

Spontaneous CP violation and flavor changing neutral currents in minimal $SO(10)$

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We explore spontaneous CP violation in the minimal non-super-symmetric $SO(10)$ grand unified theory, with a scalar sector comprising a CP -even 45_H , a 126_H , and a complex 10_H . All renormalizable couplings are real due to CP symmetry, and the Kobayashi-Maskawa phase arises solely from complex electroweak vacuum expectation values. The model requires an additional Higgs doublet fine-tuned below 500 GeV and constrains new Yukawa couplings, linking certain flavor-violating (FV) processes. Future proton decay observations may reveal correlated FV decay ratios, offering insights into minimal $SO(10)$.

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I. INTRODUCTION

$SO(10)$ grand unification theory (GUT) [1] is one of the most appealing paradigms to understand the Standard Model (SM) of elementary particles and unveil new physics beyond it. Strong and electroweak interactions are successfully unified in renormalizable $SO(10)$ theories without requiring light scalars [2,3], due to the intermediate scales related to left-right (LR) [4,5] or quark-lepton (QL) [6] symmetries. Proton decay is predicted, and the upper limit of proton lifetime is calculable, at least in certain minimal scenarios [7,8]. Furthermore, $SO(10)$ is a complete theory for matter unification. A family of SM fermions plus a right-handed neutrino fit exactly into the spinor representation 16_F of $SO(10)$. That explains the seemingly mysterious anomaly cancellation and charge quantization [9,10].

Meanwhile, testing $SO(10)$ is highly challenging. Technically, the direct energy scale of all realistic experiments can never reach the unification scale of about 10^{15} GeV. An alternative approach is to search the hints at low energy because matter unification implies quarks and leptons behave somewhat similarly. Much effort is focused on numerically fitting the fermion masses and mixing angles, such as [11–14]. However, in our view, these fitted central values should be regarded as predictions because no uncertainties can be identified. For the realistic renormalizable

$SO(10)$ GUTs,¹ no robust and discriminative results were found until now. Clearly, we need more flavor observables to test $SO(10)$. The difficulty is that $SO(10)$ GUT itself does not require any new sub-TeV particles to mediate flavor transition.

We choose to specify the general $SO(10)$ theory by imposing the condition that all renormalizable couplings are real. It leads to a more concise theory with fewer free parameters. CP symmetry, the combined transform of charge conjugation C and parity P , is enhanced in this limit. CP now serves as a fundamental symmetry of nature [15], neglecting gravitational effects [16]. The CP -odd Kobayashi-Maskawa phase [17] of weak interactions derives only from the complex vacuum expectation values (VEVs) [18]. CP symmetry is spontaneously violated, in the sense that the physical solution does not respect CP symmetry although the Lagrangian does. Spontaneous CP violation (SCPV) is not possible in pure SM because one can always rotate the CPV phase of the electroweak VEV away, using $U(1)_Y$ gauge redundancy [19,20]. In unified theories, the VEVs for $SO(10)$, or intermediate symmetry breaking, sometimes get a physical phase, but the low energy theories are almost indistinguishable with the explicit CPV one. Such high scale SCPV is widely discussed in the literatures [21–25].

Interestingly, we notice high scale SCPV is impossible in minimal renormalizable $SO(10)$, whose scalar sector only contains a CP -even 45_H , an 126_H , and a complex 10_H [26,27]. The 126_H field indeed contains a large complex VEV, but similar to SM, its phase is also unphysical. SCPV is possible and *only* possible together with electroweak symmetry breaking, requiring an additional light Higgs doublet.

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¹We always take the non-super-symmetric (non-SUSY) framework.

The new doublet is already contained in 126_H or 10_H , but its mass must be fine-tuned towards the electroweak scale [28,29], just like the SM Higgs doublet. Bearing the fine-tuning, we arrive at a theory without a decouple limit. Perturbative unitarity requires the new scalars lie below about 500 GeV [30]. The low energy theory becomes very similar to what Lee originally proposed [18]. The difference is that the Yukawa-type couplings are more constrained than the general two-Higgs doublet models (2HDM) [19] because of the LR and QL symmetry of $SO(10)$.

Electroweak scale SCPV has been somehow overlooked in the past, partly due to the unavoidable flavor changing neutral currents (FCNC) [31]. But strictly speaking, the model is on the edge but not ruled out yet [29,30] because completely eliminating FCNC is not necessary. We treat FCNC as key predictions and advocate greater efforts to measure a set of flavor violating (FV) observables with higher precision, including charged lepton flavor violation (cLFV), neutral meson oscillation, and heavy resonances decaying into FV final states at colliders. These signals do not just shed light on a new Higgs doublet, but also provide more flavor observables related by $SO(10)$. Considering proton decay branching ratios, we have more observables than free parameters. If the FV and proton decay signals are both precisely measured in the near future, we will either get a strong hint on minimal CP -conserving $SO(10)$, or directly disprove it.

Our work is structured as follows. In Sec. II, we revisit the minimal realistic $SO(10)$ theory and discuss its Yukawa sector in the limit of CP symmetry. Then, we intuitively show why the scalar spectrum for electroweak scale SCPV

contains light 2HDM. Following this, we derive the low energy theory at weak scale and specify its predictions. A more detailed phenomenological analysis is provided in Sec. III. We firstly explain why the theory is not excluded and then show how to test it by comparing the relative strength of a set of experimental observables. Finally, our findings and further discussions are summarized in Sec. IV. In the Appendix, we perform a deeper analysis on the Yukawa sector, illustrate some one-loop GUT-scale corrections, and summarize values for the hadron matrix elements we have used.

II. MINIMAL CP INVARIANT $SO(10)$ GRAND UNIFICATION THEORY

Three generations of fermionic representation $16_F = (Q_L, u_R, d_R) + (\ell_L, \nu_R, e_R)$ contain all quarks, leptons, and right-handed neutrinos. Then how many new scalar fields must be included in a realistic $SO(10)$ theory? For realistic fermion masses, the simplest choice is a complexified 10_H and 126_H [26]. Neither 10_H nor 126_H can directly break $SO(10)$ down to the SM gauge group $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$. The minimal path is using an additional 45_H to firstly break $SO(10)$ to some intermediate groups, then reduce its rank with 126_H , and finally get G_{SM} [27]. Although potentially within the swampland, where the perturbative expansion lies around the edge of breaking down [32,33], we still choose one 45_H , 126_H and a complexified (two real) $10_H = (10_H^1 + i10_H^2)/\sqrt{2}$ as the minimal scalar sector for a renormalizable $SO(10)$ GUT. The symmetry breaking chain is

$$\begin{aligned}
 SO(10) \times CP &\xrightarrow[M_{GUT}]{\langle(1,1,1,0)\rangle \in 45_H} SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times CP \\
 &\xrightarrow[M_R]{\langle(1,1,3,1)\rangle \in 126_H} SU(3)_C \times SU(2)_L \times U(1)_Y \times CP \\
 &\xrightarrow[M_W]{\langle(1,2,2,0)\rangle \in 126_H, 10_H} SU(3)_c \times U(1)_{EM}.
 \end{aligned} \tag{1}$$

The $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ quantum numbers are shown in the parenthesis. Alternatively, the intermediate symmetry can also be $SU(4)_C \times SU(2)_L \times U(1)_R \times CP$, while the other Pati-Salam type breaking patterns are not possible with the minimal scalar sector [3,34], unless adding more steps. If the intermediate LR (or QL) symmetry breaking scale M_I is around 10^9 GeV (or 10^{11} GeV), then the gauge coupling unification works perfectly [3,35,36]. This result, however, can be relaxed if some of the physical states in 126_H and/or 45_H are fine-tuned light.² We therefore do not assume any specific value

for M_I but only take it as a high scale, currently not achievable by terrestrial experiments.

If one defines the CP transform for the scalar particles as [with the $SO(10)$ indices implicit],

$$45_H \rightarrow 45_H, \quad 126_H \rightarrow \overline{126}_H, \quad 10_H \rightarrow 10_H^*, \tag{2}$$

then all couplings are restricted real. Furthermore, the vacuum of the adjoint representation 45_H is real. 126_H is a complex representation, but its VEV $\langle(1, 1, 3, 1)\rangle$ can always be chosen to be real by a phase redefinition [27]. As a result, it is impossible to break CP symmetry at high scales with the scalar sector defined in Eq. (2). In this work,

²See [7,8] for a similar theory with 16_H and 45_H .

we will not discuss about other alternative definitions of CP symmetry, which may allow pure imaginary couplings. While as a remark, it has been discussed in [21] that with an extended scalar of a complex (one CP even, one CP odd) 45_H , C , and CP break together at GUT scale. Whatever, within the minimal $SO(10)$ scenario discussed here, the only possible VEV with a physical phase is from the Higgs $\langle(1, 2, 2, 0)\rangle$. CP is spontaneously violated, only together with electroweak symmetry breaking.

Spontaneous CP violation gives two degenerate vacua, leading to domain wall solutions, a disaster for cosmology [37]. A natural way out of the domain wall problem is requiring symmetry nonrestoration in the very early Universe, when the temperature $T \gtrsim M_W$ [38–40]. Unfortunately, such mechanism is not realistic with only two light Higgs doublets [41]. To keep CP nonrestoration at high temperature, one needs at least a third light Higgs doublet, which requires another fine tuning. A more common solution to the domain wall problem is adding a tiny CP -odd perturbation (often called a biased term) to the Lagrangian, so the domain wall is unstable and collapses quickly after its formation [37,42,43]. Such a biased term may derive from effective operators due to quantum gravity, which is expected to violate all global symmetries [16,44]. The biased term solution is what we choose in this work. In other words, we assume the renormalizable minimal $SO(10)$ theory itself respects CP symmetry exactly, while all the unknown physics beyond grand unification, such as quantum gravity, can break CP but have only negligible effects on the low energy theory, except for destabilizing domain walls. The

gravitational waves of domain wall collapse can serve as a smoking gun for electroweak scale discrete symmetry broken, but the amplitudes are, in general, quite small. For potential gravitational wave signals, we refer to [45,46].

