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The backroom assignment problem for in-store order fulfillment

Sebastian Koehler *, Felicia Theilacker

Karlsruhe Institute of Technology, Kaiserstraße 12, 76131 Karlsruhe, Germany

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ABSTRACT

Efficient in-store fulfillment is essential for today's omnichannel services, as retailers are taking on tasks previously performed by customers themselves while shopping. This paper introduces a novel backroom assignment problem for omnichannel stores, aiming to optimize the allocation of articles to a forward pick area exclusively dedicated to online demand. We present both random and dedicated storage policy formulations for the backroom assignment problem (*BAP*), determining the allocation of articles, their quantities to the forward pick area, and the selection of storage units. To achieve a balance between computational efficiency and solution quality, we introduce two decomposition methods. We evaluate the impact of our proposed BAP formulations using real data from a drug store chain and quantify the effects of an increasing online demand ratio and different forward pick area sizes on the in-store logistical effort. Results from a three-store use case demonstrate that backroom assignments can substantially reduce in-store logistical effort compared to a scenario without backroom usage, especially as demand shifts increasingly towards the online channel. The results also show that our decomposition methods are effective in handling problem instances in most cases, equipping retailers to evaluate the influence of backroom assignments. We conclude with managerial implications and explore future research opportunities.

1. Introduction

Delivering omnichannel services that provide a seamless customer experience is a major challenge facing the entire retail industry today. This has led to a trend where stores, particularly in the daily consumer goods sector (e.g., grocery and drugstores), shift from traditional brick-and-mortar to buy-online-pick-up-in-store (*BOPIS*) concepts (Chou, Pietri, Loske, Klumpp, & Montemanni, 2021). As traditional retail supply chain structures are unable to meet the demands for shorter window or same day delivery, inventory must be moved closer to the customer (Pazour & Furmans, 2023). While adding additional warehouses to the supply chain results in high capital costs, leveraging existing store infrastructure enables retailers to offer competitive delivery services (Ishfaq, Defee, Gibson, & Raja, 2016). To meet customer requirements, retailers need to offer customer-centric services such as home delivery and in-store pickup options. This has shifted the responsibility for order picking—traditionally performed by customers themselves—to the retailer, turning it into a service. Complicating factors are that current layouts are mostly designed to maximize shopping revenue (Ozgormus & Smith, 2020) and that manual processes of order fulfillment are done from the shopping area stock (Bhowmick et al., 2023). One approach to increasing in-store fulfillment efficiency is to implement or integrate a forward pick area, an additional storage area dedicated to order fulfillment within the backroom (Bhowmick et al., 2023; Seghezzi, Siragusa, & Mangiaracina, 2022). While it has been shown that backroom assignments can increase picking efficiency, quantifying the impact of such forward pick areas on the overall in-store logistics effort, as well as determining the optimal allocation of articles to these areas and the choice of storage units used, remains unanswered. Therefore, this study focuses on the following research questions (*RQ*):

RQ1: How can the impact of backroom assignments on in-store logistical effort be quantified when assigning a discrete number of boxes per article to the forward pick area?

RQ2: Given the limited capacity of the backroom, how can the configuration of forward pick area storage units and the assignment of SKUs be optimized?

RQ3: In both current and future retail scenarios, what are the projected savings from using a forward pick area of different sizes?

To answer these research questions, the structure of this paper is organized as follows: Section 2 provides an overview of the current relevant literature. Section 3 describes the impact of a forward pick area in the material flow and presents an analytical cost model that quantifies the effect of backroom assignments at the SKU level. Considering the

* Correspondence to: Gotthard-Franz-Straße 8, Karlsruhe, 76131, Germany.
E-mail address: Sebastian.koehler@kit.edu (S. Koehler).

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limited space in the backroom, we formulate the BAP for a dedicated and a random storage policy and develop two decomposition methods to achieve high-quality solutions within a reasonable computational time in Section 4. We then apply our models to a real-world drugstore use case using data from three different drugstores and discuss the solution quality of our methods. Subsequently, we present the managerial insights derived from the use case in Section 5 and conclude our work with a critical review and discussion of future research in Section 6.

2. Related research

To improve the customer experience, retailers are transitioning from traditional brick-and-mortar (*B&M*) to the BOPIS concepts as a result of omnichannel disruption (Chou et al., 2021). However, this shift necessitates that retailers provide additional services previously handled by customers themselves. Even within the traditional B&M environment, in-store logistics already play a crucial role, accounting for approximately 48% of operational logistics costs (Kuhn & Sternbeck, 2013). Additional omnichannel services intensify the need for efficient order fulfillment strategies, which primarily depend on two factors: store layout and the picking process itself.

While order fulfillment has been well studied in warehousing contexts (De Koster, Le-Duc, & Roodbergen, 2007), research on in-store order fulfillment design is relatively limited. The primary distinction between retail and warehouse environments lies in the methods of demand fulfillment. In warehouses, overall demand must be satisfied using various storage locations, whereas in a retail setting, demand varies, with in-store demand requiring fulfillment exclusively from stock available on the sales floor. Assigning inventory to the backroom in a retail store restricts access for in-store customers. However, online demand can be fulfilled using stock from both the backroom and the shopping area. Additionally, space is heavily restricted in retail settings compared to warehouse settings. In retail layouts, only around 10%–15% of the available space is typically allocated for the backroom (Dunne, Lusch, & Carver, 2013). In terms of general (warehouse) layout decisions to enable faster order picking, Bartholdi and Hackman (2008) examine the effect of a forward pick area. The forward pick area is a storage location within a storage system, strategically placed for easy access by order pickers. This area is replenished from bulk or reserve storage. They conclude that frequently picked SKUs should be allocated to the forward pick area to achieve an optimal result. However, Bartholdi's model presents significant drawbacks for discrete assignments (Walter, Boysen, & Scholl, 2013). Consequently, our work aligns closely with Walter et al. (2013), which formulates the discrete forward-reserve allocation and sizing problem (*DFRASP*). This approach is particularly relevant as it integrates article allocation, storage unit selection, and forward pick area sizing. The key distinction of our approach is that we also consider the influence of both the backroom, acting as a forward pick area, and the shopping area. Additionally, in a retail environment, demand is segmented such that in-store demand must be exclusively fulfilled using stock from the shopping area. This relates to the Storage Location Assignment Problem (*SLAP*), which assigns incoming products to storage locations within storage departments/zones (Gu, Goetschalckx, & McGinnis, 2007), and the Correlated Storage Assignment Problem (*CSLAP*) that aims to minimize travel distance (Xiao & Zheng, 2012). While all these works concentrate on warehouse environments, to the best of our knowledge, no published work specifically addresses a DFRASP-like problem in a retail setting.

In terms of the layout, retail facilities are usually split into a customer-facing shopping area and an operationally focused backroom area (Mou, 2022). In most retail stores, inventory for picking is only stored on shelves in the shopping area. The backroom is currently used for buffering incoming pallets or leftovers from replenishment in the backroom due to the limited shelf space in the store area. While warehouses are designed for efficiency, the shopping area is usually designed to maximize revenue (Bianchi-Aguiar, Hübner, Carravilla, & Oliveira, 2021; Ozgormus & Smith, 2020). Limited studies have been conducted on the design of store backrooms. In Bhowmick et al. (2023), a qualitative framework is presented that compares two different store concepts. In one case, the backroom is solely utilized as a buffer for incoming goods and leftovers, while in the other concept, a larger backroom with a reduced shopping area is utilized for order fulfillment. In contrast to our work, the second concept assumes a reduced shopping area, which results in a higher picking effort due to the increased demand that needs to be fulfilled using the online channel.

In Pires, Pratas, Liz, and Amorim (2017), a general qualitative framework is presented that describes the process and decisions involved in the backroom design process for grocery stores. In a sequential interrelated eight step process, the article assignment to a department is done before the sizing of the different departments and the evaluation of the entire backroom system. The authors also highlight and work out the differences and similarities between warehouse and retail backroom operations. Our BAP solves the assignment to discrete storage units and sizing problem in parallel on a micro level. In general, our approach focuses on integrating a forward pick area into the remaining space of the backroom rather than designing the entire backroom.

In Pires, Camanho, and Amorim (2020), a model for sizing different backroom departments that considers picking frequencies is presented. Unlike our approach, which focuses on the tactical/operational level, the work of Pires et al. (2020) deals with the strategic sizing of different backroom areas, adjusting the model of Heragu, Du, Mantel, and Schuur (2005). Although this model could also be applied to existing backroom layouts, the assignment of articles to the department or the choice of discrete storage units is not considered.

The study by Seghezzi et al. (2022) explores the improvement of in-store picking for e-grocery. They present the results of a simulation study that examines the effect of a forward pick area within the backroom on picking time for discrete order picking, order batching, and zone picking policies. The assignment to the forward pick area is done by applying a classical class-based storage policy for the non-food and canned products, which is only dependent on the online demand. For each of the assigned articles, it is assumed that the same space is dedicated to each article of the same category. While their assignment is based on a heuristic approach, the assignment in our work is done by an optimization approach, considering also in-store logistical efforts and the assignment to discrete storage units.

In summary, we identified a research gap regarding the comprehensive assessment of incorporating a forward area into an existing backroom and optimizing article assignments that consider all aspects of in-store logistical efforts, particularly on shopping area operations. To the best of our knowledge, there is no work related to optimizing these backroom assignments in a retail environment. This study aims to fill this gap by examining the logistical effects of backroom assignments.

