\rho'I}{\rho_0I_0} \\ \frac{k_0^2h_0^2}{12}\kappa'' & \frac{k_0^2h_0^2}{12}\tau & \frac{1}{12}\frac{\rho'I}{\rho_0I_0} & \frac{\rho_{zz}A}{\rho_0A_0} \end{pmatrix} \tag{3}$$

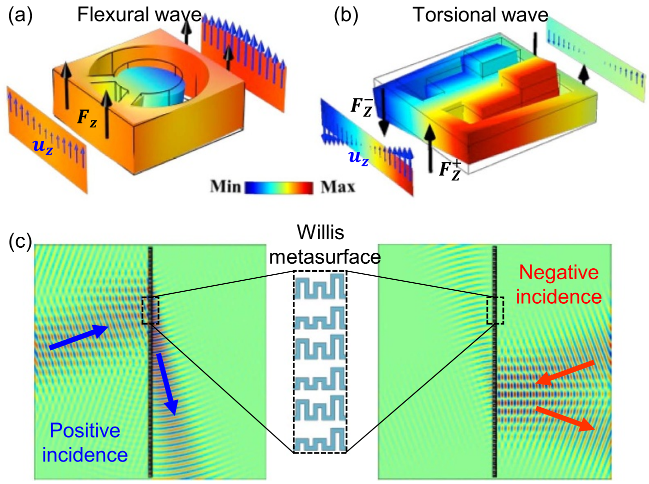

Figure 11. Elastic Willis metamaterials by breaking mirror symmetry. (a) A Willis metamaterial for elastic flexural waves (Reproduced from [103]. CC BY 4.0.). (b) A Willis metamaterial for elastic torsional waves (Reproduced from [105]. © (2021). Published by IOP Publishing Ltd. CC BY 4.0). (c) A Willis metasurface to achieve the asymmetric transmission of flexural waves on a plate. (Reprinted from [108], Copyright (2022), with permission from Elsevier).

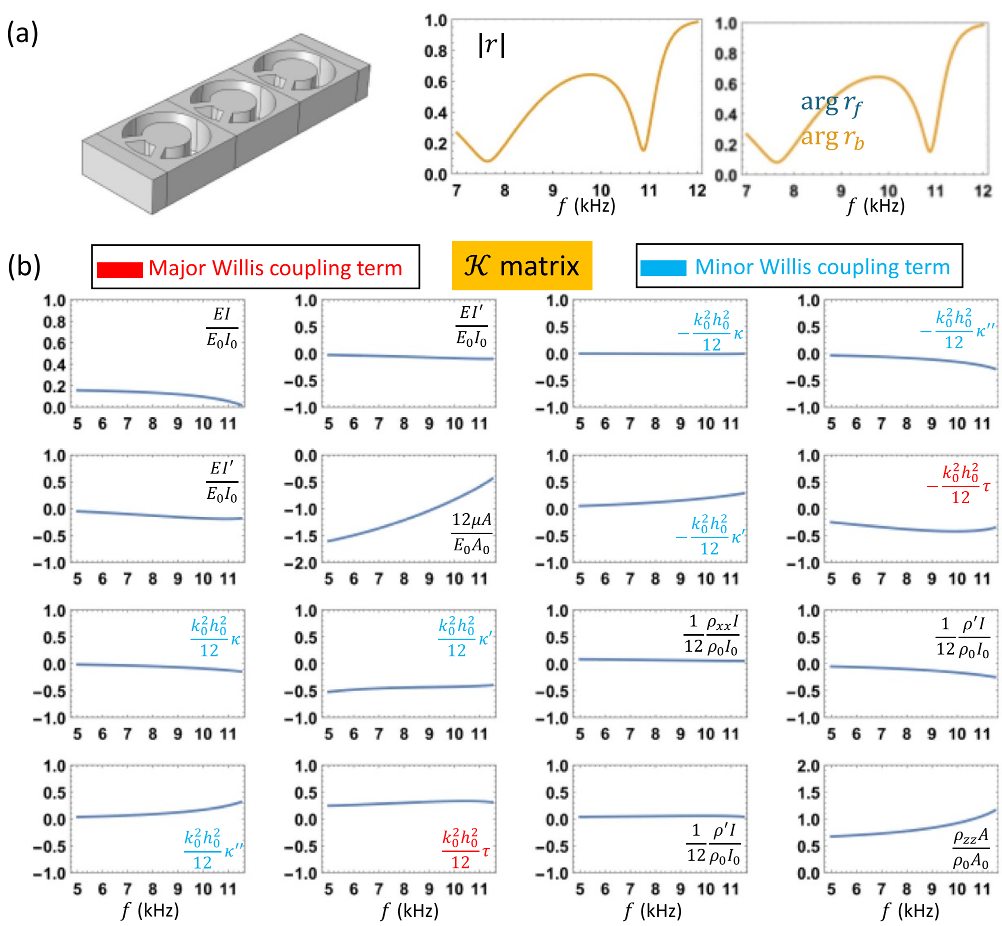

Figure 12. Extraction of the full constitutive matrix of a Willis metamaterial. (a) A typical Willis metamaterial in a beam and its reflection spectrum when the flexural wave is incident from the forward (represent by a subscript *f*) and the backward (represent by a subscript *b*) side, respectively. Reproduced from [103]. CC BY 4.0. (b) The effective constitutive matrix of the Willis metamaterial, numerically extracted based on the reflection and transmission spectrum.

is the constitutive matrix, with $\boldsymbol{\tau}$ the major Willis coupling term and $\boldsymbol{\kappa}$, $\boldsymbol{\kappa}'$, $\boldsymbol{\kappa}''$ the minor Willis coupling terms, respectively. Figure 12(b) shows the retracted effective values of $\mathcal{K}$ based on the based on the reflection and transmission spectrum. It is noted that, different from ordinary methods for retrieving effective properties, we should let waves incidence from both the forward and the backward sides, respectively. Accordingly, the reflection and transmission spectrum should be calculated twice [105, 106]. For a more complicated case of 3D Willis metamaterial, different directions of the incident waves are also required.

As for the search of large (or even maximum) Willis coupling terms, a deep learning-assisted method for inverse design may be used as proposed in acoustics [112]. In theory, the restrictions imposed by reciprocity and energy conservation limit the bounds of Willis coupling terms [113]. To break this limitation, active components may be added into metamaterials to decoupled control all constitutive parameters in a non-reciprocal regime [114].

Interestingly, prestress has shown as a feasible solution to realize the Willis coupling term, even down to the quasi-static case [115]. This strategy may be generalized to dynamic situations to broaden bandwidth. Moreover, by connecting Willis metamaterials to multi-physical effects, one may unlock opportunities to seek many intriguing applications. For example, advanced energy harvesting is expected by incorporating Willis metamaterials with piezoelectricity [116]. When Willis coupling is induced to modulate thermal conductivity and mass density in heat transfer, spatiotemporal diffusive metamaterials are expected for potential applications on directional diffusion [117].

Concluding remarks

In recent years, interest in Willis metamaterials has expanded to a large degree due to the extra degree of freedom to manipulate acoustic and elastic waves. This emerging field corresponds to bianisotropic materials in electromagnetism, and deserves to be explored further for applications. In the near future, we can explore fundamental approaches on the control of the full constitutive matrix of Willis metamaterials, the way to design significant Willis coupling terms in broadband range (or even in the quasi-static case). Moreover, the incorporation of Willis metamaterials with multi-physical effects may stimulate new ideas and research directions, together with many intriguing applications.

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10. Active and tunable phononic crystals

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Status

Since the concept of phononic crystal was proposed in 1983, there is a rapid growth of the research achievements in this field. However, most of them are about the unreconfigurable or passive phononic crystals that have unchangeable structural geometries and host material properties and therefore exhibit usually a single functionality. Since about ten years ago, tunable or active phononic crystals with changeable configurations and material properties have gradually received increasing attention because of their distinguishing feature of switchable multi-functionalities and adaptability to frequency or environment [14, 118]. The promotion of research on tunable and active phononic crystals includes four levels: reconfiguration, controllability, programmability, and intelligentize [14]. The first level is design of a reconfigurable phononic crystal whose inner structure or component material properties can be changed. In most reported studies, the reconfiguration of inner structures was realized through mechanism (e.g. bolts [119], origami [120]), large deformation [121], multimodal instability [122], phase transition [123], liquid filling [124], etc; and changes in component material properties are usually achieved through the coupling characteristics of multiple physical fields such as piezoelectric coupling, magnetomechanical coupling, thermomechanical coupling, and optomechanical coupling [118]. The second level is about control of reconfiguration by using external field stimuli. Mechanical reconfigurability of inner structures is generally realized by applying external forces. Especially, for phononic crystals made of soft materials or flexible structures, applied forces may generate large deformation or even multimodal instability [125]. Origami phononic crystals can be reshaped by applying forces [120]. Besides, a phononic crystal made of shape memory alloy or polymer may exhibit significant changes in shape when it is heated [123]. Configuration control of a phononic crystal with bolts or liquid filling must be applied to each unitcell, which is not a non-trivial task. The properties of intelligent constituent materials that make up tunable phononic crystals can be adjusted by applying external electric, magnetic, optic, and thermal fields. On the basis of reconfigurability and controllability, one can further achieve programming or coding control and even intelligent control with the aid of feedback system [126].

Tunable and active phononic crystals provide us a platform to design tunable, programmable and even intelligent devices. Besides tunable band gaps, controllable regulation of wave propagating path also received attention to achieve adjustable cavity, waveguide [119], cloak [126], switchable unidirectional propagation, etc [14, 118]. Recently, tunable or controllable topology states have aroused great interest from scientists as shown in figure 13 [127–130].

Current and future challenges

Despite more than one decade of development, tunable and active phononic crystals still face many challenges in their practical application in engineering. One of the challenges is the control method for reconfiguration. A phononic crystal is composed of unitcells. There are two ways to control the reconfiguration of a tunable phononic crystal. One way is to control each unitcell individually; and the other is to control all unitcells collaboratively. Because an actual phononic crystal, although finite in size, includes a lot of unitcells, individual control on each unitcell generally needs a complex and lumbersome control system [126], and it is difficult to implement automatic control especially for mechanical reconfiguration. Integrating the control system into a unitcell is a possible solution but is challenging technically. But miniaturization of control systems and wireless control are required. Integrated control of the entire system, including reshaping of lattice form and unitcell's structure, can be implemented through origami, large deformation, multimodal instability, and phase transition induced by external mechanical, thermal, magnetic, and optic fields. However, non-uniform reconfiguration of unitcells is difficult to achieve, which limits the function regulation. Usage of intelligent materials in tunable phononic crystals makes automatic control easier to implement. But narrow regulation range and possible time-delay in control (including feedback control) deserve attention. In addition, remote non-contact control is attractive and challenging. Light, heat, magnetic force, and electromagnetic waves are beneficial for implementing non-contact control [131]. However, materials highly sensitive with quick response to these physical fields are expected.

Besides control methods, the adjustable control of some new features or functions is also challenging. Recently, some new extraordinary properties that were found in quantum systems, e.g. topology protection, non-Hermitian, BIC (bound states in continuum), Moiré effect, have received attention [132]. Tunable and active topological phononic crystals have been reported [127–130]. Fine control on gain/loss in non-Hermitian phononic crystals can generate many anomalous behaviors, e.g. exceptional point, parity-time (PT) symmetry, topological skin effect, etc [133]. Tuning of gain can be implemented by the aid of piezoelectricity. However, adjusting loss is challenging. In addition, tunable and controllable non-linear behavior of phononic crystals is also a challenging topic.

Finally, we would like to mention another challenge, that is the customized reverse design of tunable phononic crystals in both space (3D) and time domains (1D), especially the customization in time-domain!

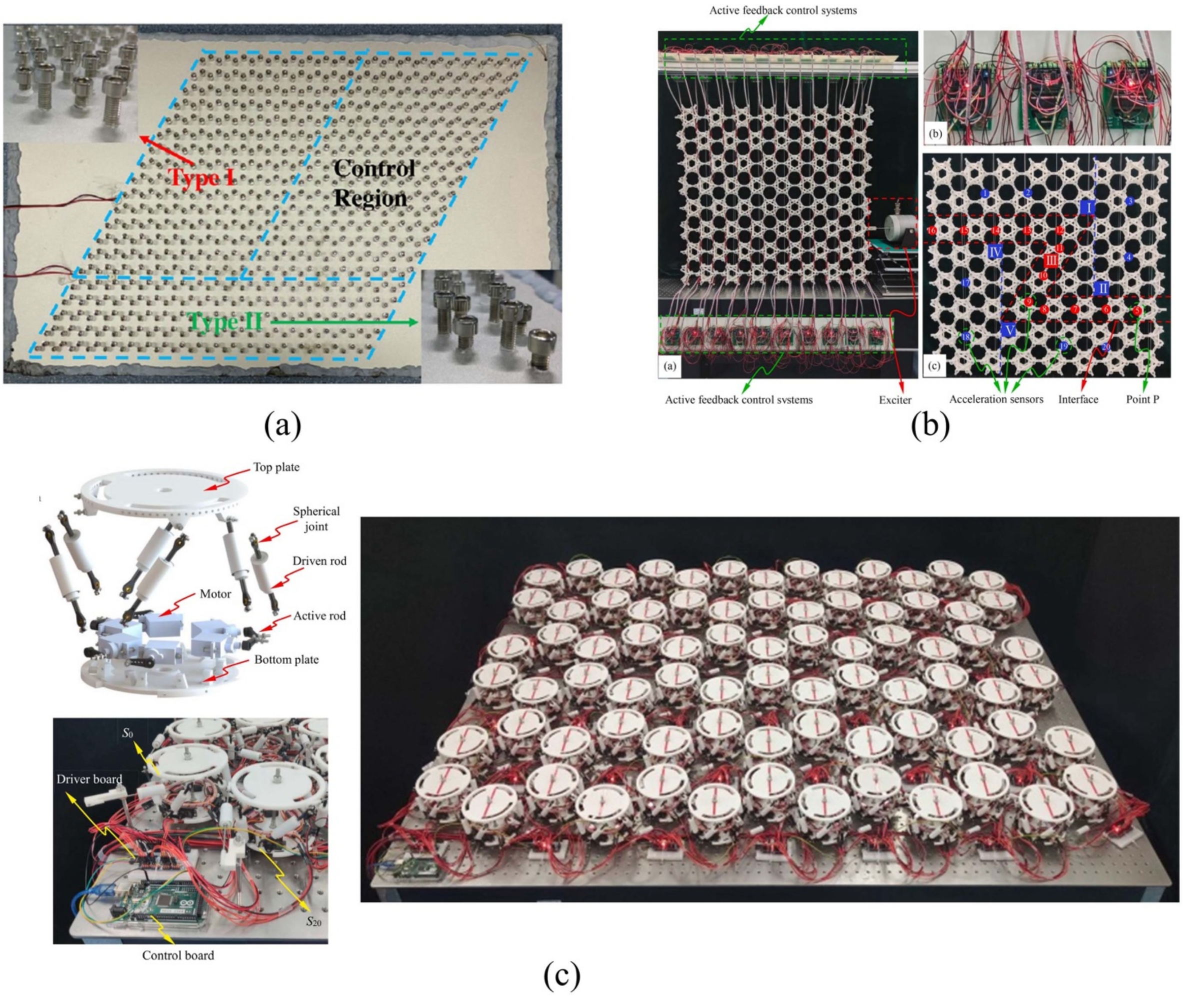

Figure 13. Tunable and active topological phononic crystals: (a) blot-aided tunable system with adjustable topological valley transmission path. Reprinted from [127], Copyright (2022), with permission from Elsevier. (b) Active nonlocal topological system with piezoelectric feedback control. Reprinted from [128], Copyright (2023), with permission from Elsevier. (c) Active topological system composed of unitcells with bistable states. Reprinted from [129], Copyright (2023), with permission from Elsevier.

Advances in science and technology to meet challenges

Further advancement in research on tunable and active phononic crystals depends on the development of physics, mechanics, material science, processing technology, electronic technology, information technology, etc. Development of mechanism theory can promote the fine design of mechanically reconfigurable structures. Tunability based on large deformation and multimodal instability of soft materials or flexible structures relies on the development of large deformation theory and numerical method for buckling and postbuckling. The multi-fields coupling large deformation theory is necessary to design soft intelligent structures [134].

Manufacturing of materials is the key to tunable phononic crystals, especially intelligent and/or soft phononic crystals. Intelligent materials that can quickly respond and produce large deformation under the action of bias fields are particularly expected. Preparation technology of elastomer with controllable stress relaxation and viscosity deserves further improvement.

Fine machining technology and micro/nano machining technology are important to miniaturization and integration of tunable and active phononic crystals, especially to those working at ultrasonic frequencies. 3D or 4D printing technology is very beneficial.

Miniaturization technology for remote wireless control and low-delay feedback control are necessary for development of active and smart phononic crystals. Optimization methods for 4D customized reverse design are extremely desired; and perhaps artificial intelligence (AI) technology can of great assistance in this aspect.

Concluding remarks

Research on tunable and active phononic crystals has become a hot topic. Some new mechanisms and methods of reconfiguration and control have been proposed. Active control on some new properties and functions has received attention. However, there are still many challenging issues that need to be addressed, which stimulates the development of science and technology. We can expect that in the foreseeable future, this topic will receive sustained attention. Besides efficient control methods, use of novel intelligent materials, miniaturization and integration, and customized design, practical application will be concerned, e.g. self-perception and adaptive intelligent equipment (for vibration isolation and noise reduction, stealth, camouflage, energy concentration and orientation, acoustic antenna, etc), programmable and adjustable acoustic devices, tunable information processing systems (storage, encryption, transmission, and simulation operations).

Acknowledgments

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11. Spatiotemporal phononic crystals

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Status

The use of time modulations of material properties, either alone or combined with spatial modulations, is gaining traction due to the added degree of freedom it offers in material design. This allows for the achievement of physical phenomena not attainable in materials solely modulated in space. Historically, research on the dynamics of time-modulated media dates to the 1950s with the development of traveling wave parametric amplifiers [135], along with studies on the dispersive properties and stability of electromagnetic waves in periodically modulated space-time environments. However, the application of these concepts in acoustics and mechanics is relatively recent and is linked to advancements in fabricating acoustic and elastic waveguides, possibly incorporating active elements.

For example, waveguides with time-modulated stiffness have shown a frequency-periodic dispersion spectrum with wavenumber gaps, namely flat dispersion branches where standing waves are parametrically amplified. Conversely, when the time-modulation is coordinated in space to generate a wave-like pattern, asymmetric Bragg scattering bandgaps in the dispersion spectrum arise [136] (see figure 14). Such one-way Bragg scattering stems from the bias induced by the spatio-temporal modulation which breaks the time-reversal symmetry.

When the wave-carrying medium supports multiple dispersive modes, such as beam- or plate-like waveguides, nonreciprocity is accompanied by frequency and mode conversion of transmitted/reflected waves. In all scenarios, the nature and strength of the observable non-reciprocal phenomena can be tuned by varying the modulation speed and amplitude. In particular, the modulation speed dictates the angle by which the dispersion curve of a spatially modulated material is tilted. Subsonic modulations yield an adiabatic regime where topological effects can be explored. Modulations with small amplitudes and medium speeds yield a Bragg scattering regime. Supersonic modulations can completely impede the propagation in a designated direction, or potentially reverse it [20].

Regarding the topological properties, when slow modulation is considered, the ratio of tilt to modulation speed can be interpreted in terms of Berry's phase and curvature and linked to a robust integer-valued Chern number [137]. Nonzero Chern numbers suggest the existence of one-way edge modes which can be transferred from one boundary of the waveguide to the other using temporal modulation as the driver.

Non-reciprocity, frequency and mode conversion, and temporal pumping are phenomena that can have direct applications in the design of devices for signal processing, communication, and information transport. However, they are not the sole physical phenomena that can be achieved using time-variant media, as discussed in the next section.

Current and future challenges

There are major research directions still open in the field of time-modulated phononic media including, among others, the investigation of non-Hermitian systems with parity-time (PT) symmetry, waveguide with time-discontinuous modulation (time interfaces), temporal waveguiding, nonlinear media modulated in time.

Non-Hermitian systems with parity-time (PT) symmetry. Non-Hermitian systems with PT symmetry are an active field of research, only partially explored in phononics. Non-Hermitian (e.g. complex) spatiotemporal modulation of the material properties [138] leading to PT symmetry, which broadly speaking stems from the proper combination of gain and loss in the material, allow to access a plethora of intriguing physical phenomena, including the emergence of exceptional points (EP). Exceptional points are degeneracies in the frequency spectra where some eigenvalues and eigenvectors coalesce. EPs support unconventional phenomena such as asymmetric scattering, unidirectional invisibility, and enhanced sensitivity. The latter property is particularly appealing in phononics for the design of sensing devices. Future efforts along this research line should be devoted to the experimental realization of NH-PT phononic systems looking at their possible integration in sensing devices.

Time-interface and temporal aiming. A time interface is a sudden change in the material properties of the medium which occurs uniformly in space. In photonics, time interfaces have been recently investigated as the temporal counterpart of spatial interface, introducing the concept of time-reflection [139]. When time reflection occurs a portion of the input signal is partially time-reversed, its frequency content transformed while the momentum remains constant due to spatial translation symmetry. Engineering of such time interfaces opens new pathways for broadband wave control. Temporal interfaces can be leveraged also to steer the direction of the input achieving the so-called temporal aiming. As of today, the adoption of time-interface and temporal wave steering in phononic crystals is still very limited including few theoretical investigations [140] (see figure 15). Future research work should be devoted to fully leveraging the additional wave control strategies provided by discontinuous time modulation and temporal aiming with an effort toward experimental realization.