A. Yukawa sector

The $SO(10)$ invariant Yukawa sector reads [23,27,47]:

$$-\mathcal{L}_Y = Y_{10} 16_F 10_H 16_F + \tilde{Y}_{10} 16_F 10_H^* 16_F + Y_{126} 16_F \overline{126}_H 16_F + \text{H.c.} \quad (3)$$

Y_{10} , \tilde{Y}_{10} , and Y_{126} are 3×3 symmetric matrices according to $SO(10)$ algebra [48] and, with the Hermitian condition imposed by CP symmetry, all the elements of Y_{10} , \tilde{Y}_{10} , and Y_{126} are real [23]. As 10_H is a complicated representation, 10_H and 10_H^* are independent degrees of freedom, and \tilde{Y}_{10} is, in general, nonzero. This is different from the scenarios with additional symmetries like $U(1)_{PQ}$ [26], relaxing the potential constraints from fermion masses and the mixing angle's fit [11]. Both 126_H and 10_H contain two Higgs doublets Φ^u and Φ^d . So, there are in total four Higgs doublets (though not all light) in the generic basis: $\Phi^i = (\Phi_{10}^d, \tilde{\Phi}_{10}^u, \Phi_{126}^d, \tilde{\Phi}_{126}^u)$, where $\tilde{\Phi} = i\sigma_2 \Phi^*$. For simplicity, absorbing all common normalization factors into the Yukawa couplings ($Y_{10} \rightarrow \frac{1}{2\sqrt{2}} Y_{10}$, $\tilde{Y}_{10} \rightarrow \frac{1}{2\sqrt{2}} \tilde{Y}_{10}$, $Y_{126} \rightarrow \frac{1}{4} \sqrt{\frac{3}{2}} Y_{126}$ [23]), the Higgs-fermions interaction reads:

$$-\mathcal{L}_Y \supset \overline{Q}_L (Y_{10} \Phi_{10}^d - \tilde{Y}_{10} \tilde{\Phi}_{10}^u + Y_{126} \Phi_{126}^d) d_R + \overline{Q}_L (Y_{10} \Phi_{10}^u + \tilde{Y}_{10} \tilde{\Phi}_{10}^d + Y_{126} \Phi_{126}^u) u_R + \overline{\ell}_L (Y_{10} \Phi_{10}^d - \tilde{Y}_{10} \tilde{\Phi}_{10}^u - 3Y_{126} \Phi_{126}^d) e_R + \overline{\ell}_L (Y_{10} \Phi_{10}^u + \tilde{Y}_{10} \tilde{\Phi}_{10}^d - 3Y_{126} \Phi_{126}^u) \nu_R + \frac{1}{2} \overline{\nu}_R^c Y_{126} \Delta_R^0 \nu_R + \frac{1}{2} \overline{\ell}_L^c Y_{126} \Delta_L \ell_L + \text{H.c.} \quad (4)$$

Φ^i can develop nonzero complex VEVs: $v_i = (v_{10}^d, v_{10}^{u*}, v_{126}^d, v_{126}^{u*})$, spontaneously breaking CP together with the electroweak symmetry. The electroweak VEV is defined as $v^2 \equiv \sum_{i=1}^4 |v_i|^2 = (246 \text{ GeV})^2$, neglecting small tadpole-induced VEVs of the scalar $SU(2)_L$ triplets ($Y = 0, 1$) in 45_H , 126_H . Δ_R^0 is the neutral component of the $SU(2)_R$ triplet in 126_H , and its large VEV $\langle \Delta_R^0 \rangle = \langle (1, 1, 3, 1) \rangle$ provides right-handed neutrino masses M_{ν_R} . Values of M_{ν_R} are commonly assumed to be around 10^{13} GeV , to give the correct light neutrino masses M_{ν_L} via the seesaw mechanism.

However, M_{ν_R} can also lie at a lower scale, when the Dirac-type neutrino mass matrix M_{ν_D} is correspondingly reduced. Smaller values for M_{ν_D} are allowed since it is not fully aligned with M_U . We will explain how to obtain the correct light neutrino mass more explicitly in Appendix A. Δ_L is the VEV for the $SU(2)_L$ triplet and also contributes to neutrino masses via type-II seesaw, if sizable.

At GUT scale, the mass matrices for down-type quarks, up-type quarks, charged leptons, and neutrinos M_D , M_U , M_E , M_{ν_L} can be derived from Eq. (4):

$$\begin{aligned} M_E &= Y_{10} v_{10}^d - \tilde{Y}_{10} v_{10}^{u*} - 3Y_{126} v_{126}^d, & M_{\nu_R} &= \frac{1}{2} Y_{126} \langle \Delta_R^0 \rangle, \\ M_D &= Y_{10} v_{10}^d - \tilde{Y}_{10} v_{10}^{u*} + Y_{126} v_{126}^d, & M_{\nu_D} &= Y_{10} v_{10}^u + \tilde{Y}_{10} v_{10}^{d*} - 3Y_{126} v_{126}^u, \\ M_U &= Y_{10}^u v_{10}^u + \tilde{Y}_{10} v_{10}^{d*} + Y_{126} v_{126}^u, & M_{\nu_L} &= -M_{\nu_D}^T M_{\nu_R}^{-1} M_{\nu_D} + \frac{1}{2} Y_{126} \langle \Delta_L \rangle. \end{aligned} \quad (5)$$

As all Yukawa couplings are real, the nonzero phases of these fermion mass matrices could only come from v_i , as the only source of CP violation. When $\tilde{Y}_{10} = 0$, M_E , M_D , M_U are linear dependent, they thus could be simultaneously diagonalized if neglecting the all quark mixing angles [49]. Numerical fitting shows that this scenario is realistic even considering neutrino oscillation data [11,13]. However, it is worthy to point out that this is not the intrinsic prediction of $SO(10)$ as it requires additional global symmetries. In general, the three fermion mass matrices (or three Yukawa couplings $Y_{10}, \tilde{Y}_{10}, Y_{126}$) are not all diagonal and at GUT scale can be parametrized with

$$\begin{aligned} M_D &= D^* m_D D^\dagger, & M_U &= U^* m_U U^\dagger, \\ M_E &= E^* m_E E^\dagger, & M_{\nu_L} &= N^* m_{\nu_L} N^\dagger, \\ V_{\text{CKM}} &= U^\dagger D, & V_{\text{PMNS}} &= E^\dagger N, & V_E &= E^\dagger D. \end{aligned} \quad (6)$$

m_D, m_U, m_E, m_{ν_L} are diagonal matrices containing the physical masses for quarks and leptons. V_{CKM} is the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix and V_{PMNS} is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix. Thanks to $SO(10)$ algebra, the symmetric $Y_{10}, \tilde{Y}_{10}, Y_{126}$ requires that the mixing matrices for left-handed or right-handed fermions are the same at GUT scale, so the unknown physical matrix is merely V_E . In comparison, the general Yukawa potential with four Higgs doublets contains 16 independent general 3×3 coupling matrices, instead of just three real symmetric ones in Eq. (4). This tells us that the flavor structure for fermion-scalar interactions of minimal realistic $SO(10)$ are strongly constrained at tree level, accounting for an important prediction of matter unification itself.

Does Eq. (5) predict anything more? One may expect so because there are only 21 physical free parameters (taking one of the VEVs real, and one of the Yukawas diagonal) determining all SM flavor observables. Numerical analysis was performed in exactly the same scenario [23], and the best-fit solutions were found. However, those should not be interpreted as predictions because no uncertainties can be quantified. Assuming one finds a million data points consistent with the SM fermion masses and mixing angles, it still only represents a negligible part of the 21-dimension parameter space (10^{-15} , conservatively assuming ten points can sample each dimension). Clearly, some (semi)analytical analysis is required, and we have explored this approach. Unfortunately, we find no robust constraints at the end, and some details are shown in Appendix A. The main difficulty is that M_{ν_L} depends on the mass matrices *nonlinearly*, as a result of the seesaw mechanism. With some approximations, we find the mixing angles of V_E can range from zero to large values similar to the neutrino mixing angles. In addition, the VEVs also appear poorly constrained. Although we cannot rule out some potential undiscovered relationships, Eq. (5) alone does not seem to

yield robust predictions. $SO(10)$ may leave clues about fermion masses and mixing angles, but they are deeply hidden.

So far, we have not included the effects of renormalization group (RG) running, so all results above are only valid at the GUT scale (tree-level). The effects from QCD and possibly scalar self-interactions are expected to be large, but they are fortunately flavor blinded. These corrections can be absorbed into the overall factors, so do not change the physical predictions. The Yukawa couplings are, in general, flavor nonuniversal, leading to corrections to some GUT-scale predictions at low energy. However, the RG effects are suppressed by the loop factor $1/(4\pi)^2$ so remain at next-to-leading order in a perturbative theory. For simplicity, all our subsequent analysis will be restricted to GUT scale (tree-level) predictions. Some quantitative estimations of the low energy deviations are provided in Appendix B.

B. Scalar spectrum

The natural mass scale of all nonchiral particles in minimal $SO(10)$ is M_{GUT} , due to the large VEV of 45_H . Clearly, the naturalness criterion fails here because the SM Higgs boson mass is 125 GeV, much lighter than M_{GUT} . It has been realized for a long time that electroweak scale SCPV requires two light Higgs doublets and double fine-tuning [28]. This is a model-independent result, and we gave a geometric illustration in Fig. 1, based on the planar diagram of Lee [18] (FIG-1). The Mexican Hat-like potential has rotation ($U(1)_Y$) and reflection (CP) symmetries, but they are broken when the spring-connected two balls fall into the valleys. As shown with the orange and dark red balls, the true vacuum of the system corresponds to the state when the balls are at the bottom of the valleys, and

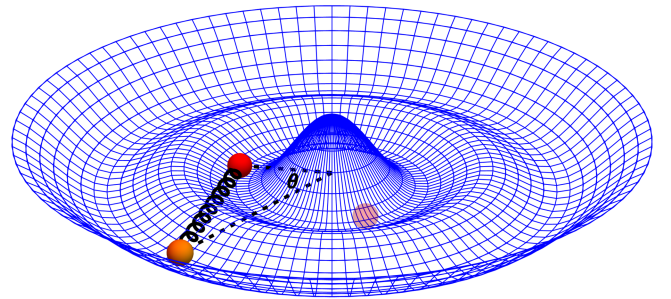


FIG. 1. Geometrical illustration of spontaneous CP violation together with $U(1)_Y$, based on the planar diagram of Lee [18] (FIG-1). The “Mexican Hat”-like potential itself has rotation ($U(1)_Y$) and reflection (CP) symmetries, but the stable physical solution does not, when the balls fall into the valleys and the spring is relaxed. The light-red ball implies there is a mirror solution $\theta \rightarrow -\theta$, and nature has to choose one as the true vacuum. The $\theta = 0$ state is invariant under reflection, but not stable. It is a local maximum because the string is contracted and the total energy of the system increases.

