Fictitious forces, fictitious fields, fictitious masses? – on how to deal with the so-called inertial forces

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Abstract:

The concept of fictitious forces has been in use in physics since the early days of mechanics. The best known is the centrifugal force. Fictitious forces are still used by many authors today, while others reject them. The idea of fictitious or inertial forces dates back to a time when there were no fields, when interactions still had to be described as action-at-a-distance effects. With the advent of electromagnetism, the field concept entered physics and gravitation could be described in a more modern way. We discuss the consequences for the fictitious forces from the perspective of a field theory. We point out an important difference between the centrifugal force and the inertial force during linear acceleration. In the first case, the third time derivative of the position is different from zero, in the second it is not. In the first case the law of momentum conservation (Newton's second law) is violated, in the second it is not.

Keywords: fictitious forces, centrifugal force, equivalence principle, gravitational field

1. Introduction

In physics we encounter fictitious or inertial forces since Huygens [1] and Newton [2]. They can be found in all later classical works on mechanics, for example [3-7], up to more recent textbooks such as PSSC [8], Feynman [9] or Cline [10]. At first there was only the centrifugal force. Then Laplace added what is now called the Coriolis force. Both occur in reference frames that rotate at a constant angular velocity relative to an inertial frame. The Euler force occurs when the angular acceleration is non- zero. There is also an inertial force in a linearly accelerated reference frame, often called d'Alembert's inertial force.

Here, what the fictitious or inertial forces have in common:

- they are proportional to the mass of the body on which they act,
- there is no body that is exerting them.

A typical definition taken from Wikipedia [11] is:

> A fictitious force, also known as an inertial force or pseudo-force, is a force that appears to act on an object when its motion is described or experienced from a non-inertial frame of reference. Unlike real forces, which result from physical interactions between objects, fictitious forces occur due to the acceleration of the observer's frame of reference rather than any actual force acting on a body. These forces are necessary for describing motion correctly within an accelerating frame, ensuring that Newton's second law of motion remains applicable.

Some authors argue that these are "real" forces [10]:

> These inertial forces are often called fictitious even though they appear real in the non-inertial frame.

However, more recent works have also criticized the concept, especially the centrifugal force.

Rogers, for example, writes [12]:

> You have often heard that name, but unfortunately, it is a misleading term when applied to the moving object. … so you will learn good physics most easily if you avoid the phrase "centrifugal force".

or Giancoli [13]:

> There is a common misconception that an object moving in a circle has an outward force acting on it, a so-called centrifugal ("center-fleeing") force. This is incorrect: there is no outward force on the revolving object.

Also Young and Freedman [14] recommend:

> Avoid using "centrifugal force".

In section 2, we look at the historical context of the concept "fictitious force". In section 3 we consider the consequences of a change from an inertial frame to a rotating frame on the field diagrams of the gravitational field and draw conclusions. In section 4, we discuss an important difference between describing a process in a linearly accelerated reference frame and in a rotating reference frame.

2. Action at a distance theory vs. field theory

Those who teach mechanics today and do not want to deal with phenomena that can only be described with the help of the general theory of relativity, essentially do so in the Newtonian way, because the general theory of relativity is mathematically complex. However, Newtonian mechanics originated at a time when there was no concept of physical fields and one had to be satisfied with invoking action-at-a-distance [15]. However, Newton himself had already considered action-at-a-distance as a provisional concept [16].

The field concept was not introduced until 150 years later by Faraday and Maxwell, initially in the context of electromagnetism. This theory could serve as a model for a more modern theory of gravitation. In fact, such a theory, Gravitoelectromagnetism (GEM), was formulated by Heaviside [17]. It is structured in a close analogy to Maxwell's theory.

Just as electromagnetism describes the electromagnetic field with the two field strengths **E** and **B**, GEM describes the gravitational field with **g** and **b**. **g** is the gravistatic field strength, often referred to as the gravitational acceleration. Its source is the mass density ρ_m:

$$\nabla \cdot \mathbf{g} = -4\pi G \rho_m \tag{1}$$

G is the gravitational constant.

This is analogous to the electric field strength **E**, whose source is the electric charge density ρ_Q:

$$\nabla \cdot \mathbf{E} = \frac{\rho_Q}{\varepsilon_0}. \tag{2}$$

ε_0 is the vacuum permittivity.

b is called gravitomagnetic field strength. Its source is the mass current density.

We will discuss the problem of fictitious forces from the perspective of this classical field theory. In our context, we only need the gravistatic field. The fields may change over time, but we restrict ourselves to quasi-static fields, thus ignoring retardation [18].