3. Material flow description and cost model

As omnichannel services like customer order picking from store inventory grow, the importance of efficient in-store logistics processes becomes increasingly critical. Typically, articles are delivered to the store on rainbow pallets or roll cages, and therefore the delivered load units need to be separated into individual articles to be presented to the customer (Kuhn & Sternbeck, 2013). Upon receipt, the articles are transferred to the shopping area for *direct replenishment* of the shelves. The articles are then unpacked from their boxes, which in retail are also called *collo* (plural: *colli*), and except for those in shelf-ready packaging *(SRP)*, each article is placed individually on the shelf (Hübner & Schaal, 2017; Kotzab & Teller, 2005). Leftover or excess stock that does not fit on the shelf must be moved to the backroom for later replenishment

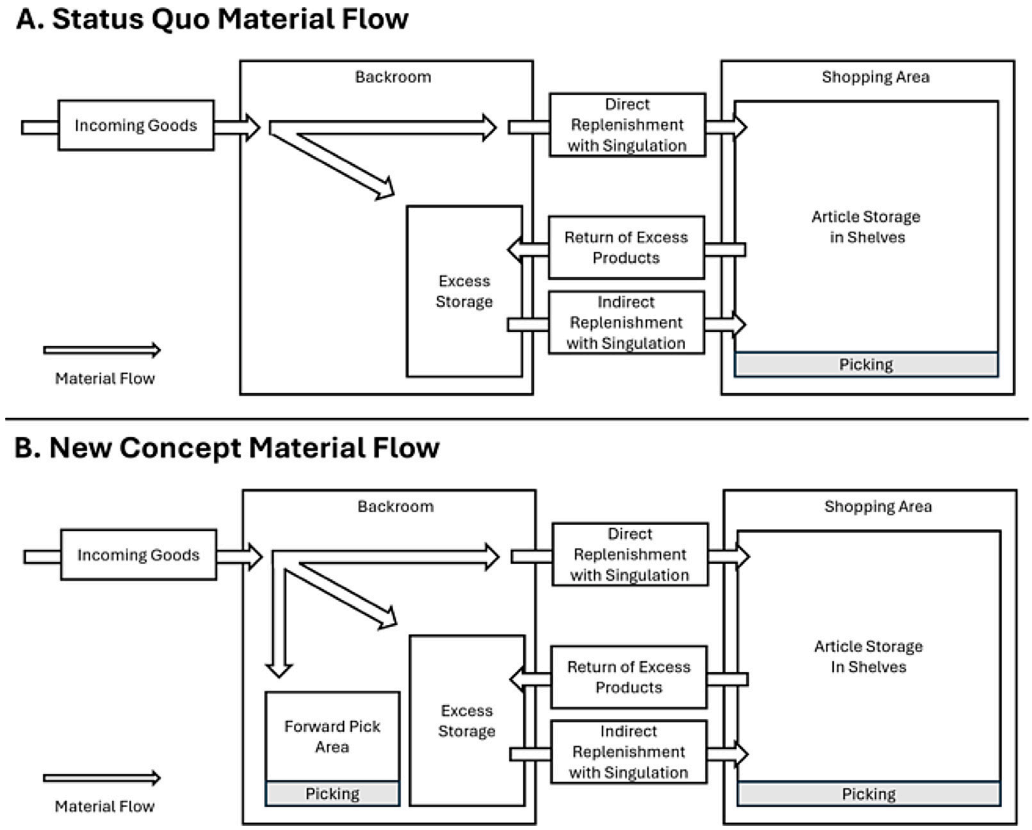

Fig. 1. Material flow comparison with and without a forward pick area.

of leftovers, which is called *indirect replenishment* (Hübner & Schaal, 2017).

The material flow captured by previous literature is shown on top in Fig. 1, where the entire inbound delivery is used to directly stock the shelves in the sales area. After receiving the goods, the replenishment of the shopping area shelves is done by separating the articles from the colli at the article level. Sometimes shelf-ready packaging reduces the effort, but the majority of products are placed on the shelf one at a time. If the shelf capacity does not match the delivered collo sizes, or if the demand until the next delivery arrives is larger than the shelf capacity, handling of leftovers and storage in the backroom are required, which is known as the backroom effect (Eroglu, Williams, & Waller, 2013). Even if inventory monitoring can be supported by RFID (Lin, Shieh, Kao, Chang, & Chen, 2008), handling leftovers is considered wasteful from a lean perspective. Picking from the customer shelf increases the replenishment effort, as more articles need to be placed on the shelves. It also increases the backroom effect because the shelf capacity must meet both uncertain in-store and online demand. In the concept of Bhowmick et al. (2023) and Seghezzi et al. (2022), the conceptual idea is to place articles in the forward pick area of the backroom to speed up the picking process. It is important to note that this allocation occurs in addition to the placement in the shopping area, ensuring that each article remains accessible to in-store customers. This means that in addition to the current standard process where retailers do not order dedicated stock for the backroom (Hübner & Schaal, 2017), additional stock is ordered only for the forward pick area. Therefore, the status quo process in Fig. 1 is adapted in such a way that colli of dedicated SKUs are stored in the backroom to cover the online demand. This approach has two main advantages: first, it minimizes the frequency of shopping area replenishment cycles and the backroom effect, as the shelf capacity is only used to meet in-store demand. Second, it simplifies handling processes because SKUs only need to be singulated when they are destined for the shopping area shelves; colli assigned to the forward pick area can be stored without additional singulation. Depending on the number of SKUs allocated to the backroom, it is possible to meet all or part of online demand from this area. However, there is a potential downside: the number of colli can increase because segregating inventory in a second location reduces the pooling effect. Another disadvantage of this concept is that it requires support by the upstream distribution center (*DC*), as the colli dedicated to the forward pick area must be labeled or sorted in-store.

Since the current literature does not consider a comprehensive quantification of the in-store logistics effort, this paper attempts to fill this gap by quantifying the impact of backroom operations. In doing so, it provides a more holistic view of the operational efficiency within in-store logistics operations. Therefore, it requires quantification of the impact of backroom operations on all in-store order fulfillment and replenishment operations.

3.1. Quantification of backroom assignments at the SKU level

Before quantifying the effect of backroom assignments, we outline key assumptions regarding their impact on in-store logistics. We assume that the demand for each $i \in A$ is uniformly distributed and known over the time horizon, ensuring that all projected demand is met. We assume that in-store demand is fulfilled exclusively from shopping area stock and that backroom quantities cannot be revoked. To model the effect of assigning x_i colli of article i to the forward pick area, the parameters of Table 1 must be introduced.

Using these parameters, the total logistical effort for article i based on the number of colli assigned to the forward pick area x_i, is calculated

Table 1
Overview of store and cost related parameters.

Store related parameters	
l	Time horizon considered in weeks.
$D_{i,\text{online}}$	Online demand of article i during l
$D_{i,\text{instore}}$	In-store demand for article i during l
$D_{i,\text{total}}$	Total demand of article i, $D_{i,\text{total}} = D_{i,\text{online}} + D_{i,\text{instore}}$
O	Average order basket size of an online order
o_i	Average number of articles i per order if article i is ordered
s_i	Number of articles i in a collo
SC_i	Shelf capacity of article i in pieces
Cost related parameters	
$c_{\text{collo, shopping area}}$	Handling costs/time of a collo per movement in the shopping area
$c_{\text{collo, backroom}}$	Handling costs/time per collo per movement in the backroom
$c_{\text{item, shopping area}}$	Handling costs/time per article movement in the shopping area
$c_{\text{item, leftover}}$	Handling costs/time per leftover article for indirect replenishment per article movement
$c_{\text{pick, backroom}}$	Handling costs/time per article pick in the backroom
$c_{\text{pick, shopping}}$	Handling costs/time per article pick in the shopping area
$c_{\text{travel backroom},i}$	Traveling costs/time picking article i in the backroom
$c_{\text{travel shopping},i}$	Traveling costs/time for picking article i in the shopping area (see Appendix A)
$c_{\text{distance shopping, i}}$	Distance from backroom door to article position i in the shopping area

as follows:

$$
\begin{aligned}
C(x_i) = {} & Collo_{i,\text{shopping area}}(x_i) \cdot c_{\text{collo, shopping area}} \quad (1.1) \\
& + Collo_{i,\text{backroom}}(x_i) \cdot c_{\text{collo, backroom}} \quad (1.2) \\
& + H_{i,\text{pick shopping area}}(x_i) \cdot c_{\text{pick, shopping area}} \quad (1.3) \\
& + H_{i,\text{pick backroom}}(x_i) \cdot c_{\text{pick, backroom}} \quad (1.4) \\
& + V_{i,\text{shopping area}}(x_i) \cdot c_{\text{travel shopping},i} \quad (1.5) \\
& + V_{i,\text{backroom}}(x_i) \cdot c_{\text{travel backroom},i} \quad (1.6) \\
& + H_{i,\text{shelf}}(x_i) \cdot c_{\text{item, shopping area}} \quad (1.7) \\
& + R_i(x_i) \cdot c_{\text{distance shopping},i} \quad (1.8) \\
& + 2 \cdot L_i(x_i) \cdot R_i(x_i) \cdot c_{\text{item, leftover}} \quad (1.9)
\end{aligned} \tag{1}
$$

Equations 1.1 and 1.2 represent the handling of colli for both in-store and online sales.The subsequent equations, 1.3 and 1.4, outline the effort involved in picking from the shopping area and the backroom, respectively. Equations 1.5 and 1.6 represent the travel effort related to order picking. Equations 1.7 and 1.8 model the direct replenishment effort, while finally, equation 1.9 describes the indirect replenishment effort. Note that indirect replenishment requires at least two handlings on article level. As illustrated in Fig. 1, adding additional storage units for online orders in the backroom affects various in-store logistical processes at multiple levels. Assignments in the backroom impact the frequency of both direct and indirect replenishment cycles, as well as the handling of boxes and articles meant for in-store and online customers. The various components of $C(x_i)$ can be calculated using:

Collo Handling:

$$
Collo_{i,\text{shopping area}}(x_i) = \begin{cases} \left\lceil \frac{D_{i,\text{instore}} + D_{i,\text{online}} - s_i \cdot x_i}{s_i} \right\rceil & \text{if } x_i \cdot s_i < D_{i,\text{online}} \\ \left\lceil \frac{D_{i,\text{instore}}}{s_i} \right\rceil & \text{otherwise} \end{cases} \tag{2}
$$

$$
Collo_{i,\text{backroom}}(x_i) = x_i \tag{3}
$$

The handling of colli is divided into two parts. Allocating colli to the backroom can reduce the total quantity needed to cover the in-store demand in the shopping area, as shown in Eq. (2) for the shopping area and Eq. (3) for the backroom. Notice that the total number of colli may increase since demand pooling is no longer possible.

Handling for picking on article level to cover the online demand:

$$
H_{i,\text{pick shopping area}}(x_i) = \begin{cases} D_{i,\text{online}} - x_i \cdot s_i & \text{if } x_i \cdot s_i < D_{i,\text{online}} \\ 0 & \text{otherwise} \end{cases} \tag{4}
$$

$$
H_{i,\text{pick backroom}}(x_i) = \begin{cases} x_i \cdot s_i & \text{if } x_i \cdot s_i < D_{i,\text{online}} \\ D_{i,\text{online}} & \text{otherwise} \end{cases} \tag{5}
$$

While Eq. (4) describes the picking effort in the shopping area, Eq. (5) does the opposite for the backroom. Depending on the assigned quantity, the online demand has to be picked either from the backroom, if assigned, or from the shelves in the shopping area.