the spring is fully relaxed. A mirror vacuum solution is illustrated with the light red ball, corresponding to $\theta \rightarrow -\theta$. These two degenerate solutions are separated by a local maximum, the $\theta = 0$ state. This is because the spring is contracted when $\theta = 0$, increasing energy of the whole system. As a result, although the $\theta = 0$ state preserves reflecting (CP) symmetry, it is not the stable solution of the system. Nature has to choose the true vacuum between the $\pm\theta$ states, spontaneously breaking CP symmetry. The rotation redundancy of the whole system corresponds to the Goldstone mode of $U(1)_Y$ broken G^0 . The massive degrees of freedom can be identified as two radical modes of oscillation in the valleys h and H , and one axial mode vibrating along the spring A . The values of the masses are the corresponding oscillation frequencies. Within a perturbative theory, that valley curvature and the spring elastic coefficient are both bounded from above by roughly the squared distance between the two peaks (in natural units). Therefore, we have a perturbative unitarity bound in analog to the single Higgs boson case [50]:

$$\sum_{h,H,A} \omega_{\text{osci}}^2 = m_h^2 + m_H^2 + m_A^2 \lesssim M_{\text{LQT}}^2 = (700 \sim 800 \text{ GeV})^2. \quad (7)$$

Although merely a geometrical illustration, Eq. (7) aligns quite well with the explicit next-to-leading order analysis. If identifying h as the discovered 125 GeV SM Higgs boson, then it is shown in [30] that m_H and m_A are individually bounded by 485 GeV and 545 GeV, so $SCPV$ with $U(1)_Y$ indeed has no decouple limit. This is clearly a consistent requirement for the low energy spectrum, so adding more heavy Higgs doublets does not change the scenario, as recently proven in [51]. Generalization to the electroweak gauge symmetry, $SU(2)_L \times U(1)_Y$, is straight forward.

Therefore, the low-energy particle spectrum for minimal CP conserving $SO(10)$ is the same as light 2HDM. But remember, we have in total four Higgs doublets in the

generic basis, two in 10_H and two in 126_H . To go to the low energy effective theory, we firstly define the following light states:

$$\begin{aligned} h^{\text{SM}} &= \frac{v_i^*}{v} \rho_i, & \tilde{H} &= \frac{u_i^*}{u} \rho_i, & G^0 &= \frac{v_i^*}{v} \eta_i, \\ \tilde{A} &= \frac{u_i^*}{u} \eta_i, & i &= 1, 2, 3, 4, \end{aligned} \quad (8)$$

where

$$\begin{aligned} \Phi_i &= \begin{pmatrix} \phi_i^+ \\ (v_i + \rho_i + i\eta_i)/\sqrt{2} \end{pmatrix}, & u_i &= v_i^* - \left(\sum_{j=1}^4 v_j^{*2} \right) \frac{v_i}{v}, \\ u &= \sum_{i=1}^4 |u_i|^2. \end{aligned} \quad (9)$$

So h^{SM} and \tilde{H} live in the plane spanned by v_i and v_i^* , the two degenerate vacuum solutions. It is straight forward to check $\sum_i^4 v_i^* u_i = 0$, so h^{SM} is orthogonal to \tilde{H} (and similarly for G^0 and \tilde{A}). In addition, there are two other linear-independent combinations of ρ_i (and η_i), accounting for the superheavy neutral states irrelevant to phenomenology. Thus, ρ_i, η_i can be represented by

$$\begin{aligned} \rho_i &= \frac{v_i}{v} h^{\text{SM}} + \frac{u_i}{u} \tilde{H} + \text{superheavy states}, \\ \eta_i &= \frac{v_i}{v} G^0 + \frac{u_i}{u} \tilde{A} + \text{superheavy states}. \end{aligned} \quad (10)$$

What follows next is the same as the general 2HDMs [19,52]. G^0 can be identified as the massless Goldstone mode, while the other two CP -even components h^{SM}, \tilde{H} and one CP -odd component \tilde{A} are still not the mass eigenstates. In general, they mix with each other, and the physical states can be parametrized with

$$\begin{pmatrix} h \\ H \\ A \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\alpha_c} & s_{\alpha_c} \\ 0 & -s_{\alpha_c} & c_{\alpha_c} \end{pmatrix} \begin{pmatrix} c_{\alpha_A} & 0 & s_{\alpha_A} \\ 0 & 1 & 0 \\ -s_{\alpha_A} & 0 & c_{\alpha_A} \end{pmatrix} \begin{pmatrix} c_{\alpha_H} & s_{\alpha_H} & 0 \\ -s_{\alpha_H} & c_{\alpha_H} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} h^{\text{SM}} \\ \tilde{H} \\ \tilde{A} \end{pmatrix}, \quad (11)$$

where $c_{\alpha_c} = \cos \alpha_c, s_{\alpha_c} = \sin \alpha_c$, etc. So, the discovered 125 GeV Higgs h may not behave exactly as SM predictions. For example, consider Higgs-Gauge boson interactions:

$$\begin{aligned} \mathcal{L}_{\text{kin}} &\supset g^2 W_\mu^+ W^{-\mu} \Phi_i^\dagger \Phi_i \supset \frac{1}{2} g^2 W_\mu^+ W^{-\mu} v h^{\text{SM}} \\ &= \frac{1}{2} c_{\alpha_H} c_{\alpha_A} g^2 W_\mu^+ W^{-\mu} v h. \end{aligned} \quad (12)$$

So the $hW_\mu^+ W^{-\mu}$ coupling strength $g_{hWW} = \frac{1}{2} c_{\alpha_H} c_{\alpha_A} g^2 v$ might be smaller than $\frac{1}{2} g^2 v$ of the SM. The ATLAS Run 2 data [53] tells $c_{\alpha_H} c_{\alpha_A} \gtrsim 0.99(0.96)$ at 68% (95%) confidence level (CL). In the limit $\alpha_H, \alpha_A \ll 1$, one can expand Eq. (11) and get a more concise expression:

$$\begin{aligned} h^{\text{SM}} &= h - |\epsilon| (c_{\tilde{a}_c} H + s_{\tilde{a}_c} A), \\ \tilde{H} + i\tilde{A} &= \epsilon h + e^{i\alpha_c} (H + iA), \end{aligned} \quad (13)$$

where $\epsilon = \alpha_H + i\alpha_A$, $\tilde{\alpha}_c = \alpha_c - \arg \epsilon$. In analog to $c_{\alpha\beta}$ in 2HDM [54], ϵ encodes the decoupling limit so that one can rewrite it as $\hat{\lambda}v^2/m_H^2$, where $\hat{\lambda}$ is a dimensionless coupling. When m_H is large enough, ϵ goes to zero so the discovered 125 GeV Higgs becomes exactly SM-like. On the other hand, since we are working within a nondecoupled theory, m_H is bounded to around 500 GeV. As a consistency requirement, $\hat{\lambda}$ must therefore be small, while m_H is tending toward its maximal possible value. The low-energy theory likely lies right at the edge where the perturbative expansion breaks down, similar to recent concerns about the UV regime [32,33]. Fortunately, these potentially

nonperturbative effects are flavor conserving and therefore do not overwhelm our predictions regarding flavor structures.

C. The low energy theory

Equations (10) and (13) allow us to rewrite Eq. (4) in physical basis concisely [by replacing $\rho_i + i\eta_i \subset \Phi_i$ with $\frac{v_i}{v}h^{\text{SM}} + \frac{u_i}{u}(\tilde{H} + i\tilde{A})$ and solving $Y_{10}, \tilde{Y}_{10}, Y_{126}$ from Eq. (5)]. After dropping superheavy states and higher-order terms, we get the tree-level interacting Lagrangian for the light neutral scalars and SM fermions:

$$\begin{aligned} -\mathcal{L}_{\Phi FF} \supset & \left(\frac{m_E}{v} + \epsilon Y_E^{\ell\ell'} \right) h \bar{\ell}_L \ell'_R + \left(\frac{m_D}{v} + \epsilon Y_D^{qq'} \right) \bar{d}_L^q d_R^{q'} h + \left(\frac{m_U}{v} + \epsilon Y_U^{qq'} \right) \bar{u}_L^q u_R^{q'} h \\ & + \mathcal{Y}_E^{\ell\ell'} (H + iA) \bar{\ell}_L \ell'_R + \mathcal{Y}_D^{qq'} (H + iA) \bar{d}_L^q d_R^{q'} + \mathcal{Y}_U^{qq'} (H + iA) \bar{u}_L^q u_R^{q'} + \text{H.c.} \end{aligned} \quad (14)$$

$$\begin{aligned} Y_E &= C_{EE} \frac{m_E}{v} + C_{ED} V_E^* \frac{m_D}{v} V_E^\dagger + C_{EU} V_E^* V_{\text{CKM}}^T \frac{m_U}{v} V_{\text{CKM}} V_E^\dagger, \\ Y_D &= C_{DE} V_E^T \frac{m_E}{v} V_E + C_{DD} \frac{m_D}{v} + C_{DU} V_{\text{CKM}}^T \frac{m_U}{v} V_{\text{CKM}}, \\ Y_U &= C_{UE} V_{\text{CKM}}^* V_E^T \frac{m_E}{v} V_E V_{\text{CKM}}^\dagger + C_{UD} V_{\text{CKM}}^T \frac{m_D}{v} V_{\text{CKM}} + C_{UU} \frac{m_U}{v}, \end{aligned} \quad (15)$$

where $\mathcal{Y}_F = e^{i\alpha_c} Y_F$, $\ell, \ell' = e, \mu, \tau, q, q' = d, s, b$ or u, c, t , running over the three families. Complex coefficients $\mathcal{C}_{FF'}(F, F' = E, D, U)$ read:

$$\begin{pmatrix} C_{EE} & C_{ED} & C_{EU} \\ C_{DE} & C_{DD} & C_{DU} \\ C_{UE} & C_{UD} & C_{UU} \end{pmatrix} = \frac{v}{u} \begin{pmatrix} u_1 & -u_2 & -3u_3 \\ u_1 & -u_2 & u_3 \\ u_2^* & u_1^* & u_4^* \end{pmatrix} \begin{pmatrix} v_{10}^d & -v_{10}^{u*} & -3v_{126}^d \\ v_{10}^d & -v_{10}^{u*} & v_{126}^d \\ v_{10}^u & v_{10}^{d*} & v_{126}^u \end{pmatrix}^{-1}. \quad (16)$$

Direct calculation shows $C_{EU} = C_{DU}$, while we have also checked numerically that there are no other correlations among the elements in $\mathcal{C}_{FF'}$. One may assume the dimensionless parameters $\mathcal{C}_{FF'}$ are, in general, all around $O(1)$, but strictly speaking, they are not predicted. Large hierarchies of two or more orders of magnitude, in principle, are possible. For a better understanding, one can take the SUSY-like 2HDM potential [19] as an analogy, where

$$C_{EE} = C_{DD} = \tan \beta, \quad C_{UU} = \cot \beta, \quad \text{others} = 0. \quad (17)$$

There is only one theoretical constraint, that $\mathcal{C}_{FF'}$ should not be too large to ensure \mathcal{Y}_F is within the limit of perturbative unitarity. The lower limit for $\mathcal{C}_{FF'}$ depends on how the fermions' mass relationship in Eq. (5) constrains v_i . It is likely that $\mathcal{C}_{FF'}$ cannot be all zero, but as discussed at the end of Sec. II A, we cannot conclude a robust lower bound.