3. The centrifugal force from the perspective of classical field theory – a fictitious field

If we describe an arrangement of massive bodies in an inertial frame, the problem is completely analogous to an electrostatic problem. We just have to replace the mass distribution with a charge distribution, where all charges must have the same sign. The field pattern is identical to that of a corresponding charge distribution. Fig. 1 shows the field of two bodies with different masses.

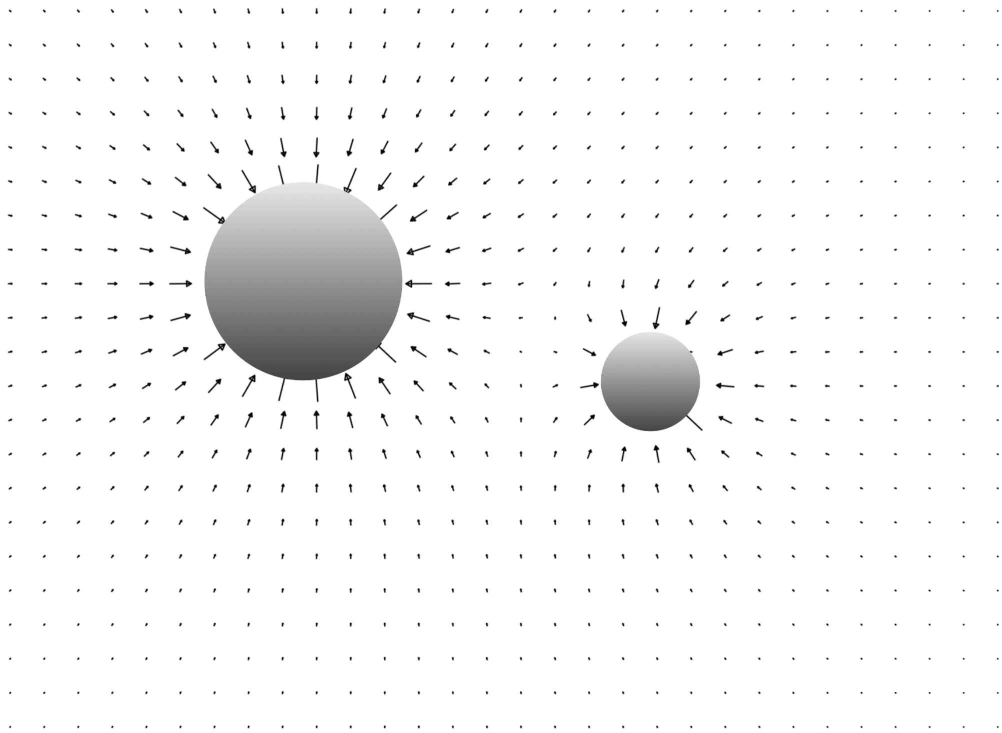

Fig. 1. Field diagram of the gravitational field of two bodies of different masses

Since there is compressive stress in the direction of the field vectors (and not tensile stress, as in the electrical case) [19], two massive bodies attract each other, while two bodies with similar electric charge repel each other.

We now consider the field in a rotating frame of reference (angular velocity $\boldsymbol{\omega}$). We can imagine that we are on a merry-go-round and want to describe the gravitational scenario around us. Recall that in this reference frame, a centrifugal force acts on any massive body. If the body is at rest in our rotating frame, this force balances the force that keeps the body on its circular path, which is called centripetal force. If there is no centripetal force acting on the body, it will be accelerated outward by the centrifugal force. For this interpretation to be possible, the centrifugal force must be proportional to the mass of the body in question.

As actions at a distance no longer fit into our modern physical world view, we will attempt to describe the appearance of centrifugal forces by means of field theory. However, we will see that this leads to inconsistencies.

Consider a small test body of mass m at rest in our rotating frame of reference. We notice that there is a force acting on it that is proportional to its mass which is called centrifugal force. Its magnitude is:

$$F = m\omega^2 r \qquad (3)$$

We conclude, that this force must be caused by a gravitational field with

$$g = \omega^2 r \qquad (4)$$

The field strength vectors would point radially outwards from the axis of rotation, and their magnitude would grow linearly with the radius. However, this means that the field should have sources everywhere in space, Fig. 2.