Number of pick location visits:

$$
V_{i,\text{shopping area}}(x_i) = \begin{cases} \left\lceil \frac{D_{i,\text{online}} - s_i \cdot x_i}{o_i} \right\rceil & \text{if } x_i \cdot s_i < D_{i,\text{online}} \\ 0 & \text{otherwise} \end{cases} \tag{6}
$$

$$
V_{i,\text{backroom}}(x_i) = \begin{cases} \left\lceil \frac{s_i \cdot x_i}{o_i} \right\rceil & \text{if } x_i \cdot s_i < D_{i,\text{online}} \\ \left\lceil \frac{D_{i,\text{online}}}{o_i} \right\rceil & \text{otherwise} \end{cases} \tag{7}
$$

In the context of picking operations, assigning colli to the backroom effectively reduces the frequency of visits to article locations in the shopping area. Therefore, Eq. (6) determines the number of visits for picking in the shopping area, while Eq. (7) models the same for the backroom.

Direct replenishment effort:

Retailers predominantly use an (R,s,nQ) policy to replenish non-perishable articles from DCs, influenced by established delivery patterns (Sternbeck & Kuhn, 2014). From a store perspective, an (s,S) policy for shelf restocking can prevent lost sales. However, due to the fixed lot sizes from the DCs, restocking can only be done in integer multiples of these sizes. Additionally, restocking occurs at set time intervals for efficiency, leading to either incomplete shelf replenishment or excess inventory, which arises from a mismatch between the available shelf space and the lot sizes of the products received from the DCs. These inefficiencies increase handling costs (Eroglu et al., 2013). To calculate the minimum number of replenishment cycles needed, we apply a $(0, SC_i)$ policy. This policy calculates the minimum number of replenishment cycles needed over a period l, based on x_i. This version of a (s, S) policy forces the system to replenish the entire shelf capacity if the stock is totally consumed, resulting in a minimum number of replenishment cycles.

$$
R_i(x_i) = \begin{cases} \left\lfloor \frac{D_{i,\text{instore}} + D_{i,\text{online}} - s_i \cdot x_i}{SC_i} \right\rfloor & \text{if } x_i \cdot s_i < D_{i,\text{online}} \\ \left\lfloor \frac{D_{i,\text{instore}}}{SC_i} \right\rfloor & \text{otherwise} \end{cases} \tag{8}
$$

$$
H_{i,\text{shelf}}(x_i) = \begin{cases} D_{i,total} - s_i \cdot x_i & \text{if } x_i \cdot s_i < D_{i,online} \\ D_{i,instore} & \text{otherwise} \end{cases} \tag{9}
$$

The minimum number of replenishment cycles is described by Eq. (8), while Eq. (9) quantifies the total replenishment effort on article level during l.

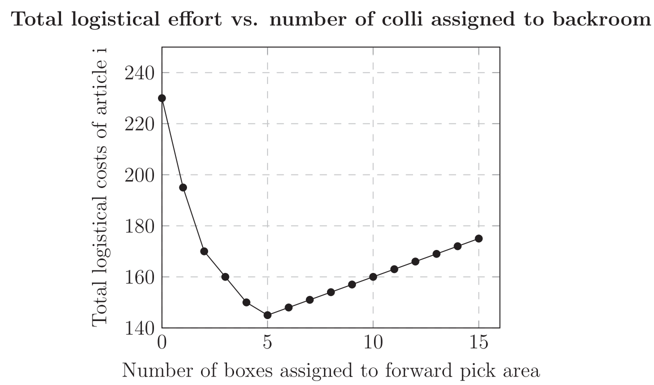

Fig. 2. Numerical example for $C(x_i)$ in general cost units.

Shopping area indirect replenishment effort:
Using our assumption that the total demand of an article is equally distributed during l we can use formula (10) to calculate the average demand that needs to be covered per replenishment cycle:

$$D_{i,R} = \begin{cases} \left\lceil \frac{D_{i,\text{instore}}+D_{i,\text{online}}-s_i \cdot x_i - SC_i}{R_i(x_i)} \right\rceil & \text{if } R_i(x_i) > 0 \text{ and } x_i \cdot s_i \leq D_{\text{instore}} \\ \left\lceil \frac{D_{i,\text{instore}}-SC_i}{R_i(x_i)} \right\rceil & \text{if } R_i(x_i) > 0 \text{ and } D_{\text{instore}} < x_i \cdot s_i \\ 0 & \text{if } R_i(x_i) = 0 \end{cases} \tag{10}$$

Given the fixed sizes of colli, we can calculate the leftovers per replenishment cycle that occur due to the backroom effect by using:

$$L_i(x_i) = \begin{cases} \left\lceil \frac{D_{i,R}}{s_i} \right\rceil \cdot s_i - D_{i,R}, & \text{if } R_i(x_i) > 0 \\ 0, & \text{else} \end{cases} \tag{11}$$

Due to rounding functions, $C(x_i)$ does not have a simple closed-form. As a result, we must use numerical methods to calculate the function values of $C(x_i)$ for each value of x_i. Since only integer values of x_i are relevant and the demand during a l is finite, we can model $C(x_i)$ as a piecewise linear function, demonstrated in a specific example in Fig. 2. Modeling $C(x_i)$ as a piecewise linear function allows for its effective integration into an optimization model. Notice that the number of segments in the optimization can be reduced to a set of breakpoints where the slope of $C(x_i)$ changes as the other (integer) points will lie on the linear segments between these breakpoints.

4. The backroom assignment model

Due to space constraints in the backroom, only a subset of products can be assigned to the forward pick area. Therefore, the assignment model has to make three major decisions. First, decide which articles and quantities shall be assigned to the forward pick area. Second, store each assigned article in appropriate storage units (SUs), as it must ensure that, for example, refrigerated products can only be stored in a refrigerator or other cooling device. Third, the storage requirements of different policies are considered, such as random storage within a suitable storage unit type or the application of a dedicated storage policy.

While offering flexibility, random storage requires technological support to ensure that articles can be located quickly, thereby increasing retrieval efficiency. Conversely, dedicated storage eliminates the need for such technological support, simplifying operations. However, it may reduce allocation efficiency due to a reduced pooling effect. Generally, a random storage policy requires technical support, leading to additional investment compared to a dedicated storage policy, which can be structured to operate without technical support systems. Comparing the outcomes of both policies can help determine if the investment in technical support systems is justified.

Before introducing an assignment model for both the random and dedicated storage policy, we will delve into the modeling of the backroom layout, as this is fundamental to the assignment model.

4.1. Backroom layout modeling approach

Retail backrooms serve three primary purposes: temporarily holding goods received from larger warehouses, storing leftover stock from direct replenishment, and managing leftover packaging materials. Usually, these backrooms are divided into different zones, each focused on a particular task (Pires et al., 2017; Seghezzi et al., 2022). As our study focuses on how to effectively add a 'forward picking area' to the current backroom layout, open spaces in particular are relevant, as shown in Fig. 3.

Each potential storage position is defined with an access point for article retrieval, which must be determined either manually or through solving a facility layout problem (Hosseini-Nasab, Fereidouni, Fatemi Ghomi, & Fakhrzad, 2018). We define the set of backroom storage locations $K = k_1, k_2, k_3 \ldots$ to construct a graph $G = (V, E)$. The vertex set V comprises K and other critical zones like the order preparation area and the door to the shopping area, which influence the choice of storage locations. Edges and their weights are established by calculating distances between these points, resulting in a shortest distance matrix $\tilde{D}$ that records the shortest paths within the graph. This matrix typically has a size of at least $(K+2)X(K+2)$ to include distances to both the order preparation zone and the backroom door. In scenarios with homogeneous storage units, detailed distinctions may not be necessary. However, with heterogeneous units, it is essential to define *storage space types* as $Tspace = 1, 2, 3, \ldots, ts$, evaluating compatibility based on factors like the presence of electrical outlets.

$$\tilde{D} = \begin{pmatrix} 0 & \min(d_{12}) & \cdots & \min(d_{1n}) \\ \min(d_{21}) & 0 & \cdots & \min(d_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ \min(d_{n1}) & \min(d_{n2}) & \cdots & 0 \end{pmatrix}$$

4.2. Problem formulation and solution approaches

Before formulating the problem for both random and dedicated policy scenarios, we introduce the multi-period capacity blocking approach. Adopting a multi-period time horizon l allows for replenishment strategies within the forward pick area, avoiding the need to transform our model into a quadratic integer programming problem, which is significantly more complex to solve. The capacity occupied by the storage unit j over the time horizon l is calculated as follows:

$$V(x_{i,j}) = \begin{cases} v_i & \text{if } x_{i,j} < r_l, \\ \left\lceil \frac{x_{i,j}}{r_l} \right\rceil \cdot v_i & \text{otherwise.} \end{cases}$$

The fundamental principle of this approach is that the number of replenishment cycles of the forward pick area is limited to r_l during l. Therefore, if the number of boxes allocated is less than r_l, it suffices to allocate space for one collo v_i. Conversely, if the allocation exceeds the number of periods, additional space must be reserved to accommodate the increased capacity demands. This condition, applicable when the allocated boxes surpass the number of replenishment cycles, ensures that the capacity of the storage unit is not exceeded. Furthermore, the model operates under the following assumptions:

- Each article type represents a set of articles with similar characteristics and storage requirements
- The demand is equally distributed for each period over the time period l for each article

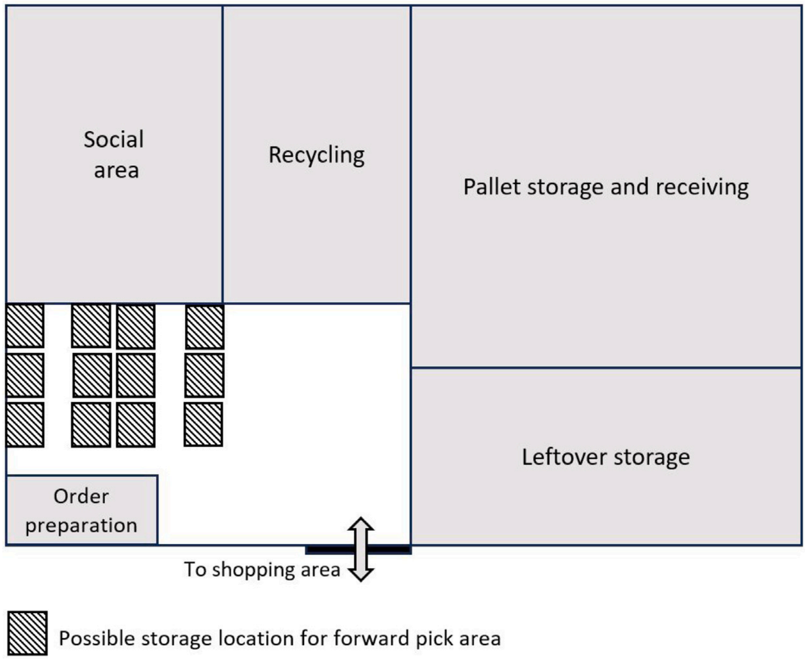

Fig. 3. Backroom layout with possible storage locations divided into four zones.