Nonlinearity and time modulation. Harnessing the combined effects of spatio-temporal modulations and nonlinearity has yet to receive adequate attention in the phononic research community. Initial research endeavors in this direction are already revealing the richer dynamics of nonlinear modulated systems, including the creation of stable, large-amplitude time-quasi-periodic solutions that can coexist with stable,

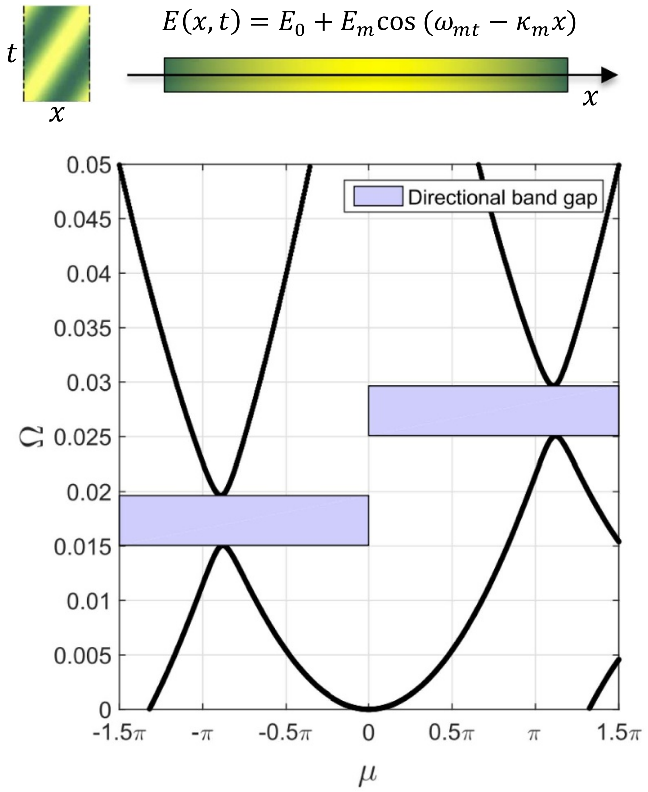

Figure 14. Dispersion curve of a space-time modulated elastic beam. Directional bandgaps highlighted in blue. Reproduced from [136].

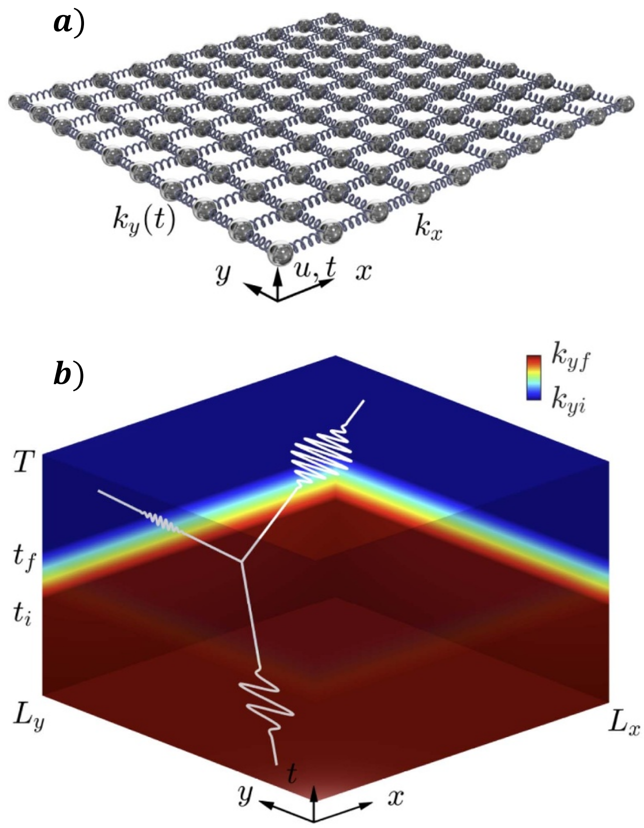

Figure 15. (a) Schematic of a 2D spring-mass lattice stiffness-modulated along the *y* direction. (b) Representation of the wave packet propagation including the frequency modulation and steering. Reproduced from [140].

small-amplitude time-periodic ones [141]. Further advancement along this research path can lay the groundwork for exploiting the balance between nonlinearity and time modulation in the design of signal processing and communication devices.

Advances in science and technology to meet challenges

From a technological perspective, significant advancements are still necessary to realize effective mechanisms for generating the desired space-time modulation pattern and integrating them into devices. Currently, the most common strategies for implementing time modulation in phononic media rely on active elements. Examples include the utilization of electromagnets and/or shunted piezoelectrics, although the realizations remain at the prototype stage [67]. Nonetheless, standard techniques in microfabrication and nanotechnology, employed for designing transducers and MEMS, are expected to bridge the gap between available lab-scale prototypes and the desired devices.

Besides the fabrication techniques, the primary drawback of active time modulation remains the continuous requirement for energy-intensive input. This raises feasibility issues, particularly for portable or energy-constrained applications. Hence, parallel design strategies seek to exploit fully mechanical, passive drivers to modulate material properties. Examples include using and tuning prestress to disrupt the time wave reciprocity in phononic crystals, employing large geometrical mechanical deformation on the waveguide achieving significant stiffness modulations due to mechanical instabilities, and adopting external rigid mechanisms (e.g. inerters) under large displacement configurations to modulate the effective mass of distributed resonant elements [142]. However, challenges remain in the required material designs to implement the desired spatial-time pattern, the limited achievable parameter tunability, and the durability and repeatability of the modulation scheme over time. Although the design of fully passive modulated phononic crystals is still in its infancy, increasing research efforts towards this goal will likely bridge this technological gap soon.

Concluding remarks

The field of space-time modulated phononic crystals holds significant potential for advancement in the foreseeable future.

The integration of time modulation with advances in microfabrication and nanotechnology promises to support the development of next-generation phononic devices with enhanced performance and versatility, paving the way for novel applications in acoustics, sensing, and information processing. There is a clear demand for such devices and future research efforts aimed at addressing these needs will undoubtedly build upon the scientific principles and advancements outlined in this roadmap.

Acknowledgments

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12. Functions and properties—hyperuniformity and correlated disorder in phononic structures to control acoustic waves

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Status

Wave propagation in discrete spatial distributions of scatterers is a venerable field of research in Physics in the broad sense [143]. In the field of acoustics, which deals with sound waves and/or vibrations, the propagation of waves in phononic structures is a hot topic that arouses intense activity covering a vast range of applications resulting from the wide range of tackled frequencies, i.e. ranging from infrasounds to radiofrequencies [144]. In this regard, phononic structures are understood as discrete spatial distributions of scatterers, with given elastic/acoustic properties, embedded in a different host material. If the host medium is a fluid, then, sometimes this complex media can be referred to as sonic structures as they affect sound waves propagating in the host. Among the several possibilities proposed in the last decades, phononic crystals and acoustic/elastic metamaterials are perhaps the most widely exploited. Effectively, they offer great potential for wave control by means of their dispersion properties imposed by periodicity due to Bragg scattering and/or by local resonance properties in the case of metamaterials. Contrary to the periodic case, random structures have no order at any scale and consequently no characteristic length drives the system. Waves that penetrate in a random material are scattered several times before emerging in random directions, producing interferences leading to interesting physical phenomena [143]. Some disordered structures can also possess ranges of frequencies with isotropic destructive scattering [145–147]. New concepts have been developed to explain the existence of these peculiar band gaps in disordered media. Simple Bragg scattering cannot serve as an explanation, as it does for periodic structures. A classification of the random structures was needed, and thus the materials with correlated disorder through the hyperuniformity emerged as an order metric, defining materials with vanishing density fluctuations in the large distance limit [148].

Imagine a N points distributed in the space. If the local number variance σ^2 (i.e. the variance in the number of points within a randomly-thrown spherical window of radius R) within a spherical observation window grows more slowly than the volume of the observation window, then the point distribution is called hyperuniform. Equivalently, hyperuniformity can be defined from the structure factor $S(\vec{q})$ in the reciprocal or Fourier space. If the N points are located at positions $\vec{r_i}$, $(i = 1,\ldots,N)$ inside a square domain of side L, the structure factor, $S(\vec{q})$ of the point pattern $\vec{r_i}$ is defined as its spatial Fourier transform and reads as follows

$$S(\vec{q}) = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N} e^{-\imath\vec{q}\left(\vec{r_j}-\vec{r_i}\right)}.$$

A point distribution the structure factor of which vanishes in the long wavelength limit, i.e. $S(\vec{q}\to 0) = 0$ where $q = |\vec{q}|$ is called hyperuniform. In the case in which the structure factor that vanishes around the origin of the wavevectors, $S(q < q_c) = 0$ with $\vec{q_c}$ the cut-off reciprocal vector defining Ω that refers to the set of the constrained reciprocal vectors $(q < q_c)$, the point pattern is called stealthy hyperuniform [149]. At this stage we need to define a parameter to characterize such type of structures. The stealthiness, χ, is the ratio of the number of constrained reciprocal lattice vectors to the number of degrees of freedom in the real space [148]. For example, the values of the stealthiness in 2D are bounded between, $\chi_{\min} = 0$ and $\chi_{\max} = \pi/4$ leading respectively to Poison's distributions and perfect crystal lattices. Figure 16(a) shows a picture of a hyperuniform acoustic material made of aluminum cylindrical (acoustically rigid) scatterers embedded in air with $\chi = 0.3$. Figures 16(b) and (c) show the structure factor corresponding to point patterns with $\chi = 0.3$ and with $\chi = 0.48$. The circumference of the constrained area imposed by the definition of stealth hyperuniform in the reciprocal space is clearly visible. In addition, the structure factor exhibits an extra isotropic region for $\chi = 0.48$, where an increase of the structure factor is visible (yellow ring region). For $\chi > 0.5$ (not shown in the Figure) the structure factor is anisotropic showing the hints of periodicity of the point pattern. It is said that the point pattern crystallizes when the stealthiness is higher than to 0.55. Experimental evidences of isotropic transparency and complete band gap formation for ultrasound propagation in stealthy hyperuniform media have been recently shown in [150].

The main wave transport characteristics of hyperuniform materials are dictated by the structure factor. The amplitude of the intensity of the scattered field is proportional to $S(\vec{q})$. Figures 16(d) and (e) respectively show the acoustic transmission the two hyperuniform materials previously analyzed. The average transmission is close to 1 in the constrained range of frequencies, showing the characteristic transparency regime of stealthy hyperuniform materials [transmission close to 1 in figures 16(d) and (e) for frequencies lower than the limit

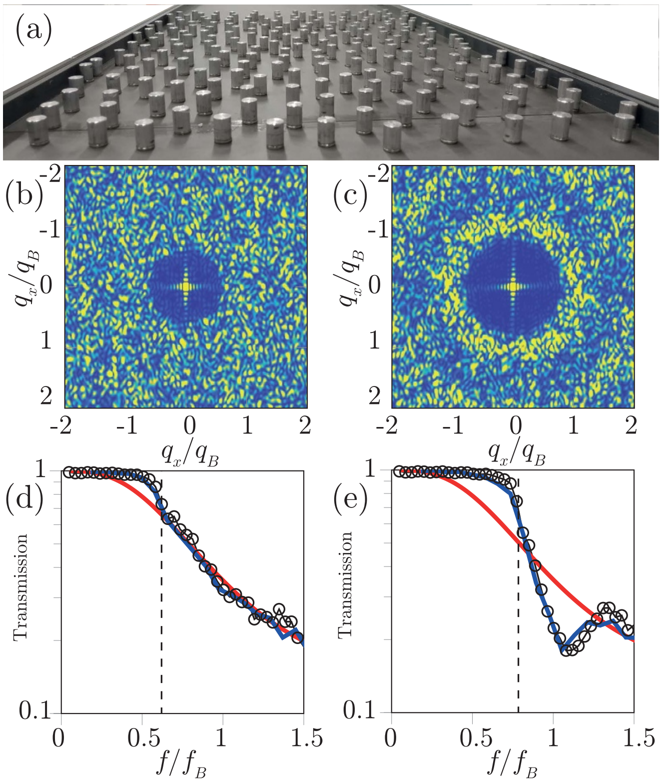

Figure 16. Acoustic hyperuniform materials. (a) Sample of a 2D system. (b)–(c) Structure factor for $\chi = 0.3$ and $\chi = 0.48$. (d)–(e) Transmission spectra for the two evaluated hyperuniform materials with $\chi = 0.3$ and $\chi = 0.48$. Red line represents the transmission for a random material, blue and black open circle lines represent the theoretical with multi-modal approach and numerical simulations respectively. Reprinted (figure) with permission from [149], Copyright (2022) by the American Physical Society.

imposed by χ (vertical dashed line)]. The behavior for frequencies higher than the limit imposed by χ, has different trend as χ increases. Therefore, the parametric analysis of the system with χ provides a very good platform to understand the transition from disordered systems to periodic ones [151]. These last features are in opposition to the quasi-periodic systems in which anisotropic band gaps are created with $\chi \gtrsim 0.5$ [152]. The concept of hyperuniformity has proven to be highly valuable as it also encompasses structures with quasiperiodic ordering.

This classification would include structures with correlated order that are neither disordered, nor periodic or random. In them, the so-called hyperuniformity has been proved to be very valuable for studying the structure factor within the long wavelength range [153].

Current and future challenges

Wave transport properties in materials made of hyperuniform distributions of scatterers [154], have recently been analyzed for electromagnetic [145, 155–157] and elastic [156] waves by combining the geometric properties of stealthy hyperuniform point patterns with the Mie resonances of the scatterers. Exploiting the possibilities of isotropic scattering by amorphous materials, waveguides with arbitrary paths have been designed [145, 155]. Moreover, and more recently, motivated by the presence of this isotropic band gaps, the localized edge modes have been observed [158] at the interfaces between two hyperuniform materials.

Nevertheless, Mie resonances are usually exploited in all these examples, although hyperuniform materials should be uniquely characterized by the spatial Fourier transform of the point pattern. In this sense, the use of non-resonant scatterers seems to be the best option for distinguishing the scattering produced by local resonances from that due solely to the arrangement of the scatterers. The analysis of wave transport properties of stealthy hyperuniform materials made of non-resonant scatterers for scalar waves have been recently discussed, ensuring that the transport properties are only due to hyperuniform point pattern distribution used to build the material. For example, when disorder is introduced into periodic systems, Bragg and Mie resonances together have been shown to play important roles: the band gap closes rapidly as disorder increases when the gap is due to Bragg scattering, while it is more robust, when it is due to Mie resonances. Recently, several works have analyzed the interplay of these two phenomena in hyperuniform materials for acoustic waves [159, 160]. It has been shown that when the local resonances are tuned in the transparent frequency band of the hyperuniform material, a dip of transmission is produced [160], while if the resonance is close to the frequency of isotropic scattering, an isotropic super band gap in hyperuniform materials can be created [159].

By using the relationship between the structure factor and the amplitude of the intensity scattered field, the challenge is the design of point scatterers distributions with target scattering properties. Disordered two-dimensional (2D) acoustic structures consisting of rigid circular cross-sectional cylinders embedded in air has been designed [161]. These structures present prescribed scattering properties when excited by a plane wave. The information on the scattering pattern is imposed in the reciprocal space and by means of an optimization procedure, the positions of scatterers can be optimized to ensure the targeted scattering properties. Figures 17(a) and (b) represents two point patterns producing respectively broadband back-scattering suppression and broadband equally intense scattering in both cases independently of the angle of incidence. Figures 17(c) and (d) show the structure factor in the reciprocal space, with the target scattering properties for the back-scattering suppression (equally intense scattering), i.e. zero (maximal) structure factor between a given range of frequencies delimited by the two white rings. Figures 17(e) and (f) represents the polar plot of the scattered field calculated by using multiple scattering.

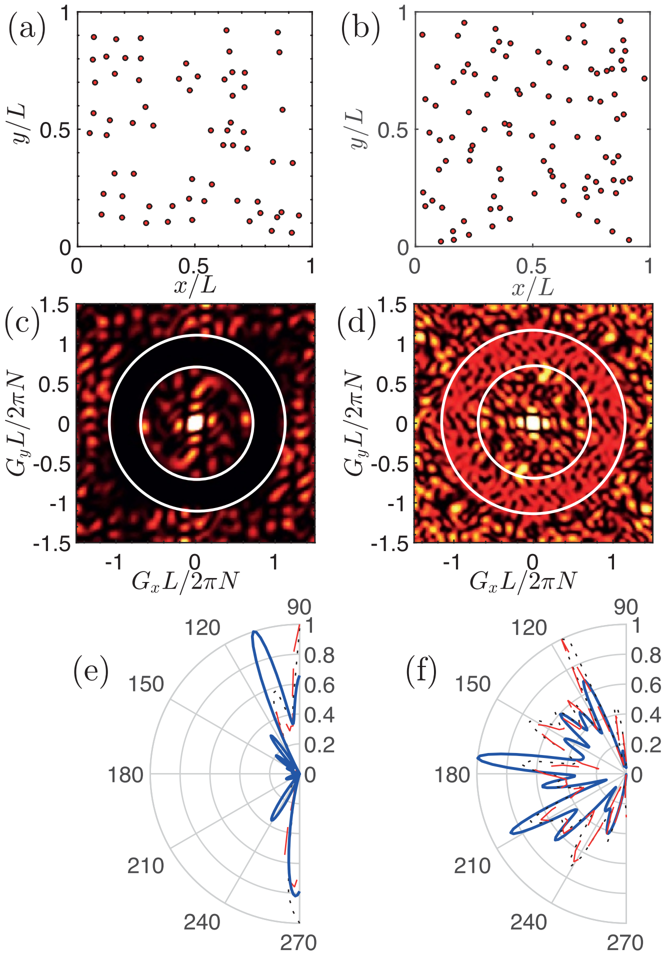

Figure 17. (a), (b) respectively show the optimized point patterns for broadband back-scattering suppression and broadband equally intense scattering in both cases independently of the angle of incidence. (c), (d) show the corresponding structure factor of (a) and (b) respectively. White circumferences limit the target range of frequencies. (e) and (f) show the polar response when a distribution of rigid acoustic scatterers is place in the distributions shown in (a) and (b) and radiated by a plane wave. [161] (2021), reprinted by permission of the publisher (Taylor & Francis Ltd, www.tandfonline.com).

Advances in science and technology to meet challenges

Correlated disorder materials often exhibit emergent phenomena for the control of waves. Advances in experimental and theoretical techniques will enable scientists to explore these phenomena in greater detail and understand the underlying mechanisms, opening new possibilities for technological applications. The advances in the design of laser vibrometers and other technologies for the characterization of the vibrations can open new venues to the understanding of these materials for the phononic community. Moreover, addressing the challenges posed by correlated disorder materials requires a multidisciplinary approach involving physicists, chemists, materials scientists, and engineers. Collaborative efforts across different disciplines facilitate a deeper understanding of these materials and can accelerate progress towards practical applications. Overall, the combined efforts of scientists and engineers utilizing advanced characterization techniques, computational modeling, synthesis methods, and interdisciplinary collaboration could drive significant advances in our understanding and utilization of correlated disorder materials, paving the way for technological breakthroughs.

Concluding remarks

Heterogeneous materials formed by a set of scatterers embedded in a host material with tailored properties represent a useful tool for the control and manipulation of acoustic, electro-magnetic and matter waves. The field of hyperuniform and correlated disorder phononic structures presents a new platform to prescribe scattering properties of the system in the reciprocal space, to later obtain the spatial distribution of scatterers with the corresponding scattering.

Acknowledgments

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13. Pentamode materials for underwater acoustics control

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Status

In the pioneer work of Milton and Cherkaev [162] in 1995, pentamode materials was mainly proposed for curiosity from the mathematical point of view. At that time, such materials mainly exist as a theoretical concept, without any anticipation for possible acoustic applications. Two decades later, Norris's transformation theory in 2008 [163] surprisingly revealed that pentamode materials show significant potential for manipulating underwater sound, with invisibility acoustic cloak as the most appealing application. This discovery quickly sparkled great interests in pentamode materials in the scientific and engineering communities. In the following years, extensive study on pentamode materials ranging from structure design, fabrication to experiments has been conducted [164–171] and continues to emerge nowadays.

Pentamode materials are mathematically defined as a class of special elastic materials whose elasticity matrix has five zero eigenvalues [162]. Each zero eigenvalue corresponds to an easy deformation mode, which does not induce stress or strain energy in materials. Generally speaking, pentamode materials require zero shear modulus but finite bulk modulus [163]. Water can be treated as an ideal pentamode material. Pentamode materials can also be approximated by solid structures and their properties can be easily tailored by geometry parameters [164–171] as examples in figure 18. For structural pentamode materials, their effective shear modulus cannot be ideally zero owing to structural stability requirement, but can be orders of magnitude smaller than their bulk modulus. It should be noted that the inevitable weak shear modulus can show prominent influences to wave control and must be carefully studied for applications [166]. Based on structure design, the long-ago anticipated pentamode acoustic cloak for underwater waves is numerically simulated in 2015 [166] and then finally verified experimentally in 2017 [167] as seen in figure 19, nearly ten years after the theory [163]. Since then, other acoustic applications are subsequently proposed and verified, like metasurfaces for focusing sound, transforming wave form [168] or insulating low frequency water sound [169]. Controlling of elastic fields by using pentamode materials is also possible yet much less explored, e.g. static elastic cloaking [164]. The study of 3D pentamode materials is limited to mechanical characterization and structural design by optimization [170], owing to the lack of reliable new models.

Current and future challenges

For wave controlling purpose, graded distribution of anisotropic pentamode materials is usually required based on transformation theory [162]. It is crucial to propose innovative pentamode structures whose effective acoustic properties, i.e. bulk modulus and in particular the stiffness anisotropy, can be tailored in a broad parameter space. Another significant challenge is to achieve impedance matching to water, which is needed for acoustic control. Pentamode structures generally require rather thin beams in order to achieve as small effective shear modulus as possible. Their effective bulk modulus is, therefore, orders of magnitude smaller than the constitute component. For 2D pentamode structures, high-stiffness metals such as Titanium or Aluminum can provide sufficient bulk modulus to match the impedance of water [164, 168]. However, to design 3D pentamode structures with good impedance match to water, the required Young's modulus of the constitute material will be as high as thousands GPa, which is obviously not acceptable, unless thicker beams are to be used but at the price of increased effective shear modulus. As a result, acoustic experiment study on 3D pentamode materials are much less behind than the 2D case [170].

