Bayesian and frequentist significance of vertical displacements from high-precision geodetic observations: case study in an earth fill dam placed in southern Spain

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Abstract

Deformation monitoring is a problem of the utmost importance in Civil Engineering, but so is the economic aspect derived from such monitoring. It is well known that to study the same magnitude, there are often different methods. In the case of deformation control of critical infrastructures, such as reservoirs, it is possible to carry out highly reliable studies (high-precision levelling) with relatively high economic costs and that can be difficult to execute or use more economical but less precise techniques supported by GNSS (Global Navigation Satellite System) observations.
The aim of this work is to detect significant deformations when the vertical displacements (settlements) are estimated from geodetic techniques with different precision and these displacements are small with respect to their precisions. To do that Bayesian and frequentist tests are compared in an earth fill dam placed in southern Spain. Different a priori distributions of the displacements are considered. The approaches are checked on data coming from seven high-precision levelling and GNSS surveys carried from 2008 to 2016 are used.

Keywords: *Bayesian, GNSS, High precision levelling, settlements, significance*

1 Introduction

Dams, as massive and essential engineering structures, hold millions of cubic meters of water and must endure considerable horizontal and vertical displacements resulting from various internal and external forces. When these displacements, surpass critical limits, they can cause serious structural damage or even total failure. Consequently, continuous monitoring of these movements is crucial due to their significant social, technical, and economic consequences. Geodetic methods are established as techniques for monitoring civil structures in engineering (Romero Andrade et al 2024; Vazquez-Ontiveros et al 2023; Biondi et al 2020; Barzaghi et al 2018; Scaioni et al 2018)
The estimation of deformations is based on appropriate models. It is crucial the quality of the observations. Inaccuracies in the model, particularly systematic and gross errors in the observations, as well as incorrectly estimated a priori variances, can distort the results and produce false deformation readings. Here, the Bayesian approach plays a significant role. In 2005, Albertella et al. introduced a novel testing procedure for deformation analysis from a Bayesian perspective. This approach incorporates prior knowledge of displacements, particularly when the estimated displacements are small compared to the measurement precision. A discussion about how Bayesian theory can help the detection of significant deformations in geodetic monitoring is presented in (Sansó and de Lacy, 2006). Tanir et al. (2008) explore the application of Bayes-Updating (BU) and Gibbs-Sampling (GS) algorithms in geodetic parameter estimation, aiming to estimate unknown parameters, establish confidence regions, and test hypotheses concerning these parameters. In 2018, Chen and Lin presented a Bayesian model for risk analysis of dam overtopping under the combined effects of flood and earthquake. Barzaghi et al. (2019) examine the use of low-cost GNSS receivers for monitoring cultural heritage structures, applying Bayesian methods to evaluate their feasibility and accuracy. Jafari et al. (2022) explores the use of Bayesian

methods to enhance the stability analysis of monitoring networks used for deformation detection in structures. Wang et al. (2023) develop a Bayesian network (BN) model for analysing historical dam failure data in China. Xiao et al. 2024 use the Bayesian approach to select explanatory variables with a significant impact on the modelling process. On this basis, robust regression analysis of dam deformation monitoring data is performed by using the least trimmed squares (LTS) estimation.

This work presents a preliminary analysis of the application of a Bayesian model to the deformation control of an embankment dam. It is organized as follows: Section 1 is a description of the data obtained through high-precision levelling (HPL) and Global Navigation Satellite System (GNSS) of the vertical displacements of a fixed set of points located at the crest of an embankment dam in southern Spain. Section 2 briefly explains the methodology used to study the significance of these displacements. Section 3 presents the results, and finally, Section 4 analyses and discusses future work.

2 Data set

The Bayesian approach is applied to study the behaviour of an earth fill dam placed in the South of Spain. The dimensions of the dam are 80 m high and 1480 m long at its crest. Its construction began in April 2004 and finished in November 2006. A vertical deformation analysis from a Bayesian point of view is presented.

In general, the geodetic monitoring of a dam is based on the control of possible displacements of its crest and principal axis. In this study, only the vertical displacements of the crest (line C) will be analysed using data from high precision levelling and GNSS (Global Navigation Satellite System) observations.

2.1 High precision levelling Observations

Seven high-precision levelling surveys were carried out: twice a year in 2008 and 2013, and once in August 2014, September 2015, and September 2016. The relative vertical displacements of the dam's control stations were measured using high-precision levelling equipment (LEICA DNA03 digital level and 3-meter Invar bar-coded staffs), with respect to fixed benchmarks. The crest length is approximately 1400 m, and the Line C is in its downstream side. This includes 64 points spaced 20 meters from each other with an approximate height of 215.60 m. These control points materialized from levelling nails protected by a hard plastic case, to prevent deterioration due to weather conditions. They are embedded 40 cm into the ground, and at the base of the casing is a layer of concrete for stability and support. At the crest, the point C00 is on a reinforced concrete wall, and therefore should be stable. It is considered as the benchmark. This serves as the reference frame for this line (de Lacy et al., 2017). Relative elevation changes for each benchmark and each survey were recorded. These measurements were used to estimate the cumulative displacements of the dam since 2008. The results indicate downstream motion of the thrust block centre of the dam during the fall and winter. The settlement reaches the maximum here, with a value of −16 cm in 2016 with respect to February 2008. The accumulative vertical displacements indicate that the magnitude of the movements decreases in time, confirming the idea that the dam tends to stabilize, (Acosta et al., 2018).