- An increase in the online order ratio affects only the fulfillment channel and does not alter the demand probability of an article
- Each storage unit requires the same amount of backroom space and capacity
- The delivery patterns are not taken into account
- A predefined set of potential storage locations is provided as input

Table 2 describes all relevant sets, variables, functions, and parameters of the assignment model. We use them to define two backroom assignment problems. On the one hand, we provide a model for a random storage policy where articles can be stored in each feasible SU. On the other hand, in the dedicated BAP, each SU can only host one article type.

4.2.1. Assignment model with random storage policy

The assignment model employs a random storage policy, permitting articles to be stored in any feasible storage unit (SU) without type restrictions. It is formulated as a Mixed Integer Program (MIP).

$$\text{Min}\left(\sum_{i\in A} C_i\left(\sum_{j\in S} x_{i,j}\right) + \sum_{j\in S} y_j \cdot k_j\right)$$

Subject to:

$$\sum_{j\in S} y_j \leq C \tag{12}$$

$$\sum_{j\in S: s_{st,j}=sp} y_j \leq max_{sp} \qquad \forall sp \in T_{space}, \tag{13}$$

$$\sum_{i\in A} V(x_{i,j}) \leq c_j \qquad \forall j \in S \tag{14}$$

$$\sum_{i\in A} x_{i,j} \leq y_j \cdot M \qquad \forall j \in S \tag{15}$$

$$\sum_{i\in A} x_{i,j} \geq min_j \cdot y_j \qquad \forall j \in S \tag{16}$$

$$x_{i,j} = 0 \qquad \text{if } t_i \neq s_j, \quad \forall i \in A, \forall j \in S \tag{17}$$

$$\sum_{j\in S: s_j=t} y_j \leq \max_t \qquad \forall t \in T_s \tag{18}$$

$$\sum_{j\in S: s_j=t} y_j \geq \min_t \qquad \forall t \in T_s \tag{19}$$

$$\sum_{j\in S} \frac{x_{i,j} \cdot s_i}{r_l} \leq \frac{D_{i,\text{online}} \cdot r_{per}}{l} \qquad \text{if } p_i = 1, \quad \forall i \in A \tag{20}$$

$$\sum_{j\in S} x_{i,j} \cdot s_i \leq (1 + f_{upper}) \cdot D_{i,\text{online}} \qquad \text{if } p_i = 0, \quad \forall i \in A \tag{21}$$

$$\sum_{j\in S} x_{i,j} \cdot s_i \geq \left(f_{lower} \cdot D_{i,\text{online}}\right) \cdot \max_{j\in\{1,\ldots,s\}} z_{i,j} \qquad \forall i \in A, \forall j \in S \tag{22}$$

$$x_{i,j} = 0 \qquad \text{if } D_{i,\text{online}} \leq min_a, \quad \forall i \in A, \forall j \in S \tag{23}$$

$$x_{i,j} \leq M \cdot z_{i,j} \qquad \forall i \in A, \forall j \in S \tag{24}$$

$$x_{i,j} \geq z_{i,j} \qquad \forall i \in A, \forall j \in S \tag{25}$$

$$\sum_{i\in A} z_{i,j} \leq \max_j \qquad \forall j \in S \tag{26}$$

$$x_{i,j} \in \mathbb{N}_0 \qquad \forall i \in A, \forall j \in S \tag{27}$$

$$y_j \in \{0,1\} \qquad \forall j \in S \tag{28}$$

$$z_{i,j} \in \{0,1\} \qquad \forall i \in A, \forall j \in S \tag{29}$$

The objective function of our problem uses the analytical model detailed in Section 3.1 to minimize total in-store logistical effort. Besides the operational effort, our model also considers the cost of each utilized storage unit j, thereby minimizing the total logistical cost. Constraint (12) ensures that the total available capacity is not exceeded by the storage units in use, while (13) avoids that the number of storage places per type is exceeded. Constraint (14) guarantees that the capacity of each individual storage unit j is adhered to. Constraints (15) and (16) stipulate that articles are assigned only to storage units that are in use, and ensure a minimum number of colli to be assigned to a storage unit, while Constraint (17) ensures the selection of storage units capable of accommodating article i. Constraints (18) and (19) define the minimum and maximum limits for each type of storage unit utilized. The constraints (20) and (21) are lazy constraints only added when the conditions are met. For each article. they limit the assigned quantity based on whether an article is perishable or not. Perishable articles are required to be consumed by the end of the horizon, while the upper quantity limit for non-perishable articles is determined by r_{per}. Constraints (22) and (23) ensure that only articles that have a minimum number of predicted online sales are considered

Table 2
Consolidated table of sets, variables, functions, and parameters for random and dedicated BAP.

Sets	
Articles	$A = \{1, 2, 3, \dots, a\}$
Article types	$T_a = \{1, 2, 3, \dots, b\}$ (dedicated storage policy)
Storage units	$S = \{1, 2, 3, \dots, s\}$
Storage unit types	$T_s = \{1, 2, 3, \dots, t\}$
Storage place types	$T_{space} = \{1, 2, 3, \dots, t_s\}$
Variables	
$x_{i,j}$	Number of collo/colli of article i assigned to in storage unit j
y_j	1 if storage unit j is used
$z_{i,j}$	1 if article i is assigned to storage unit j
Functions	
$C_i(x_{i,j})$	Total in-store logistical effort from Section 3.1
$V(x_{i,j})$	Capacity function from Section 4.2
Parameters	
a_i	Article type of article i with $a_i \in T_a$
c_j	Capacity for article storage of storage unit j
C	Available storage place capacity in the backroom
f_{upper}	Upper bound of assigned qty in % of $D_{i,\text{online}}$
f_{lower}	Lower bound of assigned qty in % of $D_{i,\text{online}}$.
k_j	Cost of storage unit j
l_j	Limit of number of article types in storage unit j
max_j	Maximum number of different article i to be stored in j
max_t	Maximum number of storage units per storage unit type
max_{sp}	Maximum number of storage places sp per space type (heterogeneous SU)
min_a	Minimum online demand assigned to the backroom per article
min_j	Minimum number of colli that shall be stored in j if used
min_t	Minimum number of storage units per storage type
M	A sufficient big number
p_i	1 if article i is perishable, 0 otherwise
r_l	Number of replenishment cycles of the forward pick area during l
r_{per}	Durability in time periods for article i (if perishable)
s_i	Collo size of article i in pieces
s_j	Type of storage unit j
$s_{st,j}$	Storage place type of storage unit j
t_i	Suitable storage type of j for article i
l	Time horizon in weeks
v_i	Volume of the collo of article i

for the optimization.

Constraints (24) and (25) establish the values for the auxiliary variable $z_{i,j}$. Constraint (26) limits the number of articles assigned to a storage unit to ensure clearly structured storage units. Constraints (27) to (29) define the limits for the decision variables, including the introduction of a new constraint for the binary decision variables associated with article types and storage units, reflecting the type restrictions in the model.

Since for large instances of this multi-knapsack-like problem the direct solution via an optimizer is either time consuming or only feasible with a potentially lower solution quality, we propose a solution approach that aims to improve either the computation time and/or the solution quality. Since the set of articles cannot be split into independent problems, we reduce the number of relevant articles by solving the problem without Constraint (26) for a large synthetic SU that has a 50% higher capacity than the sum of the storage units per type, which is suitable for all article types as shown in Fig. 4. As we want to do a pre-selection of articles, this virtual synthetic SU comes without any storage unit cost. We then use this set of pre-selected articles to solve the problem with a lower number of articles and additional type restrictions per SU and the actual capacity of each storage unit. This approach reduces the possible set of articles in the first stage. The effect of the decomposition approach is discussed in detail in Section 5.3 by comparing the MIP gap of the proposed decomposition approach with the MIP gap of a direct solution attempt with a limited amount of computation time.

Table 3
Consolidated description of sets, variables and parameters for the calculation of $\tilde{S}_{\text{opt}}$.

Symbol	Description/Definition
Sets	
T_a	Set of article types for all S_{part}
T_s	Set of storage unit types for all S_{part}
S_{part}	Set of partial solutions
Variables	
x_{spart}	1 if s_{part} is part of $\tilde{S}_{\text{opt}}$, 0 otherwise
Parameters	
$a_{s_{\text{part}}}$	The specific article of $s_{\text{part}} \in S_{\text{part}}$
C	The total capacity available
c_T	Capacity per storage unit type T available
$t_{s_{\text{part}}}$	The storage unit type of $s_{\text{part}} \in S_{\text{part}}$
$u_{s_{\text{part}}}$	The savings realized by $s_{\text{part}} \in S_{\text{part}}$
$w_{s_{\text{part}}}$	Number of used storage unit spaces of s_{part}

4.2.2. *The assignment model with dedicated storage policy*

To address the increased complexity caused by additional type restrictions in a dedicated storage policy, we use a dynamic programming approach based on the Bellman optimality principle. This approach allows us to decompose the overall problem into smaller, manageable subproblems while maintaining optimality. The general idea is to optimize the article assignment to storage units for each article type in the first step and then combine all the optimized subproblems so that the overall configuration of the forward pick area is optimal.

Our process, illustrated in Fig. 5, begins by determining a set of all viable SU combinations S_f for each article type $b \in T_a$. We define tuples $\langle s_f, A_b \rangle$, with $s_f \in S_f$ representing feasible combinations for article types b and their corresponding set of articles A_b. To avoid solving unnecessary instances, which become difficult to compute especially as the number of SUs increases, we initially assign a minimal number of SUs, $su_{a,\min}$, per article type to create an initial solution for each tuple $\langle s_f, A_b \rangle$ where $s_f \le su_{a,\min}$. We use these defined tuples to directly solve each subproblem using the mathematical problem formulation described in Section 4.2.1.