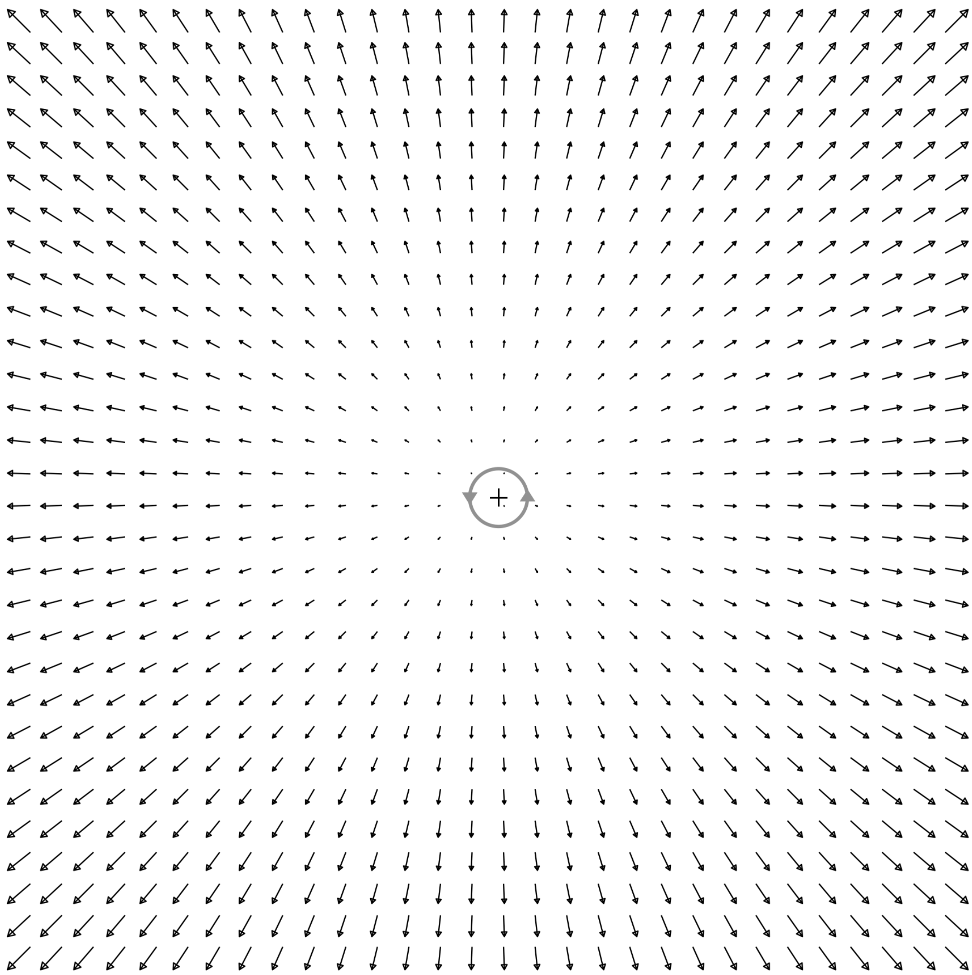

Fig. 2. Field diagram in a rotating reference frame

Let us calculate the source density of this field.

The divergence of a cylinder-symmetric vector field is

$$\nabla \cdot \mathbf{g} = \frac{1}{r}\frac{\partial}{\partial r}(rg) \tag{5}$$

With equation (4) we obtain

$$\nabla \cdot \mathbf{g} = 2\omega^2 \tag{6}$$

and with equation (1) finally

$$\rho_m = -\frac{\omega^2}{2\pi G}. \tag{7}$$

We see that

- the mass density is the same everywhere; we would have a homogeneous source distribution,
- the mass density is negative, because gravitational field lines end where (positive) mass is located. For a merry-go-round that takes 6 seconds to complete one revolution, one gets $\rho_m \approx -2.6 \cdot 10^{10}\ \mathrm{kg}/m^3$.

Since we had named the centrifugal force a fictitious force, because there is no body exerting it, we could call the field of Fig. 1 a fictitious field, and the non-existing negative mass would be called fictitious mass [20].

We will check how the field diagram of the arrangement in Fig. 1 would look like, Fig. 3. As in Fig. 2, there are sources everywhere.