Our low energy theory has more freedom than the benchmark 2HDM without tree-level FCNC [19], while it is not the most general one, either. $SO(10)$ gives

nontrivial constraints. Let us firstly try to intuitively understand it in the chiral limit. If Y_{10}, \tilde{Y}_{10} and Y_{126} are all strictly zero, then the whole theory is invariant under the chiral transform $16_F \rightarrow 16_F e^{i\theta}$ and all fermions are massless. This is partly a good symmetry because, except for the top quark, all observed fermion masses are far smaller than the electroweak scale. Clearly, the large top quark mass m_t explicitly breaks the chiral symmetry and requires large Yukawa couplings, but it is still reasonable to expect that many of the elements in Y_E, Y_D, Y_U are small. In fact, if ignoring all fermion masses except for m_t , then we approximately have a tree-level result:

$$\begin{aligned} Y_E^{\ell\ell'} &\approx C_{EU} V_E^* V_{\text{CKM}}^T \text{Diag}(0, 0, m_t) V_{\text{CKM}} V_E^\dagger \\ &\approx C_{EU} \frac{m_t}{v} (V_E^{\ell b} V_E^{\ell' b})^*, \\ Y_D^{qq'} &\approx C_{DU} V_{\text{CKM}}^T \text{Diag}(0, 0, m_t) V_{\text{CKM}} \approx C_{DU} \frac{m_t}{v} V_{\text{CKM}}^{tq} V_{\text{CKM}}^{tq'}, \\ Y_U^{qq'} &\approx C_{UU} \text{Diag}(0, 0, m_t) \approx C_{UU} \frac{m_t}{v} \delta^{qq'}. \end{aligned} \quad (18)$$

The small 2-3 and 1-3 mixing angles in the CKM matrix are assumed far smaller than the corresponding ones in V_E , to simplify $Y_E^{\ell\ell'}$. The flavor structure of Y_E is next-to-minimal flavor violation (NMFV) [55], which can be checked by experiments. Y_D then flavors minimal flavor violation (MFV) [56], and Y_U is approximately diagonal. It is clear that \mathcal{C}_{EU} must be small, in order to avoid the tree-level FCNC in the leptonic sector. Remembering $\mathcal{C}_{EU} = \mathcal{C}_{DU}$, the quark sector is then automatically free from tree-level flavor changing neutral current constraints. The theory can be made safe from FV constraints, mainly because most of the couplings are suppressed by m_τ/v , m_b/v and small mixing angles. The only specific requirement is a small \mathcal{C}_{EU} .

Equation (18) is only for illustration, and the hierarchies among $C_{FF'}$ elements might, in practice, be larger than m_t/m_b . However, there are still quantitative predictions on the nondiagonal flavor structure. Equation (5) tells that Y_{10} , \tilde{Y}_{10} , and Y_{126} (and thus Y_E , Y_D , Y_U) are linear combinations of three symmetric fermion mass matrices M_E , M_D , and M_U . Therefore, we have at tree level,

$$\begin{aligned} Y_E^{\ell\ell'} &\propto (V_E^{\ell b} V_E^{\ell' b})^* + o(\lambda^2)(V_E^{\ell b} + V_E^{\ell' b})^*, & \ell \neq \ell', \\ Y_D^{qq'} &\propto V_E^{\tau q} V_E^{\tau q'} + \frac{\mathcal{C}_{DU} m_t}{\mathcal{C}_{DE} m_\tau} V_{\text{CKM}}^{\tau q} V_{\text{CKM}}^{\tau q'}, & q \neq q', \\ Y_U^{qq'} &\propto V_E^{\tau q} V_E^{\tau q'} + \frac{\mathcal{C}_{UD} m_b}{\mathcal{C}_{UE} m_\tau} V_{\text{CKM}}^{\tau q} V_{\text{CKM}}^{\tau q'} + o(\lambda), & q \neq q'. \end{aligned} \quad (19)$$

$Y_E^{\ell\ell'}$, $Y_D^{qq'}$ is valid up to $o(m_b/m_t)$ corrections. $\lambda \sim 0.2$ is the Wolfenstein parameter, so $o(\lambda^2)$ encodes the small 2-3 and 1-3 mixing angles in the CKM matrix. It is clear that as long as $|V_E^{\ell b}|$ and $|V_E^{\ell' b}|$ are larger than $o(\lambda^2)$, the flavor structure for lepton flavor violation is indeed NMFV, shedding light on the unknown mixing matrix V_E . On the other hand, the flavor structure for down-type quark sector is not fully clear. One may expect that to be MFV, as of Eq. (18), but NMFV is also possible because \mathcal{C}_{DU} should be suppressed to avoid FCNC. $Y_U^{qq'}$ may also receive $o(\lambda)$ corrections because $V_E^T m_E V_E$ of Eq. (15) is sandwiched between V_{CKM}^* and V_{CKM}^\dagger .

III. PHENOMINOLOGY

A. Is the theory safe?

The second Higgs doublet has $SU(2)_L$ and $U(1)_Y$ charge by definition. Its interactions with weak gauge bosons change the electroweak precision data. The most sensitive one is T parameter [57]; when the new scalars' masses are nearly degenerate (custodial limit), we have approximately [58,59]

$$\begin{aligned} T &= \frac{(m_{H^+}^2 - m_H^2)(m_{H^+}^2 - m_A^2)}{48\pi s_w^2 m_W^2 m_H^2} \\ &\approx 0.18 \times \left(\frac{m_{H^+} - m_H}{100 \text{ GeV}} \right) \left(\frac{m_{H^+} - m_A}{100 \text{ GeV}} \right). \end{aligned} \quad (20)$$

Fixing $U = 0$, T should be smaller than 0.06 according to [60], excluding the CDF data [61].³ The theory is safe although the second Higgs doublet is not heavy because its contribution to the T parameter is zero in the custodial limit. Consequently, the mass spectrum of H^+ , H , A is expected to be quasidegenerate.

Shall the second Higgs be directly observed at hadron colliders? It depends on the producing cross section and decay branching ratios. Assuming $o(1)$ $H\bar{t}t$ or $H\bar{b}b$ Yukawa couplings, the total cross section at the LHC is dominated by gluon-gluon fusion, and associated production with quarks, and may reach 1–10 pb for $m_H \sim 500$ GeV [63,64]. If the $H\bar{\tau}\tau$ Yukawa coupling is also $o(1)$ (similar to minimal supersymmetric standard model with large $\tan\beta$), then the theory is excluded because the current collider constraints for $pp \rightarrow H \rightarrow \bar{\tau}\tau$ are quite tight [65]. $H\bar{\tau}\tau$ Yukawa couplings are therefore required to be smaller, leading to the final states dominated by $\bar{t}t$ and $\bar{b}b$ pairs. This scenario has large background at hadron colliders [66,67], so is not yet excluded [68,69]. Additionally, H may also be produced with the W/Z vector bosons, and/or decay to $ZZ + WW$ final states. These modes are suppressed by $|\epsilon|^2$ ($c_{\alpha\beta}^2$, if using the notation of two-Higgs doublet model benchmarks). A small ϵ is then also required by the collider constraints, along with the Higgs precision measurements discussed in Sec. II B.

FCNC cannot be completely eliminated, but they do not necessarily exceed the experimental limits. Let us start with all $\mathcal{C}_{FF'} \sim o(1)$ as a benchmark. The neutral B_s meson mixing induced by Y_D is clearly too large. Consequently, \mathcal{C}_{DU} must be suppressed by about

$$\begin{aligned} |\mathcal{C}_{DU}| &\lesssim \left(\frac{v}{m_t} \cdot \frac{1}{|V_{\text{CKM}}^{\tau s} V_{\text{CKM}}^{\tau b}|} \right) \times \frac{m_H/\sqrt{2}}{10^3 \text{ TeV}} \\ &\approx 0.013 \times \frac{m_H}{500 \text{ GeV}}, \end{aligned} \quad (21)$$

in the limit $m_H \approx m_A$. Here, 10^3 TeV is the experimental limit for the cutoff of the dimension six operator $\mathcal{O}_4 = \bar{b}_L q_R \bar{b}_R q_L$ [70]. Fortunately, no further constraints for the other $\mathcal{C}_{FF'}$ are required. As a consequence of MFV, B_d and K mixing are less constrained than B_s mixing. For the other $\Delta F = 2$ processes, D mixing is suppressed by m_τ/v and m_b/v , and the constraints from muonium-antimuonium oscillation is much weaker than neutral mesons. Given $|\mathcal{C}_{DU}| < 10^{-2}$, $\mathcal{C}_{EU} = \mathcal{C}_{DU}$, and others $= o(1)$, all the $\Delta F = 1$ processes for down-type quark and charged leptons, such as $B \rightarrow \mu^+ \mu^-$ and $\mu \rightarrow e \gamma$, are also suppressed due to the lack of sizable chirality-flipping interactions. The top quark flavor-violating decay and anomalous production could be sizable, but the experimental sensitivities are significantly poorer.

³ 1σ allowed CDF consistent range for T is $\{0.159, 0.210\}$ [62].