2.2 GNSS Observations

GNSS measurements were taken in correspondence with levelling observations (i.e., twice per year in 2008 and 2013, and once in August 2014, September 2015 and September 2016). For economic reasons, in 2008 GNSS equipment was different from the one used in 2013. To be completely homogeneous and have observations of the same accuracy, measurements from 2013 to 2016 are considered in this study. We used Leica GR10 receivers with AR10 antennas (Leica Geosystems AG, Heerbrugg, Switzerland). The surveys lasted 8 days in a static mode, with a 30-s sampling rate, and a cut off angle equal to 10◦. 10 receivers moved along all the control points. The sessions lasted 12 hours ant these control points were measured twice in order to have repeatability. GPS surveys were carried out with tripods with fixed-length legs at the control points and optical plumb. GNSS data from 2013 to 2016 were processed with Leica Geo Office Software (Leica Geo Office Software Version 7.0). Details about GPS data processing can be seen in Acosta et al. 2018. Again, the point C00 is considered as the reference point since it should be stable.

3 Methodology

In what follows, the Bayesian technique presented by Betti, Cazzaniga, and Tornatore (2011) is used. The goal is to find the posterior distributions of the vertical displacements of a set of points to calculate the probability that they are really significant, based on vague prior information. As indicated in the referenced article, the Bayesian procedure proves to be more sensitive than the frequentist methods.

The case study in this work is the settlement (vertical displacements) of the crest of an embankment dam in southern Spain. Two sets of n measurements (HPL and GNSS) are available, referring to the same periods and at the same points, for which the ellipsoidal heights at the times of the measurements are known, collected in vectors $\mathbf{h}^i$, with n components, so that $\mathbf{y}=\mathbf{h}^i-\mathbf{h}^j$ represents the relative displacements, point by point, at two different times. The values σ_0^2 of the variances used in the calculations are determined from the manufacturer's specifications of the instrument in the case of HPL measurements, and through a commercial program to which the correction suggested by Cocard et al. (1999) is applied in the case of GNSS data. It will be assumed that the descent, x, of the points is uniform between two periods sufficiently close in time and always directed downwards, so that the prior distribution of x is considered a truncated normal with parameters x_0 and σ_0:

$$f(x) = P_0\delta(x) + \frac{1}{\sqrt{2\pi}\sigma_0}\, exp\left[\frac{(x-x_0)^2}{2\sigma_0^2}\right] \cdot H(x) \qquad (1)$$

where
• P_0 is the prior probability that the displacement is zero,
• $\delta(x)$ is the Dirac delta function, and
• H(x) is the step function that is 0 in (−∞,0] and 1 in (0,+∞).
The value of P_0 can then be obtained by normalizing the prior function, leading to:

$$P_0 = 1 - \frac{1}{2} erfc\left(\frac{x_0}{2\sigma_0}\right) \qquad (2)$$

Following the development of the cited article, if we define a vector **u** with *n* components of value 1, and introduce the following simplifications in the notation:

- $\bar{x} = \dfrac{\boldsymbol{u}'C^{-1}\boldsymbol{y} + \frac{x_0}{\sigma_0^2}}{\boldsymbol{u}'C^{-1}\boldsymbol{u} + \frac{1}{\sigma_0^2}}$

- $\bar{\sigma} = \sqrt{\dfrac{1}{\boldsymbol{u}'C^{-1}\boldsymbol{u} + \frac{1}{\sigma_0^2}}}$

an expression for the posterior probability of zero displacement is obtained:

$$P(x=0|\boldsymbol{y}) = \\ = \frac{P_0}{P_0 + \frac{\bar{\sigma}}{\sigma_0}\left(1 - \frac{1}{2} erfc\left(\frac{\bar{x}}{\sqrt{2}\bar{\sigma}}\right)\right) \cdot exp\left(-\frac{1}{2}\left(\frac{x_0^2}{\sigma_0^2} - \frac{\bar{x}^2}{\bar{\sigma}^2}\right)\right)} \qquad (3)$$

consequently,

$$P(x \neq 0|\boldsymbol{y}) = 1 - P(x=0|\boldsymbol{y})$$

and movement will be accepted if $P(x \neq 0|\boldsymbol{y}) > 0.5$

4 Results

The procedure was applied to both datasets. The following figure (taken from Acosta, Luis Enrique et al., (2018)) shows the corresponding HPL measurements for the different campaigns:

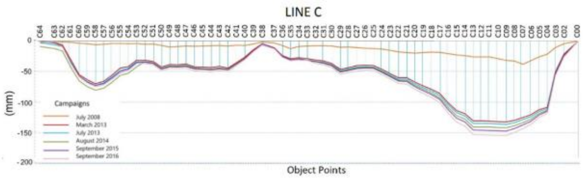

Figure 1. HPL measurements

Different pairs of campaigns were chosen for the study, although the results were not quite different in any case.

In particular, for the two campaigns of 2013 (March and July), the following results were obtained for a value of x_o=0:

Table 1

Method	σ_0^2	$P(x \neq 0\|\boldsymbol{y})$
HPL	2.04E-07	0.7499
GNSS	5,13E-04	0.7499

No difference could be found between the values of both probabilities before the seventh decimal place.

Repeating the study after grouping the points into different regions, which were studied separately, still yielded almost identical values for the posterior probabilities.

5 Analysis and conclusions

The practical equality of the results may be due to different causes. One rather optimistic interpretation is that the Bayesian technique improves the information so much that the posterior probabilities of displacement in the GNSS measurements are almost equal to those obtained from the HPL measurements. Alternatively, it may be thought that the application of the method requires a finer partitioning of the data and a better characterization of each region.

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