After calculating each initial partial solution $s_{\text{part,a}} \in S_{\text{part}}$, we proceed to define a knapsack-like problem to construct a current optimal solution $\tilde{S}_{\text{opt}}$ using the following problem formulation with parameters and variables of Table 3:

$$\max \sum_{s_{\text{part}} \in S_{\text{part}}} u_{s_{\text{part}}} \cdot x_{s_{\text{part}}}$$

Subject to:

$$\sum_{s_{\text{part}} \in S_{\text{part}}:\ a_{s_{\text{part}}} = t_a} x_{s_{\text{part}}} \le 1, \qquad \forall t_a \in T_a, \tag{30}$$

$$\sum_{s_{\text{part}} \in S_{\text{part}}} w_{s_{\text{part}}} \cdot x_{s_{\text{part}}} \le C, \tag{31}$$

$$\sum_{s_{\text{part}} \in S_{\text{part}}:\ t_{s_{part}} = b} w_{s_{\text{part}}} \cdot x_{s_{\text{part}}} \le c_T, \quad \forall b \in T_s, \tag{32}$$

$$x_{s_{\text{part}}} \in \{0, 1\}, \qquad \forall s_{\text{part}} \in S_{\text{part}}. \tag{33}$$

The objective function maximizes the savings realized if a partial solution is assigned to be part of $\tilde{S}_{\text{opt}}$. Constraint (30) ensures that for each article type only one partial solution is used. The following constraints (31) and (32) ensure that the total capacity and the capacity per SU type are not exceeded. Finally, Constraint (33) defines the value range of the binary decision variable. Note that although this problem

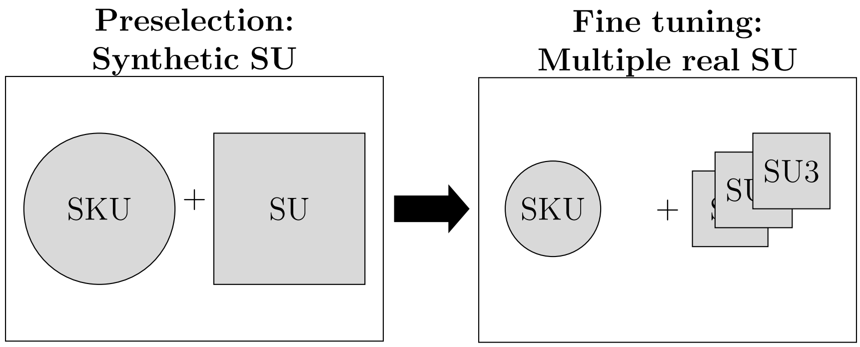

Fig. 4. Solution approach for a random storage policy using a pre-elemination phase.

is a variant of the NP-hard 0–1 knapsack problem, the limited number of article types and feasible combinations of SUs in real-world scenarios should still be optimally solvable.

After calculating the initial $\tilde{S}_{\text{opt}}$, we solve further subproblems by increasing $su_{a,\min}$. If a better solution is found, $\tilde{S}_{\text{opt}}$ has to be updated. Otherwise, the larger instance is not part of the global optimal solution. To speed up the calculation, we can use the partial solutions of $\tilde{S}_{\text{opt}}$ to define a stopping criterion. The idea is that the current calculated instance must at least outperform the current worst setting $s_{\text{worse}} \in \tilde{S}_{\text{opt}}$ which uses the same capacity as the current instance. Otherwise, according to Bellman's equation, it cannot be part of the optimal solution. There are two cases to consider:

1. The article type is already part of $\tilde{S}_{\text{opt}}$. Since each article type can occur only once, we combine its current solution with the worst partial solutions of other article types of $\tilde{S}_{\text{opt}}$ in increasing order until the same capacity is used.
2. The article type is not part of $\tilde{S}_{\text{opt}}$. We compare this solution with the combination of the worst partial solutions of $\tilde{S}_{\text{opt}}$ with the same capacity consumption.

During the calculation, we can use the sum of the objective values of S_{worse} as a stopping criterion and compare this using a progress caller with the dual problem's lower bound, lb_{dual}. If the lb_{dual} of the current instance exceeds the value of S_{worse}, the calculation can be terminated as this current instance will not improve $\tilde{S}_{\text{opt}}$. We continue as long as there are remaining instances of $\langle s_f, A_b \rangle$ to consider. If the algorithm stops, the optimal solution S_{opt} has been found.

Lemma: The set S_{opt} must consist of optimal partial solutions $s_{\text{part,a}}$.
Proof: If S_{opt} did not consist of optimal partial solutions $s_{\text{part,a}}$, we could improve S_{opt} by enhancing one of its partial solutions. This would imply that S_{opt} is not the optimal solution. Therefore, S_{opt} inherently includes the best possible configurations of $s_{\text{part,a}}$ according to Bellman's equation.

5. Real world use case

In this section, we present the results of a real data use case from a European drugstore chain where our proposed BAP models were applied. In 5.1, a detailed description of the use case and the experimental design (DOE) is given. Section 5.2 presents the results of our DOE and discusses the quality of the instances computed and the performance of the decomposition approaches in 5.3.

5.1. Use case description and design of experiments

The case study involves a European drugstore chain that provided real-world data, including shelf capacities, colli sizes and volumes, store layouts, and demand data. Notice that all stores use a free-form layout (for details, see Appendix B). We analyzed the effect of backroom assignments for three different stores:

- **Store A:** A typical urban store with bi-weekly deliveries and a monthly turnover ratio of 0.65 times its shelf capacity.
- **Store B:** A store with 50% higher demand than Store A, three deliveries per week, and a monthly turnover ratio of 0.94.
- **Store C:** A store with extremely high demand, delivering five times a week, a 300% increase in demand over Store A, and a monthly turnover ratio of 2.2.

We modeled different online versus in-store demand ratios using real in-store customer purchase probabilities over a two-month period, setting a time horizon of eight weeks. We assumed that demand patterns would remain consistent if in-store shoppers transitioned to online. The data included between 5.000 and 6.000 unique articles delivered in parcels on rainbow pallets, with the exception of specific categories such as cosmetics, which are delivered in mixed boxes in smaller units. Our study focused on the use of multi-level rolling carts for storage, ensuring compatibility with existing store operations and the non-perishable nature of the articles over the study period. It was assumed that 75% of the total storage volume of an SU could be used for backroom storage and still provide adequate handling. The model is allowed to accommodate a maximum up to $f_{upper} = 120\%$ of the anticipated online demand in the backroom for each article to consider the heterogeneous sizes of the colli. Under the implemented dedicated storage policy, each storage unit is constrained to contain only one article type. The store's assortment comprises between 45 and 55 article types. On average, each article type consists of 97.3 to 115 articles, and the sample standard deviation ranges from 107.4 to 131.2, highlighting the uneven distribution of articles. We also allow our model to assign $max_j = 50$ different articles to each SU. Given the large number of stores and the need for dynamic assortment to generate results quickly across many stores, computational limits have been established:

- For the random policy, computational time was restricted to 1 h for the pre-elimination per online ratio and 3 h for the detailed calculation per instance.
- For the direct random approach without decomposition, a computational time of 4 h per instance was considered.
- For dedicated policy cases, each partial solution computation was limited to 0.5 h.

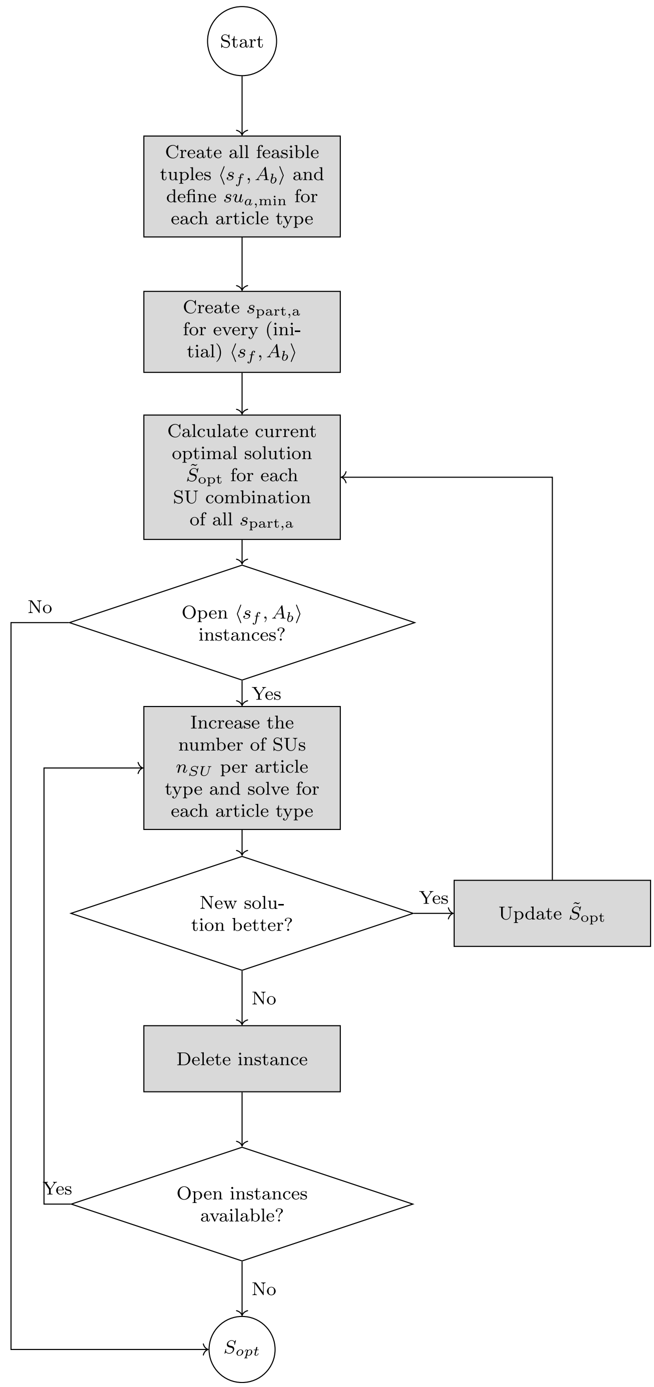

Fig. 5. Solution approach using the bellman optimality theorem for the dedicated policy case.

For both approaches, a MIP Gap of 0.05% is considered optimal. Utilizing this framework, we defined the following experimental parameters using a full factorial Design of Experiment of Table 4 with the parameter values of Table 5:

These time values represent the time required for each specific process and are based on expertise values from industry. Notably, the difference in time between backroom and shopping area activities is due to the additional effort involved in identifying article positions and managing customer interactions. For collo handling, we considered 25 s in the shopping area, which includes the identification of the article position and the opening and recycling of the collo, and 15 s for the backroom. Direct replenishment takes 2 s per article, while indirect replenishment involves 2.5 s per handling. Picking an article in the shopping area requires 20 s, whereas in the backroom it takes 10 s, reflecting the greater efficiency of backroom processes. Travel speed is also considered, with a speed of 1 m/s in the backroom and 0.5 m/s in the shopping area, accounting for the effects of customer congestion. For the travel time savings, we used the approximation from Appendix A. In the baseline scenario, we examine the impact of backroom assignments compared to a no backroom scenario. Furthermore, we seek to analyze the implications of various backroom capacities in response to an increase in the online demand ratio, necessitating more in-store order fulfillment to analyze the effect on the total in-store logistical effort in comparison with a no-backroom scenario. We divide our DOE into two different scenarios:

Table 4
Experimental factors and their respective values in the Design of Experiment (DOE).