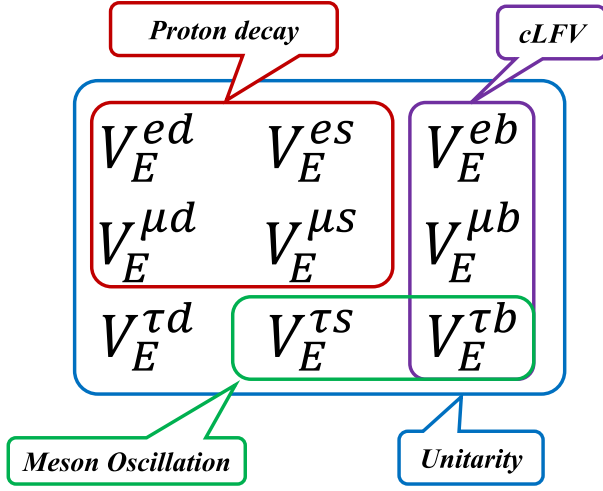


FIG. 2. Proton decay, LFV, and neutral meson oscillation are linked through a 3×3 unitary matrix V_E . Together with the unitary conditions, the theory yields more observables than free parameters, leading to a nontrivial consistency check as we will show in Eq. (34).

B. A window to check $SO(10)$

We now go to the predictions of $SO(10)$. The general idea is to demonstrate that there are more observables than free parameters. We focus on the absolute magnitudes of the unknown 3×3 mixing matrix V_E . Unitarity provides five independent constraints on the nine elements, $|V_E^{\ell q}|$, that the squared sum of any row or any column is one, minus the total. Physical predictions exist, as long as we can identify more than four direct experimental inputs. Various channels for lepton flavor violating (LFV) scalar decays, neutral meson oscillation, and proton decay are our candidates, as illustrated in Fig. 2.

Equation (19) tells that the LFV decay of H and A can decide the elements in the third column of V_E :

$$\sigma(pp \rightarrow H, A) \times \text{Br}(H, A \rightarrow \ell \ell') \propto |Y_E^{\ell \ell'}|^2 \propto |V_E^{\ell b} V_E^{\ell' b}|^2, \quad (22)$$

valid as long as $|V_E^{\ell b}|$ and $|V_E^{\ell' b}|$ are larger than $o(\lambda^2)$. At colliders, we expect signals of heavy resonances decaying into $e\tau, \mu\tau, e\mu$ final states. A recent analysis is given in [71]. There will be two peaks in the $e\mu$ invariant mass spectrum, centered at m_H and m_A . The prediction is about the relative total number of the excess events in each channel:

$$\frac{N_{e\mu}}{N_{\tau\mu}} = \frac{|V_E^{eb}|^2}{|V_E^{\tau b}|^2}, \quad \frac{N_{e\mu}}{N_{e\tau}} = \frac{|V_E^{\mu b}|^2}{|V_E^{\tau b}|^2}. \quad (23)$$

Here, $N_{\ell \ell'}$ is the total number of the excess events, normalized by the detection efficiency. If measured in the future, then the unitarity of the third column of V_E ,

$|V_E^{eb}|^2 + |V_E^{\mu b}|^2 + |V_E^{\tau b}|^2 = 1$, would allow a direct extraction of the absolute values of all these three elements.

Elements in the third row of V_E are related to quark FV. The down sector is better than the top quark one. It is partly because the top quark FV is much less precisely measured compared to neutral meson oscillations. The other reason is the nondiagonal elements of Y_U receive $o(\lambda)$ correction from the CKM matrix, while those of Y_D get only $o(\lambda^2)$ corrections. h contribution is negligible in the small ϵ limit, and the dominate new physics (NP) contribution to the $\Delta F = 2$ processes mainly comes from H and A . The nondiagonal elements in Y_D of Eq. (14) gives an effective operator:

$$\begin{aligned} H_{\text{NP}}^q &= -\frac{1}{2m_H^2} (Y_D^{bq} \bar{b}_L q_R + Y_D^{qb*} \bar{b}_R q_L)^2 \\ &\quad -\frac{1}{2m_A^2} (iY_D^{bq} \bar{b}_L q_R - iY_D^{qb*} \bar{b}_R q_L)^2 \\ &\approx -\frac{2|Y_D^{bq}|^2}{m_H^2} \bar{b}_L q_R \bar{b}_R q_L, \quad q = d, s. \end{aligned} \quad (24)$$

The Wilson coefficients for $\bar{b}_L q_R \bar{b}_L q_R$ and $\bar{b}_R q_L \bar{b}_R q_L$ vanish in the limit $m_H = m_A$ [72]. This quasidegenerate limit is implied by the electroweak precision data and can be cross checked by direct collider search. The operator $\bar{b}_L q_R \bar{b}_R q_L$ then dominates, whose coefficient $Y_D^{bq} Y_D^{qb*}$ is nearly real because Y_D is symmetric at tree-level. H_{NP}^q is therefore approximately CP conserving and connected to experimental observables via

$$\frac{\langle B_q^0 | H_{\text{NP}}^q | \bar{B}_q^0 \rangle}{\langle B_q^0 | H_{\text{SM}}^q | \bar{B}_q^0 \rangle} = \mathbf{h}_q e^{i\sigma_q} = \frac{M_{12}^{\text{Exp}}}{M_{12}^{\text{SM}}} - 1, \quad q = d, s. \quad (25)$$

$2|M_{12}^{\text{Exp}}| = \Delta m_q$ is the oscillation frequency. \mathbf{h}_q is the experimental input for our work, accounting for possible NP contributions. In pure SM, \mathbf{h}_d and \mathbf{h}_s are exactly 0 by definition. According to [73], The current 95% CL upper limits for \mathbf{h}_d and \mathbf{h}_s are, respectively, 0.26 and 0.12, and the future sensitivities at high-intensity experiments can reach about 0.03–0.04. We summarize them in Table I.

H, A also contribute to K meson mixing:

$$\begin{aligned} (\Delta M_K)_{\text{NP}} &= \frac{1}{m_K} \langle K^0 | H_{\text{NP}}^K | \bar{K}^0 \rangle \\ &\approx -\frac{2|Y_D^{sd}|^2}{m_K m_H^2} \langle K^0 | \bar{d}_L s_R \bar{d}_R s_L | \bar{K}^0 \rangle. \end{aligned} \quad (26)$$

The CP violating parameter ϵ_K receives zero contribution in the limit $m_H = m_A$. The hadronic factors, along with those for B mesons, are shown in Appendix C. The SM prediction for Δm_K is partly swamped by the long distance contributions, and the effect from the additional Higgs is not necessarily subdominant. The current measurement on

TABLE I. Current experimental bounds and future expected sensitivities on the FV processes relevant for our analysis. The constraints for \mathbf{h}_d and \mathbf{h}_s , the parameters on how much B_d and B_s meson mixing derive from SM predictions, are shown with 95% CL, using the results of CKMfit [73]. The future sensitivity comes from LHCb and Belle II, with 300/fb and 250/ab integrated luminosity, respectively. A further possible improvement with 5×10^{12} Z decays from a lepton collider like FCC-ee [74] is included in the bracket. $(\Delta M_K)_{\text{NP}}$ is the maximally allowed NP contribution to the $K_L - K_S$ mass difference, where the SM long distance contribution is based on lattice QCD [75,76]. We take the assumption of Snowmass 2021 [77] that the future lattice errors can achieve 5% level, for future sensitivity. All the observables in the Table are zero, if the SM is exact.

Observable	Current limit	Future sensitivity
\mathbf{h}_d	0.26 [73]	0.049 [73] (0.038 [74])
\mathbf{h}_s	0.12 [73]	0.044 [73] (0.031 [74])
$ (\Delta M_K)_{\text{NP}} $	5.2×10^{-15} GeV [76,78]	0.2×10^{-15} GeV [77,78]

(ΔM_K) is $3.484(6) \times 10^{-15}$ GeV and is not likely to improve much in future [78]. The latest lattice calculation shows $(\Delta M_K)_{\text{SM}} = 5.8(0.6)_{\text{stat}}(2.3)_{\text{sys}} \times 10^{-15}$ GeV [75,76], allowing sizable NP contribution with a sign opposite to the SM estimation. It is expected that reducing SM prediction uncertainty to the 5% level is possible in the future, with adequate computer resources [77].

Different from the leptonic sector, the FV structure for down-type quarks can be either MFV or NMFV. To distinguish, we notice the MFV scenario implies

$$\frac{|(\Delta M_K)_{\text{NP}}| m_K}{\langle K^0 | \bar{d}_L s_R \bar{d}_R s_L | K^0 \rangle} \approx \lambda^4 \cdot \frac{2\mathbf{h}_d |M_{12}^{\text{dSM}}| m_{B_d}}{\langle B_d^0 | \bar{b}_L d_R \bar{b}_R d_L | B_d^0 \rangle} \approx \lambda^6 \cdot \frac{2\mathbf{h}_s |M_{12}^{\text{sSM}}| m_{B_s}}{\langle B_s^0 | \bar{b}_L s_R \bar{b}_R s_L | B_s^0 \rangle}. \quad (27)$$

If no hierarchies are found, the flavor structure must be NMFV, and the mixing angles in V_E are typically larger than λ^2 . The prediction is

$$\frac{|(\Delta M_K)_{\text{NP}}|}{2\mathbf{h}_d |M_{12}^{\text{dSM}}| \xi_B} = \frac{|V_E^{ts}|^2}{|V_E^{tb}|^2}, \quad \text{with } \xi_B = \frac{m_B \langle K^0 | \bar{d}_L s_R \bar{d}_R s_L | K^0 \rangle}{m_K \langle B_d^0 | \bar{b}_L d_R \bar{b}_R d_L | B_d^0 \rangle}. \quad (28)$$

This is valid as long as the left-hand side is measured much larger than $o(\lambda^4)$. We illustrate the physically interesting region in Fig. 3. $|M_{12}^{\text{dSM}}|$ is referred from Eq. (25), by taking $M_{12}^{\text{qExp}} = 0.506 \text{ ps}^{-1}$ [79] and the central value $\mathbf{h}_d = 0.075$, $\sigma_q = -1.4$ [73]. It is clear that for any nonzero value of $|(\Delta M_K)_{\text{NP}}|$ that can be determined by near future lattice calculation, NMFV is the only possible flavor structure for Y_D . MFV requires very small $|(\Delta M_K)_{\text{NP}}|$, as shown in the