In addition to the above design issue, multiple engineering challenges, e.g. need of efficient fabrication technology and sufficient mechanical strength, must be fulfilled for realistic engineering applications of pentamode materials in controlling underwater acoustic waves. Centimeter scale metallic pentamode structures often contain thin parts with sub-millimeter thickness [167, 168]. The sophisticate structures require high-precision fabrication technology, such as the very expensive electrical discharge machining technology or micro-machining technology [168]. In addition, fabrication of small samples with hundreds of unit cells is already very time consuming, in the order of weeks. Efficient and cost-effective fabrication technique will be needed for more complicated and large-scale samples. Mechanical strength of pentamode structures should also be systematically studied if they are to be used under a deep-water or a high-pressure environment. The required small effective shear modulus for pentamode materials impose a significant challenge to achieve high structural loading capability.

Advances in science and technology to meet challenges

Topology optimization is a well exploited method in the mechanical community to find structures configurations with demanded static or dynamic mechanical properties. This method has been very efficient in optimizing 2D geometry and

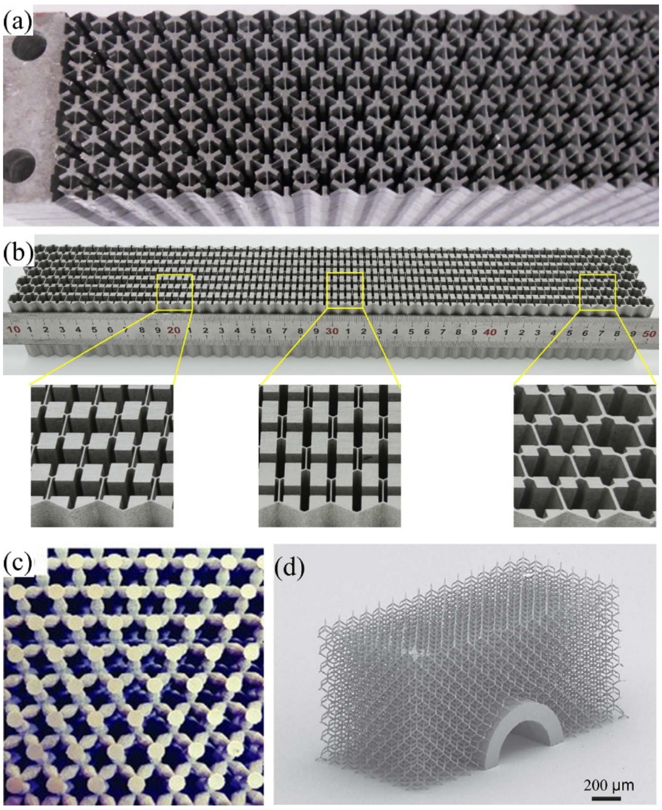

Figure 18. Pentamode metamaterial samples. (a) Macroscopic 2D pentamode materials sample fabricated by water-jet cutting. Reprinted from [164], with the permission of AIP Publishing. (b) Macroscopic 2D pentamode materials fabricated by electrical discharge machining. Reprinted (figure) with permission from [168], Copyright (2019) by the American Physical Society. (c) Additively manufactured 3D pentamode materials. Reprinted from [170], with the permission of AIP Publishing. (b) Polymer-based microscopic 3D pentamode materials. Reproduced from [165], with permission from Springer Nature.

could potentially be used to find new structures that shows the pentamode feature. Topology optimization for 3D geometry is still computation demanding, but might be combined together with neural network algorithm to find possible 3D pentamode structures. For the impedance issue of 3D pentamode structures, a possible solution is to exploit multi-phase material design, which can be manufactured via 3D printing. For joints that are crucial to effective shear modulus of pentamode structures, less stiff materials can be used, while the rest can be still made with high-stiffness metals to satisfy the impedance matching to water.

Over the years, 3D printing technology has also been frequently adopted for printing metallic structures. This method seems a suitable way for manufacturing complicated large-scale pentamode structures with the advantages of low-cost, high accuracy, and relatively good mechanical strength. Other fabrication methods suitable for massive and low-cost producing pentamode materials need to be explored, like mold technique or welding for metals. Recent preliminary study has shown that pentamode structures obtained by welding have comparable mechanical and acoustical performance as samples previously made with electrical discharge machining [171], but with orders of magnitude lower cost.

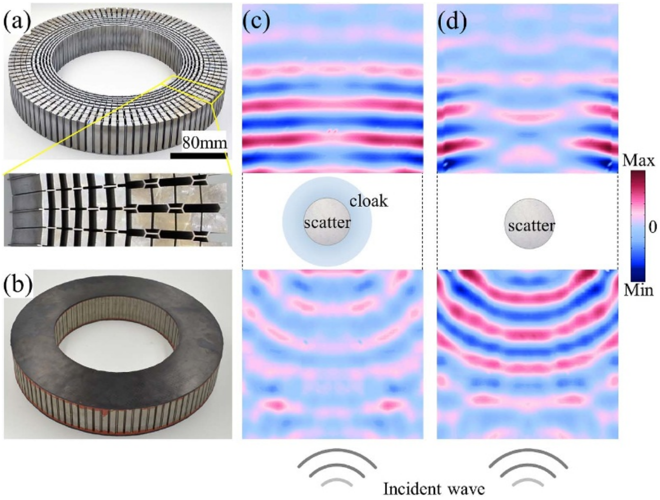

Figure 19. Experiment verification of pentamode acoustic cloak. (a) Annular pentamode cloak fabricated by electrical discharge machining. (b) Same as in (a) but the top and bottom of the cloak is covered with rubber layers to prevent water from getting into the void of the structure. (c) Measured pressure field in the forward region and backward region of an acoustic scatter enclosed by the cloak in (b) under cylindrical wave incidence. (d) The same as in (c) but the acoustic scatter alone without the cloak. Reprinted (figure) with permission from [167], Copyright (2017) by the American Physical Society.

Concluding remarks

In the past two decades, study of pentamode materials for water sound controlling has made progresses in various aspects, e.g. structure design, sample fabrication, and experiment verification. The excellent broadband controlling capability of pentamode materials for underwater acoustic waves have already been validated by proof-of-principle experiments, with the invisibility cloak as a specific example. Future research should be more focused on experiment study of 3D pentamode structures, identifying potential realistic applications, and addressing related engineering challenges, likes massive and cost-effective fabrication method. We briefly remark that other easy mode materials can be defined following the definition of pentamode materials, like unimode materials with only one zero eigenvalue or one easy mode as one example. Research into this direction might also lead to other interesting functions.

Acknowledgments

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14. Seismic phononic crystals

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Status

Low-frequency mechanical vibrations caused by traffic, machinery, or earthquakes are receiving increasing attention due to their harmful effects on infrastructures and human lives. Reducing the destructive effects of low-frequency waves has always been a challenge in scientific research and civil engineering. Inspired by phononic crystals and metamaterials, seismic phononic crystals and metamaterials aiming at attenuating or blocking surface wave propagation in the soil are proposed, and their vibration isolation performance has been demonstrated by simulations and experiments [172–176]. Theoretically, it is feasible to control the seismic or surface waves with artificial seismic phononic crystals and metamaterials, which attracted widespread attention due to the full-scale experiment designed by Brûlé *et al* in 2014 [177]. They constructed a periodic array of cylindrical holes in the soil to modify the seismic energy distribution at 50 Hz and showed that the seismic surface waves can be attenuated or blocked in this periodic structure. Since then, seismic phononic crystal and metamaterial have been widely studied. Forests could represent a widely available natural seismic metamaterial, where Rayleigh wave bandgaps are induced by local resonances of trees [178]. Besides, some studies have used metamaterials as the foundation of buildings, providing new ideas for reducing vibration at the bottom of buildings [179]. Except for the well-known Bragg and locally resonant mechanisms, many other seismic phononic crystal and metamaterial models have been proposed (figure 20), such as gradient [180], negative Poisson's ratio [181], inertial amplification [182], and anchored in bedrock [183] types.

In the studies of seismic phononic crystals and metamaterials, most works consider the soil as the linear elastic single-phase medium, which simplifies the calculations and experiments. From a practical point of view, the properties of ground soil are very complicated, such as layered and saturated. These properties have significant impacts on the seismic phononic crystals and metamaterials, which should be paid attention to in the design. Besides, material loss is inevitable in nature, which could be helpful for energy dissipation and wave attenuation [184], this should also be considered reasonably. On the other hand, the on-demand design of seismic phononic crystals and metamaterials at a given site condition is very important in practical application. The trial and error method is generally adopted to realize the designs, but a lot of time and manpower will be consumed due to the lack of a systematic design program. Thanks to the developments of computer science, machine learning, genetic algorithms, and topology optimization methods [185] have been investigated and applied to achieve fast and accurate seismic metamaterial designs, which further promote the development of seismic phononic crystals and metamaterials.

Current and future challenges

Seismic phononic crystals and metamaterials are bringing new vigor and vitality into phononic crystals. However, there are some open challenges and bottlenecks from designs to applications that still need to be tackled in the current and future (figure 20).

Most works of seismic phononic crystals and metamaterials consider the ground soil as the linear elastic single-phase medium, neglecting the practical complicated ground properties. There are some studies considering the layered [186, 187] and saturated [188, 189] properties of the ground and investigating the performance of seismic metamaterials. However, there is little research on seismic metamaterials considering the anisotropic constitutive models of ground such as the transversely isotropic constitutive model, whereas natural ground is anisotropic generally. This will greatly influence the properties of seismic metamaterials. Besides, material plasticity and large deformation are inevitable under the impact of high-energy seismic waves, which have not been studied in the area of seismic metamaterials yet. Therefore, more attention needs to be paid to the practical geological conditions and material properties in future studies.

The fast and reasonable design of seismic phononic crystals and metamaterials has always been one of the focuses. There are some advanced methods including machine learning and topology optimization [190, 191] which have been proposed to achieve the design of seismic metamaterials in different site conditions and show great advantages and potential in the on-demand design. However, these inverse design studies still consider the ground as a homogeneous and isotropic elastic medium. At present, there is a lack of further studies considering complex geological conditions and anisotropic constitutive models of the ground. Therefore, the seismic metamaterial inverse design methods which have the ability to generalization and are suitable for various geological conditions need further research.

At present, most researches on seismic phononic crystals and metamaterials are conducted on the laboratory scale, that is, scale model experiments [192], and there are few full-scale model experiments for actual engineering. Practical

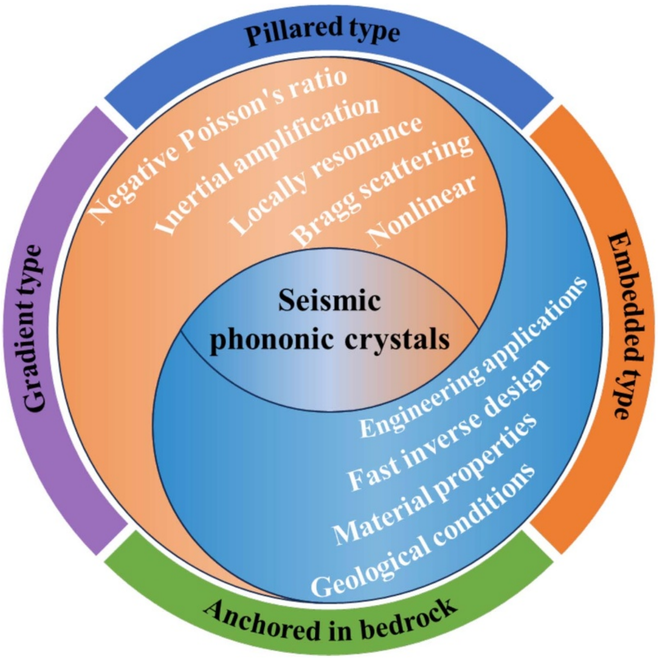

Figure 20. Diagrammatic sketch of seismic phononic crystals with various mechanisms (light orange region) and challenges (light blue region). The outer rings illustrate the common types of seismic phononic crystals.

engineering applications of seismic metamaterials have rarely been reported. One of the major obstacles that limit the practical applications of seismic metamaterials is the characteristics of large volumes, which are hard to meet in crowded urban spaces. Seismic metamaterials with a bandgap central frequency around 10 Hz usually require a lattice constant of several meters, meanwhile, the bandgap width is about a few Hz. The lower the bandgap frequency, the larger the metamaterial size. Besides, the complicated geological conditions and surrounding environments in urban also make their practical applications difficult. Therefore, promoting practical engineering applications of seismic metamaterials is a long-term and arduous challenge.

Advances in science and technology to meet challenges

The integration of advanced science and technology in material and computer science with seismic metamaterials can help resolve the above challenges, which include implementing proper theoretical models, developing new mechanisms, and improving design algorithms, etc.

There are some major characteristics of common geological ground, such as stratification, moisture content, groundwater level, etc. It is necessary to thoroughly study the effects of these factors on the properties of seismic metamaterials by analytical and numerical methods. On the other hand, the more suitable constitutive models and parameters of the ground can be chosen by *in-situ* or lab experiments, for instance, the transversely isotropic constitutive model of soil [193]. The effects of different constitutive models and parameters of the ground on the performance of seismic metamaterials can be studied, and some novel seismic metamaterial models suiting anisotropic ground should be proposed. Besides, material plasticity and large deformation must be paid attention to under the impact of high-energy seismic waves, which could be expressed by proper stress–strain relationships. In this case, the seismic metamaterials may be damaged, depending on external impact energy. The wave attenuation effects of seismic metamaterials on high-energy waves need to be investigated by numerical simulations and experiments.

The development of artificial intelligence provides novel ways for the on-demand design of seismic phononic crystals and metamaterials. Appropriate machine learning algorithms can be selected according to different situations, such as deep learning, reinforcement learning, autoencoders, etc. Besides, novel topology optimization methods such as level set topology optimization, which has the advantages of strong robustness and good adaptability, can be introduced into the seismic metamaterial design. In subsequent research, emphasis should be placed on more practical and complicated geological conditions to develop design algorithms applicable to various site conditions.

The engineering application of seismic phononic crystals and metamaterials will be the focus in the long term. The combination of laboratory and field tests, associated with numerical simulations can be used for estimating performances of seismic metamaterials in practice. Besides, novel seismic phononic crystals and metamaterial models with new low-frequency bandgap mechanisms including inertial amplification, quasi-zero or negative stiffness, and negative Poisson's ratio can be developed to reduce the size of metamaterials. On the other hand, the influences of surrounding environments and outer loads on the seismic metamaterials can be studied, which are inevitable in practical engineering applications.

Concluding remarks

Seismic phononic crystals and metamaterials have received extensive research interest due to their wave attenuation properties, simplicity in design principles, and stability in response. Various seismic metamaterial models are proposed to protect architectures susceptible to damage from seismic and surface waves, which also provides a feasible way of wave attenuation and control. Most of the existing work focuses on theoretical predictions and numerical simulations. There are still few actual full-scale experiments and even fewer further actual engineering cases. This results in many issues at the application level that are rarely considered, such as practical geological conditions, material properties, and surrounding environments. The inverse design of seismic phononic crystals and metamaterials is always a hotspot. Some artificial intelligence algorithms such as machine learning and topological optimization are applied to achieve fast and accurate seismic metamaterial design, which further promotes its developments. In summary, seismic phononic crystals and metamaterials are still in the stage of theoretical research and experimental verification, and more collaborative work should be conducted to promote their practical engineering applications in the current and future.

Acknowledgments

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15. Piezoelectric phononic crystals for vibro-acoustic control

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Status

The use of piezoelectric materials for vibro-acoustic control is an active and rapid growing field. Since the discovery of phononic bandgaps, periodic lattices of piezoelectric materials (i.e. piezoelectric phononic crystals, PPCs), have spurred broad interest and explorations for (semi-)active vibro-acoustic control applications in aerospace, automobile, shipping and underwater stealth engineering [194]. Figure 21 illustrates one representative PPC that, beyond the base shell structure, consists of an array of piezoelectric patches connected into the shunt circuit via coated metal electrodes. The piezoelectric patches, using either piezoelectric ceramic or flexible micro-fiber composite (MFC), serve as capacitors in the circuit. MFC is useful to keep conformality with the curved adhesive surfaces. In comparison with passive components like holes, heterogeneous scatterers and bulky resonant structures, the use of piezoelectric materials brings outstanding advantages in terms of tunability, reconfigurability and even programmability thanks to the self-sensing and actuation functionalities. Moreover, the combination of resonant shunt circuit and piezoelectric patches, can induce low-frequency bandgaps through electromechanical coupling, erasing the weaknesses of heavy mass, fragile and uncompact structure in traditional locally resonant metamaterials.

Great achievements have been made in the field of PPCs targeted for vibro-acoustic control at low frequencies [197]. Early efforts mostly concentrate on exploring the wave characteristics and bandgap mechanism which nowadays are well-recognized as Bragg-scattering bandgap and locally-resonant bandgap. Fueled by the demand of tunability for low frequency bandgaps, the design of piezoelectric shunted circuit attracts more attention in the past two decades. The widely used shunt circuit can be classified into two categories: one includes the resonant shunt, negative capacitance shunt and passive shunt networks, which is different from the other self-adaptive shunt circuit relying on self-sensing and built-in control laws. The resonant shunt circuit opens low-frequency phononic bandgap through the LC oscillation, so it is nature to see the higher-order resonant circuit and rainbow effect are employed to acquire multimodal vibro-acoustic control [195, 198]. The negative capacitance shunt circuit controls the wave propagation by tuning the effective stiffness of periodic structure. Recent works on shunt networks show quite broadband suppression when the vibrational modal frequencies and shapes match well with those of circuit network [199], nevertheless a dense piezoelectric array is required. More excitingly, digital synthetic circuit paves a way to programmable PPCs capable of suppressing low-frequency vibration and acoustic radiation spectrum lines of underwater structures [196] and making unnatural active media [75], showing a glimpse of the bright future of intelligent surface to tackle vibro-acoustic problems.

Current and future challenges

Despite the significant advances, huge challenges remain in aspects of bandgap widening, quasi-static bandgap formation, and reliability of control system, etc. The broadening of low-frequency bandgap is a long-lasting question in the field locally resonant metamaterials, but it is even more challenging for piezoelectric shunted circuit because the small electromechanical coupling coefficient largely narrows the bandgap. By using digital synthetic impedance techniques, higher-order resonant circuit can be constructed to generate multimodal electrical oscillation and resulting multiple phononic bandgaps, but this method is heavily limited by the circuit stability [197]. The rainbow effect utilizes spatially gradient parameters to widen the transmission gap at the cost of reduced transmittance. For shunt networks, the modal matching strategy requires a highly dense piezoelectric array for multimodal vibration damping [199], which is unpractical for large engineering structures because of the complexity in control systems. In short, new mechanism is highly required to get rid of the above limitations.

Opening bandgap at ultra-low frequencies and even quasi-static is highly desired to control the vibration of large flexible space structures, such as slender truss, cable network, inflated and membrane structures. These structures are extremely flexible in a dimension to tens or hundreds of meters, giving rise to dense intrinsic modes far lower than 1 Hz [200]. In theory, ultra-low resonant bandgap is reachable by tuning the electrical parameters; unfortunately, it requires extremely large capacitance and inductance that are hardly attainable for both analog and digital circuits. Material mismatch is another question to be considered in the application of membrane structures. It is obvious that the rigid piezoelectric patches cannot be conformal with membrane under large deformation. Besides, the attach mass is too heavy to affect the structural dynamic performance.

It is vital to improve the reliability of PPCs system in promoting its engineering application. The existing literature almost all conducted prototype experiments on regular metallic structures that are simply equipped with unpackaged home-made control circuit [194, 197]. However, the real structures may be made of anisotropic composite structures working in harsh environment, for instance, enduring flow-induced vibration in water or flutter in supersonic airflow. Undoubtedly, factors such as environment proof, welding spot vibration

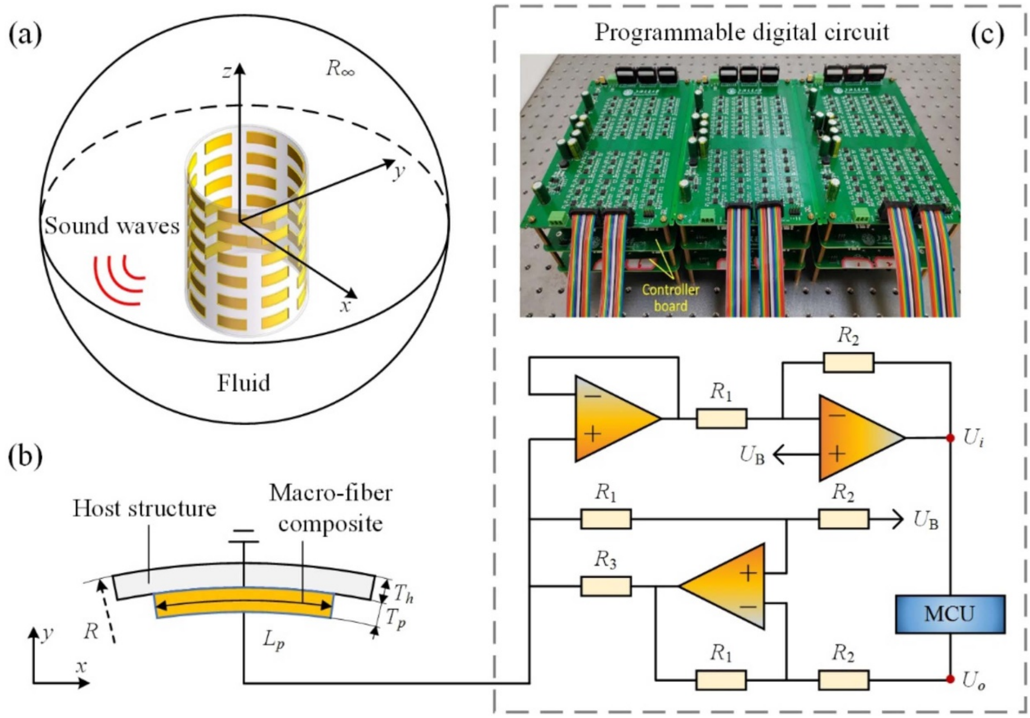

Figure 21. Programmable meta-shell for vibro-acoustic control in fluid. (a) Schematic diagram of piezoelectric patches arranged on an elastic shell. Reprinted from [195], Copyright (2023), with permission from Elsevier. (b) An amplification view of the electromechanical coupling interface and electrical connection of piezoelectric patches. Reprinted from [196], Copyright (2024), with permission from Elsevier. (c) Programmable digital shunt circuit consisting of operational amplifiers (triangular symbols) and micro-controller (MCU).

fatigue, wire crosstalk, interface debonding and so on, must be considered in the design of active piezoelectric system to guarantee the reliability during its long-time service.