Factor [values]
Time horizon l: [2 month]
Number of rolling carts n_{SU}: [3, 6, 9, 12]
Online demand ratio r_{online}: [5%, 10%, 20%, 30%, 40% 50%]
Backroom replenishment cycles r_l: [weekly]
Storage Policy: [random, dedicated]
Store: [A, B, C]

Table 5
Parameter values for in-store processes (in seconds).

Parameter	Value (s)
$c_{\text{collo, shopping area}}$	25
$c_{\text{collo, backroom}}$	15
$c_{\text{item, shopping area}}$	2
$c_{\text{item, leftover}}$	2.5
$c_{\text{pick, backroom}}$	10
$c_{\text{pick, shopping area}}$	20
$c_{\text{travel shopping area},i}$	2
$c_{\text{travel backroom},i}$	1

- **Baseline online ratio**: Varying n_{SU} with $r_{online} = 0.05$
- **Increased online ratio**: Modify the $r_{online} = [0.1, 0.2, 0.3, 0.4, 0.5]$ to anticipate future demand scenarios with increasing online demand for both the type restricted and unrestricted case.

5.2. Numerical results

In the baseline analysis for the largest backroom configuration ($n_{SU} = 12$), the effects of backroom assignments with a random storage policy are shown in Fig. 6. Our preliminary findings reveal modest savings; for Store A, these are slightly below the 1% mark, whereas Store B achieves savings marginally above 1%. These results imply that only a limited selection of high-demand articles qualify for backroom storage, and therefore the effect on the total operational effort is limited. Conversely, the demand profile of Store C allows our model to realize savings of approximately 4.5% in in-store logistical effort, thus underscoring the significance of backroom assignments for Store C even within the baseline scenario.

Following our assessment of backroom usage under current conditions, we explore the potential impact of increasing online demand ratios – up to 50% – for Stores A, B, and C in the future scenarios shown in Figs. 7, 8, and 9.

Savings for Store A peak at 11% when the online ratio reaches 0.4, but decline as it approaches 0.5. This decrease is related to the marginal increase in additional volume that can be allocated to the backroom due to the already high utilization of SU capacity. Consequently, the benefits of a higher online ratio, which typically broadens the range

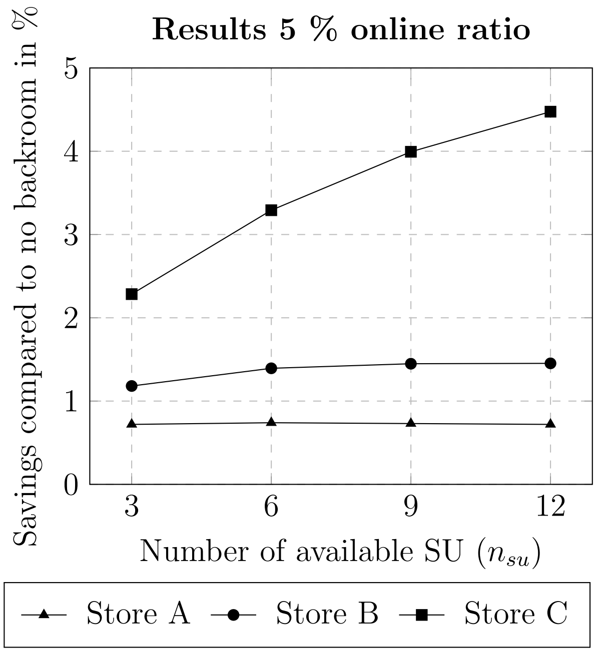

Fig. 6. Comparative analysis of in-store logistical savings for a random policy across stores versus a no-backroom baseline.

and quantities of articles suitable for backroom assignment, are counteracted by the increasing effort needed to fulfill online orders. The data also indicates that a random policy boosts savings by an average of approximately 2%. For Store B, savings of up to 16% are achievable. The data indicates that the volume used increases only marginally between an online ratio of 0.4 and 0.5. In this case, the model is able to compensate for the additional effort of more online demand. The data also shows that a random policy outperforms on average the type restricted policy also by around 2%. For Store C, savings of up to 11% are achievable. The high demand, even for small online ratios, amplifies the effect of having a backroom, in contrast to the other two stores. Investigating the results of the random policy case reveals the influence of increased complexity due to higher demand, leading to more breakpoints in our cost function. Consequently, the addition of more storage units does not automatically translate into greater savings. This can be explained by the solution quality of our time restricted instances as with the higher demand of Store C, the size of each problem instance increases due to more breakpoints in both $C_i(x_{i,j})$ and $V(x_{i,j})$, which will be discussed in detail in Section 5.3. Analyzing the total in-store logistical efforts over a two-month period, as illustrated in Fig. 10, demonstrates that backroom assignments can significantly reduce the time effort. However, it is also apparent that for all three stores, an increase in the online ratio escalates the required effort for order fulfillment during l. Specifically, a 10% rise in the online ratio leads to a linear additional 143.08 h ($R^2 = 1$) of effort in the scenario without backroom use, compared to an increase of only 113.9 h ($R^2 = 0.9985$) when backroom assignments are utilized. For Store B, these figures are 172.5 h ($R^2 = 1$) without backroom configurations and 123.5 h ($R^2 = 0.9983$) with the largest feasible backroom setup. Similarly, Store C exhibits an increase of 427.6 h ($R^2 = 1$) without backroom assignments and 350 h ($R^2 = 0.9913$) with backroom usage. Using this results a increase of 10% in online ratio increase the in-store logistical effort without backroom by around 31% (A), 26% (B) and 32% (C) while in a backroom scenario the effort increases by around 25% (A), 17% (B) and 24% (C). The results indicate that, for the given use case, only the largest store (Store C) significantly benefits from a forward pick area under the current conditions. However, if the online ratio increases, all stores would benefit from a forward pick area, even with a relatively low number of storage units. Comparing the logistical hours required to fulfill all tasks, the results show a linear correlation, leading to significant savings in operational effort.

5.3. Algorithmic performance evaluation

To assess the quality of solutions for each store and instance, we use the MIP-Gap as the primary performance indicator and create boxplots for all instances of the dedicated and random storage policy for the three stores of our use case. Fig. 11 depicts the solution quality for the dedicated policy case. Notably, this problem can be decomposed into smaller subproblems without any loss of optimality. The findings reveal, that for each store, the average solution quality of all subproblems $s_{part} \in S_{part}$ used in the instance solution is around 0.05%, which was defined as optimal for the problem instances. The plot indicates that the decomposition approach efficiently solves the BAP while maintaining high solution quality for each instance across the three stores. For Store A and Store B, even the upper whisker is smaller than 0.5%, and the worst partial solution of the global solution had a MIP-Gap of around 1%. The approach has shown its suitability for solving the BAP using a dedicated storage policy, even for large instances, within a reasonable amount of time while maintaining a good solution quality for these discrete real world problems.

Comparing the solution quality of the random policy case, we observe that the non-optimal decomposition approach surpasses the direct solution strategy where the whole problem is solved with a preselection phase for both Store A and Store B. For Store A, the decomposition approach results in an upper whisker of less than 0.5% and an upper quartile around 0.2% of the theoretical achievable MIP-Gap. In contrast, the direct solution approach for Store A shows an upper whisker of approximately 2.5% and an upper quartile of 0.4%.

Similarly, for Store B, the decomposition approach significantly outperforms the direct approach, yielding an upper whisker of about 2% and an upper quartile of 0.37%. In contrast, the direct solution approach yields an upper whisker of 15.6% and an upper quartile of 4.02%. However, as the complexity increases, the results for Store C show that the decomposition method produces results that are almost the same as the direct solution approach, but with more variability. Specifically, while the average MIP gap for Store A was 0.14% and for Store B it was 0.25%, the larger instances of Store C could only be solved with an average MIP gap of about 2%. This explains the shapes of the curves in Fig. 9, especially if an increased number of storage units makes the problem more complex and reduces solution quality within a fixed solution time. Therefore, the dedicated case can be solved with better solution quality.

5.4. Managerial insights

The application of the random and dedicated backroom assignment problem provided valuable managerial insights, offering guidance for shaping future omnichannel strategies.

- Integrating a forward pick area into the existing store infrastructure, even for a low online ratio of 5%, can yield substantial benefits. Although the per-store savings might appear relatively modest, the cumulative effect across the entire network of stores can be significant compared to a net margin of around 2% (FMI, 2024; Ring & Tigert, 2001).
- Offering omnichannel services increases in-store logistical efforts, and integrating a forward pick area can offer significant savings compared to not having a forward pick area in the backroom.
- A shift of 10% towards online channels increases the in-store logistical effort by around 25%–30%.

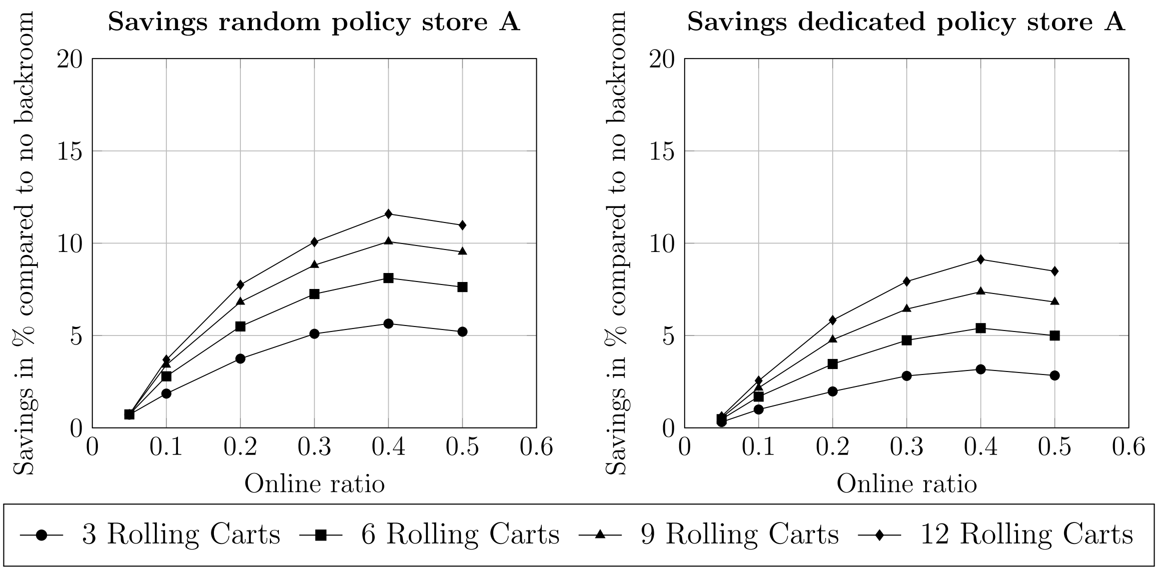

Fig. 7. Comparative analysis of the in-store logistical savings for random and dedicated storage policy store A.