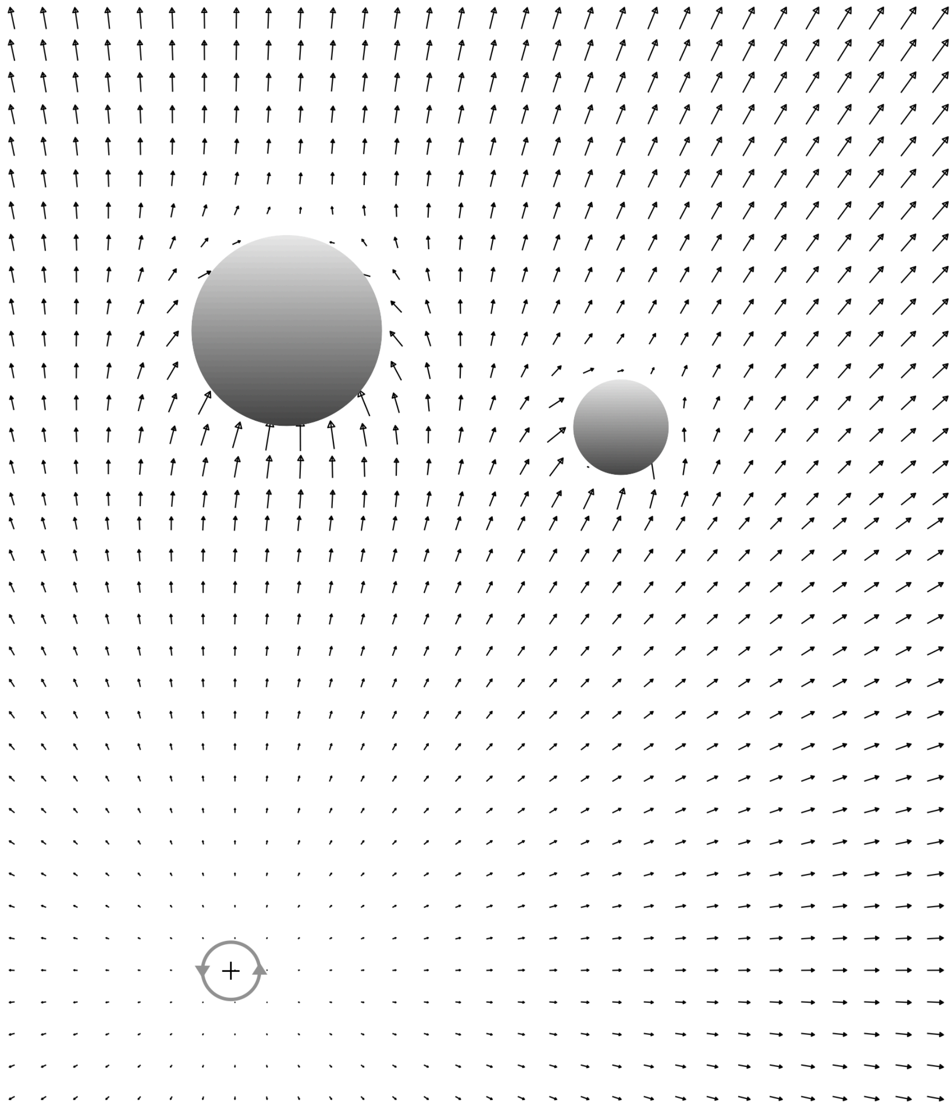

Fig. 3. Field diagram of the arrangement of Fig. 1, shown in a rotating reference frame.

This result is rather disturbing and we assume that none of our readers will agree with this attempt to save the dynamics of our bodies in the rotating reference frame. However, if one does not want to accept the existence of field sources where there is no mass, one should not accept a force whose existence presupposes such sources.

4. The third time derivative

We are familiar with Einstein's equivalence principle. If a reference frame B is moving with constant acceleration relative to an inertial frame A, then the law of conservation of momentum holds in B as well. Let us see what happens to our field diagram from Fig. 1 in the new inertial frame, Fig. 4.