Advances in science and technology to meet challenges

A lot of efforts have been paid to the optimization of PPCs bandgap width, but without too much gains. The way to the breakthrough may lies in exploring new vibro-acoustic suppression mechanism. Progress is reported using nonlinear dynamic response such as nonlinear bandgaps and nonlinear energy sink [201, 202], as well as using non-Hermitian skin effect in non-reciprocal phononic crystal [203]. Recent experiments on shunt circuit of fractional and cubic nonlinearities demonstrated the enhancement of bandgaps [201, 204], nevertheless there is one key question to be answered: how the nonlinearity works in periodic structures? Non-Hermitian skin effect describes a phenomenon when the continuum bulk bands get localized wave patterns, which is inherently caused by the band winding in the complex frequency plane. It features a broadband one-way attenuation as illustrated in figure 22(a), but there are lots work to do for vibro-acoustic control applications.

As for the quasi-static bandgaps, one possible way is using smart soft materials like piezoelectric polymer, dielectric elastomer, liquid crystal elastomer and hydrogel that are soft to be compatible with flexible structures, as illustrated in figure 22(b). This is not only beneficial for conformal integration, but also useful to lower the resonant bandgaps and simultaneously increase energy dissipation by virtue of viscoelastic properties. The use of stimuli-responsive soft materials for vibro-acoustic control is now at its infancy [205, 207], which requires more investigations on the dynamic modeling as well as computational method for high dimensional nonlinear systems.

Avenues to enhance the applicably and reliability of PPCs includes the structural-functional integration design method and flexible circuit techniques [206]. As depicted in figure 21, the PPCs typically contain four functional modules, namely the host structure, the shunt circuit, the sensing, and actuation piezoelectric patches. Each module can be designed as one functional layer, and then all the layers are integrated via the thermoplastic molding process to form a smart composite structure. Particularly, the programmable shunt circuit layer, consisting of wires and flexible electronic elements as illustrated in figure 22(c), can be embedded in between the structural layers to make a compact structure and meanwhile to get rid of environment disturbance. With the development of relevant science and technology, it is not far to see the engineering applications of PPCs composite structures to tackle vibro-acoustic problems in harsh fluid environments mentioned before.

Concluding remarks

Benefiting from the programmable digital synthetic circuit, piezoelectric phononic crystals are getting more compact

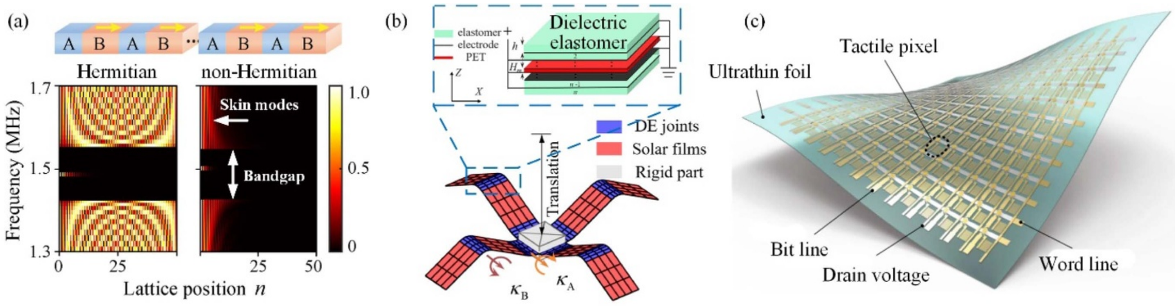

Figure 22. New developing trend to meet PPCs challenges. (a) Non-Hermitian skin effect showing localized bulk wave patterns in 1D phononic crystal made of non-reciprocal piezoelectric semiconductors. Reprinted (figure) with permission from [203], Copyright (2020) by the American Physical Society. (b) Flexible structure driven by dielectric elastomer actuators under large deformation. Reprinted from [205], Copyright (2024), with permission from Elsevier. (c) Flexible integrated circuits with excellent conformal ability. Reproduced from [206], with permission from Springer Nature.

and more intelligent than their passive counterparts. It has begun to show promising prospects in vibro-acoustic engineering for underwater and aerospace applications. But many challenges remain, the topic is now concentrated on the bandgap widening, extension of ultra-low bandgap, and design of high-reliable electromechanical systems. Progresses have been reported in terms of new suppression mechanism like nonlinear bandgaps and non-Hermitian skin effect, but the translation of capabilities from laboratory to real-word setting still requires lots of theoretical and technical efforts. On the other hand, future advance is likely to bridge different disciplines like vibro-acoustic dynamics, stimuli-responsive materials, flexible circuit, and intelligent active matter, which requires a shared vocabulary and close collaboration across multiple scientific communities.

Acknowledgments

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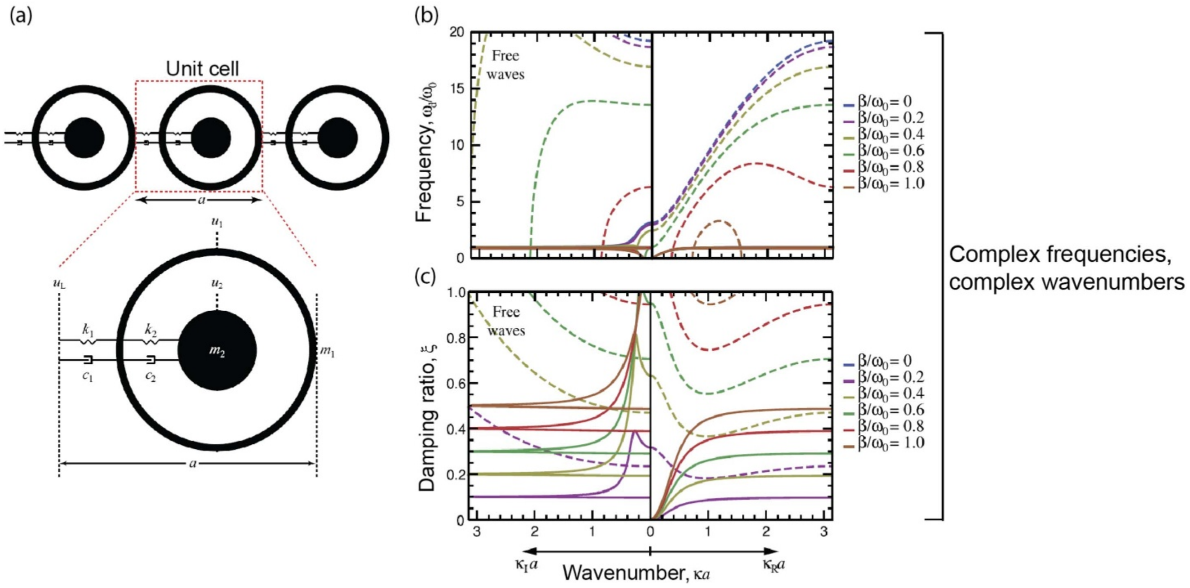

Figure 23. Damped mass-in-mass chain; the inner mass resonates rending the chain an elastic metamaterial. (a) Schematic of the chain, (b) dispersion diagram for damped free waves in the form of frequencies and wavenumbers that are simultaneously complex. Similar analysis may done an a standard mass-spring phononic-crystal chain. Reprinted from [215], Copyright (2016), with permission from Elsevier.

16. Damped phononic crystals and the concept of metadamping

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Status

The study of waves in periodic media is a prime topic in condensed matter physics since it allows for a formal analysis of the motion of atoms in crystalline materials [208]. Similarly, elastodynamic analysis of periodic materials and structures has been a key topic in vibration engineering especially with the advent of composites in aerospace applications [209]. These two key areas formed a historical foundation from which the advent of phononic crystals [1] and acoustic/elastic metamaterials [3] emerged, ushering a new era in the fields of vibrations and acoustics. Inspired by crystal physics, the dynamical properties of phononic crystals and metamaterials are usually analyzed and designed *intrinsically* at the material (i.e. unit-cell) level, as opposed to only the finite structural level [210]. While most investigations in the literature assume conservative models with no dissipation, the treatment of damping allows for more realistic representation [211–215] and also offers opportunities for design [216–220]. Damping may be incorporated in models in numerous ways, the most common being in the form of viscous [212, 214, 215, 217, 219] or viscoelastic damping [211–213, 218]. A simple form of viscous damping is proportional, damping—whereby the matrix of damping coefficients is assumed to be proportional to the mass and/or stiffness matrices [221]. Absent of the proportionality condition, a generally damped model is assumed [222]. Experiments are used to determine an appropriate damping model for a given material or structure [223]. When considering damping in the modeling of phononic crystals and metamaterials, the assumption of real frequencies enforces dissipation to take place only spatially, whereas the freedom of allowing the frequencies to be complex allows dissipation to take place in a temporal manner; in all cases the wavenumber (or wave vector) may be complex, see figure 23 [215]. The former case corresponds to a material subjected to a sustained driving frequency, whereas the latter case corresponds to a material experiencing free wave motion—such as that arises due to impulse loading, for example. As shown in figure 23, the form of the wavenumber-dependent frequency dispersion relation changes depending on which of these two cases is adopted. From the perspective of design, it was shown that elastic metamaterials comprising internal resonating substructures display enhanced dissipation under certain conditions (i.e. beyond what may be attributed to the sum of the individual material constituents) [217–220], see figure 24. This unique property, which is described as *metadamping*, is beneficial particularly when enhanced dissipation in a structure is desired but without coming at the expense of the stiffness as illustrated in figures 24(c) and (d).

Current and future challenges

The treatment of damping in phononic crystal and elastic metamaterials models by phenomenological viscous and viscoelastic laws can be tailored to a specific problem by curve fitting to experiments [218]. However, in future applications as the constituent material becomes softer, or the characteristic dimensions become smaller, more advanced damping constituent models will be needed. In some cases, microstructural models of dissipative behavior may be necessary. In approaching the atomic scale, phenomenological models may

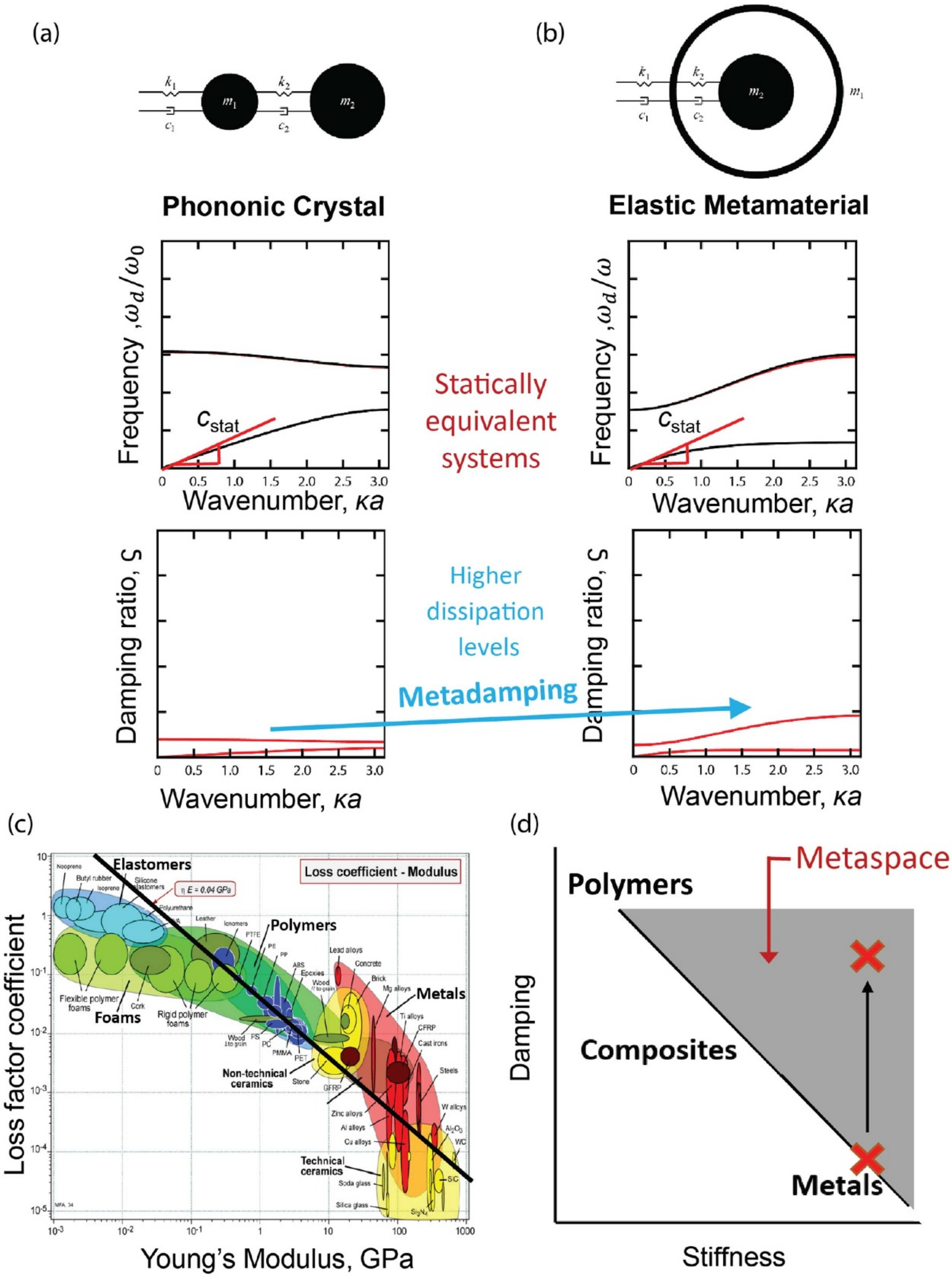

Figure 24. Statically-equivalent mass-spring chain depicting a (a) phononic crystals and (b) elastic metamaterial with corresponding damped frequency and damping ratio diagrams. Both models have identical values of viscous damping in the dashpots. The damping ratio of the elastic metamaterial is higher, indicating an enhancement of overall dissipation. This phenomenon is referred to as metadamping. With metadamping, the top-right corner of the Ashby diagram shown in (c) could be filled—as demonstrated by the schematic of (d). Reprinted from [217], Copyright (2013), with permission from Elsevier.

be replaced with phonon-phonon scattering models to account for dissipation in the form of heat loss. This type of multiscale analysis has still not been fully developed to produce robust and efficient ways to upscale. Concerning the metadamping problem, current studies based on damped spring-mass systems have demonstrated a broad spectrum enhanced dissipation effect. However, when realized in the form of more traditional engineering components, such as rods or beams [218, 219], the influence of metadamping may occur over only portions of the spectrum [218]. Future work will aim to discover metamaterial unit-cell designs that increases the metadamping capacity. Another aspect of the problem, which presents both a challenge and an opportunity is the realization of negative metadamping, i.e. a metamaterial with diminished losses. Such material systems are of importance in sensing applications. Lastly, the interplay between dissipation at the material level and structural level is a topic that is open for future investigation.

Advances in science and technology to meet challenges

The treatment of damping in realistic phononic crystal and elastic metamaterial models may be further improved by the development of tailored frequency-dependent models derived from microstructural descriptions, potentially reaching the atomic scale. In addition to more elaborate theoretical investigations, the area of damped phononic crystals and metamaterials stands to benefit greatly from upcoming advances in contemporary computations and experiments. In particular, advances in machine learning and its application to phononics [25] will accelerate the search for more accurate damping models and for more strongly metadamped metamaterial designs that exhibit engineered dissipation over broader frequency ranges. Equally important, advances in additive manufacturing will enable these designs to be realized. The current rise of the use of composite materials in the aerospace industry provides a growing infrastructure for the design and realization of damped phononic materials.

Concluding remarks

The presence of damping in phononic crystals and elastic metamaterials fundamentally alters the material's frequency dispersion, and does so in different ways depending on where the motion is driven (e.g. harmonically excited) or free (e.g. impulsively excited). Metadamping is an emerging paradigm that enables the realization of superior dissipation properties, allowing for simultaneously high damping and stiffness, or, in the other extreme, for materials with exceptionally low loss. Advances in modeling, machine learning, and additive manufacturing will further enrich the this discipline in the years to come.

Acknowledgments

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17. Thermal transport in phononic crystals

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Status

Heat transport at the nano/micro scale has attracted considerable attention due to its intriguing physical properties, such as the peculiar behavior of thermal phonons arising from their ballistic and wave-like properties. Additionally, nanoscale heat transport holds promise for alleviating heat dissipation problems in electronic and optical devices and enabling thermoelectric energy harvesting. Over the past few decades, researchers have made significant progress in elucidating the thermal transport properties of various nanostructured materials, including phononic crystals (PnCs). PnCs serve as excellent platforms for investigating thermal transport. Moreover, the ability to control thermal transport via interference, as demonstrated in the low-frequency regime with elastic waves and sounds, represents an exciting avenue for further exploration.

PnCs offer a well-defined periodic structure, making them an ideal platform for systematically investigating both coherent and incoherent thermal transport mechanisms. In the coherent regime, thermal transport is primarily governed by the wave nature of phonons. High-frequency elastic wave propagation within the PnC undergoes Bragg scattering, which alters the thermal transport properties. Two key consequences of Bragg scattering are the formation of phononic bandgaps and the reduction of the group velocity. While a perfectly designed PnC can theoretically achieve a complete phononic bandgap, this approach is limited in practice. The inherent broad spectrum of thermal phonons often requires a wider bandgap than achievable. Alternatively, a more practical approach focuses on the group velocity reduction observed near highly symmetric points in k-space. This phenomenon impedes thermal transport over a wide spectral range and plays a dominant role in reducing thermal conductivity. In contrast, the incoherent thermal transport regime offers a more practical avenue for exploration using established materials such as silicon. At room temperature, the thermal phonon mean free path (MFP) in Si can exceed a few hundred nanometers. This allows for the design of PnCs with larger periodicities, enabling the study of thermal transport in the ballistic regime. Thermal transport deviates from the classical Fourier law typically employed for diffusive heat transfer. Instead, boundary scattering of phonons becomes the dominant factor influencing thermal transport. Consequently, the thermal conductivity becomes highly dependent on the structural parameters of the PnC, highlighting the ability of nanostructuring to manipulate thermal properties in materials. This concept has been extensively explored and there are some review articles on this topic [224, 225].

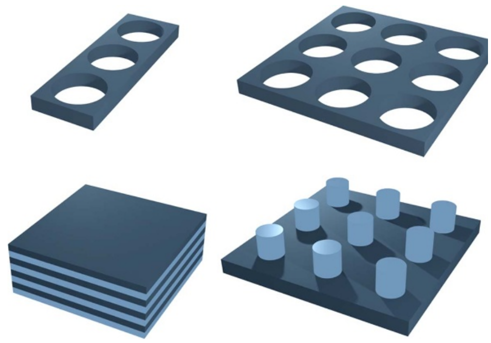

Figure 25. Schematics of the PnC structures, which have been the subject of many studies.

Current and future challenges

The Physical exploration of nanoscale thermal transport using a variety of types of PnCs (figure 25) and the challenge of developing technology to control thermal transport have been carried out [83, 226–228]. Technical problems are often discrepancies between theoretical calculations and experimental results because the fabricated structure may differ slightly from the design, there may be fluctuations in structural dimensions, and the surface conditions may not be ideal. In fact, surface roughness has a significant impact on the specularity and makes experimentation difficult, especially in the coherent regime. Coherent manipulation of thermal transport at room temperature remains challenging due to two key factors. First, the periodicity of a typical PnC must be comparable to the thermal phonon wavelength. Second, the surface roughness within the structure must be sufficiently smaller than the thermal phonon wavelength for efficient coherent thermal transport. As an example, the thermal phonon wavelength in solids at room temperature typically falls within the few-nanometer range. Even in the incoherent regimes, where the effect of surface roughness appears to be minimal, surface roughness is actually known to significantly alter thermal conductivity.

Advances in science and technology to meet challenges

Thermal transport has been simulated by the kinetic theory based on the Boltzmann transport equation of phonons, and atomistic approaches including molecular dynamics and non-equilibrium Green's function methods. In recent years, simulations have had some success in reproducing experimental results. However, in extremely fine systems, some elements of the real system have not been fully incorporated into the computational model, resulting in misalignments. 1D PnC structures on a few-atom scale, such as oxide semiconductors [228] and semiconductor superlattices [83], have been successfully thermally controlled in coherent systems at room temperature. In addition, even in 2D PnC systems, microfabrication techniques have succeeded in thermal control at ultra-low temperatures [229] and around 10 K, with a degree of design freedom [230]. For PnCs, a number of data have been reported for systematically varying the lattice constant, pore arrangement, shape and size, but most of works do not provide the surface roughness data. However, even when all the structural parameters defining a PnC structure are the same, different surface roughnesses can lead to significant differences in thermal conductivity. In both regimes, it is necessary to be able to precisely process materials with long phonon MFPs and clean interfaces for achieving robust coherent control at ambient conditions.

Concluding remarks

PnCs are the systems with rich physics of thermal transport, where both coherent and incoherent thermal transport can be observed. In thermal transport simulations, both the discontinuities at the interface between atomistic and continuum simulations and the discrepancies between simulation results and experimental data due to the limited representation of real-world elements will be progressively addressed. In experiments, the nanofabrication technology and materials science will offer an ideal periodic system for fundamental research. Additionally, high-rate growth and deposition technology for material growth, and high-throughput nanopatterning technology are important for the realization of PnC devices of practical size. Based on the basic

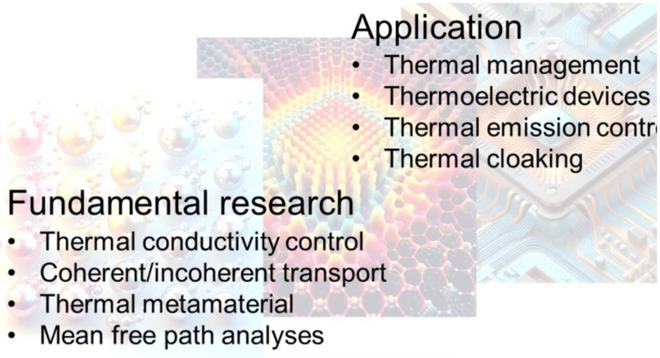

Figure 26. Fundamental research topics and examples of application using PnC nano/micro structures.

research shown in figure 26, applications for thermoelectrics [231], semiconductor thermal management and various thermally functional materials and devices [232] would be developed.