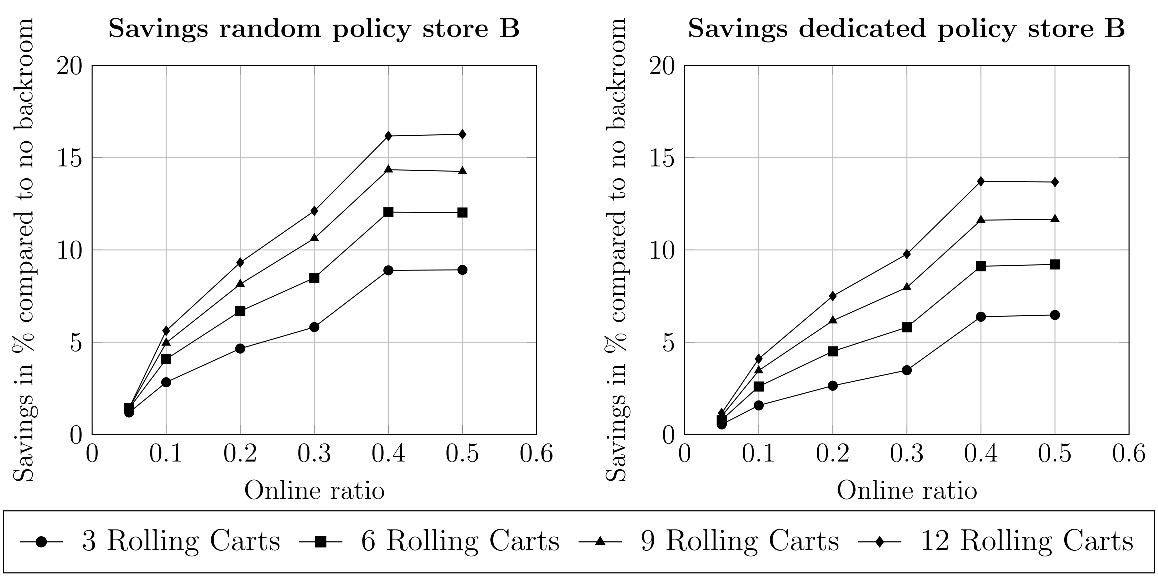

Fig. 8. Comparative analysis of the in-store logistical savings for random and dedicated storage policy store B.

- Implementing a random policy storage system can enhance the efficiency of a forward pick area. Our proposed approach can be used by retailers to analyze the outcomes of both random and dedicated storage strategies, allowing retailers to determine whether the technical investment required for a random storage system is justified by the benefits it offers.
- The BAP formulations and the analytical effort quantification model equip retailers with the tools to accurately evaluate the influence of an increasing online ratio on their in-store logistical efforts, aiding in the evaluation of different omnichannel order fulfillment strategies.

6. Discussion and conclusion

We have introduced a novel backroom assignment model and quantified its effect using real-world data at the SKU level, determining the articles, quantities, and storage units used within a forward pick area in the backroom. We developed two decomposition approaches that generate high-quality solutions in a reasonable computation time for normal or slightly increased store demands, applicable to both random and dedicated storage policies. Our findings demonstrate that backroom assignments can result in significant effort and cost savings, even for relatively low online ratios, regardless of the storage policy employed. However, as online demand increases, so does the in-store logistical effort, even with the use of backrooms. This presents notable challenges for retailers.

Using the proposed model formulations, retailers can quantitatively evaluate the cost-effectiveness of investing in advanced technical support systems to implement a random storage policy. Although our analysis is based on a small sample of three stores, the results suggest that backroom assignments can play a crucial role in omnichannel order fulfillment. It is important to note that our model relies on certain assumptions. While it currently assumes a known static demand without uncertainty, there is potential for employing fuzzy or robust optimization techniques to handle uncertainties of the environment. Our decomposition approaches perform well, particularly in managing large-scale problem sizes. However, further refinement through advanced decomposition techniques or the integration of meta-heuristic

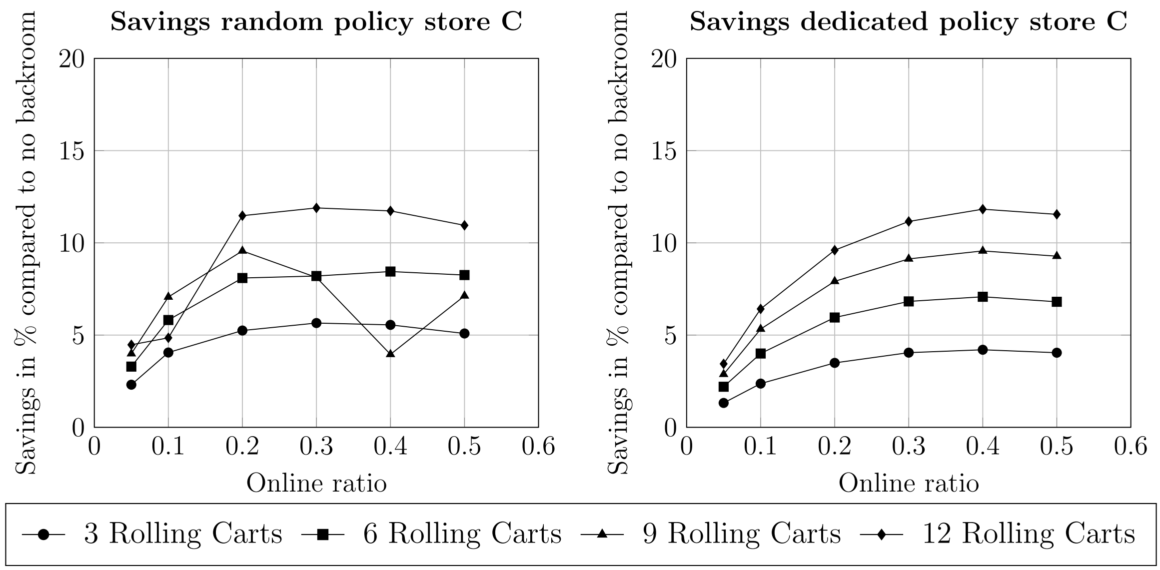

Fig. 9. Comparative analysis of the in-store logistical savings for random and dedicated storage policy store C.

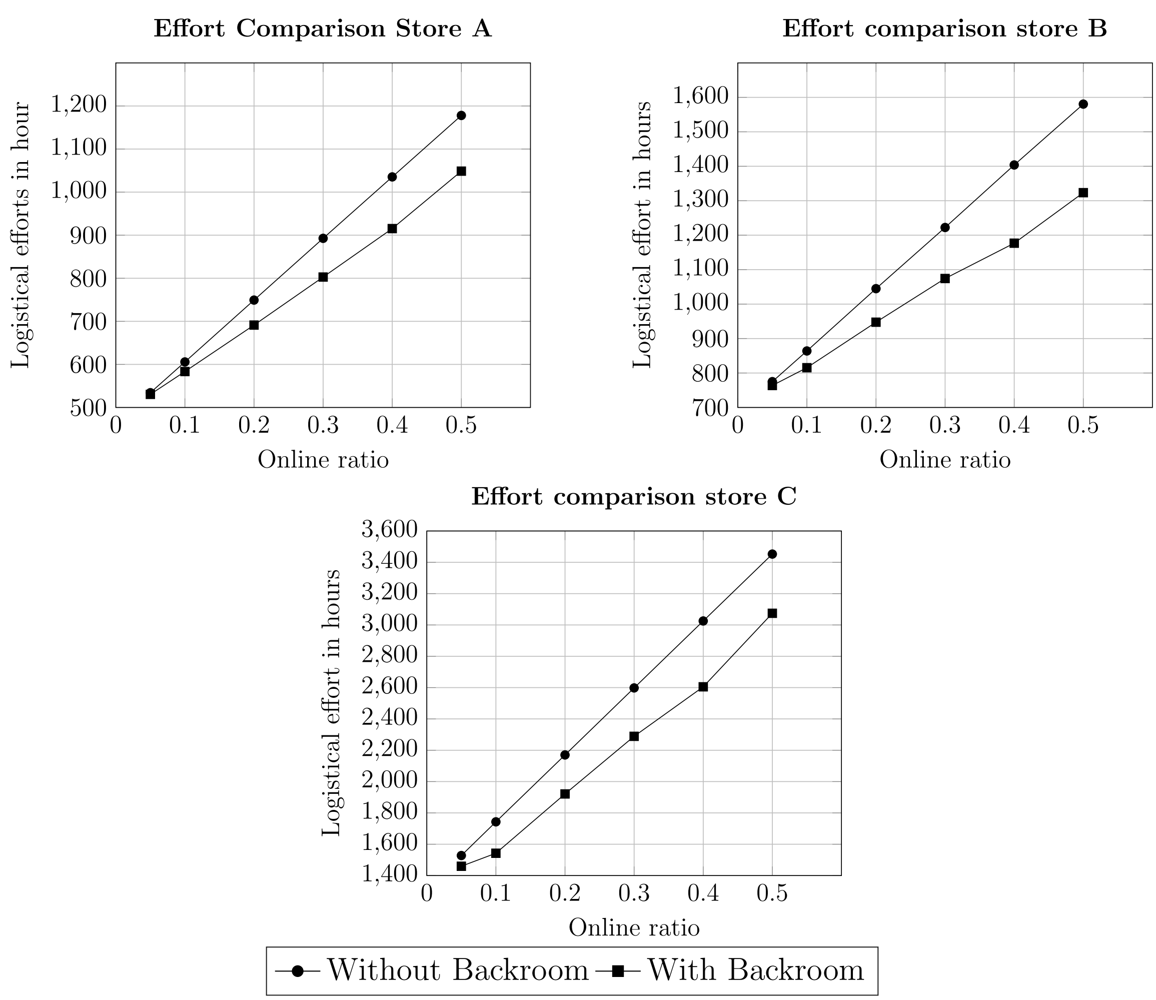

Fig. 10. Comparative analysis of the in-store logistical efforts without and with backroom assignment (n_{su} = 12).