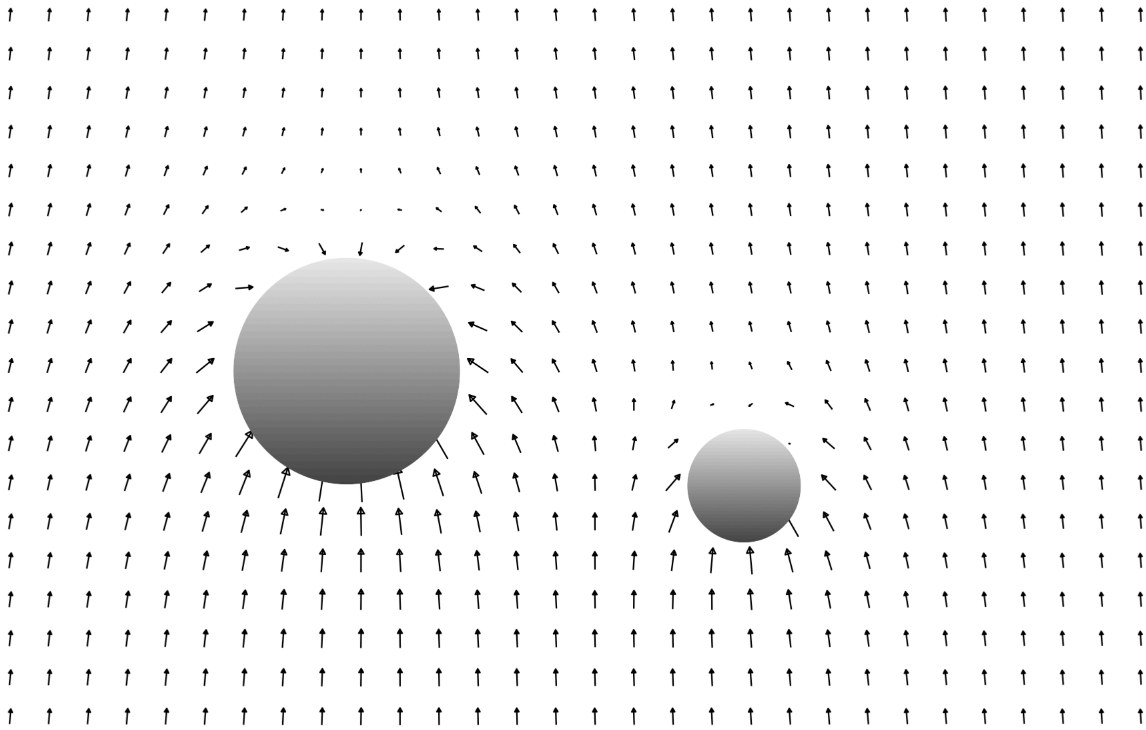

Fig. 4. Field diagram of an arrangement of bodies as in Fig. 1, shown in a reference frame moving with constant acceleration with respect to that in Fig. 1.

It looks different from that of Fig. 1, but all is well. There are no mysterious fictitious sources as in Fig. 3. The description is completely equivalent to that in the original inertial frame, and the new reference frame is again inertial. Therefore, there is no reason to give the force that occurs in the new reference frame a name of its own, namely inertial force.

It merely shows that what appears as gravity in one frame of reference appears as inertia in the other – and vice versa.

But don't the arguments we used in section 3 against introducing fictitious forces in rotating reference frames also apply to this so-called inertial force? We will see that there is a significant difference.

We have become accustomed to the fact that we can make certain reference frame changes while maintaining the validity of an important physical law: the law of conservation of momentum.

When we pass from one frame of reference to another whose zero point of position is displaced from that of the first frame, the laws of mechanics remain valid. We can say that we are free to choose the origin of the position. The same is true if we change to a reference frame that moves with constant velocity relative to the original one. The zero point of velocity (the zero point of the 1st derivative of position) can also be chosen arbitrarily. Now there is an observation, which we call the equivalence principle, which tells us that we can also freely choose the zero point of the acceleration (the second time derivative of the position). The new reference frame is allowed to move with constant acceleration relative to the original one.

All three types of reference frame changes result in changes of the values of physical quantities. However, the laws of physics, in particular the law of conservation of momentum remain valid.

But what about rotational motion? Although a rotational motion is an accelerated motion, the law of momentum conservation no longer applies in the rotating frame. To discover the reason for this, it is sufficient to look at the next, i.e. the third time derivative of position. While it was assumed to be zero when changing to a linearly accelerated frame of reference, this is no longer the case for rotational motion. For the conservation of momentum to hold, the acceleration of the new reference frame must be constant in time.

5. Conclusion

When changing the description of a mechanical process from an inertial frame to a reference frame that is accelerating against the original frame, it is common to introduce fictitious or inertial forces.

We recommend distinguishing between two cases: those in which the acceleration is constant in time and those in which it is not.

If it is constant in time, the law of conservation of momentum remains valid. The two reference frames are equivalent. This is the content of Einstein's equivalence principle. There is no reason to qualify the forces that appear in the new reference frame as fictitious.

If the acceleration is not constant, consequences arise that are hardly acceptable: Newton's laws no longer apply and gravitational fields occur that have sources at places where there are no masses. We argue that one should refrain from such reference frame changes, and thus also from introducing centrifugal forces, as well as Coriolis and Euler forces.

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