Acknowledgments

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18. Phononic crystals for sensing applications

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Status

The introduction of mixed phononic crystals (PnC) [233], gathering solids and fluids in a same structure has demonstrated the dependence of resonance frequencies on the constituent fluid. This possibility opened the way to determining the physical nature of a fluid [234], defined as the analyte, which subsequently led to a wide class of *Phononic crystal sensors.*

Firstly, the transmission spectrum of a PnC sensor must display well-defined features that are highly sensitive to the acoustic velocity of the infiltrating analyte. A prerequisite for this is the existence of a band gap, enabling mode localization and acoustic wave confinement. Besides, the features should be isolated inside the gap to allow the sensing of the probed parameter on a sufficiently broad frequency range. The dependence of the transmission feature is then corelated to the material properties, specifically the sound velocity, that can depend on several parameters of practical interest, such as temperature or analyte concentration.

Over the last decade, several designs have been proposed for determining the properties of liquids and gases. In addition to the theoretical concept, illustrated with a 1D arrangement of solid and liquid layers as a unit cell [234], Lucklum *et al* [235] showed, both theoretically and experimentally, eigenfrequencies sensitivity of a slot inserted in a 2D PnC, as a function of the liquid mixture composition. Furthermore, it has been demonstrated that coupling a PnC cavity and a Helmholtz resonator in the same structure can overcome the detection limit of the conventional acoustic sensing system and can be useful for acoustic enhancement and directional gas detection (see figure 27(a) [236]). Also, another way of sensing a liquid or gas has been achieved by designing PnCs using open resonators. Jin *et al* [237] proposed a hollow pillared structure filled with a liquid, where localized modes depend on the physical properties and height level of the liquid. A recent experiment confirmed the theoretical predictions (see figure 27(b) [238]). For liquids flowing in a tube, the concept of tubular PnC has been introduced [239]. In contrast to the above concepts, Gueddida *et al* [240] proposed a PnC membrane consisting of an array of ridges immersed in the fluid. The observation of the cavity mode resonances can be achieved either outside the area of the PnC or just above the cavity.

Current and future challenges

Regarding the sensing of fluids with PnC, two main issues emerge from the above papers. The first is the need for a high quality-factor of the features, peaks or dips, in the transmission spectrum, and the second is the effect of losses from both the fluid and the solid constituents that need to be reduced as much as possible.

Enhancing the Q-factor can be achieved by using the concept of extraordinary acoustic transmission (EAT) [242], based on 1D acoustic gratings with sub-wavelengths units or 2D panels periodically perforated with holes. Therefore, the transmission exhibits an asymmetric sharp peak, known as Fano resonance, resulting from the coherent coupling between the diffracted waves excited at the surface of a grating and the Fabry–Perot resonant modes inside the apertures. Ke *et al* [243] investigated both theoretically and experimentally the properties of an acoustic liquid sensor based on PnC steel plate drilled with an array of holes. They achieved a high sensitivity of the Fano resonant frequency to the speed of sound of the liquid surrounding the phononic plate and filling the holes. The interplay between an incident wave and a resonant standing mode has also been considered in the frame of Bonhomme *et al* work [244]. In their paper, the authors investigated a multilayered phononic pillar-based structure for Love wave manipulation to achieve mass sensitivity to the detection of single particle with a functionalization of the upper surface of the pillar. Also, this work belongs to a new class of PnC sensor using the interaction of surface acoustic waves with a PnC at the surface of a sample.

However, the sensing experimental demonstration has to face the physical reality of damping effects, introduced recently in a 2D PnC proposed as a temperature sensitive biosensor [241]. The viscosity is introduced in the Navier–Stokes equation of acoustic waves propagation in a compressible Newtonian fluid and in the boundary conditions. It is shown that, compared to the longitudinal viscosity, the shear viscosity leads to an important energy dissipation due to the shear motion in the thin interfacial liquid/solid layer (see figure 27(c)).

Advances in science and technology to meet challenges

To meet the challenges of future PnC sensors, it would appear highly fruitful to consider multiphysics devices coupling the basic properties of PnC with other physical mechanisms. For instance, Kaya *et al* [245] proposed a Mach–Zehnder interferometer implemented in a 2D PnC made of steel cylinders in water (see figure 28(a)). They investigated the variations of the fraction of the analyte in mass in a supercell on the path of one of the two splitted beams, producing the phase difference. A combination of PnC with a 3D gradient acoustic metamaterial has been realized to overcome the difficulty of sensing weak acoustic signals. The resulting higher amplification capability is verified through numerical calculation and experimental demonstrations [246].

For fluid sensing, several papers have shown the capability of photonic crystals to detect small variations of refractive index of gases and liquids. Ten years ago, it has been proposed to emphasis the potentiality of gathering the phononic

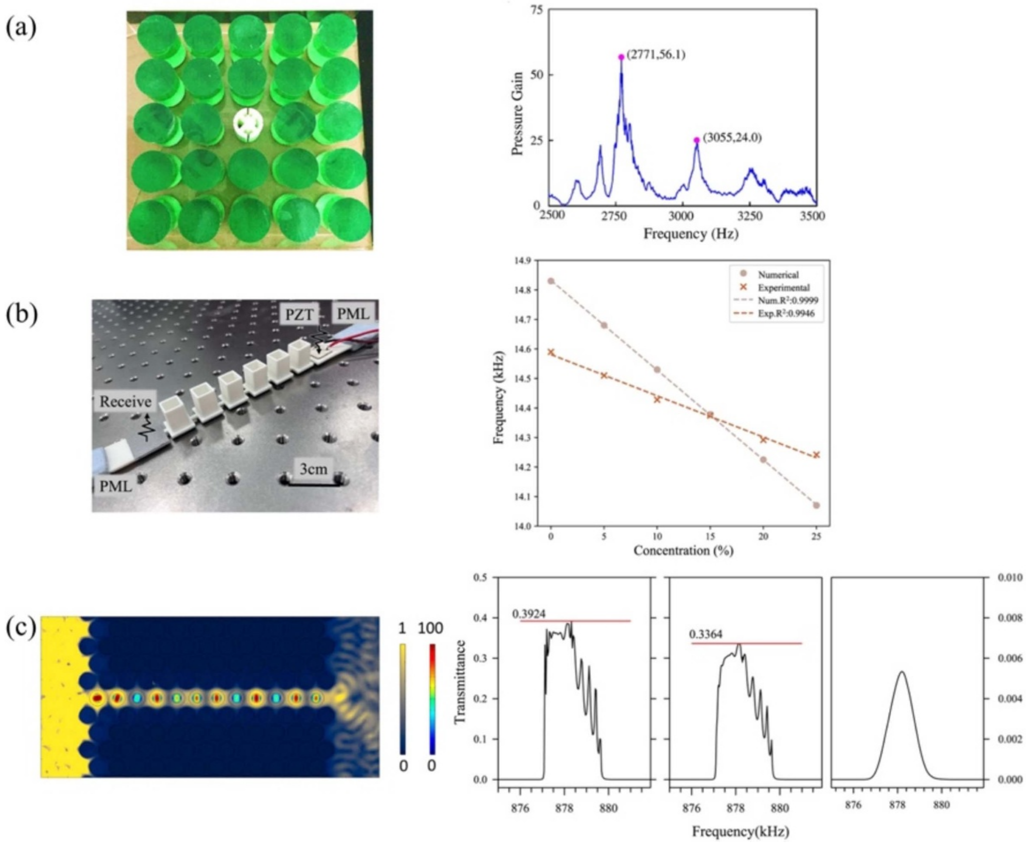

Figure 27. (a) *Left:* Top view of the coupled PnC with the Helmholtz resonator. *Right:* Pressure gain spectra against frequency with the resonances of the coupled cavity. Reprinted from [236], Copyright (2022), with permission from Elsevier. (b) *Left:* Photograph of the fabricated PnC. *Right* Relationship between the resonance frequency and the concentration of the NaCl mixture filling the hollow cylinders. Reproduced from [238]. © IOP Publishing Ltd. All rights reserved. (c) *Left:* Calculation of the displacement field along the PnC waveguide. *Right:* Zoom-in view of the transmission curves for non-lossy, lossy with longitudinal and lossy with both longitudinal and shear viscosity. Reproduced from [241]. CC BY 4.0.

and photonic properties in a same dual structure, known as phoXonic crystal. It has been showed that inserting a cavity inside a phoXonic crystal represents an efficient tool for sensing the light and sound velocities of unknown liquids with a high sensitivity of the acoustic/optical parameters. The concept has been first demonstrated in a 2D infinite crystal made of air holes drilled inside a silicon matrix, in which one row of holes is filled with the analyte [249]. Later on, the Fano-like resonances in a dual phononic and photonic crystal plate were demonstrated (see figure 28(b), [247]). More recently a slot nanobeam with gradient cavity confining the two types of physical waves was considered for the cross examination of the analyte, reducing the sensing characterization uncertainty [250].

Other opportunities for detecting PnC parameters are still open such as their use in acoustic velocimetry [248]. The model presented figure 28(c) is applied to measure the velocity of moving objects based on Doppler effect. The model is based on the coupling of surface modes of two 2D PnCs when they are brought to close proximity. The comparison is done on the phase agreement between the two surfaces where one of them is affected by the moving object. It can be used to measure blood flow rate or material flow in pipes for microfluidic applications.

Concluding remarks

PnC sensors represent a compact structure useful for bio-chemical applications [251], in which the amount of analyte can be often limited. PnC and acoustic metamaterials appear suitable for the determination of many different physical parameters of a fluid. Indeed, starting from the frequency dependence of the acoustic wave velocity it seems feasible from the transmission spectrum to extract parameters of practical interest, such as the concentration of an analyte for bio-chemical analyses or to estimate the temperature of a fluid. As reported in this review, many promising devices have been investigated, different kind of waves, from bulk acoustic waves to surface acoustic or Lamb waves. The

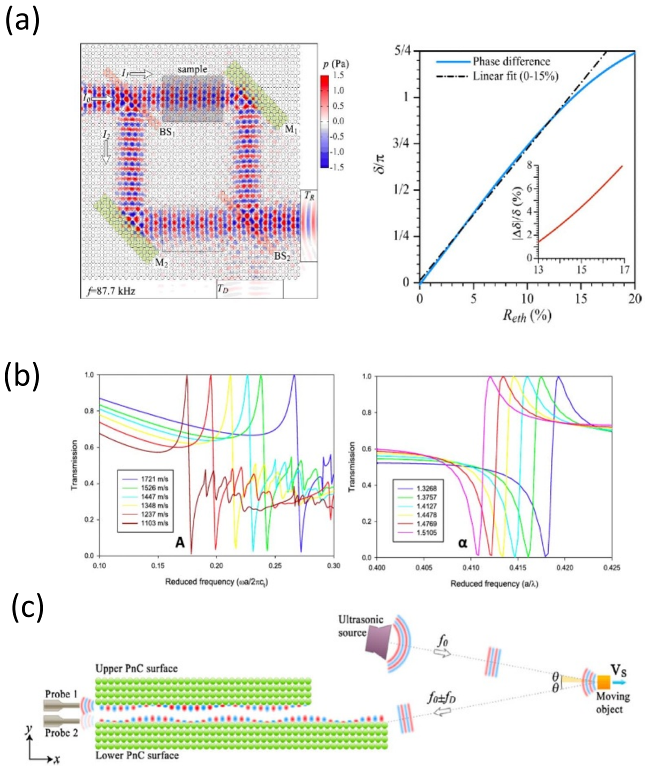

Figure 28. (a) *Left:* Simulation of the Mach–Zehnder operation. *Right:* Variation of phase difference due to ethanol-water mixture. Inset demonstrates the deviation from the linearity around 15% ethanol. Reprinted from [245], Copyright (2014), with permission from Elsevier. (b) Evolution of the phononic (resp. photonic) transmission spectrum as a function of the acoustic velocity (resp. refractive index) of the embedded medium of the probed analyte. Reprinted from [247], with the permission of AIP Publishing (c) Sketch of the proposed miniature Doppler velocimeter. Reprinted from [248], Copyright (2016), with permission from Elsevier.

viscosity of the fluid, at the origin of damping, can be evaluated from the amplitude of the resonant features in the transmission spectrum [252]. Nevertheless, from the physical reality, it remains that a balance between the quality factor and the damping due to shear viscosity needs to be considered.

To reach these objectives, different PnC sensors have been designed based on cavity modes, guided modes or Fano resonances, as well as more sophisticated devices such as the one using Mach Zehnder mechanism. The challenge now is to push the coupled properties, to combining multiphysics waves and devices to ensure the development of the PnC sensors domain.

19. Phonic crystals for acoustic tweezers

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Status

Acoustic tweezers have garnered increasing attention as a non-contact method capable of manipulating objects in a controllable manner. The underlying physical mechanism involves the absorbing, scattering, and reflecting of acoustic waves by objects, leading to an exchange of momentum and energy between the objects and the waves, then generating an acoustic radiation force on the objects. Thus, the performance of acoustic tweezers is determined by the properties of the field, the objects, and their interactions (scattering or absorption). Phononic crystals, along with metamaterials and metasurfaces, represent a burgeoning class of artificially engineered materials designed to control, direct, and shape acoustic waves [253]. The field of phononic crystal engineering advances the capabilities of acoustic tweezers, offering innovative solutions for enhancing acoustic manipulation techniques.

In the past decade, various acoustic fields shaped by phononic crystals have been created to enable versatile particle manipulation, thereby advancing acoustic tweezer technology. For example, the distinctive bandgap of phononic crystal has been used for reflection waves and enhancing the intensity of acoustic energy in a tunable manner, benefiting activities such as drop manipulation and jet enhancement [254]. The structure-induced excitation of Lamb wave in phononic crystal plate generating enhanced localized field has proven for trapping, levitation, and transporting microparticles, as well as realizing repairable sonoporation for massive cells [255, 256], as shown in figure 29. The metamaterial bricks forming with a designed passive phase array can construct a bottle-shaped beam to achieve single-sided air-borne acoustic levitation [257]. Recently, based on the particle scattering-induced mode conversion in the phononic crystal waveguide, the forward momentum of the acoustic beam can be amplified, resulting in the particle pulling toward the source [258].

Aside from shaping incident acoustic fields for particle manipulation, metamaterials can also be as anomalous acoustic scattering (particles) for shaping acoustic radiation force to realize complex manipulation phenomena. Exploiting anomalous metasurface scattering, a metasurface object can self-guide in response to the acoustic wave or be pulled toward the acoustic source [259]. These anomalous phenomena stem from the subwavelength patterns on the metasurface, which induce scattering asymmetry and ensure the necessary momentum balance.

It is worth noting that acoustic tweezers can also be utilized to manipulate individual particles for the fabrication of photonic crystals. By applying a high-amplitude acoustic standing wave field, a 3D colloidal phononic crystal can be dynamically reconfigured in real-time, showcasing clear band-pass and band-stop frequencies in the transmission spectrum [260]. Additionally, by utilizing a phononic crystal composed of periodically patterned bubbles, the acoustic radiation force can be augmented through bubble resonance. This amplification enables the effective use of the bubble array-based phononic crystal to assemble macrostructures [261].

Current and future challenges

Despite the powerful acoustic field-shaping ability of phononic crystals, they have not yet emerged as a competitive alternative to traditional acoustic components used in acoustic tweezers. This is partly due to several remaining challenges that need to be addressed, including, but not limited to:

- Identification of suitable materials or structures: While phononic crystals possess exceptional physical properties through the optimization of their structure and materials, these optimized structures or materials may not be ideal for acoustic tweezer devices. Research is needed to identify materials or structures with superior properties specifically tailored for acoustic tweezers.
- Computational complexity: Achieving device optimization requires a comprehensive, full-wave model that takes into account all components of the acoustic tweezer system, including the acoustic source, channel, finite-size phononic crystal, and more. However, the current theoretical frameworks face challenges in accurately simulating the complete system, resulting in a less-than-ideal match with experimental observations. This gap hampers the effectiveness of the design-build-test-repeat loop.
- Ensuring device performance stability: In traditional designs, the acoustic source, chamber, and passive phononic crystal are designed and manufactured separately. While this allows for the phononic crystal to be considered as a disposable chip, the separated design can result in less stable device performance. To address this, it is crucial to consider stability, compactness, and low-cost manufacturing as critical factors in the design of acoustic tweezer devices.
- Unique application scenario: Acoustic tweezers require a compact design and simple components for shaping the field. Therefore, traditional acoustic components are preferred for this purpose. However, in certain cases where unique application scenarios are being explored, the distinctive properties of phononic crystals cannot be replicated by traditional acoustic components. In such cases, the use of phononic crystals is preferred in acoustic tweezers systems.

Addressing these challenges will pave the way for further advancements in the field of acoustic tweezers, enabling more precise manipulation of particles and opening up exciting possibilities for various applications. Further research and technological developments are essential to overcome these obstacles and unlock the full potential of acoustic tweezers.

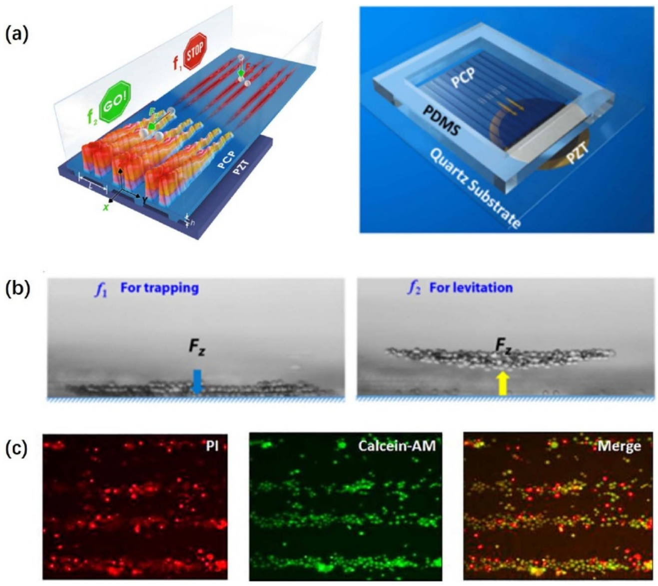

Figure 29. Dynamical control of microparticles and cells in an acoustofluidic channel using phononic crystal technology. (a) (Left) Schematic of the stop-and-go manipulation of particles in a microfluidic channel built on a phononic crystal plate (PCP), (Right) configuration of the PCP-based microfluidic device. (b) Trapping and levitation of polystyrene (PS) particles by frequency switching between two resonant frequencies. Reprinted (figure) with permission from [255], Copyright (2020) by the American Physical Society. (c) Double staining assay with PI/calcein-AM to verify cell permeability and viability after repairable sonoporation on the surface of PCP. Reprinted from [256], with the permission of AIP Publishing.

Advances in science and technology to meet challenges

To overcome the challenges associated with the use of phononic crystals in acoustic tweezers, several important advances in science and technology are being developed. These advancements are expected to have a significant impact on the field in the short to medium term. Some key areas of progress include:

- New physical mechanisms: Recent theoretical works have demonstrated the potential of utilizing the anomalous dispersion curves of phononic crystals for anomalous scattering and novel acoustic radiation force generation [259, 262]. These findings open up exciting possibilities for innovative applications of phononic crystals in acoustic tweezers.
- Novel materials: The advent of new materials with lower loss and externally controllable acoustic properties, such as those with extremely higher acoustic impedance or matching acoustic impedance in water or air, is anticipated to contribute to the development of advanced acoustic components for acoustic tweezers. Additionally, the integration of active materials, such as piezoelectric phononic crystals and soft piezoelectric films, allows for the integration of the device components and further enhances the performance and functionality of acoustic tweezers.
- Advanced computational methods: More powerful computational techniques, including those based on artificial intelligence (such as deep learning) and inverse optimization methods, are expected to offer improved optimization capabilities for phononic crystal design and the modeling of acoustic tweezer systems. These methods can provide enhanced optimization capabilities and enable more accurate simulations and predictions.
- Refined fabrication methods: The development of more precise and cost-effective fabrication methods is crucial for the practical implementation of phononic crystals in acoustic tweezers. Integrated manufacturing approaches, including the fabrication of phononic crystals combined with acoustofluidic chips, can streamline the manufacturing process and enable efficient integration of components.

Concluding remarks

The field of phononic crystal has emerged as a highly active research areas in both fundamental and applied acoustics. Building on foundations in phononic crystal, metamaterials, and metasurfaces offer exciting prospects for anomalous scattering, as well as the development of compact, scalable, and versatile acoustic components and devices. Integrating these advancements into acoustic tweezer systems holds tremendous potential for enhancing functionality while reducing size, complexity, and cost, making them more accessible for a wide range of applications. In addition, the use of acoustic tweezers for structure assembly offers an opportunity to produce phononic crystals with tailored properties. Given the rapid progress in the field and the immense flexibility in designing phononic crystal structures, there is significant room for innovative applications and further refinement of concepts and devices for acoustic tweezers.

Acknowledgments

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20. Lattice structures for sound absorption

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Status

In modern engineering, there is a need for advanced materials optimized for mass and volume, providing superior acoustic properties and customizable design options. The rise of additive manufacturing has introduced new possibilities for creating functional materials based on structures rather than chemistry. One notable class of advanced materials that caters to the prior-mentioned needs is lattice structures. These three-dimensional arrangements consist of interconnected struts, shells, plates, or hybrids forming a repeating pattern (figure 30(A)) [263–269]. Lattice structures offer extensive design freedom with their customizable feature-pore morphology and interconnectivity, making them highly adaptable for achieving specific physical properties. Research has extensively explored the physical properties of lattice structures, showcasing their potential applications as lightweight materials, energy absorbers, thermal and electromagnetic metamaterials [270].