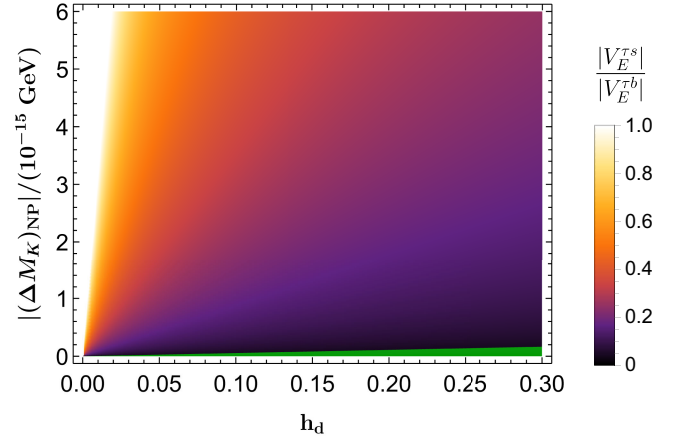


FIG. 3. $|(\Delta M_K)_{\text{NP}}|$ versus \mathbf{h}_d , where the dependence of $|V_E^{ts}|/|V_E^{tb}|$ is represented by the color scale on the right. The white color indicates $|V_E^{ts}|/|V_E^{tb}| \gtrsim 1$, implying large mixing angles in V_E . The relationship of Eq. (28) generally holds, except for the narrow green band of $|V_E^{ts}|/|V_E^{tb}| \lesssim \lambda^2 \sim 0.04$. Parameter extraction becomes challenging when $\mathbf{h}_d \rightarrow 0$ and $|(\Delta M_K)_{\text{NP}}| \rightarrow 0$, since SM is revealed in this limit.

narrow green band. If both $|(\Delta M_K)_{\text{NP}}|$ and \mathbf{h}_d are sizable, then one can extract the third row of V_E using the unitarity condition of $|V_E^{tb}|^2 + |V_E^{ts}|^2 + |V_E^{td}|^2 = 1$, assuming $|V_E^{td}|$ is already extracted from the LFV decay of H or A .

The remaining unconstrained elements lie within the 2×2 top-left submatrix of V_E . At leading order, these terms do not appear in Eq. (19) due to the suppression of m_μ/m_τ and m_s/m_b . Fortunately, the fermion mass hierarchies are not related to the proton decay amplitudes mediated by the gauge bosons. In addition, it has long been known [80] that the proton decay flavor structure can be significantly simplified when the Yukawa couplings are symmetric, as in our scenario. Therefore, using the symbols defined in Eq. (6), the general mixing matrices $V_1 \sim V_4, V_{UD}, V_{EN}$ of [81] reduce to

$$V_1 = V_4 = \mathbb{1}, \quad V_2 = V_3^\dagger = V_E, \quad V_{UD} = V_{\text{CKM}}, \quad V_{EN} = V_{\text{PMNS}}, \quad (29)$$

at GUT scale. The neutrino channels are more predictive, as all three generations of neutrinos are summed over and V_E is eliminated [80]. If the intermediate symmetry of $SO(10)$ breaking is left-right or quark-lepton, then the tree-level masses for superheavy gauge bosons are degenerate [27]. So, the width for $p \rightarrow \pi^+ \bar{\nu}$ reads:

$$\Gamma(p \rightarrow \pi^+ \bar{\nu}) = \frac{g_{\text{GUT}}^2 m_p}{2\pi M_{\text{GUT}}^4} A^2 \langle \pi^+ | (du)_R d_L | p \rangle^2. \quad (30)$$

Here, g_{GUT} is the $SO(10)$ gauge coupling at M_{GUT} , and A is the QCD renormalization factor. Related hadronic matrix elements are listed in Appendix C. Similarly, the decay

$p \rightarrow K^+\bar{\nu}$ also does not depend on the flavor mixings, and the ratio,

$$\frac{\Gamma(p \rightarrow \pi^+\bar{\nu})}{\Gamma(p \rightarrow K^+\bar{\nu})} = \frac{4(1 - m_K^2/m_p^2)^{-2} \langle \pi^+ | (du)_R d_L | p \rangle^2}{\langle K^+ | (us)_R d_L | p \rangle^2 + \lambda^2 \langle K^+ | (ud)_R s_L | p \rangle^2} \approx 81.2, \quad (31)$$

is fixed. This relation helps us distinguish whether proton decay processes are dominated by vector gauge bosons or

scalar leptoquarks. Compared with gauge interactions, scalar interactions maximally violate flavor symmetry, so verifying Eq. (31) implies the scalar contributions are negligible.

The branching ratios of proton decay to charged leptons are fixed up to V_E . Normalizing them with $\Gamma(p \rightarrow \pi^+\bar{\nu})$ and $\Gamma(p \rightarrow K^+\bar{\nu})$ dropping the $o(\lambda^2)$ terms in V_{CKM} , we have

$$\begin{aligned} \frac{\Gamma(p \rightarrow \pi^0 e^+)}{\Gamma(p \rightarrow \pi^+\bar{\nu})} &= \left| V_E^{ed} + \frac{\lambda}{2} V_E^{es} \right|^2, & \frac{\Gamma(p \rightarrow \pi^0 \mu^+)}{\Gamma(p \rightarrow \pi^+\bar{\nu})} &= \left| V_E^{\mu d} + \frac{\lambda}{2} V_E^{\mu s} \right|^2, \\ \frac{\Gamma(p \rightarrow K^0 e^+)}{\xi_K \Gamma(p \rightarrow K^+\bar{\nu})} &= |V_E^{es} + \lambda V_E^{ed}|^2, & \frac{\Gamma(p \rightarrow K^0 \mu^+)}{\xi_K \Gamma(p \rightarrow K^+\bar{\nu})} &= |V_E^{\mu s} + \lambda V_E^{\mu d}|^2, \\ \text{with } \xi_K &= \frac{2 \langle K^0 | (us)_R u_L | p \rangle^2}{\langle K^+ | (us)_R d_L | p \rangle^2 + \lambda^2 \langle K^+ | (ud)_R s_L | p \rangle^2} \approx 6.4. \end{aligned} \quad (32)$$

The $SU(2)_L$ isospin symmetry for u, d quarks is applied to simplify the hadronic elements related to neutral and charged pions. The latest measurement of different proton decay branching ratios are listed in Table II. Hyper-Kamiokande is expected to start operating as early as 2027 and improve the sensitivity by around 1 order of magnitude [82]. In Fig. 4, we show the partial lifetimes of various proton decay channels, setting the benchmarks of $\tau(p \rightarrow \pi^+\bar{\nu}) = 3.9 \times 10^{32}$ years (current limit) or $\tau(p \rightarrow \pi^+\bar{\nu}) = 3.9 \times 10^{33}$ years. The bounds from Super-Kamiokande and expected limits of Hyper-Kamiokande experiments are, respectively, indicated with solid and dashed gray lines. The diagrams suggest that, if $p \rightarrow \pi^+\bar{\nu}$ is observed in near future, the lifetimes of proton decaying into charged leptons are generally within the limit of Hyper-Kamiokande, assuming elements in the 2×2 top-left submatrix of $|V_E|$ are sizable.

Eliminating the $o(\lambda)$ terms of Eq. (32), we have

$$\begin{aligned} \frac{2\Gamma(p \rightarrow \pi^0 e^+)}{\Gamma(p \rightarrow \pi^+\bar{\nu})} - \frac{\Gamma(p \rightarrow K^0 e^+)}{\xi_K \Gamma(p \rightarrow K^+\bar{\nu})} &= 2|V_E^{ed}|^2 - |V_E^{es}|^2, \\ \frac{2\Gamma(p \rightarrow \pi^0 \mu^+)}{\Gamma(p \rightarrow \pi^+\bar{\nu})} - \frac{\Gamma(p \rightarrow K^0 \mu^+)}{\xi_K \Gamma(p \rightarrow K^+\bar{\nu})} &= 2|V_E^{\mu d}|^2 - |V_E^{\mu s}|^2. \end{aligned} \quad (33)$$

TABLE II. 90% CL limits from proton decay searches on $\tau(p \rightarrow X) \equiv 1/\Gamma(p \rightarrow X)$. Future sensitivities are expected to be improved by about 1 order of magnitude at Hyper-Kamiokande [82].

Decay mode	$\ell = e^+$ (yr)	$\ell = \mu^+$ (yr)	$\ell = \bar{\nu}$ (yr)
$p \rightarrow \pi \ell$	$> 2.4 \times 10^{34}$ [83]	$> 1.6 \times 10^{34}$ [83]	$> 3.9 \times 10^{32}$ [84]
$p \rightarrow K \ell$	$> 1.0 \times 10^{33}$ [85]	$> 3.6 \times 10^{33}$ [86]	$> 5.9 \times 10^{33}$ [87]

This result is valid up to $o(\lambda^2)$. So similar to what we did before, assuming $|V_E^{eb}|, |V_E^{\mu b}|$ are extracted from cLFV processes, we can calculate all the elements in the 2×2 top-left submatrix of V_E , using the unitarity relation of the first and second row, $|V_E^{ed}|^2 + |V_E^{es}|^2 + |V_E^{eb}|^2 = 1$ and $|V_E^{\mu d}|^2 + |V_E^{\mu s}|^2 + |V_E^{\mu b}|^2 = 1$.