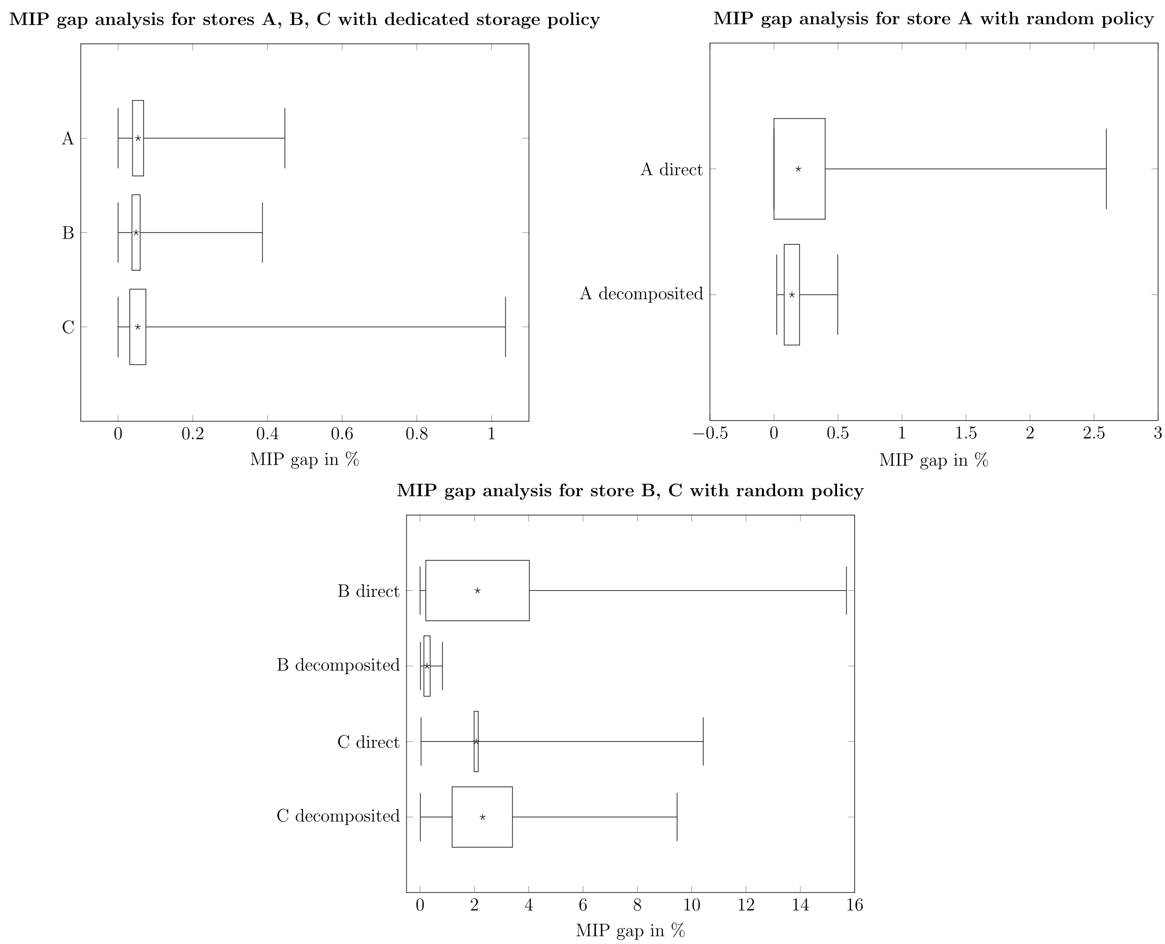

Fig. 11. MIP gap analysis for the dedicated and random storage policy instances of store A,B,C.

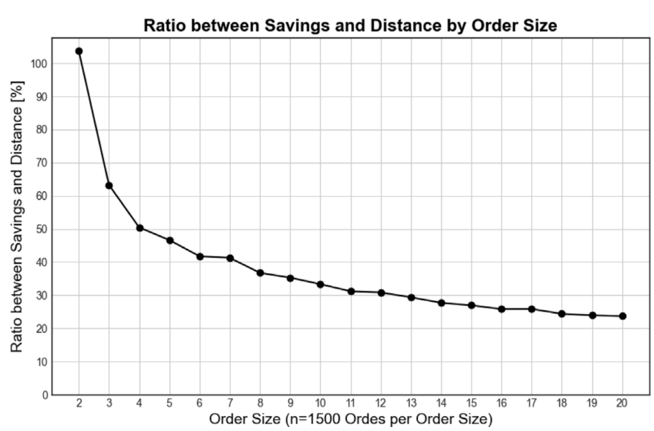

Fig. 12. Ratio between savings and distance to the removed article i for different order sizes.

methods could enhance solution quality and speed, especially for large problem instances.

Looking ahead, we aim to test our model with a broader range of retailers and factor in demand uncertainties to strengthen our findings. From a managerial perspective, comparing the implementation of forward pick areas in each store with the utilization of dark stores for more efficient order picking is worthwhile. Additionally, exploring the optimal balance between shopping and backroom areas is an intriguing direction for future research, as is considering discrete delivery patterns as discussed by Sternbeck and Kuhn (2014). Furthermore, evaluating automation solutions in retail holds promise. Although the immediate business case for in-store automation might not be apparent today, researching this topic is valuable. It can help define the necessary requirements and limitations for when the use of automation in stores becomes beneficial. Future work also should be focused on integrating a broader supply chain optimization. As new backroom processes also requires the integration of upstream supply chain factors, such as distribution and delivery logistics.

CRediT authorship contribution statement

Sebastian Koehler: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Formal analysis, Data curation, Conceptualization. **Felicia Theilacker:** Writing – original draft, Visualization, Software.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the author(s) used ChatGPT 3.5 to correct language and formatting issues and DeepL Write to correct grammar and language. After using these tools/services, the author(s) reviewed and edited the content as needed and take full responsibility for the content of the publication.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Distance savings approximation

For classical warehouse operations and order picking, several exact and heuristic approaches are known and well studied (De Koster et al., 2007). While these approaches, such as Hall (1993) and Le-Duc* and De Koster (2005), focus on the total length of a picking task, for our work it is important to express the savings in travel time when an SKU is cloned to the backroom. Another challenge is that the heuristics are usually applied to well structured layouts such as a single or multi-block layout (De Koster et al., 2007). This is not directly applicable to our use case, which is a freefrom layout. Therefore, we will now derive a numerical approach that covers the free-form layout of the shopping area. In the best case, cloning an article can lead to a situation where a shelf does not need to be visited, while on the other hand, if an article is located between two other articles of an order on the same shelf, cloning will not reduce the distance at all. Therefore, after testing several approaches, we estimate the travel savings using the following approximation:

$$2 * \lceil \frac{D_{i,online}}{o_i} \rceil * \frac{c_{distance,i}}{O} \tag{34}$$

The general idea of this approximation is that it takes into account the order size and multiple picking of the same article within an order. If an article is typically picked multiple times, the distance ratio must be reduced by that amount. As the average order size increases, so does the probability of picking articles from the same shelf. Knowing that this is only a simplified approximation, we compare our solution to a real-world drugstore scenario to determine the gap between our estimate and real-world travel distance savings. We simulated a data set of 30.000 generated orders using a discrete picking policy. The order baskets for these orders ranged from 2 to 20 visited shelves in a store approximately 30 m × 15 m in size. To ensure the realism of our analysis, we incorporated real purchase probabilities associated with each shelf meter and used an authentic store layout. The reduction in distance (in meters) was determined by iteratively removing each pick i from the picking tour and then calculating the savings achieved by applying a 2-opt algorithm to the in-store picking tour without picking i. The savings rate decreased as the number of shelves visited increased, as shown in 12. This is because of removing a pick from the route has less impact on reducing the total distance if there are fewer shelves to visit; the more shelves visited, the smaller the savings from removing a pick from the tour.

Table 6
Evaluation of simulated results.

Formula	Avg. error per pick [m]	Weighted Avg. error per pick [m]
$2 * \lceil \frac{D_{i,online}}{o_i} \rceil * \frac{c_{distance,i}}{O}$	1.67	1.82
Avg. order distance	Delta (%)	Delta (%)
136.98 m	1.2%	1.3%

Over 30.000 orders our estimation formula produced the results shown in Fig. 12 and the results are merged in Table 6.

Our results show that the calculated formula has an average error of less than 2 m per pick for both simple and weighted calculations, considering how often each article appears in orders. As our main goal is to create an optimization strategy for backroom assignments, we do not seek to make this more general at this time, and we rely on the empirical findings. This approach may only be viable for this specific use case

Appendix B. Different shopping area layouts

Next to the processes itself the design of the shopping area influences the in-store picking process. While the backroom can be designed with a focus on efficiency, operations in the shopping area must adhere to a customer-centric layout that prioritizes a positive in-store shopping experience over efficient logistics processes, which presents a significant challenge. Store layouts can be categorized into different types, such as freeform, grid, and racetrack layouts, as shown in Fig. 13 (Ijaz, Rhee, Lee, & Alfian, 2014; Nguyen, Le, Martin, Cil, & Fookes, 2022). Although a Grid Layout is most similar to the design of a warehouse, freeform Layouts are becoming more and more relevant as customers spend more time inside the stores (Ijaz et al., 2014), which matches the trend of customer group-specific store designs. Racetrack layouts have the benefit that they maximize the display area and that every customer has to pass each shelf which increases the probability of impulse purchases.

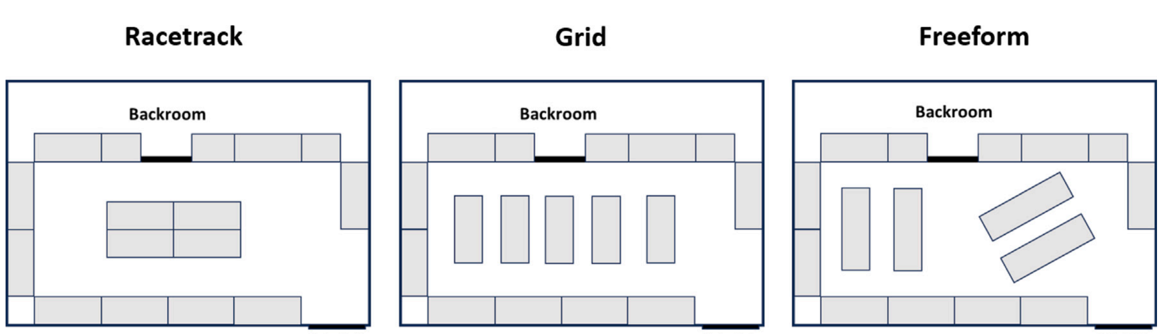

Fig. 13. Schematic store layout concepts.

Data availability

The data that has been used is confidential.

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