Given the potential of lattice structures, researchers have also delved into investigating the sound absorption properties of 3D printed lattice structures. These investigations encompass lattice structures made up solely of trusses, plates, a combination of trusses and plates, tubes, tubes combined with plates, smooth skeletal structures, triply periodic minimal surface, and bioinspired features [263]. Beyond the individual cell level, diversities such as parallel placement of heterogeneously-structured unit cells [271] and the introduction of additional acoustic meta-features have been suggested [272]. At present, the unit cell sizes studied extend up to 10 mm, while the overall sample thickness, influenced by both the unit cell size and the number of layers, ranges from approximately 20–60 mm [263]. Lattice structures with homogeneous unit cells exhibit either distinct resonance peaks or gradual absorption curves, depending on their structural morphology (figure 30(B)). In contrast, lattices composed of heterogeneously structured cells display broadband absorption curves (figure 30(B)), with effective bandwidths having absorption coefficients >0.5 typically spanning the frequency range of 1000–6000 Hz [271, 272]. Several analytical models have been suggested to anticipate absorption properties, encompassing mechanisms ranging from multi-layered Helmholtz resonance (MLHR), cavity resonance, effective acoustical properties, and slow-sound effects [263]. However, for most lattices, the most effective representation involves modeling them as a series of Helmholtz resonators using the MLHR model. These models have demonstrated high fidelity in accurately predicting sound absorption coefficients.

Lattice structures demonstrated advantages over conventional absorbers. First, they display comparable absorption characteristics, owing to them having similar acoustical geometries and thus adhering to the same sound absorption principles. Second, they are highly customizable, for instance, they can be designed with heterogeneous features and optimized for broadband absorption (figure 30(B)). Additionally, lattice structures display potential for multifunctionalities, for instance, various works demonstrated that highly sound-absorbing lattice structures also serve as lightweight structures [272–274] or tough materials [271, 275]. The broad design flexibility inherent in lattice structures not only provides avenues for customization and optimization but also opens possibilities for multifunctionality.

Current and future challenges

The primary challenge lies with analytical modeling. Thus far, most lattices are modeled using the MLHR method (figure 31(A)). One important aspect of the MLHR model lies with the resistance and reactance end corrections. Traditionally, end correction values are based on values proposed by Maa and other scientists [276]. These correction values work perfectly when modeling microperforated panel absorbers. However, when used in lattice structures, recent works reveal that the end correction factors are highly dependent on the acoustical geometries, including both the pore and cavity morphologies. For instance, works have revealed that if the pores are placed closer to the cavity wall, dissipation is enhanced and thus both the resistance and reactance end correction factors should increase [275, 277]. These works further proposed that these end corrections are functions of both the pore diameter and thickness through empirical. The scientific backing behind this dependence is largely unknown. Thus, how to derive end correction factors remains unknown. Also, there are few published works on modeling lattices through effective methods, and only one work each on modeling lattice absorption using cavity resonance and the sonic black-hole method. The potentials of these methods remain uncertain, and their reliability could benefit from further exploration. If analytical modeling can be better comprehended, there is a promising outlook for enhancing the design potentials of sound-absorbing lattices through structural manipulation.

Another challenge is associated with a lack of systematic understanding of lattice geometry on sound absorption. Unlike mechanical properties, where lattice stiffness and strength follow Gibson–Ashby relationships, there is no such systematic classification for sound absorption properties. This lack of classification applies not only across different structures but also within the same structure, where the influence of relative density, cell size, and the number of cells along sound incidence is largely unknown. For instance, as shown in [272], even

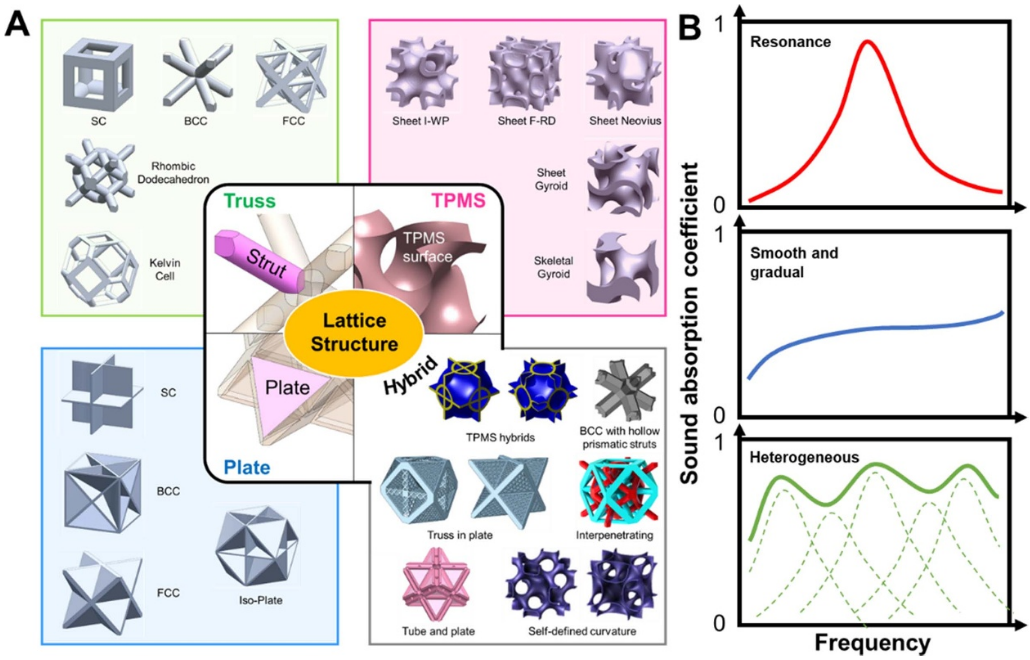

Figure 30. (A) Overview on the different types of lattice structures, revealing structures with configurations such as truss, TPMS, plate, and hybrid combinations. Figure is reproduced and adapted with permission from [263–269]. (B) Schematic of three distinct types of sound absorption coefficient curves. Reproduced from [263]. CC BY 4.0. Reproduced with permission from [264]. CC BY-NC-ND 4.0. Reprinted from [265], Copyright (2023), with permission from Elsevier. [266] John Wiley & Sons. [Copyright © 2023 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim]. Reproduced from [267]. CC BY 4.0. Reprinted with permission from [268]. Copyright (2019) American Chemical Society. Reprinted from [269], Copyright (2019), with permission from Elsevier.

for the same truss arrangement, at different relative densities, sound absorption behavior is drastically different. More studies are imperative to comprehensively characterize sound absorption properties based on lattice design. However, there are also challenges associated with undertaking such studies. As mentioned earlier, there are distinct types of lattice structures (figure 30), making it potentially difficult to quantify properties across different types of lattices. Moreover, beyond homogeneous unit cells, the sound absorption properties of lattices associated with the inhomogeneous assembly of units, configurations such as heterogeneous, fractal, hierarchical, and functionally grading, are also largely unknown.

Advances in science and technology to meet challenges

Advancements in science and technology is crucial to addressing the challenges associated with 3D printed lattice structures for sound absorption. Firstly, there is a need for more sophisticated analytical methods to enhance the modeling of lattice sound absorption properties. For example, in MLHR-type structures, this entails developing a method to determine end correction factors beyond empirical means obtained through extensive experiments. This may necessitate collaboration among experts from diverse fields, particularly involving physicists and fluid mechanics specialists, to delve into the physics underlying extended fluid vibration beyond the pore. Additionally, the development of other analytical acoustics models, such as cavity resonance, effective properties, and slow-sound effects, requires more experimental work to further refine the models. At the moment, there is an increasing number of studies adopting finite element analysis (FEA) using COMSOL Multiphysics to predict sound absorption behavior and observe sound dissipation through lattices (figure 31(B)) [274, 278]. Success have been demonstrated thus far. However, achieving high-fidelity modeling through COMSOL often demands substantial computational resources. The development of improved FEA methods or algorithms specifically for acoustics modeling could significantly enhance the efficiency of modeling lattice structures. Lastly, there is a critical need for more experimental data. Given the diverse array of lattice structures, each with its unique sound absorption mechanism, a substantial dataset is essential for gaining a deeper understanding of the structural-property relationship between lattice structure and acoustic properties. This empirical data will contribute significantly to refining and validating analytical and FEA models, ultimately advancing our comprehension of the intricate interplay between lattice design and acoustic performance.

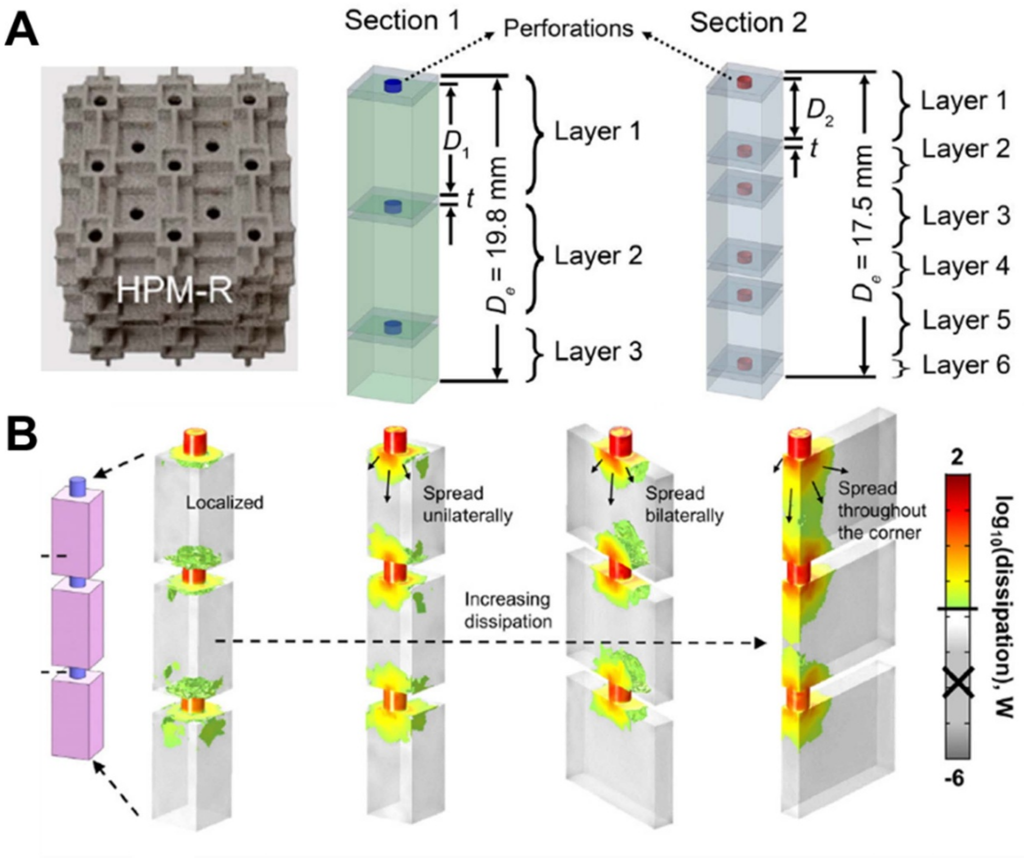

Figure 31. (A) Illustration of the discretizing and modeling of a lattice into an MLHR. [274] (2022), reprinted by permission of the publisher (Taylor & Francis Ltd, www.tandfonline.com). (B) Illustration of FEA modeling of sound dissipation through different lattice structure arrangements. Reproduced from [277] with permission from the Royal Society of Chemistry.

Concluding remarks

Overall, the integration of lattice structures, enabled by additive manufacturing, presents a promising avenue for addressing these challenges. The three-dimensional arrangements of interconnected struts, shells, and plates not only offer superior acoustic properties but also exhibit a broad design flexibility that opens possibilities for multifunctionality. However, amidst the progress, challenges such as complexities in analytical modeling complexities, especially regarding end correction factors in MLHR models, demand collaborative efforts across disciplines to unravel the intricacies of airflow mechanisms within lattice structures. Another significant challenge is associated with the lack of systematic study on lattice structures for sound absorption, lacking a classification. This knowledge gap hinders comprehensive understanding and necessitates further exploration, requiring studies in the diverse lattice types and inhomogeneous unit assemblies. The need for a substantial experimental dataset to comprehend the structural-property relationship between different lattice designs and acoustic properties remains crucial. The journey toward comprehensive solutions requires ongoing research, innovation, and a synthesis of expertise across various scientific and engineering disciplines.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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Reference

[1] Kushwaha M S, Halevi P, Dobrzynski L and Djafari-Rouhani B 1993 Acoustic band structure of periodic elastic composites *Phys. Rev. Lett.* **71** 2022–5
[2] Sigalas M and Economou E N 1993 Band structure of elastic waves in two dimensional systems *Solid State Commun.* **86** 141–3
[3] Liu Z, Zhang X, Mao Y, Zhu Y Y, Yang Z, Chan C T and Sheng P 2000 Locally resonant sonic materials *Science* **289** 1734–6
[4] Jin Y, Djafari-Rouhani B and Torrent D 2019 Gradient index phononic crystals and metamaterials *Nanophotonics* **8** 685–701
[5] Assouar B, Liang B, Wu Y, Li Y, Cheng J-C and Jing Y 2018 Acoustic metasurfaces *Nat. Rev. Mater.* **3** 460–72
[6] Jin Y, Wang W, Khelif A and Djafari-Rouhani B 2021 Elastic metasurfaces for deep and robust subwavelength focusing and imaging *Phys. Rev. Appl.* **15** 024005
[7] Gu T, Wen Z, He L, Yu M, Li Y, Li Y and Jin Y 2023 A lightweight metastructure for simultaneous low-frequency broadband sound absorption and vibration isolation *J. Acoust. Soc. Am.* **153** 96–104
[8] He L, Li Y, Torrent D, Zhuang X, Rabczuk T and Jin Y 2023 Machine learning assisted intelligent design of meta structures: a review *Microstructures* **3** 2023037
[9] Zhu W *et al* 2023 Topological phononic metamaterials *Rep. Prog. Phys.* **86** 106501
[10] Kai Y, Dhulipala S, Sun R, Lem J, DeLima W, Pezeril T and Portela C M 2023 Dynamic diagnosis of metamaterials through laser-induced vibrational signatures *Nature* **623** 514–21
[11] Jin Y, Pennec Y, Bonello B, Honarvar H, Dobrzynski L, Djafari-Rouhani B and Hussein M I 2021 Physics of surface vibrational resonances: pillared phononic crystals, metamaterials, and metasurfaces *Rep. Prog. Phys.* **84** 086502
[12] Bertoldi K and Boyce M C 2008 Mechanically triggered transformations of phononic band gaps in periodic elastomeric structures *Phys. Rev.* B **77** 052105
[13] Oudich M, Gerard N J R K, Deng Y and Jing Y 2023 Tailoring structure-borne sound through bandgap engineering in phononic crystals and metamaterials: a comprehensive review *Adv. Funct. Mater.* **33** 2206309
[14] Wang Y F, Wang Y Z, Wu B, Chen W and Wang Y S 2020 Tunable and active phononic crystals and metamaterials *Appl. Mech. Rev.* **72** 040801
[15] Holliman J E Jr, Schaef H T, Peter Mcgrail B and Miller Q R S 2022 Review of foundational concepts and emerging directions in metamaterial research: design, phenomena, and applications *Mater. Adv.* **3** 8390
[16] Cang Y, Jin Y, Djafari-Rouhani B and Fytas G 2022 Fundamentals, progress and perspectives on high-frequency phononic crystals *J. Appl. Phys.* **55** 193002
[17] Ni X, Yves S, Krasnok A and Alù A 2023 Topological metamaterials *Chem. Rev.* **123** 7585–654
[18] Wang H, Zhang X, Hua J, Lei D, Lu M and Chen Y 2021 Topological physics of non-Hermitian optics and photonics: a review *J. Opt.* **23** 123001
[19] Lu J, Deng W, Huang X, Ke M and Li Z 2023 Non-Hermitian toological phononic metamaterials *Adv. Mater.* 2307998
[20] Nassar H, Yousefzadeh B, Fleury R, Ruzzene M, Alù A, Daraio C, Norris A N, Huang G and Haberman M R 2020 Nonreciprocity in acoustic and elastic materials *Nat. Rev. Mater.* **5** 667
[21] Chen A L, Wang Y S, Wang Y F, Zhou H T and Yuan S M 2022 Design of acoustic/elastic phase gradient metasurfaces: principles, functional elements, tunability, and coding *Appl. Mech. Rev.* **74** 020801
[22] Korovin A V, Pennec Y, Stocchi M, Mencarelli D, Pierantoni L, Makkonen T, Ahopelto J and Djafari Rouhani B 2019 Conversion between surface acoustic waves and guided modes of a quasi-periodic structured nanobeam *J. Phys. D: Appl. Phys.* **52** 32LT01
[23] Navarro-Urrios D, Colombano M F, Arregui G, Madiot G, Pitanti A, Griol A, Makkonen T, Ahopelto J, Sotomayor-Torres C M and Martínez A 2022 Room-temperature silicon platform for GHz-frequency nanoelectro-opto-mechanical systems *ACS Photonics* **9** 413
[24] Lee D, Chen W, Wang L, Chan Y C and Chen W 2024 Data-driven design for metamaterials and multiscale systems: a review *Adv. Mater.* **36** 2305254
[25] Jin Y, He L, Wen Z, Mortazavi B, Guo H, Torrent D, Djafari-Rouhani B, Rabczuk T, Zhuang X and Li Y 2022 Intelligent on-demand design of phononic metamaterials *Nanophotonics* **11** 439
[26] Milton G W and Sawicki A T 2003 Theory of composites. Cambridge monographs on applied and computational mathematics *Appl. Mech. Rev.* **56** B27–8
[27] Torrent D and Sánchez-Dehesa J 2008 Anisotropic mass density by two-dimensional acoustic metamaterials *New J. Phys.* **10** 023004
[28] Milton G W, Briane M and Willis J R 2006 On cloaking for elasticity and physical equations with a transformation invariant form *New J. Phys.* **8** 248
[29] Cummer S A and Schurig D 2007 One path to acoustic cloaking *New J. Phys.* **9** 45
[30] Fang N, Xi D, Xu J, Ambati M, Srituravanich W, Sun C and Zhang X 2006 Ultrasonic metamaterials with negative modulus *Nat. Mater.* **5** 452–6
[31] Li J and Chan C T 2004 Double-negative acoustic metamaterial *Phys. Rev.* E **70** 055602
[32] Milton G W and Willis J R 2007 On modifications of Newton's second law and linear continuum elastodynamics *Proc. R. Soc.* A **463** 855–80
[33] Olivier C, Poignand G, Malléjac M, Romero-García V, Penelet G, Merkel A, Groby J P, Li J, Christensen J and Groby J-P 2021 Nonreciprocal and even Willis couplings in periodic thermoacoustic amplifiers *Phys. Rev.* B **104** 184109
[34] Norris A N, Shuvalov A L and Kutsenko A A 2012 Analytical formulation of three-dimensional dynamic homogenization for periodic elastic systems *Proc. R. Soc.* A **468** 1629–51
[35] Xiao S, Wang T, Liu T, Zhou C, Jiang X and Zhang J 2020 Active metamaterials and metadevices: a review *J. Phys. D: Appl. Phys.* **53** 503002
[36] Fang X, Wen J, Bonello B, Yin J and Yu D 2017 Ultra-low and ultra-broad-band nonlinear acoustic metamaterials *Nat. Commun.* **8** 1288
[37] Chen Y, Li X, Nassar H, Norris A N, Daraio C and Huang G 2019 Nonreciprocal wave propagation in a continuum-based metamaterial with space-time modulated resonators *Phys. Rev. Appl.* **11** 064052