In summary, we have identified five direct experimental observables, as shown on the left-hand side of Eqs. (23), (28), and (33). They only depend on the absolute magnitudes of the elements of V_E , which can be parametrized by four free parameters. Therefore, if replacing $|V_E^{eb}|, |V_E^{\mu b}|, |V_E^{\tau b}|$, and $|V_E^{cs}|$ in Eqs. (23) and (28) by $|V_E^{ed}|, |V_E^{es}|, |V_E^{\mu d}|, |V_E^{\mu s}|$, a concrete tree-level prediction emerges at leading order:

$$\begin{aligned} \frac{2\Gamma(p \rightarrow \pi^0 \ell^+)}{\Gamma(p \rightarrow \pi^+\bar{\nu})} - \frac{\Gamma(p \rightarrow K^0 \ell^+)}{\xi_K \Gamma(p \rightarrow K^+\bar{\nu})} &= \left(\frac{3|(\Delta M_K)_{\text{NP}}|}{2\mathbf{h}_d |M_{12}^{\text{dSM}}| \xi_B} - \frac{N_{e\mu}}{N_{\tau\mu}} - \frac{N_{e\mu}}{N_{e\tau}} + 1 \right) \left(\frac{N_{e\mu}}{N_{\tau\mu}} + \frac{N_{e\mu}}{N_{e\tau}} + 1 \right)^{-1}, \end{aligned} \quad (34)$$

where

$$\begin{aligned} \Gamma(p \rightarrow \pi^0 \ell^+) &= \Gamma(p \rightarrow \pi^0 e^+) + \Gamma(p \rightarrow \pi^0 \mu^+), \\ \Gamma(p \rightarrow K^0 \ell^+) &= \Gamma(p \rightarrow K^0 e^+) + \Gamma(p \rightarrow K^0 \mu^+) \end{aligned} \quad (35)$$

represents the total decay rate for both electron and muon final states. It is worth noting that this relation serves only as an illustration of an approximate constraint. Given various FCNC and proton decay modes are indeed measured in the future, we suggest fitting all the observables within the whole theory, for numerical robustness. The masses of the second

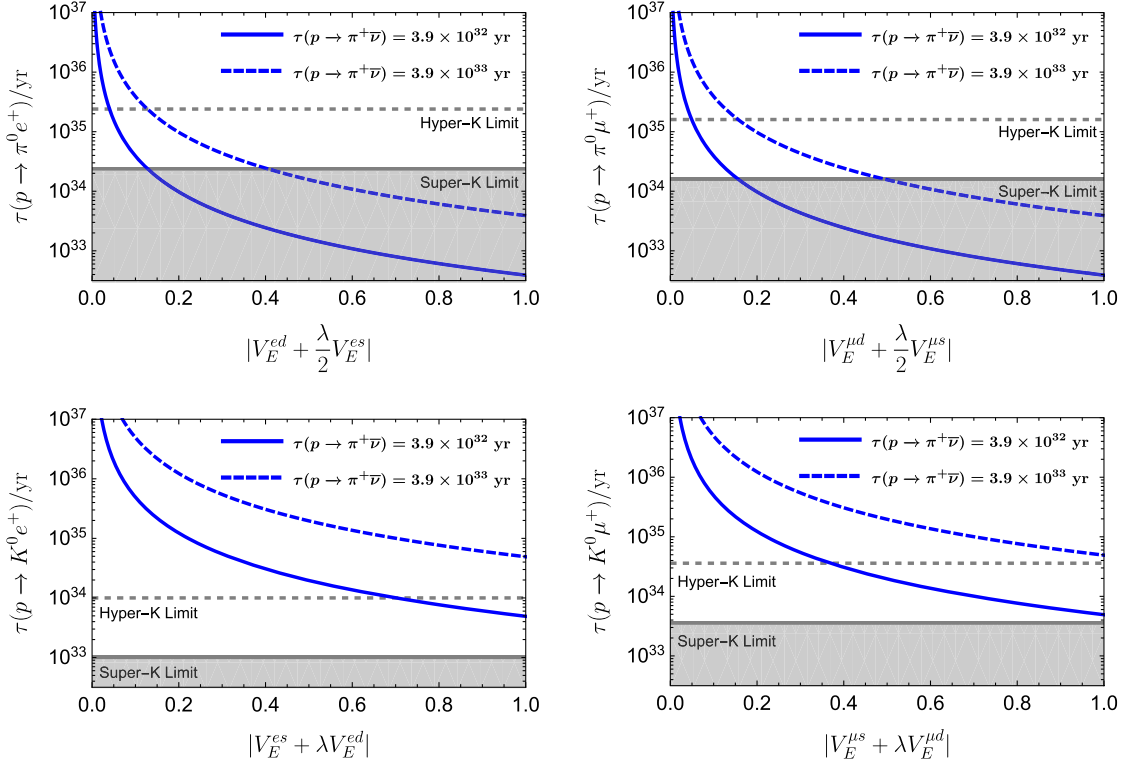


FIG. 4. Lifetimes for various proton decay modes as a function of the magnitudes of elements in the 2×2 top-left submatrix of $|V_E|$. The blue dashed and solid lines represent the lifetimes in the benchmark of setting $\tau(p \rightarrow \pi^+ \bar{\nu}) = 3.9 \times 10^{32}$ years (current limit) or $\tau(p \rightarrow \pi^+ \bar{\nu}) = 3.9 \times 10^{33}$ years, respectively. The gray regions are excluded by Super-Kamiokande, as reported in Table II. Prospective Hyper-Kamiokande bounds are shown with the dashed gray lines, assuming an order of magnitude improvement in sensitivity.

generation fermions, and the mixing angles in the CKM matrix, should then also be included. To achieve better precision, it is also important to account for the RG effects. As shown in Appendix B, the running toward, at least, the intermediate scale M_I is well defined since the light two-Higgs doublet model parameters are directly measurable for near future experiments. Verifying Eq. (34) in future experiments would indicate that all next-to-leading order corrections are indeed small, and thus provide a hint for minimal $SO(10)$. If minimal $SO(10)$ does not hold or the next-to-leading order corrections are overlooked, then the left- and right-hand sides of Eq. (34) can differ by orders of magnitude.

IV. CONCLUSION AND DISCUSSION

In this article, we have analyzed the predictions of the minimal realistic version of $SO(10)$ grand unified theory, that only one 45_H , one 126_H , and a complex 10_H are contained in the scalar sector. Additionally, we assumed CP symmetry for all renormalized interactions, which in our scenario can only be spontaneously broken. An additional light Higgs doublet is required, as a consistency requirement. Different from general two-Higgs doublet models, the flavor structure of the predicted flavor

changing neutral current processes are connected to proton decay branching ratios, as a nontrivial constraint from matter unification.

The main uncertainty is the vacuum configuration of the Higgs doublets. They can not be clearly predicted from the scalar potential of $SO(10)$ and may receive large loop corrections due to the Higgs self-couplings. Perhaps, one can find some clues from fitting the charged fermion masses and mixing angles, but we do not get any reliable prediction. Therefore, we cannot estimate the absolute strengths of the flavor changing neutral current signals. But fortunately, we can predict the relative strengths, provided various proton decay branching ratios are measured since the flavor structure of minimal $SO(10)$ is quite constrained.

We acknowledge the widely known limit that the renormalizable $SO(10)$ GUT, if realistic, lacks predictive power. It does not allow direct calculation of the proton decay branching ratios. Meanwhile, there are no clear hints from the measured fermion masses and mixing angles. One possibility is that unification is simply not achieved, but leaving charge quantization as an unexplained coincidence. Alternatively, $SO(10)$ GUT might be nonminimal and all predictions are deeply hidden, so we can never prove it. However, we find minimal $SO(10)$ GUT may give a hint

for electroweak scale physics, if GUT is slightly extended—by additionally assuming CP is a fundamental symmetry, much like the common assumption of Lorentz invariance.

The trouble—and possibly the reason this approach is overlooked—is the requirement of an additional fine-tuning. But, SM itself is also fine-tuned. The electroweak scale $v \ll M_{\text{GUT}}$ leads to $m_h \sim v \ll M_{\text{GUT}}$. The weak-scale SCPV requires two degenerate vacuum $v, v^* \ll M_{\text{GUT}}$, implying double fine-tuning and two light Higgs doublets. As long as one accepts non-SUSY GUT, there is no reason to reject the weak-scale SCPV. The fine-tuning puzzle does not necessarily change the low energy theory [88]. Naturalness might be just hidden, for instance, via cosmological relaxation [89]. If one considers the possibility that naturalness could be implicit, a class of well-motivated and potentially predictive models might remain unexplored. These theories, overlooked due to the naturalness criteria, or swamped in the supersymmetric models, could provide new insights. We thus advocate that theories without explicit naturalness also deserve thorough study in the future.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: MIXING MATRIX

There are only three independent Yukawa couplings $Y_{10}, \tilde{Y}_{10}, Y_{126}$ in $SO(10)$ generic basis, so the mass matrices in Eq. (5) are connected via

$$M_{\nu_L} = k_1(M_E - M_D) + k_2 M_U + k_3 M_U(M_E - M_D)^{-1} M_U, \quad (\text{A1})$$

where

$$k_1 = \frac{8(v_{126}^u)^2}{v_{126}^d \langle \Delta_R^0 \rangle} - \frac{\langle \Delta_L \rangle}{8v_{126}^d}, \quad k_2 = \frac{16v_{126}^u}{\langle \Delta_R^0 \rangle}, \quad k_3 = \frac{8v_{126}^d}{\langle \Delta_R^0 \rangle}. \quad (\text{A2})$$

If rewriting Eq. (A1) with physical masses and mixing matrices, it reads:

$$V_E^T (V_{\text{PMNS}}^* m_{\nu_L}^{\text{diag}} V_{\text{PMNS}}^\dagger - k_1 m_E^{\text{diag}}) V_E = -k_1 m_D^{\text{diag}} + k_2 (V_{\text{CKM}}^T m_U^{\text{diag}} V_{\text{CKM}}) + k_3 M_T, \quad (\text{A3})$$

where

$$M_T = V_{\text{CKM}}^T m_U^{\text{diag}} V_{\text{CKM}} (V_E^T m_E^{\text{diag}} V_E - m_D^{\text{diag}})^{-1} V_{\text{CKM}}^T m_U^{\text{diag}} V_{\text{CKM}}. \quad (\text{A4})$$

The complexity of M_T is the main barrier to solve V_E from Eq. (A3). However, some rough approximation can shed light on its magnitudes. Neglecting m_u, m_d, m_s , and all mixings angles in the CKM matrix, we get:

$$\begin{aligned} M_T &\approx \frac{m_\tau}{\det M_T} \begin{pmatrix} m_u^2 m_b \mathcal{D}_{11} & m_u m_c m_b \mathcal{D}_{12} & m_u m_t m_\mu \mathcal{D}_{13} \\ m_u m_c m_b \mathcal{D}_{21} & m_c^2 m_b \mathcal{D}_{22} & m_c m_t m_\mu \mathcal{D}_{23} \\ m_u m_t m_\mu \mathcal{D}_{31} & m_c m_t m_\mu \mathcal{D}_{32} & m_t^2 m_\mu \mathcal{D}_{33} \end{pmatrix}, \\ \mathcal{D}_{11} &= -(V_E^{\tau s})^2 + o\left(\frac{m_\mu}{m_b}\right), \quad \mathcal{D}_{22} = -(V_E^{\tau d})^2 + o\left(\frac{m_\mu}{m_b}\right), \\ \mathcal{D}_{33} &= (V_E^{\mu s} V_E^{\tau d} - V_E^{\mu d} V_E^{\tau s})^2 + o\left(\frac{m_s}{m_\mu}\right), \quad \mathcal{D}_{12} = \mathcal{D}_{21} = V_E^{\tau d} V_E^{\tau s} + o\left(\frac{m_\mu}{m_b}\right), \\ \mathcal{D}_{13} &= \mathcal{D}_{31} = (V_E^{\mu s} V_E^{\tau d} - V_E^{\mu d} V_E^{\tau s})(V_E^{\mu s} V_E^{\tau b} - V_E^{\mu b} V_E^{\tau s}) + o\left(\frac{m_s}{m_\mu}\right), \\ \mathcal{D}_{23} &= \mathcal{D}_{32} = (V_E^{\mu s} V_E^{\tau d} - V_E^{\mu d} V_E^{\tau s})(V_E^{\mu b} V_E^{\tau d} - V_E^{\mu d} V_E^{\tau b}) + o\left(\frac{m_s}{m_\mu}\right). \end{aligned} \quad (\text{A5})$$

Equation (A1) is valued at M_{GUT} , so we take the hierarchy $m_\mu \approx 5m_s$ at M_{GUT} . This simplifies the leading order expression of M_T . Another important GUT scale relation is $m_t m_\mu \gg m_b m_c$. These relations give a clear hierarchy structure among the elements of M_T , assuming no elements of \mathcal{D}_{ij} are accidentally small.