[38] Jordan M I and Mitchell T M 2015 Machine learning: trends, perspectives, and prospects *Science* **349** 255–60
[39] Ma W, Liu Z, Kudyshev Z A, Boltasseva A, Cai W and Liu Y 2020 Deep learning for the design of photonic structures *Nat. Photon.* **15** 77–90
[40] Liu C-X and Yu G-L 2023 Deep learning for the design of phononic crystals and elastic metamaterials *J. Comput. Des. Eng.* **10** 602–14
[41] Zhang J, Li Y, Zhao T, Zhang Q, Zuo L and Zhang K 2021 Machine-learning based design of digital materials for elastic wave control *Extrem. Mech. Lett.* **48** 101372
[42] Li X, Liu Z, Cui S, Luo C, Li C and Zhuang Z 2019 Predicting the effective mechanical property of heterogeneous materials by image based modeling and deep learning *Comput. Methods Appl. Mech. Eng.* **347** 735–53
[43] Merchant A, Batzner S, Schoenholz S S, Aykol M, Cheon G and Cubuk E D 2023 Scaling deep learning for materials discovery *Nature* **624** 80–85
[44] He L, Wen Z, Jin Y, Torrent D, Zhuang X and Rabczuk T 2021 Inverse design of topological metaplates for flexural waves with machine learning *Mater. Des.* **199** 109390
[45] He L, Guo H, Jin Y, Zhuang X, Rabczuk T and Li Y 2022 Machine-learning-driven on-demand design of phononic beams *Sci. China Phys. Mech. Astron.* **65** 214612
[46] Li X, Ning S, Liu Z, Yan Z, Luo C and Zhuang Z 2020 Designing phononic crystal with anticipated band gap through a deep learning based data-driven method *Comput. Methods Appl. Mech. Eng.* **361** 112737
[47] Luo C, Ning S, Liu Z and Zhuang Z 2020 Interactive inverse design of layered phononic crystals based on reinforcement learning *Extrem. Mech. Lett.* **36** 100651
[48] Hughes T W, Williamson I A D, Minkov M and Fan S 2019 Wave physics as an analog recurrent neural network *Sci. Adv.* **5** eaay6946
[49] Chen J, Chen Y, Xu X, Zhou W and Huang G 2022 A physics-guided machine learning for multifunctional wave control in active metabeams *Extrem. Mech. Lett.* **55** 101827
[50] Weng J, Ding Y, Hu C, Zhu X-F, Liang B, Yang J and Cheng J 2020 Meta-neural-network for real-time and passive deep-learning-based object recognition *Nat. Commun.* **11** 6309
[51] Moghaddaszadeh M, Mousa M, Aref A and Nouh M 2024 Mechanical intelligence via fully reconfigurable elastic neuromorphic metasurfaces *APL Mater.* **12** 051117
[52] Dubček T *et al* 2024 In-sensor passive speech classification with phononic metamaterials *Adv. Funct. Mater.* **34** 2311877
[53] Muhammad G, Kennedy J and Lim C W 2022 Machine learning and deep learning in phononic crystals and metamaterials—A review *Mater. Today Commun.* **33** 104606
[54] Zhu R *et al* 2021 Phase-to-pattern inverse design paradigm for fast realization of functional metasurfaces via transfer learning *Nat. Commun.* **12** 2974
[55] Hospedales T M, Antoniou A, Micaelli P and Storkey A J 2021 Meta-learning in neural networks: a survey *IEEE Trans. Pattern Anal. Mach. Intell.* **44** 5149–69
[56] Rafiei A, Moore R, Jahromi S, Hajati F and Kamaleswaran R 2024 Meta-learning in healthcare: a survey *SN Comput. Sci.* **5** 791
[57] Su H, Xiang L, Hu A, Xu Y and Yang X 2022 A novel method based on meta-learning for bearing fault diagnosis with small sample learning under different working conditions *Mech. Syst. Signal Process.* **169** 108765
[58] Xue H, Yang Y and Zhang B 2022 Topological acoustics *Nat. Rev. Mater.* **7** 974–90
[59] Ma G, Xiao M and Chan C 2019 Topological phases in acoustic and mechanical systems *Nat. Rev. Phys.* **1** 281–94
[60] Yang Z, Gao F, Shi X, Lin X, Gao Z, Chong Y and Zhang B 2015 Topological acoustics *Phys. Rev. Lett.* **114** 114301
[61] Ni X, Weiner M, Alu A and Khanikaev A 2019 Observation of higher-order topological acoustic states protected by generalized chiral symmetry *Nat. Mater.* **18** 113–20
[62] Wang M, Ye L, Christensen J and Liu Z 2018 Valley physics in non-Hermitian artificial acoustic boron nitride *Phys. Rev. Lett.* **120** 246601
[63] Gao H, Xue H, Gu Z, Liu T, Zhu J and Zhang B 2021 Non-Hermitian route to higher-order topology in an acoustic crystal *Nat. Commun.* **12** 1888
[64] Gu Z, Gao H, Xue H, Li J, Su Z and Zhu J 2022 Transient non-Hermitian skin effect *Nat. Commun.* **13** 7668
[65] Aurégan Y and Pagneux V 2017 PT-symmetric scattering in flow duct acoustics *Phys. Rev. Lett.* **118** 174301
[66] Ingard U and Ising H 1967 Acoustic nonlinearity of an orifice *J. Acoust. Soc. Am.* **42** 6–17
[67] Marconi J, Riva E, Di Ronco M, Cazzulani G, Braghin F and Ruzzene M 2020 Experimental observation of nonreciprocal band gaps in a space-time-modulated beam using a shunted piezoelectric array *Phys. Rev. Appl.* **13** 031001
[68] Wang Y, Yousefzadeh B, Chen H, Nassar H, Huang G and Daraio C 2018 Observation of nonreciprocal wave propagation in a dynamic phononic lattice *Phys. Rev. Lett.* **121** 194301
[69] Xu X C, Wu Q, Chen H, Nassar H, Chen Y Y, Norris A N, Haberman M R and Huang G L 2020 Physical observation of a robust acoustic pumping in waveguides with dynamic boundary *Phys. Rev. Lett.* **125** 253901
[70] Attarzadeh M A, Callanan J and Nouh M 2020 Experimental observation of nonreciprocal waves in a resonant metamaterial beam *Phys. Rev. Appl.* **13** 021001
[71] Shao L B, Zhu D, Colangelo M, Lee D, Sinclair N, Hu Y W, Rakich P T, Lai K J, Berggren K K and Lončar M 2022 Electrical control of surface acoustic waves *Nat. Electron.* **5** 348–55
[72] Wu Q, Zhang X D, Shivashankar P, Chen Y Y and Huang G L 2022 Independent flexural wave frequency conversion by a linear active metalayer *Phys. Rev. Lett.* **128** 244301
[73] Brandenbourger M, Locsin X, Lerner E and Coulais C 2019 Non-reciprocal robotic metamaterials *Nat. Commun.* **10** 4608
[74] Chen Y Y, Li X P, Scheibner C, Vitelli V and Huang G L 2021 Realization of active metamaterials with odd micropolar elasticity *Nat. Commun.* **12** 5935
[75] Wu Q, Xu X C, Qian H H, Wang S Y, Zhu R, Yan Z, Ma H B, Chen Y Y and Huang G L 2023 Active metamaterials for realizing odd mass density *Proc. Natl Acad. Sci.* **120** e2209829120
[76] Riva E, Rosa M I and Ruzzene M 2020 Edge states and topological pumping in stiffness-modulated elastic plates *Phys. Rev.* B **101** 094307
[77] Einstein A 1905 Über die von der molekularkinetischen Theorie der Wärme geforderte Bewegung von in ruhenden Flüssigkeiten suspendierten Teilchen *Ann. Phys.* **322** 549
[78] Hönl S, Popoff Y, Caimi D, Beccari A, Kippenberg T J and Seidler P 2022 Microwave-to-optical conversion with a gallium phosphide photonic crystal cavity *Nat. Commun.* **13** 2065
[79] Ding S W, Pingault B, Shao L, Sinclair N, Machielse B, Chia C, Maity S and Lončar M 2024 Integrated phononic waveguides in diamond *Phys. Rev. Appl.* **21** 014034
[80] Florez O, Arregui G, Albrechtsen M, Ng R C, Gomis-Bresco J, Stobbe S, Sotomayor-Torres C M and Garcia P D 2022 Engineering nanoscale hypersonic phonon transport *Nat. Nanotechnol.* **17** 947

[81] Peng X *et al* 2024 MoS_2 phononic crystals for advanced thermal management *Sci. Adv.* **10** 13
[82] Vasileiadis T, Varghese J, Babacic V, Gomis-Bresco J, Navarro Urrios D and Graczykowski B 2021 Progress and perspectives on phononic crystals *J. Appl. Phys.* **129** 160901
[83] Luckyanova M N *et al* 2012 Coherent phonon heat conduction in superlattices *Science* **338** 936
[84] Zhang Z, Guo Y, Bescond M, Chen J, Nomura M and Volz S 2021 Coherent thermal transport in nano-phononic crystals: an overview *APL Mater.* **9** 081102
[85] Weaver M J, Duivestein P, Bernasconi A C, Scharmer S, Lemang M, Thiel T C V, Hijazi F, Hensen B, Gröblacher S and Stockill R 2024 An integrated microwave-to-optics interface for scalable quantum computing *Nat. Nanotechnol.* **19** 166
[86] Meesala S, Wood S, Lake D, Chiappina P, Zhong C, Beyer A D, Shaw M D, Jiang L and Painter O 2024 Non-classical microwave–optical photon pair generation with a chip-scale transducer *Nat. Phys.* **20** 871–7
[87] Zhao H, Bozkurt A and Mirhosseini M 2023 Electro-optic transduction in silicon via gigahertz-frequency nanomechanics *Optica* **10** 790–6
[88] Chen M *et al* 2024 Phonon engineering of atomic-scale defects in superconducting quantum circuits[J] *Sci. Adv.* **10** 37
[89] Kuruma K *et al* 2023 Engineering phonon-qubit interactions using phononic crystals (arXiv:2310.06236 [quant-ph])
[90] Odeh M, Godeneli K, Li E, Tangirala R, Zhou H, Zhang X, Zhang Z-H and Sipahigil A 2023 Non-Markovian dynamics of a superconducting qubit in a phononic bandgap (arXiv:2312.01031 [quant-ph])
[91] Lindman K F 1920 Über eine durch ein isotropes System von spiralförmigen Resonatoren erzeugte Rotationspolarisation der elektromagnetischen Wellen *Ann. Phys.* **368** 621
[92] Pine A S 1970 Direct observation of acoustical activity in n quartz *Phys. Rev.* B **2** 2049
[93] Eringen A C 1966 Linear theory of micropolar elasticity *J. Math. Mech.* **15** 909
[94] Frenzel T, Kadic M and Wegener M 2017 Three-dimensional mechanical metamaterials with a twist *Science* **358** 1072
[95] Chen Y, Frenzel T, Guenneau S, Kadic M and Wegener M 2020 Mapping acoustical activity in 3D chiral mechanical metamaterials onto micropolar continuum elasticity *J. Mech. Phys. Solids* **137** 103877
[96] Frenzel T, Köpfler J, Jung E, Kadic M and Wegener M 2019 Ultrasound experiments on acoustical activity in chiral mechanical metamaterials *Nat. Commun.* **10** 3384
[97] Chen Y, Kadic M, Guenneau S and Wegener M 2020 Isotropic chiral acoustic phonons in 3D quasicrystalline metamaterials *Phys. Rev. Lett.* **124** 235502
[98] Chen Y, Kadic M and Wegener M 2021 Chiral triclinic metamaterial crystals supporting isotropic acoustical activity and isotropic chiral phonons *Proc. R. Soc.* A **477** 20200764
[99] Chen Y, Frenzel T, Zhang Q, Kadic M and Wegener M 2021 Cubic metamaterial crystal supporting broadband isotropic chiral phonons *Phys. Rev. Mater.* **5** 25201
[100] Groß M F, Schneider J L G, Wei Y, Chen Y, Kalt S, Kadic M, Liu X, Hu G and Wegener M 2023 Tetramode metamaterials as phonon polarizers *Adv. Mater.* **35** 2211801
[101] Willis J 1981 Variational principles for dynamic problems for inhomogeneous elastic media *Wave Motion* **3** 1–11
[102] Muhlestein M, Sieck C, Wilson P and Haberman M 2017 Experimental evidence of Willis coupling in a one-dimensional effective material element *Nat. Commun.* **8** 15625
[103] Liu Y *et al* 2019 Willis metamaterial on a structured beam *Phys. Rev.* X **9** 011040
[104] Sieck C F, Alù A and Haberman M R 2017 Origins of Willis coupling and acoustic bianisotropy in acoustic metamaterials through source-driven homogenization *Phys. Rev.* B **96** 104303
[105] Meng Y, Hao Y, Guenneau S, Wang S and Li J 2021 Willis coupling in water waves *New J. Phys.* **23** 073004
[106] Hao Y, Shen Y, Groby G and Li J 2022 Experimental demonstration of Willis coupling for elastic torsional waves *Wave Motion* **112** 102931
[107] Chen Y, Li X, Hu G, Haberman M and Huang G 2020 An active mechanical Willis meta-layer with asymmetric polarizabilities *Nat. Commun.* **11** 3681
[108] Su G, Du Z, Jiang P and Liu Y 2022 High-efficiency wavefront manipulation in thin plates using elastic metasurfaces beyond the generalized Snell's law *Mech. Syst. Signal Process.* **179** 109391
[109] Li J, Shen C, Díaz-Rubio A, Tretyakov S and Cummer S 2018 Systematic design and experimental demonstration of bianisotropic metasurfaces for scattering-free manipulation of acoustic wavefronts *Nat. Commun.* **9** 1342
[110] Asadchy V, Díaz-Rubio A and Tretyakov S 2018 Bianisotropic metasurfaces: physics and applications *Nanophotonics* **7** 1069–94
[111] Quan L, Yves S, Peng Y, Esfahlani H and Alù A 2021 Odd Willis coupling induced by broken time-reversal symmetry *Nat. Commun.* **12** 2615
[112] Gao H, Wang Z, Xu Z, Yang J, Liang B and Cheng J 2022 Deep learning-assisted multifunctional wavefront modulation with Willis coupling *Appl. Phys. Lett.* **121** 113501
[113] Quan L, Ra'di Y, Sounas D and Alù A 2018 Maximum Willis coupling in acoustic scatterers *Phys. Rev. Lett.* **120** 254301
[114] Cho C, Wen X, Park N and Li J 2021 Acoustic Willis meta-atom beyond the bounds of passivity and reciprocity *Commun. Phys.* **4** 82
[115] Yao R, Gao H, Sun Y, Yuan X and Xiang Z 2018 An experimental verification of the one-dimensional static Willis-form equations *Int. J. Solids Struct.* **134** 283–92
[116] Pernas-Salomón R, Haberman M, Norris A and Shmuel G 2021 The electromomentum effect in piezoelectric Willis scatterers *Wave Motion* **106** 102797
[117] Xu L, Xu G, Li J, Li Y, Huang J and Qiu C W 2022 Thermal Willis coupling in spatiotemporal diffusive metamaterials *Phys. Rev. Lett.* **129** 155901
[118] Wu B, Jiang W, Jiang J Q, Zhao Z N, Tang Y Q, Zhou W J and Chen W Q 2024 Wave manipulation in intelligent metamaterials: recent progress and prospects *Adv. Funct. Mater.* **34** 2316745
[119] Wang Y F, Yang L, Wang T T, Chen A L, Laude V and Wang Y S 2021 Guided Lamb waves in reconfigurable phononic crystal waveguides *APL Mater.* **9** 081110
[120] Jiang T J, Han Q and Li C L 2023 Topologically tunable local-resonant origami metamaterials for wave transmission and impact mitigation *J. Sound Vib.* **548** 117548
[121] Li J, Wang Y T, Chen W Q, Wang Y S and Bao R H 2019 Harnessing inclusions to tune post-buckling deformation and bandgaps of soft porous periodic structures *J. Sound Vib.* **459** 114848
[122] Bertoldi K 2017 Harnessing instabilities to design tunable architected cellular materials *Annu. Rev. Mater. Res.* **47** 51–61
[123] Chuang K C, Lv X F and Wang D F 2019 A tunable elastic metamaterial beam with flat-curved shap memory alloy resonators *Appl. Phys. Lett.* **114** 051903

[124] Wang T T, Wang Y F, Deng Z C, Laude V and Wang Y S 2022 Reconfigurable waveguides by selective fluid filling in two-dimensional phononic metaplates *Mech. Syst. Signal Process.* **165** 108392
[125] Bertoldi K, Vitelli V, Christensen J and van Hecke M 2017 Flexible mechanical metamaterials *Nat. Rev. Mater.* **2** 17066
[126] Ning L, Wang Y Z and Wang Y S 2022 Broadband square cloak in elastic wave metamaterial plate with active control *J. Acoust. Soc. Am.* **150** 4343–52
[127] Ma Z, Liu Y, Xie Y X and Wang Y S 2022 A simple elastic phononic crystal plate with adjustable topological valley transmission paths *Extrem. Mech. Lett.* **57** 101910
[128] Wang S and Wang Y Z 2023 Nonlocal topological states in elastic wave metamaterials with active feedback control *Int. J. Solids Struct.* **281** 112430
[129] Cui L F and Wang Y Z 2023 Dual-band topological states in actively convertible metamaterials with parallel platforms *Int. J. Solids Struct.* **284** 112511
[130] Yu G L, Wang W H, Lai L Y, Peng T R, Jiang C and Li Y G 2023 Thermally reconfigurable topological protection routes for the 2D phononic system *Int. J. Mech. Sci.* **241** 107989
[131] Gliozzi A S, Miniaci M, Chiappone A, Bergamini A, Morin B and Descrovi E 2020 Tunable photo-responsive elastic metamaterials *Nat. Commun.* **11** 2576
[132] Deng Y, Oudich M, Gerard N J R K, Ji J, Lu M and Jing Y 2020 Magic-angle bilayer phononic graphene *Phys. Rev.* B **102** 180304
[133] Gu Z, Gao H, Cao P C, Liu T, Zhu X F and Zhu J 2021 Controlling sound in non-Hermitian acoustic systems *Phys. Rev. Appl.* **16** 057001
[134] Zhao Z A, Chen Y J, Hu X Y, Bao R H, Wu B and Chen W Q 2023 Vibrations and waves in soft dielectric elastomer structures *Int. J. Mech. Sci.* **239** 107885
[135] Cullen A L 1958 A travelling-wave parametric amplifier *Nature* **181** 332
[136] Trainiti G and Ruzzene M 2016 Non-reciprocal elastic wave propagation in spatiotemporal periodic structures *New J. Phys.* **18** 083047
[137] Nassar H, Chen H, Norris A N and Huang G L 2018 Quantization of band tilting in modulated phononic crystals *Phys. Rev.* B **97** 14305
[138] Moghaddaszadeh M, Attarzadeh M A, Aref A and Nouh M 2022 Complex spatiotemporal modulations and non-Hermitian degeneracies in PT-symmetric phononic materials *Phys. Rev. Appl.* **18** 44013
[139] Moussa H, Xu G, Yin S, Galiffi E, Ra'di Y and Alù A 2023 Observation of temporal reflection and broadband frequency translation at photonic time interfaces *Nat. Phys.* **19** 863–
[140] Santini J and Riva E 2023 Elastic temporal waveguiding *New J. Phys.* **25** 013031
[141] Kim B L, Chong C, Hajarolasvadi S, Wang Y and Daraio C 2023 Dynamics of time-modulated, nonlinear phononic lattices *Phys. Rev.* E **107** 34211
[142] Celli P and Palermo A 2024 Time-modulated inerters as building blocks for nonreciprocal mechanical devices *J. Sound Vib.* **572** 118178
[143] Torquato S 2002 *Random Heterogeneous Materials: Microstructure and Macroscopic Properties* (Springer)
[144] Romero-García V and Hennion A-C H 2019 *Fundamentals and Applications of Acoustic Metamaterials: From Seismic to Radio Frequency* (ISTE Ltd)
[145] Man W, Florescu M, Williamson E P, He Y, Hashemizad S R, Leung B Y C, Liner D R, Torquato S, Chaikin P M and Steinhardt P J 2013 Isotropic band gaps; freeform waveguides observed in hyperuniform disordered photonic solids *Proc. Natl Acad. Sci.* **110** 15886–91
[146] Leseur O, Pierrat R and Carminati R 2016 High-density hyperuniform materials can be transparent *Optica* **3** 763
[147] Chéron E, Groby J, Pagneux V and Félix S R-G V 2022 *Phys. Rev.* B **106** 064206
[148] Torquato S, Zhang G and Stillinger F H 2015 Ensemble theory for stealthy hyperuniform disordered ground states *Phys. Rev.* X **5** 021020
[149] Uche O U, Stillinger F H and Torquato A S 2004 Constraints on collective density variables: two dimensions *Phys. Rev.* E **70** 046122
[150] Alhaïtz L, Conoir J-M and Valier-Brasier T 2023 Experimental evidence of isotropic transparency and complete band gap formation for ultrasound propagation in stealthy hyperuniform media *Phys. Rev.* E **108** 065001
[151] Chéron É, Félix S, Groby J-P, Pagneux V and Romero-García V 2022 Wave transport in stealth hyperuniform materials: the diffusive regime and beyond *Appl. Phys. Lett.* **121** 061702
[152] Froufe-Pérez L S, Engel M, Damasceno P F, Muller N, Haberko J, Glotzer S C and Scheffold F 2016 Role of short-range order and hyperuniformity in the formation of band gaps in disordered photonic materials *Phys. Rev. Lett.* **117** 053902
[153] Oğuz E, Socolar J E S, Steinhardt P J and Torquato S 2017 Hyperuniformity of quasicrystals *Phys. Rev.* B **95** 054119
[154] Kim J and Torquato S 2020 Multifunctional composites for elastic and electromagnetic wave propagation *Proc. Natl Acad. Sci.* **117** 8764
[155] Man W, Florescu M, Matsuyama K, Yadak P, Nahal G, Hashemizad S, Williamson E, Steinhardt P, Torquato S and Chaikin P 2013 Photonic band gap in isotropic hyperuniform disordered solids with low dielectric contrast *Opt. Express* **21** 19972
[156] Torquato S and Kim J 2021 Nonlocal effective electromagnetic wave characteristics of composite media: beyond the quasistatic regime *Phys. Rev.* X **11** 021002
[157] Aubry G J, Froufe-Pérez L S, Kuhl U, Legrand O, Scheffold F and Mortessagne F 2020 Experimental tuning of transport regimes in hyperuniform disordered photonic materials *Phys. Rev. Lett.* **125** 127402
[158] Mitchell N P, Nash L M, Hexner D, Turner A M and Irvine W T M 2018 Amorphous topological insulators constructed from random point sets *Nat. Phys.* **14** 380
[159] Romero-García V, Chéron É, Kuznetsova S, Groby J, Félix S, Pagneux V and Garcia-Raffi L 2021 *APL Mater.* **9** 101101
[160] Rohfritsch A, Conoir J-M, Valier-Brasier T and Marchiano R 2020 Impact of particle size and multiple scattering on the propagation of waves in stealthy-hyperuniform media *Phys. Rev.* E **102** 053001
[161] Kuznetsova S, Groby J-P, Garcia-Raffi L and Romero-García V 2021 Stealth and equiluminous materials for scattering cancellation and wave diffusion *Waves in Random and Complex Media* (Taylor & Francis, Abingdon) (https://doi.org/10.1080/17455030.2021.1948630)
[162] Milton G W and Cherkaev A V 1995 Which elasticity tensors are realizable? *J. Eng. Mater. Technol.* **117** 483
[163] Norris A N 2008 Acoustic cloaking theory *Proc. R. Soc.* A **464** 2411
[164] Hladky-Hennion A C, Vasseur J O, Haw G, Croenne C, Haumesser L and Norris A N 2013 Negative refraction of acoustic waves using a foam-like metallic structure *Appl. Phys. Lett.* **102** 144103
[165] Bückmann T, Thiel M, Kadic M, Schittny R and Wegener M 2014 An elasto-mechanical unfeelability cloak made of pentamode metamaterials *Nat. Commun.* **5** 1
[166] Chen Y, Liu X N and Hu G K 2015 Latticed pentamode acoustic cloak *Sci. Rep.* **5** 15745

[167] Chen Y, Zheng M, Liu X, Bi Y, Sun Z, Xiang P, Yang J and Hu G 2017 Broadband solid cloak for underwater acoustics *Phys. Rev.* B **95** 180104

[168] Chen Y and Hu G 2019 Broadband and high-transmission metasurface for converting underwater cylindrical waves to plane waves *Phys. Rev. Appl.* **12** 044046

[169] Chen Y, Zhao B, Liu X and Hu G 2020 Highly anisotropic hexagonal lattice material for low frequency water sound insulation *Extrem. Mech. Lett.* **40** 100916

[170] Cushing C W, Kelsten M J, Su X, Wilson P S, Haberman M R and Norris A N 2022 Design and characterization of a three-dimensional anisotropic additively manufactured pentamode material *J. Acoust. Soc. Am.* **151** 168

[171] Zhao A, Liu C, Zou H, Jia H, Zhang M, Wu T, Chen H, Zhang X and Wang Z 2023 Massive and fast fabrication of pentamode devices through a multiphase honeycomb-corrugation configuration *Mater. Des.* **228** 111816

[172] Mu D, Shu H, Zhao L and An S 2020 A review of research on seismic metamaterials *Adv. Eng. Mater.* **22** 1901148

[173] Chen Z, Wang G and Lim C 2023 Artificially engineered metaconcrete with wide bandgap for seismic surface wave manipulation *Eng. Struct.* **276** 115375

[174] Zeng Y, Zhang S-Y, Zhou H-T, Wang Y-F, Cao L, Zhu Y, Du Q-J, Assouar B and Wang Y-S 2021 Broadband inverted T-shaped seismic metamaterial *Mater. Des.* **208** 109906

[175] Palermo A, Vitali M and Marzani A 2018 Metabarriers with multi-mass locally resonating units for broad band Rayleigh waves attenuation *Soil Dyn. Earthq. Eng.* **113** 265–77

[176] Cai R, Pennec Y, Carpentier L, Jin Y, Rabczuk T, Zhuang X and Djafari-Rouhani B 2024 Topological elastic interface states in hyperuniform pillared metabeams *APL Mater.* **12** 011121

[177] Brûlé S, Javelaud E, Enoch S and Guenneau S 2014 Experiments on seismic metamaterials: molding surface waves *Phys. Rev. Lett.* **112** 133901

[178] Colombi A, Roux P, Guenneau S, Gueguen P and Craster R V 2016 Forests as a natural seismic metamaterial: Rayleigh wave bandgaps induced by local resonances *Sci. Rep.* **6** 1–7

[179] Gupta A, Sharma R, Thakur A and Gulia P 2023 Metamaterial foundation for seismic wave attenuation for low and wide frequency band *Sci. Rep.* **13** 2293

[180] Wu X, Wen Z, Jin Y, Rabczuk T, Zhuang X and Djafari-Rouhani B 2021 Broadband Rayleigh wave attenuation by gradient metamaterials *Int. J. Mech. Sci.* **205** 106592

[181] Luo Y M, He C, Tao Z, Hao J, Xu H H, Zhang Y, Zhang F and Ren X 2024 A surface-wave seismic metamaterial filled with auxetic foam *Int. J. Mech. Sci.* **262** 108715

[182] Zeng Y, Cao L, Wan S, Guo T, Wang Y-F, Du Q-J, Assouar B and Wang Y-S 2022 Seismic metamaterials: generating low-frequency bandgaps induced by inertial amplification *Int. J. Mech. Sci.* **221** 107224

[183] Chen Y, Qian F, Scarpa F, Zuo L and Zhuang X 2019 Harnessing multi-layered soil to design seismic metamaterials with ultralow frequency band gaps *Mater. Des.* **175** 107813

[184] Cai R, Jin Y, Rabczuk T, Zhuang X and Djafari-Rouhani B 2021 Propagation and attenuation of Rayleigh and pseudo surface waves in viscoelastic metamaterials *J. Appl. Phys.* **129** 124903

[185] Liu Z, Dong H-W, Yu G-L and Cheng L 2022 Achieving ultra-broadband and ultra-low-frequency surface wave bandgaps in seismic metamaterials through topology optimization *Compos. Struct.* **295** 115863

[186] Pu X, Palermo A, Cheng Z, Shi Z and Marzani A 2020 Seismic metasurfaces on porous layered media: surface resonators and fluid-solid interaction effects on the propagation of Rayleigh waves *Int. J. Eng. Sci.* **154** 103347

[187] Pu X and Shi Z 2018 Surface-wave attenuation by periodic pile barriers in layered soils *Constr. Build. Mater.* **180** 177–87

[188] Cai R, Jin Y, Djafari-Rouhani B, Zhou S, Chen P, Rabczuk T, Zhu H and Zhuang X 2024 Attenuation of Rayleigh and pseudo surface waves in saturated soil by seismic metamaterials *Comput. Geotech.* **165** 105854

[189] Meng L, Cheng Z and Shi Z 2020 Vibration mitigation in saturated soil by periodic in-filled pipe pile barriers *Comput. Geotech.* **124** 103633

[190] Liu C-X and Yu G-L 2022 Inverse design of locally resonant metabarrier by deep learning with a rule-based topology dataset *Comput. Methods Appl. Mech. Eng.* **394** 114925

[191] Liu C X, Yu G L and Liu Z 2024 Fast topology optimization of phononic crystal-based metastructures for vibration isolation by deep learning *Comput. Aided Civ. Infrastruct. Eng.* **39** 776–90

[192] Zeng Y, Peng P, Du Q-J, Wang Y-S and Assouar B 2020 Subwavelength seismic metamaterial with an ultra-low frequency bandgap *J. Appl. Phys.* **128** 014901

[193] Ji D-X and Yu G-L 2022 Shielding performance of T-shaped periodic barrier for surface waves in transversely isotropic soil *Proc. Inst. Mech. Eng.* L **236** 2242–54

[194] Gripp J A B and Rade D A 2018 Vibration and noise control using shunted piezoelectric transducers: a review *Mech. Syst. Signal Process.* **112** 359–83

[195] Dai S, Zheng Y, Mao J and Qu Y 2023 Vibro-acoustic control of a programmable meta-shell with digital piezoelectric shunting *Int. J. Mech. Sci.* **255** 108475

[196] Zheng Y, Qu Y, Dai S, Chen B and Mao J 2024 Mitigating vibration and sound radiation with a digital piezoelectric meta-shell in heavy fluids *J. Sound Vib.* **573** 118221

[197] Yi K, Chen Y, Zhu R and Huang G 2022 Electromechanical active metamaterials and their applications in controlling elastic wave propagation *Chin. Sci. Bull.* **67** 1290–304

[198] Sugino C, Ruzzene M and Erturk A 2020 Digitally programmable resonant elastic metamaterials *Phys. Rev. Appl.* **13** 061001,06/05/

[199] Luo A, Lossouarn B and Erturk A 2024 Multimodal vibration damping of a three-dimensional circular ring coupled to analogous piezoelectric networks *J. Sound Vib.* **581** 118385

[200] Zhou W, Luo K, Tian Q and Hu H 2022 Distributed actuation optimization of in-orbit membrane structures based on clustering *AIAA J.* **61** 444–53

[201] Alfahmi O and Erturk A 2024 Programmable hardening and softening cubic inductive shunts for piezoelectric structures: harmonic balance analysis and experiments *J. Sound Vib.* **571** 118029

[202] Chen B, Zheng Y, Dai S and Qu Y 2024 Bandgap enhancement of a piezoelectric metamaterial beam shunted with circuits incorporating fractional and cubic nonlinearities *Mech. Syst. Signal Process.* **212** 111262

[203] Gao P, Willatzen M and Christensen J 2020 Anomalous topological edge states in non-Hermitian piezophononic media *Phys. Rev. Lett.* **125** 206402,11/13/

[204] Zhang X, Yu H, He Z, Huang G, Chen Y and Wang G 2021 A metamaterial beam with inverse nonlinearity for broadband micro-vibration attenuation *Mech. Syst. Signal Process.* **159** 107826

[205] Tao B, Luo K, Tian Q and Hu H 2024 Inverse dynamic design for motion control of soft machines driven by dielectric elastomer actuators *Int. J. Mech. Sci.* **273** 109213

[206] Myny K 2018 The development of flexible integrated circuits based on thin-film transistors *Nat. Electron.* **1** 30–39

[207] Garnell E, Rouby C and Doaré O 2019 Dynamics and sound radiation of a dielectric elastomer membrane *J. Sound Vib.* **459** 114836
[208] Brillouin L 1946 *Wave Propagation in Periodic Structures, Electric Filters and Crystal Lattices* (Dover Publications, Inc.)
[209] Mead D M 1996 Wave propagation in continuous periodic structures: research contributions from Southampton, 1964–1995 *J. Sound Vib.* **190** 495–524
[210] Hussein M I, Leamy M J and Ruzzene M 2014 Dynamics of phononic materials and structures: historical origins, recent progress and future outlook *Trans. ASME, Appl. Mech. Rev.* **66** 040802
[211] Merheb B, Deymier P A, Jain M, Aloshyna-Lesuffleur M, Mohanty S, Baker A and Greger R W 2008 Elastic and viscoelastic effects in rubber/air acoustic band gap structures: a theoretical and experimental study *J. Appl. Phys.* **104** 064913
[212] Hussein M I and Frazier M J 2010 Band structure of phononic crystals with general damping *J. Appl. Phys.* **108** 093506
[213] Moiseyenko R P and Laude V 2011 Material loss influence on the complex band structure and group velocity in phononic crystals *Phys. Rev.* B **83** 064301
[214] Farzbod F and Leamy M J 2011 Analysis of Bloch's method in structures with energy dissipation *Trans. ASME, J. Vib. Acoust.* **133** 051010
[215] Frazier M J and Hussein M I 2016 Generalized Bloch's theorem for viscous metamaterials: dispersion and effective properties based on frequencies and wavenumbers that are simultaneously complex *C. R. Phys.* **17** 565–77
[216] Lakes R S, Lee T, Bersie A and Wang Y C 2001 Extreme damping in composite materials with negative-stiffness inclusions *Nature* **410** 565–7
[217] Hussein M I and Frazier M J 2013 Metadamping: an emergent phenomenon in dissipative metamaterials *J. Sound Vib.* **332** 4767–74
[218] Bacquet C L and Hussein M I 2018 Dissipation engineering in metamaterials by localized structural dynamics (arXiv:1809.04509)
[219] Aladwani A and Nouh M 2020 Mechanics of metadamping in flexural dissipative metamaterials: analysis and design in frequency and time domains *Int. J. Mech. Sci.* **173** 105459
[220] Al Ba'ba'a H H, Lin Z and Tol S 2011 Metadamping enhancement and tunability via scissor-like electromechanical metamaterials *J. Appl. Phys.* **130** 184901
[221] Caughey T K and O'Kelly M E J 1965 Classical normal modes in damped linear dynamic systems *Trans. ASME, J. Appl. Mech.* **32** 583–8
[222] Woodhouse J 1998 Linear damping models for structural vibration *J. Sound Vib.* **215** 547–69
[223] Phani A S and Woodhouse J 2007 Viscous damping identification in linear vibration *J. Sound Vib.* **303** 475–500
[224] Nomura M, Anufriev R, Zhang Z, Maire J, Guo Y, Yanagisawa R and Volz S 2022 *Mater. Today Phys.* **22** 100613
[225] Sledzinska M, Graczykowski B, Maire J, Chavez-Angel E, Sotomayor-Torres C M and Alzina F 2020 *Adv. Funct. Mater.* **30** 1904434
[226] Yu J K, Mitrovic S, Tham D, Varghese J and Heath J R 2010 *Nat. Nanotechnol.* **5** 718–21
[227] Davis B L and Hussein M I 2014 *Phys. Rev. Lett.* **112** 055505
[228] Ravichandran J *et al* 2014 *Nat. Mater.* **13** 168
[229] Zen N, Puurtinen T A, Isotalo T J, Chaudhuri S and Maasilta I J 2014 *Nat. Commun.* **5** 3435
[230] Maire J, Anufriev R, Yanagisawa R, Ramiere A, Volz S and Nomura M 2017 *Sci. Adv.* **3** e1700027
[231] Nomura M, Kage Y, Nakagawa J, Hori T, Miare J, Shiomi J, Anufriev R, Moser D and Paul O 2015 *Phys. Rev.* B **91** 205422
[232] Li Y, Li W, Han T, Zheng X, Li J, Li B, Fan S and Qiu C W 2021 *Nat. Rev. Mater.* **6** 488
[233] Pennec Y, Djafari-Rouhani B, Vasseur J O, Khelif A and Deymier P A 2004 Tunable filtering and demultiplexing in phononic crystals with hollow cylinders *Phys. Rev.* E **69** 046608
[234] Lucklum R and Li J 2009 Phononic crystals for liquid sensor applications *Meas. Sci. Technol.* **20** 124014
[235] Lucklum R, Ke M and Zubtsov M 2012 Two-dimensional phononic crystal sensor based on a cavity mode *Sens. Actuators* B **171** 271–7
[236] Chen T, Jiao J and Yu D 2022 Strongly coupled phononic crystals resonator with high energy density for acoustic enhancement and directional sensing *J. Sound Vib.* **529** 116911
[237] Jin Y, Pennec Y, Pan Y and Djafari-Rouhani B 2016 Phononic crystal plate with hollow pillars actively controlled by fluid filling *Crystals* **6** 64
[238] Zhu T, Wang T T, Zhou H T, Wang Y F and Wang Y S 2024 Reconfigurable phononic crystal sensor for liquid detection *Smart Mater. Struct.* **33** 035016
[239] Gueddida A, Pennec Y, Zhang V, Lucklum F, Vellekoop M, Mukhin N, Lucklum R, Bonello B and Djafari Rouhani B 2021 Tubular phononic crystal sensor *J. Appl. Phys.* **130** 105103
[240] Gueddida A, Zhang V, Carpentier L, Bonhomme J, Bonello B, Pennec Y and Djafari-Rouhani B 2023 Phononic crystal made of silicon ridges on a membrane for liquid sensing *Sensors* **23** 2080
[241] Zaremanesh M, Carpentier L, Gharibi H, Bahrami A, Mehaney A, Gueddida A, Lucklum R, Djafari-Rouhani B and Pennec Y 2021 Temperature biosensor based on triangular lattice phononic crystals *APL Mater.* **9** 061114
[242] Lu M H, Liu X-K, Feng L, Li J, Huang C-P, Chen Y-F, Zhu Y-Y, Zhu S-N and Ming N-B 2007 Extraordinary acoustic transmission through a 1D grating with very narrow apertures *Phys. Rev. Lett.* **99** 174301
[243] Ke M, Zubtsov M and Lucklum R 2011 Sub-wavelength phononic crystal liquid sensor *J. Appl. Phys.* **110** 026101
[244] Bonhomme J, Oudich M, Djafari-Rouhani B, Sarry F, Pennec Y, Bonello B, Beyssen D and Charette P G 2019 Love waves dispersion by phononic pillars for nano-particle mass sensing *Appl. Phys. Lett.* **114** 013501
[245] Kaya O A, Cicek A, Salman A and Ulug B 2014 Acoustic Mach–Zehnder interferometer utilizing self-collimated beams in a two-dimensional phononic crystal *Sens. Actuators* B **203** 197–203
[246] Zhao X, Hao G, Shang Y and Han J 2023 Three-dimensional gradient metamaterial devices coupled with phononic crystals for acoustic enhancement sensing *Crystals* **13** 1191
[247] Amoudache S, Moiseyenko R, Pennec Y, Djafari-Rouhani B, Khater A, Lucklum R and Tigrine R 2016 Optical and acoustic sensing using Fano-like resonances in dual phononic and photonic crystal plate *J. Appl. Phys.* **119** 114502
[248] Cicek A, Salman A, Kaya O A and Ulug B 2016 Phononic crystal surface mode coupling and its use in acoustic Doppler velocimetry *Ultrasonics* **65** 78–86
[249] Amoudache S, Pennec Y, Djafari Rouhani B, Khater A, Lucklum R and Tigrine R 2014 Simultaneous sensing of light and sound velocities of fluids in a two-dimensional phoXonic crystal with defects *J. Appl. Phys.* **115** 134503
[250] Huang N N, Chung Y C, Chiu H T, Hsu J C, Lin Y F, Kuo C T, Chang Y W, Chen C Y and Lin T R 2020 Dual

photonic–phononic crystal slot nanobeam with gradient cavity for liquid sensing *Crystals* **10** 421
[251] Lucklum R, Mukhin N, Djafari Rouhani B and Pennec Y 2021 Phononic crystal sensors: a new class of resonant sensors—chances and challenges for the determination of liquid properties *Front. Mech. Eng.* **7** 705194
[252] Gueddida A, Pennec Y, Silveira Fiates A L, Vellekoop M J, Bonello B and Djafari-Rouhani B 2022 Acoustic sensor based on a cylindrical resonator for monitoring a liquid flow *Crystals* **12** 1398
[253] Li Y, Jiang X, Liang B, Cheng J-C and Zhang L 2015 Metascreen-based acoustic passive phased array *Phys. Rev. Appl.* **4** 024003
[254] Wilson R, Reboud J, Bourquin Y, Neale S L, Zhang Y and Cooper J M 2011 Phononic crystal structures for acoustically driven microfluidic manipulations *Lab Chip* **11** 323–8
[255] Li F, Cai F, Zhang L, Liu Z, Li F, Meng L, Wu J, Li J, Zhang X and Zheng H 2020 Phononic-crystal-enabled dynamic manipulation of microparticles and cells in an acoustofluidic channel *Phys. Rev. Appl.* **13** 044077
[256] Li F *et al* 2018 Phononic crystal-enhanced near-boundary streaming for sonoporation *Appl. Phys. Lett.* **113** 083701
[257] Memoli G, Caleap M, Asakawa M, Sahoo D R, Drinkwater B W and Subramanian S 2017 Metamaterial bricks and quantization of meta-surfaces *Nat. Commun.* **8** 14608
[258] Gao Y *et al* 2023 Stable acoustic pulling in two-dimensional phononic crystal waveguides based on mode manipulation *Photonics* **10** 1325
[259] Stein M, Keller S, Luo Y J and Ilic O 2022 Shaping contactless radiation forces through anomalous acoustic scattering *Nat. Commun.* **13** 6533
[260] Caleap M and Drinkwater B W 2014 Acoustically trapped colloidal crystals that are reconfigurable in real time *Proc. Natl Acad. Sci. USA* **111** 6226–30
[261] Goyal R, Athanassiadis A G, Ma Z C and Fischer P 2022 Amplification of acoustic forces using microbubble arrays enables manipulation of centimeter-scale objects *Phys. Rev. Lett.* **128** 254502
[262] Wang N, Zhang R Y and Chan C T 2021 Robust acoustic pulling using chiral surface waves *Phys. Rev. Appl.* **15** 024034
[263] Li X, Chua J W, Yu X, Li Z, Zhao M, Wang Z and Zhai W 2023 3D-printed lattice structures for sound absorption: current progress, mechanisms and models, structural-property relationships, and future outlook *Adv. Sci.* **11** 2305232
[264] Chen Z, Xie Y M, Wu X, Wang Z, Li Q and Zhou S 2019 On hybrid cellular materials based on triply periodic minimal surfaces with extreme mechanical properties *Mater. Des.* **183** 108109
[265] Zhao M, Li X, Zhang D Z and Zhai W 2022 Design, mechanical properties and optimization of lattice structures with hollow prismatic struts *Int. J. Mech. Sci.* **238** 107842
[266] Wang Y, Xu F, Gao H and Li X 2023 Elastically isotropic truss-plate-hybrid hierarchical microlattices with enhanced modulus and strength *Small* **19** 2206024
[267] White B C, Garland A, Alberdi R and Boyce B L 2021 Interpenetrating lattices with enhanced mechanical functionality *Addit. Manuf.* **38** 101741
[268] Li Z, Wang X, Li X, Wang Z and Zhai W 2023 New class of multifunctional bioinspired microlattice with excellent sound absorption, damage tolerance, and high specific strength *ACS Appl. Mater. Interfaces* **15** 9940–52
[269] Bonatti C and Mohr D 2019 Smooth-shell metamaterials of cubic symmetry: anisotropic elasticity, yield strength and specific energy absorption *Acta Mater.* **164** 301–21
[270] Su R, Chen J, Zhang X, Wang W, Li Y, He R and Fang D 2023 3D-printed micro/nano-scaled mechanical metamaterials: fundamentals, technologies, progress, applications, and challenges *Small* **19** 2206391
[271] Li X, Yu X and Zhai W 2021 Additively manufactured deformation-recoverable and broadband sound-absorbing microlattice inspired by the concept of traditional perforated panels *Adv. Mater.* **33** 2104552
[272] Li X, Yu X and Zhai W 2022 Less is more: hollow-truss microlattice metamaterials with dual sound dissipation mechanisms and enhanced broadband sound absorption *Small* **18** 2204145
[273] Zhang J, Chen X, Sun Y, Wang Y and Bai L 2023 Sound-absorption and mechanical properties of multisheet gyroid lattice structures by stereolithography *Compos. Struct.* **325** 117589
[274] Li Z, Zhai W, Li X, Yu X, Guo Z and Wang Z 2022 Additively manufactured dual-functional metamaterials with customisable mechanical and sound-absorbing properties *Virtual Phys. Prototyp.* **17** 864–80
[275] Li X, Yu X, Zhao M, Li Z, Wang Z and Zhai W 2022 Multi-level bioinspired microlattice with broadband sound-absorption capabilities and deformation-tolerant compressive response *Adv. Funct. Mater.* **33** 2210160
[276] Maa D-Y 1998 Potential of microperforated panel absorber *J. Acoust. Soc. Am.* **104** 2861–6
[277] Li X, Yu X, Chua J W and Zhai W 2023 Harnessing cavity dissipation for enhanced sound absorption in Helmholtz resonance metamaterials *Mater. Horiz.* **10** 2892–903
[278] Li L, Guo Z, Yang F, Li P, Zhao M and Zhong Z 2024 Additively manufactured acoustic-mechanical multifunctional hybrid lattice structures *Int. J. Mech. Sci.* **269** 109071