M_T can be diagonalized with $M_T = V_T^* m_T^{\text{diag}} V_T^\dagger$. This leads to $V_T^{ij} \sim 1$ when $i = j$ and $V_T^{ij} \ll 1$ when $i \neq j$. Therefore, the right-hand side of Eq. (A3) can be diagonalized with a 3×3 unitary matrix closed to identity, assuming no accidental cancellations. The same flavor structure must also hold for the left-hand side, so the pattern of V_E is clear:

(i) $k_1 m_\tau \gg m_{\nu_L} \sim 0.1$ eV, $V_E \sim 1$.

(ii) $k_1 m_\tau \lesssim m_{\nu_L} \sim 0.1$ eV, $V_E \sim V_{\text{PMNS}}$.

These two scenarios can be understood intuitively. Taking the limit that neutrinos are massless ($M_{\nu_L} = 0$), Eq. (A1) becomes a constraint for M_E , M_D , M_U . Ignoring the nondiagonal elements of the CKM matrix, both M_D , M_U can be taken as diagonal, and then M_E is automatically diagonal in this limit. Taking into account the neutrino masses, the nondiagonal elements of V_E can be large, potentially as large as ones in the maximal flavor mixing matrix V_{PMNS} .

It is important to note that the discussion above is not robust. A careful reader may have already noticed that even with moderate cancellation, the predictions could change significantly. In fact, such cancellation is somehow motivated since $\langle \Delta_R^0 \rangle$ is expected to lie around 10^{11} GeV (or 10^9 GeV) for successful gauge coupling unification without fine-tuned light scalars. But the correct light neutrino mass implies a larger value, $\langle \Delta_R^0 \rangle \sim 10^{13}$ GeV. If v_{126}^μ and v_{126}^d are not simultaneously suppressed, then one need some accidental cancellation among the (3, 3) elements of the three matrices on the right-hand side of Eq. (A1). We discuss the potential structure of V_E here, mainly to give

readers a feeling about how much information is available from the fermion masses and mixing angles.

APPENDIX B: RENORMALIZATION GROUP EFFECTS

We illustrate how the RG effects change the symmetric structure of the Yukawa matrices here. The running equations between the GUT scale M_{GUT} and the intermediate scale M_I remain indeterminate because the physics around M_I is not directly accessible by experiments. Although the minimal scalar spectrum around M_I is well defined, the related Yukawa couplings remain unknown. Numerical fits may provide some insights, but these couplings cannot be faithfully derived from theory or directly measured by the experiments. Only the running equations from M_I down to the electroweak scale (SM with light 2HDM) are well defined, assuming no other particles much lighter than M_I .

The explicit running equations for general 2HDM can be found in [19]. While our low energy theory is somehow specified, significant simplifications are possible. Firstly, terms that only change the Yukawa couplings by an overall factor can be dropped at one loop. Moreover, within the parameter space of interest, only the (3,3) elements of Y_U and M_U/v are large as of $o(1)$, while all other Yukawa couplings are suppressed. Furthermore, the 2-3, 1-3 mixing angles of V_{CKM} and the mass ratios (m_c/m_t) , (m_s/m_b) , (m_μ/m_τ) can all be neglected as an approximation. As a result at one loop, we conclude that at the electroweak scale M_{EW} ,⁴

$$\begin{aligned} (Y_E^{\ell\ell} - Y_E^{\ell\tau})|_{M_{\text{EW}}} &= (Y_E^{\tau\ell} - Y_E^{\ell\tau})|_{M_I}, (Y_U^{tq'} - Y_U^{q't})|_{M_{\text{EW}}} = (Y_U^{tq'} - Y_U^{q't})|_{M_I}, \\ (Y_D^{bq} - Y_D^{qb})|_{M_{\text{EW}}} &= (Y_D^{bq} - Y_D^{qb})|_{M_I} + Y_D^{bq} \times \frac{y_t^2}{16\pi^2} \log(M_I/M_{\text{EW}}) \left(\frac{3}{2} |C_{UU}|^2 - \frac{1}{2} \right), \\ (M_D^{\tau\ell} - M_D^{\ell\tau})|_{M_{\text{EW}}} &= (M_D^{\tau\ell} - M_D^{\ell\tau})|_{M_I}, (M_D^{tq'} - M_D^{q't})|_{M_{\text{EW}}} = (M_D^{tq'} - M_D^{q't})|_{M_I}, \\ (M_D^{bq} - M_D^{qb})|_{M_{\text{EW}}} &= (M_D^{bq} - M_D^{qb})|_{M_I} + (m_b V_E^{\tau b} V_E^{\tau q}) \times \frac{y_t^2}{16\pi^2} \log(M_I/M_{\text{EW}}) \left(C_{UU} C_{\text{DE}} \frac{2m_\tau}{m_b} \right). \end{aligned} \quad (\text{B1})$$

Here $\ell = e, \mu, q = d, s, q' = u, c$, and for simplicity we took NMFV, a necessary condition to extract $V_E^{\tau q}$, as the flavor structure of Y_D . Notably, both Y_D , M_D are always not exactly symmetric at the low scale, even if all the Yukawa couplings are accidentally symmetric at M_I .⁵ Taking $M_I \sim 10^{11}$ GeV and $y_t = \frac{\sqrt{2}m_t}{v} \approx 0.5$ at M_I , we estimate

⁴As an illustration, we take the masses of the electroweak gauge bosons, top quark, and all physical states of light 2HDM (supposed to be smaller than 500 GeV) equal to M_{EW} .

⁵Without specifying the physics at M_I , one cannot quantify the deviations of Y_U , Y_E , M_U , M_E . Those may be similar to Y_D , M_D , or fully negligible.

$$\frac{y_t^2}{16\pi^2} \log(M_I/M_{\text{EW}}) \approx 3\%. \quad (\text{B2})$$

The unknown parameters C_{UU} and C_{DE} must be smaller than $o(1)$ due to the flavor changing neutral current constraints. Consequently, we expect percent-level corrections to the Wilson coefficients of $\overline{b_L} q_R \overline{b_R} q_L$. Likewise, some mixing angles relevant for proton decay, zero at GUT scale, are shifted by a few percent of $V_E^{\tau b} V_E^{\tau q}$ at low energy. These corrections should be added to Eq. (34) once future experiments measure the parameters C_{UU} and C_{DE} .

It is worth mentioning that the Yukawa textures at M_I , in general, still differ with those at the M_{GUT} . And more corrections such as QCD ($1 + o(\alpha_s/4\pi)$) should be multiplied to $(Y_F - Y_F^T)|_{M_I}$ and $(M_F - M_F^T)|_{M_I}$ of Eq. (B1), as two loop order corrections to the GUT scale result. However, we believe that the one-loop running effects from M_I down to M_{EW} are already a good illustration, in practice, due to the logarithmic enhancement $\log(M_I/M_{\text{EW}})$. Nevertheless, it

seems quite unlikely for the full RG effects to be large. If the minimal CP conserving $SO(10)$ is indeed the UV theory, then we expect the GUT scale predictions hold reasonably well at low energy.

APPENDIX C: HADRONIC ELEMENTS

The hadronic matrix elements related to proton decay are given in [90]:

$$\begin{aligned}\langle K^0|(us)_R u_L|p\rangle &= 0.103(3)(11) \text{ GeV}^2, & \langle K^0|(us)_L u_L|p\rangle &= 0.057(2)(6) \text{ GeV}^2, \\ \langle K^+|(us)_R d_L|p\rangle &= -0.049(2)(5) \text{ GeV}^2, & \langle K^+|(ud)_R s_L|p\rangle &= -0.134(4)(14) \text{ GeV}^2, \\ \langle \pi^+|(du)_R d_L|p\rangle &= -0.186(6)(18) \text{ GeV}^2.\end{aligned}\tag{C1}$$

For neutral meson oscillation, the corresponding hadronic matrix elements are commonly parametrized with bag parameters [91]. We translate the latest lattice results [92,93] into the following explicit expressions:

$$\begin{aligned}\langle K^0|\overline{d}_L s_R \overline{d}_L s_R|\overline{K}^0\rangle &= -0.039 \text{ GeV}^4, & \langle K^0|\overline{d}_L s_R \overline{d}_R s_L|\overline{K}^0\rangle &= 0.088 \text{ GeV}^4, \\ \langle B_d^0|\overline{b}_L d_R \overline{b}_L d_R|\overline{B}_d^0\rangle &= -0.52 \text{ GeV}^4, & \langle B_d^0|\overline{b}_R d_L \overline{b}_L d_R|\overline{B}_d^0\rangle &= 0.96 \text{ GeV}^4, \\ \langle B_s^0|\overline{b}_L s_R \overline{b}_L s_R|\overline{B}_s^0\rangle &= -0.84 \text{ GeV}^4, & \langle B_s^0|\overline{b}_R s_L \overline{b}_L s_R|\overline{B}_s^0\rangle &= 1.40 \text{ GeV}^4.\end{aligned}\tag{C2}$$

The hadronic elements for B mesons are at the physical point, and those for kaons are at $\mu = 3 \text{ GeV}$. The uncertainties are typically at a few percent level. We do not renormalize them to $m_H \sim 500 \text{ GeV}$ because only the ratios contribute to the physical prediction, which are independent of the renormalization scale.

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